Multi-agent learning

Teaching strategies

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Monday 14th June, 2021

Plan for Today

Part I: Preliminaries

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1. Teacher possesses memory of $k = 0$ rounds: Bully

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2. Teacher possesses memory of $k = 1$ round: Godfather

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2. Core idea of SPaM: combine teacher and follower capabilities.

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1. Three points of criticism to Godfather++.
2. Core idea of SPaM: combine teacher and follower capabilities.
3. Notion of guilt to trigger switches between teaching and following.

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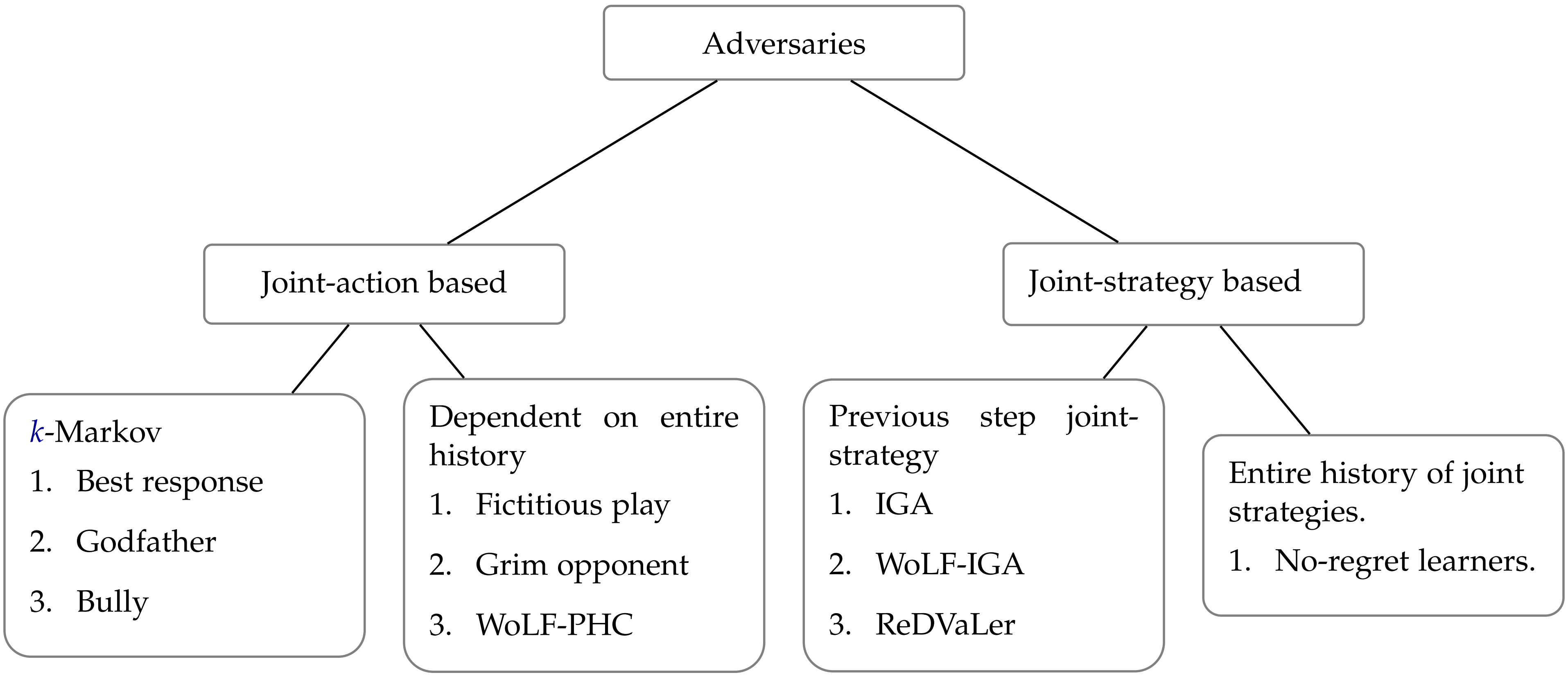
Doran Chakraborty and Peter Stone (2008). "Online Multiagent Learning against Memory Bounded Adversaries," *Machine Learning and Knowledge Discovery in Databases*, Lecture Notes in Artificial Intelligence Vol. 5212, pp. 211-26

Taxonomy of possible adversaries

(Taken from Chakraborty and Stone, 2008):



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$$\begin{array}{cc} & \begin{array}{ccc} L & M & R \end{array} \\ \begin{array}{c} T \\ C \\ B \end{array} & \begin{pmatrix} 3,6 & 8,6 & 7,3 \\ 8,1 & 6,3 & 7,3 \\ 3,5 & 9,2 & 7,5 \end{pmatrix} \end{array}$$

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1. Find, for every action of yourself, the best response(s) of your opponent.

   This yields L and M for T, M and R for C and L and R for B.

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   3 and 8 for T, 6 and 7 for C and 3 and 9 for B.

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   3 and 8 for T, 6 and 7 for C and 3 and 9 for B.

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Consider matching pennies:

	H	T
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A more complicated case. Suppose Row's strategy α is $(\alpha_1, \alpha_2, \alpha_3)$:

		L	C	R
α_1	T	$(0,1)$	$(0,0)$	$(4,0)$
α_2	M	$(8,0)$	$(1,1)$	$(8,0)$
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- Max. payoff for pure Bully is 1

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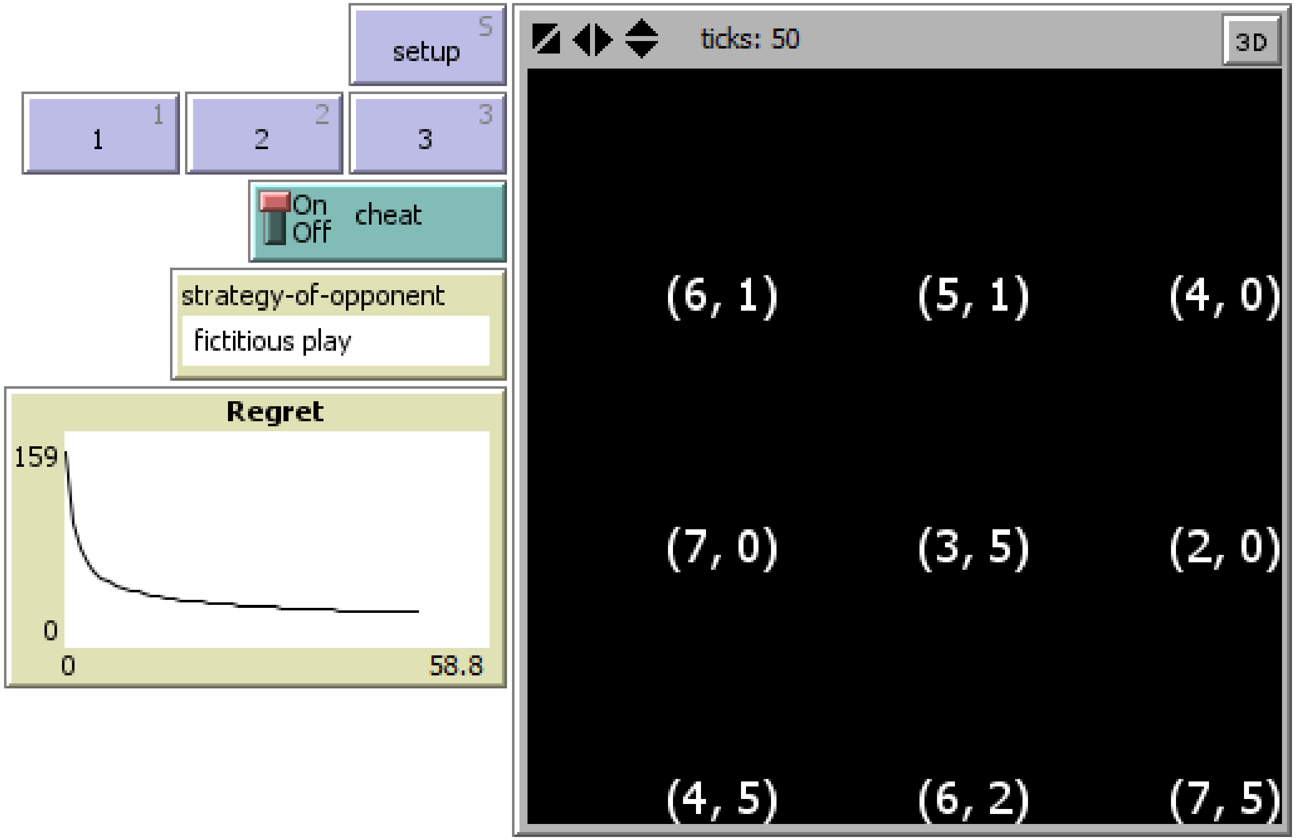
- Row's $E[\$]$ for mixed $\alpha = (0.3334, 0.3332, 0.3334)$ is $2 \cdot 0.3334 \cdot 2 + 0.3332 \cdot 8$, which is almost 4. Not bad!

Idea for an app to learn to play {against} Bully



Play against the computer. At the outset, the computer initializes to either Bully (with a probability of 50%) or pure fictitious play, the choice of which you can't see. After that, the computer won't change strategy. Try to press regret down as within few rounds as possible.

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Play any strategy that gives you the highest payoff, assuming that your opponent is a mindless follower.

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- Right most inner part (green): best response of opponent to s_i.

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- Middle inner part (as from min): guaranteed payoff for bullying opponent with s_i.

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- Entire formula: choose s_i that maximises this guaranteed payoff.

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- Right most inner part (green): best response of opponent to s_i.
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- Entire formula: choose s_i that maximises this guaranteed payoff.

  Recognise the maxmin = the security value in this formula!

Bully: precise definition (in parts)

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- Let $\mathrm{BR}(s_i)$ be the set of all best responses to i's strategy s_i:

$$\mathrm{BR}(s_i) =_{Def} \operatorname{argmax}_{s_{-i}} \{u_{-i}(s_i, s_{-i}) \mid s_{-i} \in S_{-i}\}.$$

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- Let $\mathrm{Bully}_i(s_i)$ be the payoff guaranteed for playing s_i against mindless followers (i.e, best responders):

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- The set of bully strategies is formed by:

$$\mathrm{Bully}_i =_{Def} \mathrm{argmax}_{s_i \in S_i} \mathrm{Bully}_i(s_i)$$

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- The set of bully strategies is formed by:

$$\mathrm{Bully}_i =_{Def} \mathrm{argmax}_{s_i \in S_i} \mathrm{Bully}_i(s_i)$$

- Bully is stateless (a.k.a. memoryless, i.e, memory of $k = 0$ rounds), hence keeps playing the same action throughout.

Godfather

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- Godfather needs a memory of $k = 1$ (one round).

Folk theorem for NE in repeated games

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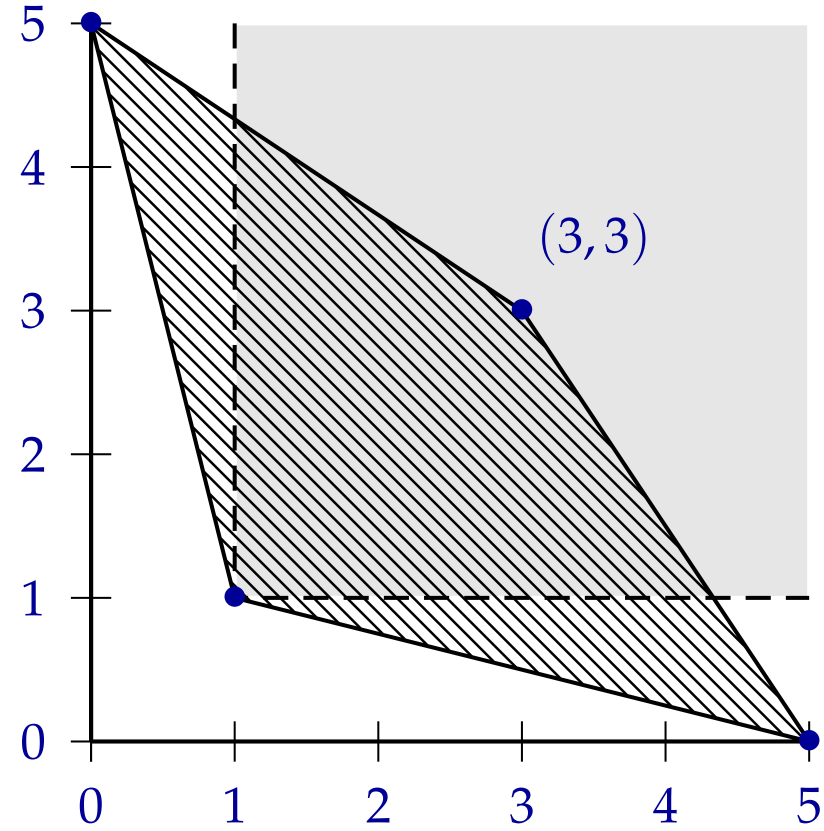
- **Enforceable payoffs** (shaded): no one goes below their minmax.

Theorem. If (x, y) is both feasible and enforceable, then (x, y) is the payoff in a Nash equilibrium of the infinitely repeated G with average payoffs.
Conversely, if (x, y) is the payoff in any Nash equilibrium of the infinitely repeated G with average payoffs, then (x, y) is enforceable.



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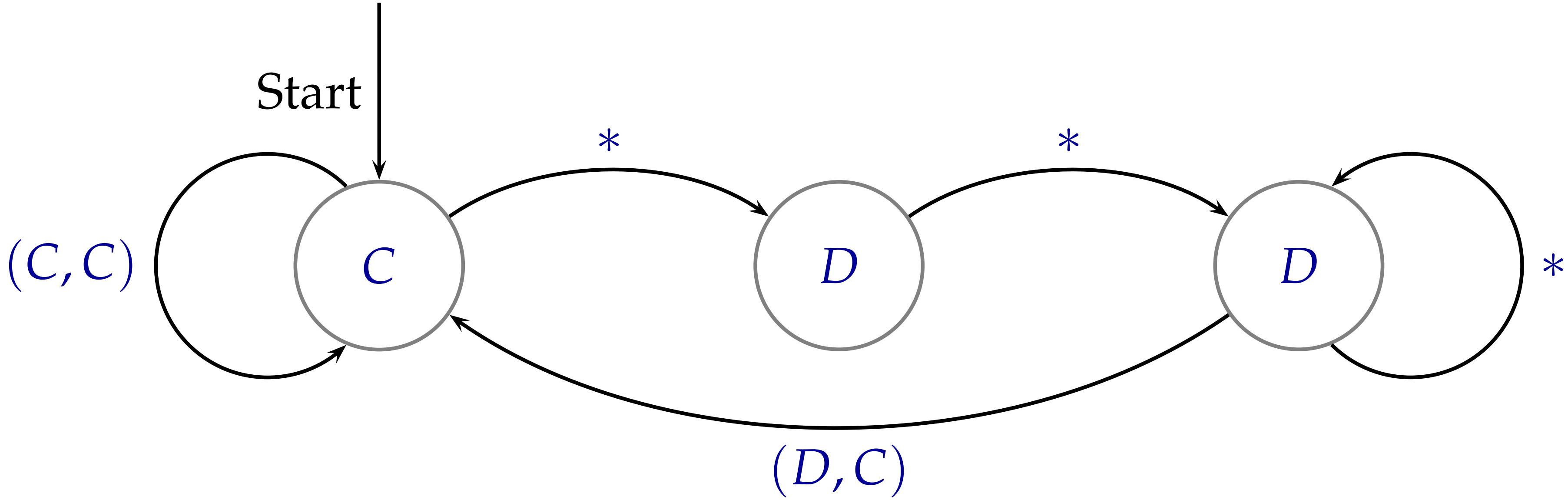
Michael L. Littman and Peter Stone (2005). "A polynomial-time Nash equilibrium algorithm for repeated games". In *Decision Support Systems* Vol. 39, pp. 55-66.

Finite machine for "two tits for tat"



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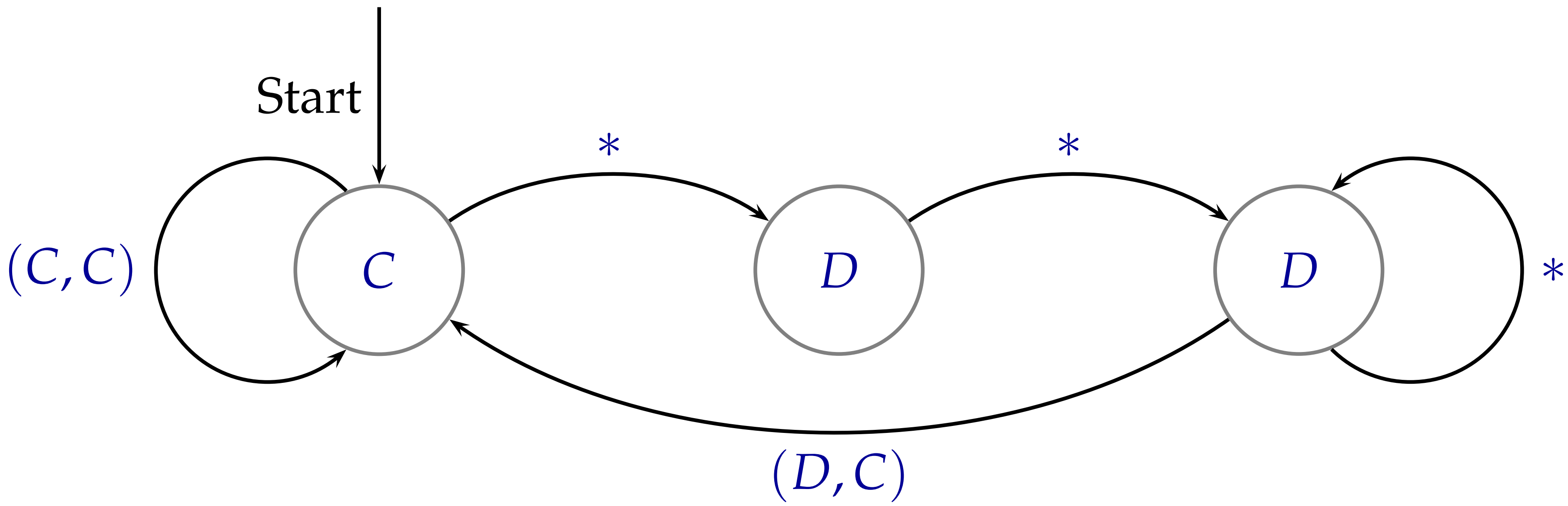

- Finite state machine for the **Prisoners' dilemma**.

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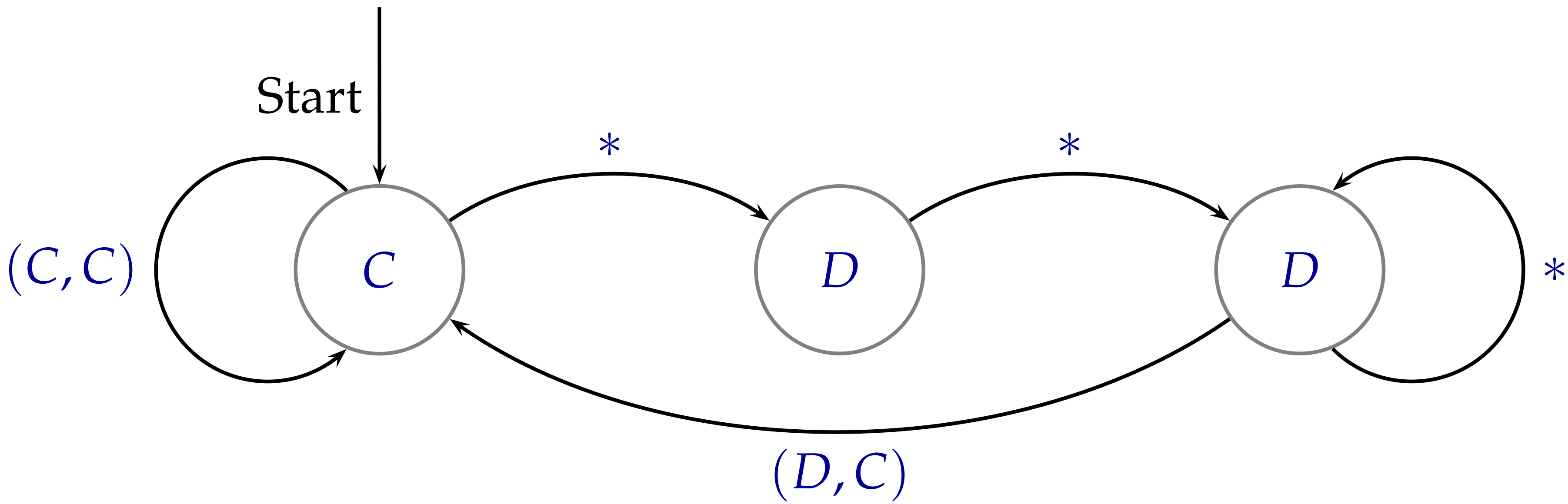

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- **Personal actions** determine states.

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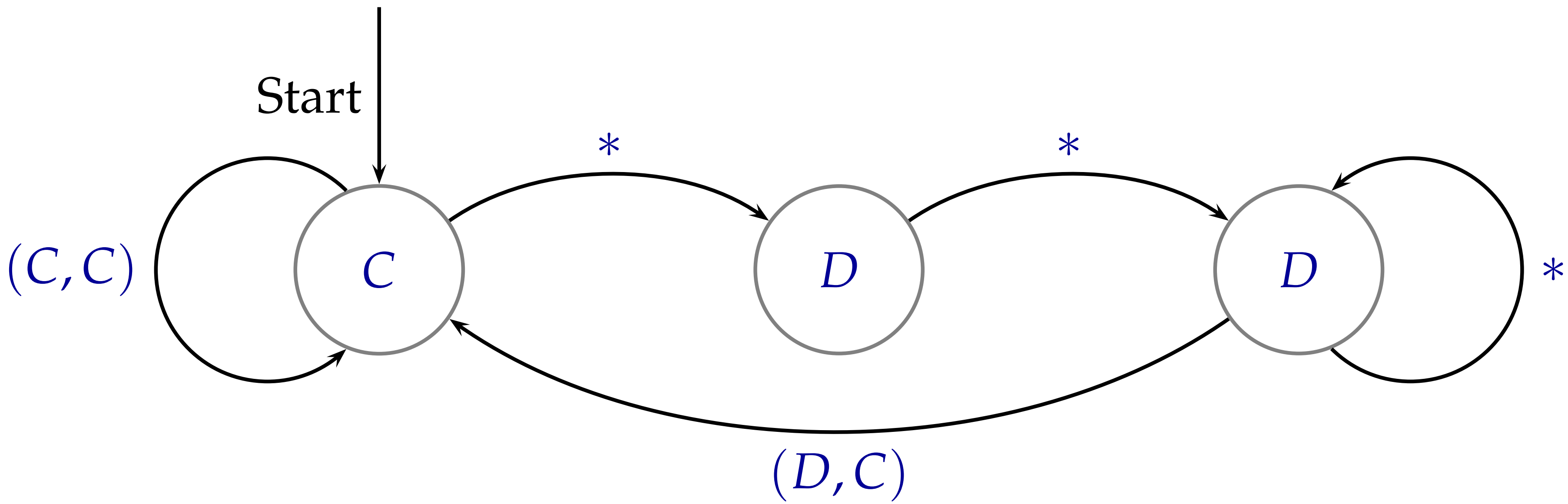
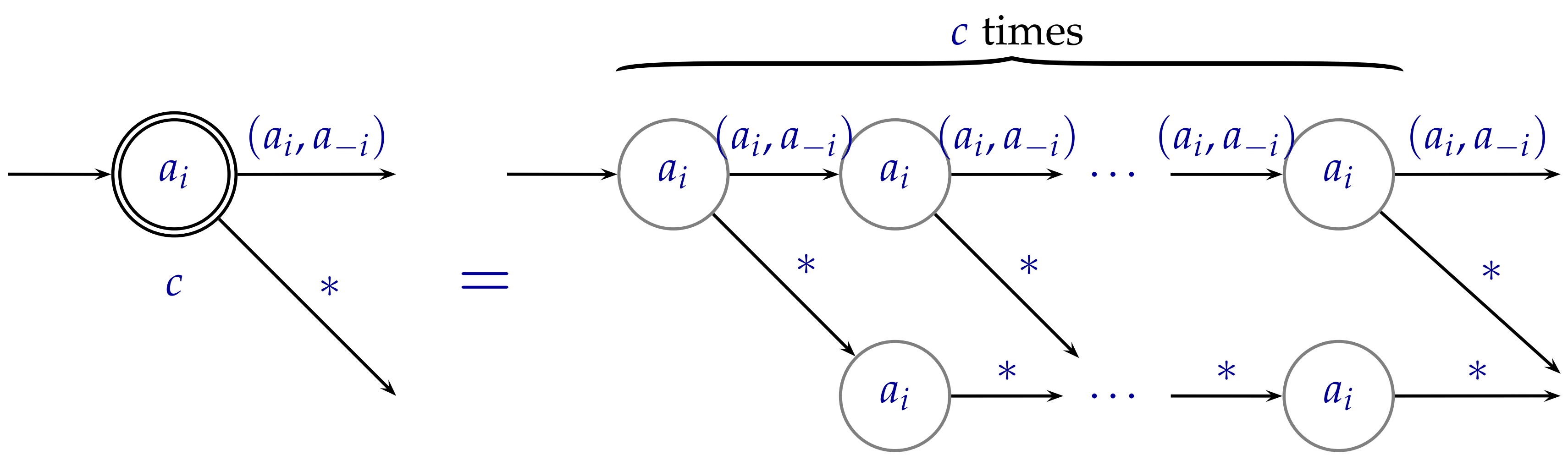


- Finite state machine for the **Prisoners' dilemma**.
- **Personal actions** determine states.
- **Action profiles** determine transitions between states.

  The "$*$" represents an "else," in the sense of "all other action profiles".

The use of counting nodes



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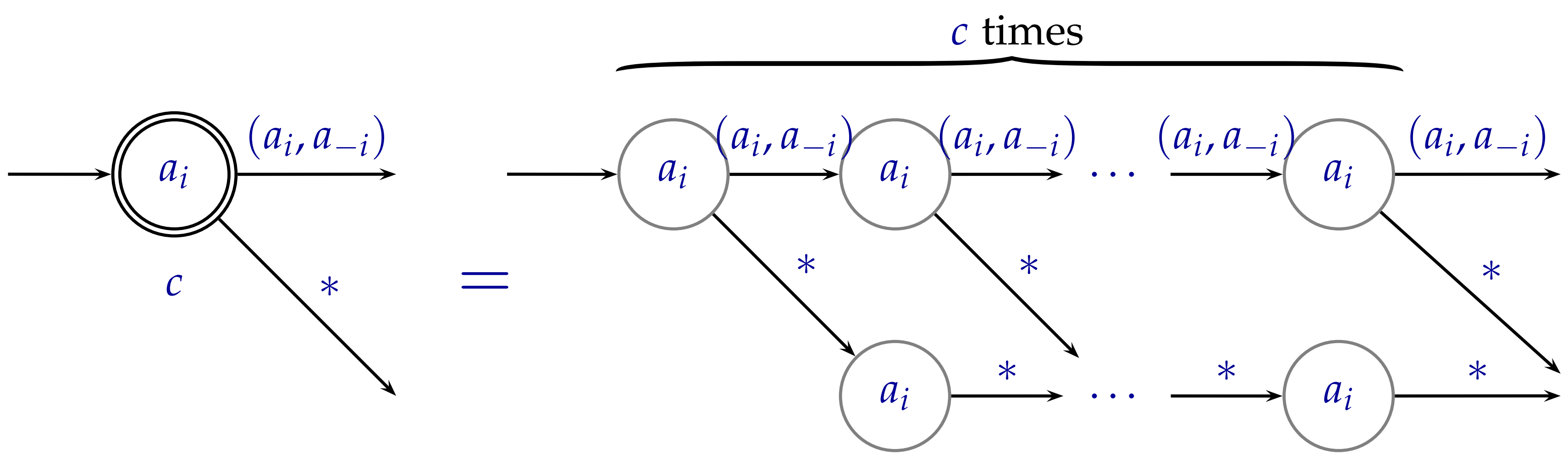

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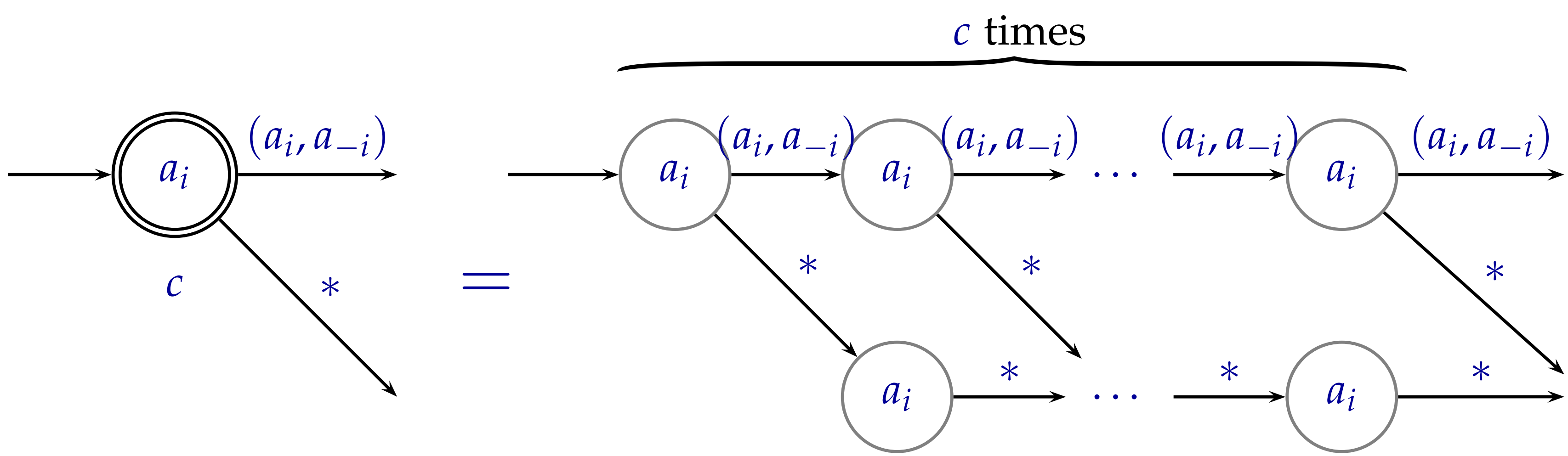

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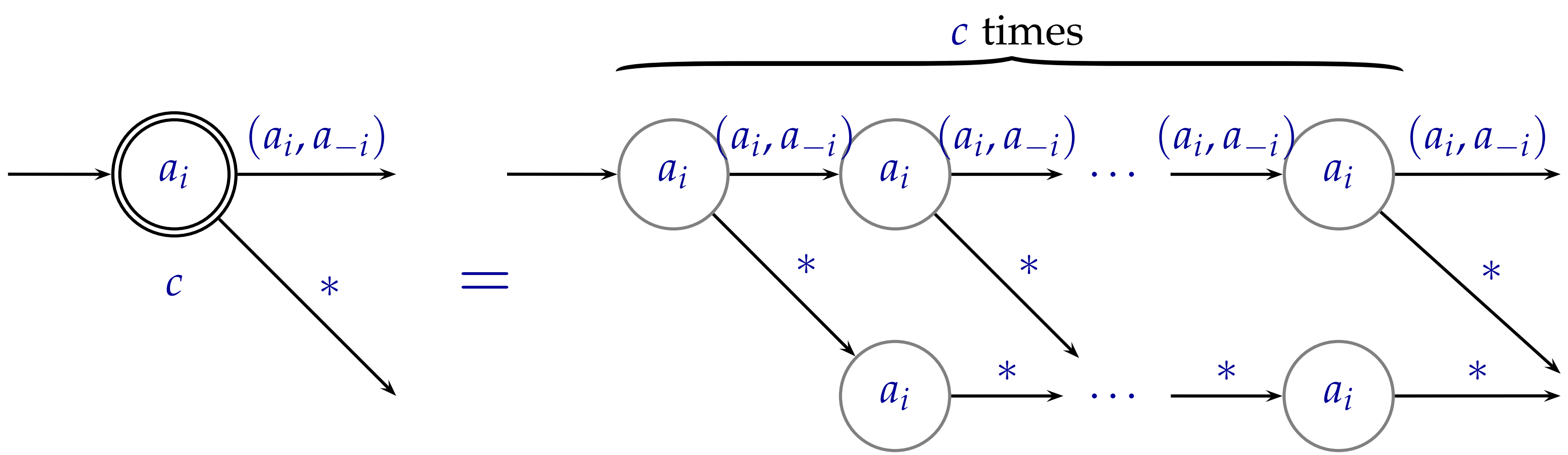

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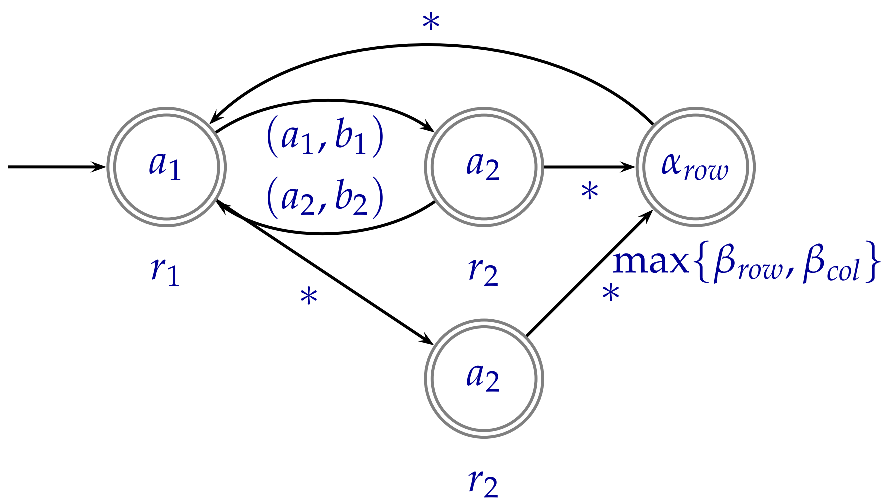
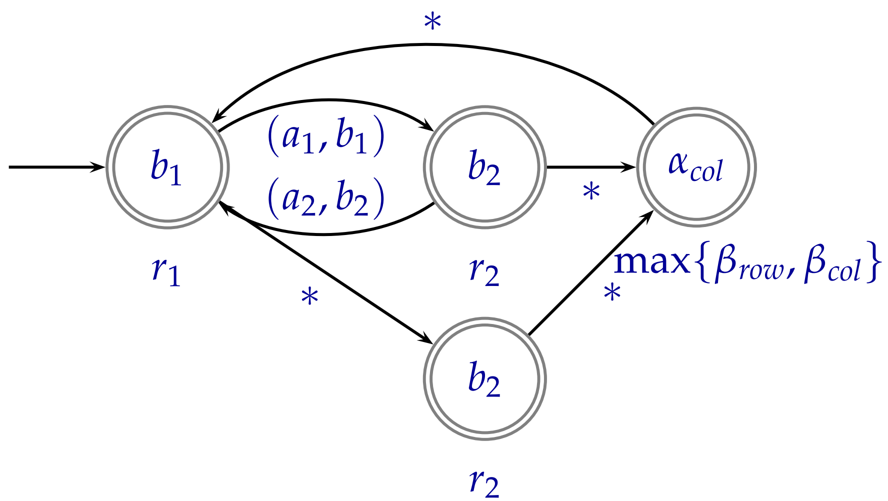
- Because integers up to c can be expressed in $\log c$ bits (roughly), size of finite machine is polynomial in $\log c$.

Pair of strategies that is a Nash equilibrium in a repeated game





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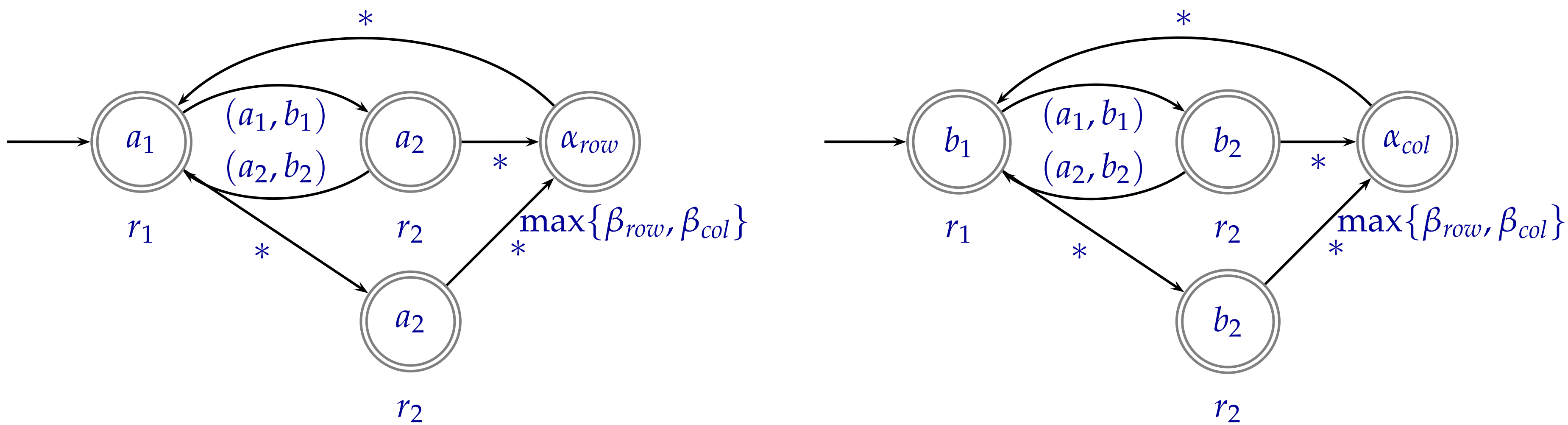

- Node a_1 and a_2 are the actions that row must play (in sync with col). First $r_1 \times a_1$, then $r_2 \times a_2$, then $r_1 \times a_1$, etc.

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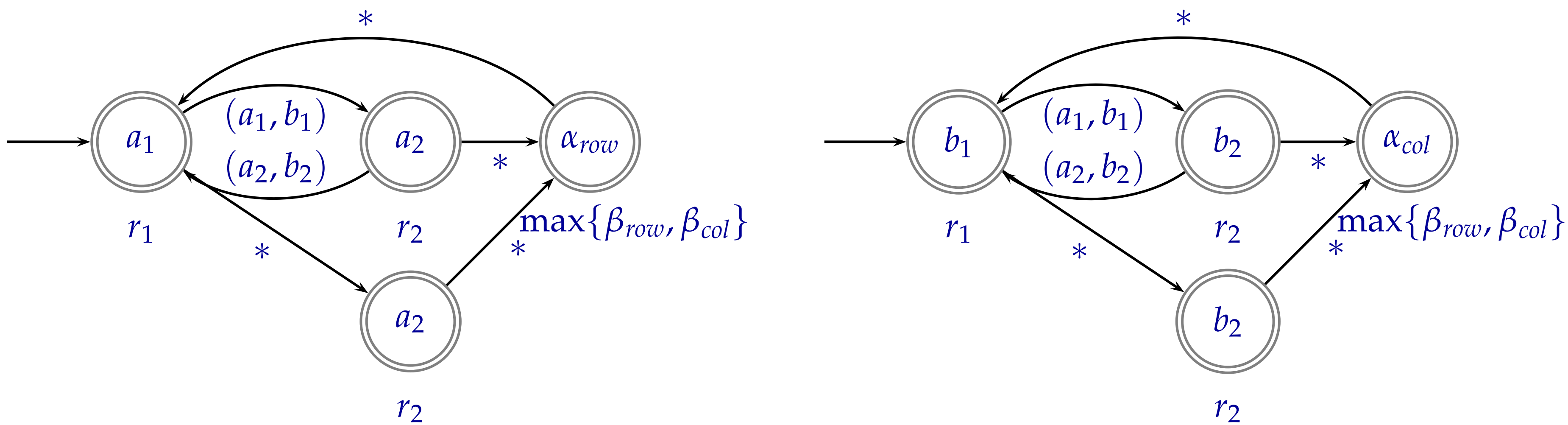

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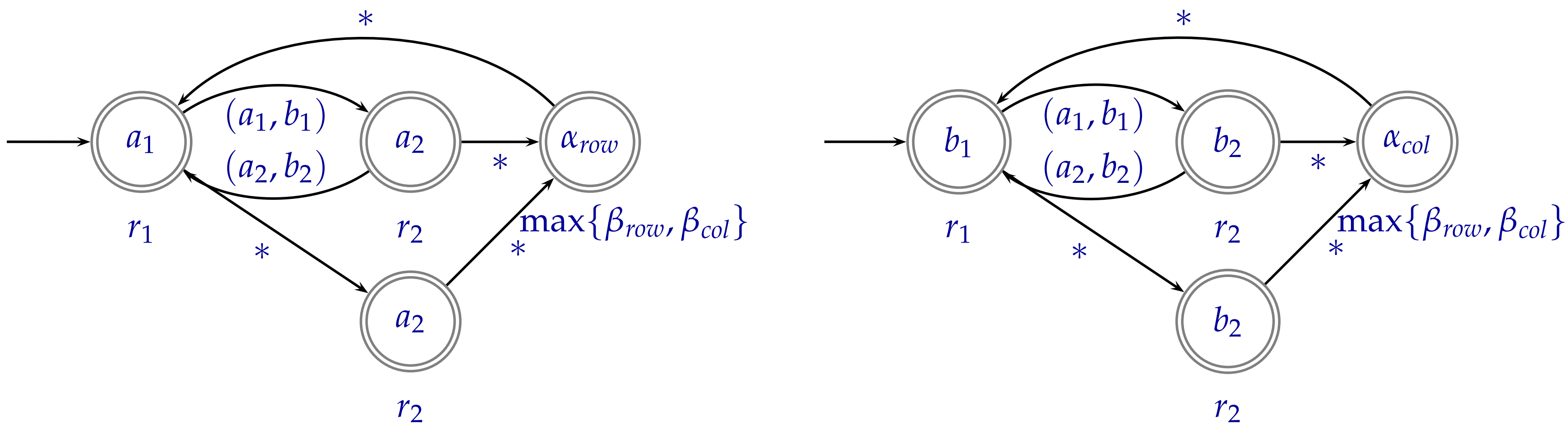

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- If opponent deviates, then retaliate with α_{row} for $\max\{\beta_{row}, \beta_{col}\}$ rounds.
- The two automata *always* run in sync, no matter who deviates first. It can (easily) be deduced that, for each player, deviating at any node is detrimental $\Rightarrow$ Nash equilibrium in repeated game.

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1. **The coordinated action profiles (a_1, b_1), (a_2, b_2) and their duration of play r_1, r_2.**

   Nash says: take strategy pair (s_1, s_2) that maximises the product of players' advantages. This pair can be obtained (or at least approximated) by playing convex

   $$\frac{r_1}{r_1 + r_2}(a_1, b_1) + \frac{r_2}{r_1 + r_2}(a_2, b_2)$$

   for r_1, r_2 not too large.

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- α_{row} and α_{col} are the minmax strategies of the stage game.
- β_{row} and β_{col} depend on turning points to "get even". These are determined by (i) the average payoff for cooperating (ii) upper bound on largest possible value for a single round of freeriding.

Part II:
Crandall & Goodrich (2005)