

# Linear Algebra

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# 1 Linear Algebra Done Right

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## 1.1 Vector Spaces

### 1.1.1 Exercises 1.A

1

$$\frac{1}{a + bi} = c + di$$

$$(a + bi)(c + di) = 1$$

$$ac + adi + bci - bd = 1$$

$$(ac - bd) + (ad + bc)i = 1$$

Which leads to

$$ac - bd = 1$$

$$ad + bc = 0$$

Because 1 has no imaginary part.

$$c = \frac{1 + bd}{a}$$

$$d = \frac{-bc}{a}$$

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$$c = \frac{1 + b \frac{-bc}{a}}{a}$$

$$c = \frac{1}{a} - \frac{b^2 c}{a^2}$$

$$c + \frac{b^2 c}{a^2} = \frac{1}{a}$$

$$c \left( 1 + \frac{b^2}{a^2} \right) = \frac{1}{a}$$

$$c = \frac{1}{a \left( 1 + \frac{b^2}{a^2} \right)}$$

$$c = \frac{1}{a + \frac{b^2}{a}}$$

$$c = \frac{1}{\frac{a^2 + b^2}{a}}$$

$$c = \frac{a}{a^2 + b^2}$$

Now that we have  $c$ , we can pick out  $d$  too.

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$$d = \frac{-bc}{a}$$

$$d = \frac{-b \left( \frac{a}{a^2+b^2} \right)}{a}$$

$$d = \frac{\frac{-ab}{a^2+b^2}}{a}$$

$$d = \frac{-b}{a^2+b^2}$$

So now we have  $d$  and  $c$  as real numbers, since  $a$  and  $b$  are real.

$$c = \frac{a}{a^2+b^2}$$

$$d = \frac{-b}{a^2+b^2}$$

.....

2

$$\frac{-1 + \sqrt{3}i}{2} =$$

$$\frac{-1}{2} + \frac{\sqrt{3}}{2}i =$$

$$\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} =$$

$$e^{i\frac{2\pi}{3}}$$

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Which means that

$$\left(e^{i\frac{2\pi}{3}}\right)^3 =$$

$$e^{i\frac{6\pi}{3}} =$$

$$e^{i2\pi} =$$

$$1$$

Hence, the cube of  $\frac{-1+\sqrt{3}i}{2}$  is equal to 1, which was to be shown.

### 1.1.2 Exercises 1.B

#### Problem 1

$$-(-v) = v$$

Add the additive inverse of  $-(-v)$  to both sides

$$0 = v + (-v)$$

$$0 = 1v + (-1)v$$

$$0 = (1 + (-1))v$$

$$0v = 0$$

Which is true because

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$$0v = (0 + 0)v = 0v + 0v$$

And if we then add the additive inverse of  $0v$  to both sides and switch the sides around the equal sign we get

$$0v = 0$$

Which shows that  $0v = 0$  and therefore  $-(-v) = v$  is true for each  $v \in V$ .

So the statement is true, which was to be proved.

**Problem 2** Suppose that  $a \neq 0$ . We can then multiply both sides by the multiplicative inverse of  $a$ .

$$v = 0 \frac{1}{a}$$

$$v = 0$$

Suppose that  $a = 0$ . Since  $0v = 0$  the value of  $v$  doesn't matter for the statement to be true.

In total, this means that  $v = 0$  or  $a = 0$ , or both  $v = 0$  and  $a = 0$ , which was to be proved.

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## 2 Lectures