Probing the Neutrino-Dark Energy Degeneracy in Cold Dark Matter Model using Baryonic Acoustic Oscillation Data

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(Dated: 28^{th} November 2024)

Lambda cold dark matter (ΛCDM) model has been very successful in explaining many cosmological observations; however, it still encounters severe issues like Hubble Tension. w_0w_aCDM and ΛCDMξ are two extended ΛCDM models attempting to address these issues by assuming varying dark energy and non-zero neutrino degeneracy factors respectively. This project aims to testify the hypothesis that dark energy parameters in w_0w_aCDM can replicate the effect of neutrino parameters in ΛCDMξ on the evolution of the universe. In other words, there is a degeneracy between dark energy and neutrino. A general cosmological model that includes both dark energy and neutrino parameters was fitted to baryonic acoustic oscillation (BAO) data using Markov Chain Monte Carlo (MCMC). A perturbative approach was introduced to study how the effect of neutrino parameters can be imitated by that of dark energy parameters. However, neutrino parameters are poorly constrained in the fitting. Attempt to study their degeneracy was unsuccessful. The results suggest that BAO observables are insensitive to neutrino at low redshift in our general model.

I. INTRODUCTION

In the early universe, photon and plasma were tightly coupled due to Thomson and Coulomb scattering. Sound wave of this photon-baryon fluid was generated in overdense region, where gravity and radiation pressure competed against each other, leading to acoustic oscillations. The sound wave of photon and baryon stopped propagating during the time of recombination and drag epoch, and the fluctuations in photon and baryon density were imprinted in the cosmic microwave background (CMB) map and the distribution of galaxy clusters we see today, which provide important measurements to testify different cosmological models [1, 2].

Despite a great success in describing the cosmic microwave background anisotropy and other cosmological observations [3–5], ΛCDM model continues to encounter various challenges, the largest of which is the Hubble tension, which refers to the discrepancy between direct measurements of the Hubble constant H_0 and the best fit value of ΛCDM model [6, 7]. Different extended ΛCDM models have been proposed to resolve all these issues. One of the most renowned model w_0w_aCDM, which assumes varying dark energy, was found to be strongly preferred over the standard ΛCDM at the 2.6σ level using Planck CMB and Dark Energy Spectroscopic Instrument (DESI) BAO data [8] while another promising model namely ΛCDMξ, which assumes finite neutrino mass and degeneracy factors ξ_{ν_i}, is able to resolve the Hubble tension using CMB and BAO data with the mean value of H_0 only 0.8σ away from the one obtained by the Supernovae H0 for the Equation of State (SH0ES) collaboration [9, 10]. Both extended models of ΛCDM fit data remarkably well and introduce only two new parameters in addition to the fundamental parameters of ΛCDM.

Given that both models depict a universe with a very similar history, it is tempting to study the relation between them i.e. how dark energy parameters in w_0w_aCDM may imitate the effect of neutrino parameters in ΛCDMξ on the evolution of the universe. To do this, a generalized cold dark matter model, which is applicable within a short range of redshift and which allows both varying dark energy and non-zero neutrino degeneracy, was used to fit to BAO data from DESI and Sloan Digital Sky Survey (SDSS) using MCMC [8, 11]. A perturbative approach that can derive the relation between dark energy and neutrino parameters semi-analytically was introduced. With this relation, we can understand how neutrino are mimicked by dark energy. Lamentably, neutrino parameters are poorly constrained in the fitting and the study of neutrino-dark energy degeneracy cannot proceed. The results imply that BAO observables are insensitive to neutrino at low redshift and also suggest that a model that is applicable to all redshift should be used instead.

II. THEORY

The w_0w_aCDM and ΛCDMξ are described in details below. Equations are written in natural units, in which the speed of light c, the Boltzmann constant k and the Planck's constant $\hbar$ are set to one.

A. Introduction to w_0w_aCDM Model

The w_0w_aCDM assumes an equation of state (EOS) of dark energy described by the Chevalier-Polarski-Linder parametrization [12, 13]:

$$P = (w_0 + (1 - a)w_a)\,\rho \tag{1}$$

where P is the pressure, ρ is the energy density, a is the cosmic scale factor, w_0 and w_a are two parameters in this model.

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The term $w_0+(1-a)w_a$ can be understood as a Taylor expansion of a more general function $w(a)$ at $a=1$ up to the first order. The EOS leads to a slight modification of the standard Friedmann equation of ΛCDM, which describes the expansion rate of a flat universe governed by the Friedmann-Lemaitre-Robertson-Walker (FLRW) metric, as shown below

$$\frac{H(a)}{H_0} = \sqrt{\Omega_m a^{-3} + \Omega_\gamma a^{-4} + \Omega_\Lambda a^{-3(1+w_0+w_a)} e^{3w_a(a-1)}} \tag{2}$$

where Ω_γ, Ω_Λ and Ω_m are present-day energy density (in terms of critical density) of relativistic particles, dark energy and matter including both baryon and cold dark matter. One of the feature of this model is that dark energy density varies with the scale factor a.

B. Introduction to ΛCDMξ Model

In Planck ΛCDM and many other cosmological models, the sum of neutrino masses is assumed to be $\sum m_\nu = 0.06$ eV, which is the lower bound of total neutrino mass in normal hierachy, and only one eigenstate is assumed to be massive [5, 8]. Besides, the degeneracy factors $\xi_i = \mu_i/T_\nu$, where μ_i and T_ν are chemical potential and neutrino decoupling temperature, are all assumed to be zero [5, 8]. These assumptions lack physical motivation and are discarded in ΛCDMξ model, which separates relic neutrino from radiation and introduces total neutrino mass M_ν and the degeneracy factor of the third mass eigenstate ξ_{ν_3} as fitting parameters [9, 10].

After decoupling, relic neutrino remains in the same mass eigenstate as time evolves. The total neutrino and anti-neutrino energy density of the three mass eigenstates are given by

$$\rho_\nu(a) \propto \sum_i \int_0^\infty \left(\frac{\sqrt{p^2 + (\frac{M_\nu}{3})^2}}{e^{\frac{pa}{T_\nu} - \xi_{\nu_i}} + 1} + \frac{\sqrt{p^2 + (\frac{M_\nu}{3})^2}}{e^{\frac{pa}{T_\nu} + \xi_{\nu_i}} + 1} \right) p^2 dp \tag{3}$$

where p is the momentum. The neutrino mass in the exponential function is neglected given $M_\nu/T_\nu \ll 1$ and each mass eigenstate is assumed to share the same mass as adopted in [5]. To further reduce the number of free parameters, we should note that $(\xi_{\nu_1}, \xi_{\nu_2}.\xi_{\nu_3})$ and $(\xi_{\nu_e}, \xi_{\nu_\mu}.\xi_{\nu_\tau})$ are connected by the following relations

$$\boldsymbol{L_m} = U_{PMNS} \boldsymbol{L_f} U^\dagger_{PMNS} \tag{4}$$

$$L_i = \frac{1}{12\varsigma(3)} \left(\frac{T_\nu}{T_\gamma}\right)^3 (\pi^2 \xi_{\nu_i} + \xi^2_{\nu_i}) \tag{5}$$

where $\boldsymbol{L_m}$ and $\boldsymbol{L_f}$ are the neutrino lepton number asymmetry matrix of mass eigenstate and flavor eigenstate respectively, U_{PMNS} is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix and T_γ is the CMB temperature. By assuming $\xi_{\nu_\mu} = \xi_{\nu_\tau}$ and adopting the value of ξ_{ν_e} in [14], ξ_{ν_1} and ξ_{ν_2} can be expressed as a function of ξ_{ν_3} as depicted in figure 1. The total neutrino energy density can now be expressed as $\rho(a; M_\nu, \xi_{\nu_3})$. An empirical formula of the density is presented in **Appendix A**. The Friedmann equation of this model is given by

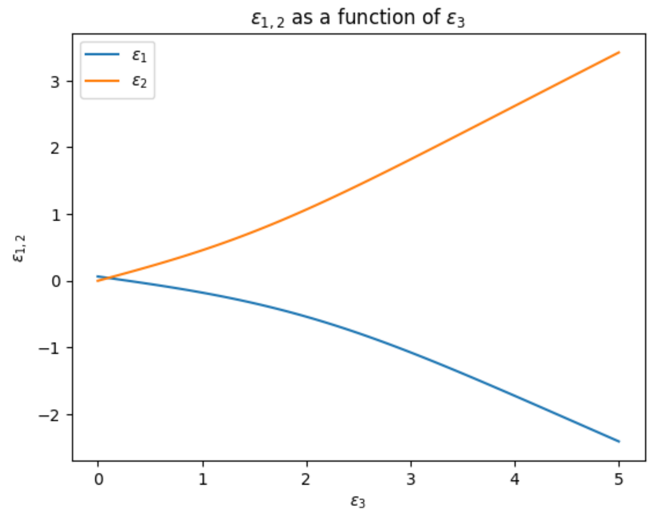

Figure 1. ξ_{ν_1} and ξ_{ν_2} as a function of ξ_{ν_3}

$$\frac{H(a)}{H_0} = \sqrt{\Omega_m a^{-3} + \Omega_\gamma a^{-4} + \Omega_\Lambda + \frac{\rho(a; M_\nu, \xi_{\nu_3})}{\rho_{crit}}} \tag{6}$$

The feature of this model is that neutrino transitioned from ultra-relativistic regime to non-relativistic regime at redshift $z \sim 200$ for $M_\nu \sim 0.1\, eV$.

C. Comparison of w_0w_aCDM and ΛCDMξ Model

Figure 2 shows the numerical solution of the Friedmann equation of Planck ΛCDM, w_0w_aCDM and ΛCDMξ Models using the fitting results in [5, 8, 10]. While their universe will evolve very differently in the next ten billion years, the cosmological history resembles one another. Notably, ΛCDMξ slightly deviates from the other two, with the age of the universe being 13.21 billion years. Still, given their resemblance, it is tempting to investigate the relation between them, particularly how w_0w_aCDM and ΛCDMξ may imitate one another .

III. METHOD

To study the degeneracy between neutrino and dark energy parameters, a more general cold dark matter model that allows both varying dark energy and non-zero neutrino degeneracy was used whose Friedmann equation is given by

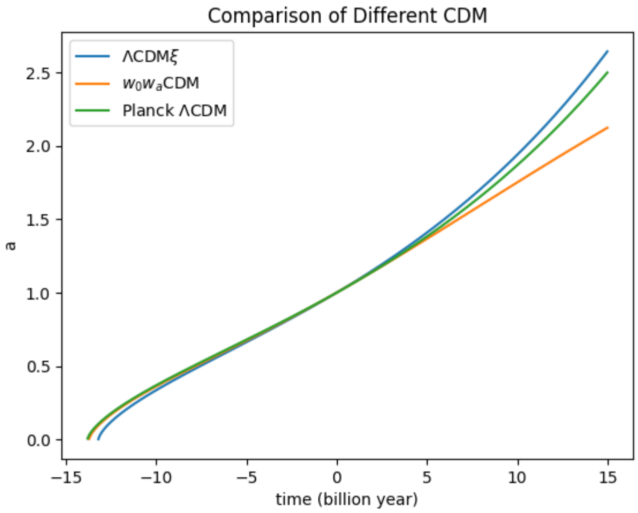

Figure 2. Cosmic scale factor a as a function of time according to different cosmological models

$$\frac{H(a)}{H_0} = \sqrt{\Omega_b a^{-3} + \Omega_c a^{-3} + \Omega_\Lambda a^{-\gamma_\Lambda} + \Omega_\nu a^{-\gamma_\nu}} \quad (7)$$

This model is only applicable at low redshift at which radiation energy density is neglected and only applicable within a short range of redshift during which the a dependence of neutrino and dark energy density are approximately constant.

To determine the unknown parameters, **Cobaya** [15, 16] was used to perform MCMC fitting based on BAO data from DESI and SDSS [8, 11]. The measurements contain three types of observables $\beta_\perp$, $\beta_{||}$ and β_{avg} at redshift of order of magnitude 0 to -1. They are defined as

$$\beta_\perp(z) = \int_0^z \frac{cdz'}{r_d H(z')} \quad (8)$$

$$\beta_{||}(z) = \frac{c}{r_d H(z)} \quad (9)$$

$$\beta_{avg}(z) = \left(z(\beta_\perp(z))^2 \beta_{||}(z)\right)^{\frac{1}{3}} \quad (10)$$

where r_d is the comoving distance of baryon sound horizon. Obtaining r_d however requires a cosmological model describing the evolution of $H(z)$ at high redshift. As such, apart from $\Omega_b h^2$, $\Omega_c h^2$, $\Omega_\Lambda h^2$, γ_Λ, $\Omega_\nu h^2$ and γ_ν where $h \equiv H_0/100\text{kms}^{-1}\text{Mpc}^{-1}$, r_d was also introduced as a fitting parameter. The degeneracy between $\Omega_b h^2$ and $\Omega_c h^2$ was broken with a Gaussian prior on $\Omega_b h^2$ using value obtained from big bang nucleosynthesis (BBN) in [17]. Besides, the degeneracy between dark energy and neutrino can be broken with an uniform prior of different ranges on γ_Λ and γ_ν, which are $-4 \leq \gamma_\Lambda \leq 4$ and $3 \leq \gamma_\nu \leq 4$ respectively.

Previous research was conducted by fitting ΛCDM model with parameters r_d, H_0 and $\Omega_m h^2$ to BAO data and r_d is well-constrained only when $\Omega_m h^2$ is constrained by a Gaussian prior [18]. Therefore, with merely a Gaussian prior on $\Omega_b h^2$, BAO data probably cannot constrain r_d in our model. Hence, I also performed MCMC fitting with a Gaussian prior on $\Omega_c h^2$ based on the ΛCDM value.

After obtaining the fitting parameters, I was expected to adopt a perturbative approach suggested in [9] to study the correlation between neutrino and dark energy parameters. The idea is illustrated below using the observable $\beta_\perp$ as an example. According to our model, $\beta_\perp$ can be written as

$$\beta_\perp = \beta_\perp(\Omega_b h^2, \Omega_c h^2, \Omega_\Lambda h^2, \gamma_\Lambda, \Omega_\nu h^2, \gamma_\nu, r_d) \quad (11)$$

Assuming we had a BAO data β_{obs} at z_{obs} and $\beta_\perp|_{ref} \approx \beta_{obs}$ at a reference point in the 7 parameters space which we choose to be those of Planck ΛCDM, we can obtain the relation between the 7 parameters by evaluating $d\beta_\perp|_{ref} = 0$. The equation simply means we are varying different parameters without changing the value of $\beta_\perp$ because it matches the data. This is to mimic how MCMC works semi-analytically. After that, we should find that $\beta_\perp$ is more sensitive to some parameters than others. We drop those terms that have a small first derivative of $\beta_\perp$ for example here all Ωh^2 and r_d. Then, we are left with

$$\left.\frac{d\beta_\perp}{d\gamma_\Lambda}\right|_{ref} \gamma_\Lambda + \left.\frac{d\beta_\perp}{d\gamma_\nu}\right|_{ref} \gamma_\nu = 0 \quad (12)$$

which suggests a degeneracy between dark energy and neutrino because the effect of varying γ_Λ can be compensated by varying γ_ν. As long as γ_Λ and γ_ν satisfies Equation 12 and do not deviate too much from the reference point, we should be able to obtain the same BAO observables and hence the same evolution history.

IV. RESULT

The fitting results are presented in Table I and Figure 3, which is plotted by **Getdist** [19]. The result based on BAO data with only a Gaussian prior on $\Omega_b h^2$ is compared with the one with an additional Gaussian prior on $\Omega_c h^2$. In both cases, $\Omega_b h^2$, $\Omega_\Lambda h^2$ and γ_Λ are well constrained of which $\Omega_b h^2$ is in perfect agreement with BBN and ΛCDM value due to the Gaussian prior. However, without a Gaussian prior on $\Omega_c h^2$, the ΛCDM value of γ_Λ lies right on the 68% confidence level, which suggests that BAO data barely prefers a constant dark energy.

The original idea of imposing a Gaussian prior on $\Omega_c h^2$ is to constrain r_d. Nevertheless, in both cases, r_d is poorly constrained by data. Worse still, $\Omega_\nu h^2$ and γ_ν are poorly constrained, which simply implies that BAO observables are not sensitive to neutrino parameters at low redshift at which BAO data is measured. In other words, varying $\Omega_\nu h^2$ and γ_ν around their actual value has little effect on β. Although these observables supposedly include information in the early universe in r_d, which is very sensitive to neutrino parameters, r_d

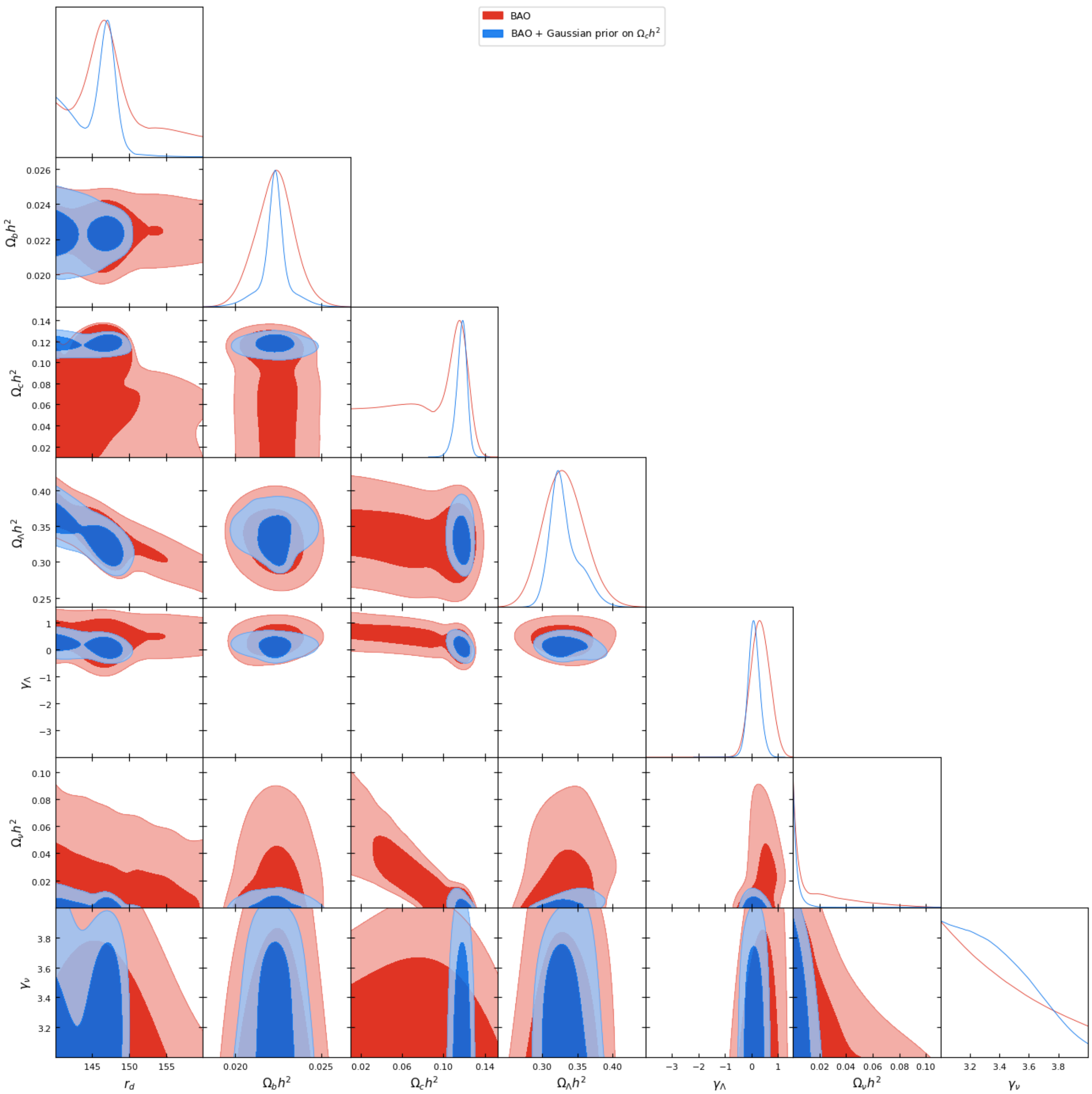

Figure 3. 2D contours (for 68% and 95% confidence level) and 1D marginalized posterior pdfs of the 7 parameters

BAO		BAO + Gaussian Prior on $\Omega_c h^2$	
Parameter	68% limits	Parameter	68% limits
r_d	< 148	r_d	< 147
$\Omega_b h^2$	0.0223 ± 0.0011	$\Omega_b h^2$	0.02230 ± 0.00081
$\Omega_c h^2$	$0.082^{+0.046}_{-0.041}$	$\Omega_c h^2$	$0.1176^{+0.0056}_{-0.0043}$
$\Omega_\Lambda h^2$	0.330 ± 0.028	$\Omega_\Lambda h^2$	$0.331^{+0.013}_{-0.025}$
γ_Λ	0.34 ± 0.34	γ_Λ	0.099 ± 0.21
$\Omega_\nu h^2$	< 0.0255	$\Omega_\nu h^2$	< 0.00317
γ_ν	< 3.51	γ_ν	< 3.49
H_0	$67.4^{+2.8}_{-2.4}$	H_0	$68.8^{+1.1}_{-1.9}$

Table I. Mean values and 68% confidence level of the 7 fitting parameters as well as the Hubble constant

was treated as a fitting parameter so this information cannot be used to constrain neutrino parameters. As a result, I cannot proceed with the study of the correlation between dark energy and neutrinos using the perturbative method. The hypothesis remains inconclusive.

V. CONCLUSION

To study how dark energy parameters in w_0w_aCDM may imitate neutrino parameters in ΛCDMξ, a general cold dark matter model which allows varying dark energy and non-zero neutrino degeneracy was fitted to BAO data using MCMC method. However, neutrino parameters are poorly constrained, suggesting that BAO observables are insensitive to neutrino at low redshift. r_d is also poorly constrained by the data, although a Gaussian prior to $\Omega_c h^2$. As such, I failed to study the neutrino-dark energy degeneracy. Therefore, the hypothesis that dark energy can replicate the effect of a neutrino remains inconclusive.

VI. FUTURE WORK

Given that treating r_d as a fitting parameter reduces the constraining power of neutrino parameters, future research should adopt a model that is applicable to any redshift so that we can calculate r_d analytically. We can simply modify Equation 5 by adding $a^{-\gamma_\Lambda}$ on the dark energy term. If everything converges nicely in the MCMC fitting, we can try to replace $a^{-\gamma_\Lambda}$ by the Chevalier-Polarski-Linder parametrization in $w_0 w_a$CDM which is $a^{-3(1+w_0+w_a)}e^{3w_a(a-1)}$. In addition, CMB data can also be utilized, which should considerably enhance the constraining power.

VII. ACKNOWLEDGEMENETS

I would like to thank Prof. Chu for his guidance throughout the project and also his Phd and Mphil students Wangzheng Zhang and Boris Ng who patiently answered my questions and provided useful advices.

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Appendix A: Empirical Formula of Total Neutrino Energy Density

The following idea was first introduced by Mr. Ka-Tung Lau but his work was incomplete. I am only responsible for finishing it. We begin with the continuity equation

$$\frac{d\rho}{dt} + \frac{3}{a}\frac{da}{dt}(\rho + P) = 0 \tag{A1}$$

where $P = w\rho$ is the equation of state of neutrino. After rearranging the equation, it is transformed into

$$-\frac{dln(\rho)}{dln(a)} = 3(1+w) \tag{A2}$$

The LHS was then fitted into a 3-variables function

$$-\frac{dln(\rho)}{dln(a)} = 3 + \frac{1}{2}\left(1 + tanh\left(-\frac{ln(M_\nu a) - 0.02148\xi_{\nu_3}^2 + 0.00086\xi_{\nu_3} + 6.44172}{1.1003}\right)\right) \tag{A3}$$

Integrating both sides, we have

$$\rho(a; M_\nu, \xi_{\nu_3}) = \rho(1; M_\nu, \xi_{\nu_3}) a^{-\frac{7}{2}} \left(\frac{cosh\left(-\frac{ln(M_\nu a) - 0.02148\xi_{\nu_3}^2 + 0.00086\xi_{\nu_3} + 6.44172}{1.1003}\right)}{cosh\left(-\frac{ln(M_\nu) - 0.02148\xi_{\nu_3}^2 + 0.00086\xi_{\nu_3} + 6.44172}{1.1003}\right)} \right)^{\frac{1.1003}{2}} \tag{A4}$$

where $\rho(1; M_\nu, \xi_{\nu_3})$ was fitted separately

$$\rho(1; M_\nu, \xi_{\nu_3}) = \left(0.048146 e^{0.067935 {\xi_{\nu_3}}^{2.0638}} - 0.027286\right) M_\nu + 0.1428417773 e^{2.273099427\times10^{-6} {\xi_{\nu_3}}^{2.35038797}} - 0.1428399747 \tag{A5}$$

Both Equation A4 and A5 were obtained by assuming $0 \le \xi_{\nu_3} \le 2$ and $0\ eV < M_\nu \le 1\ eV$. The percentage error of the formula against $ln(a)$ is plotted in figure 4. Overall, the percentage error remains below 1% and peaks at the time when neutrino transitioned from ultra-relativistic regime to non-relativistic regime . We should note that neutrino energy density scales as a^{-4} at high redshift and hence, even though the percentage error at high redshift remains around 0.6%, the absolute error increases rapidly and the formula becomes unreliable. To address this issue, a hybrid function ρ_{hyb} is needed

$$\rho_{hyb}(a; M_\nu, \xi_{\nu_3}) = \begin{cases} \rho(a_{crit}; M_\nu, \xi_{\nu_3}) \left(\frac{a}{a_{crit}}\right)^{-4} & \text{if } a < a_{crit} \\ \\ \rho(a; M_\nu, \xi_{\nu_3}) & \text{if } a \ge a_{crit} \end{cases} \tag{A6}$$

where a_{crit} is left to be tested.

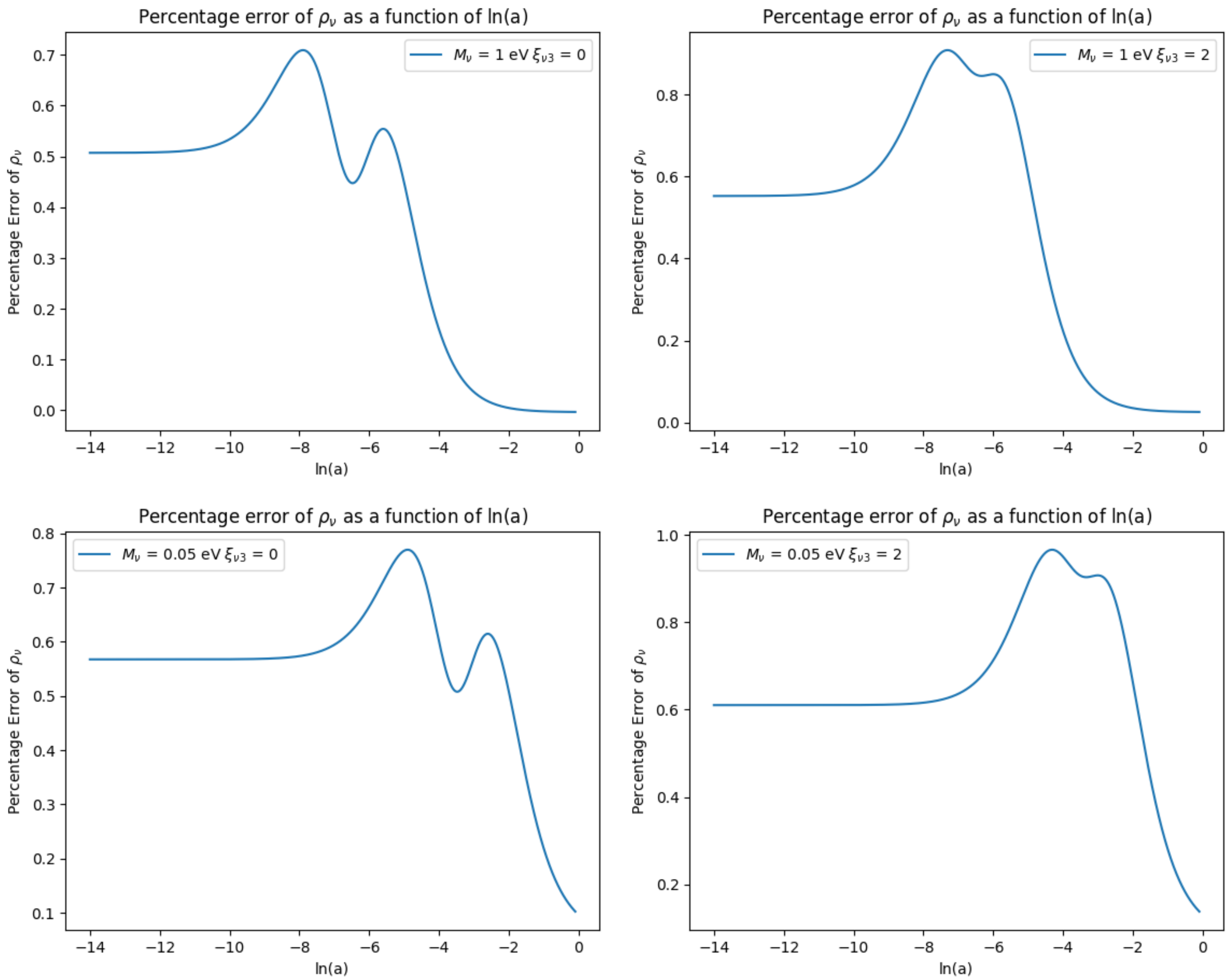

Figure 4. Percentage error of $\rho(a; M_\nu, \xi_{\nu_3})$ as a function of $ln(a)$ for different M_ν and ξ_{ν_3}