The result of the proposition now follows by inserting the last three displayed expressions into (5.13). □

The approximation of the hypergeometric distribution by the binomial is illustrated graphically in Figure 5.7. We note at this point that the approximation is remarkably good if both parameters a and b are much larger than n. For example, if $a = 200, b = 200$, and $n = 6$, the exact values from the hypergeometric distribution for $X = 3$, $X = 4$, and $X = 5$ are

$$P(X = 3) = 0.314\,867\,5, \quad P(X = 4) = 0.234\,957\,9, \quad P(X = 5) = 0.092\,566\,3,$$

while those from a binomial approximation using $p = a/(a + b) = 0.5$ are

$$P(X = 3) \cong 0.3125, \quad P(X = 4) \cong 0.234\,375, \quad P(X = 5) \cong 0.093\,75,$$

respectively. The relative percentage error for these probabilities is 0.75%, 0.25%, and 1.28%, respectively.

The next example, however, illustrates that the binomial approximation can be poor if a and/or b is of the same magnitude as n.

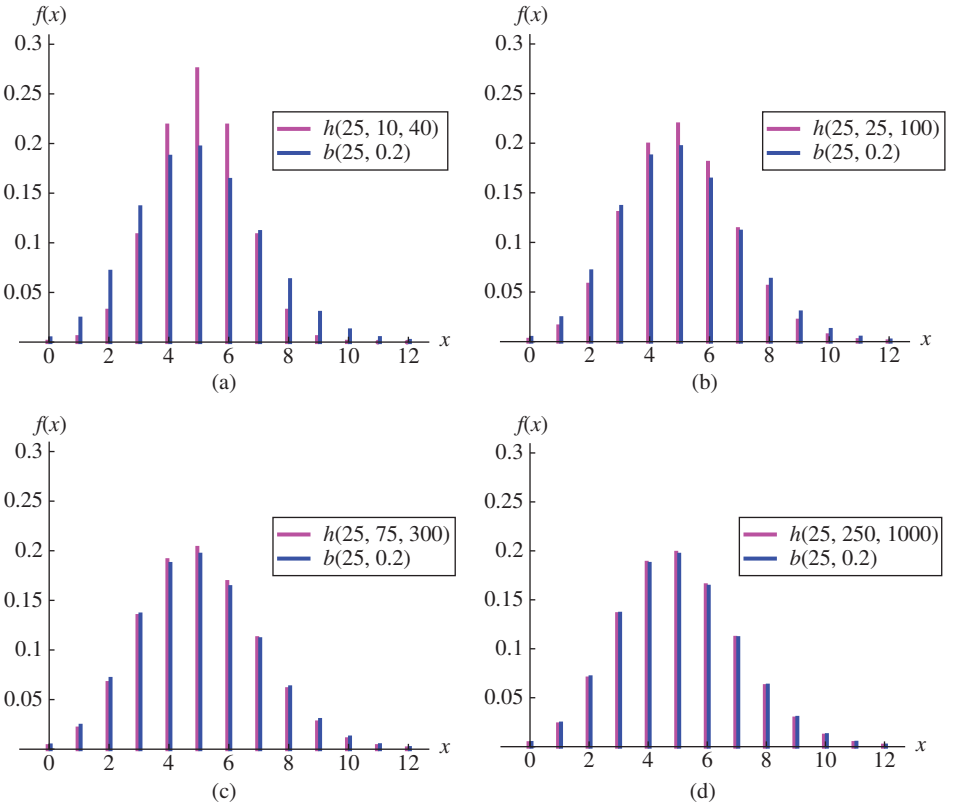

Figure 5.7 Approximation of the hypergeometric distribution by the binomial distribution for some choices of n, a, and b.