Electro-Mechanical System: Swept Sine Input

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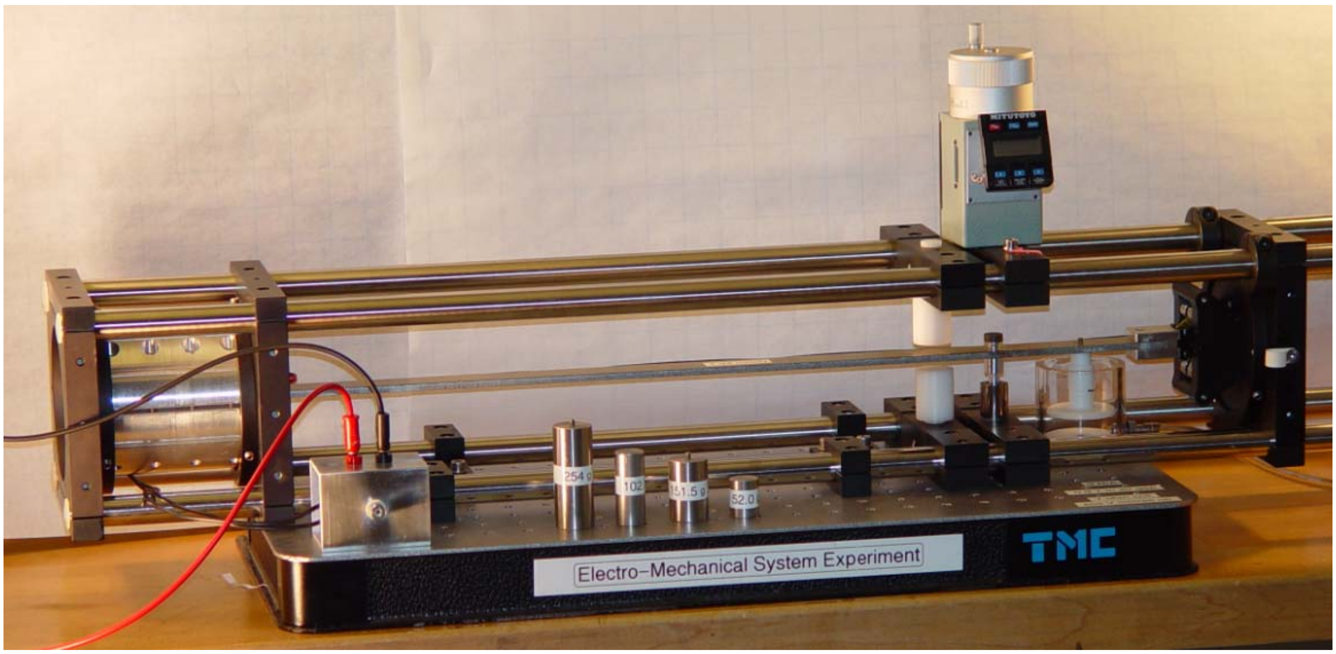
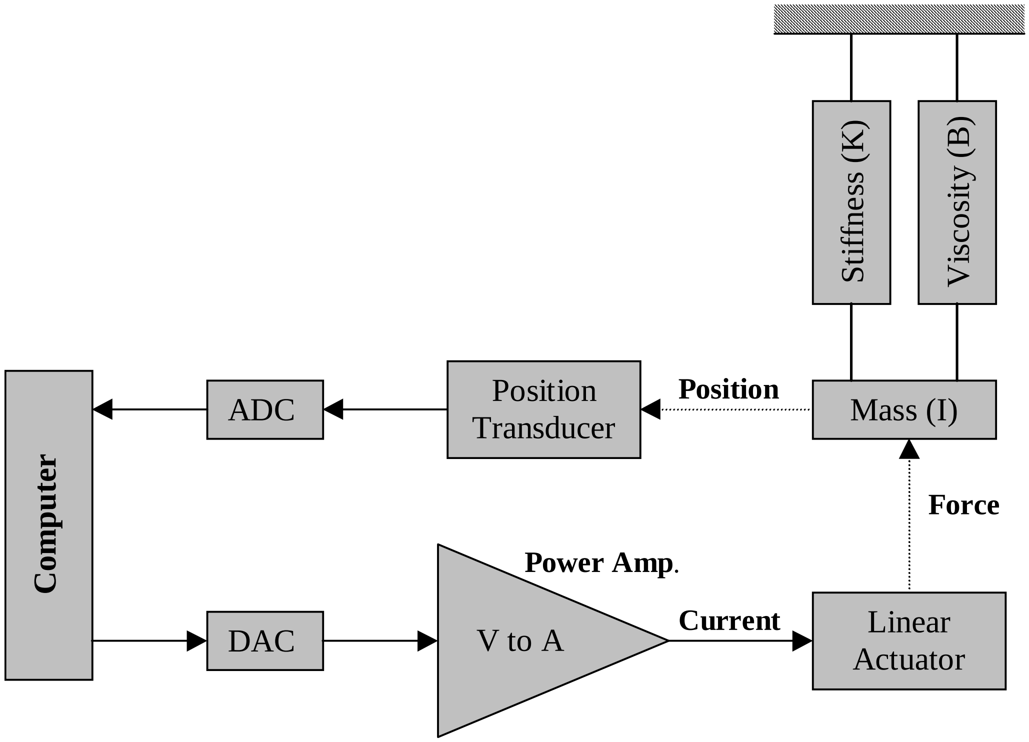
Introduction

A copper-wire coil (acting as a linear actuator) is attached to the free end of a 25 mm wide by 4 mm thick stainless steel beam which protrudes 615 mm from a large clamp (see Figutre below). The coil moves vertically by about 12 mm (peak to peak) within an air gap formed by two pairs of permanent magnets. The top pair of permanent magnets create a 0.9 T magnetic field in the gap. The bottom pair of permanent magnets produce a -0.9 T magnetic field. These magnetic fields are at right angles to current flowing in the coil. The Lorentz force generated by the coil current interacting with the magnetic field is at right angles to both the current and magnetic field. The beam is consequently deflected by this force (a positive current produces an upward force and deflects the beam upward). The beam deflection is measured by an inductive position sensor (Fastar) mounted 500 mm from the clamp. The position sensor may be calibrated using a Mitutoyo Digital Micrometer (1 μm resolution) mounted above the position sensor.



A dashpot type damper, mounted 560 mm from the clamp, may be filled with a liquid (e.g. water) to provide viscous damping. A set of calibrated masses (approximately 50 g increments) may be mounted above the dashpot to provide known forces to deflect the beam for measurement of the static beam stiffness and for determination of the coil current to force relation.



Static Characteristics

Some important static characteristics of the system (obtained using the Mathcad module titled "ElectroMechanicalSystemStaticMeasurements.mcd") are listed below.

$\text{CoilCurrentMax} := 0.8 \cdot A \qquad \text{CoilResistance} := 7 \cdot \text{ohm}$

$\text{CoilVoltageMax} := \text{CoilResistance} \cdot \text{CoilCurrentMax} \qquad \text{CoilVoltageMax} = 5.6 \cdot V$

To be safe always limit the voltage to about 4 V and the current to 0.6 A (i.e. set the power supply current limit to 0.6 A).

$$\text{PositionSensorVoltageToPosition} := 2.5 \cdot 10^{-3} \cdot \frac{m}{V}$$

$$\text{BeamStiffness} := 750 \cdot \frac{N}{m}$$

$$\text{CoilCurrentToForce} := 9 \cdot \frac{N}{A}$$

$$\text{CoilVoltageToForce} := \frac{\text{CoilCurrentToForce}}{\text{CoilResistance}} \qquad \text{CoilVoltageToForce} = 1.28571 \cdot \frac{N}{V}$$

$$\text{CoilForceToVoltage} := \frac{1}{\text{CoilVoltageToForce}} \qquad \text{CoilForceToVoltage} = 0.77778 \cdot \frac{V}{N}$$

Generate Sinusoidal Log Sweep

We know from other experiments that the resonant frequency of the beam is in the order of 8 Hz. We will therefore subject it to a series of sinusoidal force inputs (each one is called a trial) starting at 0.1 Hz and increment to around 30 Hz with approximately logarithmic increments. The sampling rate is 500 Hz in all cases.

$\Delta t := 0.002$ s interval between samples

$J := 50$ number of trials

$j := 0 .. J$

$f_j := 10^{\frac{j}{20}-1}$ logarithmically spaced frequencies

Each sinusoidal force is applied np times per trial.

$$np_j := \begin{cases} 1 & \text{if } f_j < 1 \\ 10 & \text{if } f_j \geq 1 \wedge f_j < 10 \\ 100 & \text{if } f_j \geq 10 \end{cases}$$

$n_j := \mathrm{round}\left(\frac{np_j}{f_j \cdot \Delta t}\right)$ number of samples per trial

The frequency in each trail is

$f_j := \frac{np_j}{n_j \cdot \Delta t}$ Hz

The total experimental time is therefore

$$\sum_{j=1}^{J} \left(n_j \cdot \Delta t\right) = 221.556 \quad \text{s}$$

$go = 0$

$$\begin{pmatrix} Gain \\ Phase \end{pmatrix} := \left| \begin{array}{l} \text{if } go = 1 \\ \quad \left| \begin{array}{l} time \leftarrow timer(\Delta t) \\ \text{for } j \in 0..J \\ \quad \left| \begin{array}{l} \text{for } i \in 1..n_j \\ \quad \left| \begin{array}{l} timer(0) \\ x \leftarrow \sin(2\cdot\pi\cdot f_j\cdot i\cdot\Delta t) \\ err \leftarrow da(0, 0.77778\cdot x) \\ y_i \leftarrow 2.5\cdot 10^{-3}\cdot ad(0) \end{array} \right. \\ \mu \leftarrow \frac{1}{n_j}\cdot\sum_{i=1}^{n_j} y_i \\ \text{for } i \in 1..n_j \\ \quad y_i \leftarrow y_i - \mu \\ Csin_j \leftarrow \frac{1}{n_j}\cdot\sum_{i=1}^{n_j} (y_i\cdot\sin(2\cdot\pi\cdot f_j\cdot i\cdot\Delta t)) \\ Ccos_j \leftarrow \frac{1}{n_j}\cdot\sum_{i=1}^{n_j} (y_i\cdot\cos(2\cdot\pi\cdot f_j\cdot i\cdot\Delta t)) \\ Gain_j \leftarrow 2\cdot\sqrt{(Csin_j)^2 + (Ccos_j)^2} \\ Phase_j \leftarrow angle(Csin_j, Ccos_j) - 2\pi \end{array} \right. \\ \begin{pmatrix} Gain \\ Phase \end{pmatrix} \end{array} \right. \\ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \text{otherwise} \end{array} \right.$$

The sinusoidal force amplitude is 1 N

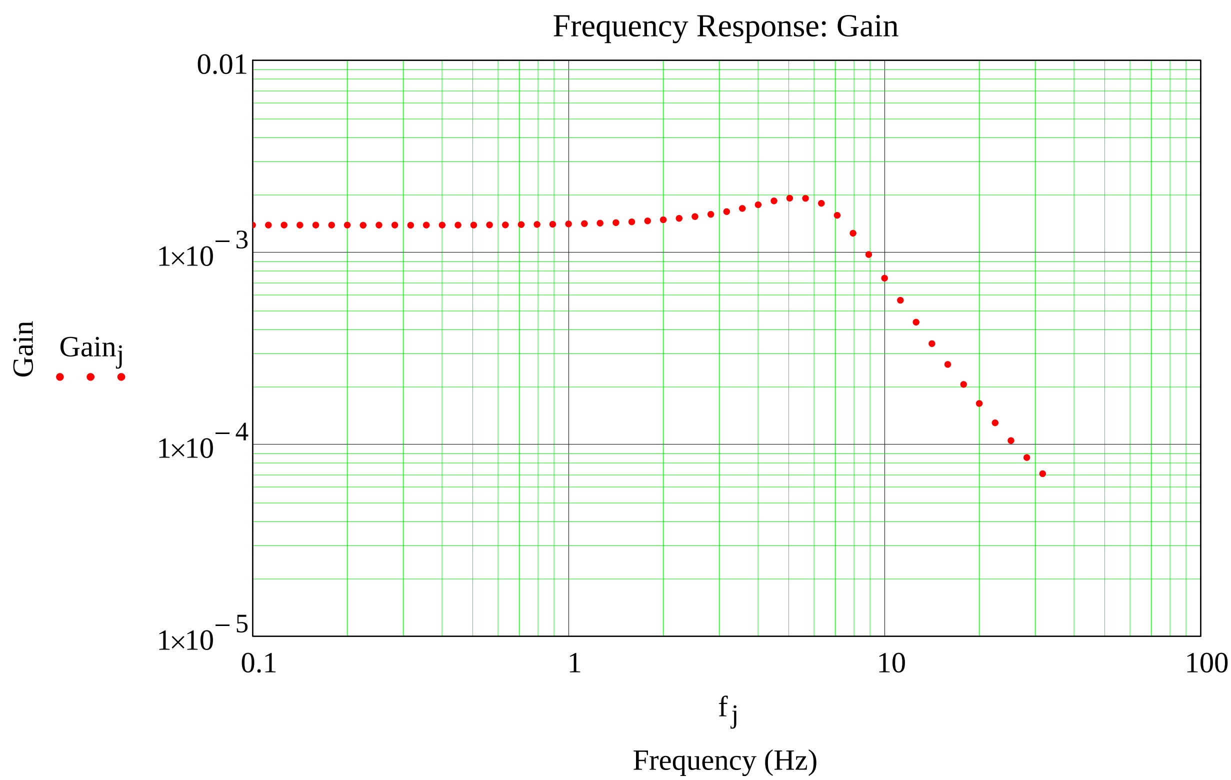
$CoilForceToVoltage = 0.77778\cdot\frac{V}{N}$

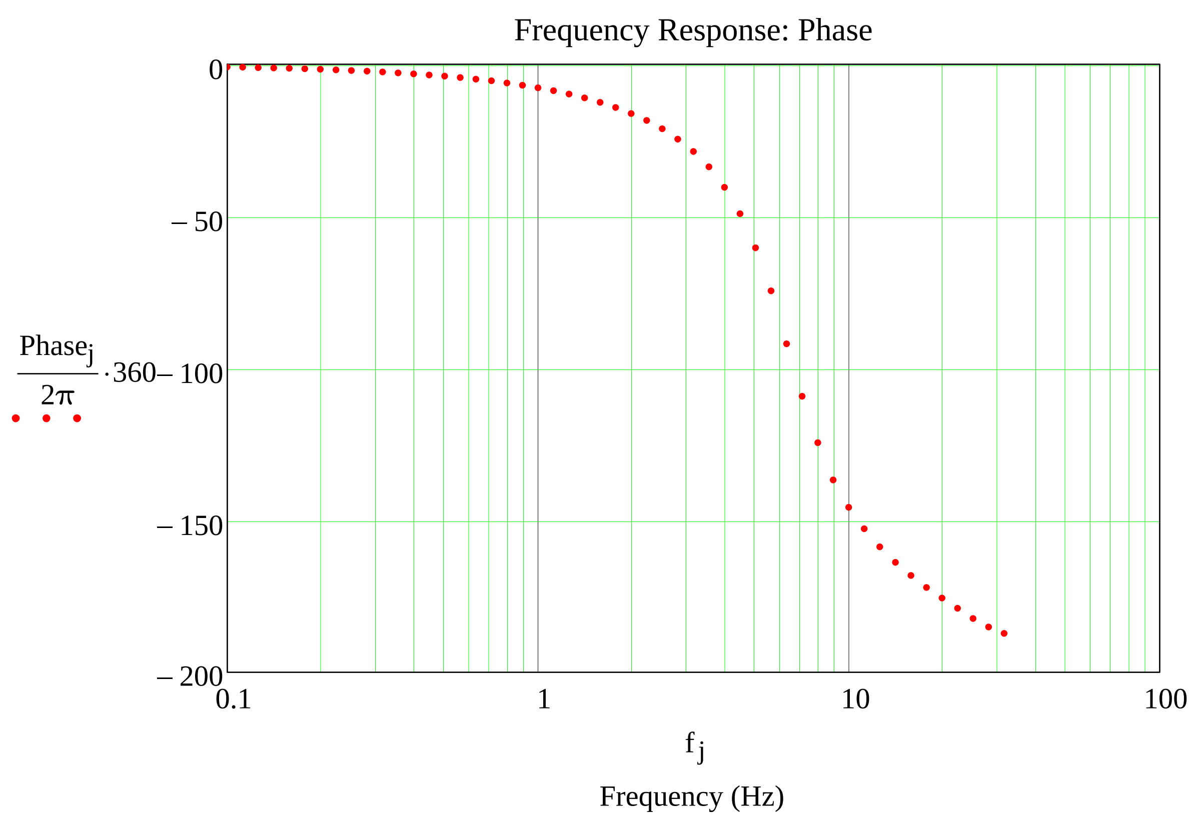
$PositionSensorVoltageToPosition := 2.5\cdot 10^{-3}\cdot\frac{m}{V}$

compute mean

set mean = 0

Results


Frequency Response: Gain
0.01
1×10−3
1×10−4
1×10−5
Gain
Gainj
0.1
1
10
100
fj
Frequency (Hz)


Frequency Response: Phase
0
− 50
− 100
− 150
− 200
Phasej / 2π ·360
0.1
1
10
100
fj
Frequency (Hz)

Fit Transfer Function to Frequency Response Data

$$H(s, A, \omega_n, \zeta) := \frac{A\cdot\omega_n^{\,2}}{s^2 + 2\cdot\zeta\cdot\omega_n\cdot s + \omega_n^{\,2}}$$

Second-order Low-pass Transfer Function

we replace s with i.ω $\omega \equiv \omega$

$$\left|H(i\cdot\omega, A, \omega_n, \zeta)\right| \text{ simplify } \rightarrow |A|$$

$$SS(A, \omega_n, \zeta) := \sum_{j=1}^{J} \left(\left|H(i\cdot 2\pi\cdot f_j, A, \omega_n, \zeta)\right| - Gain_j\right)^2$$

defines the sum of squares objective function

Provide initial estimates

$A := 0.001$ $\omega_n := 2\cdot\pi\cdot 5$ $\zeta := 0.2$

$TOL := 0.001$

$$\begin{pmatrix} A \\ \omega_n \\ \zeta \end{pmatrix} := \text{Minimize}(SS, A, \omega_n, \zeta)$$

Mathcads minimization operator.

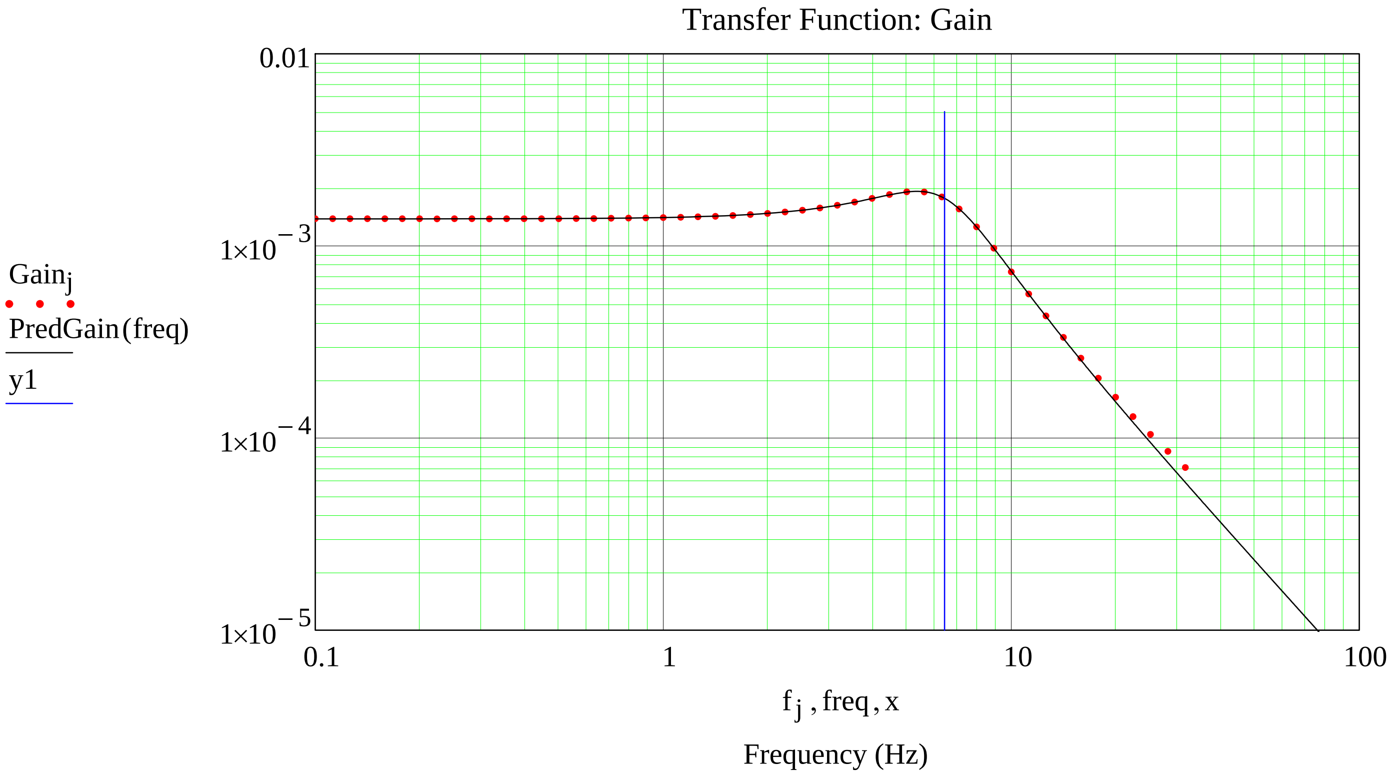
Results

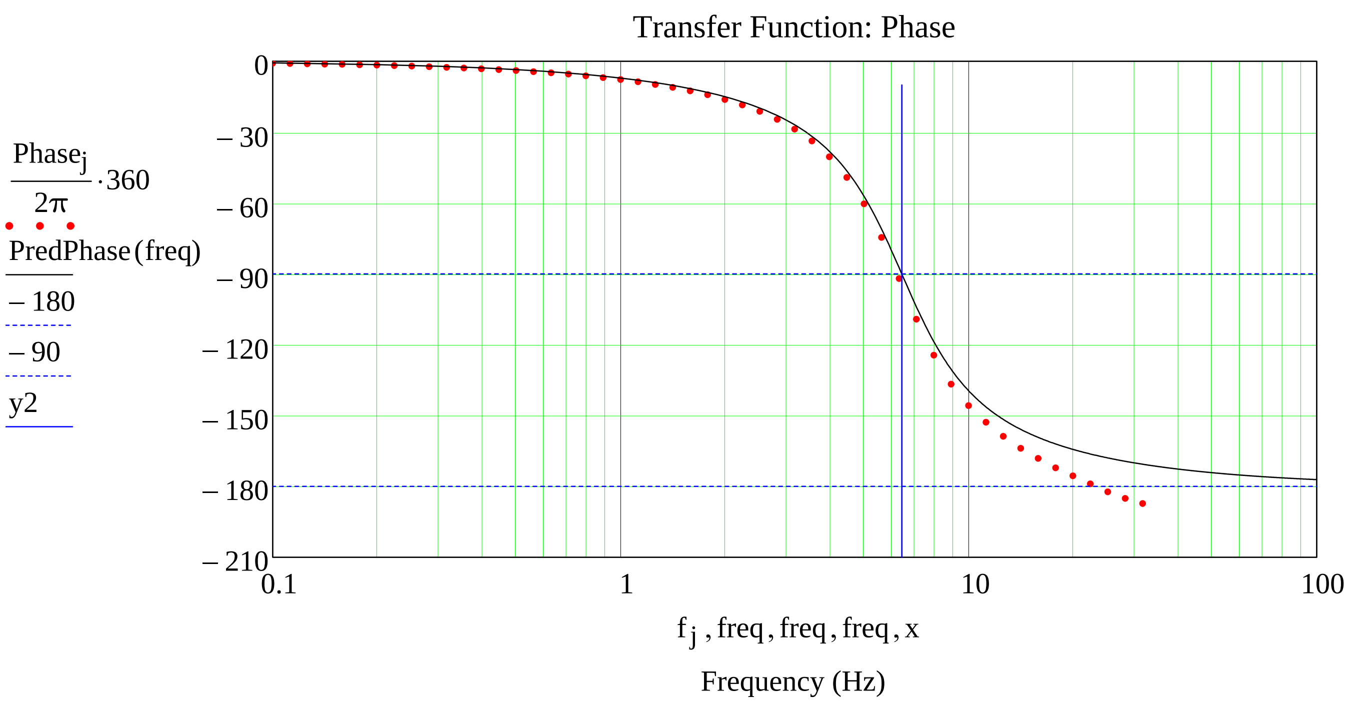
$A = 0.00138 \ \frac{m}{N}$ $\frac{\omega_n}{2\pi} = 6.4283$ Hz $\zeta = 0.39041$

$$\text{PredGain}(f) := \left|H(i\cdot 2\pi\cdot f, A, \omega_n, \zeta)\right|$$

$$\text{PredPhase}(f) := \frac{\arg\left(H(i\cdot 2\pi\cdot f, A, \omega_n, \zeta)\right)}{2\pi}\cdot 360 \quad \text{deg}$$

Compare Data with Model





It is important to note that, as expected for a second-order low-pass system, the phase shift is -90 at the natural frequency, ω_n (vertical blue line). However the measured phase continues past -180 indicating that the system exhibits some addtional behavior not predictable by the simple second-order model. We could (but will not) fit a higher-order model (or a model with a pure delay) to account for this.