Multi-agent learning

Equilibria

Gerard Vreeswijk, Intelligent Software Systems, Computer Science Department, Faculty of Sciences, Utrecht University, The Netherlands.

Tuesday 25th May, 2021

Equilibria: motivation

Equilibria: motivation

- The concept of Nash equilibrium is important in game theory.

Equilibria: motivation

- The concept of Nash equilibrium is important in game theory.
- However, if players form strategies through learning, their strategies generally do not converge to Nash equilibria.

Equilibria: motivation

- The concept of Nash equilibrium is important in game theory.
- However, if players form strategies through learning, their strategies generally do not converge to Nash equilibria.
- The concept of Nash equilibrium neglects coordination and common history in learning.

Equilibria: motivation

- The concept of Nash equilibrium is important in game theory.
- However, if players form strategies through learning, their strategies generally do not converge to Nash equilibria.
- The concept of Nash equilibrium neglects coordination and common history in learning.
- If players learn, their strategies generally do converge to other, more general, types of equilibria

Equilibria: motivation

- The concept of Nash equilibrium is important in game theory.
- However, if players form strategies through learning, their strategies generally do not converge to Nash equilibria.
- The concept of Nash equilibrium neglects coordination and common history in learning.
- If players learn, their strategies generally do converge to other, more general, types of equilibria, such as correlated equilibria

Equilibria: motivation

- The concept of Nash equilibrium is important in game theory.
- However, if players form strategies through learning, their strategies generally do not converge to Nash equilibria.
- The concept of Nash equilibrium neglects coordination and common history in learning.
- If players learn, their strategies generally do converge to other, more general, types of equilibria, such as correlated equilibria and coarse correlated equilibria.

Plan for today

Plan for today

1. **Some preparation**

Plan for today

1. **Some preparation**
    - Rehearse terminology.

Plan for today

1. **Some preparation**
    - Rehearse terminology.
    - Redefine Nash equilibrium.

Plan for today

1. **Some preparation**
    - Rehearse terminology.
    - Redefine Nash equilibrium.
    - Probability distributions over the strategy space.

Plan for today

1. **Some preparation**
   - Rehearse terminology.
   - Redefine Nash equilibrium.
   - Probability distributions over the strategy space.
2. **Correlated equilibrium**

Plan for today

1. **Some preparation**
   - Rehearse terminology.
   - Redefine Nash equilibrium.
   - Probability distributions over the strategy space.
2. **Correlated equilibrium**
   - Intuition.

Plan for today

1. **Some preparation**
   - Rehearse terminology.
   - Redefine Nash equilibrium.
   - Probability distributions over the strategy space.
2. **Correlated equilibrium**
   - Intuition.
   - Definition.

Plan for today

1. **Some preparation**
    - Rehearse terminology.
    - Redefine Nash equilibrium.
    - Probability distributions over the strategy space.
2. **Correlated equilibrium**
    - Intuition.
    - Definition.
    - Examples.

Plan for today

1. **Some preparation**
    - Rehearse terminology.
    - Redefine Nash equilibrium.
    - Probability distributions over the strategy space.
2. **Correlated equilibrium**
    - Intuition.
    - Definition.
    - Examples. (Many.)

Plan for today

1. **Some preparation**
    - Rehearse terminology.
    - Redefine Nash equilibrium.
    - Probability distributions over the strategy space.
2. **Correlated equilibrium**
    - Intuition.
    - Definition.
    - Examples. (Many.)
3. **Hierarchy of equilibria**:

Plan for today

1. **Some preparation**
    - Rehearse terminology.
    - Redefine Nash equilibrium.
    - Probability distributions over the strategy space.
2. **Correlated equilibrium**
    - Intuition.
    - Definition.
    - Examples. (Many.)
3. **Hierarchy of equilibria**: NE $\Rightarrow$ CE $\Rightarrow$ CCE.

Plan for today

1. **Some preparation**
   - Rehearse terminology.
   - Redefine Nash equilibrium.
   - Probability distributions over the strategy space.
2. **Correlated equilibrium**
   - Intuition.
   - Definition.
   - Examples. (Many.)
3. **Hierarchy of equilibria**: NE $\Rightarrow$ CE $\Rightarrow$ CCE.
4. **Summary**

Recap of notation

Notation

Notation

- Players are denoted by numbers: $I = \{1, \ldots, n\}$.

Notation

- Players are denoted by numbers: $I = \{1, \ldots, n\}$.
- The set of actions available to player i is denoted by X_i.

Notation

- Players are denoted by numbers: $I = \{1, \ldots, n\}$.
- The set of actions available to player i is denoted by X_i.

  Example: $X_1 = \{\text{left}, \text{right}, \text{up}, \text{down}\}$.

Notation

- Players are denoted by numbers: $I = \{1, \ldots, n\}$.
- The set of actions available to player i is denoted by X_i.

  Example: $X_1 = \{\text{left}, \text{right}, \text{up}, \text{down}\}$.
- $X = X_1 \times \cdots \times X_n$ is the set of all action profiles.

Notation

- Players are denoted by numbers: $I = \{1, \ldots, n\}$.
- The set of actions available to player i is denoted by X_i.

  Example: $X_1 = \{\text{left}, \text{right}, \text{up}, \text{down}\}$.
- $X = X_1 \times \cdots \times X_n$ is the set of all action profiles. (Typical: $x, x', \ldots$.)

Notation

- Players are denoted by numbers: $I = \{1, \dots, n\}$.
- The set of actions available to player i is denoted by X_i.

  Example: $X_1 = \{\text{left}, \text{right}, \text{up}, \text{down}\}$.
- $X = X_1 \times \dots \times X_n$ is the set of all action profiles. (Typical: $x, x', \dots$.)
- $X_{-i} = X_1 \times \dots \times X_{i-1} \times X_{i+1} \times \dots \times X_n$ is the set of all counter-action profiles.

Notation

- Players are denoted by numbers: $I = \{1, \dots, n\}$.
- The set of actions available to player i is denoted by X_i.

  Example: $X_1 = \{\text{left}, \text{right}, \text{up}, \text{down}\}$.
- $X = X_1 \times \cdots \times X_n$ is the set of all action profiles. (Typical: $x, x', \dots$.)
- $X_{-i} = X_1 \times \cdots \times X_{i-1} \times X_{i+1} \times \cdots \times X_n$ is the set of all counter-action profiles. (Typical elements: $x_{-i}, \dots$.)

Notation

- Players are denoted by numbers: $I = \{1, \ldots, n\}$.
- The set of actions available to player i is denoted by X_i.

  Example: $X_1 = \{\mathrm{left}, \mathrm{right}, \mathrm{up}, \mathrm{down}\}$.
- $X = X_1 \times \cdots \times X_n$ is the set of all action profiles. (Typical: $x, x', \ldots$.)
- $X_{-i} = X_1 \times \cdots \times X_{i-1} \times X_{i+1} \times \cdots \times X_n$ is the set of all counter-action profiles. (Typical elements: $x_{-i}, \ldots$.)
- $u_i : X \to \mathbb{R}$ is the utility function of player i.

Notation

- Players are denoted by numbers: $I = \{1, \ldots, n\}$.
- The set of actions available to player i is denoted by X_i.

  Example: $X_1 = \{\text{left}, \text{right}, \text{up}, \text{down}\}$.
- $X = X_1 \times \cdots \times X_n$ is the set of all action profiles. (Typical: $x, x', \ldots$.)
- $X_{-i} = X_1 \times \cdots \times X_{i-1} \times X_{i+1} \times \cdots \times X_n$ is the set of all counter-action profiles. (Typical elements: $x_{-i}, \ldots$.)
- $u_i : X \to \mathbb{R}$ is the utility function of player i.
- $S_i = \Delta(X_i)$ is the set of all strategies available to player i.

Notation

- Players are denoted by numbers: $I = \{1, \dots, n\}$.
- The set of actions available to player i is denoted by X_i.

  Example: $X_1 = \{\text{left}, \text{right}, \text{up}, \text{down}\}$.
- $X = X_1 \times \cdots \times X_n$ is the set of all action profiles. (Typical: $x, x', \dots$.)
- $X_{-i} = X_1 \times \cdots \times X_{i-1} \times X_{i+1} \times \cdots \times X_n$ is the set of all counter-action profiles. (Typical elements: $x_{-i}, \dots$.)
- $u_i : X \to \mathbb{R}$ is the utility function of player i.
- $S_i = \Delta(X_i)$ is the set of all strategies available to player i. (Typical elements: $s_i, \dots$.)

Notation

- Players are denoted by numbers: $I = \{1, \dots, n\}$.
- The set of actions available to player i is denoted by X_i.

  Example: $X_1 = \{\text{left}, \text{right}, \text{up}, \text{down}\}$.
- $X = X_1 \times \cdots \times X_n$ is the set of all action profiles. (Typical: $x, x', \dots$.)
- $X_{-i} = X_1 \times \cdots \times X_{i-1} \times X_{i+1} \times \cdots \times X_n$ is the set of all counter-action profiles. (Typical elements: $x_{-i}, \dots$.)
- $u_i : X \to \mathbb{R}$ is the utility function of player i.
- $S_i = \Delta(X_i)$ is the set of all strategies available to player i. (Typical elements: $s_i, \dots$.)
- $S = S_1 \times \cdots \times S_n$ is the set of all possible strategy profiles.

Notation

- Players are denoted by numbers: $I = \{1, \dots, n\}$.
- The set of actions available to player i is denoted by X_i.

  Example: $X_1 = \{\text{left}, \text{right}, \text{up}, \text{down}\}$.
- $X = X_1 \times \cdots \times X_n$ is the set of all action profiles. (Typical: $x, x', \dots$.)
- $X_{-i} = X_1 \times \cdots \times X_{i-1} \times X_{i+1} \times \cdots \times X_n$ is the set of all counter-action profiles. (Typical elements: $x_{-i}, \dots$.)
- $u_i : X \to \mathbb{R}$ is the utility function of player i.
- $S_i = \Delta(X_i)$ is the set of all strategies available to player i. (Typical elements: $s_i, \dots$.)
- $S = S_1 \times \cdots \times S_n$ is the set of all possible strategy profiles.
- Profile s is sometimes written as $s = (s_i, s_{-i})$, where s_{-i} is s_i's counter-strategy profile.

Notation

Notation

- Define $s(x)$ as the probability that *action profile* x is played when players follow *strategy profile* s

Notation

- Define $s(x)$ as the probability that *action profile* x is played when players follow *strategy profile* s:

$$s(x) =_{Def} s_1(x_1) \times \cdots \times s_n(x_n).$$

Notation

- Define $s(x)$ as the probability that *action profile* x is played when players follow *strategy profile* s:

$$s(x) =_{Def} s_1(x_1) \times \cdots \times s_n(x_n).$$

- Define $u_i(s)$ as player i's utility when players follow strategy profile s

Notation

- Define $s(x)$ as the probability that *action profile* x is played when players follow *strategy profile* s:

$$s(x) =_{Def} s_1(x_1) \times \cdots \times s_n(x_n).$$

- Define $u_i(s)$ as player i's utility when players follow strategy profile s:

$$u_i(s) =_{Def} \sum_{x \in X} s(x) u_i(x).$$

Notation

- Define $s(x)$ as the probability that *action profile* x is played when players follow *strategy profile* s:

$$s(x) =_{Def} s_1(x_1) \times \cdots \times s_n(x_n).$$

- Define $u_i(s)$ as player i's utility when players follow strategy profile s:

$$u_i(s) =_{Def} \sum_{x \in X} s(x) u_i(x).$$

- Summary:

Notation

- Define $s(x)$ as the probability that *action profile* x is played when players follow *strategy profile* s:

$$s(x) =_{Def} s_1(x_1) \times \cdots \times s_n(x_n).$$

- Define $u_i(s)$ as player i's utility when players follow strategy profile s:

$$u_i(s) =_{Def} \sum_{x \in X} s(x) u_i(x).$$

- Summary: the expected utility u of a strategy profile s for player i can be expressed as:

$$u_i : S \to \mathbb{R} :$$

Notation

- Define $s(x)$ as the probability that *action profile* x is played when players follow *strategy profile* s:

$$s(x) =_{Def} s_1(x_1) \times \cdots \times s_n(x_n).$$

- Define $u_i(s)$ as player i's utility when players follow strategy profile s:

$$u_i(s) =_{Def} \sum_{x \in X} s(x) u_i(x).$$

- Summary: the expected utility u of a strategy profile s for player i can be expressed as:

$$u_i : S \to \mathbb{R} : s \ \mapsto \textstyle\sum_x \left[\, s_1(x_1) \times \cdots \times s_n(x_n) \right] u_i(x)$$

Notation

- Define $s(x)$ as the probability that *action profile* x is played when players follow *strategy profile* s:

$$s(x) =_{Def} s_1(x_1) \times \cdots \times s_n(x_n).$$

- Define $u_i(s)$ as player i's utility when players follow strategy profile s:

$$u_i(s) =_{Def} \sum_{x \in X} s(x) u_i(x).$$

- Summary: the expected utility u of a strategy profile s for player i can be expressed as:

$$u_i : S \to \mathbb{R} : s \mapsto \underset{\sum_x}{\sum_x} \underset{s(x)}{\left[s_1(x_1) \times \cdots \times s_n(x_n) \right]} \underset{u_i(x)}{u_i(x)}$$

Notation

- Define $s(x)$ as the probability that *action profile* x is played when players follow *strategy profile* s:

$$s(x) =_{Def} s_1(x_1) \times \cdots \times s_n(x_n).$$

- Define $u_i(s)$ as player i's utility when players follow strategy profile s:

$$u_i(s) =_{Def} \sum_{x \in X} s(x) u_i(x).$$

- Summary: the expected utility u of a strategy profile s for player i can be expressed as:

$$\begin{array}{lllll} u_i : S \to \mathbb{R} : s & \mapsto & \sum_x \big[& s_1(x_1) \times \cdots \times s_n(x_n) \big] & u_i(x) \\ & & \sum_x & s(x) & u_i(x) \\ & & u_i(s) & & \end{array}$$

Example

Battle of the sexes:

	L (0.2)	R (0.8)
U (0.6)	$(2, 1)$	$(0, 0)$
D (0.4)	$(0, 0)$	$(2, 1)$

Example

Battle of the sexes:

	L (0.2)	R (0.8)
U (0.6)	$(2,1)$	$(0,0)$
D (0.4)	$(0,0)$	$(2,1)$

Then:

Example

Battle of the sexes:

	L (0.2)	R (0.8)
U (0.6)	$(2,1)$	$(0,0)$
D (0.4)	$(0,0)$	$(2,1)$

Then:

- All action profiles: $X = \{(U,L),(U,R),(D,L),(D,R)\}$.

Example

Battle of the sexes:

	L (0.2)	R (0.8)
U (0.6)	$(2, 1)$	$(0, 0)$
D (0.4)	$(0, 0)$	$(2, 1)$

Then:

- All action profiles: $X = \{(U, L), (U, R), (D, L), (D, R)\}$.
- The current strategy profile: $s = ((0.6, 0.4), (0.2, 0.8)) \in S$.

Example

Battle of the sexes:

	L (0.2)	R (0.8)
U (0.6)	(2, 1)	(0, 0)
D (0.4)	(0, 0)	(2, 1)

Then:

- All action profiles: $X = \{(U, L), (U, R), (D, L), (D, R)\}$.
- The current strategy profile: $s = ((0.6, 0.4), (0.2, 0.8)) \in S$.
-

Example

Battle of the sexes:

	L (0.2)	R (0.8)
U (0.6)	$(2, 1)$	$(0, 0)$
D (0.4)	$(0, 0)$	$(2, 1)$

Then:

- All action profiles: $X = \{(U, L), (U, R), (D, L), (D, R)\}$.
- The current strategy profile: $s = ((0.6, 0.4), (0.2, 0.8)) \in S$.
- $$u_1(s) = \sum_x [s_1(x_1)s_2(x_2)] u_1(x)$$

Example

Battle of the sexes:

	L (0.2)	R (0.8)
U (0.6)	(2, 1)	(0, 0)
D (0.4)	(0, 0)	(2, 1)

Then:

- All action profiles: $X = \{(U, L), (U, R), (D, L), (D, R)\}$.
- The current strategy profile: $s = ((0.6, 0.4), (0.2, 0.8)) \in S$.
- $$\begin{aligned} u_1(s) &= \sum_x \left[s_1(x_1) s_2(x_2) \right] u_1(x) \\ &= s_1(U) s_2(L) u_1(U, L) \end{aligned}$$

Example

Battle of the sexes:

	L (0.2)	R (0.8)
U (0.6)	(2, 1)	(0, 0)
D (0.4)	(0, 0)	(2, 1)

Then:

- All action profiles: $X = \{(U, L), (U, R), (D, L), (D, R)\}$.
- The current strategy profile: $s = ((0.6, 0.4), (0.2, 0.8)) \in S$.
- $$\begin{aligned} u_1(s) &= \sum_x \left[s_1(x_1) s_2(x_2) \right] u_1(x) \\ &= s_1(U)s_2(L)u_1(U, L) + s_1(U)s_2(R)u_1(U, R) \\ &\quad + \ s_1(D)s_2(L)u_1(D, L) + s_1(D)s_2(R)u_1(D, R) \end{aligned}$$

Example

Battle of the sexes:

	L (0.2)	R (0.8)
U (0.6)	$(2, 1)$	$(0, 0)$
D (0.4)	$(0, 0)$	$(2, 1)$

Then:

- All action profiles: $X = \{(U, L), (U, R), (D, L), (D, R)\}$.
- The current strategy profile: $s = ((0.6, 0.4), (0.2, 0.8)) \in S$.
-
$$\begin{aligned} u_1(s) &= \sum_x \left[s_1(x_1) s_2(x_2) \right] u_1(x) \\ &= s_1(U)s_2(L)u_1(U,L) + s_1(U)s_2(R)u_1(U,R) \\ &\quad + \; s_1(D)s_2(L)u_1(D,L) + s_1(D)s_2(R)u_1(D,R) \\ &= 0.6 \times 0.2 \times 2 + \cancel{0.6 \times 0.8 \times 0} + \cancel{0.4 \times 0.2 \times 0} + 0.4 \times 0.8 \times 2 \end{aligned}$$

Example

Battle of the sexes:

	L (0.2)	R (0.8)
U (0.6)	(2, 1)	(0, 0)
D (0.4)	(0, 0)	(2, 1)

Then:

- All action profiles: $X = \{(U, L), (U, R), (D, L), (D, R)\}$.
- The current strategy profile: $s = ((0.6, 0.4), (0.2, 0.8)) \in S$.
- $$\begin{aligned} u_1(s) &= \sum_x \left[s_1(x_1) s_2(x_2) \right] u_1(x) \\ &= s_1(U)s_2(L)u_1(U, L) + s_1(U)s_2(R)u_1(U, R) \\ &\quad + s_1(D)s_2(L)u_1(D, L) + s_1(D)s_2(R)u_1(D, R) \\ &= 0.6 \times 0.2 \times 2 + \cancel{0.6 \times 0.8 \times 0} + \cancel{0.4 \times 0.2 \times 0} + 0.4 \times 0.8 \times 2 \\ &= 0.88. \end{aligned}$$

Nash equilibria defined in terms of pure strategies

Best response

Definition (Best response). Strategy s_i is said to be a best response to the counterprofile s_{-i} if

$$\text{for all } s_i' \in S_i : u_i(s_i', s_{-i}) \leq u_i(s_i, s_{-i}).$$

Best response

Definition (Best response). Strategy s_i is said to be a best response to the counterprofile s_{-i} if

$$\text{for all } s_i' \in S_i : u_i(s_i', s_{-i}) \leq u_i(s_i, s_{-i}).$$

A best response is not necessarily unique.

Best response

Definition (Best response). Strategy s_i is said to be a best response to the counterprofile s_{-i} if

$$\text{for all } s_i' \in S_i : u_i(s_i', s_{-i}) \leq u_i(s_i, s_{-i}).$$

A best response is not necessarily unique. Let $B(s_{-i})$ be the set of best responses to s_{-i}.

Best response

Definition (Best response). Strategy s_i is said to be a best response to the counterprofile s_{-i} if

$$\text{for all } s_i' \in S_i : u_i(s_i', s_{-i}) \leq u_i(s_i, s_{-i}).$$

A best response is not necessarily unique. Let $B(s_{-i})$ be the set of best responses to s_{-i}.

- If two or more pure actions are best responses, then any mix of them also is a best response.

Best response

Definition (Best response). Strategy s_i is said to be a best response to the counterprofile s_{-i} if

$$\text{for all } s_i' \in S_i : u_i(s_i', s_{-i}) \leq u_i(s_i, s_{-i}).$$

A best response is not necessarily unique. Let $B(s_{-i})$ be the set of best responses to s_{-i}.

- If two or more pure actions are best responses, then any mix of them also is a best response.
- When the support (or carrier) of a best response includes two or more actions, the agent must be indifferent among them.

Best response

Definition (Best response). Strategy s_i is said to be a best response to the counterprofile s_{-i} if

$$\text{for all } s_i' \in S_i : u_i(s_i', s_{-i}) \leq u_i(s_i, s_{-i}).$$

A best response is not necessarily unique. Let $B(s_{-i})$ be the set of best responses to s_{-i}.

- If two or more pure actions are best responses, then any mix of them also is a best response.
- When the support (or carrier) of a best response includes two or more actions, the agent must be indifferent among them. (If not, then put all weight on the best action.)

Best response

Definition (Best response). Strategy s_i is said to be a best response to the counterprofile s_{-i} if

$$\text{for all } s_i' \in S_i : u_i(s_i', s_{-i}) \leq u_i(s_i, s_{-i}).$$

A best response is not necessarily unique. Let $B(s_{-i})$ be the set of best responses to s_{-i}.

- If two or more pure actions are best responses, then any mix of them also is a best response.
- When the support (or carrier) of a best response includes two or more actions, the agent must be indifferent among them. (If not, then put all weight on the best action.)
- Therefore, any mix of these actions must also be a best response.

Expected utility of an alternative action or strategy

Expected utility of an alternative action or strategy

- Recall that the expected utility u of a strategy profile s for player i, denoted by $u_i(s)$, is

$$u_i(s) = \sum_x \qquad s(x) \qquad\qquad u_i(x)$$

Expected utility of an alternative action or strategy

- Recall that the expected utility u of a strategy profile s for player i, denoted by $u_i(s)$, is

$$\begin{aligned} u_i(s) &= \textstyle\sum_x \quad s(x) \quad u_i(x) \\ &= \textstyle\sum_{x_i, x_{-i}} \quad s(x_i, x_{-i}) \quad u_i(x_i, x_{-i}) \end{aligned}$$

Expected utility of an alternative action or strategy

- Recall that the expected utility u of a strategy profile s for player i, denoted by $u_i(s)$, is

$$\begin{aligned} u_i(s) &= \textstyle\sum_x && s(x) && u_i(x) \\ &= \textstyle\sum_{x_i, x_{-i}} && s(x_i, x_{-i}) && u_i(x_i, x_{-i}) \\ &= \textstyle\sum_{x_i, x_{-i}} && s_i(x_i)s_{-i}(x_{-i}) && u_i(x_i, x_{-i}) \end{aligned}$$

Expected utility of an alternative action or strategy

- Recall that the expected utility u of a strategy profile s for player i, denoted by $u_i(s)$, is

$$\begin{aligned} u_i(s) &= \textstyle\sum_x s(x) u_i(x) \\ &= \textstyle\sum_{x_i, x_{-i}} s(x_i, x_{-i}) u_i(x_i, x_{-i}) \\ &= \textstyle\sum_{x_i, x_{-i}} s_i(x_i) s_{-i}(x_{-i}) u_i(x_i, x_{-i}) \\ &= \textstyle\sum_{x_i} \sum_{x_{-i}} s_i(x_i) s_{-i}(x_{-i}) u_i(x_i, x_{-i}) \end{aligned}$$

Expected utility of an alternative action or strategy

- Recall that the expected utility u of a strategy profile s for player i, denoted by $u_i(s)$, is

$$\begin{aligned} u_i(s) &= \textstyle\sum_x s(x) u_i(x) \\ &= \textstyle\sum_{x_i, x_{-i}} s(x_i, x_{-i}) u_i(x_i, x_{-i}) \\ &= \textstyle\sum_{x_i, x_{-i}} s_i(x_i) s_{-i}(x_{-i}) u_i(x_i, x_{-i}) \\ &= \textstyle\sum_{x_i} \sum_{x_{-i}} s_i(x_i) s_{-i}(x_{-i}) u_i(x_i, x_{-i}) \\ &= \textstyle\sum_{x_{-i}} \sum_{x_i} s_i(x_i) s_{-i}(x_{-i}) u_i(x_i, x_{-i}) \end{aligned}$$

Expected utility of an alternative action or strategy

- Recall that the expected utility u of a strategy profile s for player i, denoted by $u_i(s)$, is

$$\begin{aligned} u_i(s) &= \sum_x s(x) u_i(x) \\ &= \sum_{x_i, x_{-i}} s(x_i, x_{-i}) u_i(x_i, x_{-i}) \\ &= \sum_{x_i, x_{-i}} s_i(x_i) s_{-i}(x_{-i}) u_i(x_i, x_{-i}) \\ &= \sum_{x_i} \sum_{x_{-i}} s_i(x_i) s_{-i}(x_{-i}) u_i(x_i, x_{-i}) \\ &= \sum_{x_{-i}} \sum_{x_i} s_i(x_i) s_{-i}(x_{-i}) u_i(x_i, x_{-i}) \end{aligned}$$

- With alternative action x_i':

$$u_i(x_i', s_{-i}) = \sum_{x_{-i}} s_{-i}(x_{-i}) u_i(x_i', x_{-i}).$$

Expected utility of an alternative action or strategy

- Recall that the expected utility u of a strategy profile s for player i, denoted by $u_i(s)$, is

$$\begin{aligned} u_i(s) &= \textstyle\sum_x s(x) u_i(x) \\ &= \textstyle\sum_{x_i, x_{-i}} s(x_i, x_{-i}) u_i(x_i, x_{-i}) \\ &= \textstyle\sum_{x_i, x_{-i}} s_i(x_i) s_{-i}(x_{-i}) u_i(x_i, x_{-i}) \\ &= \textstyle\sum_{x_i} \sum_{x_{-i}} s_i(x_i) s_{-i}(x_{-i}) u_i(x_i, x_{-i}) \\ &= \textstyle\sum_{x_{-i}} \sum_{x_i} s_i(x_i) s_{-i}(x_{-i}) u_i(x_i, x_{-i}) \end{aligned}$$

- With alternative action x'_i:

$$u_i(x'_i, s_{-i}) = \textstyle\sum_{x_{-i}} s_{-i}(x_{-i}) u_i(x'_i, x_{-i}).$$

- With alternative strategy s'_i:

$$u_i(s'_i, s_{-i}) = \textstyle\sum_{x_i, x_{-i}} s'_i(x_i) s_{-i}(x_{-i}) u_i(x_i, x_{-i}).$$

Nash equilibrium

All i maintain some strategy s_i. The strategy profile s is a Nash equilibrium if no one can profit by changing s_i unilaterally.

Nash equilibrium

All i maintain some strategy s_i. The strategy profile s is a Nash equilibrium if no one can profit by changing s_i unilaterally.

Definition (Nash equilibrium). A strategy profile s is said to be a Nash equilibrium if all strategies in it are best responses:

$$\text{for all } i : s_i \in B(s_{-i}).$$

Nash equilibrium

All i maintain some strategy s_i. The strategy profile s is a Nash equilibrium if no one can profit by changing s_i unilaterally.

Definition (Nash equilibrium). A strategy profile s is said to be a Nash equilibrium if all strategies in it are best responses:

$$\text{for all } i : s_i \in B(s_{-i}).$$

The "pure action way" to define a NE:

Nash equilibrium

All i maintain some strategy s_i. The strategy profile s is a Nash equilibrium if no one can profit by changing s_i unilaterally.

Definition (Nash equilibrium). A strategy profile s is said to be a Nash equilibrium if all strategies in it are best responses:

$$\text{for all } i : s_i \in B(s_{-i}).$$

The "pure action way" to define a NE: No alternative action $x_i' \in X_i$ can do better than any pure best response $x_i \in X_i$

Nash equilibrium

All i maintain some strategy s_i. The strategy profile s is a Nash equilibrium if no one can profit by changing s_i unilaterally.

Definition (Nash equilibrium). A strategy profile s is said to be a Nash equilibrium if all strategies in it are best responses:

$$\text{for all } i : s_i \in B(s_{-i}).$$

The "pure action way" to define a NE: No alternative action $x_i' \in X_i$ can do better than any pure best response $x_i \in X_i$:

For all players i, pure best responses $x_i \in X_i \cap B(s_{-i})$ and alternative $x_i' \in X_i$:

Nash equilibrium

All i maintain some strategy s_i. The strategy profile s is a Nash equilibrium if no one can profit by changing s_i unilaterally.

Definition (Nash equilibrium). A strategy profile s is said to be a Nash equilibrium if all strategies in it are best responses:

$$\text{for all } i : s_i \in B(s_{-i}).$$

The "pure action way" to define a NE: No alternative action $x_i' \in X_i$ can do better than any pure best response $x_i \in X_i$:

For all players i, pure best responses $x_i \in X_i \cap B(s_{-i})$ and alternative $x_i' \in X_i$:

$$\sum_{x_{-i}} s_{-i}(x_{-i}) u_i(x_i', x_{-i}) \leq \sum_{x_{-i}} s_{-i}(x_{-i}) u_i(x_i, x_{-i}).$$

Probability distributions over the strategy space

Strategies ⇒ strategy profile ⇒ joint distribution

Strategies ⇒ strategy profile ⇒ joint distribution

- Suppose n players, strategies $s_1, \dots, s_n$ are given:

$$
\begin{array}{cc|cccc}
 & s_{-i} & y_1^{-i} & y_2^{-i} & \dots & y_n^{-i} \\
 & s_i & q_1 & q_2 & \dots & q_n \\
\hline
x_1^i & p_1 & p_1 q_1 & p_1 q_2 & \dots & p_1 q_n \\
x_2^i & p_2 & p_2 q_1 & p_2 q_2 & \dots & p_2 q_n \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
x_m^i & p_m & p_m q_1 & p_m q_2 & \dots & p_m q_n
\end{array}
$$

Strategies ⇒ strategy profile ⇒ joint distribution

- Suppose n players, strategies $s_1, \dots, s_n$ are given:

$$
\begin{array}{cc|cccc}
 & s_{-i} & y_1^{-i} & y_2^{-i} & \dots & y_n^{-i} \\
s_i & & q_1 & q_2 & \dots & q_n \\
\hline
x_1^i & p_1 & p_1 q_1 & p_1 q_2 & \dots & p_1 q_n \\
x_2^i & p_2 & p_2 q_1 & p_2 q_2 & \dots & p_2 q_n \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
x_m^i & p_m & p_m q_1 & p_m q_2 & \dots & p_m q_n
\end{array}
$$

where n is the number of different counter-profiles.

Strategies ⇒ strategy profile ⇒ joint distribution

- Suppose n players, strategies $s_1, \dots, s_n$ are given:

$$
\begin{array}{cc|cccc}
 & s_{-i} & y_1^{-i} & y_2^{-i} & \dots & y_n^{-i} \\
\multicolumn{2}{c|}{s_i} & q_1 & q_2 & \dots & q_n \\
\hline
x_1^i & p_1 & p_1 q_1 & p_1 q_2 & \dots & p_1 q_n \\
x_2^i & p_2 & p_2 q_1 & p_2 q_2 & \dots & p_2 q_n \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
x_m^i & p_m & p_m q_1 & p_m q_2 & \dots & p_m q_n
\end{array}
$$

  where n is the number of different counter-profiles.

- Players act independently.

Strategies ⇒ strategy profile ⇒ joint distribution

- Suppose n players, strategies $s_1, \dots, s_n$ are given:

$$\begin{array}{cc|cccc}
 & s_{-i} & y_1^{-i} & y_2^{-i} & \dots & y_n^{-i} \\
 & s_i & q_1 & q_2 & \dots & q_n \\
\hline
x_1^i & p_1 & p_1q_1 & p_1q_2 & \dots & p_1q_n \\
x_2^i & p_2 & p_2q_1 & p_2q_2 & \dots & p_2q_n \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
x_m^i & p_m & p_mq_1 & p_mq_2 & \dots & p_mq_n
\end{array}$$

  where n is the number of different counter-profiles.

- Players act independently.

- The strategy $s_i = (p_1, \dots, p_m)$ and the counter strategy profile $s_{-i} = (q_1, \dots, q_n)$ together define a product distribution $s \in \Delta(X)$:

$$s(x_1, \dots, x_n) =_{Def} s(x_1) \times \dots \times s(x_n).$$

Joint distribution $\not\Rightarrow$ marginal strategies

Suppose a (possibly non-product) distribution $q \in \Delta(X)$ is given.

$$\begin{array}{cc|cccc}
 & q_{-i} & y_1^{-i} & y_2^{-i} & \cdots & y_n^{-i} \\
 & q_i & q_{11} \cdots q_{m1} & q_{12} \cdots q_{m2} & \cdots & q_{1n} \cdots q_{mn} \\
\hline
x_1^i & q_{11} \cdots q_{1n} & q_{11} & q_{12} & \cdots & q_{1n} \\
x_2^i & q_{21} \cdots q_{2n} & q_{21} & q_{22} & \cdots & q_{2n} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
x_m^i & q_{m1} \cdots q_{mn} & q_{m1} & q_{m2} & \cdots & q_{mn}
\end{array}$$

Joint distribution $\not\Rightarrow$ marginal strategies

Suppose a (possibly non-product) distribution $q \in \Delta(X)$ is given.

$$\begin{array}{cc|ccccc}
 & q_{-i} & y_1^{-i} & y_2^{-i} & \cdots & y_n^{-i} \\
 & q_i & q_{11}\cdots q_{m1} & q_{12}\cdots q_{m2} & \cdots & q_{1n}\cdots q_{mn} \\
\hline
x_1^i & q_{11}\cdots q_{1n} & q_{11} & q_{12} & \cdots & q_{1n} \\
x_2^i & q_{21}\cdots q_{2n} & q_{21} & q_{22} & \cdots & q_{2n} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
x_m^i & q_{m1}\cdots q_{mn} & q_{m1} & q_{m2} & \cdots & q_{mn}
\end{array}$$

- If players follow q, they need not act independently. (Example: off-diagonal is zero.)

Joint distribution $\not\Rightarrow$ marginal strategies

Suppose a (possibly non-product) distribution $q \in \Delta(X)$ is given.

	q_{-i}	y_1^{-i}	y_2^{-i}	$\cdots$	y_n^{-i}
	q_i	$q_{11} \cdots q_{m1}$	$q_{12} \cdots q_{m2}$	$\cdots$	$q_{1n} \cdots q_{mn}$
x_1^i	$q_{11} \cdots q_{1n}$	q_{11}	q_{12}	$\cdots$	q_{1n}
x_2^i	$q_{21} \cdots q_{2n}$	q_{21}	q_{22}	$\cdots$	q_{2n}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
x_m^i	$q_{m1} \cdots q_{mn}$	q_{m1}	q_{m2}	$\cdots$	q_{mn}

- If players follow q, they need not act independently. (Example: off-diagonal is zero.)
- The marginals form strategies: $s_i = q_i$, $s_{-i} = q_{-i}$.

Joint distribution $\not\Rightarrow$ marginal strategies

Suppose a (possibly non-product) distribution $q \in \Delta(X)$ is given.

$$\begin{array}{cc|cccc}
 & q_{-i} & y_1^{-i} & y_2^{-i} & \cdots & y_n^{-i} \\
 & q_i & q_{11}\cdots q_{m1} & q_{12}\cdots q_{m2} & \cdots & q_{1n}\cdots q_{mn} \\
\hline
x_1^i & q_{11}\cdots q_{1n} & q_{11} & q_{12} & \cdots & q_{1n} \\
x_2^i & q_{21}\cdots q_{2n} & q_{21} & q_{22} & \cdots & q_{2n} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
x_m^i & q_{m1}\cdots q_{mn} & q_{m1} & q_{m2} & \cdots & q_{mn}
\end{array}$$

- If players follow q, they need not act independently. (Example: off-diagonal is zero.)
- The marginals form strategies: $s_i = q_i$, $s_{-i} = q_{-i}$.
- But now generally

$$s(x_i, x_{-i}) \neq s(x_i)s(x_{-i}).$$

Joint distribution vs. joint strategy profile

Example. Consider:

	L (0.2)	R (0.8)
U (0.6)		
D (0.4)		

Joint distribution vs. joint strategy profile

Example. Consider:

	L (0.2)	R (0.8)
U (0.6)	0.12	
D (0.4)		

Joint distribution vs. joint strategy profile

Example. Consider:

	L (0.2)	R (0.8)
U (0.6)	0.12	0.48
D (0.4)		

Joint distribution vs. joint strategy profile

Example. Consider:

	L (0.2)	R (0.8)
U (0.6)	0.12	0.48
D (0.4)	0.08	

Joint distribution vs. joint strategy profile

Example. Consider:

	L (0.2)	R (0.8)
U (0.6)	0.12	0.48
D (0.4)	0.08	0.32

Joint distribution vs. joint strategy profile

Example. Consider:

	L (0.2)	R (0.8)
U (0.6)	0.12	0.48
D (0.4)	0.08	0.32

In this case the joint distribution, namely $q = (0.12, 0.48, 0.08, 0.32)$, is induced by marginal distributions $s_1 = (0.6, 0.4)$ and $s_2 = (0.2, 0.8)$: $q = s_1 \times s_2$.

Joint distribution vs. joint strategy profile

Example. Consider:

	L (0.2)	R (0.8)
U (0.6)	0.12	0.48
D (0.4)	0.08	0.32

In this case the joint distribution, namely $q = (0.12, 0.48, 0.08, 0.32)$, is induced by marginal distributions $s_1 = (0.6, 0.4)$ and $s_2 = (0.2, 0.8)$: $q = s_1 \times s_2$.

Contrast this with q':

	L	R
U	0.13	0.47
D	0.07	0.33

Joint distribution vs. joint strategy profile

Example. Consider:

	L (0.2)	R (0.8)
U (0.6)	0.12	0.48
D (0.4)	0.08	0.32

In this case the joint distribution, namely $q = (0.12, 0.48, 0.08, 0.32)$, is induced by marginal distributions $s_1 = (0.6, 0.4)$ and $s_2 = (0.2, 0.8)$: $q = s_1 \times s_2$.

Contrast this with q':

	L (??)	R (??)
U (??)	0.13	0.47
D (??)	0.07	0.33

Joint distribution vs. joint strategy profile

Example. Consider:

	L (0.2)	R (0.8)
U (0.6)	0.12	0.48
D (0.4)	0.08	0.32

In this case the joint distribution, namely $q = (0.12, 0.48, 0.08, 0.32)$, is induced by marginal distributions $s_1 = (0.6, 0.4)$ and $s_2 = (0.2, 0.8)$: $q = s_1 \times s_2$.

Contrast this with q':

	L (??)	R (??)
U (??)	0.13	0.47
D (??)	0.07	0.33

No marginal distributions exist that induce the joint distribution.

Joint distribution vs. joint strategy profile

Example. Consider:

	L (0.2)	R (0.8)
U (0.6)	0.12	0.48
D (0.4)	0.08	0.32

In this case the joint distribution, namely $q = (0.12, 0.48, 0.08, 0.32)$, is induced by marginal distributions $s_1 = (0.6, 0.4)$ and $s_2 = (0.2, 0.8)$: $q = s_1 \times s_2$.

Contrast this with q':

	L $(??)$	R $(??)$
U $(??)$	0.13	0.47
D $(??)$	0.07	0.33

No marginal distributions exist that induce the joint distribution. In particular, s_1 and s_2 don't do it, i.e, $q' \neq s_1 \times s_2$.

Correlated equilibrium

Correlated equilibrium (Intuition)

Chicken game

	Other:	
You:	Dare	Sway
Dare	$(-10, -10)$	$(5, 0)$
Sway	$(0, 5)$	$(-1, -1)$

Correlated equilibrium (Intuition)

Chicken game

	Other:	
You:	Dare	Sway
Dare	$(-10, -10)$	$(5, 0)$
Sway	$(0, 5)$	$(-1, -1)$

Three Nash equilibria:

Correlated equilibrium (Intuition)

Chicken game

	Other:	
You:	Dare	Sway
Dare	$(-10, -10)$	$(5, 0)$
Sway	$(0, 5)$	$(-1, -1)$

Three Nash equilibria:

- $((1, 0), (0, 1))$

Correlated equilibrium (Intuition)

Chicken game

	Other: Dare	Sway
You:		
Dare	$(-10, -10)$	$(5, 0)$
Sway	$(0, 5)$	$(-1, -1)$

Three Nash equilibria:

- $((1, 0), (0, 1))$
- $((0, 1), (1, 0))$

Correlated equilibrium (Intuition)

Chicken game

	Other:	
You:	Dare	Sway
Dare	$(-10, -10)$	$(5, 0)$
Sway	$(0, 5)$	$(-1, -1)$

Three Nash equilibria:

- $((1, 0), (0, 1))$
- $((0, 1), (1, 0))$
- $((3/8, 5/8), (3/8, 5/8))$

Correlated equilibrium (Intuition)

Chicken game

	Other:	
You:	Dare	Sway
Dare	$(-10,-10)$	$(5,0)$
Sway	$(0,5)$	$(-1,-1)$

Three Nash equilibria:

- $((1,0),(0,1))$
- $((0,1),(1,0))$
- $((3/8,5/8),(3/8,5/8))$

Expected payoff $-5/8$ for both in the last equilibrium.

Correlated equilibrium (Intuition)

Chicken game

	Other:	
You:	Dare	Sway
Dare	$(-10, -10)$	$(5, 0)$
Sway	$(0, 5)$	$(-1, -1)$

Three Nash equilibria:

- $((1, 0), (0, 1))$
- $((0, 1), (1, 0))$
- $((3/8, 5/8), (3/8, 5/8))$

Expected payoff $-5/8$ for both in the last equilibrium.

Correlated equilibrium (Idea). Let a probability distribution

$$q : X \rightarrow [0, 1]$$

be given. This q can be seen as a coordinating device.

Correlated equilibrium (Intuition)

Chicken game

You:	*Other:* Dare	Sway
Dare	$(-10,-10)$	$(5,0)$
Sway	$(0,5)$	$(-1,-1)$

Three Nash equilibria:

- $((1,0),(0,1))$
- $((0,1),(1,0))$
- $((3/8,5/8),(3/8,5/8))$

Expected payoff $-5/8$ for both in the last equilibrium.

Correlated equilibrium (Idea). Let a probability distribution

$$q : X \to [0,1]$$

be given. This q can be seen as a coordinating device.

Think of a traffic light:

Correlated equilibrium (Intuition)

Chicken game

You:	*Other:* Dare	Sway
Dare	$(-10, -10)$	$(5, 0)$
Sway	$(0, 5)$	$(-1, -1)$

Three Nash equilibria:

- $((1, 0), (0, 1))$
- $((0, 1), (1, 0))$
- $((3/8, 5/8), (3/8, 5/8))$

Expected payoff $-5/8$ for both in the last equilibrium.

Correlated equilibrium (Idea). Let a probability distribution

$$q : X \to [0, 1]$$

be given. This q can be seen as a coordinating device.

Think of a traffic light:

$q =$

You:	*Other:* Green	Red
Green	0.00	0.55
Red	0.40	0.05

Correlated equilibrium (Intuition)

Chicken game

You:	*Other:* Dare	Sway
Dare	$(-10,-10)$	$(5,0)$
Sway	$(0,5)$	$(-1,-1)$

Three Nash equilibria:

- $((1,0),(0,1))$
- $((0,1),(1,0))$
- $((3/8,5/8),(3/8,5/8))$

Expected payoff $-5/8$ for both in the last equilibrium.

Correlated equilibrium (Idea). Let a probability distribution

$$q : X \to [0,1]$$

be given. This q can be seen as a coordinating device.

Think of a traffic light:

$q =$

You:	*Other:* Green	Red
Green	0.00	0.55
Red	0.40	0.05

Each time, the system is in one of these four states.

Correlated equilibrium (definition)

$$q = \begin{array}{r|cc|} & \multicolumn{2}{c|}{\textit{Other:}} \\ \text{You:} & \text{Green} & \text{Red} \\ \hline \text{Green} & 0.00 & 0.55 \\ \text{Red} & 0.40 & 0.05 \\ \hline \end{array}$$

Correlated equilibrium (definition)

$q =$

	Other:	
You:	Green	Red
Green	0.00	0.55
Red	0.40	0.05

- With joint probability, q, the system is in each of these four states (action profiles) $x \in X$.

Correlated equilibrium (definition)

$q =$

	Other:	
You:	Green	Red
Green	0.00	0.55
Red	0.40	0.05

- With joint probability, q, the system is in each of these four states (action profiles) $x \in X$.
- Players know q.

Correlated equilibrium (definition)

$q =$

	Other:	
You:	Green	Red
Green	0.00	0.55
Red	0.40	0.05

- With joint probability, q, the system is in each of these four states (action profiles) $x \in X$.
- Players know q.
- At each realisation of q, every party i comes to know only its coordinate (i.e., action, Green or Red), x_i, of the system state x.

Correlated equilibrium (definition)

$q =$

	Other:	
You:	Green	Red
Green	0.00	0.55
Red	0.40	0.05

- With joint probability, q, the system is in each of these four states (action profiles) $x \in X$.
- Players know q.
- At each realisation of q, every party i comes to know only its coordinate (i.e., action, Green or Red), x_i, of the system state x.

Definition. A distribution $q \in \Delta(X)$ is called a correlated equilibrium if no party has an incentive to deviate from its own coordinate x_i, assuming that others do not deviate from x_{-i} as well.

Correlated equilibrium (formula)

Idea:

Correlated equilibrium (formula)

Idea:

Suppose $q \in \Delta(X)$ is given.

Correlated equilibrium (formula)

Idea:

Suppose $q \in \Delta(X)$ is given. Suppose everyone knows q.

Correlated equilibrium (formula)

Idea:

Suppose $q \in \Delta(X)$ is given. Suppose everyone knows q. Let x be a realisation of q.

Correlated equilibrium (formula)

Idea:

Suppose $q \in \Delta(X)$ is given. Suppose everyone knows q. Let x be a realisation of q. Inform everyone i about x_i, but not about x_{-i}.

Correlated equilibrium (formula)

Idea:

Suppose $q \in \Delta(X)$ is given. Suppose everyone knows q. Let x be a realisation of q. Inform everyone i about x_i, but not about x_{-i}.

Now, in a CE, no one wants to change:

Correlated equilibrium (formula)

Idea:

Suppose $q \in \Delta(X)$ is given. Suppose everyone knows q. Let x be a realisation of q. Inform everyone i about x_i, but not about x_{-i}.

Now, in a CE, no one wants to change:

For all i, x_i and x'_i:

Correlated equilibrium (formula)

Idea:

Suppose $q \in \Delta(X)$ is given. Suppose everyone knows q. Let x be a realisation of q. Inform everyone i about x_i, but not about x_{-i}.

Now, in a CE, no one wants to change:

For all i, x_i and x'_i:

$$\sum_{x_{-i}} q(x_{-i}|x_i)u_i(x'_i, x_{-i}) \leq \sum_{x_{-i}} q(x_{-i}|x_i)u_i(x_i, x_{-i}).$$

Correlated equilibrium (formula)

Idea:

Suppose $q \in \Delta(X)$ is given. Suppose everyone knows q. Let x be a realisation of q. Inform everyone i about x_i, but not about x_{-i}.

Now, in a CE, no one wants to change:

For all i, x_i and x'_i:

$$\sum_{x_{-i}} q(x_{-i}|x_i)u_i(x'_i, x_{-i}) \leq \sum_{x_{-i}} q(x_{-i}|x_i)u_i(x_i, x_{-i}).$$

Multiplying by $q(x_i)$ gives, for all i, x_i and x'_i:

Correlated equilibrium (formula)

Idea:

Suppose $q \in \Delta(X)$ is given. Suppose everyone knows q. Let x be a realisation of q. Inform everyone i about x_i, but not about x_{-i}.

Now, in a CE, no one wants to change:

For all i, x_i and x'_i:

$$\sum_{x_{-i}} q(x_{-i}|x_i)u_i(x'_i, x_{-i}) \leq \sum_{x_{-i}} q(x_{-i}|x_i)u_i(x_i, x_{-i}).$$

Multiplying by $q(x_i)$ gives, for all i, x_i and x'_i:

$$\sum_{x_{-i}} q(x_i, x_{-i})u_i(x'_i, x_{-i}) \leq \sum_{x_{-i}} q(x_i, x_{-i})u_i(x_i, x_{-i}).$$

Correlated equilibrium (formula)

Idea:

Suppose $q \in \Delta(X)$ is given. Suppose everyone knows q. Let x be a realisation of q. Inform everyone i about x_i, but not about x_{-i}.

Now, in a CE, no one wants to change:

For all i, x_i and x'_i:

$$\sum_{x_{-i}} q(x_{-i}|x_i)u_i(x'_i, x_{-i}) \leq \sum_{x_{-i}} q(x_{-i}|x_i)u_i(x_i, x_{-i}).$$

Multiplying by $q(x_i)$ gives, for all i, x_i and x'_i:

$$\sum_{x_{-i}} q(x_i, x_{-i})u_i(x'_i, x_{-i}) \leq \sum_{x_{-i}} q(x_i, x_{-i})u_i(x_i, x_{-i}).$$

The latter is often used as the formula to verify a CE.

How to verify a correlated equilibrium

To verify a correlated equilibrium

We will show that

$$q = \begin{array}{r|cc|} & \multicolumn{2}{c|}{\textit{Other:}} \\ \textit{Player 1:} & \text{Green} & \text{Red} \\ \hline \text{Green} & 0.00 & 0.55 \\ \text{Red} & 0.40 & 0.05 \\ \hline \end{array}$$

To verify a correlated equilibrium

We will show that

q =

Player 1:	*Other:* Green	Red
Green	0.00	0.55
Red	0.40	0.05

is a correlated equilibrium of

Player 1:	*Other:* Green	Red
Green	$(-10, -10)$	$(5, 0)$
Red	$(0, 5)$	$(-1, -1)$

To verify a correlated equilibrium

We will show that

$q =$

	Other:	
Player 1:	Green	Red
Green	0.00	0.55
Red	0.40	0.05

is a correlated equilibrium of

	Other:	
Player 1:	Green	Red
Green	$(-10,-10)$	$(5,0)$
Red	$(0,5)$	$(-1,-1)$

- Suppose Player 1 sees Green. Would it be better for him to act as if he sees Red?

To verify a correlated equilibrium

We will show that

q =

	Other:	
Player 1:	Green	Red
Green	0.00	0.55
Red	0.40	0.05

$$\text{Green}: \frac{0}{0.55}(-10) + \frac{0.55}{0.55}5 = 5$$

$$\text{Red}: \frac{0}{0.55}0 + \frac{0.55}{0.55}(-1) = -1$$

is a correlated equilibrium of

	Other:	
Player 1:	Green	Red
Green	$(-10, -10)$	$(5, 0)$
Red	$(0, 5)$	$(-1, -1)$

- Suppose Player 1 sees Green. Would it be better for him to act as if he sees Red?

To verify a correlated equilibrium

We will show that

$q =$

Player 1:	Other: Green	Red
Green	0.00	0.55
Red	0.40	0.05

is a correlated equilibrium of

Player 1:	Other: Green	Red
Green	$(-10, -10)$	$(5, 0)$
Red	$(0, 5)$	$(-1, -1)$

- Suppose Player 1 sees Green. Would it be better for him to act as if he sees Red?

$$\text{Green}: \frac{0}{0.55}(-10) + \frac{0.55}{0.55}5 = 5$$

$$\text{Red}: \frac{0}{0.55}0 + \frac{0.55}{0.55}(-1) = -1$$

- Suppose Player 1 sees Red. Would it be better for him to act as if he sees Green?

To verify a correlated equilibrium

We will show that

q =

Player 1:	*Other:* Green	Red
Green	0.00	0.55
Red	0.40	0.05

is a correlated equilibrium of

Player 1:	*Other:* Green	Red
Green	$(-10, -10)$	$(5, 0)$
Red	$(0, 5)$	$(-1, -1)$

- Suppose Player 1 sees Green. Would it be better for him to act as if he sees Red?

$$\text{Green} : \frac{0}{0.55}(-10) + \frac{0.55}{0.55}5 = 5$$

$$\text{Red} : \frac{0}{0.55}0 + \frac{0.55}{0.55}(-1) = -1$$

- Suppose Player 1 sees Red. Would it be better for him to act as if he sees Green?

$$\text{Red} : \frac{0.40}{0.45}0 + \frac{0.05}{0.45}(-1) = -0.11$$

$$\text{Green} : \frac{0.40}{0.45}(-10) + \frac{0.05}{0.45}5 = -8.35$$

To verify a correlated equilibrium

We will show that

q =

	Other:	
Player 1:	Green	Red
Green	0.00	0.55
Red	0.40	0.05

is a correlated equilibrium of

	Other:	
Player 1:	Green	Red
Green	$(-10, -10)$	$(5, 0)$
Red	$(0, 5)$	$(-1, -1)$

- Suppose Player 1 sees Green. Would it be better for him to act as if he sees Red?

$$\text{Green}: \frac{0}{0.55}(-10) + \frac{0.55}{0.55}5 = 5$$

$$\text{Red}: \frac{0}{0.55}0 + \frac{0.55}{0.55}(-1) = -1$$

- Suppose Player 1 sees Red. Would it be better for him to act as if he sees Green?

$$\text{Red}: \frac{0.40}{0.45}0 + \frac{0.05}{0.45}(-1) = -0.11$$

$$\text{Green}: \frac{0.40}{0.45}(-10) + \frac{0.05}{0.45}5 = -8.35$$

- $(5 + (-0.11))/2 = 2.45 >$ payoffs from two out of three NE.

The problem to find all correlated equilibria

Find all correlated equilibria

Problem: find all correlated equilibria for

	Other:	
You:	Green	Red
Green	$(-10,-10)$	$(5,0)$
Red	$(0,5)$	$(-1,-1)$

Find all correlated equilibria

Problem: find all correlated equilibria for

	Other:	
You:	Green	Red
Green	$(-10, -10)$	$(5, 0)$
Red	$(0, 5)$	$(-1, -1)$

Solution: set

$q =$

	Other:	
You:	Green	Red
Green	α	β
Red	γ	δ

Find all correlated equilibria

Problem: find all correlated equilibria for

	Other:	
You:	Green	Red
Green	$(-10, -10)$	$(5, 0)$
Red	$(0, 5)$	$(-1, -1)$

Of course, first:

Solution: set

$$q = \begin{array}{r|cc} & \multicolumn{2}{c}{\textit{Other:}} \\ \text{You:} & \text{Green} & \text{Red} \\ \hline \text{Green} & \alpha & \beta \\ \text{Red} & \gamma & \delta \\ \hline \end{array}$$

Find all correlated equilibria

Problem: find all correlated equilibria for

You:	*Other:* Green	Red
Green	$(-10, -10)$	$(5, 0)$
Red	$(0, 5)$	$(-1, -1)$

Solution: set

$$q = \begin{array}{r|cc} & \multicolumn{2}{c}{\textit{Other:}} \\ \text{You:} & \text{Green} & \text{Red} \\ \hline \text{Green} & \alpha & \beta \\ \text{Red} & \gamma & \delta \\ \hline \end{array}$$

Of course, first:

- $0 \leq \alpha, \beta, \gamma, \delta \leq 1$

Find all correlated equilibria

Problem: find all correlated equilibria for

	Other:	
You:	Green	Red
Green	$(-10, -10)$	$(5, 0)$
Red	$(0, 5)$	$(-1, -1)$

Solution: set

$$q = \begin{array}{r|cc|} & \multicolumn{2}{c|}{\textit{Other:}} \\ \text{You:} & \text{Green} & \text{Red} \\ \hline \text{Green} & \alpha & \beta \\ \text{Red} & \gamma & \delta \\ \hline \end{array}$$

Of course, first:

- $0 \leq \alpha, \beta, \gamma, \delta \leq 1$
- $\alpha + \beta + \gamma + \delta = 1$

Find all correlated equilibria

Problem: find all correlated equilibria for

	Other:	
You:	Green	Red
Green	$(-10, -10)$	$(5, 0)$
Red	$(0, 5)$	$(-1, -1)$

Solution: set

$q =$

	Other:	
You:	Green	Red
Green	α	β
Red	γ	δ

Of course, first:

- $0 \leq \alpha, \beta, \gamma, \delta \leq 1$
- $\alpha + \beta + \gamma + \delta = 1$

But also:

Find all correlated equilibria

Problem: find all correlated equilibria for

	Other:	
You:	Green	Red
Green	$(-10, -10)$	$(5, 0)$
Red	$(0, 5)$	$(-1, -1)$

Solution: set

$$q = \begin{array}{r|cc|} & \multicolumn{2}{c|}{\textit{Other:}} \\ \text{You:} & \text{Green} & \text{Red} \\ \hline \text{Green} & \alpha & \beta \\ \text{Red} & \gamma & \delta \\ \hline \end{array}$$

Of course, first:

- $0 \leq \alpha, \beta, \gamma, \delta \leq 1$
- $\alpha + \beta + \gamma + \delta = 1$

But also:

- $u_1(\text{act like } G \mid \text{signal } G) \geq u_1(\text{act like } R \mid \text{signal } G)$.

Find all correlated equilibria

Problem: find all correlated equilibria for

	Other:	
You:	Green	Red
Green	$(-10, -10)$	$(5, 0)$
Red	$(0, 5)$	$(-1, -1)$

Solution: set

$$q = \begin{array}{r|cc} \text{You:} & \text{Green} & \text{Red} \\ \hline \text{Green} & \alpha & \beta \\ \text{Red} & \gamma & \delta \end{array}$$

(*Other:* over columns Green, Red)

Of course, first:

- $0 \leq \alpha, \beta, \gamma, \delta \leq 1$
- $\alpha + \beta + \gamma + \delta = 1$

But also:

- $u_1(\text{act like } G \mid \text{signal } G) \geq u_1(\text{act like } R \mid \text{signal } G)$.
- $u_1(\text{act like } R \mid \text{signal } R) \geq u_1(\text{act like } G \mid \text{signal } R)$.

Find all correlated equilibria

Problem: find all correlated equilibria for

	Other:	
You:	Green	Red
Green	$(-10, -10)$	$(5, 0)$
Red	$(0, 5)$	$(-1, -1)$

Solution: set

$$q = \begin{array}{r|cc|} & \multicolumn{2}{c|}{\textit{Other:}} \\ \text{You:} & \text{Green} & \text{Red} \\ \hline \text{Green} & \alpha & \beta \\ \text{Red} & \gamma & \delta \\ \hline \end{array}$$

Of course, first:

- $0 \leq \alpha, \beta, \gamma, \delta \leq 1$
- $\alpha + \beta + \gamma + \delta = 1$

But also:

- $u_1(\text{act like } G \mid \text{signal } G) \geq u_1(\text{act like } R \mid \text{signal } G)$.
- $u_1(\text{act like } R \mid \text{signal } R) \geq u_1(\text{act like } G \mid \text{signal } R)$.
- Similarly for u_2 (the column player).

Find all correlated equilibria

$u_1(\text{act like } G \mid \text{signal } G)$

Find all correlated equilibria

$$u_1(\text{act like } G \mid \text{signal } G) \geq u_1(\text{act like } R \mid \text{signal } G)$$

Find all correlated equilibria

$$u_1(\text{act like } G \mid \text{signal } G) \geq u_1(\text{act like } R \mid \text{signal } G)$$

$$\frac{\alpha}{\alpha+\beta}(-10) + \frac{\beta}{\alpha+\beta}5$$

Find all correlated equilibria

$$u_1(\text{act like } G \mid \text{signal } G) \geq u_1(\text{act like } R \mid \text{signal } G)$$

$$\frac{\alpha}{\alpha+\beta}(-10) + \frac{\beta}{\alpha+\beta}5 \geq \frac{\alpha}{\alpha+\beta}0 + \frac{\beta}{\alpha+\beta} - 1$$

Find all correlated equilibria

$$u_1(\text{act like } G \mid \text{signal } G) \geq u_1(\text{act like } R \mid \text{signal } G)$$

$$\frac{\alpha}{\alpha+\beta}(-10) + \frac{\beta}{\alpha+\beta}5 \geq \frac{\alpha}{\alpha+\beta}0 + \frac{\beta}{\alpha+\beta} - 1$$

$$-10\alpha + 5\beta$$

Find all correlated equilibria

$$u_1(\text{act like } G \mid \text{signal } G) \geq u_1(\text{act like } R \mid \text{signal } G)$$

$$\frac{\alpha}{\alpha+\beta}(-10) + \frac{\beta}{\alpha+\beta}5 \geq \frac{\alpha}{\alpha+\beta}0 + \frac{\beta}{\alpha+\beta} - 1$$

$$-10\alpha + 5\beta \geq 0\alpha + -1\beta$$

Find all correlated equilibria

$$u_1(\text{act like } G \mid \text{signal } G) \geq u_1(\text{act like } R \mid \text{signal } G)$$

$$\frac{\alpha}{\alpha+\beta}(-10) + \frac{\beta}{\alpha+\beta}5 \geq \frac{\alpha}{\alpha+\beta}0 + \frac{\beta}{\alpha+\beta} - 1$$

$$-10\alpha + 5\beta \geq 0\alpha + -1\beta$$

$$-5\alpha + 3\beta$$

Find all correlated equilibria

$$
\begin{aligned}
u_1(\text{act like } G \mid \text{signal } G) &\geq u_1(\text{act like } R \mid \text{signal } G) \\
\frac{\alpha}{\alpha+\beta}(-10) + \frac{\beta}{\alpha+\beta}5 &\geq \frac{\alpha}{\alpha+\beta}0 + \frac{\beta}{\alpha+\beta} - 1 \\
-10\alpha + 5\beta &\geq 0\alpha + -1\beta \\
-5\alpha + 3\beta &\geq 0.
\end{aligned}
$$

Find all correlated equilibria

$$
\begin{aligned}
u_1(\text{act like } G \mid \text{signal } G) &\geq u_1(\text{act like } R \mid \text{signal } G) \\
\frac{\alpha}{\alpha+\beta}(-10) + \frac{\beta}{\alpha+\beta}5 &\geq \frac{\alpha}{\alpha+\beta}0 + \frac{\beta}{\alpha+\beta} - 1 \\
-10\alpha + 5\beta &\geq 0\alpha + -1\beta \\
-5\alpha + 3\beta &\geq 0.
\end{aligned}
$$

Further,

Find all correlated equilibria

$$
\begin{aligned}
u_1(\text{act like } G \mid \text{signal } G) &\geq u_1(\text{act like } R \mid \text{signal } G) \\
\frac{\alpha}{\alpha+\beta}(-10) + \frac{\beta}{\alpha+\beta}5 &\geq \frac{\alpha}{\alpha+\beta}0 + \frac{\beta}{\alpha+\beta} - 1 \\
-10\alpha + 5\beta &\geq 0\alpha + -1\beta \\
-5\alpha + 3\beta &\geq 0.
\end{aligned}
$$

Further,

$$u_1(\text{act like } R \mid \text{signal } R)$$

Find all correlated equilibria

$$
\begin{aligned}
u_1(\text{act like } G \mid \text{signal } G) &\geq u_1(\text{act like } R \mid \text{signal } G) \\
\frac{\alpha}{\alpha+\beta}(-10) + \frac{\beta}{\alpha+\beta}5 &\geq \frac{\alpha}{\alpha+\beta}0 + \frac{\beta}{\alpha+\beta} - 1 \\
-10\alpha + 5\beta &\geq 0\alpha + -1\beta \\
-5\alpha + 3\beta &\geq 0.
\end{aligned}
$$

Further,

$$u_1(\text{act like } R \mid \text{signal } R) \geq u_1(\text{act like } G \mid \text{signal } R)$$

Find all correlated equilibria

$$u_1(\text{act like } G \mid \text{signal } G) \geq u_1(\text{act like } R \mid \text{signal } G)$$

$$\frac{\alpha}{\alpha+\beta}(-10) + \frac{\beta}{\alpha+\beta}5 \geq \frac{\alpha}{\alpha+\beta}0 + \frac{\beta}{\alpha+\beta} - 1$$

$$-10\alpha + 5\beta \geq 0\alpha + -1\beta$$

$$-5\alpha + 3\beta \geq 0.$$

Further,

$$u_1(\text{act like } R \mid \text{signal } R) \geq u_1(\text{act like } G \mid \text{signal } R)$$

$$\frac{\gamma}{\gamma+\delta}0 + \frac{\delta}{\gamma+\delta} - 1$$

Find all correlated equilibria

$$
\begin{aligned}
u_1(\text{act like } G \mid \text{signal } G) &\geq u_1(\text{act like } R \mid \text{signal } G)\\
\frac{\alpha}{\alpha+\beta}(-10) + \frac{\beta}{\alpha+\beta}5 &\geq \frac{\alpha}{\alpha+\beta}0 + \frac{\beta}{\alpha+\beta} - 1\\
-10\alpha + 5\beta &\geq 0\alpha + -1\beta\\
-5\alpha + 3\beta &\geq 0.
\end{aligned}
$$

Further,

$$
\begin{aligned}
u_1(\text{act like } R \mid \text{signal } R) &\geq u_1(\text{act like } G \mid \text{signal } R)\\
\frac{\gamma}{\gamma+\delta}0 + \frac{\delta}{\gamma+\delta} - 1 &\geq \frac{\gamma}{\gamma+\delta}(-10) + \frac{\delta}{\gamma+\delta}5
\end{aligned}
$$

Find all correlated equilibria

$$
\begin{aligned}
u_1(\text{act like } G \mid \text{signal } G) &\geq u_1(\text{act like } R \mid \text{signal } G) \\
\frac{\alpha}{\alpha+\beta}(-10) + \frac{\beta}{\alpha+\beta}5 &\geq \frac{\alpha}{\alpha+\beta}0 + \frac{\beta}{\alpha+\beta} - 1 \\
-10\alpha + 5\beta &\geq 0\alpha + -1\beta \\
-5\alpha + 3\beta &\geq 0.
\end{aligned}
$$

Further,

$$
\begin{aligned}
u_1(\text{act like } R \mid \text{signal } R) &\geq u_1(\text{act like } G \mid \text{signal } R) \\
\frac{\gamma}{\gamma+\delta}0 + \frac{\delta}{\gamma+\delta} - 1 &\geq \frac{\gamma}{\gamma+\delta}(-10) + \frac{\delta}{\gamma+\delta}5 \\
0\gamma + -1\delta &
\end{aligned}
$$

Find all correlated equilibria

$$
\begin{aligned}
u_1(\text{act like } G \mid \text{signal } G) &\geq u_1(\text{act like } R \mid \text{signal } G) \\
\frac{\alpha}{\alpha+\beta}(-10) + \frac{\beta}{\alpha+\beta}5 &\geq \frac{\alpha}{\alpha+\beta}0 + \frac{\beta}{\alpha+\beta} - 1 \\
-10\alpha + 5\beta &\geq 0\alpha + -1\beta \\
-5\alpha + 3\beta &\geq 0.
\end{aligned}
$$

Further,

$$
\begin{aligned}
u_1(\text{act like } R \mid \text{signal } R) &\geq u_1(\text{act like } G \mid \text{signal } R) \\
\frac{\gamma}{\gamma+\delta}0 + \frac{\delta}{\gamma+\delta} - 1 &\geq \frac{\gamma}{\gamma+\delta}(-10) + \frac{\delta}{\gamma+\delta}5 \\
0\gamma + -1\delta &\geq -10\gamma + 5\delta
\end{aligned}
$$

Find all correlated equilibria

$$u_1(\text{act like } G \mid \text{signal } G) \geq u_1(\text{act like } R \mid \text{signal } G)$$

$$\frac{\alpha}{\alpha+\beta}(-10) + \frac{\beta}{\alpha+\beta}5 \geq \frac{\alpha}{\alpha+\beta}0 + \frac{\beta}{\alpha+\beta} - 1$$

$$-10\alpha + 5\beta \geq 0\alpha + -1\beta$$

$$-5\alpha + 3\beta \geq 0.$$

Further,

$$u_1(\text{act like } R \mid \text{signal } R) \geq u_1(\text{act like } G \mid \text{signal } R)$$

$$\frac{\gamma}{\gamma+\delta}0 + \frac{\delta}{\gamma+\delta} - 1 \geq \frac{\gamma}{\gamma+\delta}(-10) + \frac{\delta}{\gamma+\delta}5$$

$$0\gamma + -1\delta \geq -10\gamma + 5\delta$$

$$5\gamma - 3\delta$$

Find all correlated equilibria

$$
\begin{aligned}
u_1(\text{act like } G \mid \text{signal } G) &\geq u_1(\text{act like } R \mid \text{signal } G) \\
\frac{\alpha}{\alpha+\beta}(-10) + \frac{\beta}{\alpha+\beta}5 &\geq \frac{\alpha}{\alpha+\beta}0 + \frac{\beta}{\alpha+\beta} - 1 \\
-10\alpha + 5\beta &\geq 0\alpha + -1\beta \\
-5\alpha + 3\beta &\geq 0.
\end{aligned}
$$

Further,

$$
\begin{aligned}
u_1(\text{act like } R \mid \text{signal } R) &\geq u_1(\text{act like } G \mid \text{signal } R) \\
\frac{\gamma}{\gamma+\delta}0 + \frac{\delta}{\gamma+\delta} - 1 &\geq \frac{\gamma}{\gamma+\delta}(-10) + \frac{\delta}{\gamma+\delta}5 \\
0\gamma + -1\delta &\geq -10\gamma + 5\delta \\
5\gamma - 3\delta &\geq 0.
\end{aligned}
$$

Find all correlated equilibria

$$
\begin{aligned}
u_1(\text{act like } G \mid \text{signal } G) &\geq u_1(\text{act like } R \mid \text{signal } G) \\
\frac{\alpha}{\alpha+\beta}(-10) + \frac{\beta}{\alpha+\beta}5 &\geq \frac{\alpha}{\alpha+\beta}0 + \frac{\beta}{\alpha+\beta} - 1 \\
-10\alpha + 5\beta &\geq 0\alpha + -1\beta \\
-5\alpha + 3\beta &\geq 0.
\end{aligned}
$$

Further,

$$
\begin{aligned}
u_1(\text{act like } R \mid \text{signal } R) &\geq u_1(\text{act like } G \mid \text{signal } R) \\
\frac{\gamma}{\gamma+\delta}0 + \frac{\delta}{\gamma+\delta} - 1 &\geq \frac{\gamma}{\gamma+\delta}(-10) + \frac{\delta}{\gamma+\delta}5 \\
0\gamma + -1\delta &\geq -10\gamma + 5\delta \\
5\gamma - 3\delta &\geq 0.
\end{aligned}
$$

Similarly for u_2 (the column player).

Find all correlated equilibria

We end up with:

$$\left\{ \begin{array}{rr} 0 \leq \alpha, \beta, \gamma, \delta \leq 1 & 5\gamma - 3\delta \geq 0 \\ \alpha + \beta + \gamma + \delta = 1 & -5\alpha + 3\gamma \geq 0 \\ -5\alpha + 3\beta \geq 0 & 5\beta - 3\delta \geq 0 \end{array} \right.$$

Find all correlated equilibria

We end up with:

$$\begin{cases} 0 \leq \alpha, \beta, \gamma, \delta \leq 1 & 5\gamma - 3\delta \geq 0 \\ \alpha + \beta + \gamma + \delta = 1 & -5\alpha + 3\gamma \geq 0 \\ -5\alpha + 3\beta \geq 0 & 5\beta - 3\delta \geq 0 \end{cases}$$

This is a solid convex polyhedron in $\mathbb{R}^3$:

Find all correlated equilibria

We end up with:

$$\begin{cases} 0 \leq \alpha, \beta, \gamma, \delta \leq 1 & 5\gamma - 3\delta \geq 0 \\ \alpha + \beta + \gamma + \delta = 1 & -5\alpha + 3\gamma \geq 0 \\ -5\alpha + 3\beta \geq 0 & 5\beta - 3\delta \geq 0 \end{cases}$$

This is a solid convex polyhedron in $\mathbb{R}^3$:

$$\Leftrightarrow \quad \begin{cases} 1 \geq \alpha, \beta, \gamma \geq 0 \\ -5\alpha + 3\beta \geq 0 \\ 5\gamma - 3(1 - \alpha - \beta - \gamma) \geq 0 \\ -5\alpha + 3\gamma \geq 0 \\ 5\beta - 3(1 - \alpha - \beta - \gamma) \geq 0 \end{cases}$$

Find all correlated equilibria

We end up with:

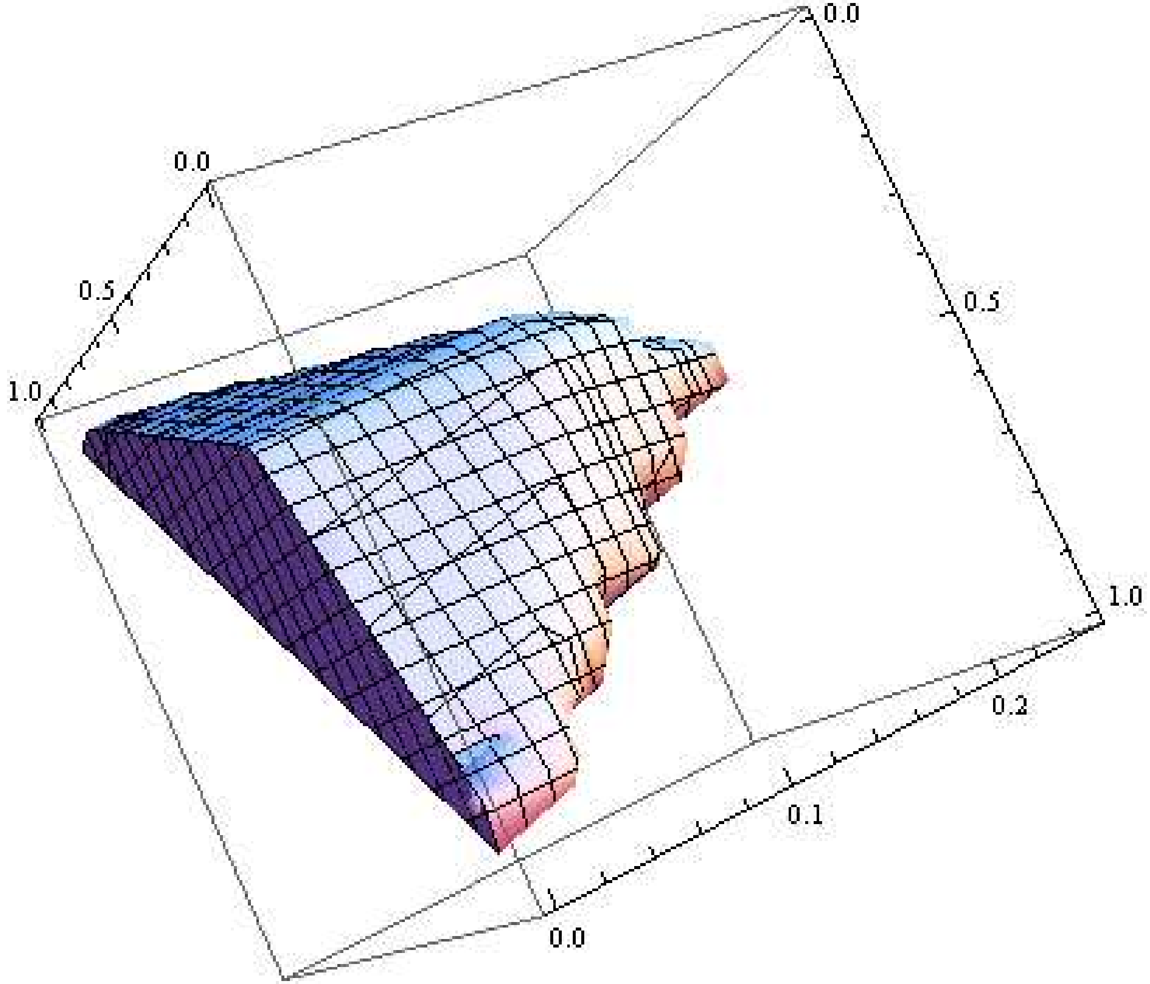
$$\begin{cases} 0 \le \alpha, \beta, \gamma, \delta \le 1 & 5\gamma - 3\delta \ge 0 \\ \alpha + \beta + \gamma + \delta = 1 & -5\alpha + 3\gamma \ge 0 \\ -5\alpha + 3\beta \ge 0 & 5\beta - 3\delta \ge 0 \end{cases}$$

This is a solid convex polyhedron in $\mathbb{R}^3$:

$$\Leftrightarrow \quad \begin{cases} 1 \ge \alpha, \beta, \gamma \ge 0 \\ -5\alpha + 3\beta \ge 0 \\ 5\gamma - 3(1 - \alpha - \beta - \gamma) \ge 0 \\ -5\alpha + 3\gamma \ge 0 \\ 5\beta - 3(1 - \alpha - \beta - \gamma) \ge 0 \end{cases} \quad \Leftrightarrow \quad \begin{cases} 1 \ge \alpha, \beta, \gamma \ge 0 \\ -5\alpha + 3\beta \ge 0 \\ 3\alpha + 3\beta + 8\gamma \ge 3 \\ -5\alpha + 3\gamma \ge 0 \\ 3\alpha + 8\beta + 3\gamma \ge 3. \end{cases}$$

Correlated equilibrium

Admissible values for α, β and γ in the traffic light problem:



Find specific correlated equilibria

Find specific correlated equilibria

What is the longest proportion of time both traffic lights can be red simultaneously before drivers start to ignore them?

Find specific correlated equilibria

What is the longest proportion of time both traffic lights can be red simultaneously before drivers start to ignore them?

$$\text{Maximize:} \quad \delta$$

$$\text{Subject to:} \begin{cases} \alpha \geq 0, \beta \geq 0, \gamma \geq 0, \delta \geq 0 & 5\gamma - 3\delta \geq 0 \\ \alpha + \beta + \gamma + \delta = 1 & -5\alpha + 3\gamma \geq 0 \\ -5\alpha + 3\beta \geq 0 & 5\beta - 3\delta \geq 0 \end{cases}$$

Find specific correlated equilibria

What is the longest proportion of time both traffic lights can be red simultaneously before drivers start to ignore them?

$$\text{Maximize:} \quad \delta$$

$$\text{Subject to:} \begin{cases} \alpha \geq 0, \beta \geq 0, \gamma \geq 0, \delta \geq 0 & 5\gamma - 3\delta \geq 0 \\ \alpha + \beta + \gamma + \delta = 1 & -5\alpha + 3\gamma \geq 0 \\ -5\alpha + 3\beta \geq 0 & 5\beta - 3\delta \geq 0 \end{cases}$$

Gives:

Find specific correlated equilibria

What is the longest proportion of time both traffic lights can be red simultaneously before drivers start to ignore them?

$$\text{Maximize:} \quad \delta$$

$$\text{Subject to:} \begin{cases} \alpha \geq 0, \beta \geq 0, \gamma \geq 0, \delta \geq 0 & 5\gamma - 3\delta \geq 0 \\ \alpha + \beta + \gamma + \delta = 1 & -5\alpha + 3\gamma \geq 0 \\ -5\alpha + 3\beta \geq 0 & 5\beta - 3\delta \geq 0 \end{cases}$$

Gives:

$$(\alpha, \beta, \gamma, \delta) = \left(0, \frac{3}{11}, \frac{3}{11}, \frac{5}{11}\right).$$

Find specific correlated equilibria

What is the longest proportion of time both traffic lights can be red simultaneously before drivers start to ignore them?

$$\text{Maximize:} \quad \delta$$

$$\text{Subject to:} \begin{cases} \alpha \geq 0, \beta \geq 0, \gamma \geq 0, \delta \geq 0 & 5\gamma - 3\delta \geq 0 \\ \alpha + \beta + \gamma + \delta = 1 & -5\alpha + 3\gamma \geq 0 \\ -5\alpha + 3\beta \geq 0 & 5\beta - 3\delta \geq 0 \end{cases}$$

Gives:

$$(\alpha, \beta, \gamma, \delta) = \left(0, \frac{3}{11}, \frac{3}{11}, \frac{5}{11}\right).$$

Answer: at most $5/11 = 45\%$ of the time.

Find specific correlated equilibria

Is it possible to let the row driver wait all the time without compromising a correlated equilibrium?

Find specific correlated equilibria

Is it possible to let the row driver wait all the time without compromising a correlated equilibrium?

$$\text{Minimize:} \quad \beta$$

$$\text{Subject to:} \begin{cases} \alpha \geq 0, \beta \geq 0, \gamma \geq 0, \delta \geq 0 & 5\gamma - 3\delta \geq 0 \\ \alpha + \beta + \gamma + \delta = 1 & -5\alpha + 3\gamma \geq 0 \\ -5\alpha + 3\beta \geq 0 & 5\beta - 3\delta \geq 0 \end{cases}$$

Find specific correlated equilibria

Is it possible to let the row driver wait all the time without compromising a correlated equilibrium?

$$\text{Minimize:} \quad \beta$$

$$\text{Subject to:} \left\{ \begin{array}{rr} \alpha \geq 0, \beta \geq 0, \gamma \geq 0, \delta \geq 0 & 5\gamma - 3\delta \geq 0 \\ \alpha + \beta + \gamma + \delta = 1 & -5\alpha + 3\gamma \geq 0 \\ -5\alpha + 3\beta \geq 0 & 5\beta - 3\delta \geq 0 \end{array} \right.$$

Gives:

$$(\alpha, \beta, \gamma, \delta) = (0, 0, 1, 0)\,.$$

Find specific correlated equilibria

Is it possible to let the row driver wait all the time without compromising a correlated equilibrium?

$$\text{Minimize:} \quad \beta$$

$$\text{Subject to:} \begin{cases} \alpha \geq 0, \beta \geq 0, \gamma \geq 0, \delta \geq 0 & 5\gamma - 3\delta \geq 0 \\ \alpha + \beta + \gamma + \delta = 1 & -5\alpha + 3\gamma \geq 0 \\ -5\alpha + 3\beta \geq 0 & 5\beta - 3\delta \geq 0 \end{cases}$$

Gives:

$$(\alpha, \beta, \gamma, \delta) = (0, 0, 1, 0)\,.$$

Answer: yes

Find specific correlated equilibria

Is it possible to let the row driver wait all the time without compromising a correlated equilibrium?

$$\text{Minimize:} \quad \beta$$

$$\text{Subject to:} \begin{cases} \alpha \geq 0, \beta \geq 0, \gamma \geq 0, \delta \geq 0 & 5\gamma - 3\delta \geq 0 \\ \alpha + \beta + \gamma + \delta = 1 & -5\alpha + 3\gamma \geq 0 \\ -5\alpha + 3\beta \geq 0 & 5\beta - 3\delta \geq 0 \end{cases}$$

Gives:

$$(\alpha, \beta, \gamma, \delta) = (0, 0, 1, 0) \, .$$

Answer: yes, in that case $\gamma = 1$, i.e., the column driver then has to be given green light all of the time.

Find specific correlated equilibria

Is it possible to let the row driver wait all the time while letting the column driver pass no more than 50% of the time

Find specific correlated equilibria

Is it possible to let the row driver wait all the time while letting the column driver pass no more than 50% of the time, without compromising a correlated equilibrium?

Find specific correlated equilibria

Is it possible to let the row driver wait all the time while letting the column driver pass no more than 50% of the time, without compromising a correlated equilibrium?

$$\text{Minimize:} \quad \beta$$

$$\text{Subject to:} \begin{cases} \alpha \geq 0, \beta \geq 0, \gamma \geq 0, \delta \geq 0, \gamma \leq 1/2 & 5\gamma - 3\delta \geq 0 \\ \alpha + \beta + \gamma + \delta = 1 & -5\alpha + 3\gamma \geq 0 \\ -5\alpha + 3\beta \geq 0 & 5\beta - 3\delta \geq 0 \end{cases}$$

Find specific correlated equilibria

Is it possible to let the row driver wait all the time while letting the column driver pass no more than 50% of the time, without compromising a correlated equilibrium?

$$\text{Minimize:} \quad \beta$$

$$\text{Subject to:} \begin{cases} \alpha \geq 0, \beta \geq 0, \gamma \geq 0, \delta \geq 0, \gamma \leq 1/2 & 5\gamma - 3\delta \geq 0 \\ \alpha + \beta + \gamma + \delta = 1 & -5\alpha + 3\gamma \geq 0 \\ -5\alpha + 3\beta \geq 0 & 5\beta - 3\delta \geq 0 \end{cases}$$

Gives:

$$(\alpha, \beta, \gamma, \delta) = \left(\frac{9}{98}, \frac{15}{98}, \frac{1}{2}, \frac{25}{98} \right).$$

Find specific correlated equilibria

Is it possible to let the row driver wait all the time while letting the column driver pass no more than 50% of the time, without compromising a correlated equilibrium?

$$\text{Minimize:} \quad \beta$$

$$\text{Subject to:} \left\{ \begin{array}{cr} \alpha \geq 0, \beta \geq 0, \gamma \geq 0, \delta \geq 0, \gamma \leq 1/2 & 5\gamma - 3\delta \geq 0 \\ \alpha + \beta + \gamma + \delta = 1 & -5\alpha + 3\gamma \geq 0 \\ -5\alpha + 3\beta \geq 0 & 5\beta - 3\delta \geq 0 \end{array} \right.$$

Gives:

$$(\alpha, \beta, \gamma, \delta) = \left(\frac{9}{98}, \frac{15}{98}, \frac{1}{2}, \frac{25}{98} \right).$$

Answer: no.

Find specific correlated equilibria

Is it possible to let the row driver wait all the time while letting the column driver pass no more than 50% of the time, without compromising a correlated equilibrium?

$$\text{Minimize:} \quad \beta$$

$$\text{Subject to:} \begin{cases} \alpha \geq 0, \beta \geq 0, \gamma \geq 0, \delta \geq 0, \gamma \leq 1/2 & 5\gamma - 3\delta \geq 0 \\ \alpha + \beta + \gamma + \delta = 1 & -5\alpha + 3\gamma \geq 0 \\ -5\alpha + 3\beta \geq 0 & 5\beta - 3\delta \geq 0 \end{cases}$$

Gives:

$$(\alpha, \beta, \gamma, \delta) = \left(\frac{9}{98}, \frac{15}{98}, \frac{1}{2}, \frac{25}{98} \right).$$

Answer: no. To maintain an equilibrium, the row driver has to give way $15/98 \approx 15\%$ of the time.

Coarse correlated equilibria

Coarse correlated equilibrium (definition)

$q =$

	Other:	
You:	Green	Red
Green	0.00	0.55
Red	0.40	0.05

Coarse correlated equilibrium (definition)

$$q = \begin{array}{r|cc|} & \multicolumn{2}{c|}{\textit{Other:}} \\ \text{You:} & \text{Green} & \text{Red} \\ \hline \text{Green} & 0.00 & 0.55 \\ \text{Red} & 0.40 & 0.05 \\ \hline \end{array}$$

- With joint probability, q, the system is in each of these four states (action profiles) $x \in X$.

Coarse correlated equilibrium (definition)

$q =$

	Other:	
You:	Green	Red
Green	0.00	0.55
Red	0.40	0.05

- With joint probability, q, the system is in each of these four states (action profiles) $x \in X$.
- Players know q.

Coarse correlated equilibrium (definition)

$$q \quad = \quad \begin{array}{r|cc|} & \multicolumn{2}{c|}{\textit{Other:}} \\ \text{You:} & \text{Green} & \text{Red} \\ \hline \text{Green} & 0.00 & 0.55 \\ \text{Red} & 0.40 & 0.05 \\ \hline \end{array}$$

- With joint probability, q, the system is in each of these four states (action profiles) $x \in X$.
- Players know q.
- At each realisation of q, every party i comes to know only its coordinate (i.e., action, Green or Red), x_i, of the system state x.

Coarse correlated equilibrium (definition)

$q =$

	Other:	
You:	Green	Red
Green	0.00	0.55
Red	0.40	0.05

- With joint probability, q, the system is in each of these four states (action profiles) $x \in X$.
- Players know q.
- At each realisation of q, every party i comes to know only its coordinate (i.e., action, Green or Red), x_i, of the system state x.

Definition. A distribution $q \in \Delta(X)$ is called a coarse correlated equilibrium or Hannan set, if, prior to announcing $x \in X$, no party has an incentive to deviate from its own coordinate x_i, assuming that others do not deviate from x_{-i} as well.

Coarse correlated equilibrium (formula)

Idea:

Coarse correlated equilibrium (formula)

Idea:

Suppose $q \in \Delta(X)$ is given

Coarse correlated equilibrium (formula)

Idea:

Suppose $q \in \Delta(X)$ is given, and everyone knows q.

Coarse correlated equilibrium (formula)

Idea:

Suppose $q \in \Delta(X)$ is given, and everyone knows q. Let x be a realisation of q.

Coarse correlated equilibrium (formula)

Idea:

Suppose $q \in \Delta(X)$ is given, and everyone knows q. Let x be a realisation of q. No one i is informed about x_i.

Coarse correlated equilibrium (formula)

Idea:

Suppose $q \in \Delta(X)$ is given, and everyone knows q. Let x be a realisation of q. No one i is informed about x_i.

Now, in a CEE, everyone blindly accepts what is given.

Coarse correlated equilibrium (formula)

Idea:

Suppose $q \in \Delta(X)$ is given, and everyone knows q. Let x be a realisation of q. No one i is informed about x_i.

Now, in a CEE, everyone blindly accepts what is given. (Think traffic light.)

Coarse correlated equilibrium (formula)

Idea:

Suppose $q \in \Delta(X)$ is given, and everyone knows q. Let x be a realisation of q. No one i is informed about x_i.

Now, in a CEE, everyone blindly accepts what is given. (Think traffic light.)

For all players i and alternative actions x'_i:

$$\begin{aligned}\sum_{x_{-i}} q_{-i}(x_{-i}) u_i(x'_i, x_{-i}) &\leq \sum_{x_i, x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i}) \\ &= \sum_{x} q(x) u_i(x) \\ &= u_i(q).\end{aligned}$$

Coarse correlated equilibrium (formula)

Idea:

Suppose $q \in \Delta(X)$ is given, and everyone knows q. Let x be a realisation of q. No one i is informed about x_i.

Now, in a CEE, everyone blindly accepts what is given. (Think traffic light.)

For all players i and alternative actions x'_i:

$$\begin{aligned}\sum_{x_{-i}} q_{-i}(x_{-i}) u_i(x'_i, x_{-i}) &\leq \sum_{x_i, x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i}) \\ &= \sum_x q(x) u_i(x) \\ &= u_i(q).\end{aligned}$$

This is the same formula as for a Nash equilibrium, only the joint distribution q is not necessarily a distribution induced by strategies $\{s_i\}_i$.

Find CCE for the traffic light problem

For all i and x_i': $\sum_{x_{-i}} q_{-i}(x_{-i}) u_i(x_i', x_{-i}) \leq u_i(q)$.

Find CCE for the traffic light problem

For all i and x'_i: $\sum_{x_{-i}} q_{-i}(x_{-i})u_i(x'_i, x_{-i}) \leq u_i(q)$. So:

$$\begin{cases} \sum_{x_{-1}} q_{-1}(x_{-1})u_1(G, x_{-1}) \leq u_1(q), \\ \sum_{x_{-1}} q_{-1}(x_{-1})u_1(R, x_{-1}) \leq u_1(q), \\ \sum_{x_{-2}} q_{-2}(x_{-2})u_2(x_{-2}, G) \leq u_2(q), \\ \sum_{x_{-2}} q_{-2}(x_{-2})u_2(x_{-2}, R) \leq u_2(q). \end{cases}$$

Find CCE for the traffic light problem

For all i and x'_i: $\sum_{x_{-i}} q_{-i}(x_{-i})u_i(x'_i, x_{-i}) \leq u_i(q)$. So:

$$\begin{cases} \sum_{x_{-1}} q_{-1}(x_{-1})u_1(G, x_{-1}) \leq u_1(q), \\ \sum_{x_{-1}} q_{-1}(x_{-1})u_1(R, x_{-1}) \leq u_1(q), \\ \sum_{x_{-2}} q_{-2}(x_{-2})u_2(x_{-2}, G) \leq u_2(q), \\ \sum_{x_{-2}} q_{-2}(x_{-2})u_2(x_{-2}, R) \leq u_2(q). \end{cases}$$

Which is

$$\begin{cases} q_{-1}(G)u_1(G, G) + q_{-1}(R)u_1(G, R) \leq u_1(q), \\ q_{-1}(G)u_1(R, G) + q_{-1}(R)u_1(R, R) \leq u_1(q), \\ q_{-2}(G)u_2(G, G) + q_{-2}(R)u_2(R, G) \leq u_2(q), \\ q_{-2}(G)u_2(G, R) + q_{-2}(R)u_2(R, R) \leq u_2(q). \end{cases}$$

Find CCE for the traffic light problem

For all i and x'_i: $\sum_{x_{-i}} q_{-i}(x_{-i}) u_i(x'_i, x_{-i}) \leq u_i(q)$. So:

$$\begin{cases} \sum_{x_{-1}} q_{-1}(x_{-1}) u_1(G, x_{-1}) \leq u_1(q), \\ \sum_{x_{-1}} q_{-1}(x_{-1}) u_1(R, x_{-1}) \leq u_1(q), \\ \sum_{x_{-2}} q_{-2}(x_{-2}) u_2(x_{-2}, G) \leq u_2(q), \\ \sum_{x_{-2}} q_{-2}(x_{-2}) u_2(x_{-2}, R) \leq u_2(q). \end{cases}$$

Which is

$$\begin{cases} q_{-1}(G) u_1(G, G) + q_{-1}(R) u_1(G, R) \leq u_1(q), \\ q_{-1}(G) u_1(R, G) + q_{-1}(R) u_1(R, R) \leq u_1(q), \\ q_{-2}(G) u_2(G, G) + q_{-2}(R) u_2(R, G) \leq u_2(q), \\ q_{-2}(G) u_2(G, R) + q_{-2}(R) u_2(R, R) \leq u_2(q). \end{cases}$$

We will have to solve

$$\begin{cases} (\alpha + \gamma) \cdot -10 + (\beta + \delta) \cdot 5 \leq -10\alpha + 5\beta + 0\gamma - 1\delta, \\ (\alpha + \gamma) \cdot 0 + (\beta + \delta) \cdot -1 \leq -10\alpha + 5\beta + 0\gamma - 1\delta, \\ (\alpha + \beta) \cdot -10 + (\gamma + \delta) \cdot 5 \leq -10\alpha + 0\beta + 5\gamma - 1\delta, \\ (\alpha + \beta) \cdot 0 + (\gamma + \delta) \cdot -1 \leq -10\alpha + 0\beta + 5\gamma - 1\delta. \end{cases}$$

Find CCE for the traffic light problem (continued)

We end up with the same system of inequalities as with the computation of all correlated equilibria, see Slide 25.

Find CCE for the traffic light problem (ctd)

And the same polyhedron, see Slide 26.

For 2x2 games, CE and CCE coincide

For 2x2 games, CE and CCE coincide

For 2x2 games, CE and CCE coincide

Suppose

	Left	Right
Up	(a, a')	(b, c')
Down	(c, b')	(d, d')

For 2x2 games, CE and CCE coincide

Suppose

	Left	Right
Up	(a, a')	(b, c')
Down	(c, b')	(d, d')

and $q =$

	Left	Right
Up	α	β
Down	γ	δ

.

For 2x2 games, CE and CCE coincide

Suppose

$$\begin{array}{r|cc} & \text{Left} & \text{Right} \\ \hline \text{Up} & (a, a') & (b, c') \\ \text{Down} & (c, b') & (d, d') \end{array} \qquad \text{and } q = \qquad \begin{array}{r|cc} & \text{Left} & \text{Right} \\ \hline \text{Up} & \alpha & \beta \\ \text{Down} & \gamma & \delta \end{array} \quad .$$

CE means for all players i, actions x_i and alternative actions x_i':

$$\sum_{x_{-i}} q(x_i, x_{-i}) u_i(x_i', x_{-i}) \leq \sum_{x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i}).$$

For 2x2 games, CE and CCE coincide

Suppose

	Left	Right
Up	(a, a')	(b, c')
Down	(c, b')	(d, d')

and $q =$

	Left	Right
Up	α	β
Down	γ	δ

.

CE means for all players i, actions x_i and alternative actions x'_i:

$$\sum_{x_{-i}} q(x_i, x_{-i}) u_i(x'_i, x_{-i}) \leq \sum_{x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i}).$$

So

For 2x2 games, CE and CCE coincide

Suppose

$$\begin{array}{r|cc} & \text{Left} & \text{Right} \\ \hline \text{Up} & (a, a') & (b, c') \\ \text{Down} & (c, b') & (d, d') \end{array} \qquad \text{and } q = \qquad \begin{array}{r|cc} & \text{Left} & \text{Right} \\ \hline \text{Up} & \alpha & \beta \\ \text{Down} & \gamma & \delta \end{array} .$$

CE means for all players i, actions x_i and alternative actions x'_i:

$$\sum_{x_{-i}} q(x_i, x_{-i}) u_i(x'_i, x_{-i}) \leq \sum_{x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i}).$$

So

$$\left\{ \begin{array}{l} \\ \\ \\ \end{array} \right.$$

For 2x2 games, CE and CCE coincide

Suppose

$$\begin{array}{r|cc} & \text{Left} & \text{Right} \\ \hline \text{Up} & (a, a') & (b, c') \\ \text{Down} & (c, b') & (d, d') \end{array} \quad \text{and } q = \quad \begin{array}{r|cc} & \text{Left} & \text{Right} \\ \hline \text{Up} & \alpha & \beta \\ \text{Down} & \gamma & \delta \end{array} .$$

CE means for all players i, actions x_i and alternative actions x'_i:

$$\sum_{x_{-i}} q(x_i, x_{-i}) u_i(x'_i, x_{-i}) \leq \sum_{x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i}).$$

So

$$\begin{cases} D \text{ for } U\text{:} \quad q(U, L)u_1(D, L) + q(U, R)u_1(D, R) \leq q(U, L)u_1(U, L) + q(U, R)u_1(U, R) \\ \\ \\ \end{cases}$$

For 2x2 games, CE and CCE coincide

Suppose

$$\begin{array}{r|cc} & \text{Left} & \text{Right} \\ \hline \text{Up} & (a, a') & (b, c') \\ \text{Down} & (c, b') & (d, d') \end{array} \qquad \text{and } q = \begin{array}{r|cc} & \text{Left} & \text{Right} \\ \hline \text{Up} & \alpha & \beta \\ \text{Down} & \gamma & \delta \end{array} .$$

CE means for all players i, actions x_i and alternative actions x_i':

$$\sum_{x_{-i}} q(x_i, x_{-i}) u_i(x_i', x_{-i}) \leq \sum_{x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i}).$$

So

$$\left\{ \begin{array}{ll} D \text{ for } U: & q(U,L)u_1(D,L) + q(U,R)u_1(D,R) \leq q(U,L)u_1(U,L) + q(U,R)u_1(U,R) \\ U \text{ for } D: & q(D,L)u_1(U,L) + q(D,R)u_1(U,R) \leq q(D,L)u_1(D,L) + q(D,R)u_1(D,R) \\ & \\ & \end{array} \right.$$

For 2x2 games, CE and CCE coincide

Suppose

$$\begin{array}{r|cc} & \text{Left} & \text{Right} \\ \hline \text{Up} & (a, a') & (b, c') \\ \text{Down} & (c, b') & (d, d') \end{array} \quad \text{and } q = \quad \begin{array}{r|cc} & \text{Left} & \text{Right} \\ \hline \text{Up} & \alpha & \beta \\ \text{Down} & \gamma & \delta \end{array} .$$

CE means for all players i, actions x_i and alternative actions x'_i:

$$\sum_{x_{-i}} q(x_i, x_{-i}) u_i(x'_i, x_{-i}) \leq \sum_{x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i}).$$

So

$$\left\{ \begin{array}{rl} D \text{ for } U: & q(U,L)u_1(D,L) + q(U,R)u_1(D,R) \leq q(U,L)u_1(U,L) + q(U,R)u_1(U,R) \\ U \text{ for } D: & q(D,L)u_1(U,L) + q(D,R)u_1(U,R) \leq q(D,L)u_1(D,L) + q(D,R)u_1(D,R) \\ R \text{ for } L: & q(U,L)u_2(U,R) + q(D,L)u_1(D,R) \leq q(U,L)u_2(U,L) + q(D,L)u_2(D,L) \\ & \end{array} \right.$$

For 2x2 games, CE and CCE coincide

Suppose

$$\begin{array}{r|cc} & \text{Left} & \text{Right} \\ \hline \text{Up} & (a, a') & (b, c') \\ \text{Down} & (c, b') & (d, d') \end{array} \qquad \text{and } q = \qquad \begin{array}{r|cc} & \text{Left} & \text{Right} \\ \hline \text{Up} & \alpha & \beta \\ \text{Down} & \gamma & \delta \end{array} \; .$$

CE means for all players i, actions x_i and alternative actions x_i':

$$\sum_{x_{-i}} q(x_i, x_{-i}) u_i(x_i', x_{-i}) \leq \sum_{x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i}).$$

So

$$\left\{ \begin{array}{rl} D \text{ for } U\text{:} & q(U,L)u_1(D,L) + q(U,R)u_1(D,R) \leq q(U,L)u_1(U,L) + q(U,R)u_1(U,R) \\ U \text{ for } D\text{:} & q(D,L)u_1(U,L) + q(D,R)u_1(U,R) \leq q(D,L)u_1(D,L) + q(D,R)u_1(D,R) \\ R \text{ for } L\text{:} & q(U,L)u_2(U,R) + q(D,L)u_1(D,R) \leq q(U,L)u_2(U,L) + q(D,L)u_2(D,L) \\ L \text{ for } R\text{:} & q(U,R)u_2(U,L) + q(D,R)u_1(D,L) \leq q(U,R)u_2(U,R) + q(D,R)u_2(D,R) \end{array} \right.$$

For 2x2 games, CE and CCE coincide

For 2x2 games, CE and CCE coincide

So

For 2x2 games, CE and CCE coincide

So

$$\left\{ \vphantom{\begin{array}{c} a \\ b \\ c \\ d \\ e \end{array}} \right.$$

For 2x2 games, CE and CCE coincide

So

$$\begin{cases} \alpha c + \beta d & \leq \alpha a + \beta b \\ & \\ & \end{cases}$$

For 2x2 games, CE and CCE coincide

So

$$\begin{cases} \alpha c + \beta d & \leq \alpha a + \beta b \\ \gamma a + \delta b & \leq \gamma c + \delta d \\ & \\ & \end{cases}$$

For 2x2 games, CE and CCE coincide

So

$$\begin{cases} \alpha c + \beta d & \leq \alpha a + \beta b \\ \gamma a + \delta b & \leq \gamma c + \delta d \\ \alpha c' + \gamma d' & \leq \alpha a' + \gamma b' \\ \end{cases}$$

For 2x2 games, CE and CCE coincide

So

$$\begin{cases} \alpha c + \beta d & \leq \alpha a + \beta b \\ \gamma a + \delta b & \leq \gamma c + \delta d \\ \alpha c' + \gamma d' & \leq \alpha a' + \gamma b' \\ \beta a' + \delta b' & \leq \beta c' + \delta d' \end{cases}$$

For 2x2 games, CE and CCE coincide

So

$$\begin{cases} \alpha c + \beta d & \leq \alpha a + \beta b \\ \gamma a + \delta b & \leq \gamma c + \delta d \\ \alpha c' + \gamma d' & \leq \alpha a' + \gamma b' \\ \beta a' + \delta b' & \leq \beta c' + \delta d' \end{cases} \qquad \Leftrightarrow$$

For 2x2 games, CE and CCE coincide

So

$$\left\{\begin{array}{ll} \alpha c + \beta d & \leq \alpha a + \beta b \\ \gamma a + \delta b & \leq \gamma c + \delta d \\ \alpha c' + \gamma d' & \leq \alpha a' + \gamma b' \\ \beta a' + \delta b' & \leq \beta c' + \delta d' \end{array}\right. \qquad \Leftrightarrow \qquad \left\{\begin{array}{l} \\ \\ \\ \\ \end{array}\right.$$

For 2x2 games, CE and CCE coincide

So

$$\left\{\begin{array}{ll} \alpha c + \beta d & \leq \alpha a + \beta b \\ \gamma a + \delta b & \leq \gamma c + \delta d \\ \alpha c' + \gamma d' & \leq \alpha a' + \gamma b' \\ \beta a' + \delta b' & \leq \beta c' + \delta d' \end{array}\right. \quad \Leftrightarrow \quad \left\{\begin{array}{lll} \alpha(c-a) & +\beta(d-b) & \leq 0 \\ & & \\ & & \\ & & \end{array}\right.$$

For 2x2 games, CE and CCE coincide

So

$$\begin{cases} \alpha c + \beta d & \leq \alpha a + \beta b \\ \gamma a + \delta b & \leq \gamma c + \delta d \\ \alpha c' + \gamma d' & \leq \alpha a' + \gamma b' \\ \beta a' + \delta b' & \leq \beta c' + \delta d' \end{cases} \quad \Leftrightarrow \quad \begin{cases} \alpha(c-a) & + \beta(d-b) & \leq 0 \\ \gamma(a-c) & + \delta(b-d) & \leq 0 \\ & & \\ & & \end{cases}$$

For 2x2 games, CE and CCE coincide

So

$$\begin{cases} \alpha c + \beta d & \leq \alpha a + \beta b \\ \gamma a + \delta b & \leq \gamma c + \delta d \\ \alpha c' + \gamma d' & \leq \alpha a' + \gamma b' \\ \beta a' + \delta b' & \leq \beta c' + \delta d' \end{cases} \Leftrightarrow \begin{cases} \alpha(c-a) & + \beta(d-b) & \leq 0 \\ \gamma(a-c) & + \delta(b-d) & \leq 0 \\ \alpha(c'-a') & + \gamma(d'-b') & \leq 0 \\ & & \end{cases}$$

For 2x2 games, CE and CCE coincide

So

$$\begin{cases} \alpha c + \beta d & \leq \alpha a + \beta b \\ \gamma a + \delta b & \leq \gamma c + \delta d \\ \alpha c' + \gamma d' & \leq \alpha a' + \gamma b' \\ \beta a' + \delta b' & \leq \beta c' + \delta d' \end{cases} \Leftrightarrow \begin{cases} \alpha(c-a) & + \beta(d-b) & \leq 0 \\ \gamma(a-c) & + \delta(b-d) & \leq 0 \\ \alpha(c'-a') & + \gamma(d'-b') & \leq 0 \\ \beta(a'-c') & + \delta(b'-d') & \leq 0. \end{cases}$$

For 2x2 games, CE and CCE coincide

So

$$\begin{cases} \alpha c + \beta d & \leq \alpha a + \beta b \\ \gamma a + \delta b & \leq \gamma c + \delta d \\ \alpha c' + \gamma d' & \leq \alpha a' + \gamma b' \\ \beta a' + \delta b' & \leq \beta c' + \delta d' \end{cases} \Leftrightarrow \begin{cases} \alpha(c-a) & + \beta(d-b) & \leq 0 \\ \gamma(a-c) & + \delta(b-d) & \leq 0 \\ \alpha(c'-a') & + \gamma(d'-b') & \leq 0 \\ \beta(a'-c') & + \delta(b'-d') & \leq 0. \end{cases}$$

CCE means for all players i and alternative actions x_i':

$$\sum_{x_{-i}} q_{-i}(x_{-i}) u_i(x_i', x_{-i}) \leq \sum_{x_i, x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i})$$

For 2x2 games, CE and CCE coincide

So

$$\begin{cases} \alpha c + \beta d & \leq \alpha a + \beta b \\ \gamma a + \delta b & \leq \gamma c + \delta d \\ \alpha c' + \gamma d' & \leq \alpha a' + \gamma b' \\ \beta a' + \delta b' & \leq \beta c' + \delta d' \end{cases} \Leftrightarrow \begin{cases} \alpha(c-a) & + \beta(d-b) & \leq 0 \\ \gamma(a-c) & + \delta(b-d) & \leq 0 \\ \alpha(c'-a') & + \gamma(d'-b') & \leq 0 \\ \beta(a'-c') & + \delta(b'-d') & \leq 0. \end{cases}$$

CCE means for all players i and alternative actions x_i':

$$\sum_{x_{-i}} q_{-i}(x_{-i}) u_i(x_i', x_{-i}) \leq \sum_{x_i, x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i})$$

So

For 2x2 games, CE and CCE coincide

So

$$\begin{cases} \alpha c + \beta d & \leq \alpha a + \beta b \\ \gamma a + \delta b & \leq \gamma c + \delta d \\ \alpha c' + \gamma d' & \leq \alpha a' + \gamma b' \\ \beta a' + \delta b' & \leq \beta c' + \delta d' \end{cases} \Leftrightarrow \begin{cases} \alpha(c-a) & + \beta(d-b) & \leq 0 \\ \gamma(a-c) & + \delta(b-d) & \leq 0 \\ \alpha(c'-a') & + \gamma(d'-b') & \leq 0 \\ \beta(a'-c') & + \delta(b'-d') & \leq 0. \end{cases}$$

CCE means for all players i and alternative actions x'_i:

$$\sum_{x_{-i}} q_{-i}(x_{-i}) u_i(x'_i, x_{-i}) \leq \sum_{x_i, x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i})$$

So

$$\begin{cases} \\ \\ \\ \end{cases}$$

For 2x2 games, CE and CCE coincide

So

$$\begin{cases} \alpha c + \beta d & \leq \alpha a + \beta b \\ \gamma a + \delta b & \leq \gamma c + \delta d \\ \alpha c' + \gamma d' & \leq \alpha a' + \gamma b' \\ \beta a' + \delta b' & \leq \beta c' + \delta d' \end{cases} \Leftrightarrow \begin{cases} \alpha(c-a) & + \beta(d-b) & \leq 0 \\ \gamma(a-c) & + \delta(b-d) & \leq 0 \\ \alpha(c'-a') & + \gamma(d'-b') & \leq 0 \\ \beta(a'-c') & + \delta(b'-d') & \leq 0. \end{cases}$$

CCE means for all players i and alternative actions x_i':

$$\sum_{x_{-i}} q_{-i}(x_{-i}) u_i(x_i', x_{-i}) \leq \sum_{x_i, x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i})$$

So

$$\begin{cases} \text{always } D\text{:} \quad q(L)u_1(D,L) + q(R)u_1(D,R) \leq q(U,L)u_1(U,L) + \cdots + q(D,R)u_1(D,R) \\ \\ \\ \end{cases}$$

For 2x2 games, CE and CCE coincide

So

$$\left\{\begin{array}{ll} \alpha c + \beta d & \leq \alpha a + \beta b \\ \gamma a + \delta b & \leq \gamma c + \delta d \\ \alpha c' + \gamma d' & \leq \alpha a' + \gamma b' \\ \beta a' + \delta b' & \leq \beta c' + \delta d' \end{array}\right. \quad \Leftrightarrow \quad \left\{\begin{array}{lll} \alpha(c-a) & +\beta(d-b) & \leq 0 \\ \gamma(a-c) & +\delta(b-d) & \leq 0 \\ \alpha(c'-a') & +\gamma(d'-b') & \leq 0 \\ \beta(a'-c') & +\delta(b'-d') & \leq 0. \end{array}\right.$$

CCE means for all players i and alternative actions x_i':

$$\sum_{x_{-i}} q_{-i}(x_{-i}) u_i(x_i', x_{-i}) \leq \sum_{x_i, x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i})$$

So

$$\left\{\begin{array}{lll} \text{always } D\text{:} & q(L)u_1(D,L) + q(R)u_1(D,R) & \leq q(U,L)u_1(U,L) + \cdots + q(D,R)u_1(D,R) \\ \text{always } U\text{:} & q(L)u_1(U,L) + q(R)u_1(U,R) & \leq q(U,L)u_1(U,L) + \cdots + q(D,R)u_1(D,R) \\ & & \\ & & \end{array}\right.$$

For 2x2 games, CE and CCE coincide

So

$$\begin{cases} \alpha c + \beta d & \leq \alpha a + \beta b \\ \gamma a + \delta b & \leq \gamma c + \delta d \\ \alpha c' + \gamma d' & \leq \alpha a' + \gamma b' \\ \beta a' + \delta b' & \leq \beta c' + \delta d' \end{cases} \Leftrightarrow \begin{cases} \alpha(c - a) & + \beta(d - b) & \leq 0 \\ \gamma(a - c) & + \delta(b - d) & \leq 0 \\ \alpha(c' - a') & + \gamma(d' - b') & \leq 0 \\ \beta(a' - c') & + \delta(b' - d') & \leq 0. \end{cases}$$

CCE means for all players i and alternative actions x'_i:

$$\sum_{x_{-i}} q_{-i}(x_{-i}) u_i(x'_i, x_{-i}) \leq \sum_{x_i, x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i})$$

So

$$\begin{cases} \text{always } D\text{:} & q(L)u_1(D, L) + q(R)u_1(D, R) & \leq q(U, L)u_1(U, L) + \cdots + q(D, R)u_1(D, R) \\ \text{always } U\text{:} & q(L)u_1(U, L) + q(R)u_1(U, R) & \leq q(U, L)u_1(U, L) + \cdots + q(D, R)u_1(D, R) \\ \text{always } R\text{:} & q(U)u_2(U, R) + q(D)u_2(D, R) & \leq q(U, L)u_2(U, L) + \cdots + q(D, R)u_2(D, R) \\ & & \end{cases}$$

For 2x2 games, CE and CCE coincide

So

$$\begin{cases} \alpha c + \beta d & \leq \alpha a + \beta b \\ \gamma a + \delta b & \leq \gamma c + \delta d \\ \alpha c' + \gamma d' & \leq \alpha a' + \gamma b' \\ \beta a' + \delta b' & \leq \beta c' + \delta d' \end{cases} \Leftrightarrow \begin{cases} \alpha(c - a) & + \beta(d - b) & \leq 0 \\ \gamma(a - c) & + \delta(b - d) & \leq 0 \\ \alpha(c' - a') & + \gamma(d' - b') & \leq 0 \\ \beta(a' - c') & + \delta(b' - d') & \leq 0. \end{cases}$$

CCE means for all players i and alternative actions x_i':

$$\sum_{x_{-i}} q_{-i}(x_{-i}) u_i(x_i', x_{-i}) \leq \sum_{x_i, x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i})$$

So

$$\begin{cases} \text{always } D\text{:} & q(L)u_1(D, L) + q(R)u_1(D, R) & \leq q(U, L)u_1(U, L) + \cdots + q(D, R)u_1(D, R) \\ \text{always } U\text{:} & q(L)u_1(U, L) + q(R)u_1(U, R) & \leq q(U, L)u_1(U, L) + \cdots + q(D, R)u_1(D, R) \\ \text{always } R\text{:} & q(U)u_2(U, R) + q(D)u_2(D, R) & \leq q(U, L)u_2(U, L) + \cdots + q(D, R)u_2(D, R) \\ \text{always } L\text{:} & q(U)u_2(U, L) + q(D)u_2(D, L) & \leq q(U, L)u_2(U, L) + \cdots + q(D, R)u_2(D, R) \end{cases}$$

For 2x2 games, CE and CCE coincide

For 2x2 games, CE and CCE coincide

CCE means for all players i and alternative actions x_i':

$$\sum_{x_{-i}} q_{-i}(x_{-i}) u_i(x_i', x_{-i}) \leq \sum_{x_i, x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i})$$

For 2x2 games, CE and CCE coincide

CCE means for all players i and alternative actions x'_i:

$$\sum_{x_{-i}} q_{-i}(x_{-i}) u_i(x'_i, x_{-i}) \leq \sum_{x_i, x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i})$$

So

For 2x2 games, CE and CCE coincide

CCE means for all players i and alternative actions x_i':

$$\sum_{x_{-i}} q_{-i}(x_{-i}) u_i(x_i', x_{-i}) \leq \sum_{x_i, x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i})$$

So

$$\left\{ \begin{array}{l} \\ \\ \\ \end{array} \right.$$

For 2x2 games, CE and CCE coincide

CCE means for all players i and alternative actions x_i':

$$\sum_{x_{-i}} q_{-i}(x_{-i}) u_i(x_i', x_{-i}) \leq \sum_{x_i, x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i})$$

So

$$\begin{cases} (\alpha + \gamma)c + (\beta + \delta)d \quad \leq \alpha a + \beta b + \gamma c + \delta d \\ \\ \end{cases}$$

For 2x2 games, CE and CCE coincide

CCE means for all players i and alternative actions x'_i:

$$\sum_{x_{-i}} q_{-i}(x_{-i}) u_i(x'_i, x_{-i}) \leq \sum_{x_i, x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i})$$

So

$$\begin{cases} (\alpha + \gamma)c + (\beta + \delta)d & \leq \alpha a + \beta b + \gamma c + \delta d \\ (\alpha + \gamma)a + (\beta + \delta)b & \leq \alpha a + \beta b + \gamma c + \delta d \\ & \\ & \end{cases}$$

For 2x2 games, CE and CCE coincide

CCE means for all players i and alternative actions x_i':

$$\sum_{x_{-i}} q_{-i}(x_{-i}) u_i(x_i', x_{-i}) \leq \sum_{x_i, x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i})$$

So

$$\begin{cases} (\alpha+\gamma)c + (\beta+\delta)d & \leq \alpha a + \beta b + \gamma c + \delta d \\ (\alpha+\gamma)a + (\beta+\delta)b & \leq \alpha a + \beta b + \gamma c + \delta d \\ (\alpha+\beta)c' + (\gamma+\delta)d' & \leq \alpha a' + \beta c' + \gamma b' + \delta d' \\ & \end{cases}$$

For 2x2 games, CE and CCE coincide

CCE means for all players i and alternative actions x_i':

$$\sum_{x_{-i}} q_{-i}(x_{-i}) u_i(x_i', x_{-i}) \leq \sum_{x_i, x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i})$$

So

$$\begin{cases} (\alpha+\gamma)c + (\beta+\delta)d & \leq \alpha a + \beta b + \gamma c + \delta d \\ (\alpha+\gamma)a + (\beta+\delta)b & \leq \alpha a + \beta b + \gamma c + \delta d \\ (\alpha+\beta)c' + (\gamma+\delta)d' & \leq \alpha a' + \beta c' + \gamma b' + \delta d' \\ (\alpha+\beta)a' + (\gamma+\delta)b' & \leq \alpha a' + \beta c' + \gamma b' + \delta d' \end{cases}$$

For 2x2 games, CE and CCE coincide

CCE means for all players i and alternative actions x'_i:

$$\sum_{x_{-i}} q_{-i}(x_{-i})u_i(x'_i, x_{-i}) \leq \sum_{x_i, x_{-i}} q(x_i, x_{-i})u_i(x_i, x_{-i})$$

So

$$\begin{cases} (\alpha+\gamma)c + (\beta+\delta)d & \leq \alpha a + \beta b + \gamma c + \delta d \\ (\alpha+\gamma)a + (\beta+\delta)b & \leq \alpha a + \beta b + \gamma c + \delta d \\ (\alpha+\beta)c' + (\gamma+\delta)d' & \leq \alpha a' + \beta c' + \gamma b' + \delta d' \\ (\alpha+\beta)a' + (\gamma+\delta)b' & \leq \alpha a' + \beta c' + \gamma b' + \delta d' \end{cases}$$

So

For 2x2 games, CE and CCE coincide

CCE means for all players i and alternative actions x'_i:

$$\sum_{x_{-i}} q_{-i}(x_{-i}) u_i(x'_i, x_{-i}) \leq \sum_{x_i, x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i})$$

So

$$\begin{cases} (\alpha+\gamma)c + (\beta+\delta)d & \leq \alpha a + \beta b + \gamma c + \delta d \\ (\alpha+\gamma)a + (\beta+\delta)b & \leq \alpha a + \beta b + \gamma c + \delta d \\ (\alpha+\beta)c' + (\gamma+\delta)d' & \leq \alpha a' + \beta c' + \gamma b' + \delta d' \\ (\alpha+\beta)a' + (\gamma+\delta)b' & \leq \alpha a' + \beta c' + \gamma b' + \delta d' \end{cases}$$

So

$$\begin{cases} \\ \\ \\ \end{cases}$$

For 2x2 games, CE and CCE coincide

CCE means for all players i and alternative actions x_i':

$$\sum_{x_{-i}} q_{-i}(x_{-i})u_i(x_i', x_{-i}) \leq \sum_{x_i, x_{-i}} q(x_i, x_{-i})u_i(x_i, x_{-i})$$

So

$$\begin{cases} (\alpha+\gamma)c + (\beta+\delta)d & \leq \alpha a + \beta b + \gamma c + \delta d \\ (\alpha+\gamma)a + (\beta+\delta)b & \leq \alpha a + \beta b + \gamma c + \delta d \\ (\alpha+\beta)c' + (\gamma+\delta)d' & \leq \alpha a' + \beta c' + \gamma b' + \delta d' \\ (\alpha+\beta)a' + (\gamma+\delta)b' & \leq \alpha a' + \beta c' + \gamma b' + \delta d' \end{cases}$$

So

$$\begin{cases} \alpha(c-a) \quad + \beta(d-b) \quad \leq 0 \\ \\ \\ \end{cases}$$

For 2x2 games, CE and CCE coincide

CCE means for all players i and alternative actions x_i':

$$\sum_{x_{-i}} q_{-i}(x_{-i}) u_i(x_i', x_{-i}) \leq \sum_{x_i, x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i})$$

So

$$\begin{cases} (\alpha+\gamma)c + (\beta+\delta)d & \leq \alpha a + \beta b + \gamma c + \delta d \\ (\alpha+\gamma)a + (\beta+\delta)b & \leq \alpha a + \beta b + \gamma c + \delta d \\ (\alpha+\beta)c' + (\gamma+\delta)d' & \leq \alpha a' + \beta c' + \gamma b' + \delta d' \\ (\alpha+\beta)a' + (\gamma+\delta)b' & \leq \alpha a' + \beta c' + \gamma b' + \delta d' \end{cases}$$

So

$$\begin{cases} \alpha(c-a) & + \beta(d-b) & \leq 0 \\ \gamma(a-c) & + \delta(b-d) & \leq 0 \\ & & \\ & & \end{cases}$$

For 2x2 games, CE and CCE coincide

CCE means for all players i and alternative actions x'_i:

$$\sum_{x_{-i}} q_{-i}(x_{-i}) u_i(x'_i, x_{-i}) \leq \sum_{x_i, x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i})$$

So

$$\begin{cases} (\alpha+\gamma)c + (\beta+\delta)d & \leq \alpha a + \beta b + \gamma c + \delta d \\ (\alpha+\gamma)a + (\beta+\delta)b & \leq \alpha a + \beta b + \gamma c + \delta d \\ (\alpha+\beta)c' + (\gamma+\delta)d' & \leq \alpha a' + \beta c' + \gamma b' + \delta d' \\ (\alpha+\beta)a' + (\gamma+\delta)b' & \leq \alpha a' + \beta c' + \gamma b' + \delta d' \end{cases}$$

So

$$\begin{cases} \alpha(c-a) & + \beta(d-b) & \leq 0 \\ \gamma(a-c) & + \delta(b-d) & \leq 0 \\ \alpha(c'-a') & + \gamma(d'-b') & \leq 0 \\ \end{cases}$$

For 2x2 games, CE and CCE coincide

CCE means for all players i and alternative actions x'_i:

$$\sum_{x_{-i}} q_{-i}(x_{-i}) u_i(x'_i, x_{-i}) \leq \sum_{x_i, x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i})$$

So

$$\begin{cases} (\alpha+\gamma)c + (\beta+\delta)d & \leq \alpha a + \beta b + \gamma c + \delta d \\ (\alpha+\gamma)a + (\beta+\delta)b & \leq \alpha a + \beta b + \gamma c + \delta d \\ (\alpha+\beta)c' + (\gamma+\delta)d' & \leq \alpha a' + \beta c' + \gamma b' + \delta d' \\ (\alpha+\beta)a' + (\gamma+\delta)b' & \leq \alpha a' + \beta c' + \gamma b' + \delta d' \end{cases}$$

So

$$\begin{cases} \alpha(c-a) & + \beta(d-b) & \leq 0 \\ \gamma(a-c) & + \delta(b-d) & \leq 0 \\ \alpha(c'-a') & + \gamma(d'-b') & \leq 0 \\ \beta(a'-c') & + \delta(b'-d') & \leq 0. \end{cases}$$

For 2x2 games, CE and CCE coincide

CCE means for all players i and alternative actions x_i':

$$\sum_{x_{-i}} q_{-i}(x_{-i}) u_i(x_i', x_{-i}) \leq \sum_{x_i, x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i})$$

So

$$\begin{cases} (\alpha+\gamma)c + (\beta+\delta)d & \leq \alpha a + \beta b + \gamma c + \delta d \\ (\alpha+\gamma)a + (\beta+\delta)b & \leq \alpha a + \beta b + \gamma c + \delta d \\ (\alpha+\beta)c' + (\gamma+\delta)d' & \leq \alpha a' + \beta c' + \gamma b' + \delta d' \\ (\alpha+\beta)a' + (\gamma+\delta)b' & \leq \alpha a' + \beta c' + \gamma b' + \delta d' \end{cases}$$

So

$$\begin{cases} \alpha(c-a) & + \beta(d-b) & \leq 0 \\ \gamma(a-c) & + \delta(b-d) & \leq 0 \\ \alpha(c'-a') & + \gamma(d'-b') & \leq 0 \\ \beta(a'-c') & + \delta(b'-d') & \leq 0. \end{cases}$$

So for 2x2 games, CE and CCE coincide.

CCE that aren’t CE

CCE that aren't CE

- CE consists of all $q \in \Delta(X)$ that give equal probability, say x, to green profiles, and probability $y_i \leq x$ to blue profiles, such that $6x + y_1 + y_2 + y_3 = 1$.
- CCE consists of much more, viz. all $q \in \Delta(X)$ where each column sum does not supersede row's payoff

$$q(R,R) + q(Y,Y) + q(B,B)$$

and each row sum does not supersede cols's payoff

$$q(R,B) + q(Y,R) + q(B,Y).$$

See also PY, SLaiL, Ch. 3, p. 34.

CCE that aren't CE

- With only two actions, CE and CCE coincide

- CE consists of all $q \in \Delta(X)$ that give equal probability, say x, to green profiles, and probability $y_i \leq x$ to blue profiles, such that $6x + y_1 + y_2 + y_3 = 1$.

- CCE consists of much more, viz. all $q \in \Delta(X)$ where each column sum does not supersede row's payoff

$$q(R, R) + q(Y, Y) + q(B, B)$$

and each row sum does not supersede cols's payoff

$$q(R, B) + q(Y, R) + q(B, Y).$$

See also PY, SLaiL, Ch. 3, p. 34.

CCE that aren't CE

- With only two actions, CE and CCE coincide, because probabilities of counter strategy profiles are complementary.

- CE consists of all $q \in \Delta(X)$ that give equal probability, say x, to green profiles, and probability $y_i \leq x$ to blue profiles, such that $6x + y_1 + y_2 + y_3 = 1$.

- CCE consists of much more, viz. all $q \in \Delta(X)$ where each column sum does not supersede row's payoff

$$q(R, R) + q(Y, Y) + q(B, B)$$

and each row sum does not supersede cols's payoff

$$q(R, B) + q(Y, R) + q(B, Y).$$

See also PY, SLaiL, Ch. 3, p. 34.

CCE that aren't CE

- With only two actions, CE and CCE coincide, because probabilities of counter strategy profiles are complementary.
- Consider the fashion game

	Red	Yellow	Blue
Red	$(1,0)$	$(0,0)$	$(0,1)$
Yellow	$(0,1)$	$(1,0)$	$(0,0)$
Blue	$(0,0)$	$(0,1)$	$(1,0)$

where row wants to copy col, and col wants to be one color ahead of row: Y $<$ R $<$ B $<$ Y.

- CE consists of all $q \in \Delta(X)$ that give equal probability, say x, to green profiles, and probability $y_i \leq x$ to blue profiles, such that $6x + y_1 + y_2 + y_3 = 1$.
- CCE consists of much more, viz. all $q \in \Delta(X)$ where each column sum does not supersede row's payoff

$$q(R,R) + q(Y,Y) + q(B,B)$$

and each row sum does not supersede cols's payoff

$$q(R,B) + q(Y,R) + q(B,Y).$$

See also PY, SLaiL, Ch. 3, p. 34.

CCE that aren't CE

- With only two actions, CE and CCE coincide, because probabilities of counter strategy profiles are complementary.
- Consider the fashion game

	Red	Yellow	Blue
Red	$(1,0)$	$(0,0)$	$(0,1)$
Yellow	$(0,1)$	$(1,0)$	$(0,0)$
Blue	$(0,0)$	$(0,1)$	$(1,0)$

  where row wants to copy col, and col wants to be one color ahead of row: Y $<$ R $<$ B $<$ Y.
- One NE, viz., $\{(1/3, 1/3, 1/3)\}^2$.

CCE that aren't CE

- With only two actions, CE and CCE coincide, because probabilities of counter strategy profiles are complementary.
- Consider the fashion game

	Red	Yellow	Blue
Red	$(1,0)$	$(0,0)$	$(0,1)$
Yellow	$(0,1)$	$(1,0)$	$(0,0)$
Blue	$(0,0)$	$(0,1)$	$(1,0)$

  where row wants to copy col, and col wants to be one color ahead of row: Y $<$ R $<$ B $<$ Y.
- One NE, viz., $\{(1/3, 1/3, 1/3)\}^2$.
- CE consists of all $q \in \Delta(X)$ that give equal probability, say x, to green profiles, and probability $y_i \leq x$ to blue profiles, such that $6x + y_1 + y_2 + y_3 = 1$.

CCE that aren't CE

- With only two actions, CE and CCE coincide, because probabilities of counter strategy profiles are complementary.
- Consider the fashion game

	Red	Yellow	Blue
Red	$(1,0)$	$(0,0)$	$(0,1)$
Yellow	$(0,1)$	$(1,0)$	$(0,0)$
Blue	$(0,0)$	$(0,1)$	$(1,0)$

where row wants to copy col, and col wants to be one color ahead of row: Y $<$ R $<$ B $<$ Y.

- One NE, viz., $\{(1/3, 1/3, 1/3)\}^2$.
- CE consists of all $q \in \Delta(X)$ that give equal probability, say x, to green profiles, and probability $y_i \leq x$ to blue profiles, such that $6x + y_1 + y_2 + y_3 = 1$.
- CCE consists of much more

CCE that aren't CE

- With only two actions, CE and CCE coincide, because probabilities of counter strategy profiles are complementary.
- Consider the fashion game

	Red	Yellow	Blue
Red	$(1,0)$	$(0,0)$	$(0,1)$
Yellow	$(0,1)$	$(1,0)$	$(0,0)$
Blue	$(0,0)$	$(0,1)$	$(1,0)$

  where row wants to copy col, and col wants to be one color ahead of row: Y $<$ R $<$ B $<$ Y.

- One NE, viz., $\{(1/3, 1/3, 1/3)\}^2$.
- CE consists of all $q \in \Delta(X)$ that give equal probability, say x, to green profiles, and probability $y_i \leq x$ to blue profiles, such that $6x + y_1 + y_2 + y_3 = 1$.
- CCE consists of much more, viz. all $q \in \Delta(X)$ where each column sum does not supersede row's payoff

$$q(R,R) + q(Y,Y) + q(B,B)$$

CCE that aren't CE

- With only two actions, CE and CCE coincide, because probabilities of counter strategy profiles are complementary.
- Consider the fashion game

	Red	Yellow	Blue
Red	$(1,0)$	$(0,0)$	$(0,1)$
Yellow	$(0,1)$	$(1,0)$	$(0,0)$
Blue	$(0,0)$	$(0,1)$	$(1,0)$

  where row wants to copy col, and col wants to be one color ahead of row: Y $<$ R $<$ B $<$ Y.

- One NE, viz., $\{(1/3, 1/3, 1/3)\}^2$.
- CE consists of all $q \in \Delta(X)$ that give equal probability, say x, to green profiles, and probability $y_i \leq x$ to blue profiles, such that $6x + y_1 + y_2 + y_3 = 1$.
- CCE consists of much more, viz. all $q \in \Delta(X)$ where each column sum does not supersede row's payoff

$$q(R,R) + q(Y,Y) + q(B,B)$$

  and each row sum does not supersede cols's payoff

$$q(R,B) + q(Y,R) + q(B,Y).$$

CCE that aren't CE

- With only two actions, CE and CCE coincide, because probabilities of counter strategy profiles are complementary.
- Consider the fashion game

	Red	Yellow	Blue
Red	$(1,0)$	$(0,0)$	$(0,1)$
Yellow	$(0,1)$	$(1,0)$	$(0,0)$
Blue	$(0,0)$	$(0,1)$	$(1,0)$

  where row wants to copy col, and col wants to be one color ahead of row: Y $<$ R $<$ B $<$ Y.
- One NE, viz., $\{(1/3, 1/3, 1/3)\}^2$.
- CE consists of all $q \in \Delta(X)$ that give equal probability, say x, to green profiles, and probability $y_i \leq x$ to blue profiles, such that $6x + y_1 + y_2 + y_3 = 1$.
- CCE consists of much more, viz. all $q \in \Delta(X)$ where each column sum does not supersede row's payoff

$$q(R,R) + q(Y,Y) + q(B,B)$$

  and each row sum does not supersede cols's payoff

$$q(R,B) + q(Y,R) + q(B,Y).$$

  See also PY, SLaiL, Ch. 3, p. 34.

Hierarchy of equilibria

Nash equilibrium ⇒ correlated equilibrium

If strategies are independent, we have

$$s_{-i}(x_{-i}|x_i) = s_{-i}(x_{-i})$$

Nash equilibrium ⇒ correlated equilibrium

If strategies are independent, we have

$$s_{-i}(x_{-i}|x_i) = s_{-i}(x_{-i})$$

Immediately,

$$\text{for all } i \text{ and } x'_i : \sum_{x_{-i}} s_{-i}(x_{-i}) u_i(x'_i, x_{-i}) \leq u_i(s) \qquad \text{(Nash)}$$

$$\Rightarrow \quad \text{for all } x_i, i \text{ and } x'_i : \sum_{x_{-i}} s(x_{-i}|x_i) u_i(x'_i, x_{-i}) \leq u_i(s) \qquad \text{(CE)}$$

Nash equilibrium ⇒ correlated equilibrium

If strategies are independent, we have

$$s_{-i}(x_{-i}|x_i) = s_{-i}(x_{-i})$$

Immediately,

$$\text{for all } i \text{ and } x_i' : \sum_{x_{-i}} s_{-i}(x_{-i}) u_i(x_i', x_{-i}) \leq u_i(s) \qquad \text{(Nash)}$$

$$\Rightarrow \quad \text{for all } x_i,\ i \text{ and } x_i' : \sum_{x_{-i}} s(x_{-i}|x_i) u_i(x_i', x_{-i}) \leq u_i(s) \qquad \text{(CE)}$$

The latter is the conditional formulation of a correlated equilibrium. (See slide where formula for CE is introduced.)

Nash equilibrium ⇒ correlated equilibrium

If strategies are independent, we have

$$s_{-i}(x_{-i}|x_i) = s_{-i}(x_{-i})$$

Immediately,

$$\text{for all } i \text{ and } x_i' : \sum_{x_{-i}} s_{-i}(x_{-i}) u_i(x_i', x_{-i}) \leq u_i(s) \qquad \text{(Nash)}$$

$$\Rightarrow \quad \text{for all } x_i,\ i \text{ and } x_i' : \sum_{x_{-i}} s(x_{-i}|x_i) u_i(x_i', x_{-i}) \leq u_i(s) \qquad \text{(CE)}$$

The latter is the conditional formulation of a correlated equilibrium. (See slide where formula for CE is introduced.)

Therefore, every Nash equilibrium is a correlated equilibrium.

Summary

NE: for all i and x'_i : $\sum_{x_{-i}} s_{-i}(x_{-i})\, u_i(x'_i, x_{-i}) \leq \sum_x s(x) u_i(x)$

CE: for all x_i, i and x'_i : $\sum_{x_{-i}} q(x_i, x_{-i})\, u_i(x'_i, x_{-i}) \leq \sum_{x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i})$

CCE: for all i and x'_i : $\sum_{x_{-i}} q_{-i}(x_{-i})\, u_i(x'_i, x_{-i}) \leq \sum_x q(x) u_i(x)$

Summary

NE: for all i and x_i' : $\sum_{x_{-i}} s_{-i}(x_{-i})\, u_i(x_i', x_{-i}) \leq \sum_x s(x) u_i(x)$

CE: for all x_i, i and x_i' : $\sum_{x_{-i}} q(x_i, x_{-i})\, u_i(x_i', x_{-i}) \leq \sum_{x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i})$

CCE: for all i and x_i' : $\sum_{x_{-i}} q_{-i}(x_{-i})\, u_i(x_i', x_{-i}) \leq \sum_x q(x) u_i(x)$

- With CE and CCE there are no individual strategies.

Summary

NE: for all i and x'_i : $\sum_{x_{-i}} s_{-i}(x_{-i})\, u_i(x'_i, x_{-i}) \leq \sum_x s(x) u_i(x)$

CE: for all x_i, i and x'_i : $\sum_{x_{-i}} q(x_i, x_{-i})\, u_i(x'_i, x_{-i}) \leq \sum_{x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i})$

CCE: for all i and x'_i : $\sum_{x_{-i}} q_{-i}(x_{-i})\, u_i(x'_i, x_{-i}) \leq \sum_x q(x) u_i(x)$

- With CE and CCE there are no individual strategies.
- CCE $\Rightarrow$ exact conditions for empirical distribution of action profiles in no-regret matching.

Summary

NE: for all i and x'_i : $\sum_{x_{-i}} s_{-i}(x_{-i})\, u_i(x'_i, x_{-i}) \leq \sum_x s(x) u_i(x)$

CE: for all x_i, i and x'_i : $\sum_{x_{-i}} q(x_i, x_{-i})\, u_i(x'_i, x_{-i}) \leq \sum_{x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i})$

CCE: for all i and x'_i : $\sum_{x_{-i}} q_{-i}(x_{-i})\, u_i(x'_i, x_{-i}) \leq \sum_x q(x) u_i(x)$

- With CE and CCE there are no individual strategies.
- CCE $\Rightarrow$ exact conditions for empirical distribution of action profiles in no-regret matching. This becomes clear when no-regret learning is discussed.

Summary

$$\begin{aligned}
&\text{NE: for all} && i \text{ and } x_i' : \textstyle\sum_{x_{-i}} s_{-i}(x_{-i})\, u_i(x_i', x_{-i}) \leq \sum_x s(x) u_i(x) \\
&\text{CE: for all} && x_i, i \text{ and } x_i' : \textstyle\sum_{x_{-i}} q(x_i, x_{-i})\, u_i(x_i', x_{-i}) \leq \sum_{x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i}) \\
&\text{CCE: for all} && i \text{ and } x_i' : \textstyle\sum_{x_{-i}} q_{-i}(x_{-i})\, u_i(x_i', x_{-i}) \leq \sum_x q(x) u_i(x)
\end{aligned}$$

- With CE and CCE there are no individual strategies.
- CCE ⇒ exact conditions for empirical distribution of action profiles in no-regret matching. This becomes clear when no-regret learning is discussed.
- The formulas for Nash and CCE are identical

Summary

NE: for all i and x_i' : $\sum_{x_{-i}} s_{-i}(x_{-i})\, u_i(x_i', x_{-i}) \leq \sum_x s(x) u_i(x)$

CE: for all x_i, i and x_i' : $\sum_{x_{-i}} q(x_i, x_{-i})\, u_i(x_i', x_{-i}) \leq \sum_{x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i})$

CCE: for all i and x_i' : $\sum_{x_{-i}} q_{-i}(x_{-i})\, u_i(x_i', x_{-i}) \leq \sum_x q(x) u_i(x)$

- With CE and CCE there are no individual strategies.
- CCE $\Rightarrow$ exact conditions for empirical distribution of action profiles in no-regret matching. This becomes clear when no-regret learning is discussed.
- The formulas for Nash and CCE are identical, but for Nash the s is the product of its marginalised strategies.

Summary

NE: for all i and x_i' : $\sum_{x_{-i}} s_{-i}(x_{-i})\, u_i(x_i', x_{-i}) \leq \sum_x s(x) u_i(x)$

CE: for all x_i, i and x_i' : $\sum_{x_{-i}} q(x_i, x_{-i})\, u_i(x_i', x_{-i}) \leq \sum_{x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i})$

CCE: for all i and x_i' : $\sum_{x_{-i}} q_{-i}(x_{-i})\, u_i(x_i', x_{-i}) \leq \sum_x q(x) u_i(x)$

- With CE and CCE there are no individual strategies.
- CCE $\Rightarrow$ exact conditions for empirical distribution of action profiles in no-regret matching. This becomes clear when no-regret learning is discussed.
- The formulas for Nash and CCE are identical, but for Nash the s is the product of its marginalised strategies. Therefore, NE $\Rightarrow$ CCE.

Summary

NE: for all i and x_i' : $\sum_{x_{-i}} s_{-i}(x_{-i})\, u_i(x_i', x_{-i}) \leq \sum_x s(x) u_i(x)$

CE: for all x_i, i and x_i' : $\sum_{x_{-i}} q(x_i, x_{-i})\, u_i(x_i', x_{-i}) \leq \sum_{x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i})$

CCE: for all i and x_i' : $\sum_{x_{-i}} q_{-i}(x_{-i})\, u_i(x_i', x_{-i}) \leq \sum_x q(x) u_i(x)$

- With CE and CCE there are no individual strategies.
- CCE $\Rightarrow$ exact conditions for empirical distribution of action profiles in no-regret matching. This becomes clear when no-regret learning is discussed.
- The formulas for Nash and CCE are identical, but for Nash the s is the product of its marginalised strategies. Therefore, NE $\Rightarrow$ CCE.
- The LHS of the CCE is the x_i-sum over all LHS's of the CE.

Summary

$$
\begin{aligned}
\text{NE: for all } & i \text{ and } x'_i : \textstyle\sum_{x_{-i}} s_{-i}(x_{-i})\, u_i(x'_i, x_{-i}) \leq \textstyle\sum_x s(x) u_i(x) \\
\text{CE: for all } & x_i, i \text{ and } x'_i : \textstyle\sum_{x_{-i}} q(x_i, x_{-i})\, u_i(x'_i, x_{-i}) \leq \textstyle\sum_{x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i}) \\
\text{CCE: for all } & i \text{ and } x'_i : \textstyle\sum_{x_{-i}} q_{-i}(x_{-i})\, u_i(x'_i, x_{-i}) \leq \textstyle\sum_x q(x) u_i(x)
\end{aligned}
$$

- With CE and CCE there are no individual strategies.
- CCE $\Rightarrow$ exact conditions for empirical distribution of action profiles in no-regret matching. This becomes clear when no-regret learning is discussed.
- The formulas for Nash and CCE are identical, but for Nash the s is the product of its marginalised strategies. Therefore, NE $\Rightarrow$ CCE.
- The LHS of the CCE is the x_i-sum over all LHS's of the CE. Therefore, CE $\Rightarrow$ CCE.

Summary

NE: for all i and x'_i : $\sum_{x_{-i}} s_{-i}(x_{-i})\, u_i(x'_i, x_{-i}) \leq \sum_x s(x) u_i(x)$

CE: for all x_i, i and x'_i : $\sum_{x_{-i}} q(x_i, x_{-i})\, u_i(x'_i, x_{-i}) \leq \sum_{x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i})$

CCE: for all i and x'_i : $\sum_{x_{-i}} q_{-i}(x_{-i})\, u_i(x'_i, x_{-i}) \leq \sum_x q(x) u_i(x)$

- With CE and CCE there are no individual strategies.
- CCE ⇒ exact conditions for empirical distribution of action profiles in no-regret matching. This becomes clear when no-regret learning is discussed.
- The formulas for Nash and CCE are identical, but for Nash the s is the product of its marginalised strategies. Therefore, NE ⇒ CCE.
- The LHS of the CCE is the x_i-sum over all LHS's of the CE. Therefore, CE ⇒ CCE.
- We already derived NE ⇒ CE.

Summary

$$\text{NE: for all } \quad i \text{ and } x_i' : \textstyle\sum_{x_{-i}} s_{-i}(x_{-i})\, u_i(x_i', x_{-i}) \leq \sum_x s(x) u_i(x)$$

$$\text{CE: for all } x_i, i \text{ and } x_i' : \textstyle\sum_{x_{-i}} q(x_i, x_{-i})\, u_i(x_i', x_{-i}) \leq \sum_{x_{-i}} q(x_i, x_{-i}) u_i(x_i, x_{-i})$$

$$\text{CCE: for all } \quad i \text{ and } x_i' : \textstyle\sum_{x_{-i}} q_{-i}(x_{-i})\, u_i(x_i', x_{-i}) \leq \sum_x q(x) u_i(x)$$

- With CE and CCE there are no individual strategies.
- CCE ⇒ exact conditions for empirical distribution of action profiles in no-regret matching. This becomes clear when no-regret learning is discussed.
- The formulas for Nash and CCE are identical, but for Nash the s is the product of its marginalised strategies. Therefore, NE ⇒ CCE.
- The LHS of the CCE is the x_i-sum over all LHS's of the CE. Therefore, CE ⇒ CCE.
- We already derived NE ⇒ CE. Therefore, NE ⇒ CE ⇒ CCE.

What next?

What next?

- **No-regret learning**: this is a generalisation of reinforcement learning

  No-regret $=_{Def}$ play those actions that would have been successful in the past.

What next?

- **No-regret learning**: this is a generalisation of reinforcement learning

  No-regret $=_{Def}$ play those actions that would have been successful in the past.

- **Similarities** with reinforcement learning:

What next?

- **No-regret learning**: this is a generalisation of reinforcement learning

  No-regret $=_{Def}$ play those actions that would have been successful in the past.

- **Similarities** with reinforcement learning:

  1. Driven by past payoffs.

What next?

- **No-regret learning**: this is a generalisation of reinforcement learning

  No-regret $=_{Def}$ play those actions that would have been successful in the past.

- **Similarities** with reinforcement learning:

  1. Driven by past payoffs.
  2. Not interested in (the behaviour of) the opponent.

What next?

- **No-regret learning**: this is a generalisation of reinforcement learning

  No-regret $=_{Def}$ play those actions that would have been successful in the past.

- **Similarities** with reinforcement learning:

  1. Driven by past payoffs.
  2. Not interested in (the behaviour of) the opponent.
  3. Myopic.

What next?

- **No-regret learning**: this is a generalisation of reinforcement learning

  No-regret $=_{Def}$ play those actions that would have been successful in the past.

- **Similarities** with reinforcement learning:

  1. Driven by past payoffs.
  2. Not interested in (the behaviour of) the opponent.
  3. Myopic.

- **Differences**:

What next?

- **No-regret learning**: this is a generalisation of reinforcement learning

  No-regret $=_{Def}$ play those actions that would have been successful in the past.

- **Similarities** with reinforcement learning:

  1. Driven by past payoffs.
  2. Not interested in (the behaviour of) the opponent.
  3. Myopic.

- **Differences**:

  1. No-regret learning als learns from hypothetical payoffs.

What next?

- **No-regret learning**: this is a generalisation of reinforcement learning

  No-regret $=_{Def}$ play those actions that would have been successful in the past.

- **Similarities** with reinforcement learning:

  1. Driven by past payoffs.
  2. Not interested in (the behaviour of) the opponent.
  3. Myopic.

- **Differences**:

  1. No-regret learning als learns from hypothetical payoffs.
  2. It is more easy to obtain results regarding performance.

Exam problems

Exam problems

Exam problems

With different numbers in payoff matrices as in the examples above:

Exam problems

With different numbers in payoff matrices as in the examples above:

For $X \in \{\mathrm{CE}, \mathrm{CCE}\}$:

Exam problems

With different numbers in payoff matrices as in the examples above:

For $X \in \{\mathrm{CE}, \mathrm{CCE}\}$:

- Verify X.

Exam problems

With different numbers in payoff matrices as in the examples above:

For $X \in \{\mathrm{CE}, \mathrm{CCE}\}$:

- Verify X.
- Compute all X

Exam problems

With different numbers in payoff matrices as in the examples above:

For $X \in \{\text{CE}, \text{CCE}\}$:

- Verify X.
- Compute all X (*).

Exam problems

With different numbers in payoff matrices as in the examples above:

For $X \in \{\mathrm{CE}, \mathrm{CCE}\}$:

- Verify X.
- Compute all X (*).
- Formulate a constrained X problem

Exam problems

With different numbers in payoff matrices as in the examples above:

For $X \in \{\mathrm{CE}, \mathrm{CCE}\}$:

- Verify X.
- Compute all X (*).
- Formulate a constrained X problem (*).

Exam problems

With different numbers in payoff matrices as in the examples above:

For $X \in \{\text{CE}, \text{CCE}\}$:

- Verify X.
- Compute all X (*).
- Formulate a constrained X problem (*).

(*) In a system of inequalities, write variables on one side, and constants on the other side.

Exam problems

With different numbers in payoff matrices as in the examples above:

For $X \in \{\mathrm{CE}, \mathrm{CCE}\}$:

- Verify X.
- Compute all X (*).
- Formulate a constrained X problem (*).

(*) In a system of inequalities, write variables on one side, and constants on the other side. Of course, you are not asked to actually solve such systems of inequalities at an exam.

Exam problems

With different numbers in payoff matrices as in the examples above:

For $X \in \{\text{CE}, \text{CCE}\}$:

- Verify X.
- Compute all X (*).
- Formulate a constrained X problem (*).

(*) In a system of inequalities, write variables on one side, and constants on the other side. Of course, you are not asked to actually solve such systems of inequalities at an exam.

Good luck!