Multi-agent learning

Teaching strategies

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Thursday 18th June, 2020

Plan for Today

Part I: Preliminaries

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1. Three points of criticism to Godfather++.
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3. Notion of guilt to trigger switches between teaching and following.

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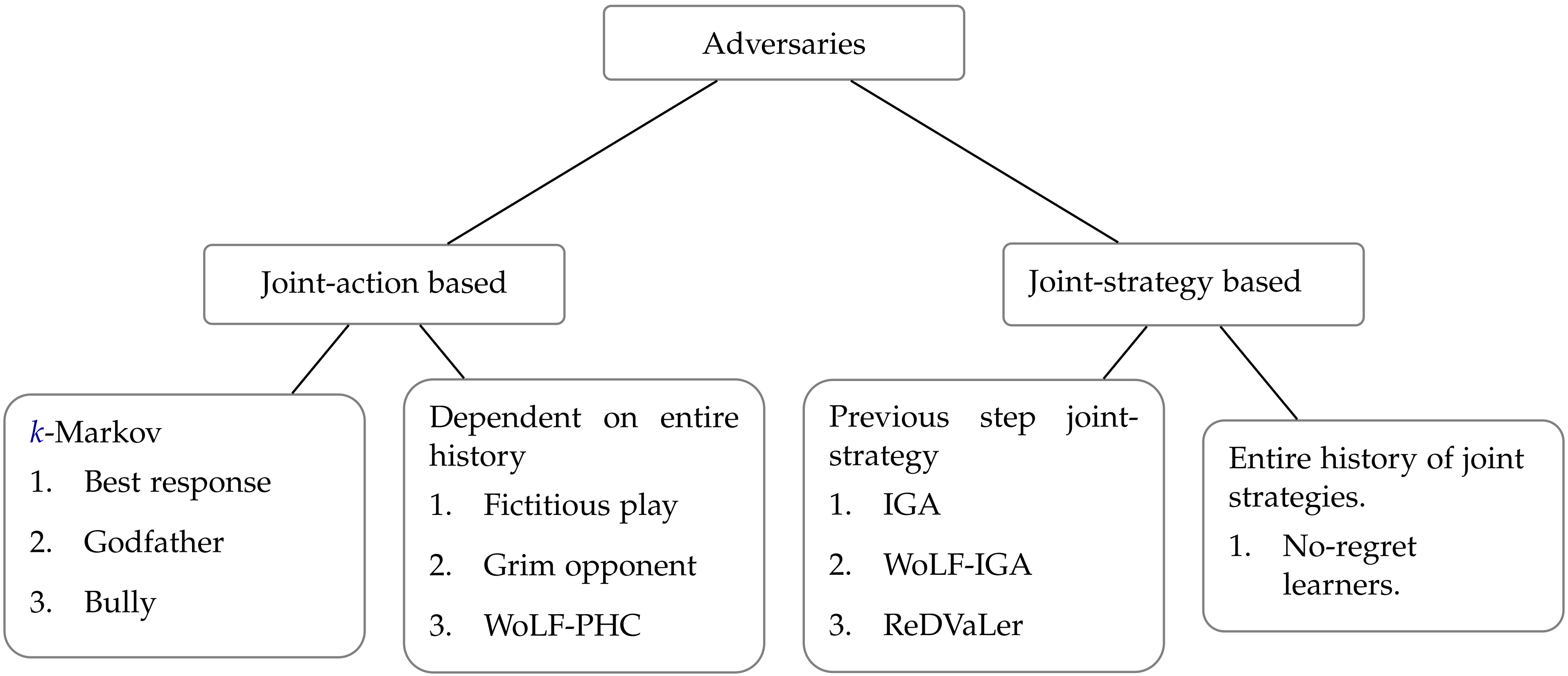
Doran Chakraborty and Peter Stone (2008). "Online Multiagent Learning against Memory Bounded Adversaries," *Machine Learning and Knowledge Discovery in Databases*, Lecture Notes in Artificial Intelligence Vol. 5212, pp. 211-26

Taxonomy of possible adversaries

(Taken from Chakraborty and Stone, 2008):



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$$\begin{array}{cc} & \begin{array}{ccc} L & M & R \end{array} \\ \begin{array}{c} T \\ C \\ B \end{array} & \begin{pmatrix} 3,6 & 8,6 & 7,3 \\ 8,1 & 6,3 & 7,3 \\ 3,5 & 9,2 & 7,5 \end{pmatrix} \end{array}$$

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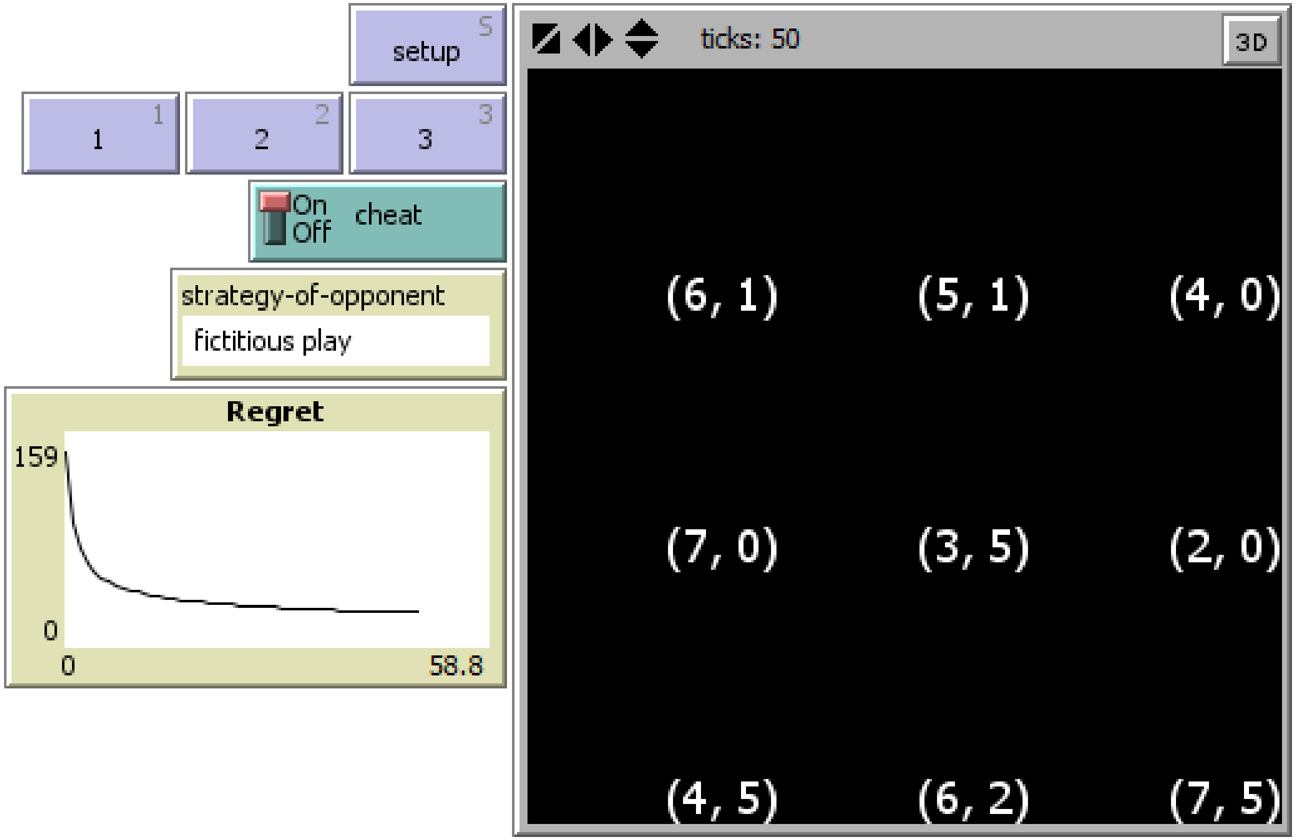
Idea for an app to learn to play {against} Bully



Play against the computer. At the outset, the computer initializes to either Bully (with a probability of 50%) or pure fictitious play, the choice of which you can't see. After that, the computer won't change strategy. Try to press regret down as within few rounds as possible.

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  Recognise the maxmin = the security value in this formula!

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- Bully is stateless (a.k.a. memoryless, i.e, memory of $k = 0$ rounds), hence keeps playing the same action throughout.

Godfather

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- Godfather needs a memory of $k = 1$ (one round).

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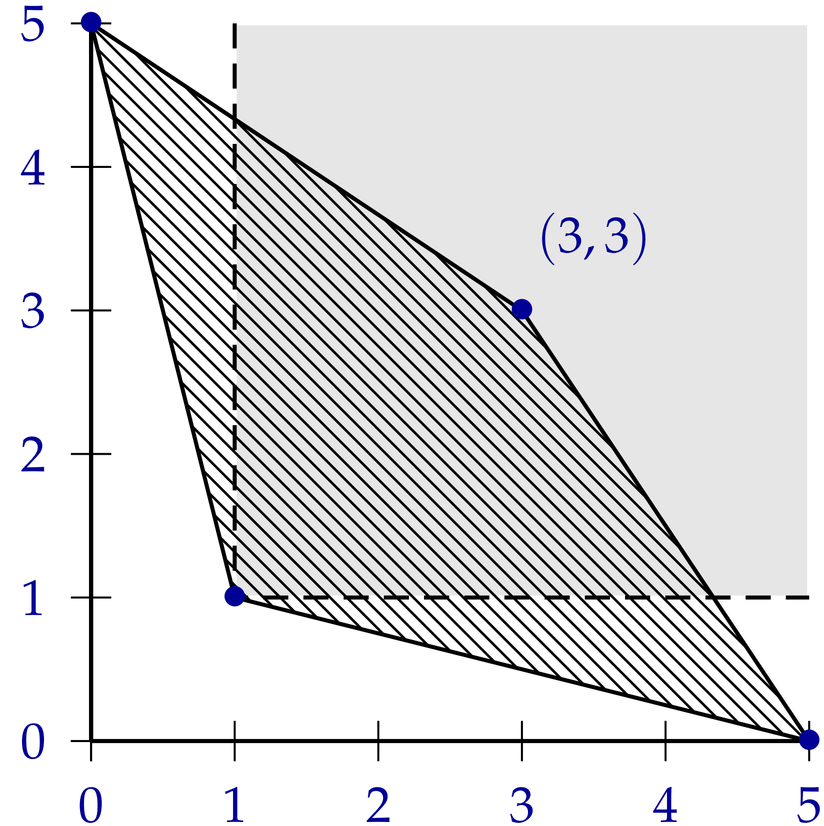
- **Enforceable payoffs** (shaded): no one goes below their minmax.

Theorem. If (x, y) is both feasible and enforceable, then (x, y) is the payoff in a Nash equilibrium of the infinitely repeated G with average payoffs.
Conversely, if (x, y) is the payoff in any Nash equilibrium of the infinitely repeated G with average payoffs, then (x, y) is enforceable.



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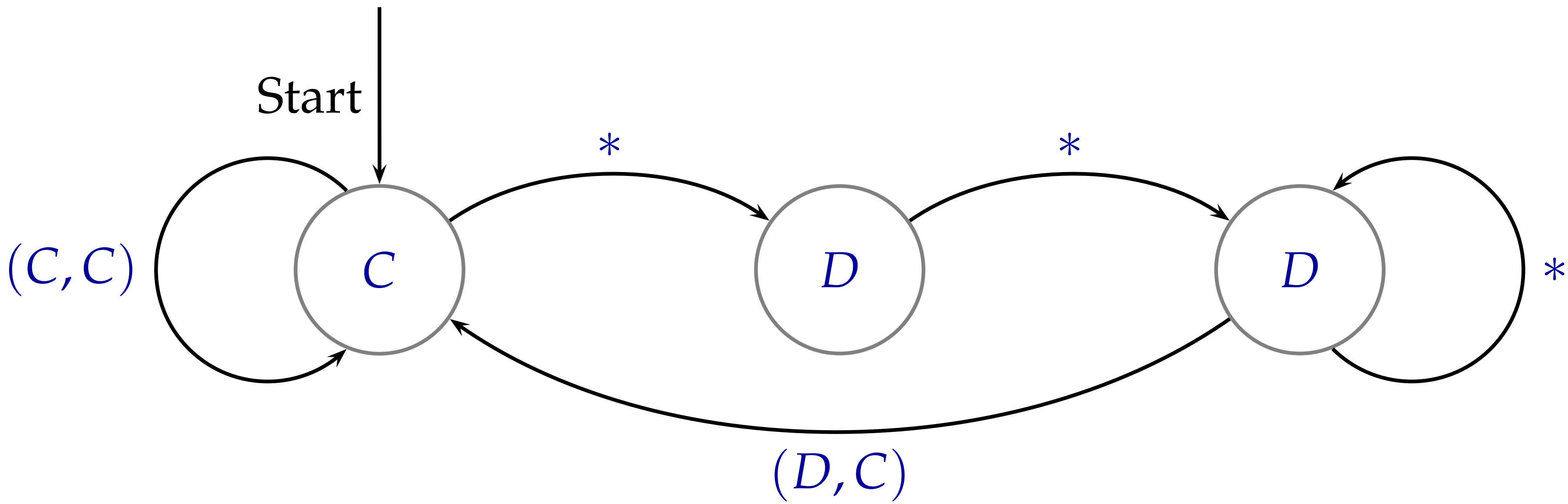
Finite machine for “two tits for tat”



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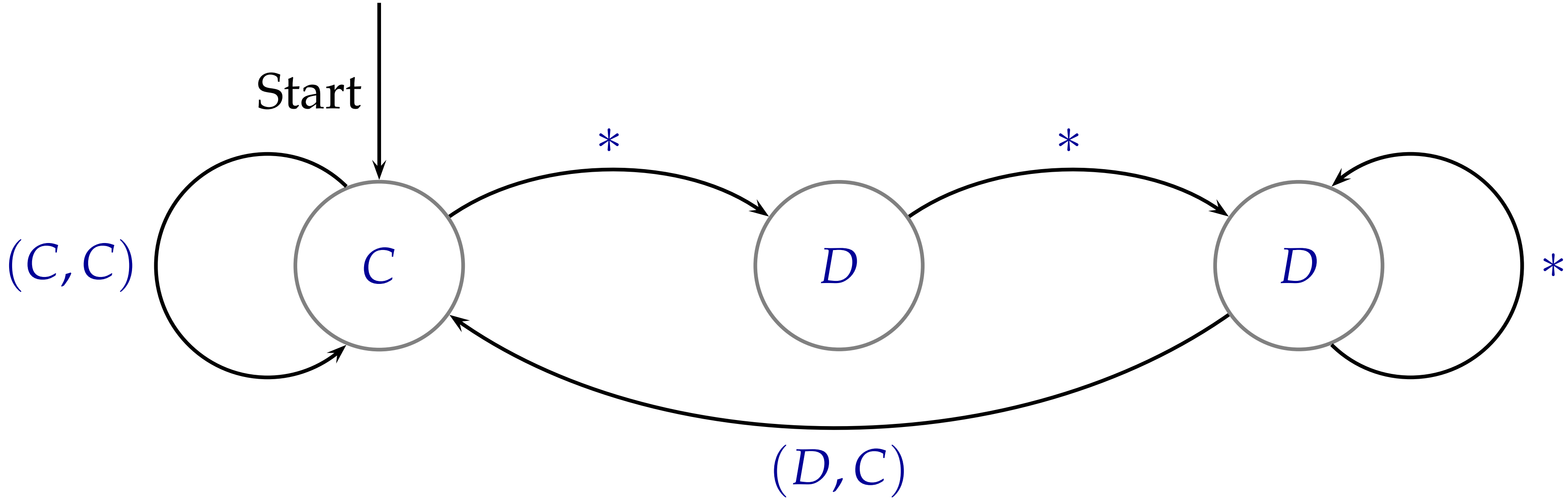

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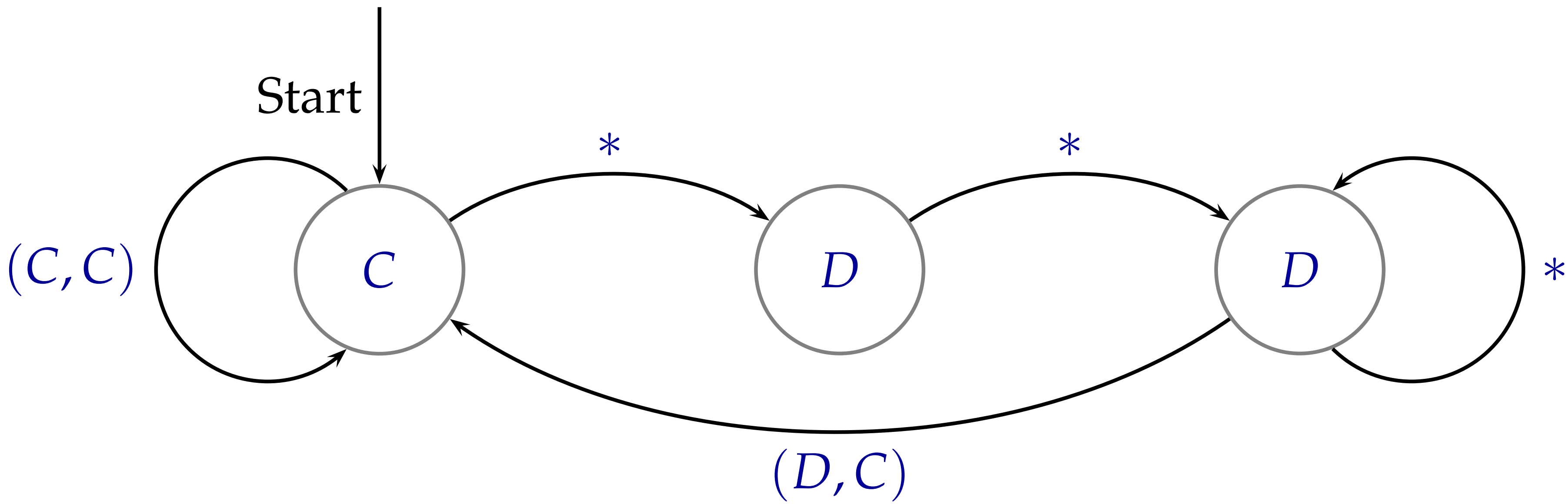
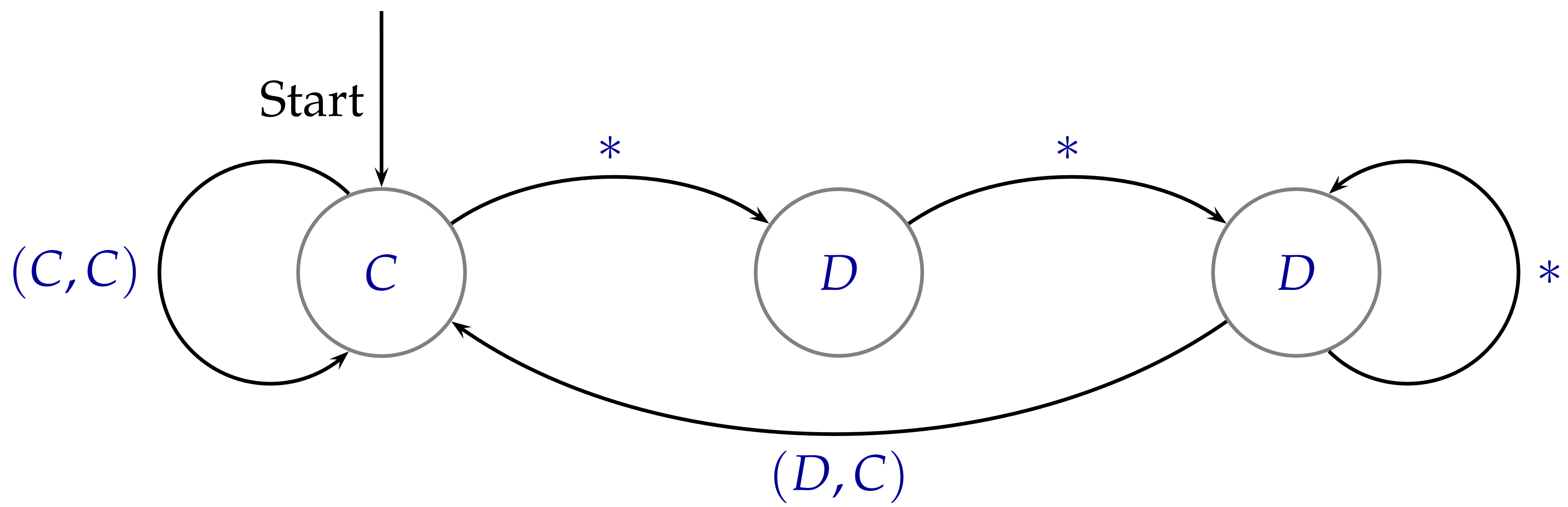


- Finite state machine for the **Prisoners' dilemma**.
- **Personal actions** determine states.

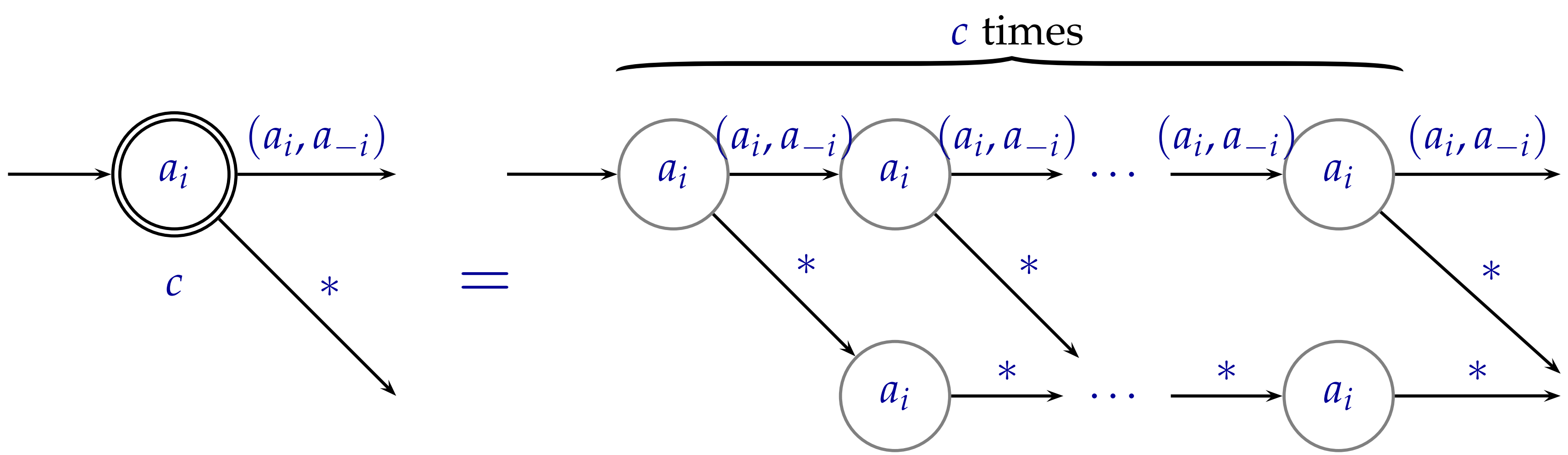
Finite machine for “two tits for tat”



- Finite state machine for the **Prisoners’ dilemma**.
- **Personal actions** determine states.
- **Action profiles** determine transitions between states.

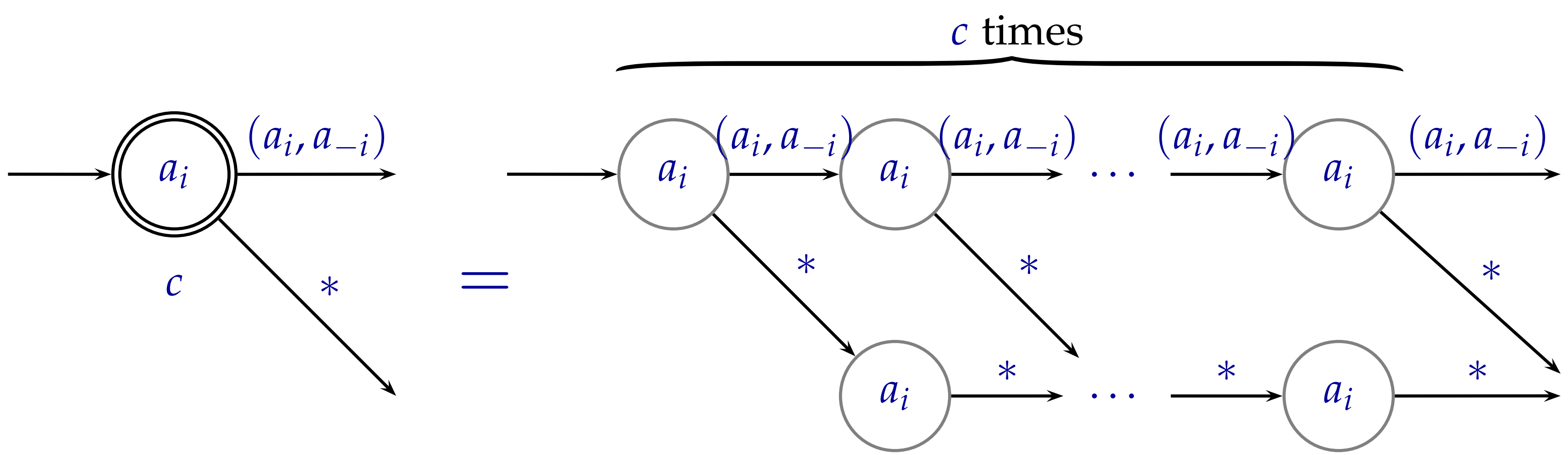
  The “$*$” represents an “else,” in the sense of “all other action profiles”.

The use of counting nodes



Upon entry:

The use of counting nodes



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- If exactly c times action profile (a_i, a_{-i}) is played, then take exit above.

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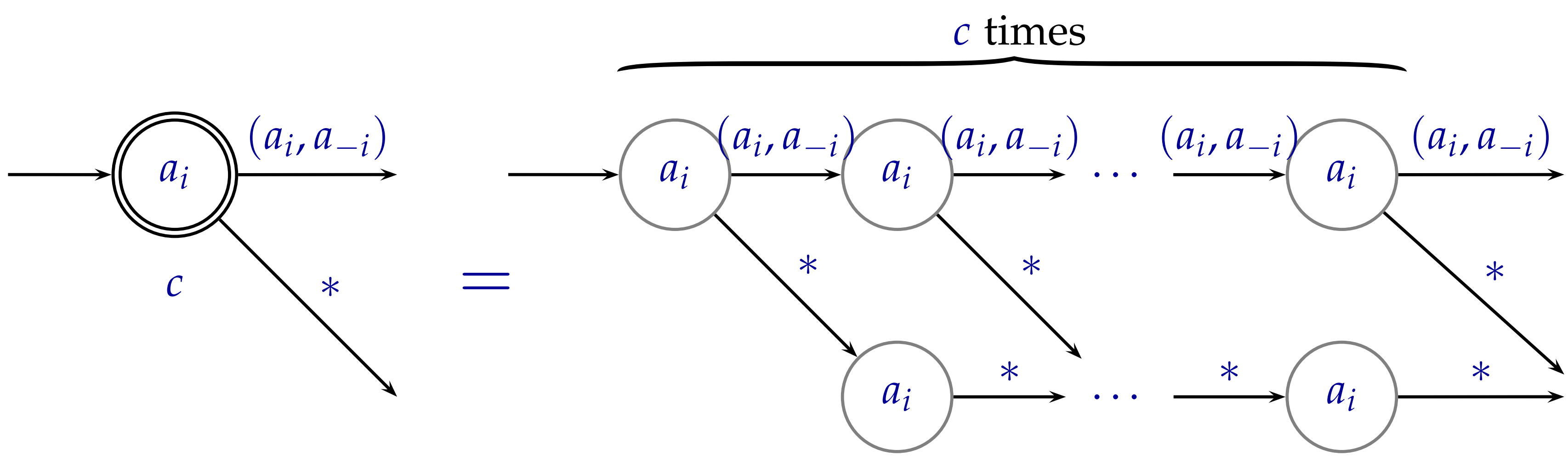

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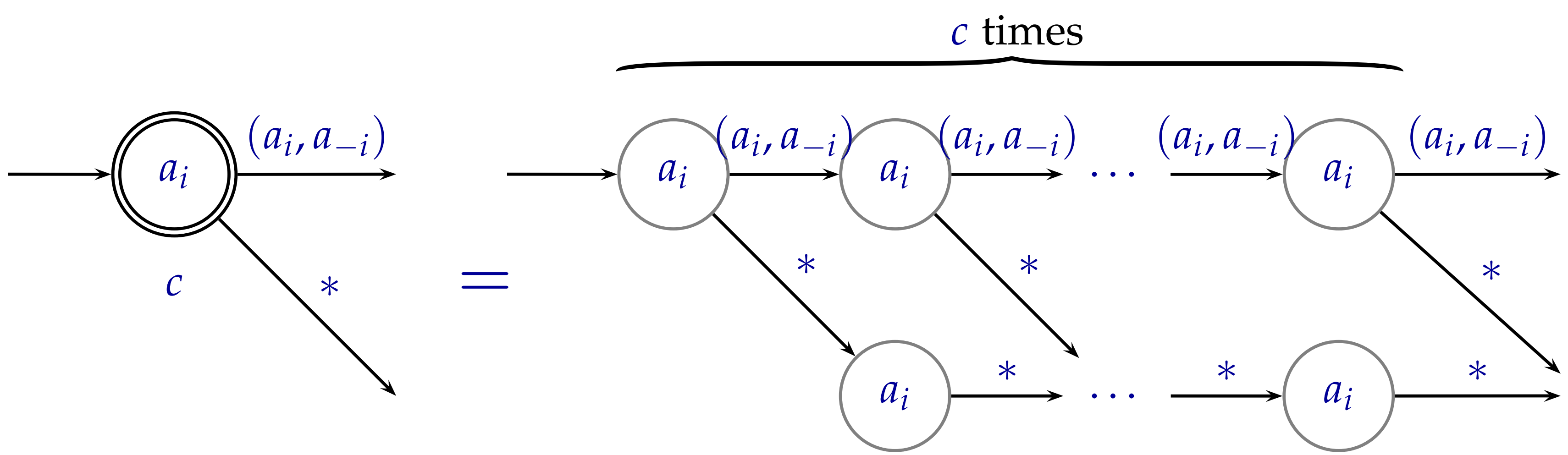

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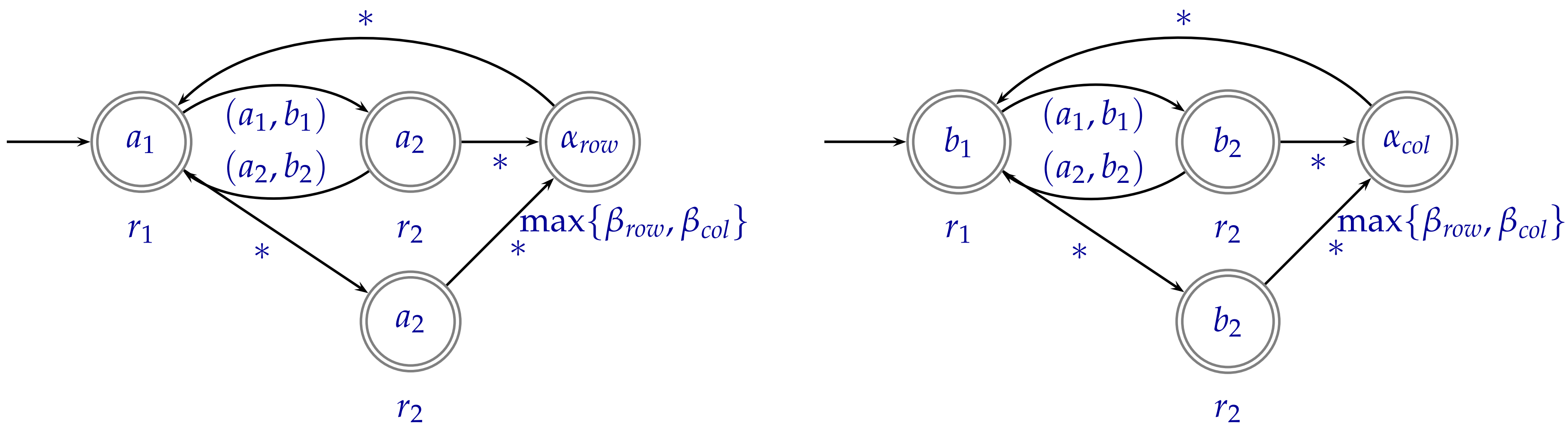
- Because integers up to c can be expressed in $\log c$ bits (roughly), size of finite machine is polynomial in $\log c$.

Pair of strategies that is a Nash equilibrium in a repeated game



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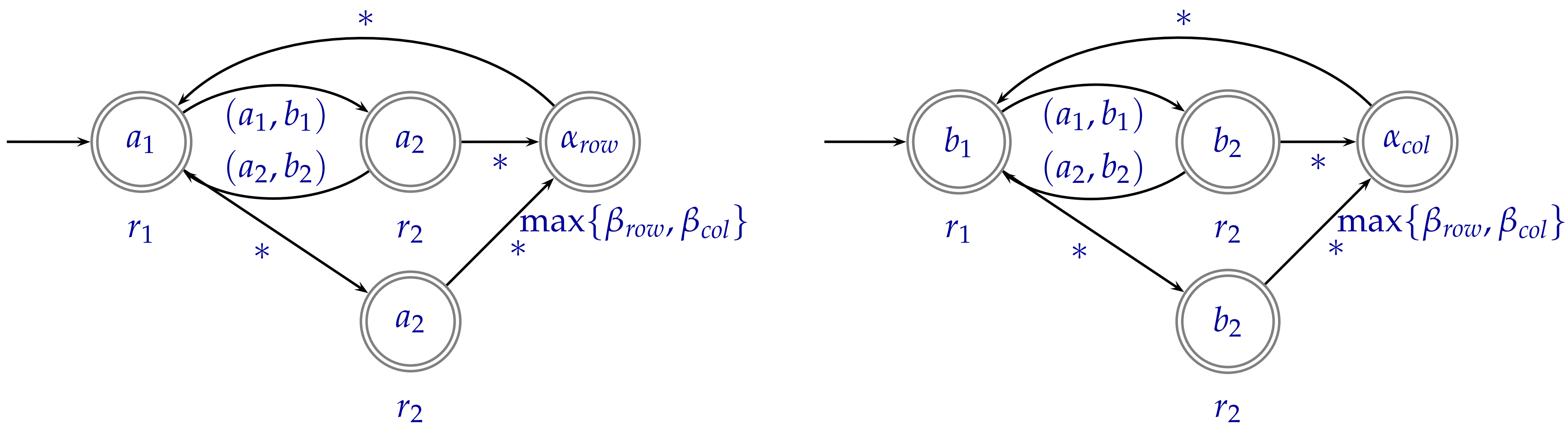

- Node a_1 and a_2 are the actions that row must play (in sync with col). First $r_1 \times a_1$, then $r_2 \times a_2$, then $r_1 \times a_1$, etc.

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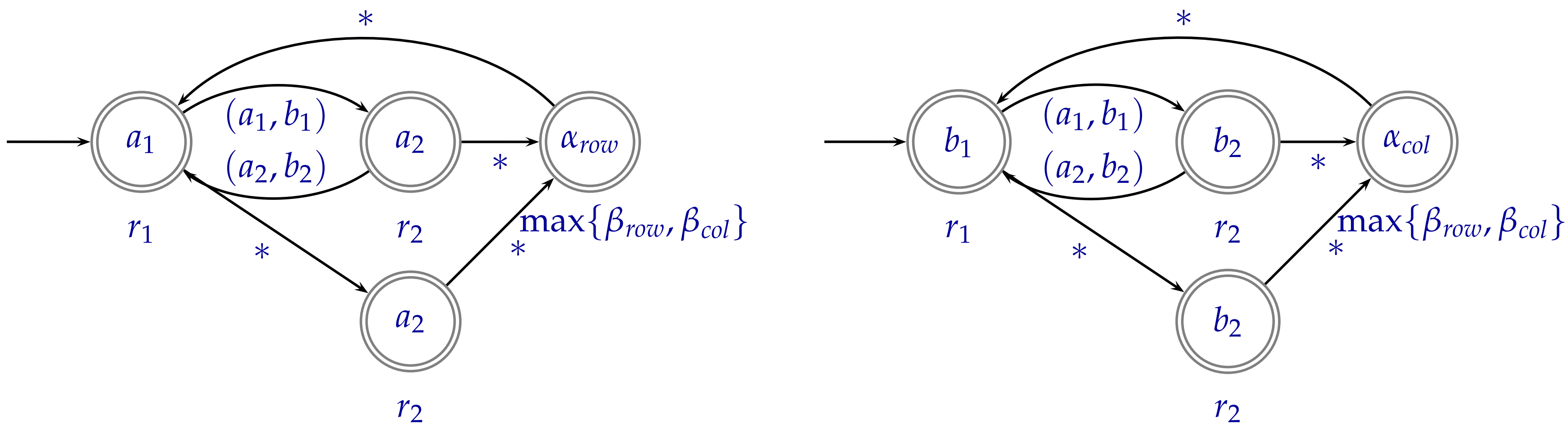

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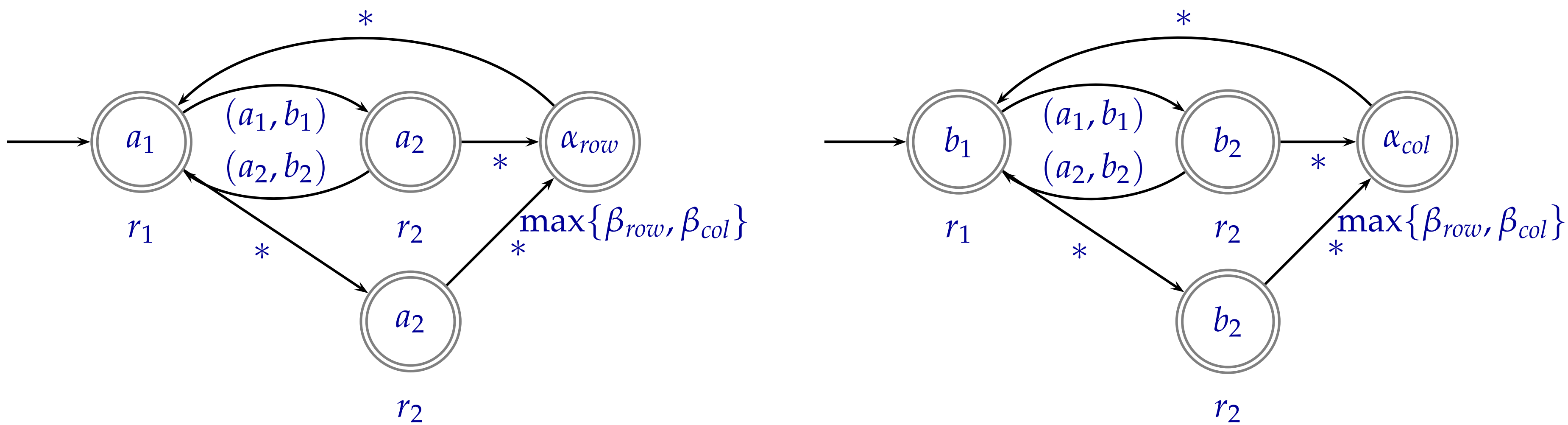

- Node a_1 and a_2 are the actions that row must play (in sync with col). First $r_1 \times a_1$, then $r_2 \times a_2$, then $r_1 \times a_1$, etc.
- If opponent deviates, then retaliate with α_{row} for $\max\{\beta_{row}, \beta_{col}\}$ rounds.
- The two automata *always* run in sync, no matter who deviates first. It can (easily) be deduced that, for each player, deviating at any node is detrimental $\Rightarrow$ Nash equilibrium in repeated game.

The devil and the details…

It should be that all parameters can be determined analytically, in polynomial time.

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1. **The coordinated action profiles (a_1, b_1), (a_2, b_2) and their duration of play r_1, r_2.**

   Nash says: take strategy pair (s_1, s_2) that maximises the product of players' advantages. This pair can be obtained (or at least approximated) by playing convex

   $$\frac{r_1}{r_1 + r_2}(a_1, b_1) + \frac{r_2}{r_1 + r_2}(a_2, b_2)$$

   for r_1, r_2 not too large.

   Pair (s_1, s_2) is obtained by looping through $(A^2)^2$ (all pairs of pairs of actions).

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   - α_{row} and α_{col} are the minmax strategies of the stage game.
   - β_{row} and β_{col} depend on turning points to "get even". These are determined by (i) the average payoff for cooperating (ii) upper bound on largest possible value for a single round of freeriding.

Part II:
Crandall & Goodrich (2005)