# 1.021, 3.021, 10.333, 22.00 Introduction to Modeling and Simulation - Spring 2018

**Problem set #2**

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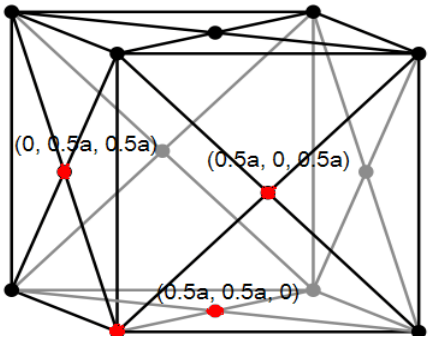
## 1. Lennard-Jones potential development (50%)




(a) The equilibrium distance in LJ potential is $r_0 = 2^{1/6}\sigma$
It equals to the nearest neighboring distance in FCC crystal as $_0 = 2^{-1/2}a_0$
Thus $\sigma = 2^{-2/3}a_0 = 0.63a_0$

(b) For LJ potential $\phi = 4\varepsilon\left[\left(\frac{\sigma}{r_{ij}}\right)^{12} - \left(\frac{\sigma}{r_{ij}}\right)^{6}\right]$

$$k = \phi''(r_{ij} = \ _0) = \frac{d^2\phi}{dr_{ij}^2}|_{r_{ij}=r_0} = \frac{4\varepsilon}{\frac{2}{0}}\left[156\left(\frac{\sigma}{\ _0}\right)^{12} - 42\left(\frac{\sigma}{\ _0}\right)^{6}\right]$$

since $_0 = 2^{1/6}\sigma$,

$$k = \frac{72\varepsilon}{2^{1/3}\sigma^2} = 57.15\frac{\varepsilon}{\sigma^2}$$

(c)
Bulk modulus:

$$K = \frac{E}{3(1-2\nu)} = \frac{2E}{3} = \frac{16\mu}{9} = \frac{8r_0^2k}{9V} = \frac{32r_0^2k}{9a_0^3}$$

Using $k = \frac{72\varepsilon}{2^{1/3}\sigma^2}$, $_0 = 2^{1/6}\sigma$, and $a_{09} = 2^{2/3}\sigma$, we have

$$K = \frac{64\varepsilon}{\sigma^3}$$

From DFT measurement $a_0 = 3.64$, Thus $\sigma = 2^{-2/3}a_0 = 2.29$ Å
K=140 GPa=140/160.2 eV/Å$^3$=0.874 eV/Å$^3$

$$\varepsilon = \frac{K\sigma^3}{64} = 0.164 \text{ eV}$$

(d)
For this calculation $\sigma = 2.29$ Å, $\varepsilon = 0.164$ eV, cut-off around $0.71a_0$
For Cleri's paper $\sigma = 2.314$ Å, $\varepsilon = 0.167$ eV, cut-off around $1.49a_0$
The result is similar as what is obtained by Cleri et. al.. The larger energy may be caused by the longer cut-off function used, and thus more interaction is included for the bond energy.

(e)

(i) Parameters found in this calculation $\sigma = 2.29$ Å, $\varepsilon = 0.164$ eV, cut-off radius as $1.1r_0 = 2.84$ Å

Simulation inputs:

Crystal geometry: lattice const. 3.64 Å, 4*4*4 unit cells

LJ parameters: cut-off 2.84 Å, epsilon 0.164, sigma, 2.29

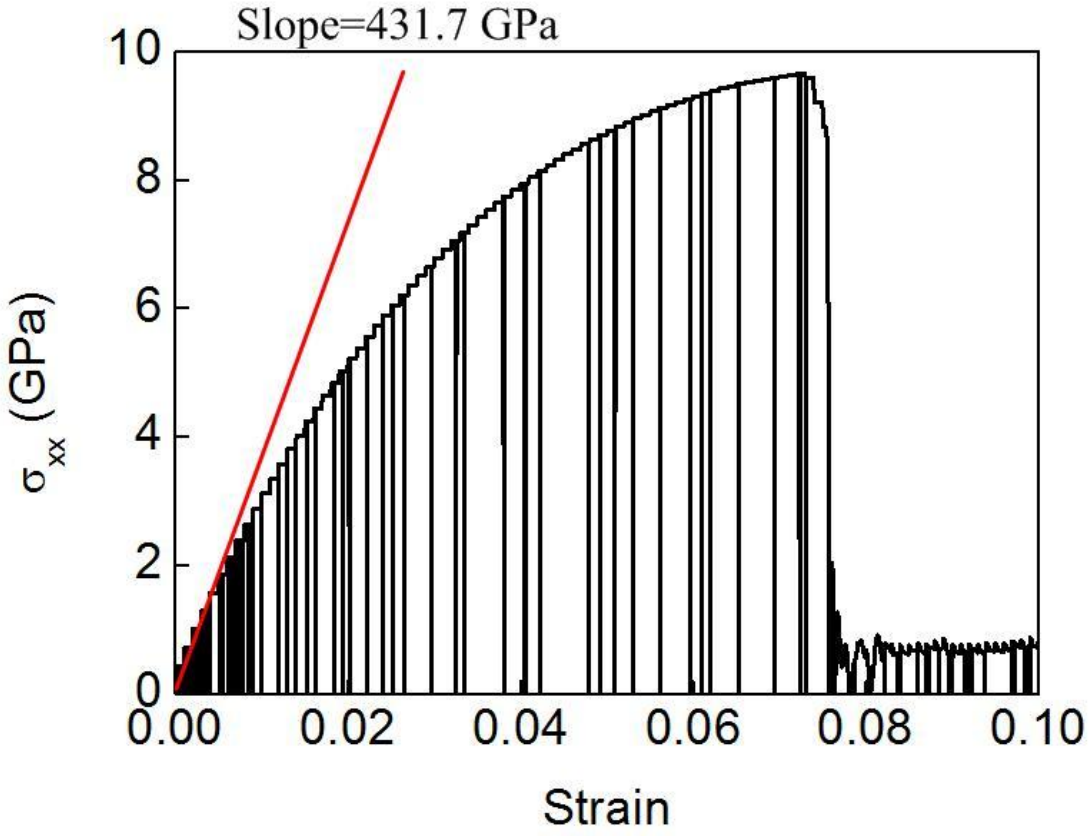
Strain increment: increment interval 100, increment in xyz directions 0.001, 0.001, 0.001

Maximum number of steps: 10000, time step 0.25

Stress strain curve:

stress (in GPa)= Press_xx (default, in eV/A$^3$)*160.2 GPa/(eV/A$^3$)

strain = Time (in output data)/0.25/100*0.001




Since for here $\sigma_{11} = \sigma_{22} = \sigma_{33}$ and $\varepsilon_{11} = \varepsilon_{22} = \varepsilon_{33}$, we have K=$\sigma_{11}/(3\varepsilon_{11})$=**143.9 GPa**

(ii) Parameters found Cleri's paper $\sigma = 2.314$ Å, $\varepsilon = 0.167$ eV, cut-off around $1.49 * 1.56\sigma = 5.38$ Å

Simulation inputs:

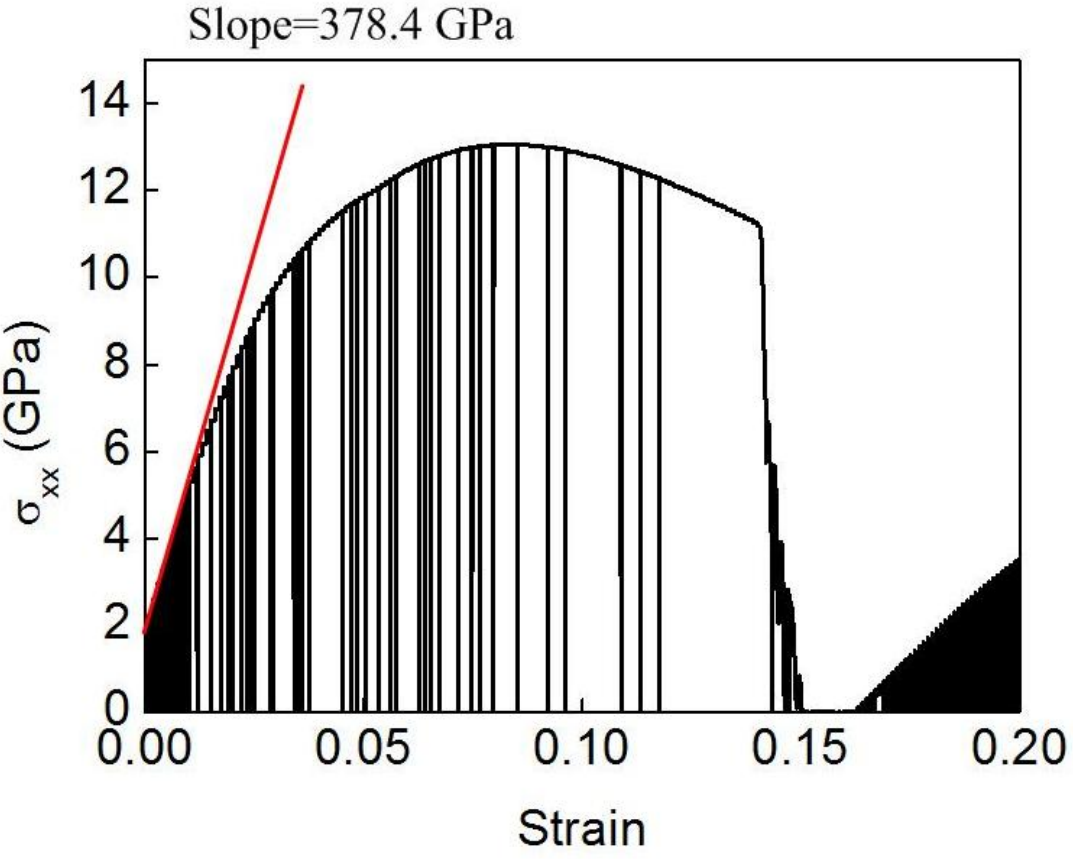
Crystal geometry: lattice const. 3.61 Å, 4*4*4 unit cells

LJ parameters: cut-off 5.38 Å, epsilon 0.167, sigma, 2.314

Strain increment: increment interval 100, increment in xyz directions 0.001, 0.001, 0.001

Maximum number of steps: 20000, time step 0.25

Stress strain curve:
stress (in GPa)= Press_xx (default, in eV/$A^3$)*160.2 GPa/(eV/$A^3$)
strain = Time (in output data)/0.25/100*0.001




Since for here $\sigma_{11} = \sigma_{22} = \sigma_{33}$ and $\varepsilon_{11} = \varepsilon_{22} = \varepsilon_{33}$, we have K=$\sigma_{11}/(3\varepsilon_{11})$=**126.1 GPa**

(f) For parameters found in this calculation, critical strain is ~0.073
For parameters found in Cleri's paper, critical strain is ~0.081

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## 2. Monte Carlo simulation of cell spreading (50%)

```
clear;
cell_occupy(1,1)=0;                                    %cell_occupy is the array to record the points occupied by cells
cell_occupy(1,2)=0;                                    %cell_occupy(n,1) for x

lattice=2;                                             %lattice space (pitch) of the nanopillars, in the unit of um.
                                                           %simulation for lattice<1.5 is slow, BE patient!
kbt=0.6;                                               %thermal effect kBT at T=300 K, with a unit of kcal/mol
N=40000;                                               %number of the total steps (attempts)
k_stiff=2.9;                                           %stiffness of the cell in spreading, kcal/mol/um^2
%Simulation starts by plotting out the original point of the cell

plot(cell_occupy(1,1)*lattice,cell_occupy(1,2)*lattice,'o','Color',[0.6 0.8 0.1], ...
'LineWidth',5,'MarkerEdgeColor','r','MarkerFaceColor','r','MarkerSize',15); hold on;
for i=1:N                                              %start of the loop
    spreadnum=size(cell_occupy);                       %compute the number of points get occupied by the cell
    moving_point=ceil(rand()*spreadnum(1));            %select a point among all the existing points
    rand_angle=rand();
    if (rand_angle<=0.25)                              %determine which direction to move, select one among {+x,+y,-x,-y}
        moving_angle=[1,0];
    elseif (rand_angle<=0.5)
        moving_angle=[0,1];
    elseif (rand_angle<=0.75)
        moving_angle=[-1,0];
    else
        moving_angle=[0,-1];
    end
    moving_dist=1;                                     %the maximum movement for each attempt is 1 lattice distance
    move_target=cell_occupy(moving_point,1:2)+moving_dist*moving_angle;
    diff_ene=0.5*(moving_dist*lattice)^2*k_stiff;
    % Energy function of single point spreading. 2.9 is a factor with the purpose of fitting the experimental results
    a=0;
    if (diff_ene<0 && moving_dist==1)
        a=1;                                           %accept and move
    else
        k=rand;
```

```
            if (k<exp(-diff_ene/kbt) && moving_dist==1)
                a=1;                                        %accept and move
            else
                a=0;                                        %reject attempt
            end
        end
        if(a==1)                                            %if accept, judge whether the target is taken
            occupy=0;
            for p=1:spreadnum(1)
                if (cell_occupy(p,1:2)==move_target)
                    occupy=1;
                end
            end
            if(occupy==0)
                cell_occupy(spreadnum(1)+1,1:2)=move_target;
            else
                push=1;
                while(push==1)
                    move_target=move_target+moving_angle;
                    push=0;
                    for p=1:spreadnum(1)
                        if (cell_occupy(p,1:2)==move_target)
                            push=1;
                        end
                    end
                end
                cell_occupy(spreadnum(1)+1,1:2)=move_target;
            end
            line(1,1:2)=cell_occupy(moving_point,1:2);      %the starting point for this spreading attempt
            line(2,1:2)=move_target;                        %the end point for this spreading attempt
            plot(line(:,1)*lattice,line(:,2)*lattice,'Color',[0.6 0.8 0.1],'LineWidth',5);
            %plot the trace of this spreading attempt
        end
    end
    plot(cell_occupy(1,1)*lattice,cell_occupy(1,2)*lattice,'o','Color',[0.6 0.8 0.1], ...
    'LineWidth',5,'MarkerEdgeColor','r','MarkerFaceColor','r','MarkerSize',15); hold off;
```

(1) The cell spreading behaviors on top of the nano pillars are governed by the geometry of the substrate. Specifically, controlled by the in-plane density of nano pillar. On a substrate composed of many nano pillars (of small inter-pillar distance), a cell spreads evenly in all directions without preference in any specific direction. For substrates with fewer nano pillars (of large inter-pillar distance), the cell does not spread as much and there is some preferred direction for cell to spread toward.

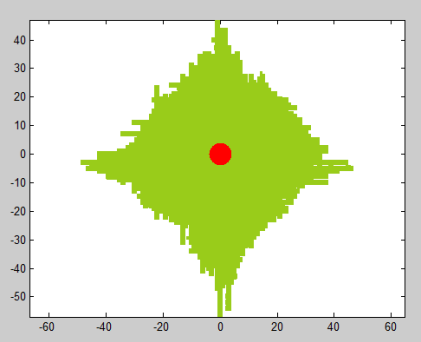
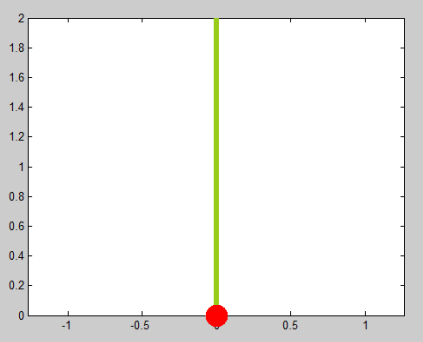
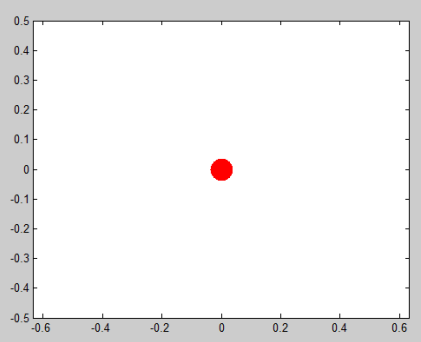
Cell spreading is a continuous process 1) it spreads by climbing on top of the nano pillar and the increasing inter-pillar distance increases the difficulty for cell to spread out; 2) the cell spreading (movement) is controlled by the cytoskeleton motion inside the cell. The complex assemble and dissemble of actin filament generates inhomogeneous stress inside the cell, making the cell move directional (as shown by the leading edge in cell movement).

(2) A simple Monte Carlo model is applied in this study to learn how a cell spreads out. We create an array to record the coordinates of all the nano pillars got occupied by the cell (with one single point initially occupied). The distance between neighboring nano pillars is given by "lattice". For every single step, we randomly pick up a point of the cell, given by the coordinate of an occupied nano pillar, and make an attempt to extrude for one single lattice distance from this point in a random direction. We simply model the energy difference before and after extrusion as elastic deformation energy given by a harmonic spring as "diff_ene=0.5*k_stiff*(moving_dist*lattice)^2" where k_stiff is a empirical parameter represents the cell stiffness. The Metropolis criterion is applied to compute the probability to accept or deny the attempt by comparing the energy difference against the thermal fluctuation energy. If the attempt is accepted and the targeting nano pillar is available, meaning the point we selected for extrusion locates at the edge, we simply include the targeting point into the list. Otherwise we consider that the cell must spread from its leading edge at the periphery and select the point at the cell edge to make the same extrusion.

(3)

(4) The MATLAB code is as given above, as well as the notes.

(5) Several parameters, including the temperature effect, lattice distance and cell stiffness can be altered to qualitatively discuss their effect. It is observed that higher temperature, smaller lattice distance and smaller cell stiffness can all contribute to the cell spreading. However, this model is still too simple to totally mimic the experiments as 1) the 2D simulation cannot include the constant volume constraint 2) the energy function is too simple that does not include the adhesion energy 3) the "push model" does not quantitatively describe the leading edge and directional preference for cell spreading. For difference lattice distance, the result is as shown below.

| Lattice=1 um | Lattice=2 um | Lattice=4 um |
| --- | --- | --- |
|  |  |  |