Big Hint for PSet Problem # 5.

Problem 5 is a S-T-R-E-T-C-H problem, leveraging your applied probability modeling knowledge. But we offer here a way of looking at the problem:

For simplicity ignore boundary effects on the destination song *j*. That is, assume that *j* is not 'too close' to 0 or *N*. Note that whatever location we are currently at in our algorithmic process, our past behavior is irrelevant. We only care about our current location, call it *i*, and our destination location *j*.

Here is the fundamental idea: There must be a threshold *T* such that if we are *T* distance units or less from *j*, then we switch from RANDOM to OSAAT. If there is no threshold, then we would use all the time either RANDOM or OSAAT. From part (b) we know that OSAAT-only means an average of *N*/3 button pushes. And elementary probability arguments (Bernoulli trials) suggest that RANDOM-only means an average of *N* trials or button pushes. But a mixed strategy using a threshold *T* results in fewer button pushes than either of the 'pure' strategies. When we are at the threshold *T*, we are ***indifferent*** between switching to OSAAT or continuing with RANDOM. By indifference, we imply that the mean number of button pushes under either alternative OSAAT or "continuing now with RANDOM" must be equal!

If we are at the threshold *T*, the number of button pushes remaining to get to destination *j* under OSAAT is equal to $|T - j|$. If we continue (temporarily) with RANDOM, then the mean total number of button pushes remaining to get to *j* is equal to the sum (1) the mean number of subsequent RANDOM button pushes to get us to *T* or closer to destination *j*, plus (2) the mean number of OSAAT button pushes to get us from the (close) RANDOM location to destination *j*. Once we land within *T* of destination *j*, we are on average *T*/2 units of distance away from *j*.

Can you work out the rest of the details? And part (b) is straightforward and maybe should be looked at first!

Good luck and have fun with this!