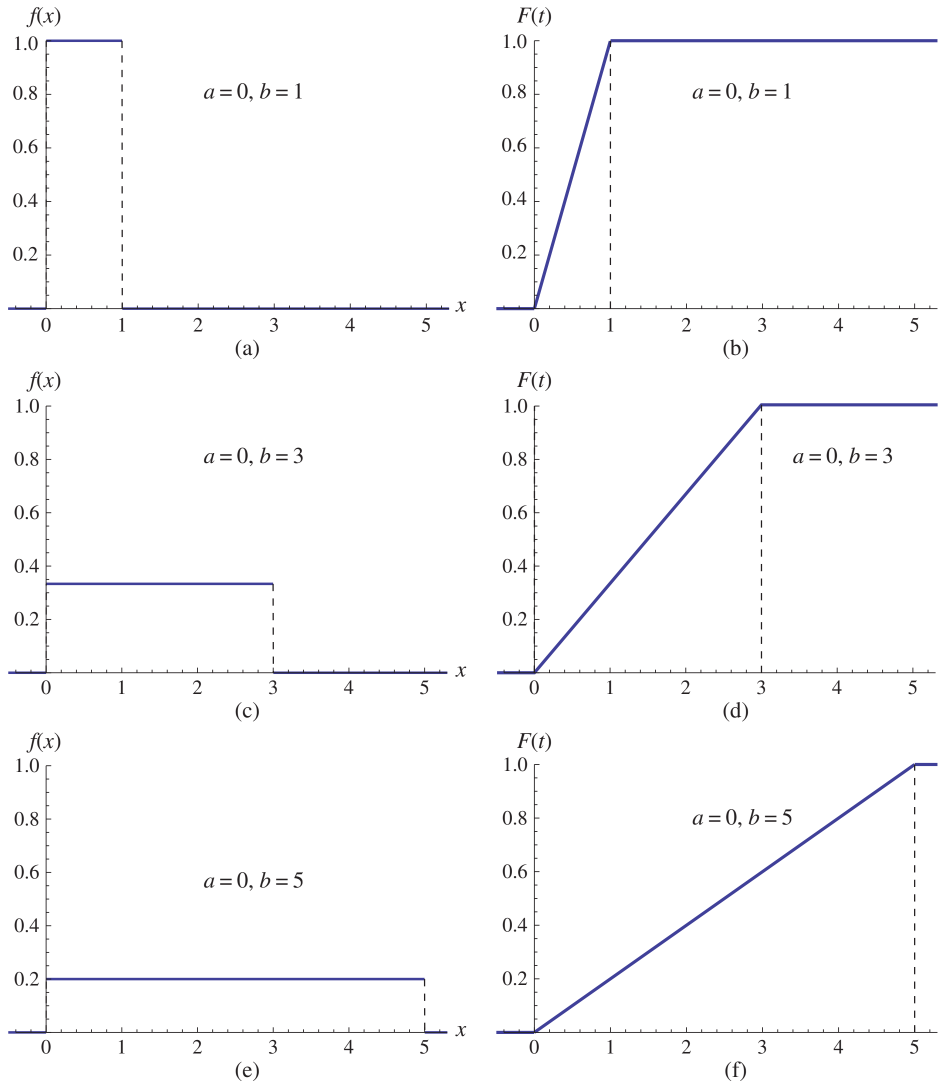

Figure 7.2 The density function (left) and the distribution function (right) of the uniform distribution for different choices of a and b.

For the variance, we calculate first the moment of second order around zero. This is

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,\mathrm{d}x = \int_a^b x^2 \cdot \frac{1}{b-a}\,\mathrm{d}x = \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b$$
$$= \frac{b^3 - a^3}{3(b-a)} = \frac{(b-a)(b^2+ab+a^2)}{3(b-a)} = \frac{b^2+ab+a^2}{3}.$$

Making use of this in the formula $\mathrm{Var}(X) = E(X^2) - [E(X)]^2$, we derive

$$\mathrm{Var}(X) = \frac{b^2+ab+a^2}{3} - \left(\frac{a+b}{2}\right)^2 = \frac{(b-a)^2}{12},$$

as required. □