Could Low-Mass-Small-Radius White Dwarfs be Fermionic Dark Matter-Admixed White Dwarfs?

Lam Ching Hui, Leung Ching Yu, Luk Cheuk Yin, Tang Suet Yi, Yau Chi Kin

Introduction

White Dwarfs are extremely dense stellar remnants that have masses similar to the sun, but only have radii close to the earth. They are too dense that they can only be supported by electron degeneracy pressure against gravitational collapse.

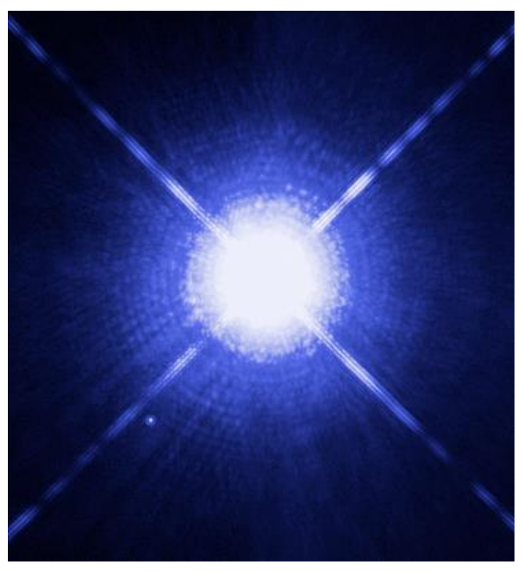


Fig. 1 Sirius, the brightest star in the night sky, and its companion white dwarf.
Source: Hubble Space Telescope

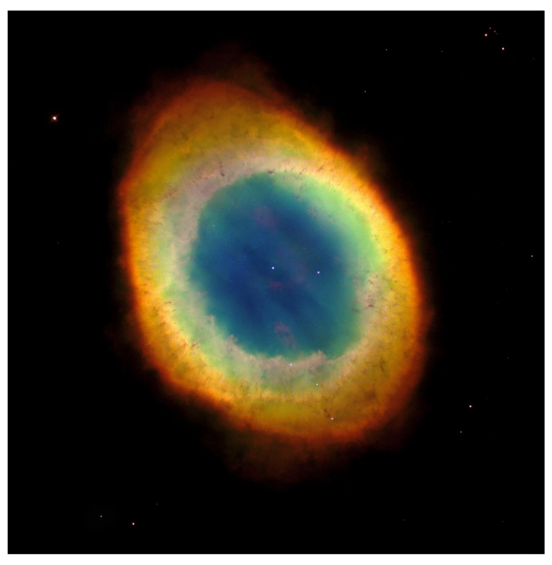


Fig. 2 M57 Ring Nebula and its central white dwarf.
Source: Hubble Space Telescope

By assuming hydrostatic equilibrium, we can plot the density profiles of white dwarfs with different central densities (Fig. 3):

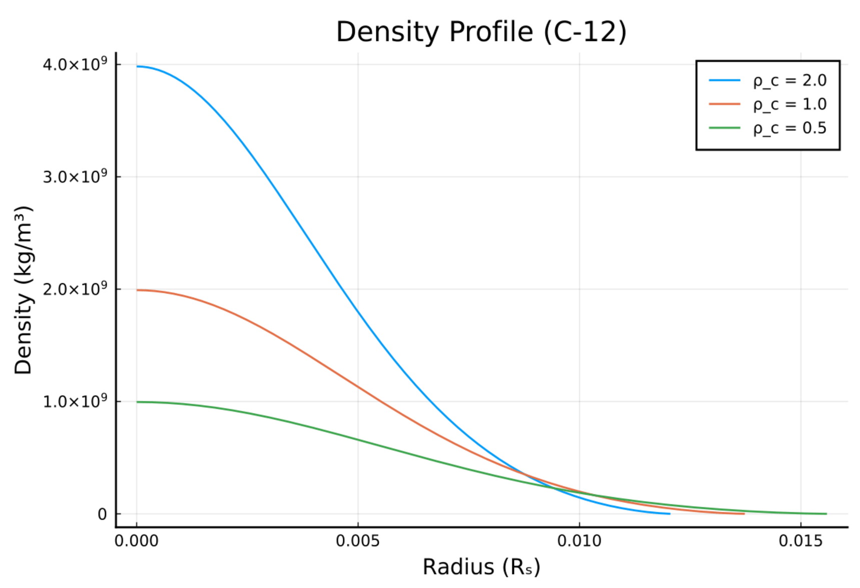


Fig. 3 Density profiles of carbon-core white dwarfs of central densities ρ_c = 2.0, 1.0 and 0.5 in scaled dimensionless unit.

Furthermore, by considering many central densities, we can plot the masses and radii to get the white dwarf mass-radius relation (Fig. 4). For further purposes, we find an empirical formula for the radii of the white dwarfs:

$$R = \frac{aY_e^{2/3}}{M^{1/3}}\left(M_{ch}(Y_e) - M\right)^{1/2} e^{\frac{d}{Y_e^2}M}$$

- M = Mass of White Dwarf
- Y_e = Z/A = Atomic-number-to-mass-number ratio
- M_{ch} = Chandrasekhar Limit of particular Y_e
- a = 0.0167462182835758
- d = 0.0331974316183360

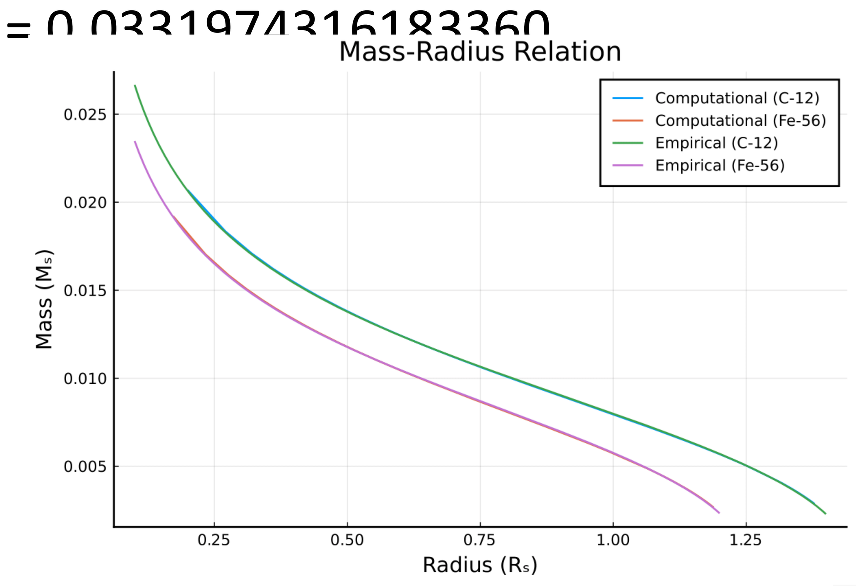


Fig. 4 Comparison between the computational and the empirical Mass-Radius relation of carbon-core and iron-core white dwarfs

However, from different databases like the Montreal, Kepler and Gaia database, we found millions of white dwarfs that are outliers from the computed majority (Fig. 5). Although the High-Mass-Large-Radius white dwarfs can be explained by thermal effects and rapid rotations, the Low-Mass-Small-Radius ones remain a mystery.

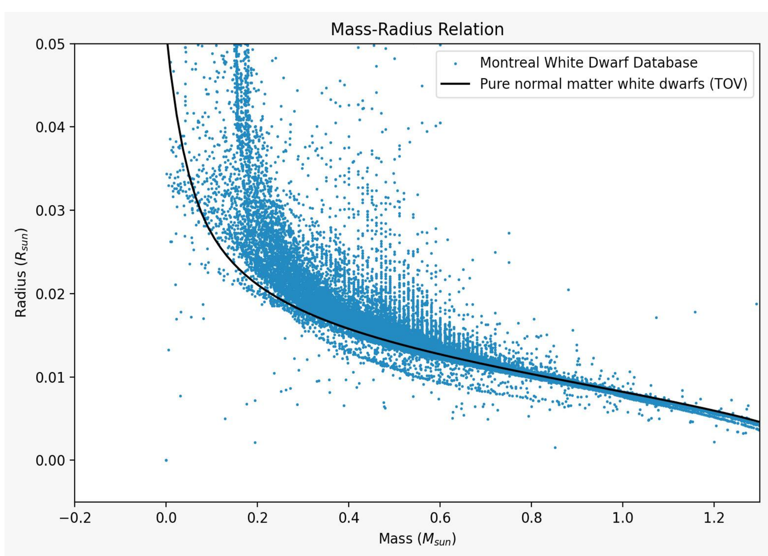


Fig. 5 *Source: www.montrealwhitedwarfdatabase.org/*

In this work, we show that these peculiar white dwarfs may be Fermionic Dark Matter-Admixed White Dwarfs. By assuming dark matter particles to be fermionic and degeneracy pressure exists between them, we are able to explain these Low-Mass-Small-Radius White Dwarfs with the Fermionic Dark Matter-Admixed Model, and further constrained the mass of a Dark Matter particle to be 0.5 GeV < m_{DM} < 10 GeV.

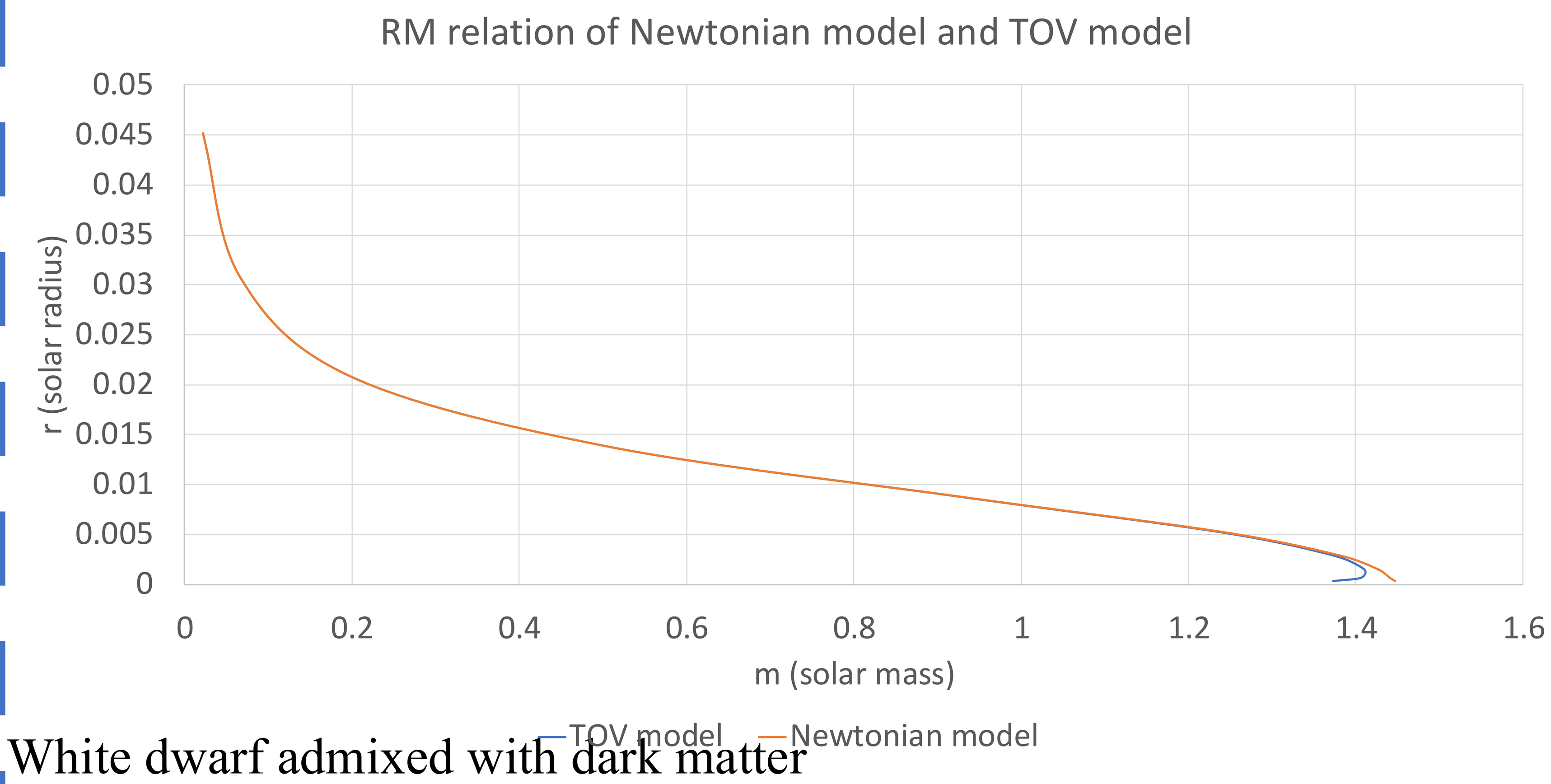
Equations

$$F_{grav} = -\frac{Gm}{r^2}\rho \qquad \frac{dP}{dr} = -\frac{Gm(r)}{r^2}\rho(r) \qquad \frac{dm}{dr} = 4\pi r^2\rho(r)$$

(left to right: gravitational force acting on a unit volume of matter, force per unit volume due to changing degeneracy pressure, differential relation between mass and density)

General relativity is important when M/R is large, correction of equation is required by applying Tolman–Oppenheimer–Volkoff (TOV) equation:

$$\frac{dP}{dr} = -\frac{Gm}{r^2}\rho\left(1+\frac{P}{\rho c^2}\right)\left(1+\frac{4\pi r^3 P}{mc^2}\right)\left(1-\frac{2Gm}{rc^2}\right)^{-1}$$



White dwarf admixed with dark matter

Normal matter equations

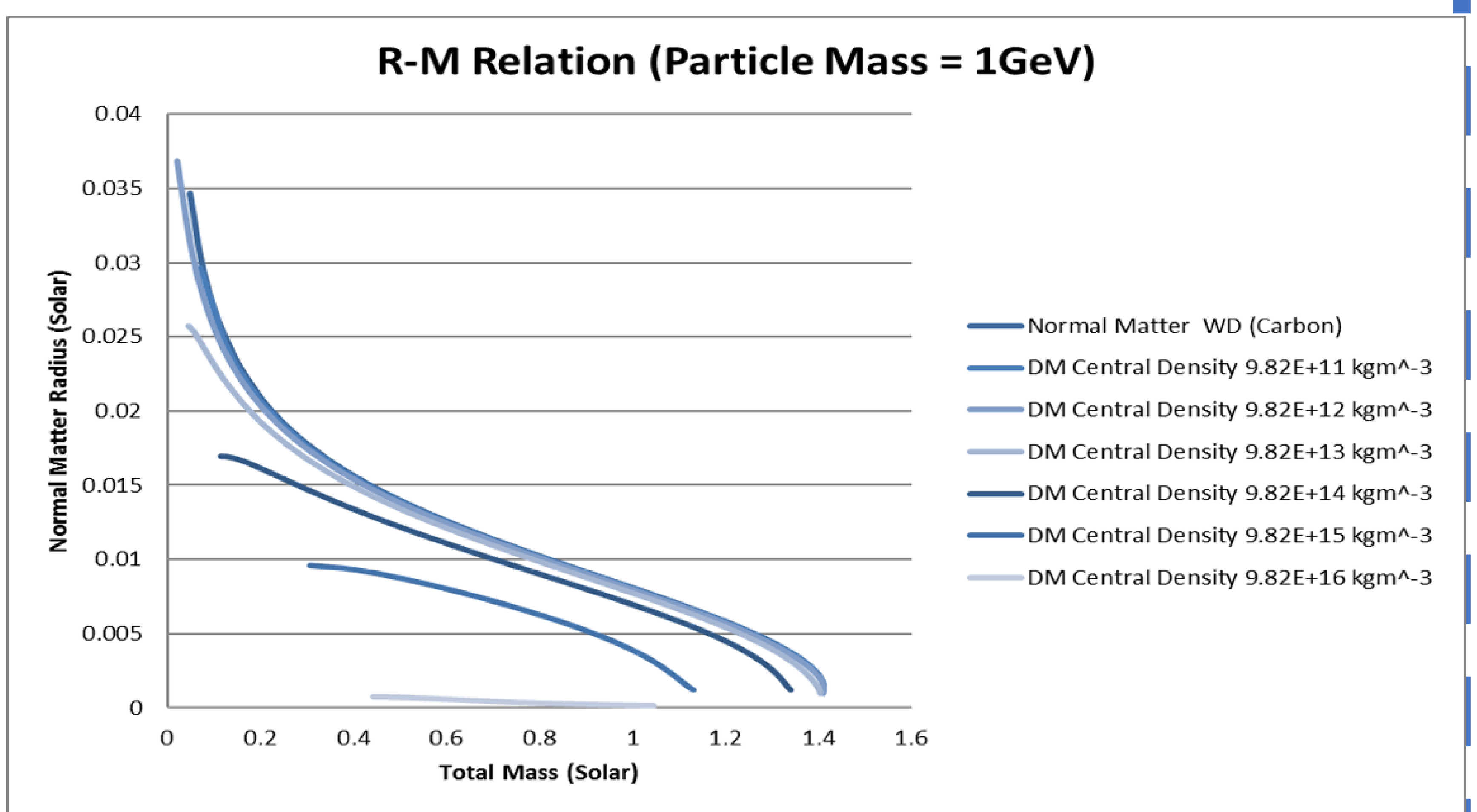
$$\frac{dm_{nm}}{dr} = 4\pi r^2\rho_{nm}$$

$$\frac{dP_{nm}}{dr} = -\frac{Gm_{tot}}{r^2}\rho_{nm}\left(1+\frac{P_{nm}}{\rho_{nm}c^2}\right)\left(1+\frac{4\pi r^3 P_{nm}}{m_{tot}c^2}\right)\left(1-\frac{2Gm_{tot}}{rc^2}\right)^{-1}$$

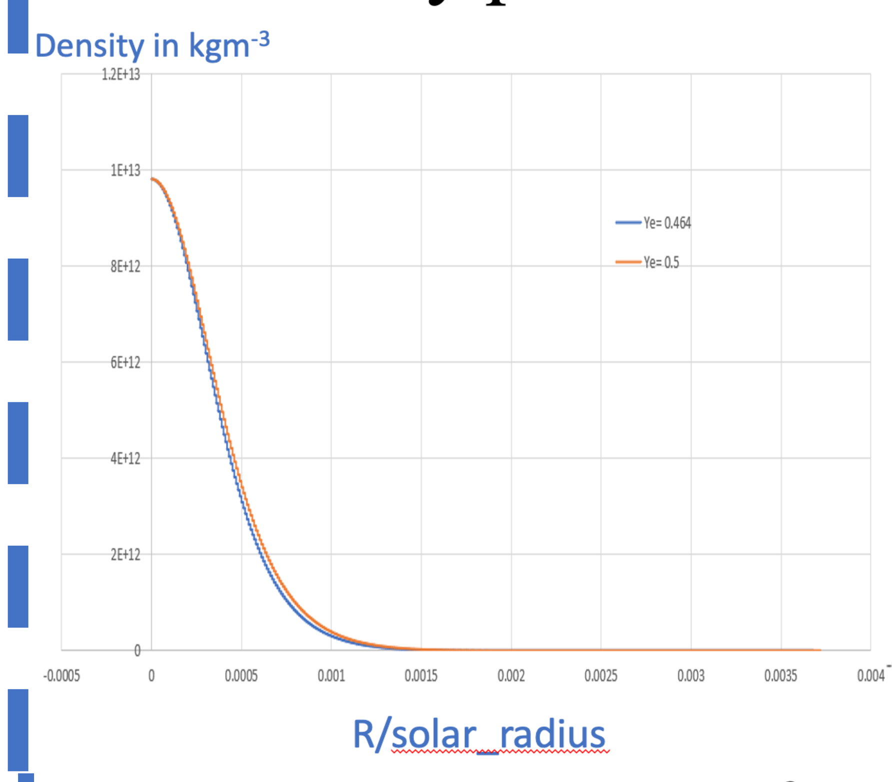
Dark matter equations

$$\frac{dm_{dm}}{dr} = 4\pi r^2\rho_{dm}$$

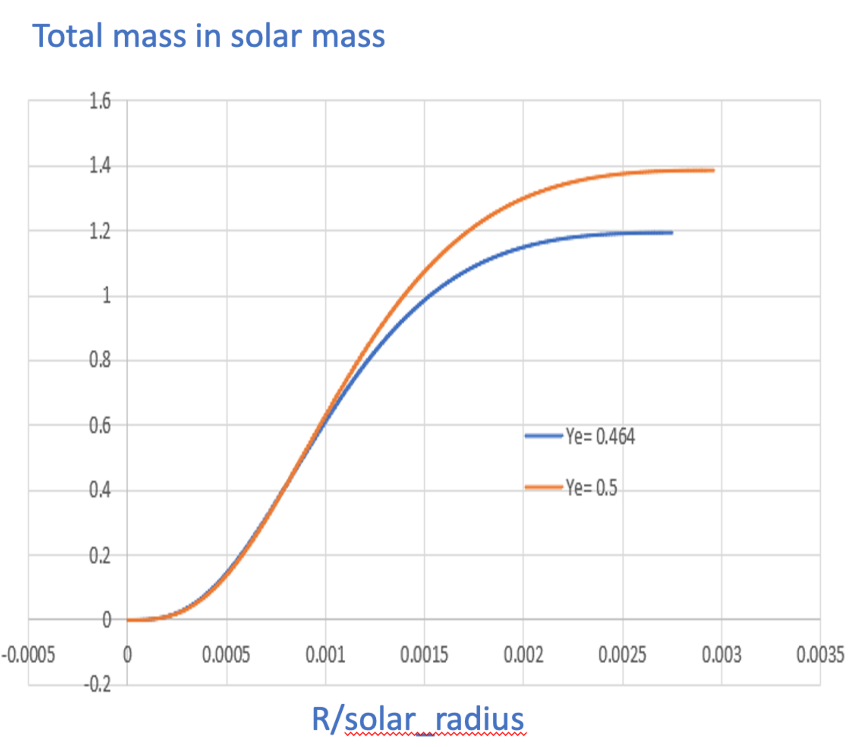
$$\frac{dP_{dm}}{dr} = -\frac{Gm_{tot}}{r^2}\rho_{dm}\left(1+\frac{P_{dm}}{\rho_{dm}c^2}\right)\left(1+\frac{4\pi r^3 P_{dm}}{m_{tot}c^2}\right)\left(1-\frac{2Gm_{tot}}{rc^2}\right)^{-1}$$



Density profile



Internal structure



TOV admixed model (ρ^c_{nm}= 9.81E12 kgm^{-3} , ρ^c_{dm}= 9.81E9 kgm^{-3})

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Results

We will use our model to acquire the normal matter mass and fraction of dark matter mass $\frac{M_d}{M_n}$ (= ε) for any observed white dwarf with known normal matter radius ($R_n(M_n,M_d)$) and total mass M_{tot}.below.

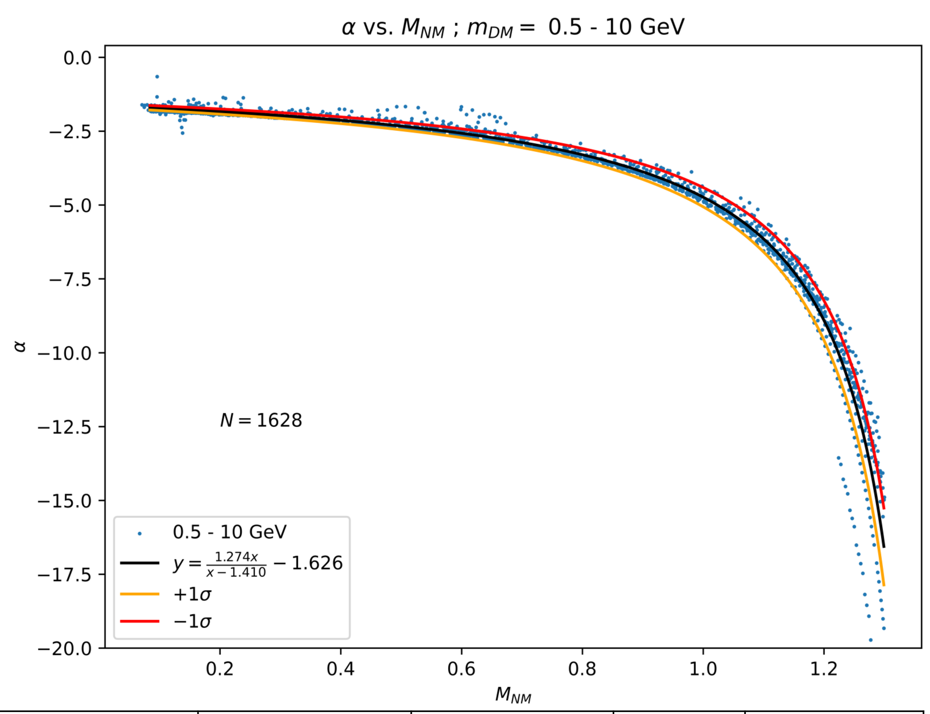
Approach 1 : Taylor Expansion

When ε < 0.05, a small addition of dark matter mass inside the white dwarf causes a small contraction of its radius. Hence, via Taylor expansion, we decided to model our data with a function $\alpha(M_n, m_{DM})$.

$$\frac{R_n(M_n, M_d)}{R_n(M_n, 0)} = 1 + \alpha\varepsilon$$

An empirical formula $\alpha = \frac{Ax}{x-B} + C$ was used to fit over 1600 data point generated with dark matter particle mass from 0.5 to 10 GeV. $R_n(M_n, 0)$ is obtained from the empirical formula from purely normal matter white dwarfs.

A negligible dependence on particle mass has been observed. Hence, one set of fit parameters (A = 1.274, B = 1.410, C = -1.626) was able to describe almost all data points up to an uncertainty of 1σ.



Examples of dark-matter admixed white dwarfs candidates identified through this scheme:

White Dwarf Name	M_{NM} (solar unit)	R (solar unit)	α	% Dark matter (ε)
2MASS J10145164+4541479	0.301	0.0170	-1.97	2.13
Wolf 28	0.696	0.0104	-2.87	3.49
2SLAQ J215903.70+001150.6	0.878	0.00742	-3.73	4.46

Working with data from the Montreal White Dwarf Database has shown (1) there are more than 300 of such white dwarfs with ε between 2% and 5%, (2) many of these white dwarfs, if they contain a dark matter core, has dark matter particle mass ranging from 0.5 to 10 GeV. Our model allows these values to be easily obtained.

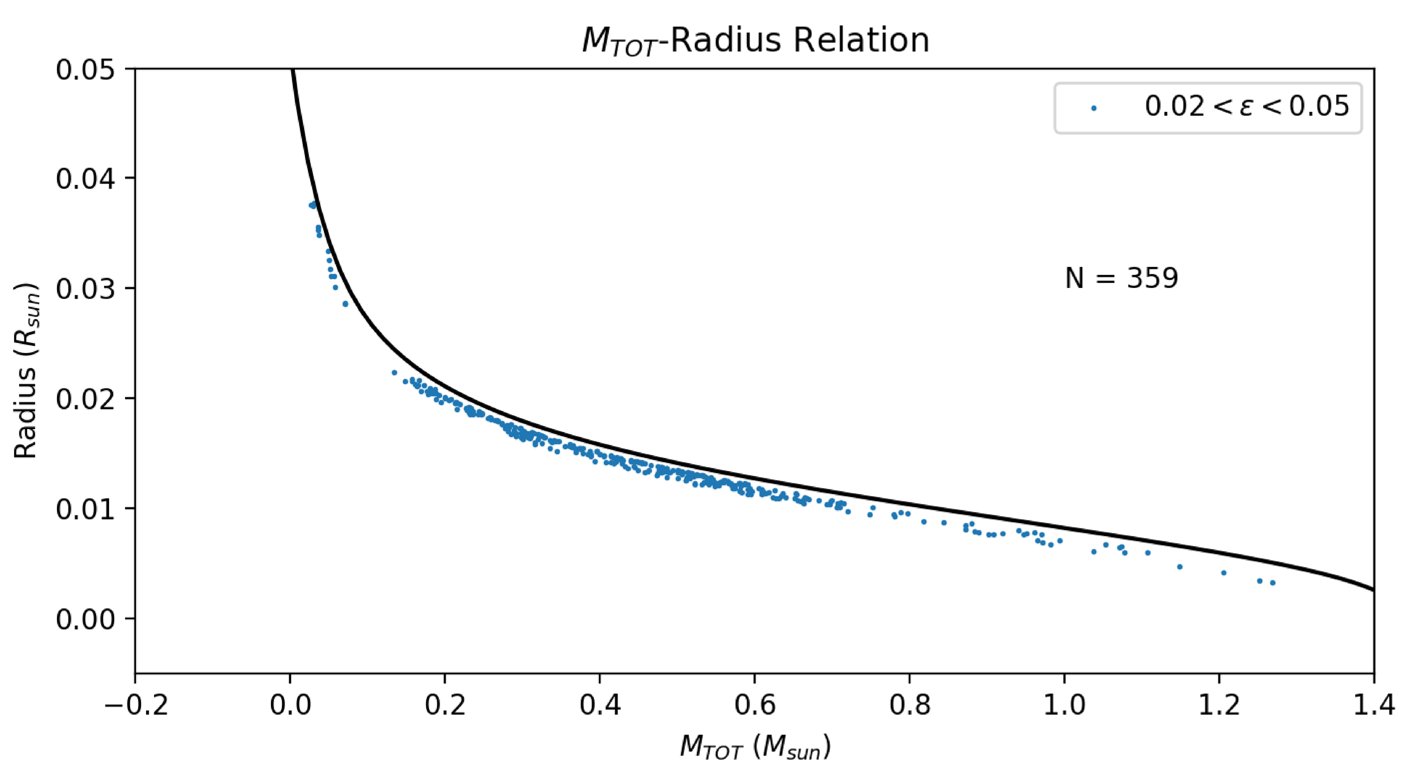
Due to the variety of data sources measured from different regimes, the mass provided by the database could be that of total mass, $M_{tot} = M_{NM}(1 + \varepsilon)$, including dark matter component in the value. To account for this, Taylor expanding alpha leads to a quadratic term in ε:

$$\frac{R_n(M_n, M_d)}{R_n(M_n, 0)} = 1 + \left(\frac{AM_{tot}}{M_{tot} - B} + C\right)\epsilon + \left(\frac{ABM_{tot}}{(M_{tot} - B)^2}\epsilon^2\right)$$

Examples of dark-matter admixed white dwarfs candidates identified through this scheme:

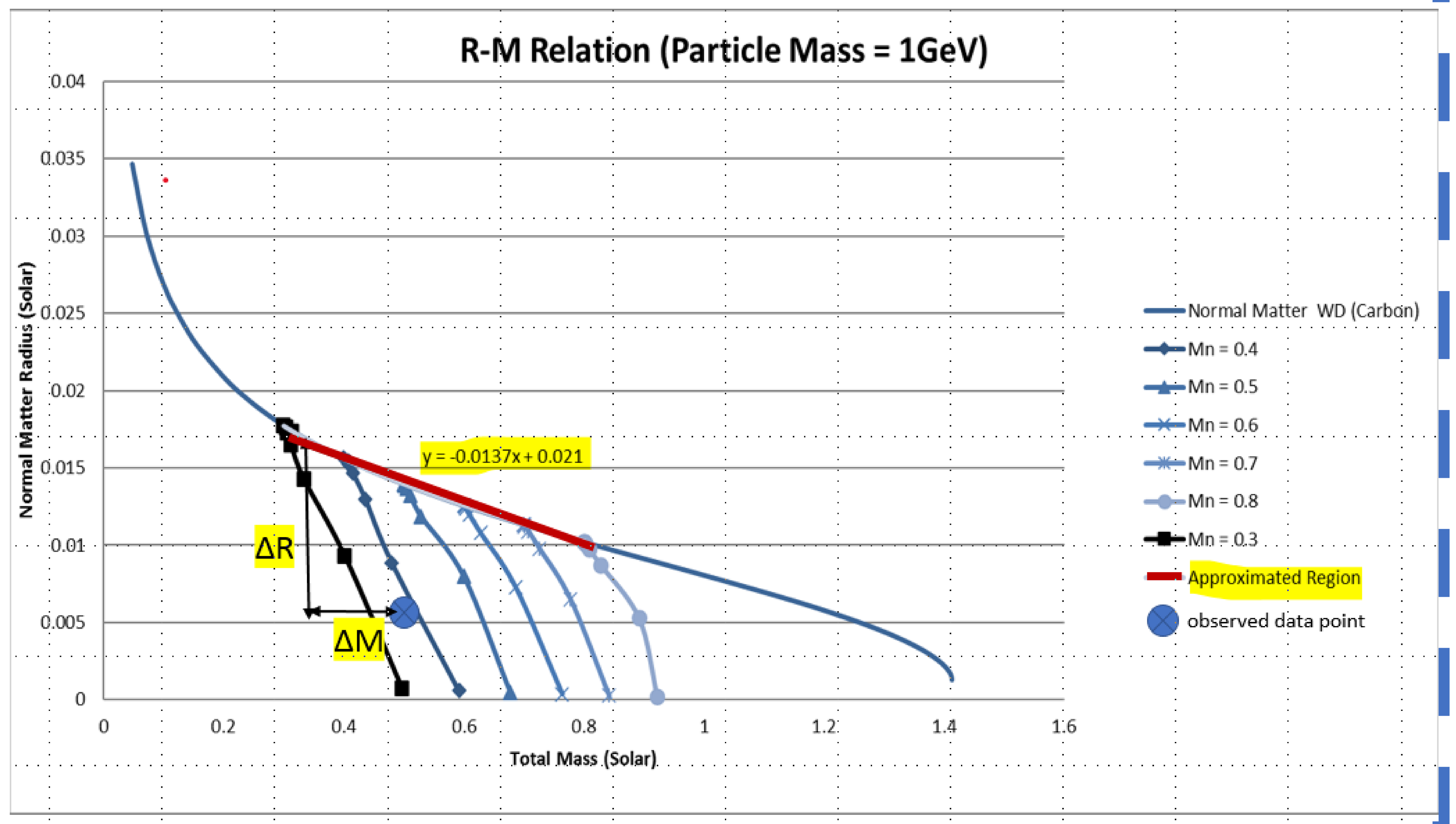
Name	MTOT (solar unit)	R (solar unit)	% Dark matter (ε)
2MASS J10145164+4541479	0.301	0.0170	2.55
Wolf 28	0.696	0.0104	2.63
2SLAQ J215903.70+001150.6	0.878	0.00742	6.50

359 white dwarfs have been identified to have ε within 2% and 5%. They are easily identifiable as lying slightly below the curve of purely normal matter white dwarfs.



Results

Approach 2 : Graphical Method (0.3<Mn<0.8)



Different branches are plotted by fixing the normal matter mass and varying the dark matter mass

Approximation within the region :

Normal matter WD follows $R_n(M_n, 0) = -0.0137M_n + 0.021$

Different branches are straight lines of the same slope −0.0769

Equation :

$$\Delta R = R_n(M_n, M_d) - R_n(M_n, 0) \qquad \Delta M = M_{tot} - M_n = \frac{\Delta R}{Slope}$$

$$M_n = M_{tot} - \frac{R_n(M_n, M_d) - (-0.0137M_n + 0.021)}{-0.0769}$$

$$\therefore M_n = \frac{M_{tot}}{0.823} + 15.823 \times R_n(M_n, M_d) - 0.332$$

$$\therefore \varepsilon = \frac{M_{tot}}{M_n} - 1$$

Summary

The dark matter admixed model can explain most of the white dwarfs that are more compact than the normal ones. By the Taylor Expansion method and the graphical method, we can obtain the normal matter and dark matter mass fraction for any observed peculiar white dwarfs with known $R_n(M_n, M_d)$ & M_{tot}

Related Projects

LSST(Observatory)
searching for DM by gravitational lensing
discovering 10 millions of white dwarfs
James Webb Telescope Mission / Gaia Mission
searching for peculiar white dwarfs

Work to Be Done in the Future

Examine the Dark Matter Dominated Region :

Degenerated Star decreases its radius when increasing the mass. To ensure that dark matter dominated stars follow the rule (i.e. they are physical), we have to examine how the dark matter radius behaves when increasing the mass.