Multi-agent learning

Comparing algorithms empirically

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  The **algorithms** themselves and the **outcomes** of the comparison are of secondary interest in our review today!

Work discussed

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- Airiau *et al.*: evolutionary dynamics.

  Airiau, Stéphane, Sabyasachi Saha, and Sandip Sen. "Evolutionary tournament-based comparison of learning and non-learning algorithms for iterated games" in: *Journal of Artificial Societies and Social Simulation* **10**.3 (2007).

Round robin tournament

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Given a pool of games $\mathcal{G}$ to test on, all approaches have in common that they have a grand table of head-to-head scores:

	A_1	A_2	$\dots$	A_{12}	avg
algorithm A_1	3.2	5.1	$\dots$	4.7	4.1
A_2	2.4	1.2	$\dots$	2.2	1.3
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
A_{12}	3.1	6.1	$\dots$	3.8	4.2

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- Often each entry is computed multiple times to even out randomness in algorithms (which are implementations of response rules).
- Sometimes there is a settling-in phase (a.k.a. burn-in phase) in which payoffs are not yet recorded.

Work of Axelrod (1980, 1984)

Axelrod receiving the National Medal of Science (2014)

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- In 2012, Alexander Stewart and Joshua Plotkin ran a variant of Axelrod's tournament with 19 strategies to test the effectiveness of the then newly discovered Zero-Determinant strategies.

Work of Zawadzki *et al.* (2014)

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- Games: a suite of 13 interesting families, $\mathcal{D} = D_1, \dots, D_{13}$: $D_1 =$ games with normal covariant random payoffs; $D_2 =$ Bertrand oligopoly; $D_3 =$ Cournot duopoly; $D_4 =$ dispersion games; $D_5 =$ grab the dollar type games; $D_6 =$ guess two thirds of the average games; ...

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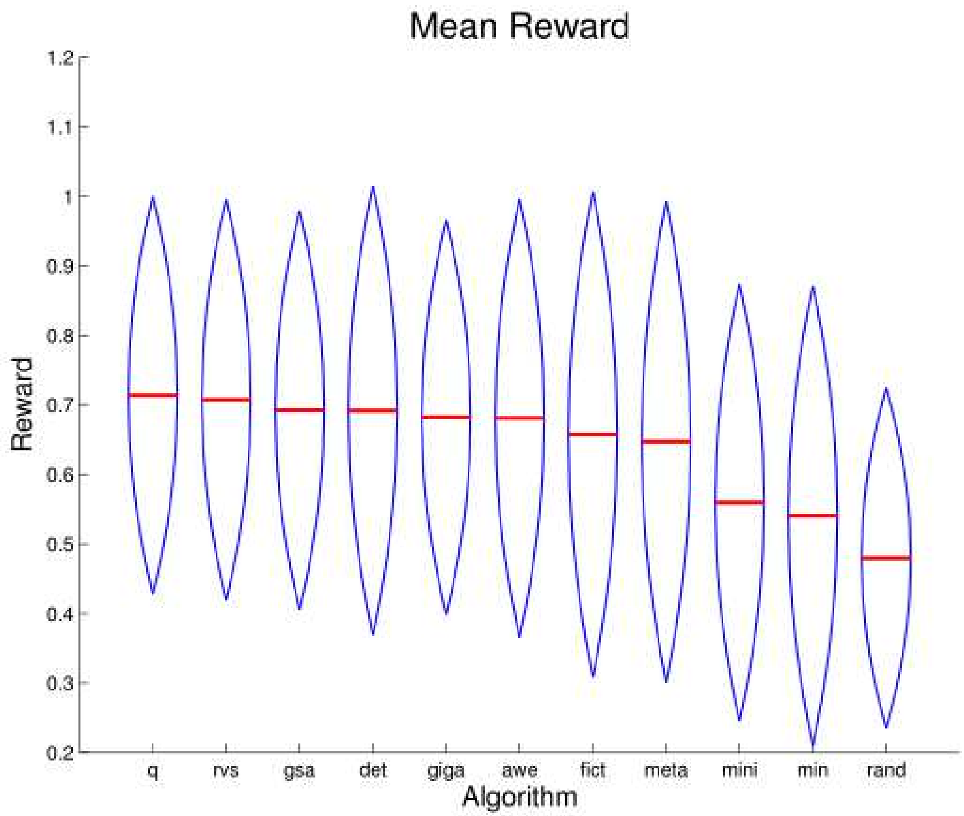
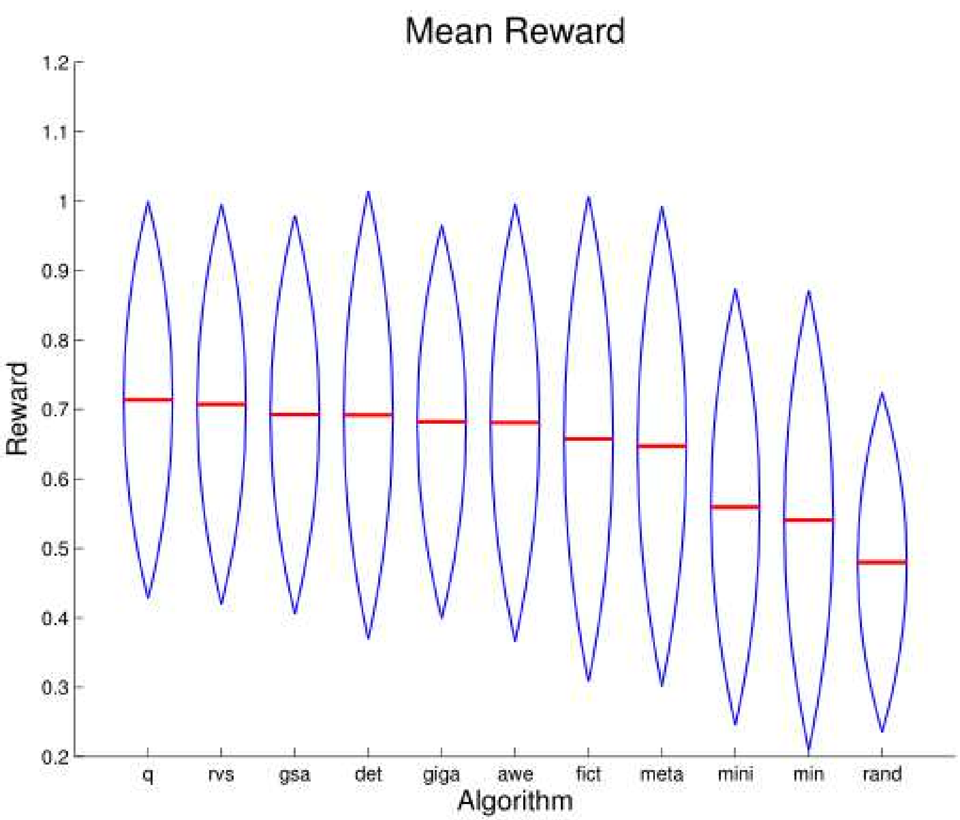
- Conclusion: Q-learning is the overall winner.

Zawadzki *et al.*



Mean reward over all opponents and games.

Zawadzki *et al.*



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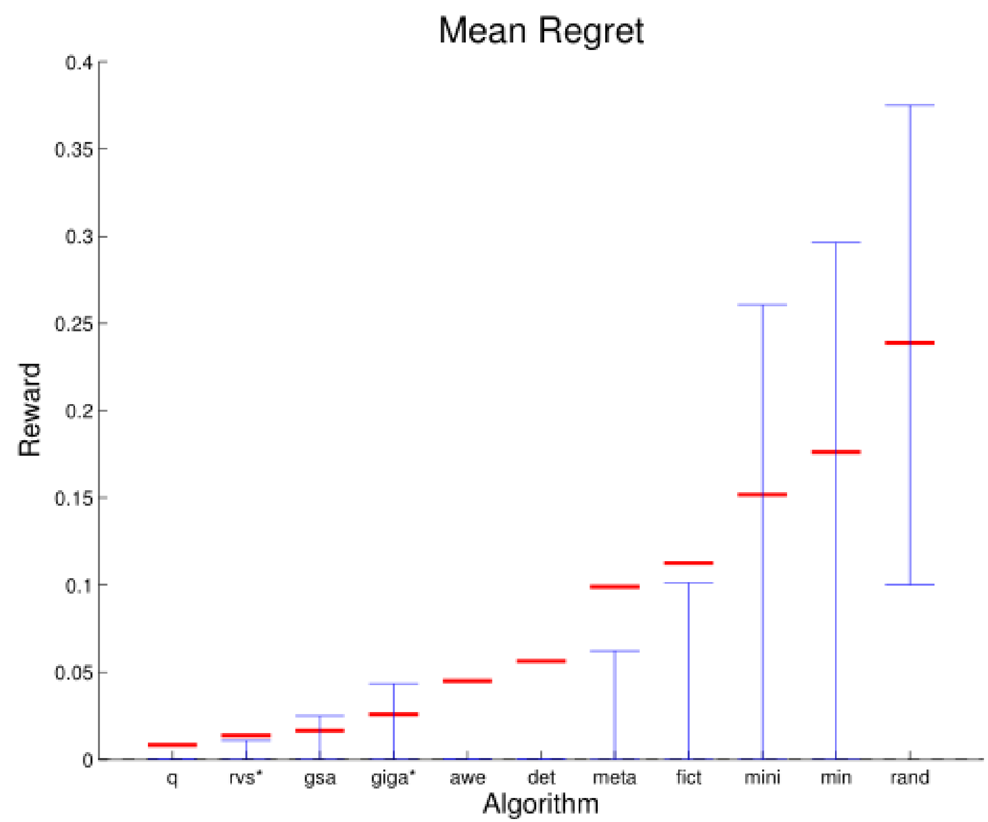

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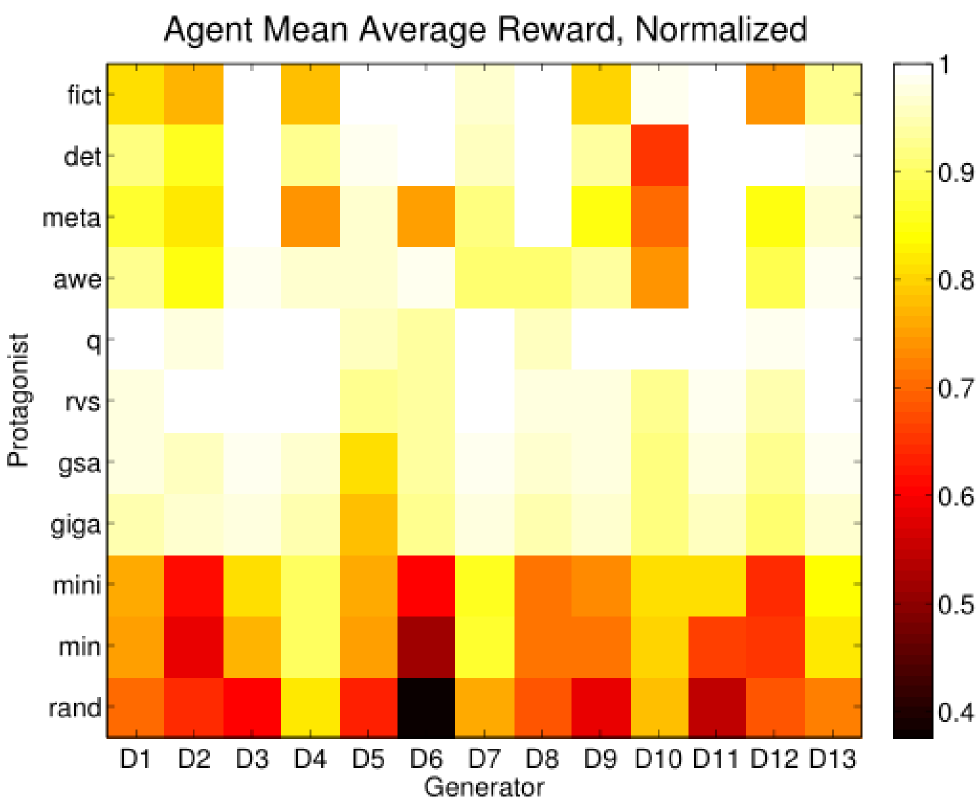

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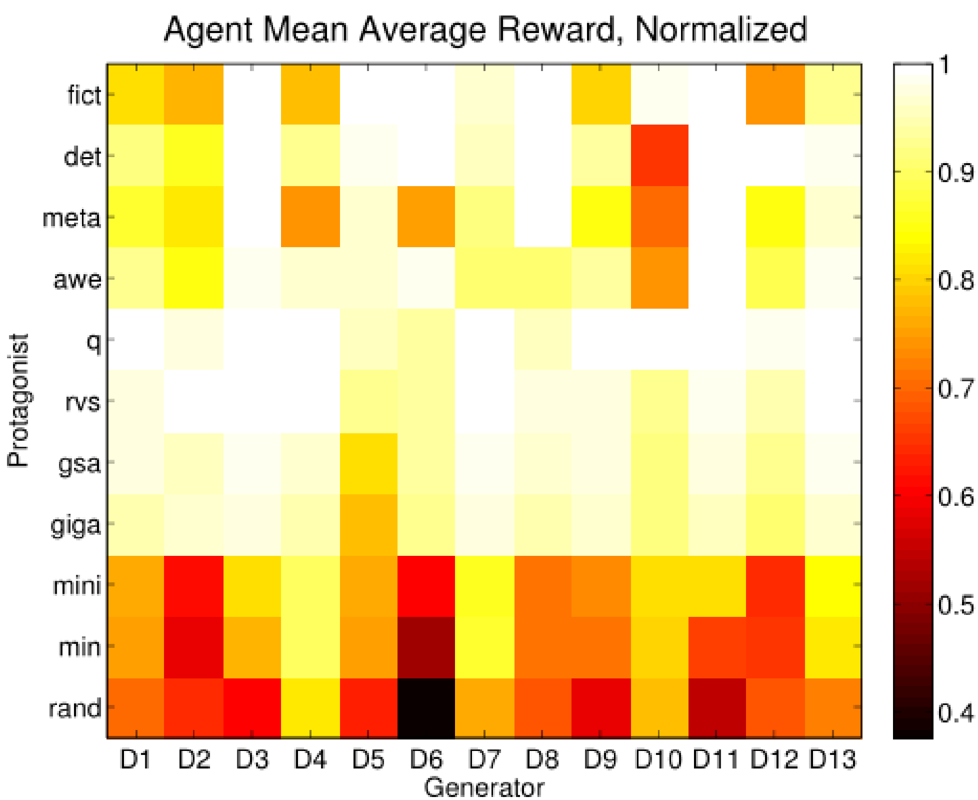

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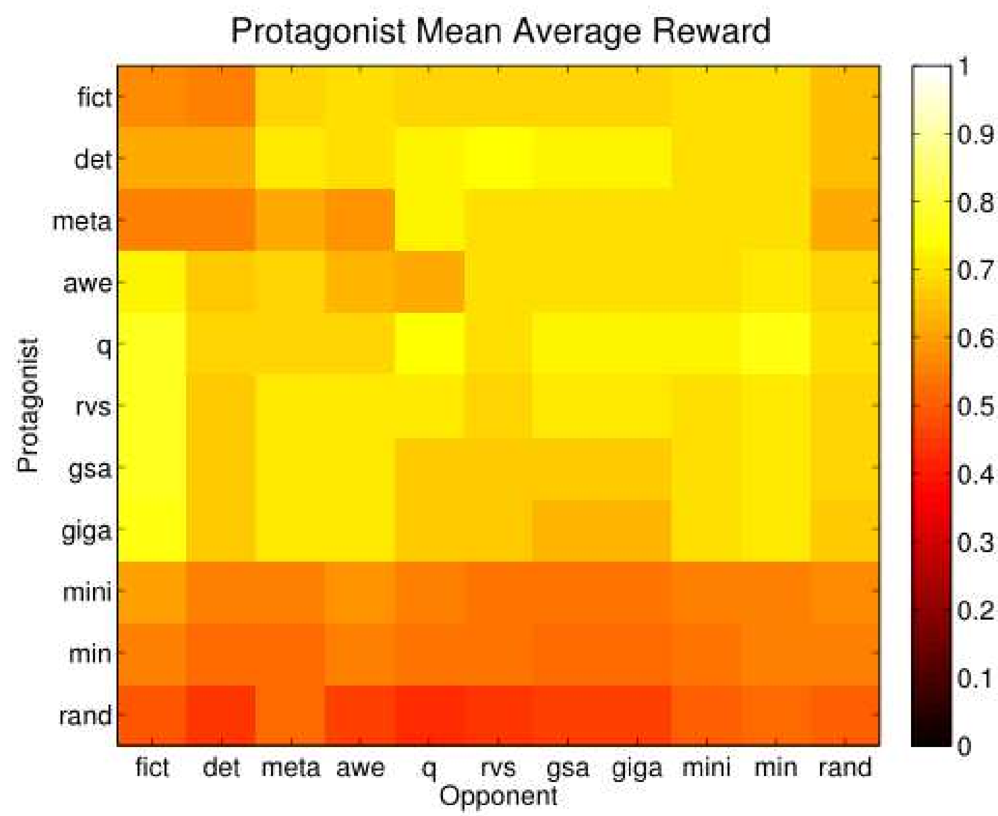

Mean reward against different opponents.

Parametric test: paired t-test

	A_1			A_2			A_3			*dots*		
A_1	G_1	G_2	G_3	G_1	G_2	G_3	G_1	G_2	G_3	...	...	...
	2.1	3.1	4.7	5.1	1.1	1.2	3.5	4.2	3.8	...	...	...
A_2	G_1	G_2	G_3	G_1	G_2	G_3	G_1	G_2	G_3	...	...	...
	2.7	3.5	4.1	4.9	0.9	1.9	3.7	4.7	4.5	...	...	...

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- If the two series are generated by the same random process, the test statistic $t = \bar{X}_D / (\bar{s}_D \sqrt{n})$ should follow the Student's t-distribution with mean 0 and $n - 1$ degrees of freedom.

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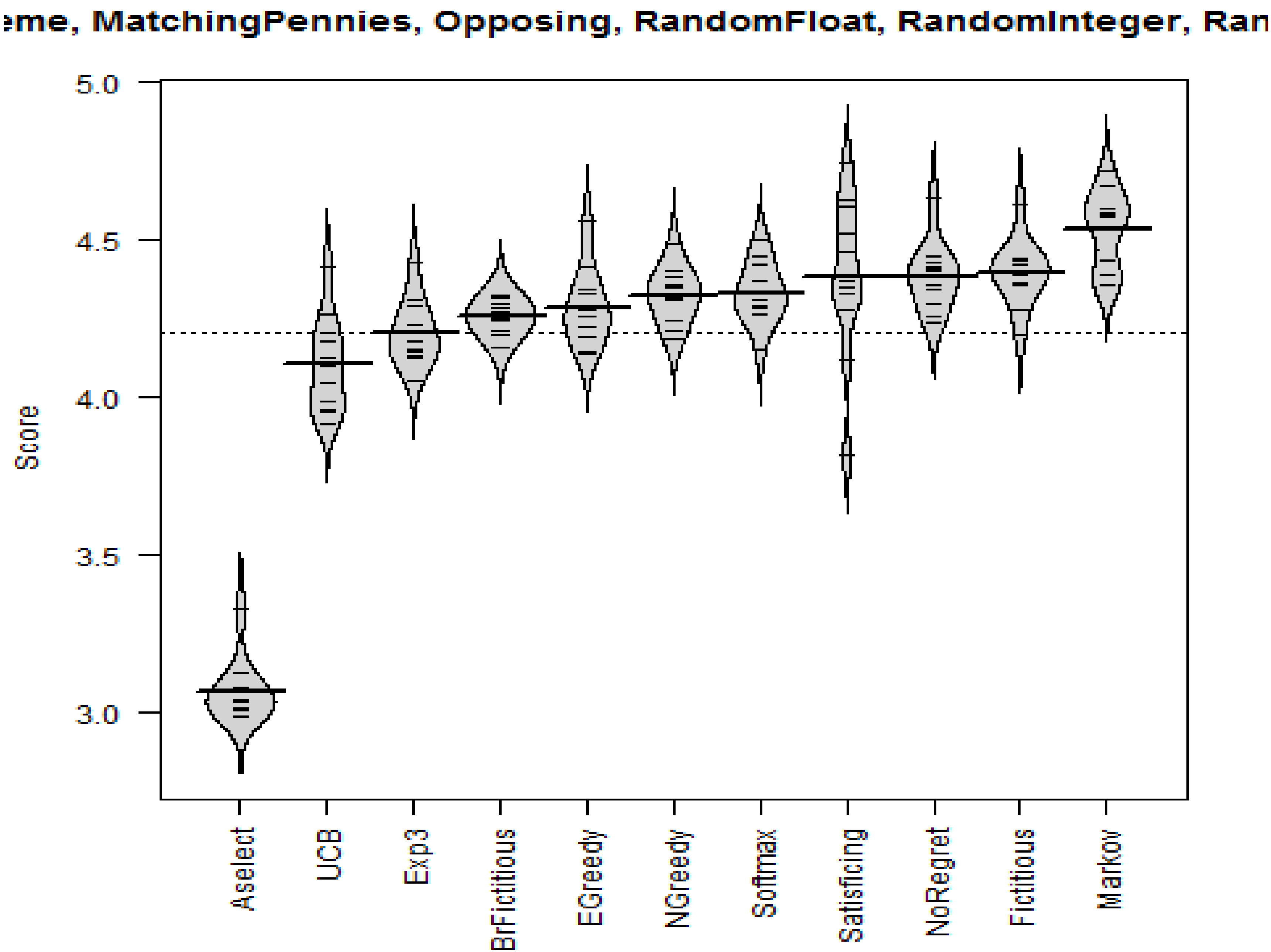
- If t is too eccentric, then we'll have to reject that possibility, since eccentric values of t are unlikely ("have a low p-value").

Parametric test: paired t-test



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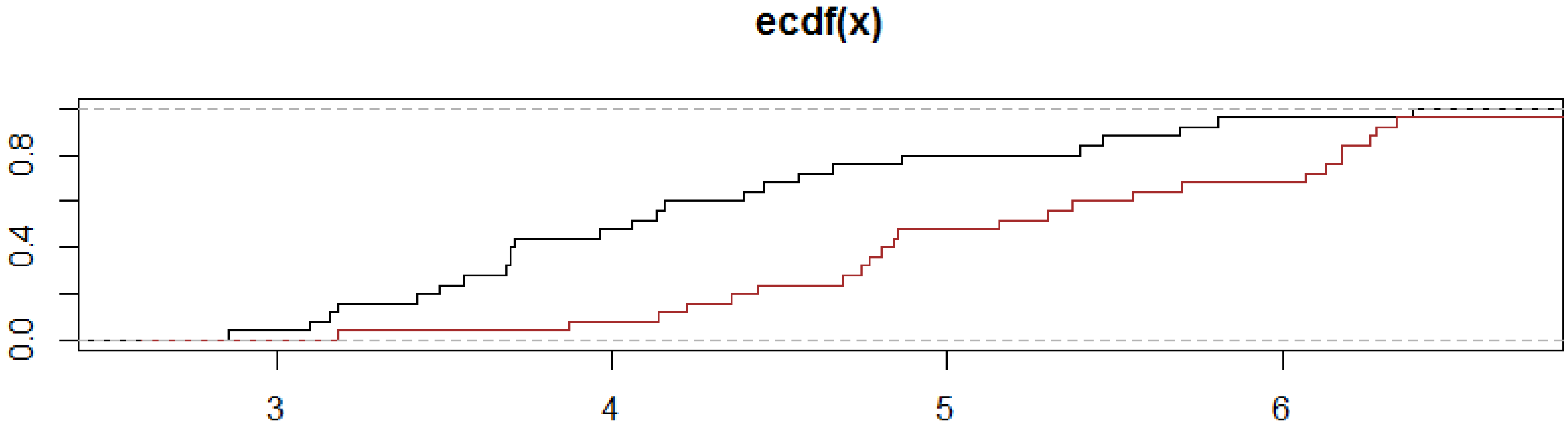

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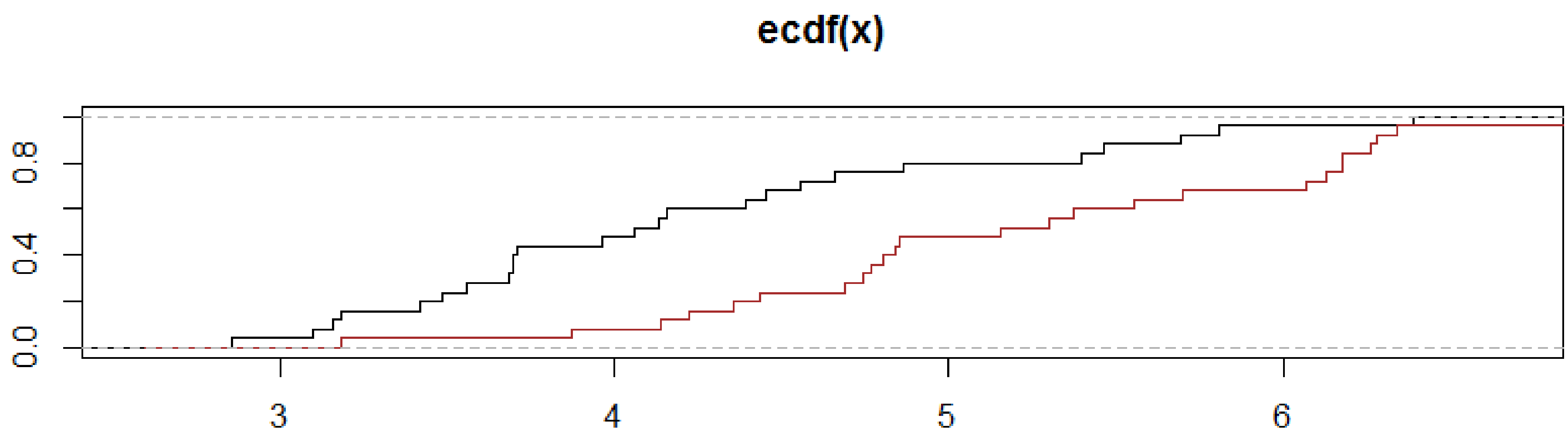

- The p-value is the probability of seeing a test statistic (i.e., max distance) as high as the one observed, under the assumption that both samples were drawn from the same distribution.

Zawadzki *et al.*

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Work of Bouzy *et al.* (2010)

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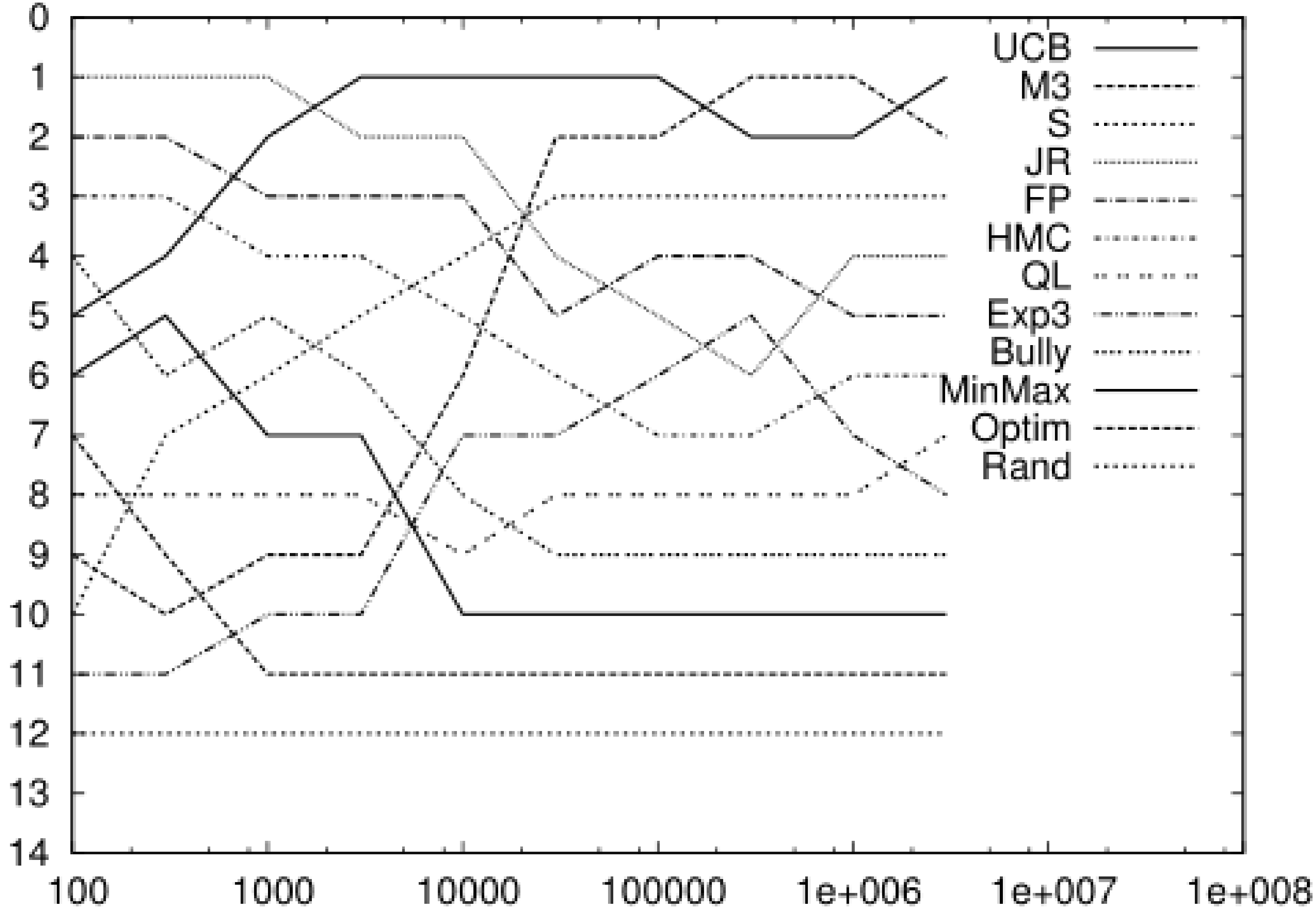

Ranking evolution according to the number of steps played in games (logscale). The key is ordered according to the final ranking.

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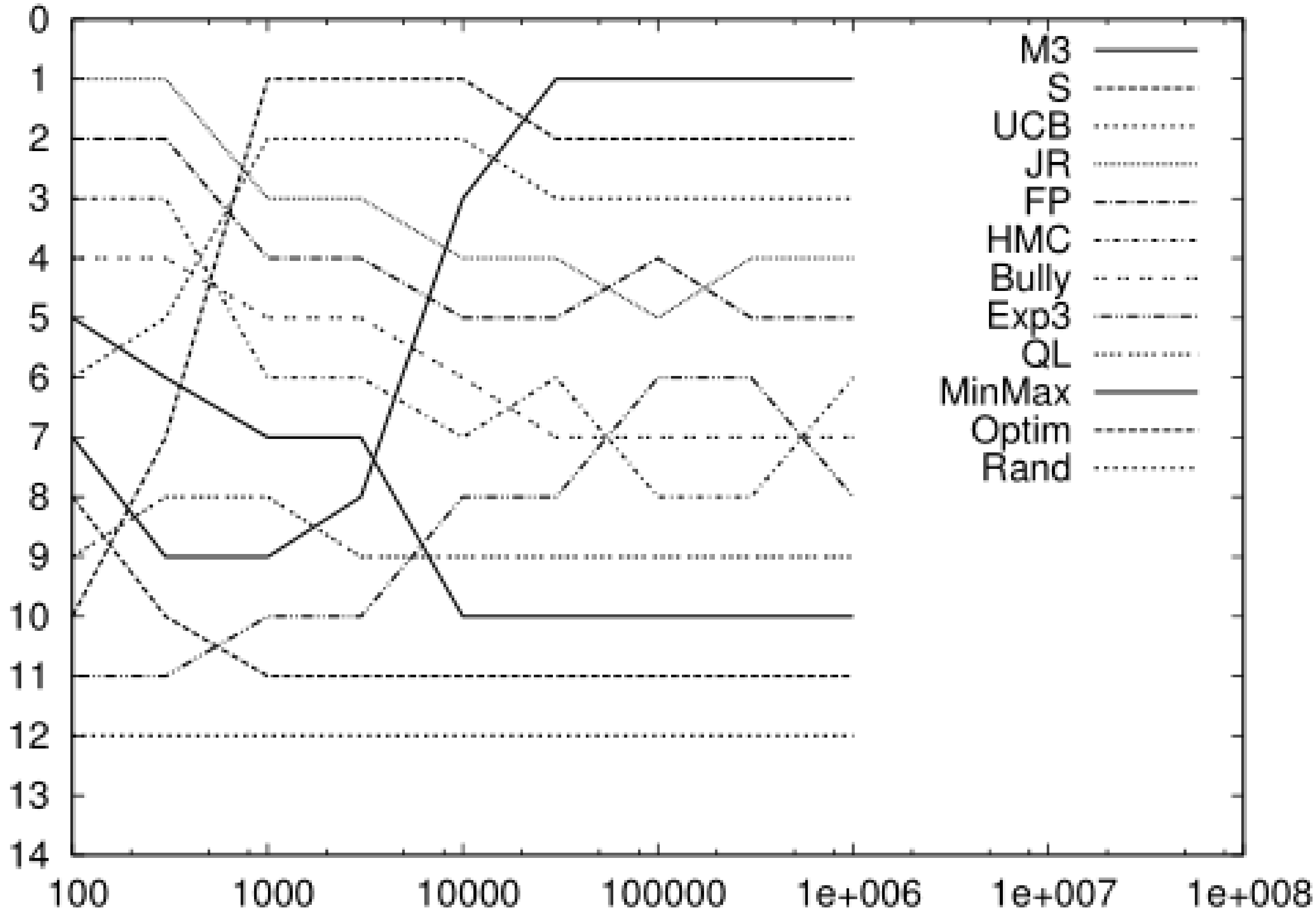

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Conclusion: M3, Sat, and UCB perform best. Do not maintain averages but geometric (decaying) averages of payoffs.

Another interesting direction is exploring why Exp3 is the best MAL player on both cooperative games and competitive games, but not on general-sum games, and to exploit this fact to design a new MAL algorithm.

Work of Airiau *et al.* (2007)

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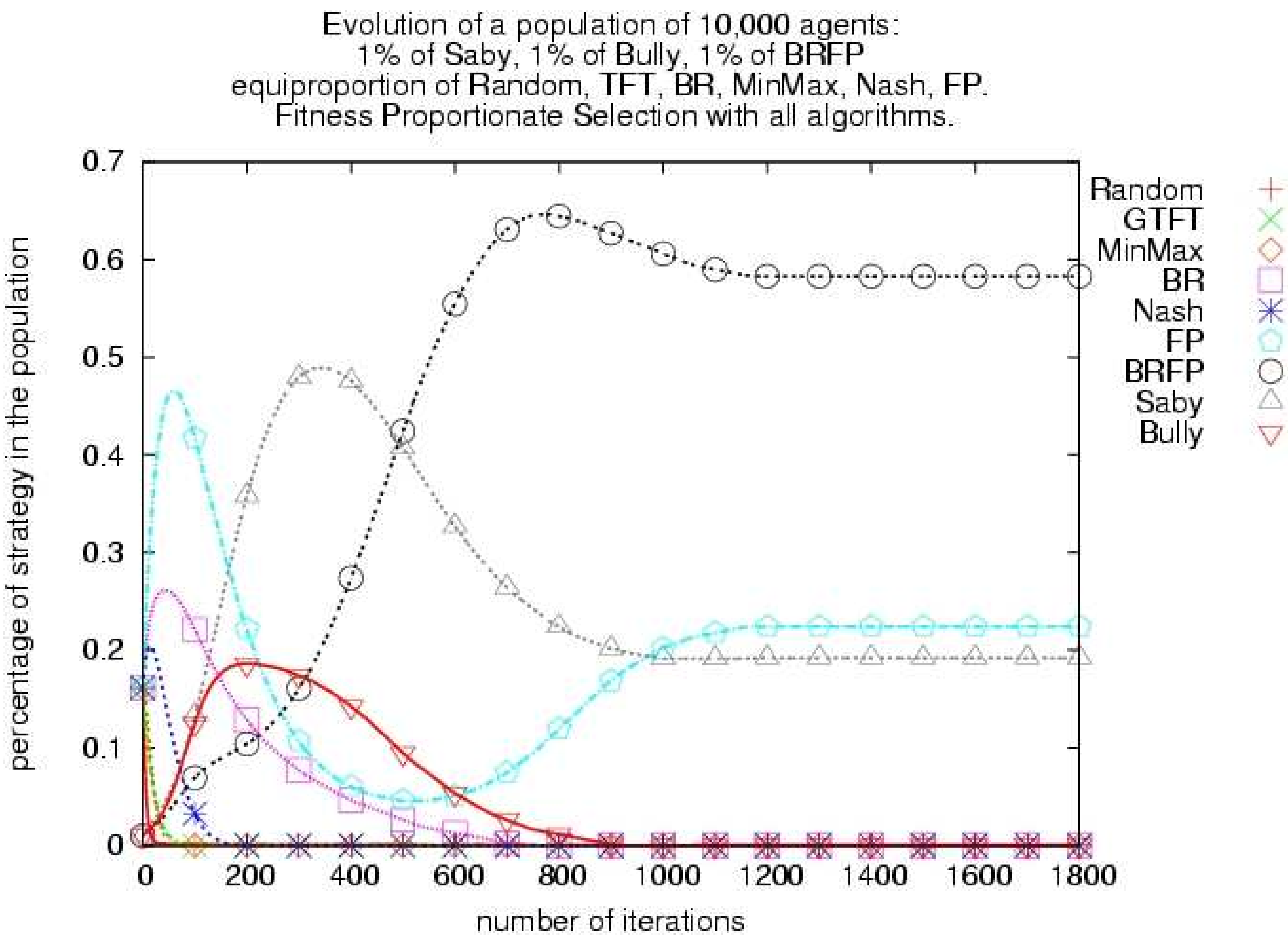
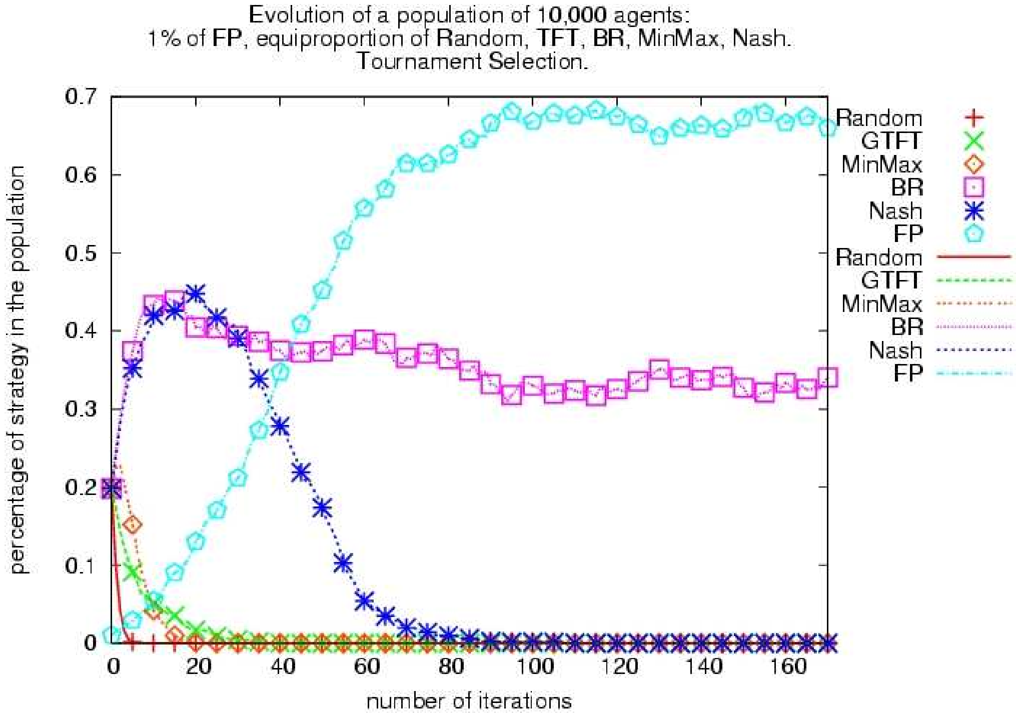
Airiau *et al.* results



Evolutionary tournament with six algorithms: 1% FP and equiproportion of R, GTFT, BR, MaxMin, Nash each.



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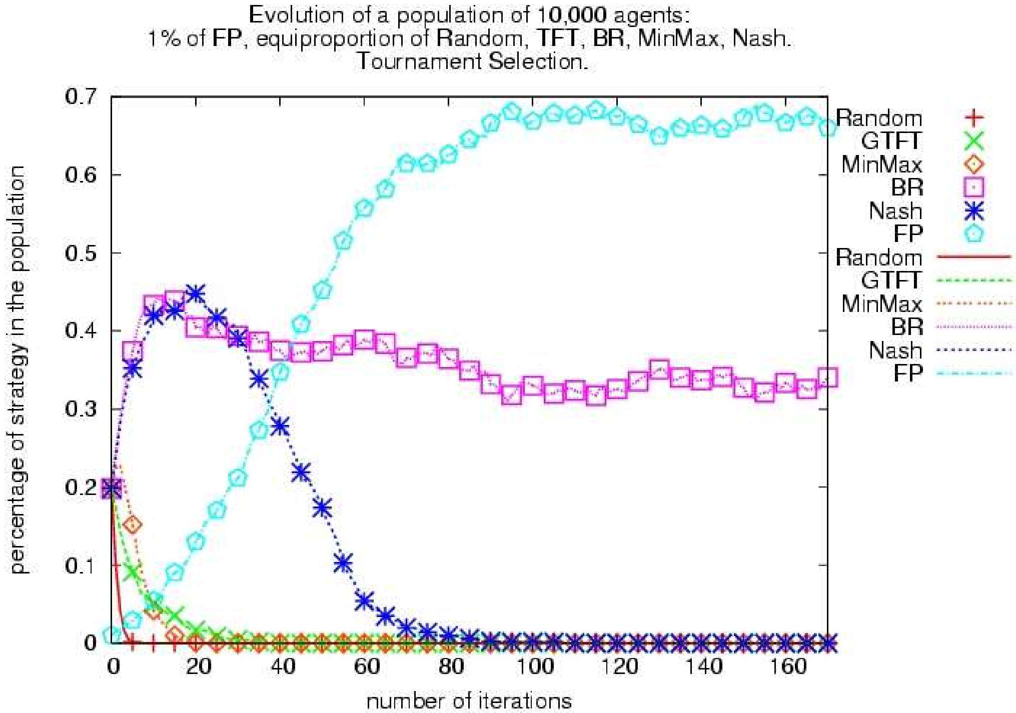

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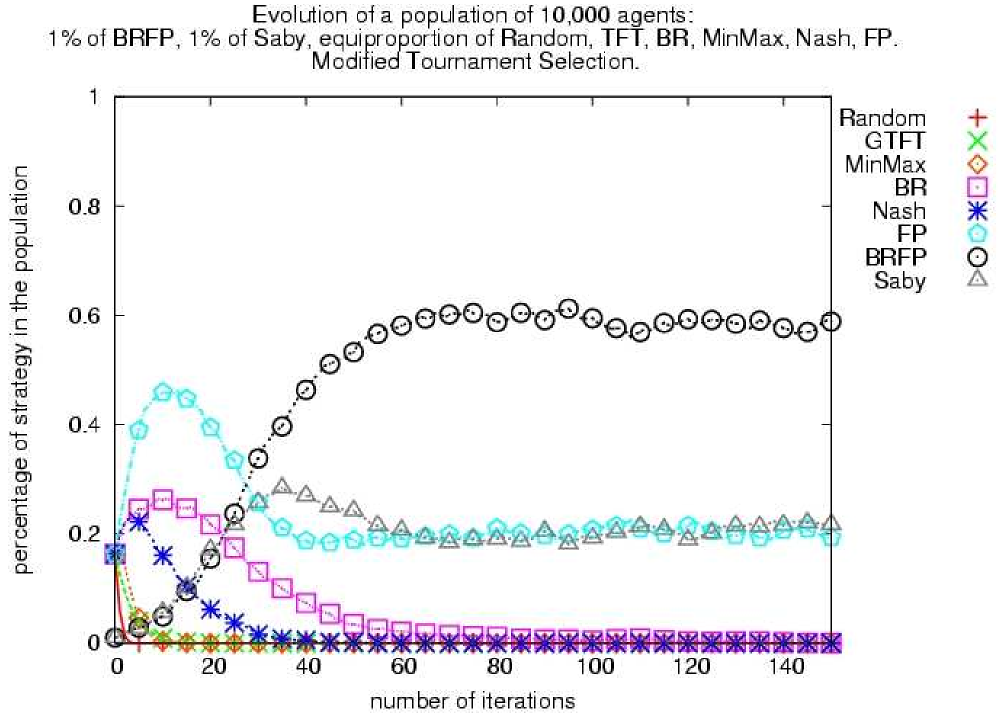
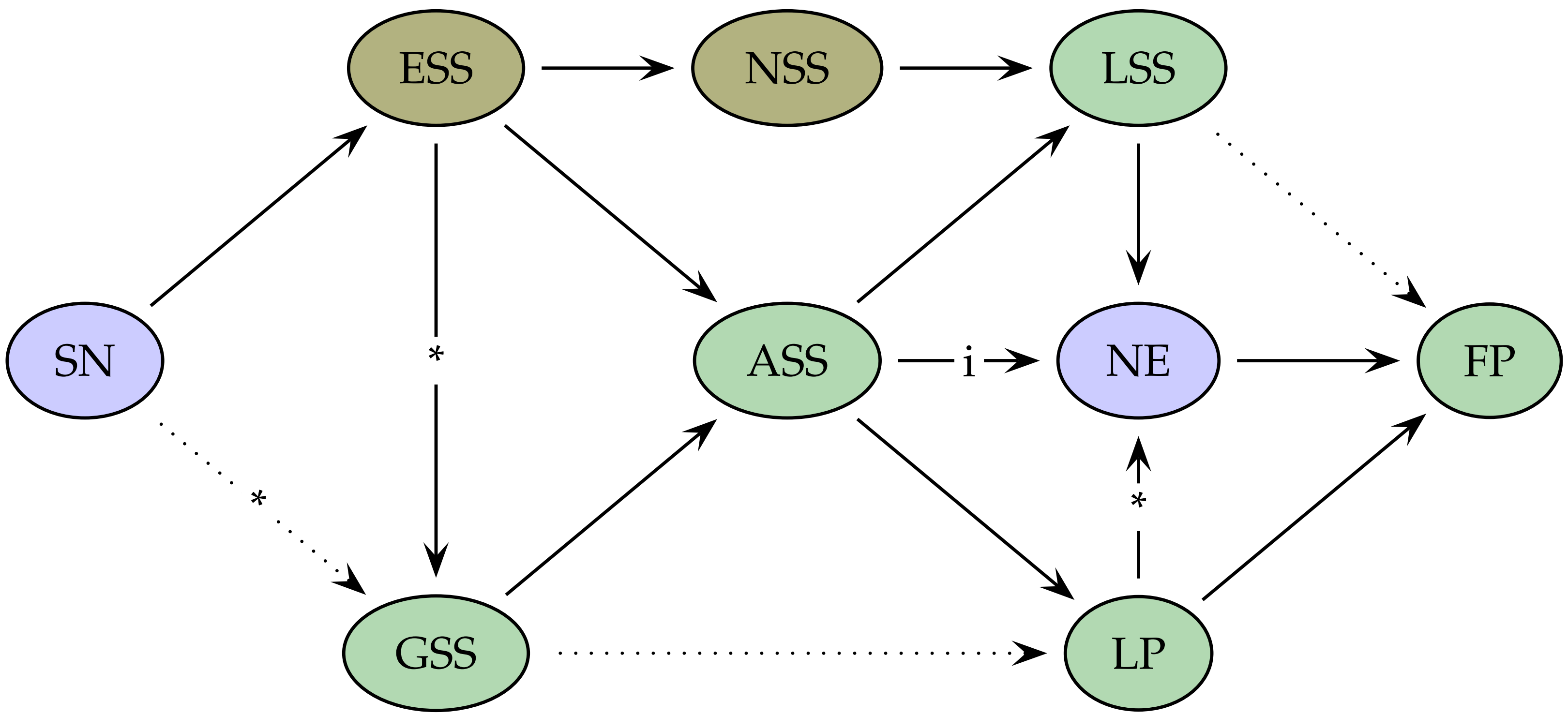


With modified tournament selection.[a]

[a] Modified tournament selection is a hybrid of fitness-proportionat selection and 2-sample tournament selection. Cf. Sec. 2.7. of Airiau *et al.*' 2007 paper.

Implications (green concerns the replicator dynamic)



SN = strict Nash, ESS - evol'y stable strategy, GSS = glob'y stable state, ASS = asymp'y stable state, NSS = neutrally stable strategy, LP = limit point, LSS = Lyapunov stable state, NE = Nash eq., FP = fixed point, * = only if fully mixed, i = isolated Nash eq. Dotted lines are indirect implications.

Evaluation by computing NE of grand table

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Reconsider the grand table:

	A_1	A_2	$\ldots$	A_{12}	avg
A_1	3.1	5.1	$\ldots$	4.7	4.1
A_2	2.4	1.2	$\ldots$	2.2	1.3
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See this as a game in normal form:

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It is interesting to interpret Nash equilibria among reply rules on the grand table.

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General evaluation

1. **Selection of learning algorithms to test on**. All approaches demonstrate arbitrariness and lack of method.

2. **Conditions on algorithms**: coupled, uncoupled, completely uncoupled. All methods give contestants full information $\Rightarrow$ unequal playing field. Further, accessibility to computationally expensive game properties (mixed Nash equilibria) is taken for granted.

   We should have different leagues: coupled (disposal of Nash equilibria), uncoupled, completely uncoupled.

3. **Parametrisation of algorithms**. The parameter search space is often in the order of $[0,1]^k$, $k \geq 1$ per algorithm (k the number of parameters), which gives uncountably many sub-algorithms.

4. **Selection of games**. Approaches demonstrate poor method. Justification of choices is weak and subjective.

General evaluation (continued)

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7. **Evolutionary selection**: How to select algorithms? Fitness-proportional selection, tournament selection, reward-based selection, stochastic universal sampling, replicator dynamic?

8. **Knock-out tournament**: How to decide when and where to drop the worst performer? The order of elimination puts a (plausible but arbitrary) bias on the ranking.

Now you know enough . . .

Now you know enough . . .
to design your own MAL algorithm . . .

Now you know enough . . .
to design your own MAL algorithm . . .
and MAL comparison methods . . .

The end

Good luck and *au revoir* ...