1

a) We use the following script to find the DFT of the discrete time series

```
% Discrete time series x
x = [0, 2, 3, 7];

% Fast fourier transform of x, then take the real part
y = fft(x);
r = real(y);

plot(r);
```

b) This results in the following plot

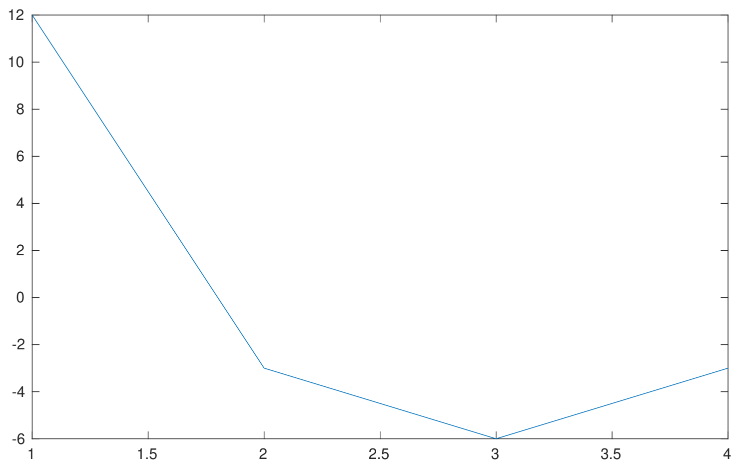

Figure 1: Matlab plot of the real part of the `fft` of the discrete time series

2

We have the system of equations

$$\begin{aligned} 5x_1 + x_2 + 2x_3 &= 19 \\ x_1 + 4x_2 - 2x_3 &= -2 \\ 2x_1 + 3x_2 + 8x_3 &= 39 \end{aligned}$$

Which can be rewritten to

$$\begin{aligned} x_1 &= -0.200x_2 - 0.400x_3 + 3.800 \\ x_2 &= -0.250x_1 + 0.500x_3 - 0.500 \\ x_3 &= -0.250x_1 - 0.375x_2 + 4.875 \end{aligned}$$

a)

We can write the system on the form $\boldsymbol{Ax} = \boldsymbol{b}$ using the matrices $\boldsymbol{A} = \boldsymbol{I} + \boldsymbol{L} + \boldsymbol{U}$ where

$$\boldsymbol{U} = \begin{bmatrix} 0 & 0.2 & 0.4 \\ 0 & 0 & -0.5 \\ 0 & 0 & 0 \end{bmatrix}, \quad \boldsymbol{L} = \begin{bmatrix} 0 & 0 & 0 \\ 0.25 & 0 & 0 \\ 0.25 & 0.375 & 0 \end{bmatrix}$$

$$\boldsymbol{A} = \boldsymbol{I} + \boldsymbol{L} + \boldsymbol{U} = \begin{bmatrix} 1 & 0.2 & 0.4 \\ 0.25 & 1 & -0.5 \\ 0.25 & 0.375 & 1 \end{bmatrix} \text{ and } \boldsymbol{b} = \begin{bmatrix} 3.8 \\ -0.5 \\ 4.875 \end{bmatrix}$$

We know the Gauss-Seidel iteration converges for $||\boldsymbol{C}|| < 1$ where the iteration matrix $\boldsymbol{C} = -(\boldsymbol{I} + \boldsymbol{L})^{-1}\boldsymbol{U}$ Calculating the $\boldsymbol{C}$ matrix we get $\boldsymbol{C} = [0, 0.125, 0.2]^T$, which means the Gauss-Seidel iteration converges as all values of $\boldsymbol{C}$ are less than 1.

The Jacobi iteration matrix is given by $\boldsymbol{I} - \boldsymbol{A}$ and the iteration converges if and only if this matrix has a spectral radius of less than 1. We can find the spectral radius of this matrix by using the matlab command `max(abs(eig(eye(3) - A)))` which outputs `0.2640`. This means the Jacobi iteration converges.

b) We apply four steps of Gauss-Seidel iteration using the following matlab script

```
% Gauss-Seidel iteration script, based on the Gauss-Seidel iteration
% example from mathworks.com

U = [0 0.2 0.4; 0 0 -0.5; 0 0 0];
L = [0 0 0; 0.25 0 0; 0.25 0.375 0];

b = [3.8 -0.5 4.875]';

I = eye(3);
A = I + L + U;

% initial vector x
x = ones(3,1);

n = size(x, 1);

GaussItr = 0;

% list of all values of x
Gausshistory=[x];

while GaussItr<4
    x_old=x;
    for i=1:n
        sigma=0;
        for j=1:i-1
                sigma=sigma+A(i,j)*x(j);
        end
        for j=i+1:n
                sigma=sigma+A(i,j)*x_old(j);
        end
        x(i)=(1/A(i,i))*(b(i)-sigma);
    end

    % append new x to Gausshistory
    Gausshistory=[Gausshistory, x];

    GaussItr=GaussItr+1;
end
fprintf('Solution of the system is : \n%f\n%f\n%f\nin %d iterations\n',x,GaussItr);
```

This outputs

```
Solution of the system is :
2.004181
1.007509
3.996139
in 4 iterations
```

The `Gausshistory` variable contains

```
Gausshistory =
  1.0000    3.2000    2.2100    2.0142    2.0042
  1.0000   -0.8000    1.1350    0.9449    1.0075
  1.0000    4.3750    3.8969    4.0171    3.9961
```

Plotting these values we get a nice plot showing the convergence.

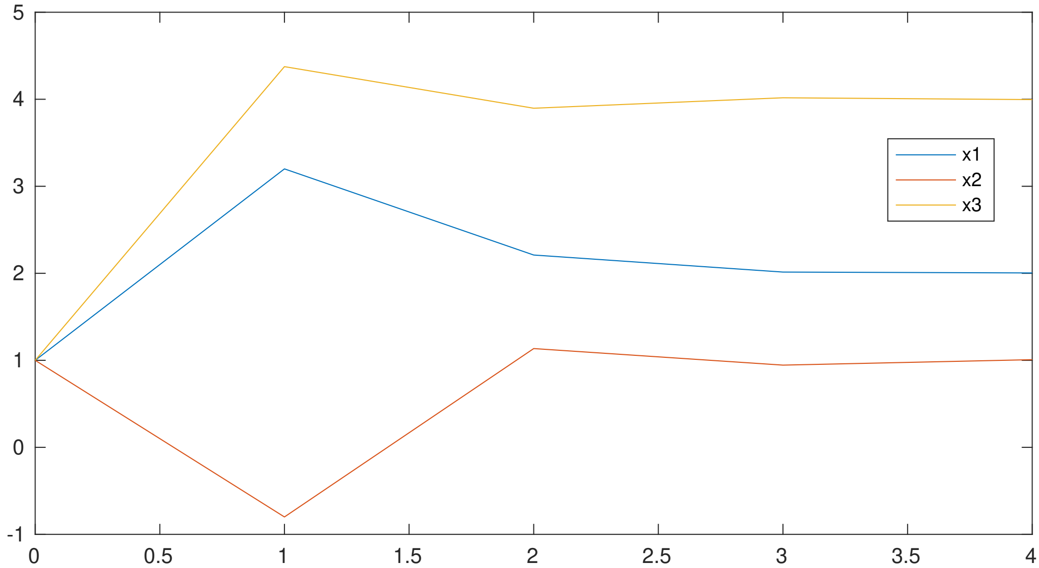

Figure 2: `>> plot(Gausshistory')`

c) We apply four steps of Jacobi iteration using the following matlab script

```matlab
% Jacobi iteration script, based on the Jacobi iteration
% example from mathworks.com

U = [0 0.2 0.4; 0 0 -0.5; 0 0 0];
L = [0 0 0; 0.25 0 0; 0.25 0.375 0];

b = [3.8 -0.5 4.875]';

I = eye(3);
A = I + L + U;

% initial vector x
x = ones(3,1);

n = size(x, 1);;

JacobItr = 0;

% list of all values of x
Jacobhistory=[x];

while JacobItr<4
    xold=x;
    for i=1:n
        sigma=0;
        for j=1:n
            if j~=i
                sigma=sigma+A(i,j)*x(j);
            end
        end
        x(i)=(1/A(i,i))*(b(i)-sigma);
    end

    JacobItr=JacobItr+1;
    Jacobhistory =[Jacobhistory, x];
end
fprintf('Solution of the system is : \n%f\n%f\n%f\nin %d iterations\n',x,JacobItr);
```

This outputs

```
Solution of the system is :
2.004181
1.007509
3.996139
in 4 iterations
```

The `Jacobhistory` variable contains the exact same results as the Gauss-seidel iteration

`Jacobhistory =`

1.0000	3.2000	2.2100	2.0142	2.0042
1.0000	-0.8000	1.1350	0.9449	1.0075
1.0000	4.3750	3.8969	4.0171	3.9961