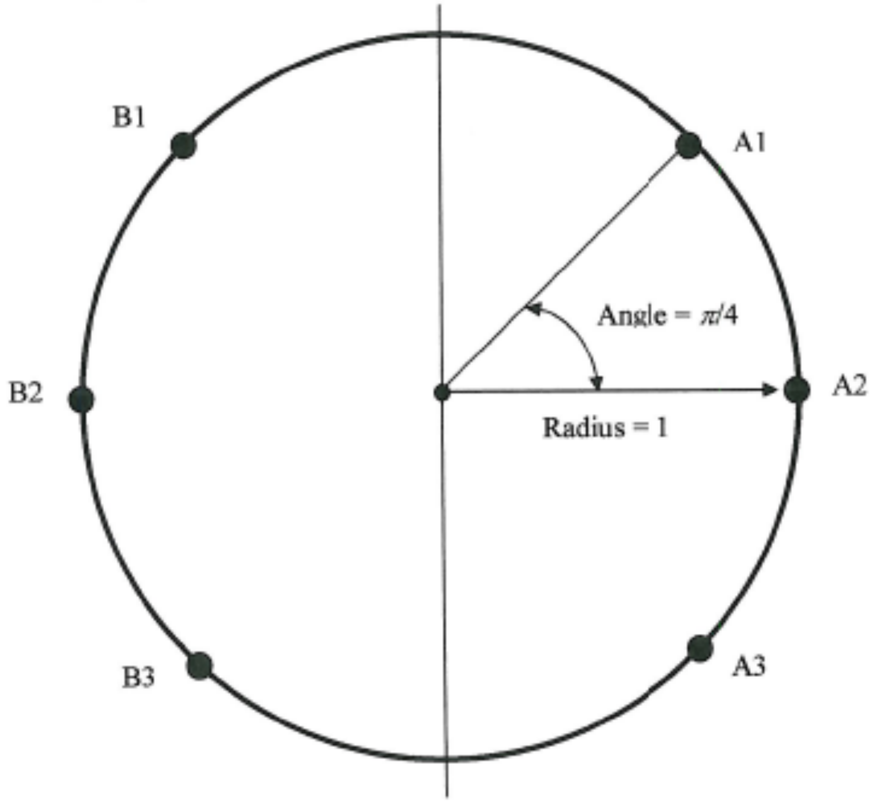
Problem 2 (42 points, with each part worth 7 points)



We consider a 2-server spatial queue for ambulances that operates like the traditional Hypercube queueing model. See the symmetric figure above to understand the geometry. Calls for ambulance service are uniformly independently distributed over the **circumference of the circle** of radius 1, as shown in the figure. There are two primary response areas, **A** and **B**. Response area **A** is the half circumference of the circle to the right of the vertical diameter drawn in the figure. Response area **B** is the half circumference to the left of the vertical diameter drawn in the figure. When idle, i.e., not serving customers, ambulance **A** is *equally likely to be stationed at any one of the three parking stations*, **A1, A2** or **A3.** When idle, ambulance **B** is *equally likely to be stationed at any one of its three parking stations*, **B1, B2** or **B3**.

Calls for ambulance service arrive in accordance with a homogeneous Poisson process with rate λ =1.0 call/hour. Travel speed is very fast, so virtually all of the service time is on-scene time. An ambulance always travels the shortest circumferential distance between its location and the call for service to which it is dispatched. Upon completion of service at the scene, the ambulance virtually immediately returns to one of its three parking stations, picking one of the three at random.

We model the service time for each customer as being distributed according to a negative exponential pdf with mean $(1/\mu)$ = 1.0 hour. The ambulance dispatcher always dispatches ambulance **A** to any customer in response area **A**, if ambulance **A** is available.

Otherwise a customer in response area **A** is serviced by ambulance **B**, if ambulance **B** is available. Symmetrically, the ambulance dispatcher always dispatches ambulance **B** to any customer in response area **B**, if ambulance **B** is available. Otherwise a customer in response area **B** is serviced by ambulance **A**, if ambulance **A** is available. Customers who arrive when both ambulances are busy are lost forever, i.e., not served. Thus, we have a zero queue capacity system, a 'loss system.'

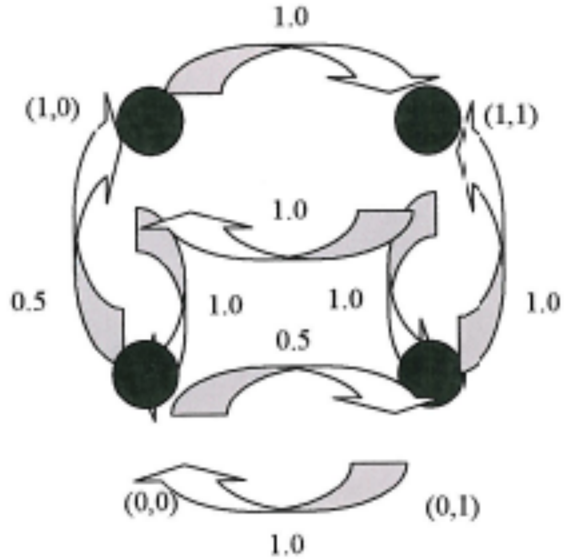
(a) Carefully draw the 2-server hypercube state space and transition rate diagram, showing all the upward and downward transition rates.
(b) Find the utilization factor (i.e., the fraction of time busy serving customers) for each ambulance.
(c) Compute the mean response distance traveled by an ambulance to a random customer.

Now assume that the dispatcher has an automatic vehicle location system so he knows the parking station at which each available ambulance is located. The dispatch strategy is now changed so that when both ambulances are free, i.e., not serving customers, the dispatcher will dispatch the closer ambulance to the customer. For instance, if a customer calls in from response area **B**, near the top of the circle and just to the left of the vertical diameter drawn, if ambulance **B** is available and located at **B2** or **B3** and if ambulance **A** is located at **A1**, then ambulance **A** is closer and is dispatched to the customer. The other parts of the dispatch strategy remain unchanged.

(d) Carefully draw the 2-server hypercube state space and transition rate diagram, showing all the upward and downward transition rates.
(e) Find the utilization factor (i.e., the fraction of time busy serving customers) for each ambulance.
(f) Compute the mean response distance traveled by an ambulance to a random customer.

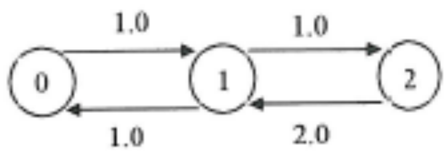
Problem 2

(a)



(b) By symmetry we note that each ambulance has the same performance statistics. Thus we can use the reduced state space M/M./2/0 model.

$P_0 + P_0 + 0.5P_0 = 1.0 \qquad P_0 = 0.4 = P_1 \quad P_2 = 0.2 \qquad \rho_1 = \rho_2 = 0.5P_1 + P_2 = 0.4$



(c) By symmetry, mean response distance to a random customer equals the mean response distance traveled by ambulance A to a random customer.
Mean A-to-A response distance is $(2/3)\{(1/2)(2\pi/8)(1/4) + (1/2)(3*2\pi/8)(3/4)\} + (1/3)(1/2)(2\pi/4) = 5\pi/24 + \pi/12 = 7\pi/24$.
Mean A-to-B response distance is $(2/3)\{2\pi/8 + 2\pi/4\} + (1/3)\{2\pi/4 + 2\pi/8\} = 3\pi/4$
Utilizing Equations (5.16) in the text, we find that $f_{11} = f_{22} = 3/8$ and $f_{12} = f_{21} = 1/8$.
Therefore 3/4 of ambulance A's customers are in primary response area **A** and 1/4 take it to primary response area **B**. Thus, the desired answer is
Mean response distance = $(3/4)\,7\pi/24 + (1/4)\,3\pi/4 = 13\pi/32$. (=1.276)

(d) The state-rate-transition diagram remains unchanged from part (a). The only possible changes are from state (00), involving the two upward transition rates. But we know that these rates must sum to one and that there is symmetry between the units. You may think of it this way: For every customer that ambulance A loses due to ambulance B being closer, ambulance B loses due to ambulance A being closer,

(e) Answers the same as in part (b) above, since we can still shrink the state space to the same M/M/2/0 model and invoke symmetry, or simply note that the 2-dimensional Hypercube models are identical with regard to workloads.

(f) We divide and conquer, very carefully. First, since $P_{00} = P_{01} + P_{10} = 0.4$, 50% of answered customers find the system in state (0,0) and 50% of the answered customers find the system with one server free and one busy. (Customers who arrive when both are busy are rejected and do not arise in the mean travel distance calculations.)

So, Answer = (Conditional Mean Travel Distance When both are Free)*0.5
+(Conditional Mean Travel Distance When one is Free)*0.5
= E[D|2 free]*0.5 + E[D|1 free]*0.5.

First consider, E[D|1 free]. We can immediately see that E[D|1 free] = (1/4)*2π. This is because a customer arriving when only one ambulance is free is uniformly distributed over the circle of circumference 2π. The free ambulance will take the shorter of the two circumferential paths to the customer, yielding a mean travel distance of (1/2)*(1/2 of the circumference) = (1/4)*2π.

Now consider E[D|2 free]. For this we must divide and conquer over all possible combinations of real time locations of ambulances A and B. In examining the circular total system, we see that there are three different configurations of interest:

1. Ambulances A and B aligned at same latitude: (A1, B1), (A2, B2), (A3, B3)
   Probability = 3/9
2. Ambulances A and B at 'diagonals': (A1, B3), (A3, B1)
   Probability = 2/9
3. 1 ambulance in center & the other not: (A1, B2), (A3, B2), (A2, B1), (A2,B3).
   Probability = 4/9.

For condition 1, each ambulance is closer to all points in its primary response area, so the real time vehicle locations information makes no difference. The conditional mean travel distance is the same as the A-to-A mean distance in part (c) and equals π

For condition 2, each ambulance serves the half circle nearest it. Thus the corresponding conditional mean travel distance is $\pi/4$.

For condition 3, suppose we have (A1, B2). Then ambulance A covers the first $\pi/8$ miles of the 'northern' part of response area B and ambulance B covers the first $\pi/8$ miles of the southern part of response area A. The conditional mean travel distance is $(3/8)*(3\pi/8)*0.5 + (5/8)*(5\pi/8))*0.5 = 19\pi/1$

Thus $E[D] = (1/4)2\pi(1/2) + \{(3/9)(7\pi/24) + (2/9)(\pi/4) + (4/9)(19\pi/128)\}(1/2)$
Or, $E[D] = \pi/4 + \{7\pi/32\}(1/2) = 23\pi/64 \approx 1.129$, approximately a 12% reduction compared to part (c) with no real time vehicle location information.