

Linear Algebra

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1 Linear Algebra Done Right

1.1 Vector Spaces

1.1.1 Exercises 1.A

1

$$\frac{1}{a + bi} = c + di$$

$$(a + bi)(c + di) = 1$$

$$ac + adi + bci - bd = 1$$

$$(ac - bd) + (ad + bc)i = 1$$

Which leads to

$$ac - bd = 1$$

$$ad + bc = 0$$

Because 1 has no imaginary part.

$$c = \frac{1 + bd}{a}$$

$$d = \frac{-bc}{a}$$

$$c = \frac{1 + b \frac{-bc}{a}}{a}$$

$$c = \frac{1}{a} - \frac{b^2 c}{a^2}$$

$$c + \frac{b^2 c}{a^2} = \frac{1}{a}$$

$$c \left(1 + \frac{b^2}{a^2} \right) = \frac{1}{a}$$

$$c = \frac{1}{a \left(1 + \frac{b^2}{a^2} \right)}$$

$$c = \frac{1}{a + \frac{b^2}{a}}$$

$$c = \frac{1}{\frac{a^2 + b^2}{a}}$$

$$c = \frac{a}{a^2 + b^2}$$

Now that we have c , we can pick out d too.

$$d = \frac{-bc}{a}$$

$$d = \frac{-b \left(\frac{a}{a^2+b^2} \right)}{a}$$

$$d = \frac{\frac{-ab}{a^2+b^2}}{a}$$

$$d = \frac{-b}{a^2+b^2}$$

So now we have d and c as real numbers, since a and b are real.

$$c = \frac{a}{a^2+b^2}$$

$$d = \frac{-b}{a^2+b^2}$$

.....

2

$$\frac{-1 + \sqrt{3}i}{2} =$$

$$\frac{-1}{2} + \frac{\sqrt{3}}{2}i =$$

$$\cos \frac{2\pi}{3} + i \sin \frac{2\pi}{3} =$$

$$e^{i\frac{2\pi}{3}}$$

Which means that

$$\left(e^{i\frac{2\pi}{3}}\right)^3 =$$

$$e^{i\frac{6\pi}{3}} =$$

$$e^{i2\pi} =$$

$$1$$

Hence, the cube of $\frac{-1+\sqrt{3}i}{2}$ is equal to 1, which was to be shown.

1.1.2 Exercises 1.B

Problem 1

$$-(-v) = v$$

Add the additive inverse of $-(-v)$ to both sides

$$0 = v + (-v)$$

The additive inverse of v added together with v is the zero vector.

$$0 = 0$$

So the statement is true, which was to be proved.

Problem 2

$$av = 0$$

Suppose that $a = 0$

2 Lectures

2.1 Lecture 1: Area of the Sphere

To calculate the area of the sphere, we begin with:

$$x^2 + y^2 = r^2 \quad (1)$$

$$y = \sqrt{r^2 - x^2} \quad (2)$$

Then use an integral to describe the length of the half circle:

$$\int_{-r}^r \sqrt{1 - \left(\frac{dy}{dx}\right)^2} dx \quad (3)$$

Now apply a revolution to that:

$$\pi \int_{-r}^r \left(1 - \left(\frac{dy}{dx}\right)^2\right) dx \quad (4)$$

Since $\frac{dy}{dx} = \frac{1}{2} (r^2 - x^2)^{-\frac{1}{2}} * (-2x) = -\frac{x}{\sqrt{r^2 - x^2}}$ Then

$$\pi \int_{-r}^r \left(1 - \left(-\frac{x}{\sqrt{r^2 - x^2}}\right)^2\right) dx \quad (5)$$

$$(6)$$

Which then is:

$$\pi \int_{-r}^r \left(1 - \left(\frac{x^2}{r^2 - x^2} \right) \right) dx = \quad (7)$$

$$\pi \int_{-r}^r \left(1 - \left(\frac{x^2}{r^2 - x^2} \right) \right) dx = \quad (8)$$

$$\pi \int_{-r}^r \left(1 - \left(\frac{x^2}{r^2 - x^2} \right) \right) dx = \quad (9)$$

2.2 Lecture 2: Title of This Other Lecture

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