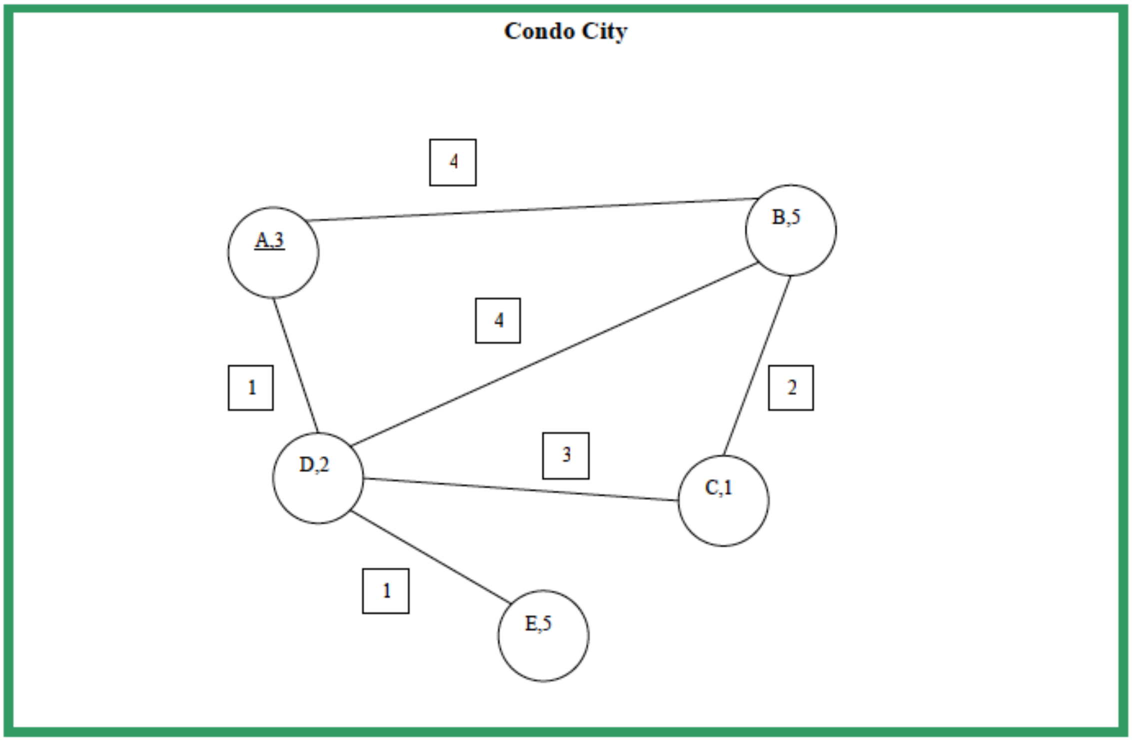
Problem 3 (42 points): Locating Facilities for Condo Complexes in a small town

Consider the 5 node network shown bellow. The nodes in condo city represent condo complexes, and the weights on each node represent the number of families living in each complex (thus the node weights must all be integral), while the arcs represent the distance between the complexes.

Thus A,3: Node A with weight 3



1. (5 Points) We are trying to locate a mail facility for the town. Find an optimal location of the mail facility such that the total weighted travel distance from the complexes to the facility is minimized.

2. (5 Points) What is the minimum number of additional families that must move into complex E in order to make E an optimal location for the mail facility? Please explain your answer briefly.

3. (9 Points) Assume we never built the mail facility in part 1. Assume also that we now wish to house 4 additional families somewhere on this network, all at one point. All 4 of these families are physically challenged and need to be located at an optimal point on the network, i.e., a point that would coincide with the mail facility. The obvious answer is, of course, to locate these 4 families at the location you identified in response to Part 1 of this problem. Unfortunately, this location happens to be the only point on the entire network where no additional families (beyond the ones originally in the complexes) can be located. With the exception of this restriction, there are two types of places where these four families can be located: either at an existing complex (e.g., if we located the four families at node A, the weight of A would be increased to 7), or at a point on some arc of the network (i.e., by building a new complex F on some arc with weight 4). Identify on the network ALL locations (nodes and points along arcs) where we can locate this complex; such that an optimal location for the mail facility will indeed co-incide with the complex where the four additional families will be housed.

4. (5 Points) Return to the original network of Part 1. Now 50 additional families would like to move into the town. However, currently the complex where you located the mail station in Part 1 is filled to capacity. However, the rest of the complexes have infinite capacity. Is it possible to allocate these additional 50 families in the remaining 4 complexes in such a way that the optimal location of the mail center is not changed? If so state one way this can be done (i.e., how many additional families should move into each complex); if not, give a short proof of why not.

Missing solution to Part 1.

Part 2:

x>=6

Thus if 6 families move into E then E becomes an optimal location for the mail center.

3.
In order to solve this part we add 4 units of weight to a node already in the network and see if the optimal facility will be located at the node. If we do this for all 4 nodes but D (which cannot be used) we find that only when we locate the families at node B do we have a tie between B and D and thus B is an optimal location, all other nodes are inferior. We then realize that we only need to perform a local analysis on the arcs that connect two optimal node locations. For the example above this turns out to be nodes B and D, thus only points along arc (B,D) need to be considered. Considering moving a very small number e units off B shows that D becomes the optimal facility as does moving a small number of units off D towards B shows that D will remain the optimal complex. Thus, the only place to locate the four new families is at complex B. It is important to note that for other weights (other than 4 families) it is indeed possible to have optimal locations be on arcs. For example, if the additional families were 10,000, any point on any arc in the network where these 10,000 families are located is optimal. It is not correct to say that the optimal location will always be at an existing node when the facility has demand units attached to it.

4. The idea here was to simply balance the weights so the facility doesn't move. The easiest way to do this is: "for every unit we locate at node A, we locate another unit at Node E" thus balancing the changes in demand weights. Thus an optimal way to locate the families without changing the location of the optimal facility is to locate 25 families at A and 25 families at E.