Linear Algebra

Otto Martinwall

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Contents

1 Linear Algebra Done Right

1.1 Vector Spaces

For proofs, we're going to use a more general field $\mathbb{F}$ to prove our theorems, so that our our proofs also apply to the fields $\mathbb{R}$ and $\mathbb{C}$.

Definition 1. *$\mathbb{F}^n$ is the set of all lists of length n of elements of $\mathbb{F}$.*

$$\mathbb{F}^n = \{(x_1, \ldots, x_n) \mid x_j \in \mathbb{F} \; \forall j = 1, \ldots, n\}$$

*For $(x_1, \ldots, x_n) \in \mathbb{F}$ and $j \in \{1, \ldots, n\}$, we say that x_j is the j^{th} **coordinate** of $(x_1, \ldots, x_n)$. $(x_1, \ldots, x_n)$ are scalars, a.k.a. numbers.*

This means that if we say $\mathbb{F} = \mathbb{R}$ then we can apply the theorems in the real n-space.

1.1.1 Exercises 1.A

1

$$\frac{1}{a + bi} = c + di$$

$$(a + bi)(c + di) = 1$$

$$ac + adi + bci - bd = 1$$

$$(ac - bd) + (ad + bc)i = 1$$

Which leads to

$$ac - bd = 1$$

$$ad + bc = 0$$

Because 1 has no imaginary part.

$$c = \frac{1+bd}{a}$$

$$d = \frac{-bc}{a}$$

$$c = \frac{1+b\frac{-bc}{a}}{a}$$

$$c = \frac{1}{a} - \frac{b^2c}{a^2}$$

$$c + \frac{b^2c}{a^2} = \frac{1}{a}$$

$$c\left(1+\frac{b^2}{a^2}\right) = \frac{1}{a}$$

$$c = \frac{1}{a\left(1+\frac{b^2}{a^2}\right)}$$

$$c = \frac{1}{a+\frac{b^2}{a}}$$

$$c = \frac{1}{\frac{a^2+b^2}{a}}$$

$$c = \frac{a}{a^2+b^2}$$

Now that we have c, we can pick out d too.

$$d = \frac{-bc}{a}$$

$$d = \frac{-b\left(\frac{a}{a^2+b^2}\right)}{a}$$

$$d = \frac{\frac{-ab}{a^2+b^2}}{a}$$

$$d = \frac{-b}{a^2+b^2}$$

So now we have d and c as real numbers, since a and b are real.

$$c = \frac{a}{a^2+b^2}$$

$$d = \frac{-b}{a^2+b^2}$$

2

$$\frac{-1+\sqrt{3}i}{2} =$$

$$\frac{-1}{2} + \frac{\sqrt{3}}{2}i =$$

$$\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3} =$$

$$e^{i\frac{2\pi}{3}}$$

Which means that

$$\left(e^{i\frac{2\pi}{3}}\right)^3 =$$

$$e^{i\frac{6\pi}{3}} =$$

$$e^{i2\pi} =$$

$$1$$

Hence, the cube of $\frac{-1+\sqrt{3}i}{2}$ is equal to 1, which was to be shown.

1.1.2 Exercises 1.B

Problem 1

$$-(-v) = v$$

Add the additive inverse of $-(-v)$ to both sides

$$0 = v + (-v)$$

The additive inverse of v added together with v is the zero vector.

$$0 = 0$$

So the statement is true, which was to be proved.

Problem 2

$$av = 0$$

Suppose that $a = 0$

2 Lectures

2.1 Lecture 1: Area of the Sphere

To calculate the area of the sphere, we begin with:

$$x^2 + y^2 = r^2 \tag{1}$$

$$y = \sqrt{r^2 - x^2} \tag{2}$$

Then use an integral to describe the length of the half circle:

$$\int_{-r}^{r} \sqrt{1 - (\frac{dy}{dx})^2} dx \tag{3}$$

Now apply a revolution to that:

$$\pi \int_{-r}^{r} \left(1 - \left(\frac{dy}{dx}\right)^2\right) dx \tag{4}$$

Since $\frac{dy}{dx} = \frac{1}{2}\left(r^2 - x^2\right)^{-\frac{1}{2}} * (-2x) = -\frac{x}{\sqrt{r^2 - x^2}}$ Then

$$\pi \int_{-r}^{r} \left(1 - \left(-\frac{x}{\sqrt{r^2 - x^2}}\right)^2\right) dx \tag{5}$$

$$\tag{6}$$

Which then is:

$$\pi \int_{-r}^{r} \left(1 - \left(\frac{x^2}{r^2 - x^2}\right)\right) dx = \tag{7}$$

$$\pi \int_{-r}^{r} \left(1 - \left(\frac{x^2}{r^2 - x^2}\right)\right) dx = \tag{8}$$

$$\pi \int_{-r}^{r} \left(1 - \left(\frac{x^2}{r^2 - x^2}\right)\right) dx = \tag{9}$$

2.2 Lecture 2: Title of This Other Lecture

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