A Subject

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Contents

1 Calculus A Complete Course

1.1 Limits and Continuity

Theorem 1 (The Squeeze Theorem). *Suppose that $f(x) \leq g(x) \leq h(x)$ holds for all x in some open interval containing c, except possibly at $x = c$. Suppose also that*

$$\lim_{x \to c} f(x) = \lim_{x \to c} h(x) = L$$

Then $\lim_{x \to c} g(x) = L$.

Proof. For this proof, the (ϵ, δ)-definition of the limit will be used.

The goal is to prove that $\lim_{x \to c} g(x) = L$, which is true if

$$\forall \epsilon > 0, \exists \delta > 0 : \forall x, (|x - a| < \delta \Rightarrow |g(x) - L| < \epsilon).$$

Since $\lim_{x \to c} f(x) = L$,

$$\forall \epsilon > 0, \exists \delta_1 > 0 : \forall x, (|x - a| < \delta_1 \Rightarrow |f(x) - L| < \epsilon) \tag{1}$$

And since $\lim_{x \to c} h(x) = L$,

$$\forall \epsilon > 0, \exists \delta_2 > 0 : \forall x, (|x - a| < \delta_2 \Rightarrow |h(x) - L| < \epsilon). \tag{2}$$

Then we have

$$f(x) \leq g(x) \leq h(x)$$

$$f(x) - L \leq g(x) - L \leq h(x) - L$$

We can choose $\delta = \min\{\delta_1, \delta_2\}$, then if $|x - a| < \delta$, and combining (1) and (2), we have

$$
\begin{aligned}
&-\epsilon < f(x) - L \leq g(x) - L \leq h(x) - L < \epsilon \\
&-\epsilon < g(x) - L < \epsilon \\
&|g(x) - L| < \epsilon
\end{aligned}
$$

So $\lim_{x \to c} g(x) = L$, which completes the proof. $\square$

1.1.1 Exercises 1.1

1.1.2 Exercises 1.2

2 Lectures

2.1 Lecture 1: Area of the Sphere

To calculate the area of the sphere, we begin with:

$$x^2 + y^2 = r^2 \tag{3}$$

$$y = \sqrt{r^2 - x^2} \tag{4}$$

Then use an integral to describe the length of the half circle:

$$\int_{-r}^{r} \sqrt{1 - (\frac{dy}{dx})^2} dx \tag{5}$$

Now apply a revolution to that:

$$\pi \int_{-r}^{r} \left(1 - \left(\frac{dy}{dx}\right)^2\right) dx \tag{6}$$

Since $\frac{dy}{dx} = \frac{1}{2}\left(r^2 - x^2\right)^{-\frac{1}{2}} * (-2x) = -\frac{x}{\sqrt{r^2 - x^2}}$ Then

$$\pi \int_{-r}^{r} \left(1 - \left(-\frac{x}{\sqrt{r^2 - x^2}}\right)^2\right) dx \tag{7}$$

$$\tag{8}$$

Which then is:

$$\pi \int_{-r}^{r} \left(1 - \left(\frac{x^2}{r^2 - x^2}\right)\right) dx = \tag{9}$$

$$\pi \int_{-r}^{r} \left(1 - \left(\frac{x^2}{r^2 - x^2}\right)\right) dx = \tag{10}$$

$$\pi \int_{-r}^{r} \left(1 - \left(\frac{x^2}{r^2 - x^2}\right)\right) dx = \tag{11}$$

2.2 Lecture 2: Title of This Other Lecture

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