## M2608.001300 기계학습 기초 및 전기정보 응용 Assignment 2: Support Vector Machines

### Setup

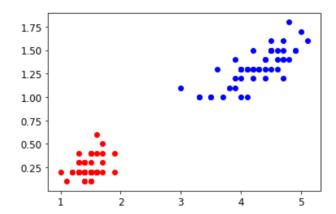
Check that Python 3.5 or later is installed (although Python 2.x may work, it is deprecated so we strongly recommend you use Python 3 instead), as well as Scikit-Learn 0.20 or later is installed.

```
In [1]: # Python >=3.5 is required
         import sys
         assert sys.version_info >= (3, 5)
         # Scikit-Learn >=0.20 is required
         import sklearn
         assert sklearn.__version__ >= "0.20"
         # Common imports
         import numpy as np
         import os
         # to make this notebook's output stable across runs
         np.random.seed(42)
         # To plot pretty figures
         %matplotlib inline
         import matplotlib as mpl
         import matplotlib.pyplot as plt
         mpl.rc('axes', labelsize=14)
mpl.rc('xtick', labelsize=12)
mpl.rc('ytick', labelsize=12)
```

## **Problem 1. Large margin classification**

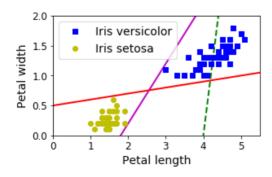
TODO: sklearn.svm library의 SVC 클래스와 SVC의 fit method를 이용하세요. SVC의 kernel로 Linear kernel을 사용하세요.

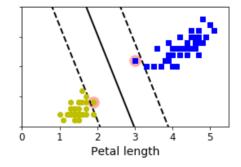
Out[2]: <matplotlib.collections.PathCollection at 0x7f4406698150>



```
In [3]: # Bad models
           x0 = np.linspace(0, 5.5, 200)
           pred 1 = 5*x0 - 20
           pred^{-}2 = x0 - 1.8
           pred^{-}3 = 0.1 * x0 + 0.5
           def plot svc decision boundary(svm clf, xmin, xmax):
                w = svm clf.coef [0]
                b = svm clf.intercept [0]
                print(w, b)
                # At the decision boundary, w0*x0 + w1*x1 + b = 0
                \# => x1 = -w0/w1 * x0 - b/w1
                x0 = np.linspace(xmin, xmax, 200)
                decision boundary = -w[0]/w[1] * x0 - b/w[1]
                margin = 1/w[1]
                gutter up = decision boundary + margin
                gutter_down = decision_boundary - margin
                svs = svm_clf.support_vectors_
plt.scatter(svs[:, 0], svs[:, 1], s=180, facecolors='#FFAAAA')
               plt.plot(x0, decision_boundary, "k-", linewidth=2)
plt.plot(x0, gutter_up, "k--", linewidth=2)
plt.plot(x0, gutter_down, "k--", linewidth=2)
           fig, axes = plt.subplots(ncols=2, figsize=(10,2.7), sharey=True)
           plt.sca(axes[0])
          plt.plot(x0, pred_1, "g--", linewidth=2)
plt.plot(x0, pred_2, "m-", linewidth=2)
plt.plot(x0, pred_3, "r-", linewidth=2)
          plt.plot(X[:, 0][y==1], X[:, 1][y==1], "bs", label="Iris versicolor")
plt.plot(X[:, 0][y==0], X[:, 1][y==0], "yo", label="Iris setosa")
          plt.xlabel("Petal length", fontsize=14)
plt.ylabel("Petal width", fontsize=14)
           plt.legend(loc="upper left", fontsize=14)
           plt.axis([0, 5.5, 0, 2])
           plt.sca(axes[1])
           plot_svc_decision_boundary(svm_clf, 0, 5.5)
           plt.plot(X[:, 0][y==1], X[:, 1][y==1], "bs")
           plt.plot(X[:, 0][y==0], X[:, 1][y==0], "yo")
           plt.xlabel("Petal length", fontsize=14)
           plt.axis([0, 5.5, 0, 2])
           plt.show()
```

#### [1.1 0.7] -3.2799999713897705



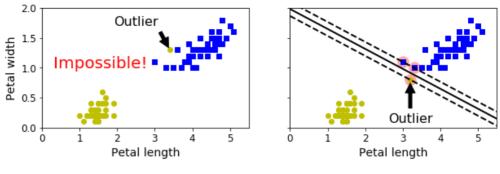


# **Problem 2. Sensitivity to outliers**

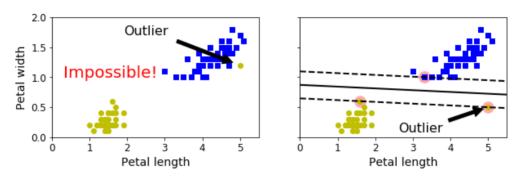
TODO: X\_outliers 값을 바꿔가며 실험해보세요.

```
In [4]: def plot outliers(X outliers):
             y_{outliers} = np.array([0, 0])
             Xo1 = np.concatenate([X, X_outliers[:1]], axis=0)
             yo1 = np.concatenate([y, y_outliers[:1]], axis=0)
             Xo2 = np.concatenate([X, X_outliers[1:]], axis=0)
             yo2 = np.concatenate([y, y_outliers[1:]], axis=0)
             svm clf2 = SVC(kernel="linear", C=10**9)
             svm clf2.fit(Xo2, yo2)
             fig, axes = plt.subplots(ncols=2, figsize=(10,2.7), sharey=True)
             plt.sca(axes[0])
             plt.plot(Xo1[:, 0][yo1==1], Xo1[:, 1][yo1==1], "bs")
plt.plot(Xo1[:, 0][yo1==0], Xo1[:, 1][yo1==0], "yo")
             plt.text(0.3, 1.0, "Impossible!", fontsize=20, color="red")
             plt.xlabel("Petal length", fontsize=14)
plt.ylabel("Petal width", fontsize=14)
             plt.annotate("Outlier",
                           xy=(X_outliers[0][0], X_outliers[0][1]),
                           xytext = (2.5, 1.7),
                           ha="center"
                           arrowprops=dict(facecolor='black', shrink=0.1),
                           fontsize=16,
             plt.axis([0, 5.5, 0, 2])
             plt.sca(axes[1])
             plt.plot(Xo2[:, 0][yo2==1], Xo2[:, 1][yo2==1], "bs")
             plt.plot(Xo2[:, 0][yo2==0], Xo2[:, 1][yo2==0], "yo")
             plot_svc_decision_boundary(svm_clf2, 0, 5.5)
             plt.xlabel("Petal length", fontsize=14)
             plt.annotate("Outlier"
                           xy=(X_outliers[1][0], X_outliers[1][1]),
                           xytext=(3.2, 0.08),
                           ha="center"
                           arrowprops=dict(facecolor='black', shrink=0.1),
                           fontsize=16,
             plt.axis([0, 5.5, 0, 2])
             plt.show()
         plot_outliers(np.array([[3.4, 1.3], [3.2, 0.8]]))
         plot_outliers(np.array([[5.0, 1.2], [5.0, 0.5]]))
        plot_outliers(np.array([[4.0, 1.0], [2.0, 0.6]]))
plot_outliers(np.array([[5.5, 2.0], [5.5, 0.0]]))
plot_outliers(np.array([[3.0, 1.1], [3.0, 1.09]]))
```

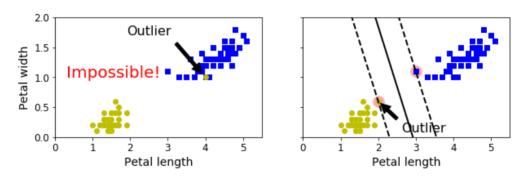
### [2.8559135 8.56781863] -16.99262620355056



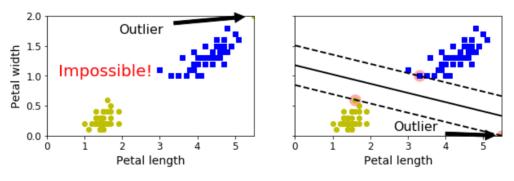
[0.13096612 4.44338648] -3.8758138402074698



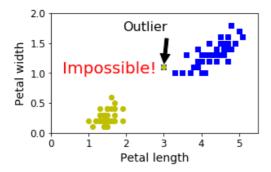
[1.59999961 0.7999998 ] -4.6799987811282255

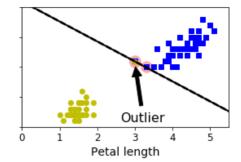


[0.46532113 3.02458357] -3.559557720483316



[ 67.22805793 201.80671668] -422.6644674289849





## Problem 3. Large margin vs margin violations

```
In [5]: | import numpy as np
         from sklearn import datasets
         from sklearn.pipeline import Pipeline
         from sklearn.preprocessing import StandardScaler
         from sklearn.svm import LinearSVC
         iris = datasets.load iris()
         X = iris["data"][:, (2, 3)] # petal length, petal width
y = (iris["target"] == 2).astype(np.float64) # Iris virginica
         svm_clf = Pipeline([
                  ("scaler", StandardScaler()),
                  ("linear svc", LinearSVC(C=1, loss="hinge", random state=42)),
             ])
         svm_clf.fit(X, y)
Out[5]: Pipeline(memory=None,
                   steps=[('scaler',
                            StandardScaler(copy=True, with mean=True, with std=Tru
         e)),
                           ('linear_svc',
                           LinearSVC(C=1, class_weight=None, dual=True,
                                      fit intercept=True, intercept scaling=1,
                                      loss='hinge', max iter=1000, multi class='ovr
                                      penalty='l2', random_state=42, tol=0.0001,
                                      verbose=0))],
                   verbose=False)
In [6]: svm_clf.predict([[5.5, 1.7]])
Out[6]: array([1.])
```

Now let's generate the graph comparing different regularization settings:

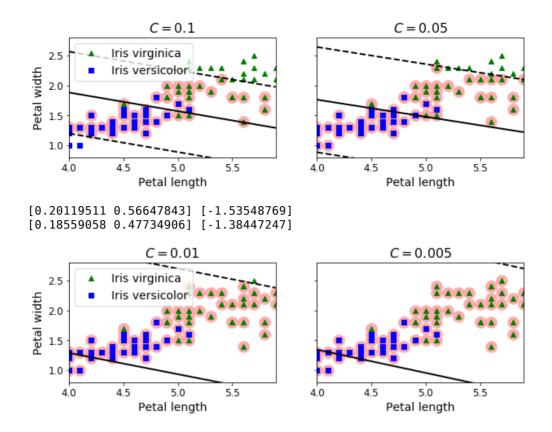
TODO: svm\_clf1 과 svm\_clf2 의 C 값을 바꿔가며 실험해보세요.

```
In [7]: def plot margin violations(svm clf1, svm clf2):
                scaler = StandardScaler()
                scaled_svm_clf1 = Pipeline([
                          ("scaler", scaler),
                          ("linear_svc", svm_clf1),
                     1)
                scaled_svm_clf2 = Pipeline([
                          ("scaler", scaler),
("linear_svc", svm_clf2),
                     1)
                scaled_svm_clf1.fit(X, y)
                scaled_svm_clf2.fit(X, y)
                # Convert to unscaled parameters
                b1 = svm_clf1.decision_function([-scaler.mean_ / scaler.scale_])
                b2 = svm_clf2.decision_function([-scaler.mean_ / scaler.scale_])
                w1 = svm_clf1.coef_[0] / scaler.scale_
                w2 = svm_clf2.coef_[0] / scaler.scale_
                svm_clf1.intercept_ = np.array([b1])
svm_clf2.intercept_ = np.array([b2])
                svm_clf1.coef_ = np.array([w1])
svm_clf2.coef_ = np.array([w2])
                # Find support vectors (LinearSVC does not do this automatically)
                t = y * 2 - 1
                svm_clf1.support_vectors_ = X[support_vectors_idx1]
                svm_clf2.support_vectors_ = X[support_vectors_idx2]
                fig, axes = plt.subplots(ncols=2, figsize=(10,2.7), sharey=True)
                plt.sca(axes[0])
                plt.plot(X[:, 0][y==1], X[:, 1][y==1], "g^", label="Iris virginica")
plt.plot(X[:, 0][y==0], X[:, 1][y==0], "bs", label="Iris versicolo
                plot_svc_decision_boundary(svm_clf1, 4, 5.9)
                plt.xlabel("Petal length", fontsize=14)
plt.ylabel("Petal width", fontsize=14)
                plt.legend(loc="upper left", fontsize=14)
                plt.title("$C = {}$".format(svm_clf1.C), fontsize=16)
                plt.axis([4, 5.9, 0.8, 2.8])
                plt.sca(axes[1])
               \label{eq:plot_x} \begin{array}{ll} \text{plt.plot}(X[:,\ 0][y==1],\ X[:,\ 1][y==1],\ "g^") \\ \text{plt.plot}(X[:,\ 0][y==0],\ X[:,\ 1][y==0],\ "bs") \\ \text{plot_svc_decision_boundary}(\text{svm_clf2},\ 4,\ 5.99) \\ \text{plt.xlabel}("Petal length",\ fontsize=14) \end{array}
                plt.title("$C = {}$".format(svm_clf2.C), fontsize=16)
                plt.axis([4, 5.9, 0.8, 2.8])
                plt.show()
```

```
In [8]:
          svm clf1 = LinearSVC(C=1000, loss="hinge", random_state=42, max_iter=1e
           5)
           svm clf2 = LinearSVC(C=500, loss="hinge", random state=42, max iter=1e5)
           plot_margin_violations(svm_clf1, svm_clf2)
          svm_clf1 = LinearSVC(C=100, loss="hinge", random_state=42, max_iter=1e5)
svm_clf2 = LinearSVC(C=50, loss="hinge", random_state=42, max_iter=1e5)
plot_margin_violations(svm_clf1, svm_clf2)
svm_clf1 = LinearSVC(C=10, loss="hinge", random_state=42, max_iter=1e5)
svm_clf2 = LinearSVC(C=5, loss="hinge", random_state=42, max_iter=1e5)
           plot margin violations(svm clf1, svm clf2)
           svm_clf1 = LinearSVC(C=1, loss="hinge", random_state=42, max_iter=1e5)
           svm_clf2 = LinearSVC(C=0.5, loss="hinge", random_state=42, max_iter=1e5)
          plot_margin_violations(svm_clf1, svm_clf2)
svm_clf1 = LinearSVC(C=0.1, loss="hinge", random_state=42, max_iter=1e5)
svm_clf2 = LinearSVC(C=0.05, loss="hinge", random_state=42, max_iter=1e
           5)
           plot margin violations(svm clf1, svm clf2)
           svm_clf1 = LinearSVC(C=0.01, loss="hinge", random_state=42, max_iter=1e
           5)
           svm clf2 = LinearSVC(C=0.005, loss="hinge", random state=42, max iter=1e
           5)
           plot_margin_violations(svm_clf1, svm_clf2)
```

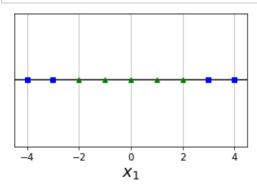
[3.9998406 7.99970047] [-32.59869574] [3.99984981 7.99968788] [-32.59871726] C = 1000C = 500Iris virginica 2.5 Iris versicolor Petal width 2.0 1.5 1.0 4.0 4.5 5.0 5.5 4.0 4.5 5.0 5.5 Petal length Petal length [3.63650373 6.36400853] [-28.27403284] [2.75873644 4.8278226 ] [-21.20779516] C = 100C = 50Iris virginica 2.5 Petal width Iris versicolor 2.0 1.5 1.0 4.5 4.5 5.0 5.5 4.0 5.0 5.5 4.0 Petal length Petal length [1.82092657 4.84598045] [-16.64530501] [1.24859957 4.16919863] [-12.87249603] C = 5C = 10Iris virginica 2.5 Iris versicolor Petal width 2.0 1.5 1.0 4.5 5.0 5.5 4.0 4.5 5.0 5.5 4.0 Petal length Petal length [0.9283666 3.14340194] [-9.76413982] [0.77625114 2.53915161] [-7.93851912] C = 1C = 0.5Iris virginica 2.5 Iris versicolor Petal width 2.0 1.5 1.0 4.5 5.0 5.5 4.0 4.5 5.0 5.5 4.0 Petal length Petal length [0.45472042 1.46436569] [-4.57613027]

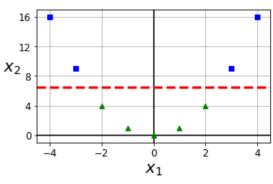
[0.32495125 1.13767445] [-3.30639774]

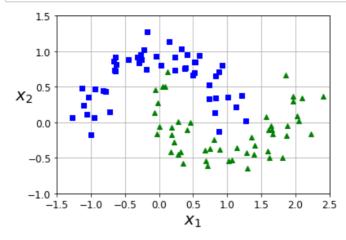


**Problem 4. Non-linear classification** 

```
In [9]: X1D = np.linspace(-4, 4, 9).reshape(-1, 1)
          X2D = np.c_{[X1D, X1D**2]}
          y = np.array([0, 0, 1, 1, 1, 1, 1, 0, 0])
           plt.figure(figsize=(10, 3))
           plt.subplot(121)
           plt.grid(True, which='both')
          plt.axhline(y=0, color='k')
          plt.plot(X1D[:, 0][y==0], np.zeros(4), "bs")
           plt.plot(X1D[:, 0][y==1], np.zeros(5), "g^")
           plt.gca().get_yaxis().set_ticks([])
          plt.xlabel(r"$x_1$", fontsize=20)
plt.axis([-4.5, 4.5, -0.2, 0.2])
          plt.subplot(122)
          plt.grid(True, which='both')
           plt.axhline(y=0, color='k')
           plt.axvline(x=0, color='k')
          plt.plot(X2D[:, 0][y==0], X2D[:, 1][y==0], "bs")
plt.plot(X2D[:, 0][y==1], X2D[:, 1][y==1], "g^")
          plt.xlabel(r"$x_1$", fontsize=20)
plt.ylabel(r"$x_2$ ", fontsize=20, rotation=0)
          plt.gca().get_yaxis().set_ticks([0, 4, 8, 12, 16])
plt.plot([-4.5, 4.5], [6.5, 6.5], "r--", linewidth=3)
plt.axis([-4.5, 4.5, -1, 17])
           plt.subplots adjust(right=1)
           plt.show()
```





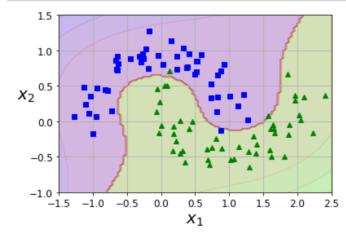


```
In [12]:

def plot_predictions(clf, axes):
    x0s = np.linspace(axes[0], axes[1], 100)
    x1s = np.linspace(axes[2], axes[3], 100)
    x0, x1 = np.meshgrid(x0s, x1s)
    X = np.c_[x0.ravel(), x1.ravel()]
    y_pred = clf.predict(X).reshape(x0.shape)
    y_decision = clf.decision_function(X).reshape(x0.shape)
    plt.contourf(x0, x1, y_pred, cmap=plt.cm.brg, alpha=0.2)
    plt.contourf(x0, x1, y_decision, cmap=plt.cm.brg, alpha=0.1)

plot_predictions(polynomial_svm_clf, [-1.5, 2.5, -1, 1.5])
    plot_dataset(X, y, [-1.5, 2.5, -1, 1.5])

plt.show()
```



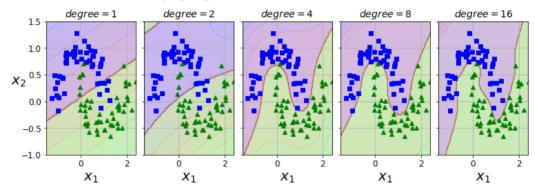
## Problem 4-1. Polynomial kernel (Non-linear classification)

TODO 1: SVC의 kernel로 Polynomial kernel을 사용하고, degree 와 coef0 의 값을 바꿔가며 실험해보세요.

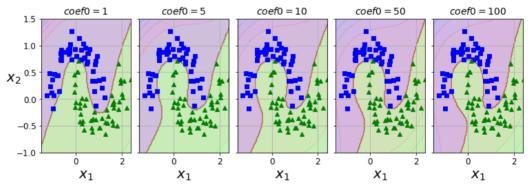
TODO\_2: SVC의 kernel로 Polynomial kernel을 사용하고, degree 와 coef0 의 값을 바꿔가며 실험해보세요.

```
In [13]:
         from sklearn.svm import SVC
         def plot_poly_kernel(svcs, variant):
             poly_kernel_svm_clfs = []
             for svc in svcs:
                 poly kernel svm clfs.append(Pipeline([
                          ("scaler", StandardScaler()),
("svm_clf", svc)
                      ]))
             for poly_kernel_svm_clf in poly_kernel_svm_clfs:
                 poly_kernel_svm_clf.fit(X, y)
             if variant == 'degree':
                 invariants = 'C = {}, coef0 = {}'.format(svcs[0].C, svcs[0].coef
         0)
             elif variant == 'coef0':
                 invariants = 'C = {}, degree = {}'.format(svcs[0].C, svcs[0].deg
         ree)
             elif variant == 'C':
                 invariants = 'degree = {}, coef0 = {}'.format(svcs[0].degree, sv
         cs[0].coef0)
             fig, axes = plt.subplots(ncols=len(svcs), figsize=(10.5, 4), sharey=
         True, constrained layout=True)
             fig.suptitle('Varying {} (invariants: {})'.format(variant, invariant
         s), fontsize=16)
             for i in range(len(svcs)):
                 plt.sca(axes[i])
                 plot_predictions(poly_kernel_svm_clfs[i], [-1.5, 2.45, -1, 1.5])
                 plot_dataset(X, y, [-1.5, 2.4, -1, 1.5])
                 if variant == 'degree':
                      axes[i].set_title(r"$degree={}$".format(svcs[i].degree), fon
         tsize=14)
                 elif variant == 'coef0':
                      axes[i].set title(r"$coef0={}$".format(svcs[i].coef0), fonts
         ize=14)
                 elif variant == 'C':
                      axes[i].set title(r"$C={}$".format(svcs[i].C), fontsize=14)
                 if i != 0:
                     plt.ylabel("")
             plt.show()
```

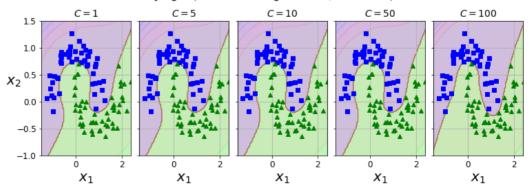
#### Varying degree (invariants: C = 5, coef0 = 1)





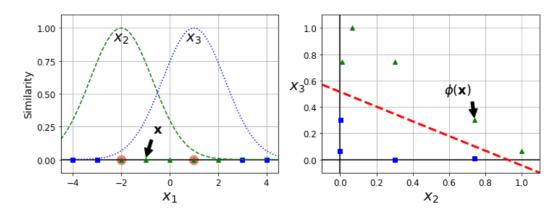


Varying C (invariants: degree = 10, coef0 = 5)



# **Problem 4-2. Gaussian RBF kernel (Non-linear classification)**

```
In [15]: def gaussian rbf(x, landmark, gamma):
                return np.exp(-gamma * np.linalg.norm(x - landmark, axis=1)**2)
           qamma = 0.3
           x1s = np.linspace(-4.5, 4.5, 200).reshape(-1, 1)
           x2s = gaussian_rbf(x1s, -2, gamma)
x3s = gaussian_rbf(x1s, 1, gamma)
           XK = np.c_[gaussian_rbf(X1D, -2, gamma), gaussian_rbf(X1D, 1, gamma)]
           yk = np.array([0, 0, 1, 1, 1, 1, 1, 0, 0])
           plt.figure(figsize=(10.5, 4))
           plt.subplot(121)
           plt.grid(True, which='both')
           plt.axhline(y=0, color='k')
           plt.scatter(x=[-2, 1], y=[0, 0], s=150, alpha=0.5, c="red")
           plt.plot(X1D[:, 0][yk==0], np.zeros(4), "bs")
plt.plot(X1D[:, 0][yk==1], np.zeros(5), "g^")
           plt.plot(x1s, x2s, "g--")
plt.plot(x1s, x3s, "b:")
           plt.gca().get_yaxis().set_ticks([0, 0.25, 0.5, 0.75, 1])
           plt.xlabel(r"$x_1$", fontsize=20)
           plt.ylabel(r"Similarity", fontsize=14)
           plt.annotate(r'$\mathbf{x}$'
                          xy=(X1D[3, 0], 0),
                          xytext=(-0.5, 0.20),
                          ha="center"
                          arrowprops=dict(facecolor='black', shrink=0.1),
                          fontsize=18,
           plt.text(-2, 0.9, "$x_2$", ha="center", fontsize=20)
plt.text(1, 0.9, "$x_3$", ha="center", fontsize=20)
           plt.axis([-4.5, 4.5, -0.1, 1.1])
           plt.subplot(122)
           plt.grid(True, which='both')
           plt.axhline(y=0, color='k')
           plt.axvline(x=0, color='k')
           plt.plot(XK[:, 0][yk==0], XK[:, 1][yk==0], "bs")
plt.plot(XK[:, 0][yk==1], XK[:, 1][yk==1], "g^")
           plt.xlabel(r"$x_2$", fontsize=20)
plt.ylabel(r"$x_3$ ", fontsize=20, rotation=0)
           plt.annotate(r'$\phi\left(\mathbf{x}\right)$',
                          xy=(XK[3, 0], XK[3, 1]),
                          xytext=(0.65, 0.50),
                          ha="center",
                          arrowprops=dict(facecolor='black', shrink=0.1),
                          fontsize=18.
           plt.plot([-0.1, 1.1], [0.57, -0.1], "r--", linewidth=3)
           plt.axis([-0.1, 1.1, -0.1, 1.1])
           plt.subplots_adjust(right=1)
           plt.show()
```



```
In [16]: 
x1_example = X1D[3, 0]
for landmark in (-2, 1):
    k = gaussian_rbf(np.array([[x1_example]]), np.array([[landmark]]), g
amma)
    print("Phi({}, {}) = {}".format(x1_example, landmark, k))

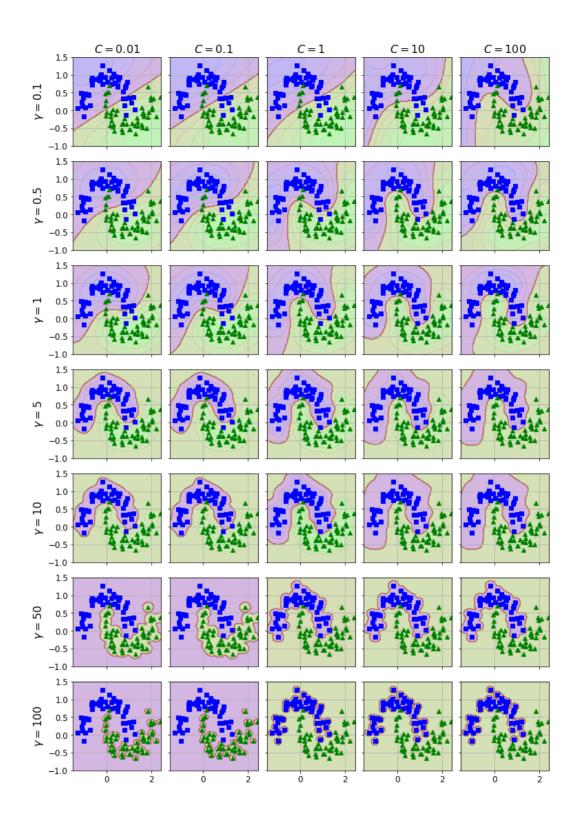
Phi(-1.0, -2) = [0.74081822]
Phi(-1.0, 1) = [0.30119421]
```

TODO 1: SVC의 kernel로 Gaussian RBF kernel을 사용하세요.

```
In [17]:
        rbf kernel svm clf = Pipeline([
               ("scaler", StandardScaler()),
("svm_clf", SVC(kernel='rbf', gamma=5, C=0.001))
        rbf_kernel_svm_clf.fit(X, y)
        Out[17]: Pipeline(memory=None,
                steps=[('scaler',
                       StandardScaler(copy=True, with mean=True, with std=Tru
        e)),
                       ('svm clf'
                       SVC(C=0.001, break_ties=False, cache_size=200,
                           class weight=None, coef0=0.0,
                           decision function shape='ovr', degree=3, gamma=5,
                           kernel='rbf', max_iter=-1, probability=False,
                           random_state=None, shrinking=True, tol=0.001,
                           verbose=False))],
                verbose=False)
```

TODO\_2: gamma1, gamma2, C1, C2 값을 바꿔가며 실험해보세요.

```
In [18]: from sklearn.svm import SVC
         def plot_rbf_kernel(gammas, Cs):
             hyperparams = []
             for i, gamma in enumerate(gammas):
                for j, C in enumerate(Cs):
                    hyperparams.append((gamma, C))
             svm clfs = []
             for gamma, C in hyperparams:
                rbf kernel svm clf = Pipeline([
                        ("scaler", StandardScaler()),
("svm_clf", SVC(kernel="rbf", gamma=gamma, C=C))
                    1)
                 rbf_kernel_svm_clf.fit(X, y)
                svm clfs.append(rbf kernel svm clf)
             fig, axes = plt.subplots(nrows=len(gammas), ncols=len(Cs), figsize=
         (10.5, 15), sharex=True, sharey=True, constrained_layout=True)
             for i, svm clf in enumerate(svm clfs):
                 row = \overline{i} // len(Cs)
                col = i % len(Cs)
                plt.sca(axes[row, col])
                plot_predictions(svm_clf, [-1.5, 2.45, -1, 1.5])
                plot_dataset(X, y, [-1.5, 2.45, -1, 1.5])
                gamma, C = hyperparams[i]
                plt.xlabel("")
                plt.ylabel("")
                if row == 0:
                    plt.title(r"$C = {}$".format(C), fontsize=16)
                if col == 0:
                    plt.ylabel(r"$\gamma = {}$".format(gamma), rotation="vertica")
         l", fontsize=16)
             plt.show()
         plot rbf kernel(gammas=[0.1, 0.5, 1, 5, 10, 50, 100], Cs=[0.01, 0.1, 1,
         10, 100]
```



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