Impact of a Proposed Table Size Rule Change on Modelled Shot Speed and Opponent Response Time

Elliott King Word count = 1564

Introduction

Table tennis is a sport that has evolved over the last half century with the International Table Tennis Federation (ITTF) implementing rule changes to make the game appealing to television audiences and have been bold in their actions despite significant effects to the game.

On 1st April 2017 the ITTF posted an 'April-fools' that they would expand the current 1.525 $\times$2.74m long table to 3$\times$6m (ITTF, 2017). They go on to explain that this would make rallies longer thereby capturing their desired larger audience. Given the rule change in 2000 increasing ball diameter from 38 to 40mm to slow the game (Xie, Teh, & Qin, 2002) the surprise of the ITTF's joke is that it would not follow through if the change would produce the desired effect.

The objective of this work is to ascertain what effect the hypothetical change would make to the velocity of four major shots and the response time required by the opponent to make an effective return.

Table 1 Symbols used in this paper and their meanings

Symbol	Meaning
A	Cross-sectional area of projectile
C_D	Dimensionless drag coefficient
C_L	Dimensionless lift coefficient
F_D	Drag force vector
F_L	Lift / Magnus force vector
F_x, F_y, F_z	Components of force
g	Acceleration due to gravity (9.81m/s^{-2} used)
m	Mass of the projectile
$\vec{V}$	Velocity vector (m/s^{-1})
V_x, V_y, V_z	Magnitude and components of velocity vector
ρ	Air Density (1.22 kg m^{-3} used)
$\omega_x, \omega_y, \omega_z$	Magnitude and components of angular velocity vector (rad/s^{-1})
μ	Viscosity of air (1.51$\times 10^{-5}$ used)

Methodology & Modelling

A programmable model made in MATLAB was used to compare shot velocities and on four common shots; a forehand drive, spin serve, a loop shot and a smash. An attempt is also made to provide a necessary response time an opponent would have to achieve in order to execute an effective return shot and a comment given on the impact on the use of the shot

Equations of Motion with Spin

Aerodynamic drag is assumed to be in opposition to the velocity vector of a projectile with a magnitude described as

$$F_D = \frac{1}{2}\rho A C_D V^2, \tag{1}$$

with the vector form

$$\vec{F}_D = \frac{1}{2}\rho A C_D V \cdot \vec{V}. \tag{2}$$

The lift force may be expressed as

$$\vec{F}_L = \frac{1}{2}\rho A C_L V^2 \sin\theta \cdot \hat{n}, \tag{3}$$

where $\hat{n}$ is a vector in the direction of $\vec{\omega} \times \vec{V}$.

Since $\sin\theta$ (the direction of the force vector) is equal to the spin and velocity vectors divided by their magnitudes, equation (3) can also be rewritten as

$$\vec{F}_L = \frac{1}{2}\rho A C_L V \cdot \left(\frac{\vec{\omega} \times \vec{V}}{\omega}\right). \tag{4}$$

When using Newton's second law resolved into components and combining drag and lift equations the equations of motion for the projectile are

$$m\ddot{x} = \frac{1}{2}\rho A \vec{V}\left[C_D v_x - C_L\left\{\frac{\omega_y v_z - \omega_z v_y}{\omega}\right\}\right] \tag{5}$$

$$m\,\ddot{y}=\frac{1}{2}\rho A\,\vec{V}\left[C_D v_y-C_L\left\{\frac{\omega_z v_x-\omega_x v_z}{\omega}\right\}\right] \quad (6)$$

$$m\,\ddot{z}=\frac{1}{2}\rho A\,\vec{V}\left[C_D v_z-C_L\left\{\frac{\omega_x v_y-\omega_y v_x}{\omega}\right\}\right]-mg \quad . \quad (7)$$

From the above equations a free body diagram can be created.

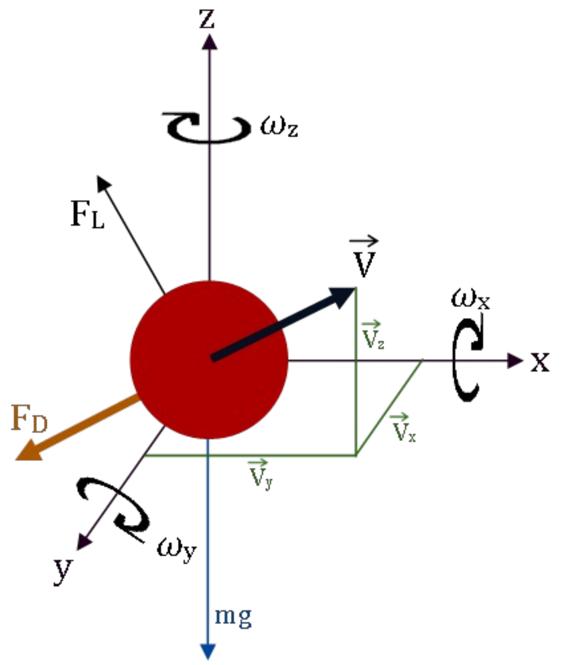

Figure 1 Free Body Diagram

Relationship between Reynolds Number and Lift and Drag

Both the C_L and C_D of smooth spheres have been shown to change with Reynolds number ($\Re=\rho dV/\mu$) and the dimensionless spin rate ($SP=\pi d\omega/V$). Miyazaki et al., 2016 conducted the most comprehensive experiment into the effects of both Re and SP on C_D and C_L.

As stated in their paper the researchers improve on the single significant figure of accuracy presented in prior works and as such are the only usable source for modelling the lift crisis region of a table tennis ball.

Sixth order polynomial equations were fitted to the coefficient versus SP data at each Re (Figure 2) and the MATLAB code applies the equation for the coefficient based on the SP specific to a band of Re. Although crude, this solution gives coefficients that are sensitive to both Re (within 1×10^4) and SP within the lift crisis region during flight.

Spin Degradation

Due to the short time scale the effect of spin degradation to the model was negligible. Neglecting spin degradation for table tennis is an approach well back in literature. (Miyazaki, Sakai, Komatsu, Takahashi, & Himeno, 2016)

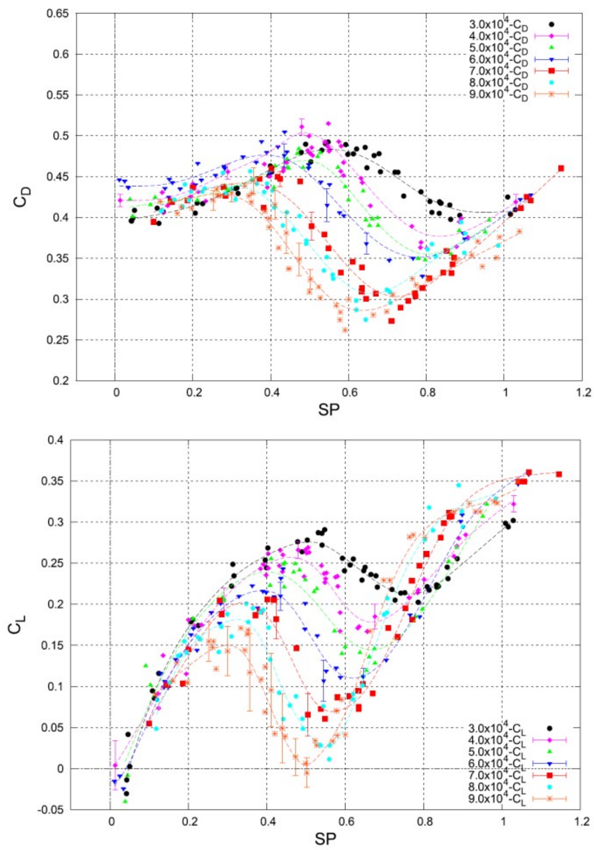

Figure 2 CD (top) and CL (bottom) for changing SP (Miyazaki, Sakai, Komatsu, Takahashi, & Himeno, 2016)

Bouncing

As the game is played indoors, atmospheric conditions are assumed to be constant meaning the only external condition to be considered is the bounce on the table.

The bounce was described using a rigid body model. This is a clear limitation but due to the relative masses of the colliding objects and a high coefficient of restitution (0.92 by Miyazaki, Sakai, Komatsu, Takahashi, & Himeno, 2016))

. Slipping and rolling conditions were applied based on the inbound conditions for the bounce.

Key Assumptions

1) Players cannot indefinitely increase velocity without impacting other shot mechanics

2) Players abilities to anticipate a trajectory is equally effective on a larger table

3) Absolute maximum shot velocity (32.2m/s (Guinness World Records, 2016)) and spin (~150rps (Miyazaki, Sakai, Komatsu, Takahashi, & Himeno, 2016) (Xie, Teh, & Qin, 2002)) are limited by the player and not the table.

4) The lift force varies smoothly with $\sin\theta$ rather than depending on features of the boundary layer (Schlichting & Gersten, 2000).

With spin assumed constant there are six boundary conditions t=0: x(0), y(0), z(0), vx(0), vy(0), vz(0). Additional launch conditions are; an elevation angle (+ve above table), an azimuth angle (cross table) and the spin components.

Table 2 Initial and Table Exit Velocities

Shot Name	Regular Table			Enlarged Table			Key Metric (change on enlarged table)
	Initial Velocity (m/s^{-1})	Time to edge of table (s)	Velocity at edge of table (m/s^{-1})	Initial Velocity (m/s^{-1})	Time to edge of table (s)	Velocity at edge of table (m/s^{-1})	
Spin Serve	8.5	0.4799	4.5218	20*	0.6264	6.1505	+0.1m starting height
Forehand Drive	18.7 (Huang, et al., 2013) (Xie, Teh, & Qin, 2002)	0.2622	9.9111	30*	0.4418	9.2895	+8° elevation required
Loop	8.1 (Xie, Teh, & Qin, 2002)	0.5244	4.4543	13.8	0.8896	4.3528	+1.6m outside table
Smash	31.248 (Guinness World Records, 2016)	0.116	18.3483	31.248	0.2998	13.209	-12° elevation required

*Shot speed increase limited by necessity to generate maximal spin

Results

Starting positions for shots and the objectives of the shots were in part taken from video of high level players and from the shot descriptions on an online forum (PingPongPedia, 2014). Shot velocities and spin rates were taken from data in literature (Xie, Teh, & Qin, 2002). Multiple values are quoted for the maximum spin rate across literature as it is subject specific so a mean value of 150rps has been used within the model.

Attempts were made to recreate the characteristics of each shot trajectory (Appendix) when comparing table sizes

Forehand Drive

A basic rally stroke for keeping pressure on the opponent, the aim of the shot is to maximise velocity without compromise to margin for error (by applying topspin).

Greater elevation is required (+8°) to maintain the topspin used to give the error margin in the shot.

Spin Serve
When spin serving the player is attempting to compromise maximising their ball velocity and spin whilst also hitting straight up the table to the opponent backhand (shown to produce the lowest return speeds (Huang, et al., 2013))

Despite a greater initial velocity and table edge velocity, the response time is greater when playing on an enlarged table, giving an opponent greater opportunity to predict and control the return shot.

Despite an increase to initial velocity and only a small change in table edge velocity the time taken increases significantly (0.2622 to 0.4418 seconds).

Loop Shot
The shot is used to recover a ball from off the side of the table using side spin, usually hit from table level or below.

When the shot is played on a larger table it can be hit from further outside the table (+1.6m / proportionally 40% further). Allowing a player more chance of recovering a wide shot.

Smash
The player is using maximum shot power to clear the ball from the table before an opponent can react.

When played on a larger table, a flatter shot (-22° to -10°) is required and as would be expected, a longer response time available.

Discussion

Implications of the findings
The findings of this work suggest an enlarged table would produce longer rallies as despite being able to increase their initial shot velocities players have more time to respond to the shots and adjust.

Although available response time is a useful tool in estimating a player's ability to return a ball it is limited as there is likely a greater distance to travel to play the shot. Therefore a more reliable conclusion is that the physicality of dominant players would tend towards maximising shot speed and covering ground the fastest.

This would appear to fit the brief of slowing the game and by ITTF logic, increase television audiences

Validity of the model and its limitations
The model possesses good face validity and 3d plots of the data show good agreement with trajectories seen in game video.

A key area of deficiency in the model is the use of a rigid body bounce model. In reality the impact is viscoelastic but as previously mentioned a rigid body solution is close to accurate. Coefficients of friction are also taken from literature (Inaba, et al., 2017), who also provide equations to model a bounce. However these equations are based upon 40 year old research values of a non-analogous impact situation.

The table tennis situation has details which are both advantageous and disadvantageous. For example, the model assumes the projectile is a smooth sphere. In other sports this decreases the accuracy of the model however in table tennis the ball is often seamless so surface effects can be neglected. This also reduces the effect of spin decay, further vindicating its absence in the model.

Table tennis is a unique sport in that lift force due to spin is often greater during match play than gravitational force due to the low mass of the ball. Thus, fully realised spin feature were essential to the model. Constructing spin around the principal axes of the model simplified the equations of motion and allowed for a single coordinate system to be used.

Further work could compare this single system model with one utilising a dual coordinate systems, one global and one local to the ball to resolve the Magnus force components. The extent to which they would be different is likely equal to the extent that assumption 4 is true, although its examination is well beyond the scope of this work and would suit a work more focussed on boundary layer theory.

A note must be made that it's assumed no change in tactics would be employed. There's a logical thought that players would simply attempt to drop shot one another leaving the ball irretrievable. The ITTF did not specify a net height for the enlarged table so during modelling a double height net was used, thereby minimising the threat of the drop shot.

References

Huang, H.-H., Hsueh, Y.-C., Chen, Y.-Y., Chang, T.-J., Pan, K.-M., & Tsai, C.-L. (2013). The Kinematics Analysis of Table Tennis Forehand and Backhand Drives. *International Society of Biomechnics .* Brazil: Brazilian Congress of Biomechanics.

Inaba, Y., Tamaki, S., Ikebukuro, H., Yamada, K., Ozaki, H., & Yoshida, K. (2017). Effect of Changing Table Tennis Ball Material from Celluloid to Plastic on the Post-Collision Ball Trajectory. *Journal of Human Kinetics*, 29-38.

ITTF. (2017, April 1 Apr 2017). *Table Regulations Changing; Bigger is Better*. Retrieved from ITTF.com: https://www.ittf.com/2017/04/01/table-regulations-changing-bigger-better/

Miyazaki, T., Sakai, W., Komatsu, T., Takahashi, N., & Himeno, R. (2016). Lift Crisis of a Spinning Table Tennis Ball. *European Journal of Physics*.

PingPongPedia. (2014, October 15 May 2017). *Types of Shots*. Retrieved from PingPongPedia: http://pingpong.wikia.com/wiki/Types_of_shots

Schlichting, H., & Gersten, K. (2000). *Boundary-Layer Theory 8th Edition.* Berlin: Springer.

Xie, W., Teh, K. C., & Qin, Z. F. (2002). Speed and Spin of 40mm Table Tennis Balle and the Effects on Elite Players. *International Symposium on Biomechanics in Sports.* Caceres: ISBS.

Appendix

Shot Trajectories

Forehand Drive

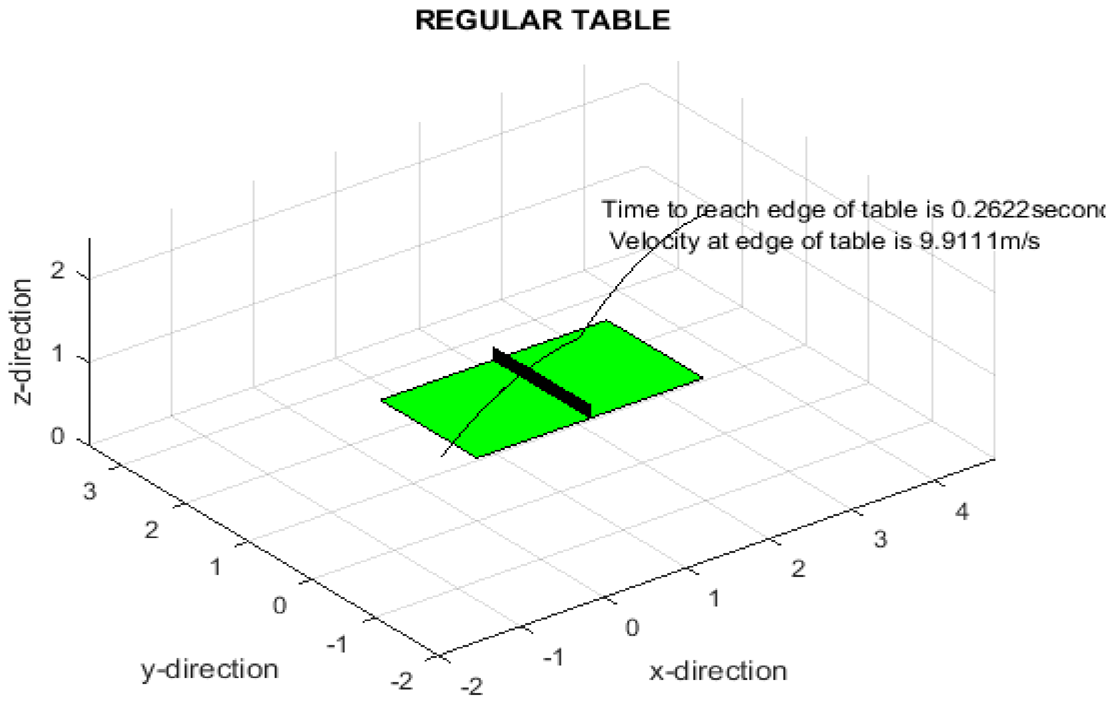

Figure 3 Regular table forehand drive

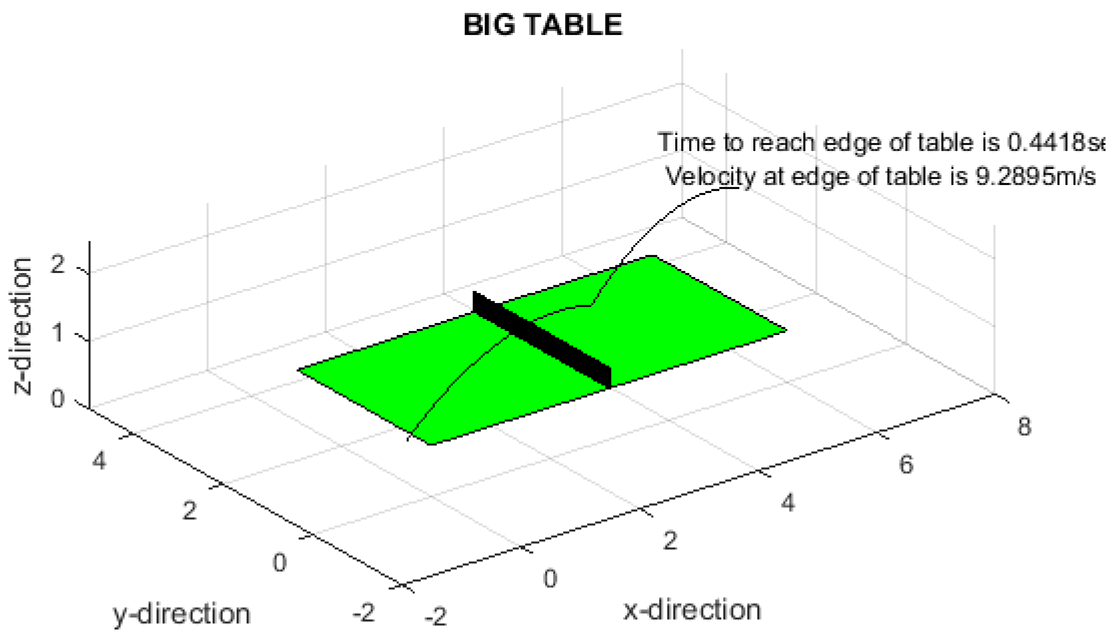

Figure 4 Big table forehand drive

Spin Serve

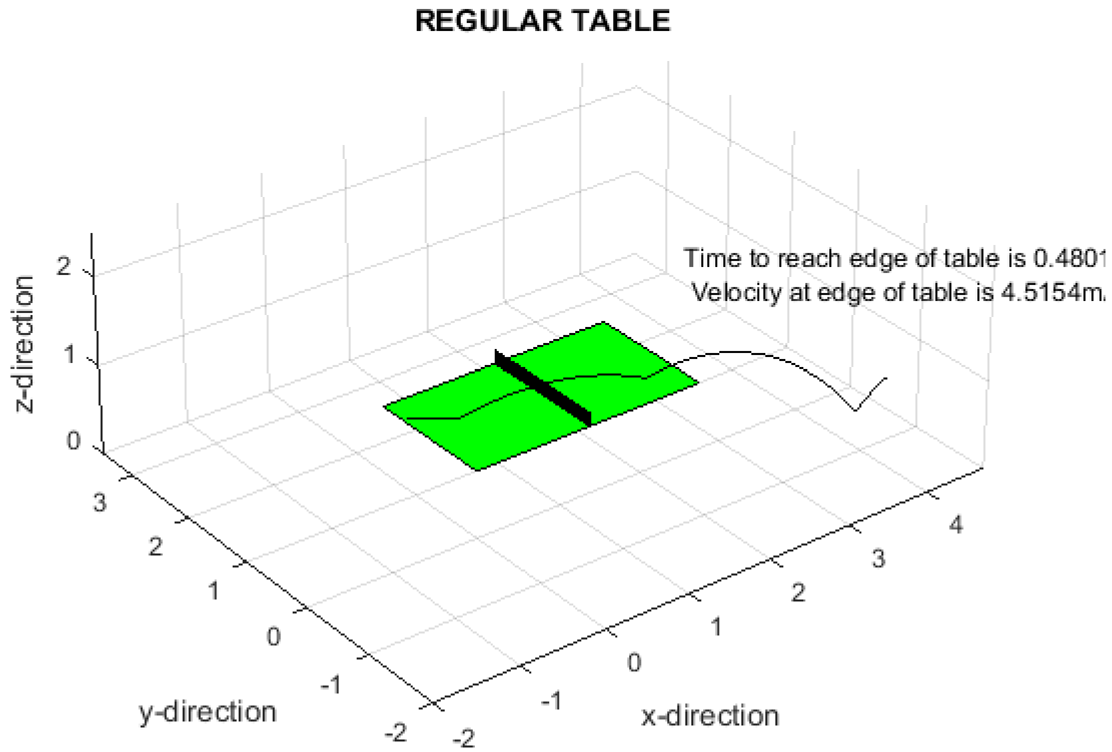

Figure 5 Regular table spin serve

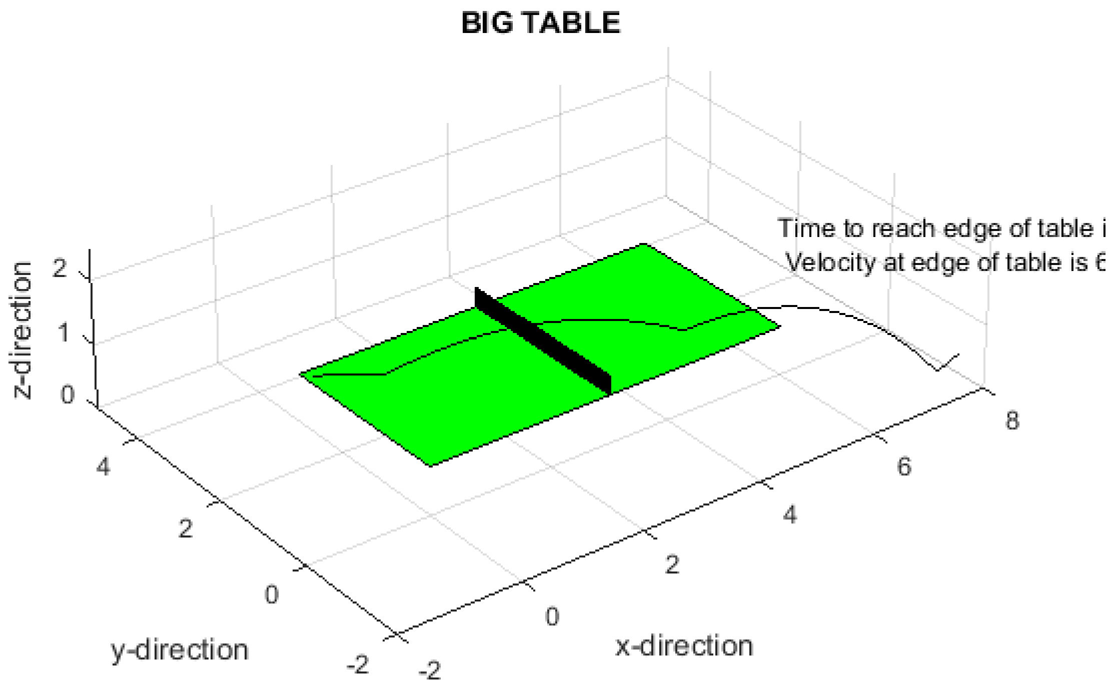

Figure 6 Big table spin serve

Loop Shot

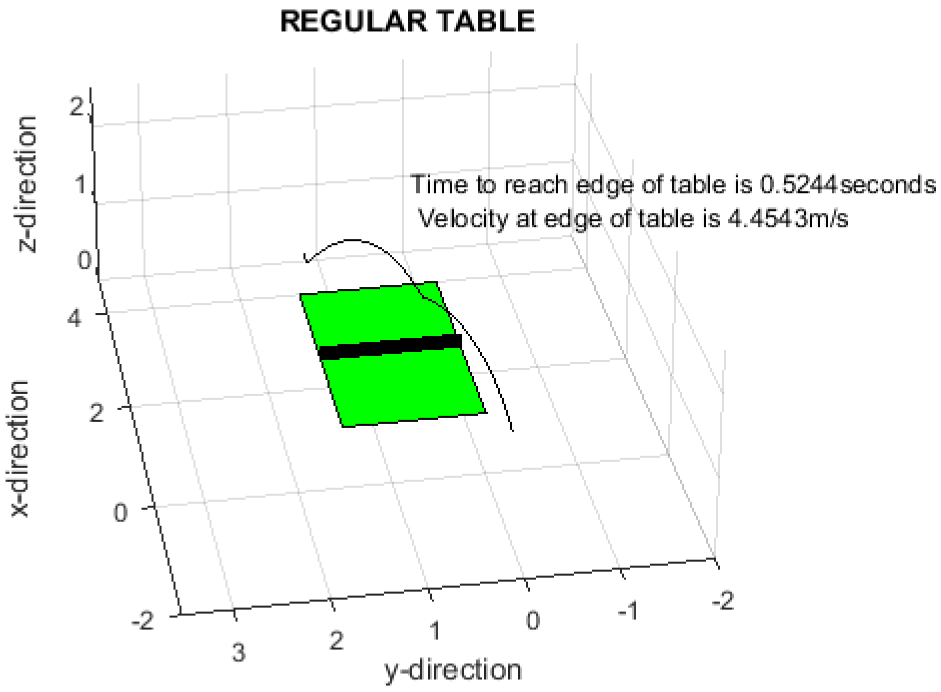

Figure 7 Regular table loop shot

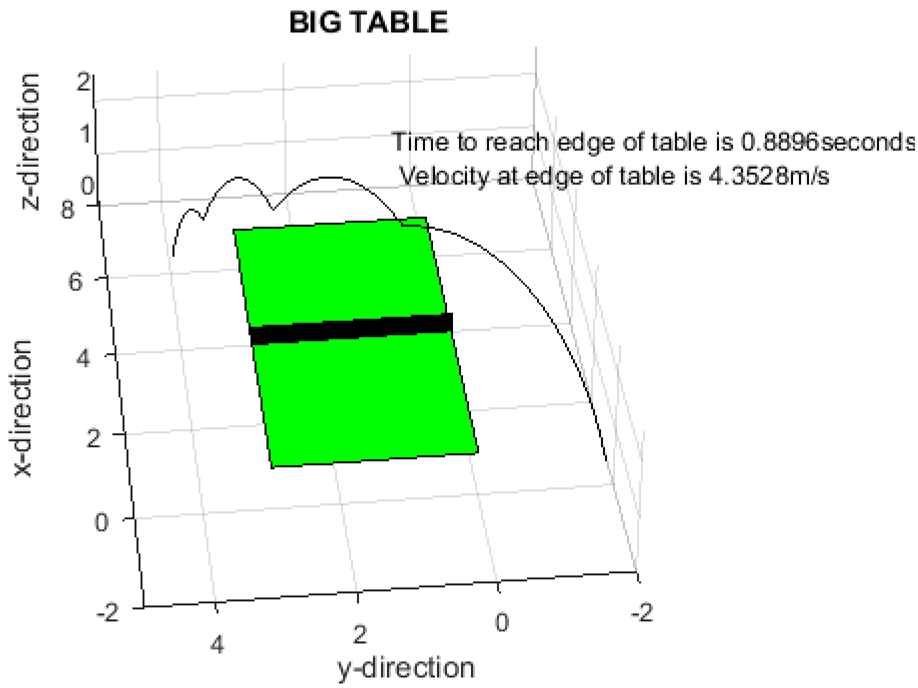

Figure 8 Big table loop shot

Smash

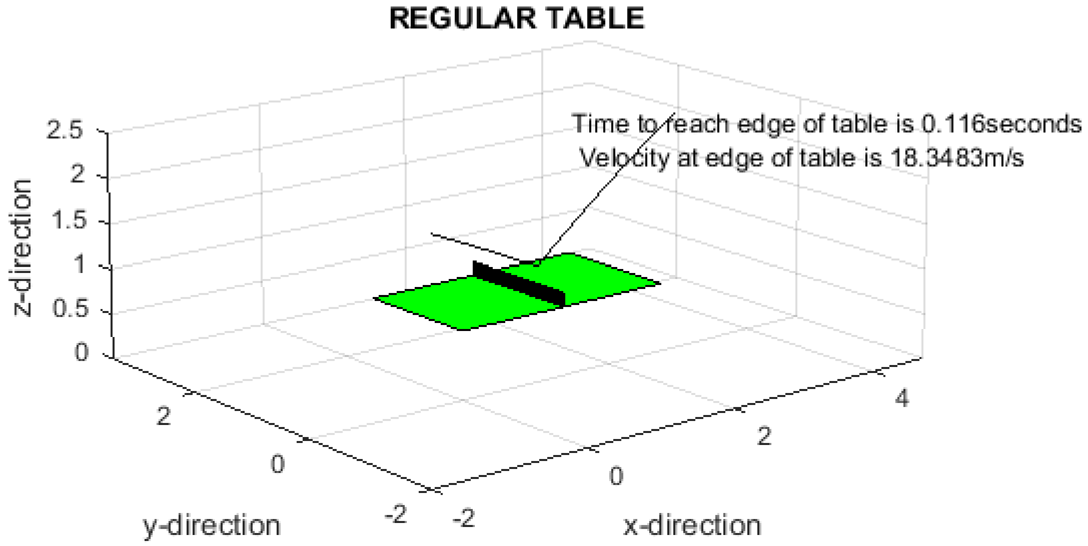

Figure 9 Regular table smash

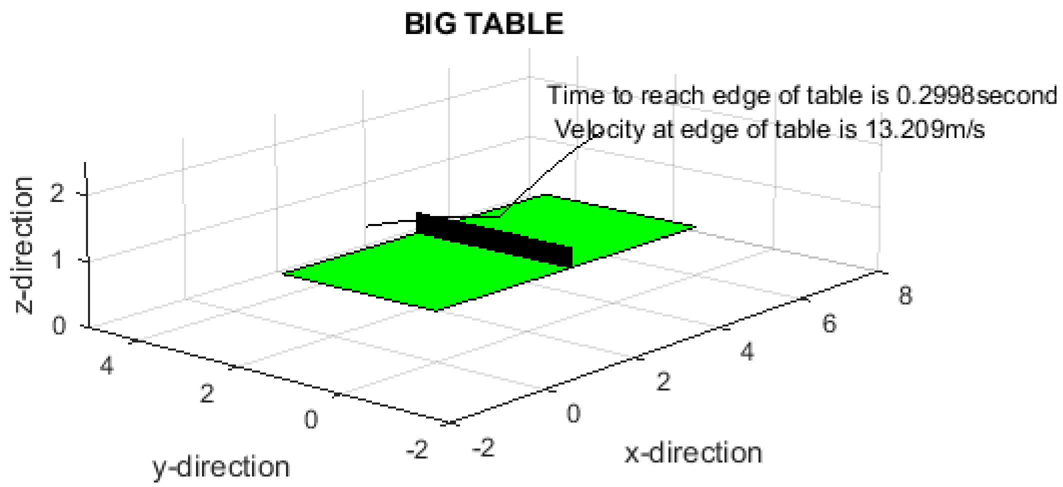

Figure 10 Big table smash