Section 10.3

Exercise 37

The graphs are isomorphic since both graphs are cyclic and have the same number of vertices all of degree 2. This means we can follow either of the graphs in its cycle and label their vertices as $u_1 = v_1$, $u_2 = v_3$, $u_3 = v_5$, ... and so on.

Exercise 42

These graphs are not isomorphic since there is an edge between u_2 and u_3, which are vertices of degree 4, while the other two vertices of degree four v_2 and v_4 does not have an edge between them.

Exercise 54

c) If we build on the set of non-isomorphic simple graphs with 3 vertices (there are two) adding a vertex can increase the degree of one, two or three existing vertices by one. The first graph in the set is a triangle. Adding a vertex to this graph can be done in three ways since the triangle is symmetric (new vertex has one edge, two edges, or three edges).

   For the other graph in the set with 3 vertices (the line) There are two ways of adding a vertex with one edge, two ways of adding a vertex with two edges, and one way of adding a vertex with three edges. These are all non-isomorphic with our existing four vertex graphs, except when the added vertex has two edges that forms a triangle. This is a total of 3 + 4 = 7 non-isomorphic graphs with four vertices.

Section 10.4

Exercise 26

b) Between c and d of length 3: From c we have an edge to b, f and d that might be part of a path to d with length 3.

   b has 3 paths with length 2 that reach d; (b, e, d), (b, c, d) and (b, a, d).

   f has two paths with length 2 that reach d; (f, c, d) and (f, e, d).

   d has 3 paths with length 2 that reach d; (d, e, d), (d, c, d) and (d, a, d).

   This is a total of 8 paths, which corresponds to $\boldsymbol{A}^{\mathbf{3}}_{(3,4)}$ where $\boldsymbol{A}$ is the adjacency matrix of the graph in figure 1.

Exercise 30

The sum of all degrees of the vertices in a simple undirected graph is always two times the number of edges (two vertices per edge). This means there are an even number of vertices of an odd degree. Since a simple undirected graph has a simple path between any two vertices it follows that a vertex of an odd degree must have a simple path to another vertex of an odd degree

Exercise 56

If $\boldsymbol{A}$ is an adjacency matrix of a graph and v and w are vertices of that graph, then the number of paths of length r is given by $\boldsymbol{A}^{r}_{(v,w)}$. Calculating this expression, and increasing r by one if the result is 0 will eventually produce non-zero result where r is the shortest lenght of a path from v to w.

Section 10.5

Exercise 3

The graphs does not have an Eurler Circuit since a and d are vertices of an odd degree. It does however have an Euler Path since the rest of the vertices are of an even degree.

An example is $a, c, e, b, e, c, d, b, a, e, d$

Exercise 30

The graph does not have a hamilton cycle since there is only one edge from the right side to the left side of the graph.

Exercise 36

The graph has several hamilton cycles. An example is $a, d, g, h, i, e, f, c, b, (a)$.

Exercise 48

Two fully connected subgraphs that share exactly one vertex is an example of a graph that does not have a Hamilton cycle since there is no way "back" from either subgraph after "crossing" the shared vertex. If both subgraphs have equal amount of vertices (and are fully connected) their vertices are of degree $(n-1)/2$ except the shared vertex of degree $n-1$. A concrete example is two triangles that share only one vertex.

Section 10.6

Exercise 3

Since all edges are weighted positively we can use Dijkstras algorithm.

node visited in step	a	b	c	d	e	f	g	z
a	0	4	3					
c	0	4	3	6	9			
b	0	4	3	6	9			
d	0	4	3	6	7	11		
e	0	4	3	6	7	11	12	
f	0	4	3	6	7	11	12	18
g	0	4	3	6	7	11	12	16
z	0	4	3	6	7	11	12	16

Exercise 14

Finding the shortest path by using a weighted graph can be done by uniformly weighting the graph.

Exercise 18

No, the sum of two unique weights in a shortest path can still be the same as the length of another path.