Section 3.1

Exercise 53

a) Total: 2 quarters, 1 penny (51c)

b) Total: 2 quarters, 1 dimes, 1 nickel, 4 pennies (69c)

c) Total: 3 quarters, 1 penny (76c)

d) Total: 2 quarters, 1 dime (60c)

Exercise 55

a) Total: 2 quarters, 1 penny (51c)

b) Total: 2 quarters, 1 dime, 9 pennies (69c)

c) Total: 3 quarters, 1 penny (76c)

d) Total: 2 quarters, 1 dime (60c)
The fewest coins are given when $(n \bmod 25) \bmod 10 \geq 5$

Exercise 56

Proof by counterexample:
15c using the greedy algorithm would draw one 12c coin and 3 pennies, which is four coins, when one dime and one nickel would suffice.

Section 3.2

Exercise 27

a) $n \log(n^2 + 1) = \mathcal{O}(n\, 2 \log n) = \mathcal{O}(n \log n)$
$\mathcal{O}(n^2 \log n + n \log n) = \mathcal{O}(n^2 \log n)$

b) $(n \log n + 1)^2 = \mathcal{O}(n^2 (\log n)^2)$ $(\log n + 1)(n^2 + 1) = \mathcal{O}(n^2 \log n)$ $\mathcal{O}(n^2 \log n) + \mathcal{O}(n^2 (\log n)^2) = \mathcal{O}(n^2 (\log n)^2)$

Exercise 30

c) $|\lfloor x + 1/2 \rfloor| \leq C|x| \forall x > k$ is true for (among others) $C = 2$ and $k = 2$
$C|\lfloor x + 1/2 \rfloor| \geq |x| \forall x > k$ is true for (among others) $C = 2$ and $k = 2$
Therefore both expressions is $\Theta(x)$

e) $\log_2(x) = \frac{\log_{10}(x)}{\log_{10}(2)} \implies C \log_{10}(x) \propto \log_2(x)$ which means they are both $\Theta(\log x)$

Exercise 34

a)

$$
\begin{aligned}
f(x) &= 3x^2 + x + 1 \\
g(x) &= 3x^2 \\
C_1|3x^2| \leq |3x^2 &+ x + 1| \leq C_2|3x^2|, \forall x > k
\end{aligned}
$$

Since k can be chosen freely we choose $k > 0$ to get rid of absolute values

$$3C_1 x^2 \leq 3x^2 + x + 1 \leq 3C_2 x^2, \forall x > k$$

We simplify and get

$$3C_1 \leq 3 + \frac{1}{x} + \frac{1}{x^2} \leq 3C_2, \forall x > k$$

The constants $C_1 = \frac{1}{2}, C_2 = 2, k = 1$ is a solution

b) `loglog` in matlab is a nice way to plot this

```
x = logspace(-1,2);
g = 3*x.^2;
f1 = 1.5*x.^2 + 0.5*x + 0.5;
f2 = 6*x.^2 + 2*x + 2;
loglog(x,f1, x, g, x, f2);
legend('C_1 f(x)','g(x)','C_2 f(x)');
```

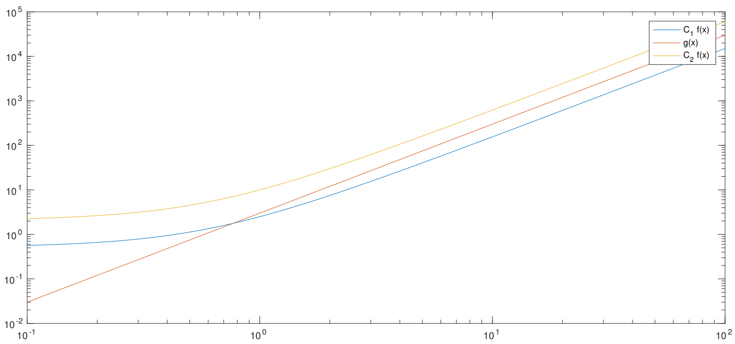

Figure 1: $f(x) = \Theta(3x^2)$ as we see here for $x > 10^0 = 1$

Exercise 42

From $f(x) = \mathcal{O}(g(x))$ we have that

$$\exists C, k \quad \text{such that} \quad C|g(x)| \geq |f(x)|, \quad \forall x > k$$

If $2^{f(x)} = \mathcal{O}(2^{g(x)})$ then the following must be true

$$C|2^{g(x)}| \geq |2^{f(x)}|$$

$2^a > 0, \forall a$ so we can ignore absolute values

$$\log_2 C + \log_2 2^{g(x)} \geq \log_2 2^{f(x)}$$
$$\log_2 C + g(x) \geq f(x)$$

For large values of x, $\log_2 C$ becomes insignificant, which means that $g(x) \geq f(x)$ must hold for large x if $2^{f(x)} = \mathcal{O}(2^{g(x)})$

Since $f(x) = \mathcal{O}(g(x))$ does not imply $g(x) \geq f(x), \forall x > k$ then it does not follow from $f(x) = \mathcal{O}(g(x))$ that $2^{f(x)} = \mathcal{O}(2^{g(x)})$

Section 4.1

Exercise 11

a) $11 + 80 \mod 12 = 7$
The clock reads 7:00

b) $12 + 40 \mod 12 = 4$
The clock reads 4:00

c) $6 + 100 \mod 12 = 10$
The clock reads 10:00