Multi-agent learning

Prediction, postdiction, and calibration

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6. Forecast rule (by agent): $p : H \to \Delta(Z)$:
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For example, if $|Z| = 3$ then $\|p^i - q^i\|^2$ might be

$$\left\| \begin{pmatrix} 0.2 \\ 0.1 \\ 0.7 \end{pmatrix} - \begin{pmatrix} 0.3 \\ 0.1 \\ 1.0 \end{pmatrix} \right\|^2 = \left\| \begin{pmatrix} 0.1 \\ 0.0 \\ 0.3 \end{pmatrix} \right\|^2 = 0.1^2 + 0.0^2 + 0.3^2 = 0.01 + 0.09 = 0.1$$

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- Having

$$\lim_{t\to\infty} \|p^t - q^t\| = 0 \text{ almost surely}$$

  is desirable. Later, it turns out this requirement *is too demanding*.

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Pessimistic result (Oakes, 1985): *For every forecasting rule f there exists a realisation ω and an $\epsilon > 0$ such that f does not ϵ-calibrate.*

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Now

$$\begin{aligned} C^t(\omega) &= \sum_{p \in \Delta(Z)} \frac{n^t(p)}{t} \|\phi^t(p) - p\|^2 \\ &= \sum_{p \geq 1/2} \frac{n^t(p)}{t} \|0 - p\|^2 + \sum_{p < 1/2} \frac{n^t(p)}{t} \|1 - p\|^2 \\ &\geq \lambda \|\frac{1}{2}\|^2 + (1-\lambda)\|\frac{1}{2}\|^2 = \lambda \frac{1}{4} + (1-\lambda)\frac{1}{4} = \frac{1}{4} > \epsilon. \end{aligned}$$

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5. F is calibrated if the average error goes to zero for every realisation almost surely.

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Theorem. (Foster and Vohra, 1997-98). *Given any finite set* Z *and any* $\epsilon > 0$*, there exist random forecasting rules that are* ϵ*-calibrated for all sequences on* Z*.*

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- $\| p' - p^* \|$ is small because the difference in conditional regrets is small. It follows that $\| \phi(p^*) - p^* \|$ is small. (Triangle inequality.)

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- If all predictors are bad, there must be a skew pair. (Check!)

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  To let p start at "the right place", setting $\phi^0(p) =_{Def} p$ is also good.

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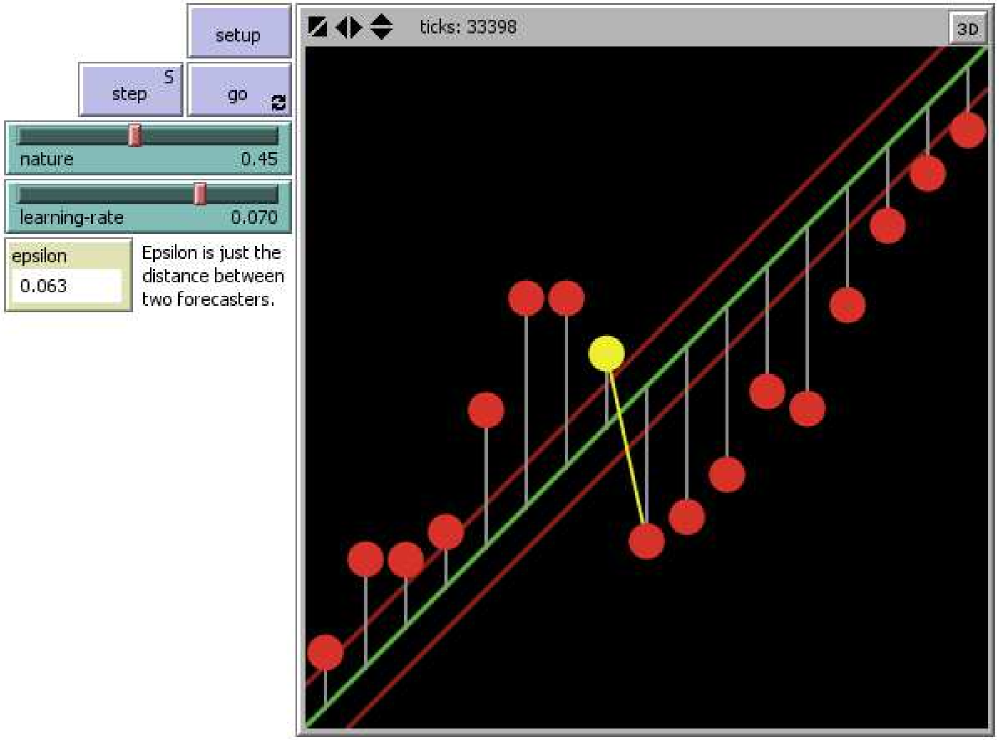
2. Else, pick a skew pair and forecast with either one of them (choose randomly).



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3. Make action vector orthogonal to error vector:

$$\underbrace{\begin{pmatrix} b^t \\ a^t \end{pmatrix}}_{\text{Actions}} \perp \underbrace{\begin{pmatrix} a^t \\ -b^t \end{pmatrix}}_{\text{Error}}.$$

Now forecast with odds $b^t : a^t$, i.e., with p a $b^t/(a^t + b^t)$ of the times.