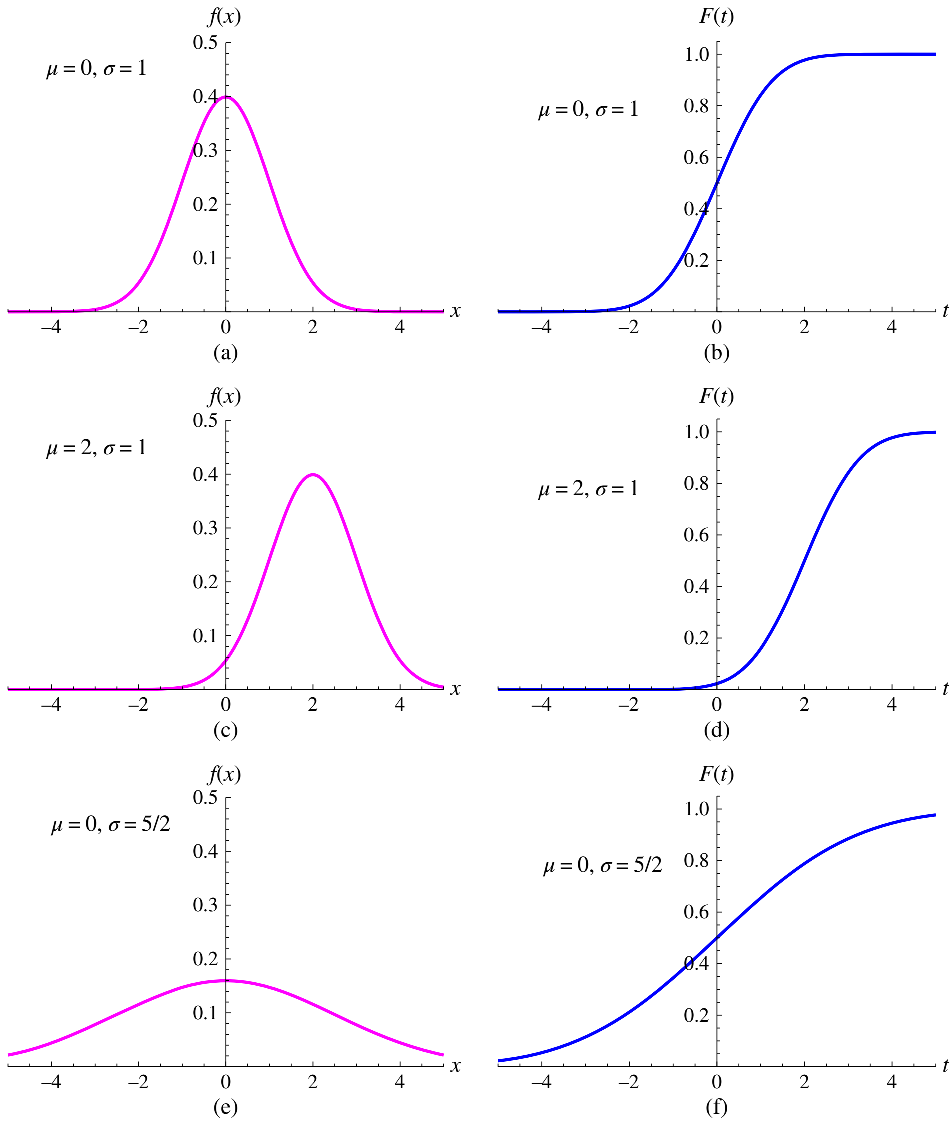

Figure 7.6 The density function (left) and the distribution function (right) of the normal distribution, for various values of μ and σ^2.

Proof:

(i) Let us denote by f and F the density function and the distribution function of the variable X, respectively. Then the distribution function of $Z = (X - \mu)/\sigma$ is given by (see also Example 6.3)

$$F_Z(z) = \left(\frac{X - \mu}{\sigma} \leq z\right) = P(X \leq \mu + \sigma z) = F(\mu + \sigma z),$$

and by differentiation we obtain the density of Z to be

$$f_Z(z) = F_Z'(z) = \sigma f(\mu + \sigma z) = \sigma \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{[(\mu+\sigma z)-\mu]^2}{2\sigma^2}} = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} = \phi(z),$$