

Problem 4-1. Probability of the Current Observation

```
##### YOUR CODE HERE #####
alpha, c = HMMfwd(pi, a, b, obs)
print('=== Probability of the Current Observation ===\n', np.sum(alpha[:, -1]))
#####

=== Probability of the Current Observation ===
2.1380223436320317e-05
```

Problem 4-2. Most Likely Sequence of States

```
##### YOUR CODE HERE #####
path, delta, phi = Viterbi(pi, a, b, obs)
print('=== Most Likely Sequence ===\n', path)
print('==== Path Reference ==== \n', phi)
#####

=== Most Likely Sequence ===
[3 0 0 0 0 1 0 2]
==== Path Reference ====
[[0 3 0 0 0 0 1 0]
 [0 3 0 0 0 0 0 0]
 [0 0 0 0 0 0 0 0]
 [0 0 2 2 0 0 1 0]]
```

Problem 4-3. Best Fitting Model Parameters

```
##### YOUR CODE HERE #####
pi, a, b = BaumWelch(obs, nStates)
print('==== Transition Probability ==== \n', a)
print('==== Emission Probability ==== \n', b)
print('=== Initial State Probability === \n', pi)
#####

==== Transition Probability ====
[[0.00000000e+000 0.00000000e+000 1.00000000e+000 0.00000000e+000]
 [7.68794637e-019 4.99994129e-001 2.05656449e-015 5.00005871e-001]
 [4.82763744e-001 1.00154433e-061 5.17236256e-001 3.53894285e-031]
 [4.09737090e-077 0.00000000e+000 1.00000000e+000 3.86291196e-100]]
==== Emission Probability ====
[[0.00000000e+00 2.59970825e-68 6.90463459e-01 3.62853588e-01]
 [1.00000000e+00 0.00000000e+00 0.00000000e+00 1.82445405e-01]
 [0.00000000e+00 8.44660960e-01 3.43704961e-26 4.16776305e-01]
 [2.34839669e-05 1.05060749e-31 0.00000000e+00 5.72138770e-01]]
=== Initial State Probability ===
[0. 1. 0. 0.]
```