

# Single Variable Calculus

Otto Martinwall

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# 1 Calculus A Complete Course

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## 1.1 Limits and Continuity

**Theorem 1** (The Squeeze Theorem). *Suppose that  $f(x) \leq g(x) \leq h(x)$  holds for all  $x$  in some open interval containing  $c$ , except possibly at  $x = c$ . Suppose also that*

$$\lim_{x \rightarrow c} f(x) = \lim_{x \rightarrow c} h(x) = L$$

*Then  $\lim_{x \rightarrow c} g(x) = L$ .*

*Proof.* For this proof, the  $(\epsilon, \delta)$ -definition of the limit will be used.

The goal is to prove that  $\lim_{x \rightarrow c} g(x) = L$ , which is true if

$$\forall \epsilon > 0, \exists \delta > 0 : \forall x, (|x - a| < \delta \Rightarrow |g(x) - L| < \epsilon).$$

Since  $\lim_{x \rightarrow c} f(x) = L$ ,

$$\forall \epsilon > 0, \exists \delta_1 > 0 : \forall x, (|x - a| < \delta_1 \Rightarrow |f(x) - L| < \epsilon) \quad (1)$$

And since  $\lim_{x \rightarrow c} h(x) = L$ ,

$$\forall \epsilon > 0, \exists \delta_2 > 0 : \forall x, (|x - a| < \delta_2 \Rightarrow |h(x) - L| < \epsilon). \quad (2)$$

Then we have

$$f(x) \leq g(x) \leq h(x)$$

$$f(x) - L \leq g(x) - L \leq h(x) - L$$

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We can choose  $\delta = \min\{\delta_1, \delta_2\}$ , then if  $|x - a| < \delta$ , and combining (1) and (2), we have

$$-\epsilon < f(x) - L \leq g(x) - L \leq h(x) - L < \epsilon$$

$$-\epsilon < g(x) - L < \epsilon$$

$$|g(x) - L| < \epsilon$$

So  $\lim_{x \rightarrow c} g(x) = L$ , which completes the proof. □

### 1.1.1 Exercises 1.1

### 1.1.2 Exercises 1.2

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## 2 Lectures