# Linear Algebra

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## Contents

# 1 Linear Algebra Done Right

## 1.1 Vector Spaces

### 1.1.1 Exercises 1.A

**1**

$$\frac{1}{a+bi} = c + di$$

$$(a+bi)(c+di) = 1$$

$$ac + adi + bci - bd = 1$$

$$(ac - bd) + (ad + bc)i = 1$$

Which leads to

$$ac - bd = 1$$

$$ad + bc = 0$$

Because 1 has no imaginary part.

$$c = \frac{1+bd}{a}$$

$$d = \frac{-bc}{a}$$

$$c = \frac{1 + b\frac{-bc}{a}}{a}$$

$$c = \frac{1}{a} - \frac{b^2 c}{a^2}$$

$$c + \frac{b^2 c}{a^2} = \frac{1}{a}$$

$$c\left(1 + \frac{b^2}{a^2}\right) = \frac{1}{a}$$

$$c = \frac{1}{a\left(1 + \frac{b^2}{a^2}\right)}$$

$$c = \frac{1}{a + \frac{b^2}{a}}$$

$$c = \frac{1}{\frac{a^2+b^2}{a}}$$

$$c = \frac{a}{a^2 + b^2}$$

Now that we have $c$, we can pick out $d$ too.

$$d = \frac{-bc}{a}$$

$$d = \frac{-b\left(\frac{a}{a^2+b^2}\right)}{a}$$

$$d = \frac{\frac{-ab}{a^2+b^2}}{a}$$

$$d = \frac{-b}{a^2+b^2}$$

So now we have $d$ and $c$ as real numbers, since $a$ and $b$ are real.

$$c = \frac{a}{a^2+b^2}$$

$$d = \frac{-b}{a^2+b^2}$$

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**2**

$$\frac{-1+\sqrt{3}i}{2} =$$

$$\frac{-1}{2} + \frac{\sqrt{3}}{2}i =$$

$$\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3} =$$

$$e^{i\frac{2\pi}{3}}$$

Which means that

$$\left(e^{i\frac{2\pi}{3}}\right)^3 =$$

$$e^{i\frac{6\pi}{3}} =$$

$$e^{i2\pi} =$$

$$1$$

Hence, the cube of $\frac{-1+\sqrt{3}i}{2}$ is equal to $1$, which was to be shown.

### 1.1.2 Exercises 1.B

**Problem 1**

$$-(-v) = v$$

Add the additive inverse of $-(-v)$ to both sides

$$0 = v + (-v)$$

The additive inverse of $v$ added together with $v$ is the zero vector.

$$0 = 0$$

So the statement is true, which was to be proved.

## Problem 2

$$av = 0$$

Suppose that $a = 0$

# 2 Lectures