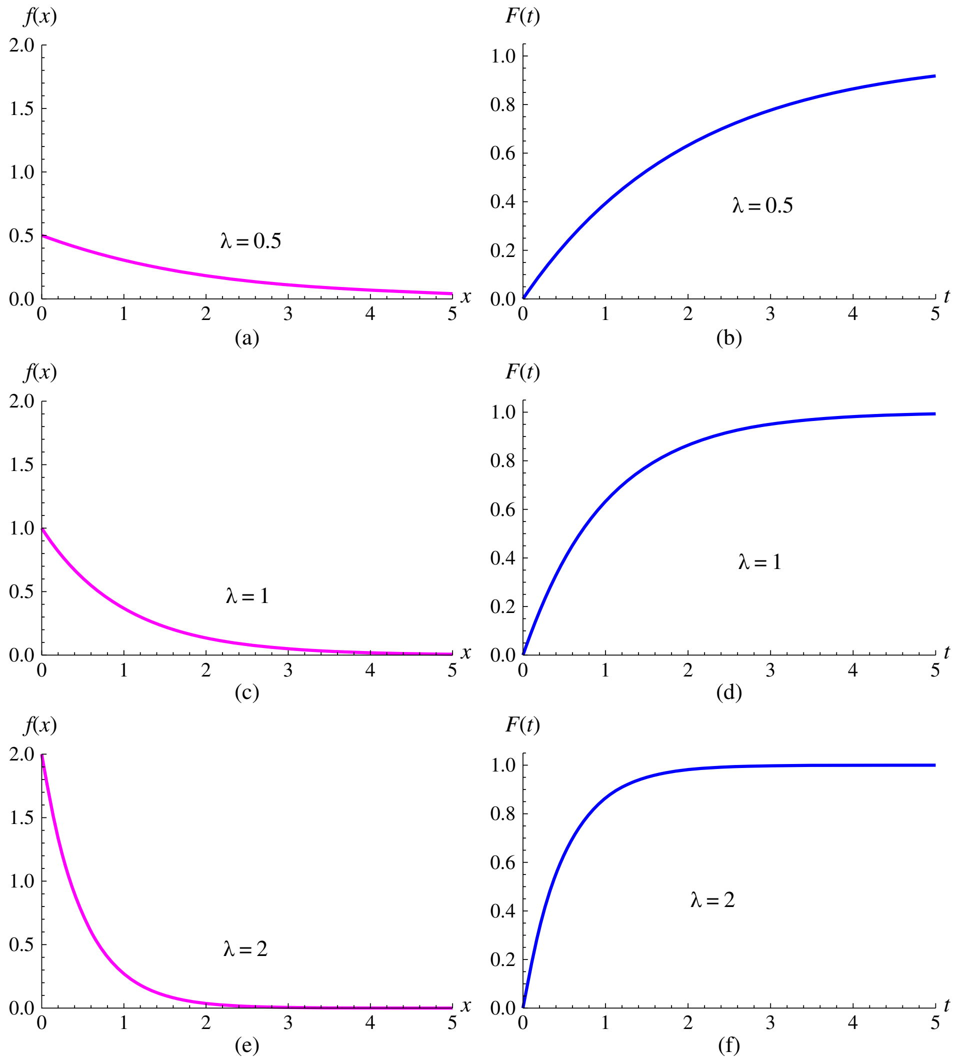

Figure 7.11 The density function (left) and the distribution function (right) of the $\mathcal{E}(\lambda)$ distribution for various choices of λ.

Proof: First for the expectation, we have

$$E(X) = \int_{-\infty}^{\infty} xf(x)\,\mathrm{d}x = \int_{0}^{\infty} x\lambda \mathrm{e}^{-\lambda x}\,\mathrm{d}x.$$

Integrating by parts, and making use of the fact the $\lambda \mathrm{e}^{-\lambda x}$ is negative of the derivative of the function $\mathrm{e}^{-\lambda x}$, we obtain

$$E(X) = \left[-x\mathrm{e}^{-\lambda x}\right]_0^{\infty} + \int_{0}^{\infty} \mathrm{e}^{-\lambda x}\,\mathrm{d}x.$$

The integral on the right equals λ^{-1} and the proof for the expectation gets completed upon noting that

$$\left[-x\mathrm{e}^{-\lambda x}\right]_0^{\infty} = -\lim_{x\to\infty} \frac{x}{\mathrm{e}^{\lambda x}} + 0 \cdot \mathrm{e}^0 = 0,$$