Single Variable Calculus

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1 Calculus A Complete Course

1.1 Limits and Continuity

Theorem 1 (The Squeeze Theorem). *Suppose that $f(x) \leq g(x) \leq h(x)$ holds for all x in some open interval containing c, except possibly at $x = c$. Suppose also that*

$$\lim_{x \to c} f(x) = \lim_{x \to c} h(x) = L$$

Then $\lim_{x \to c} g(x) = L$.

Proof. For this proof, the (ϵ, δ)-definition of the limit will be used.

The goal is to prove that $\lim_{x \to c} g(x) = L$, which is true if

$$\forall \epsilon > 0, \exists \delta > 0 : \forall x, (|x - c| < \delta \Rightarrow |g(x) - L| < \epsilon).$$

Since $\lim_{x \to c} f(x) = L$,

$$\forall \epsilon > 0, \exists \delta_1 > 0 : \forall x, (|x - c| < \delta_1 \Rightarrow |f(x) - L| < \epsilon) \tag{1}$$

And since $\lim_{x \to c} h(x) = L$,

$$\forall \epsilon > 0, \exists \delta_2 > 0 : \forall x, (|x - c| < \delta_2 \Rightarrow |h(x) - L| < \epsilon). \tag{2}$$

Then we have

$$f(x) \leq g(x) \leq h(x)$$

$$f(x) - L \leq g(x) - L \leq h(x) - L$$

We can choose $\delta = \min\{\delta_1, \delta_2\}$, then if $|x - c| < \delta$, and combining (1) and (2), we have

$$
\begin{aligned}
&-\epsilon < f(x) - L \leq g(x) - L \leq h(x) - L < \epsilon \\
&-\epsilon < g(x) - L < \epsilon \\
&|g(x) - L| < \epsilon
\end{aligned}
$$

So $\lim_{x \to c} g(x) = L$, which completes the proof. $\square$

1.1.1 Exercises 1.1

1.1.2 Exercises 1.2

2 Lectures