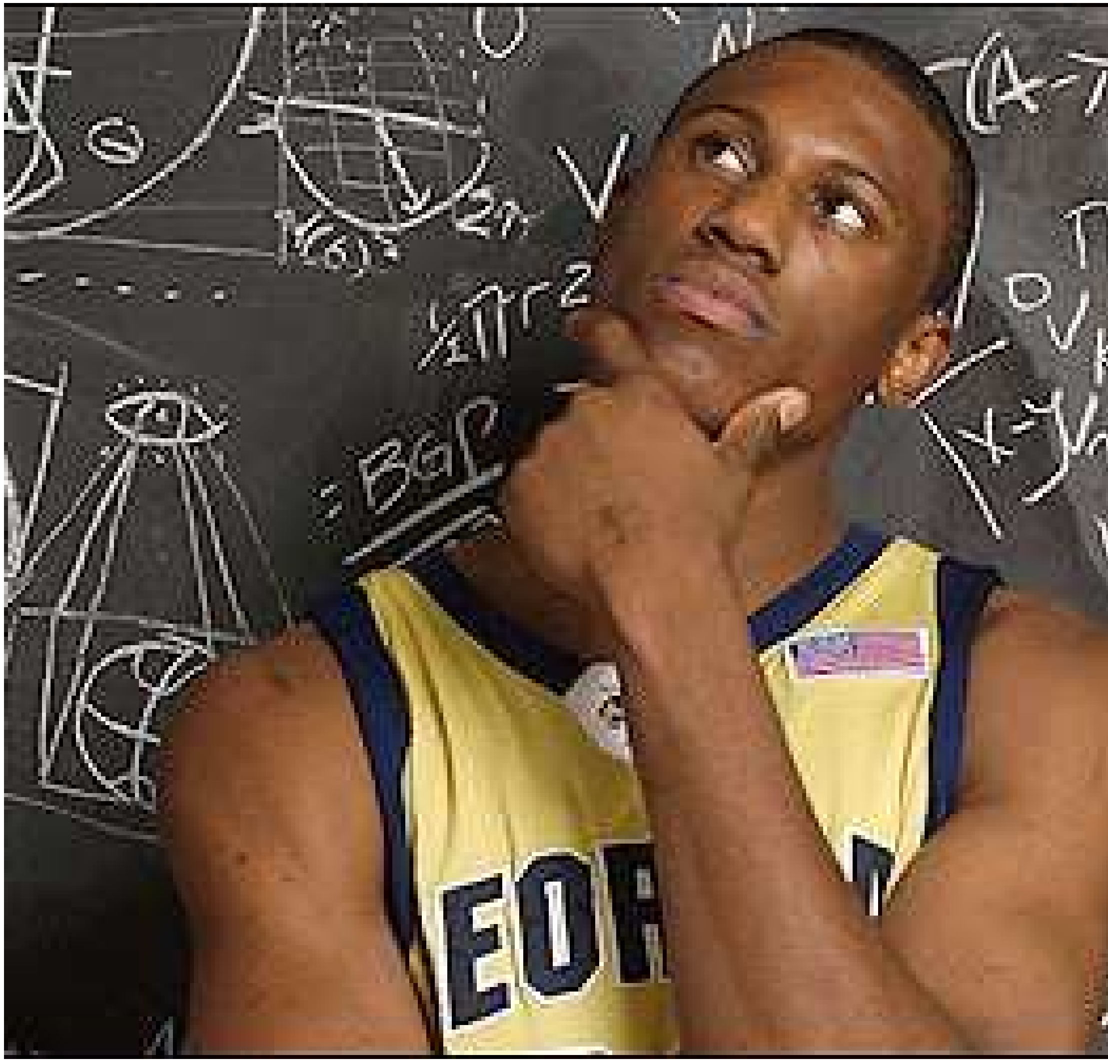
Multi-agent learning

Satisficing play

Gerard Vreeswijk, Intelligent Software Systems, Computer Science Department, Faculty of Sciences, Utrecht University, The Netherlands.

Monday 14th June, 2021

Assumptions in game playing



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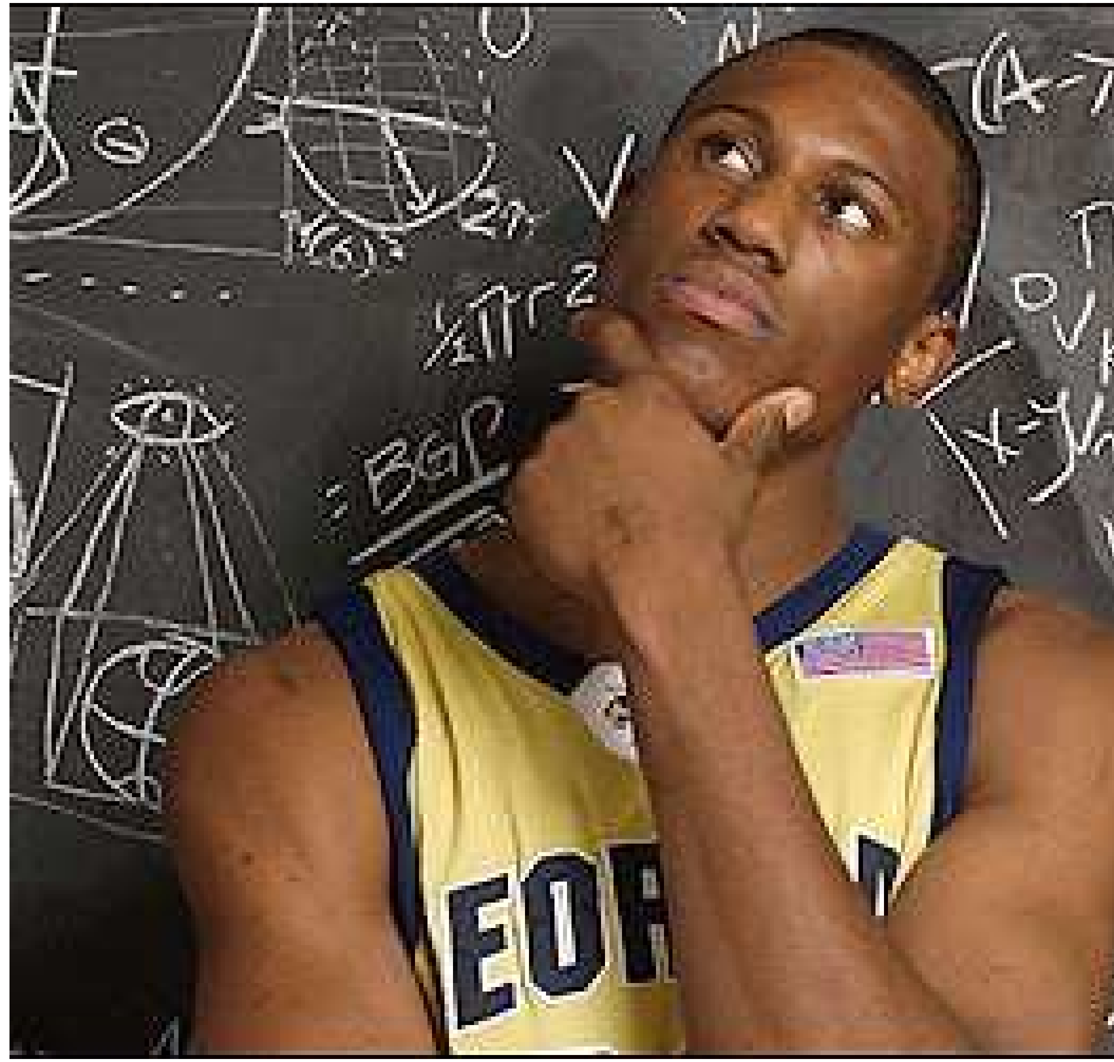

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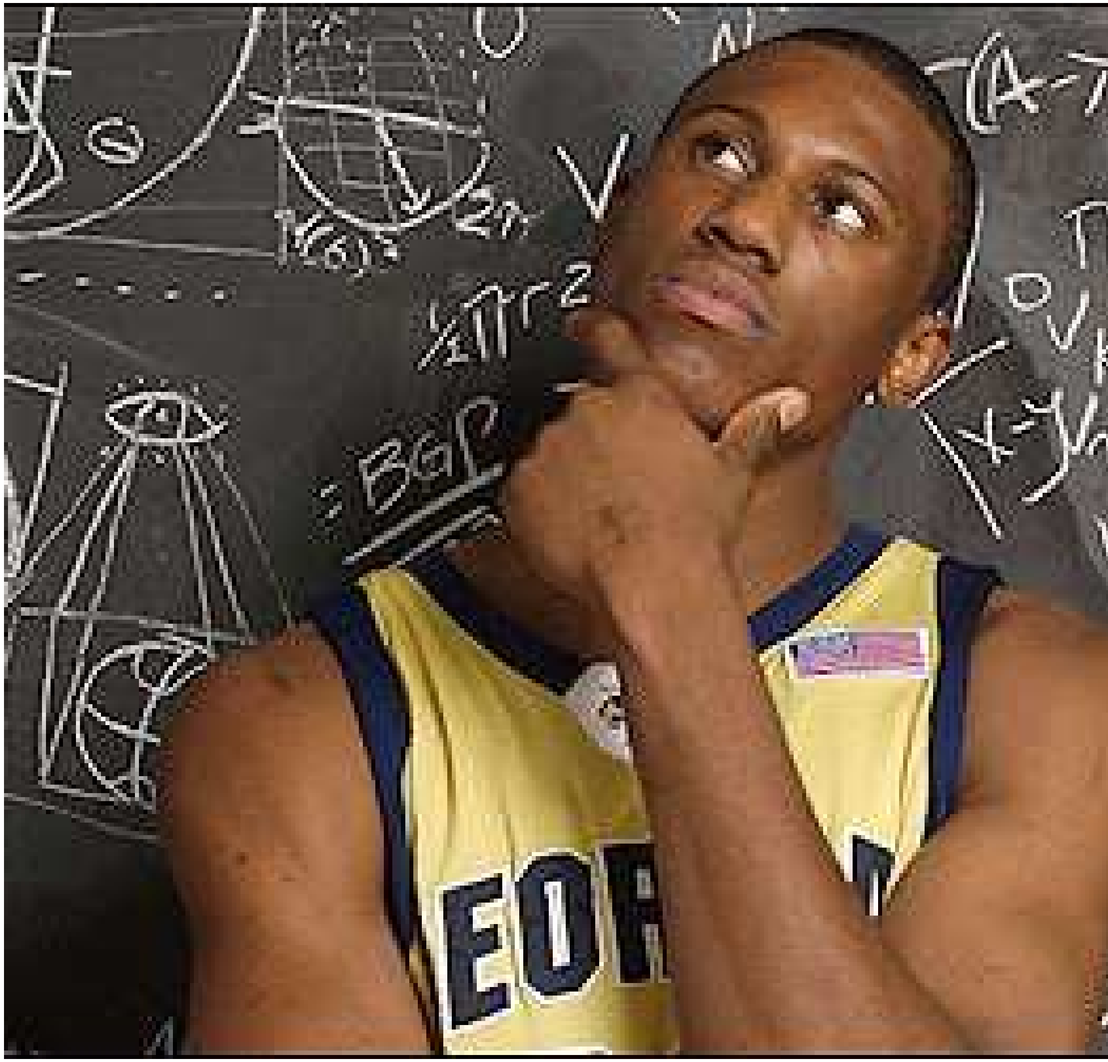

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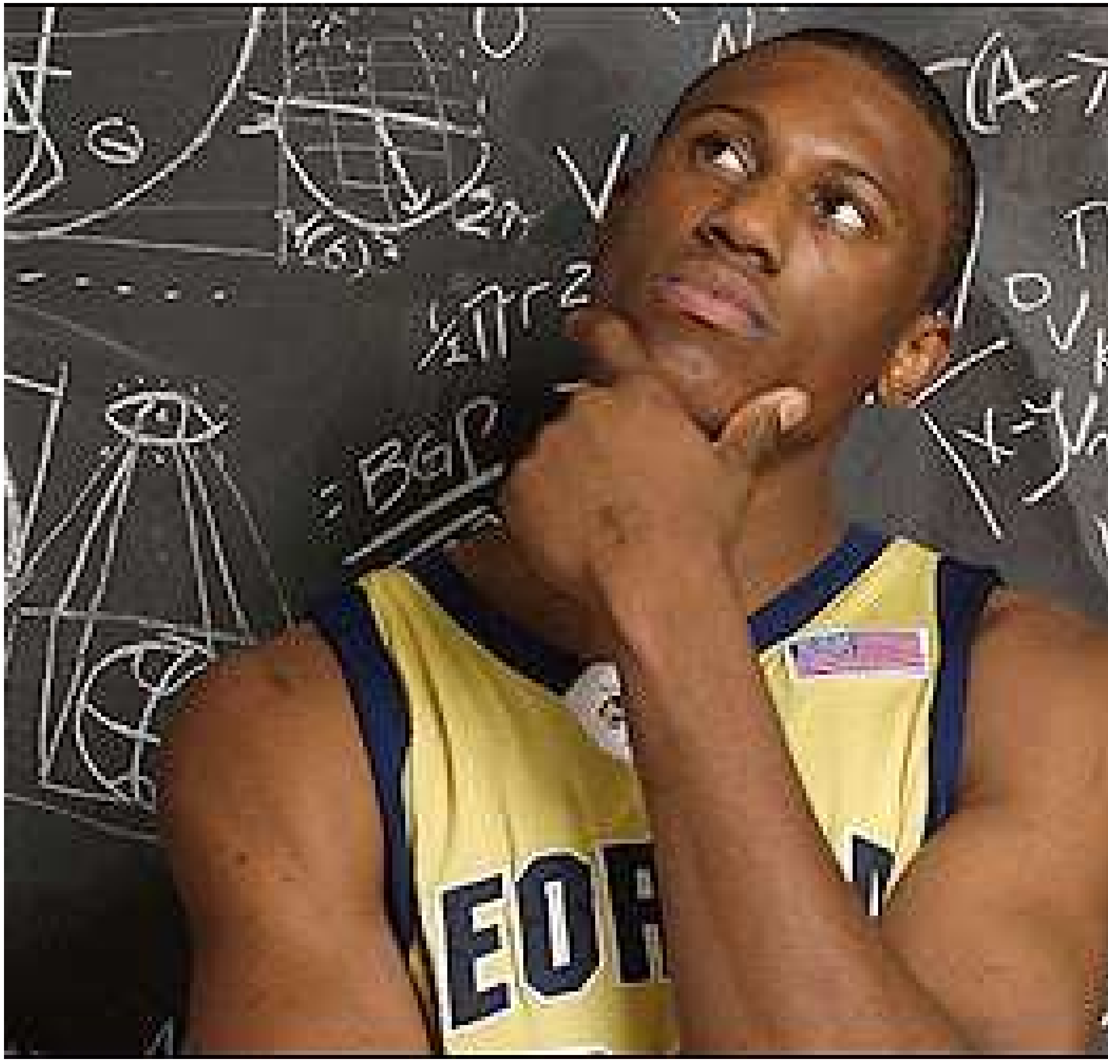

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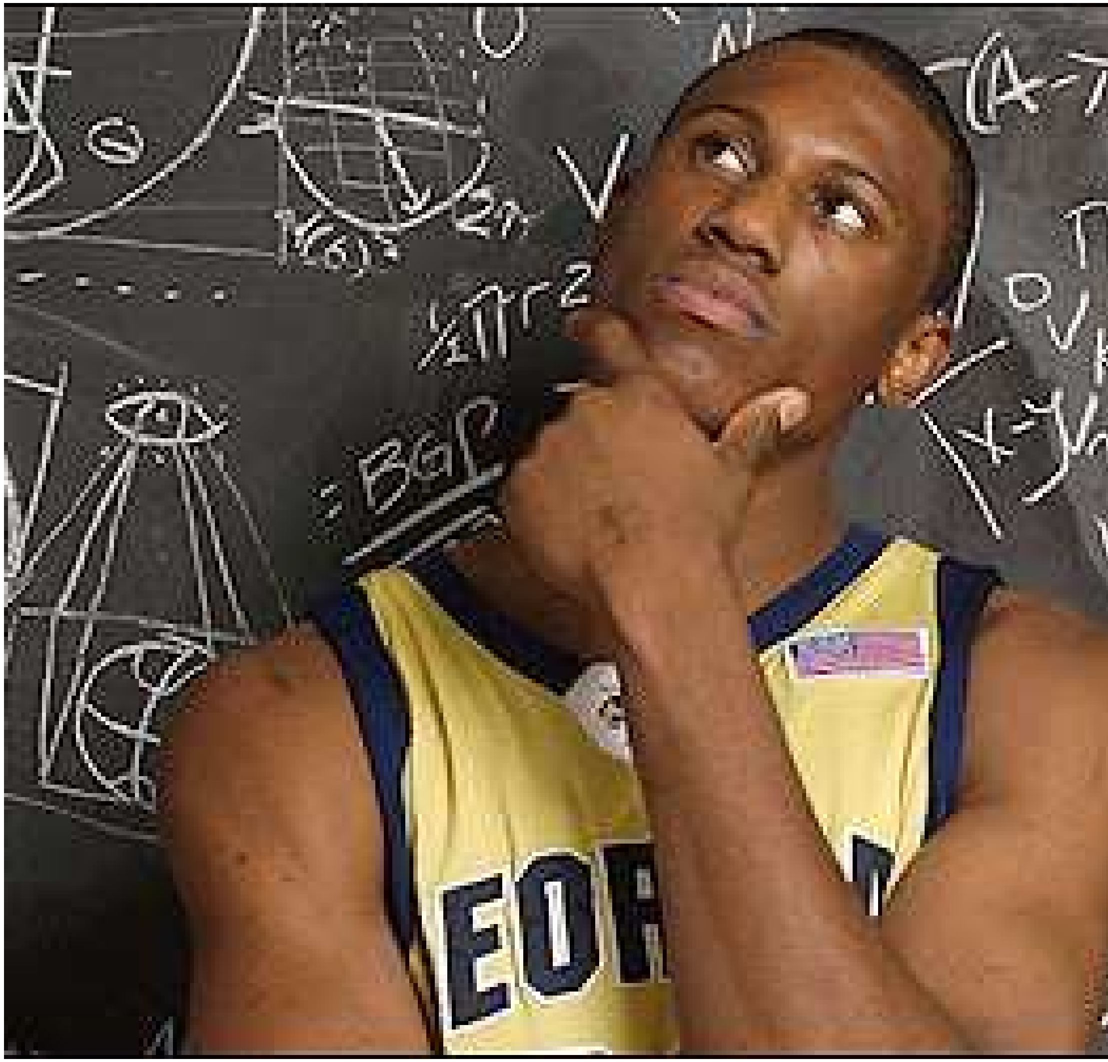

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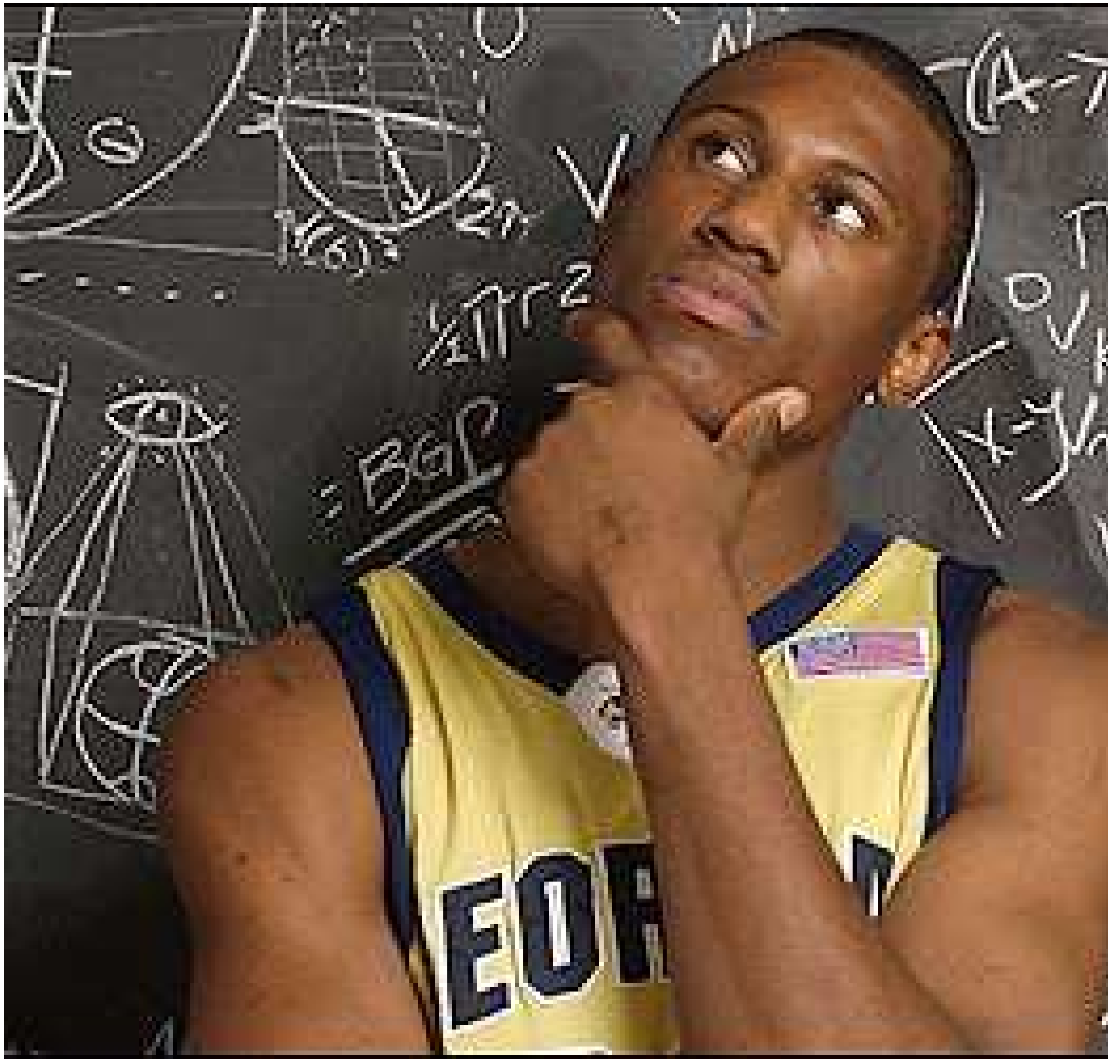

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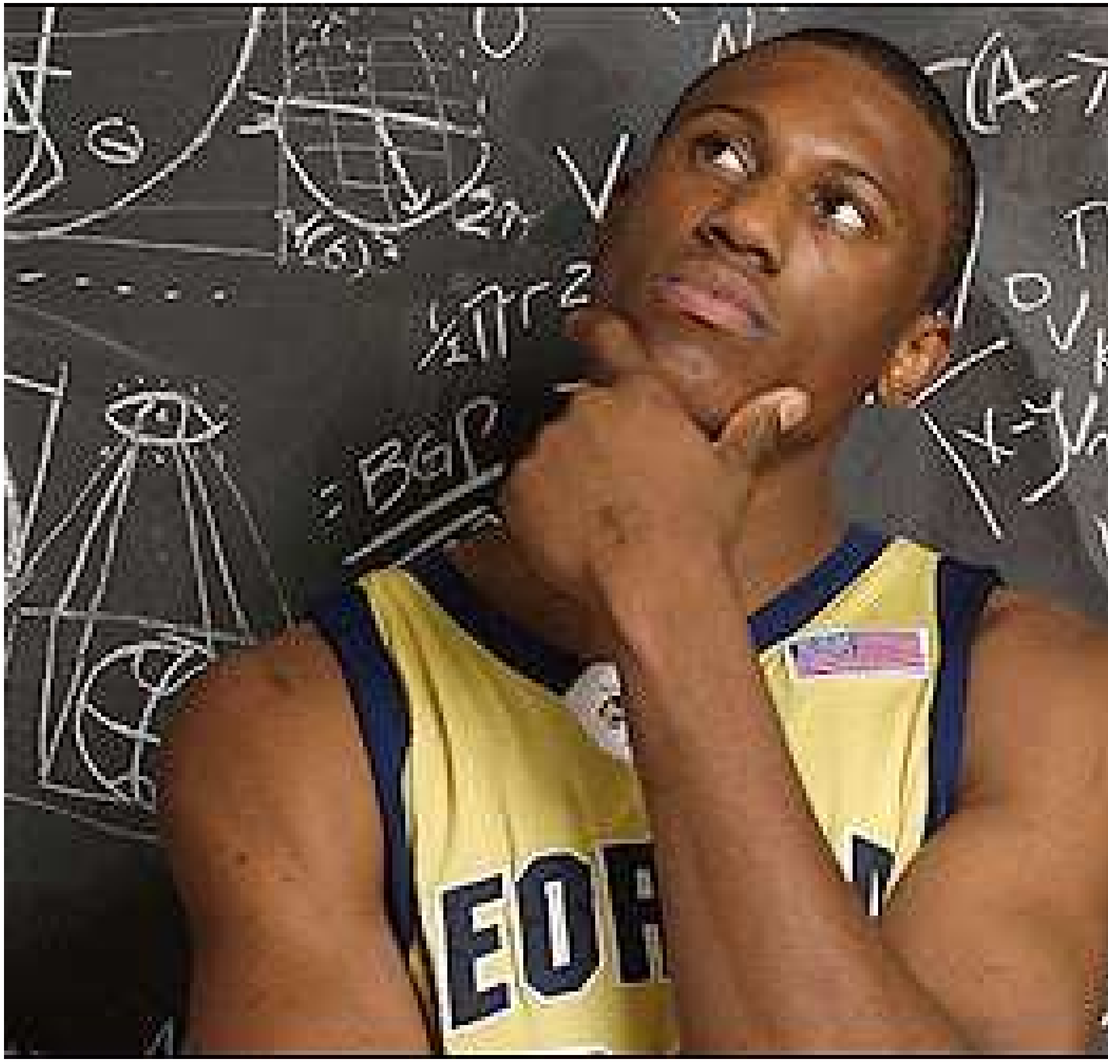
- Players are aware that they are in a game.



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Karandikar *et al.*'s algorithm for satisficing play (1989)

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$$A_{t+1} = \begin{cases} A_t & \text{if } \pi_t \geq \alpha_t, \\ \text{any other action} & \text{else.} \end{cases}$$

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  Also works if "any other action" is replaced by "any action".

Example of satisficing play

Game: prisoner's dilemma.

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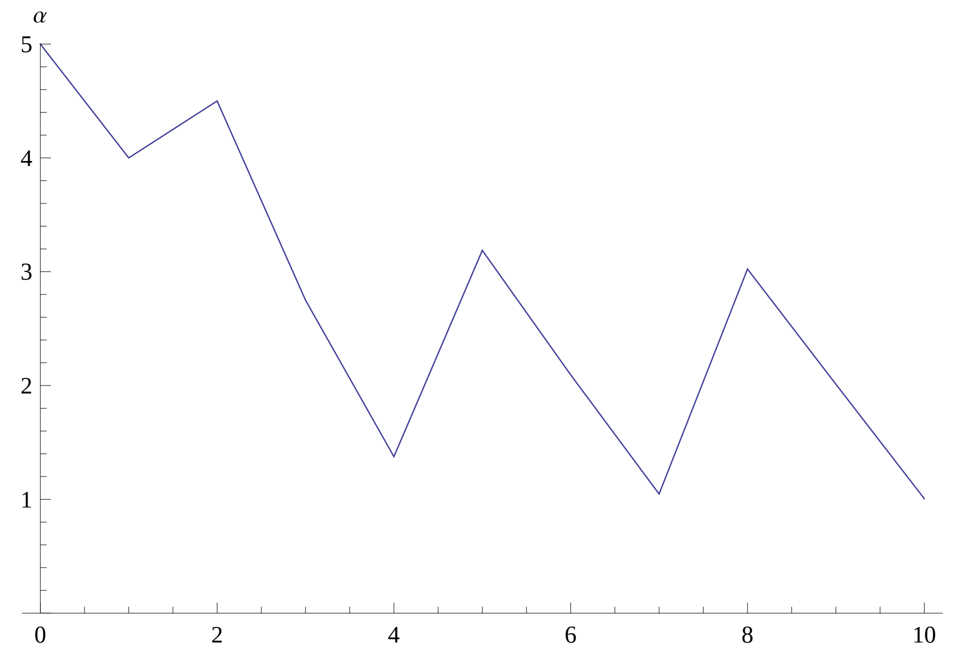
t TFT A_t π_t α_t



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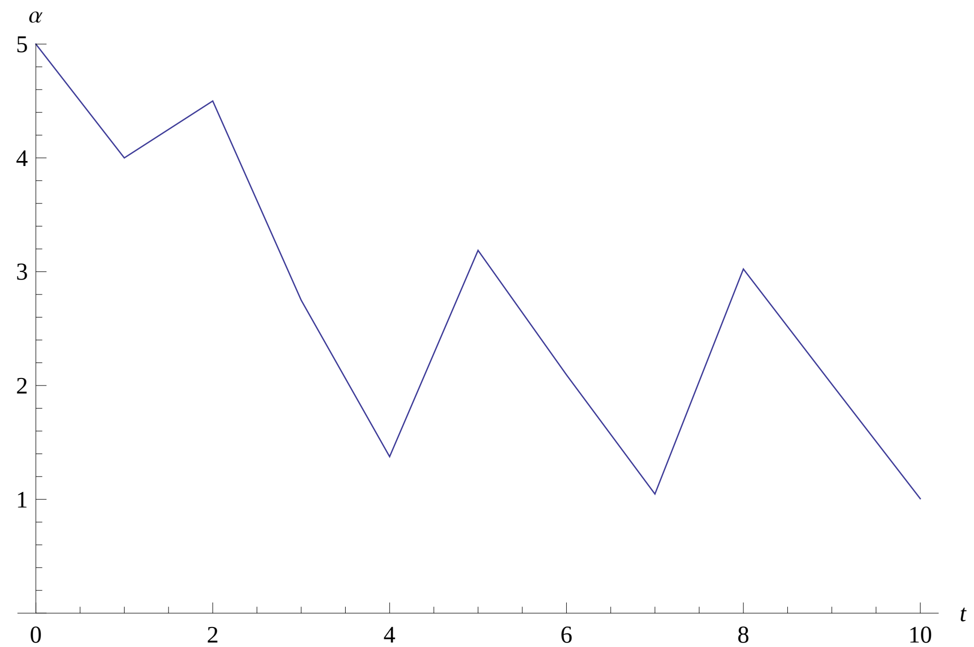

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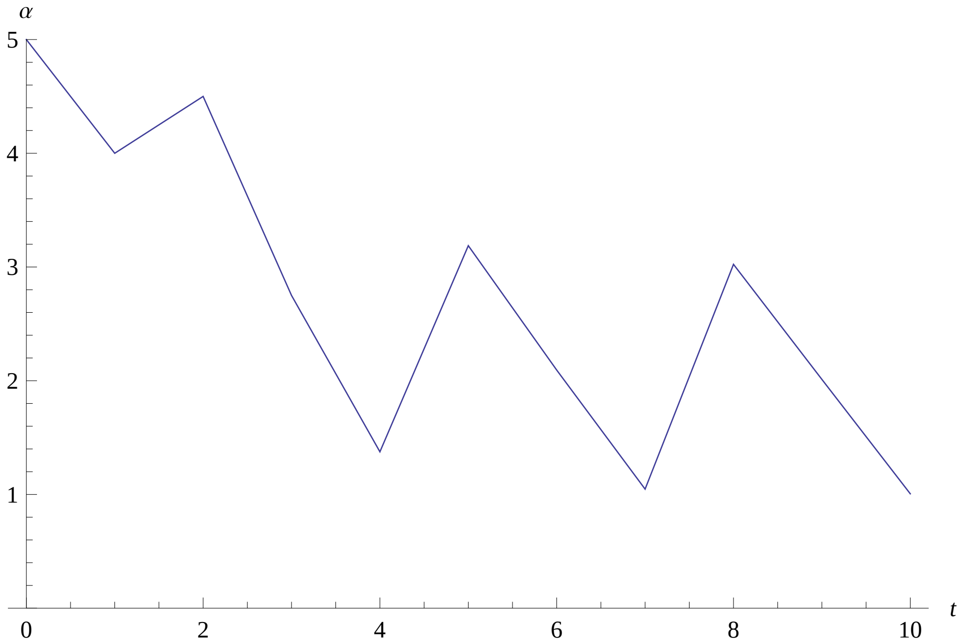
t	TFT	A_t	π_t	α_t
0				



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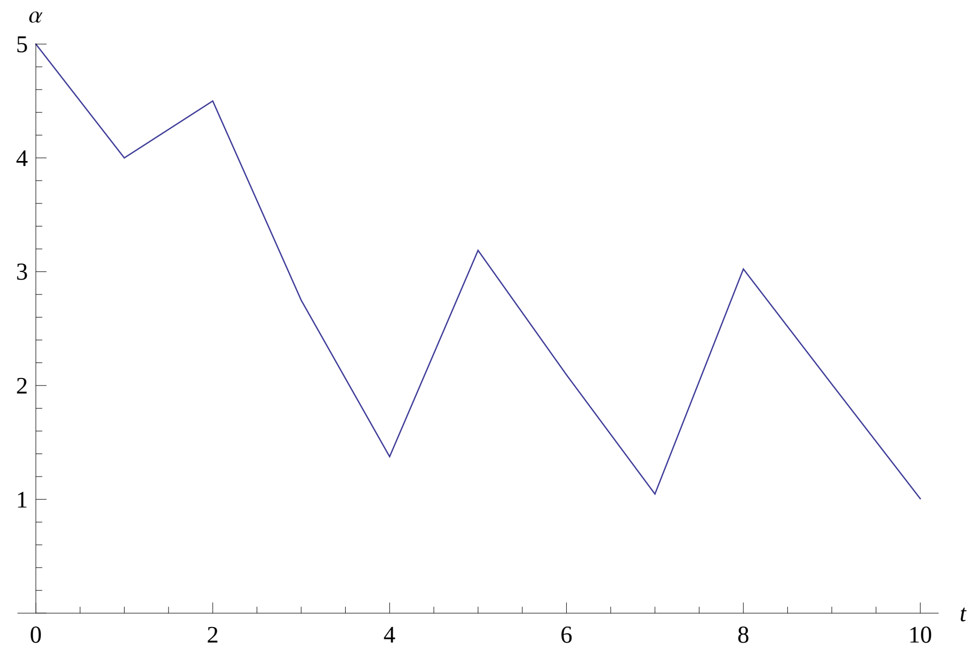
t	TFT	A_t	π_t	α_t
0	C			



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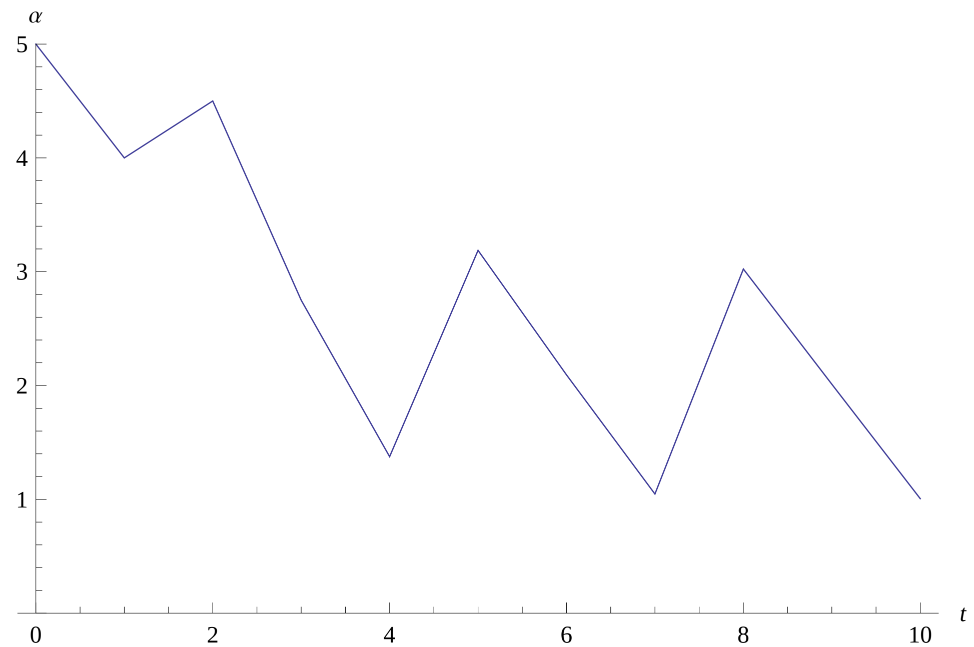
t	TFT	A_t	π_t	α_t
0	C	C		



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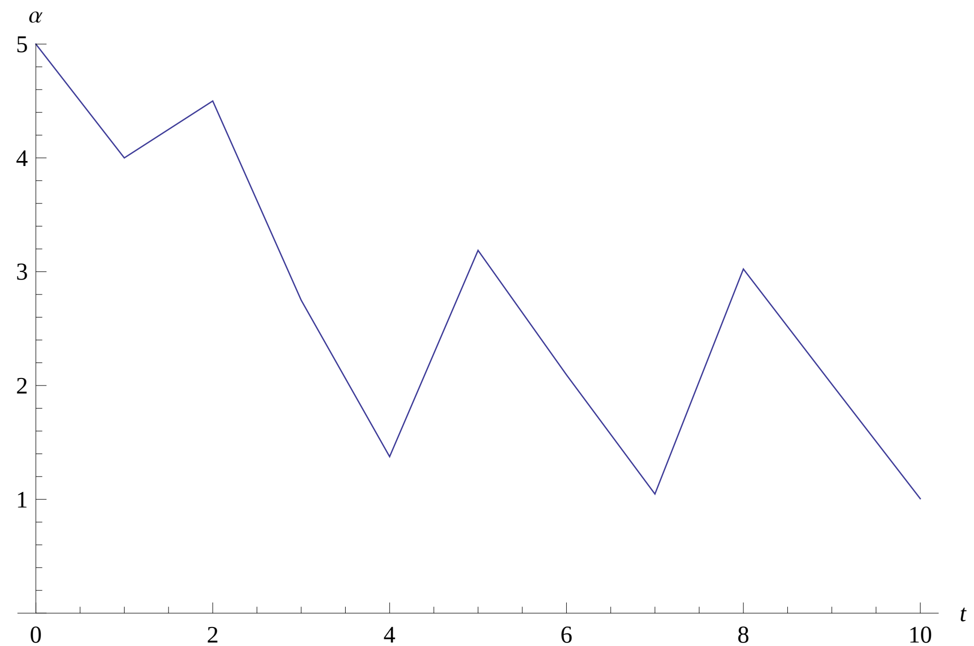
t	TFT	A_t	π_t	α_t
0	C	C	3	



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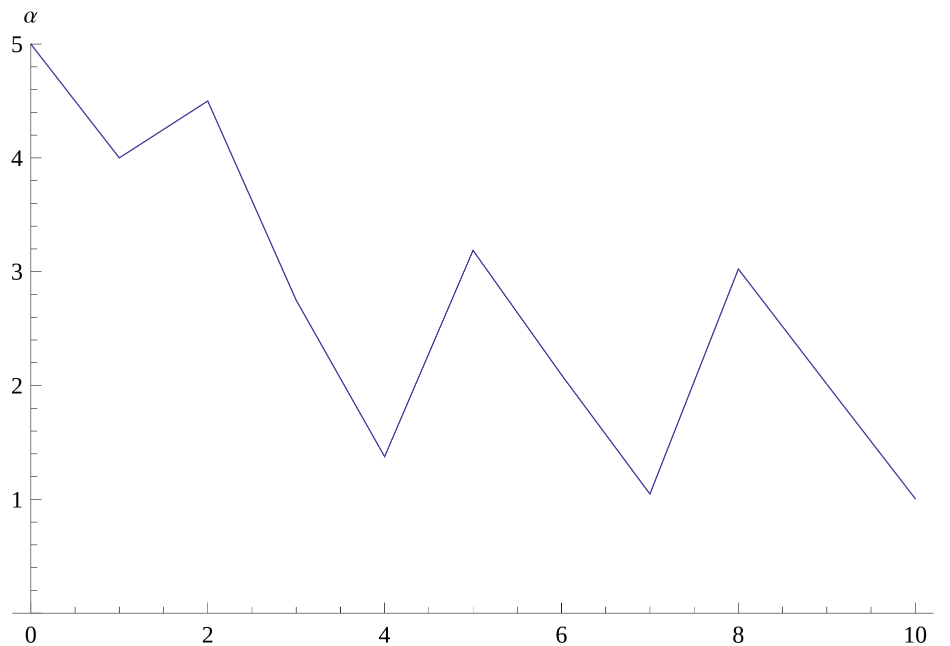
t	TFT	A_t	π_t	α_t
0	C	C	3	5



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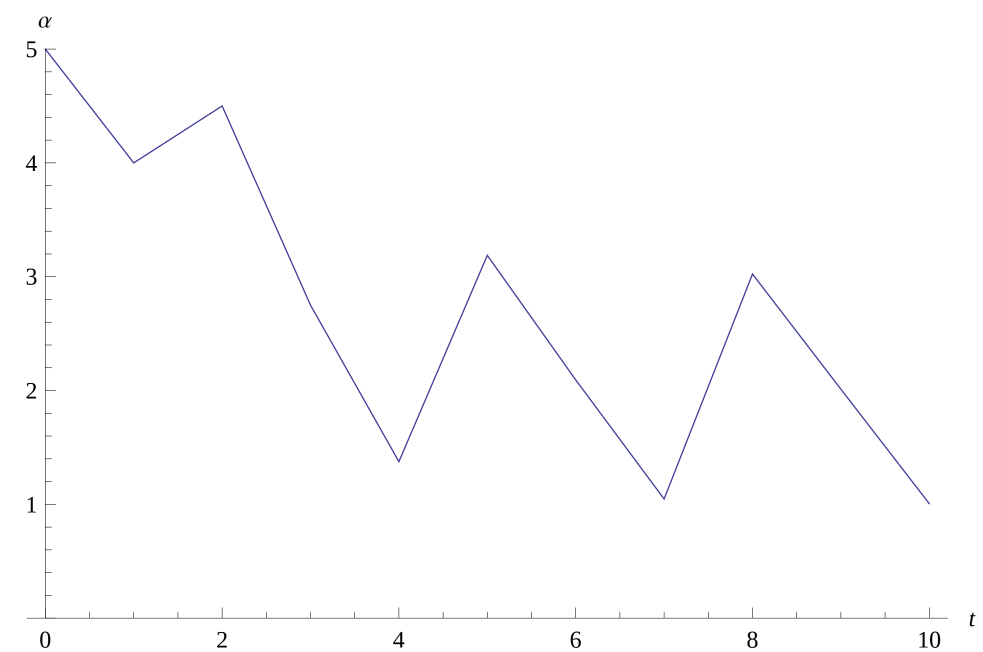
t	TFT	A_t	π_t	α_t
0	C	C	3	5
1				



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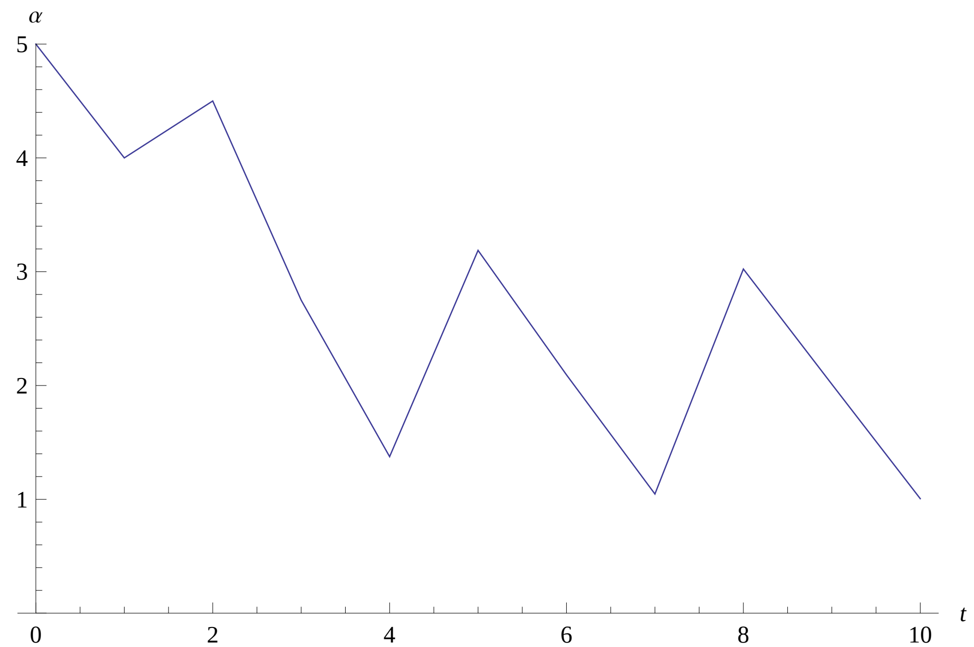
t	TFT	A_t	π_t	α_t
0	C	C	3	5
1	C			



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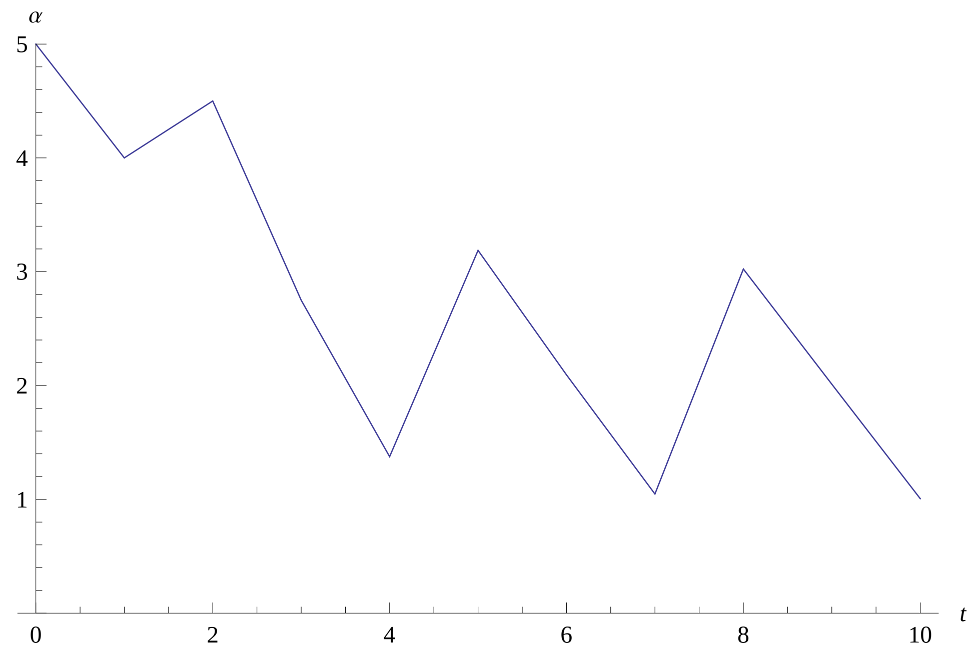
t	TFT	A_t	π_t	α_t
0	C	C	3	5
1	C	**D**		



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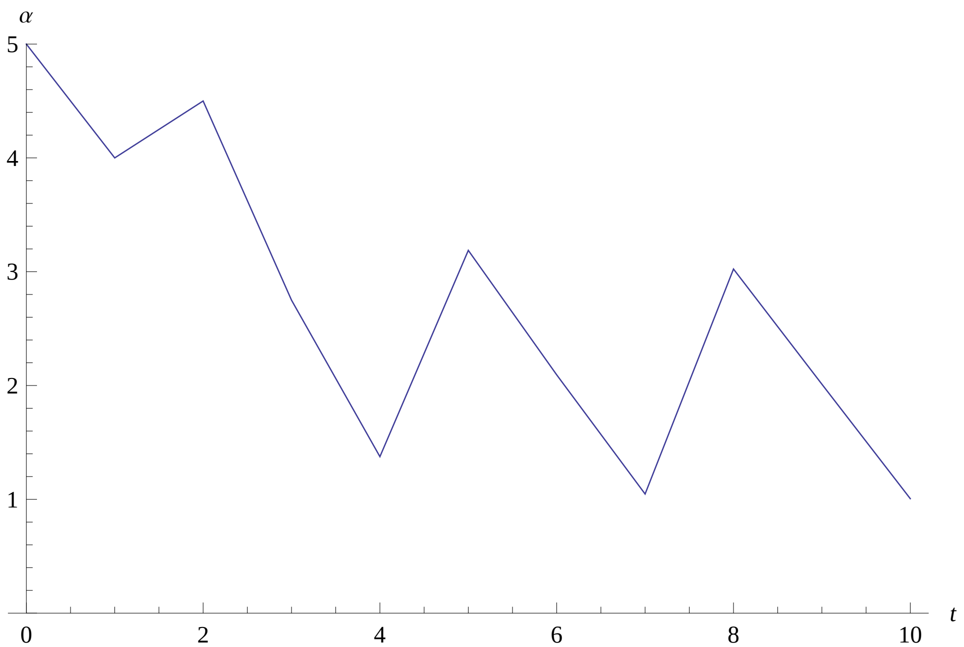
t	TFT	A_t	π_t	α_t
0	C	C	3	5
1	C	**D**	5	



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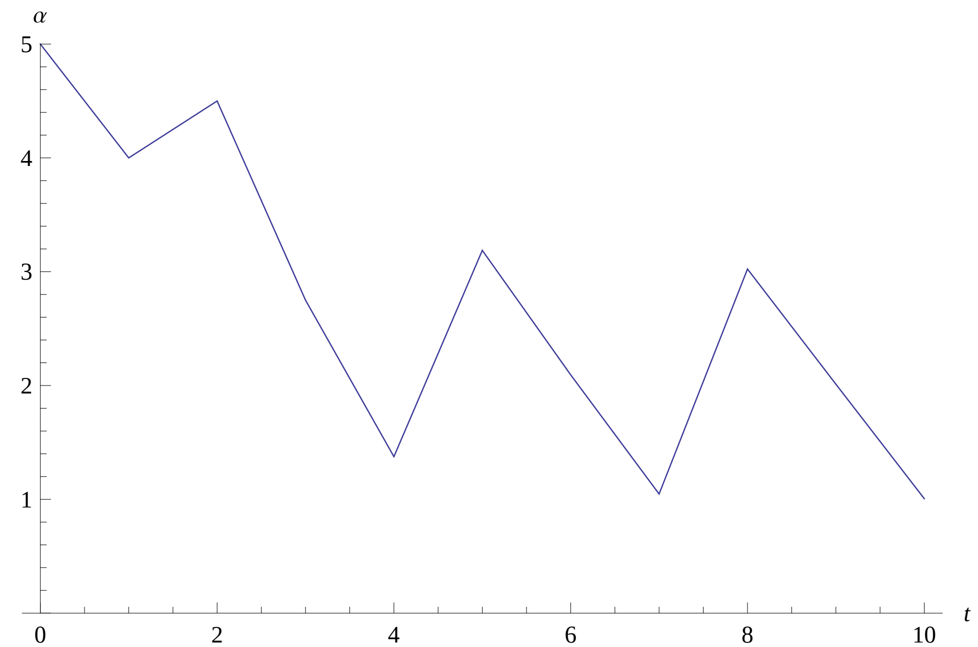
t	TFT	A_t	π_t	α_t
0	C	C	3	5
1	C	**D**	5	4



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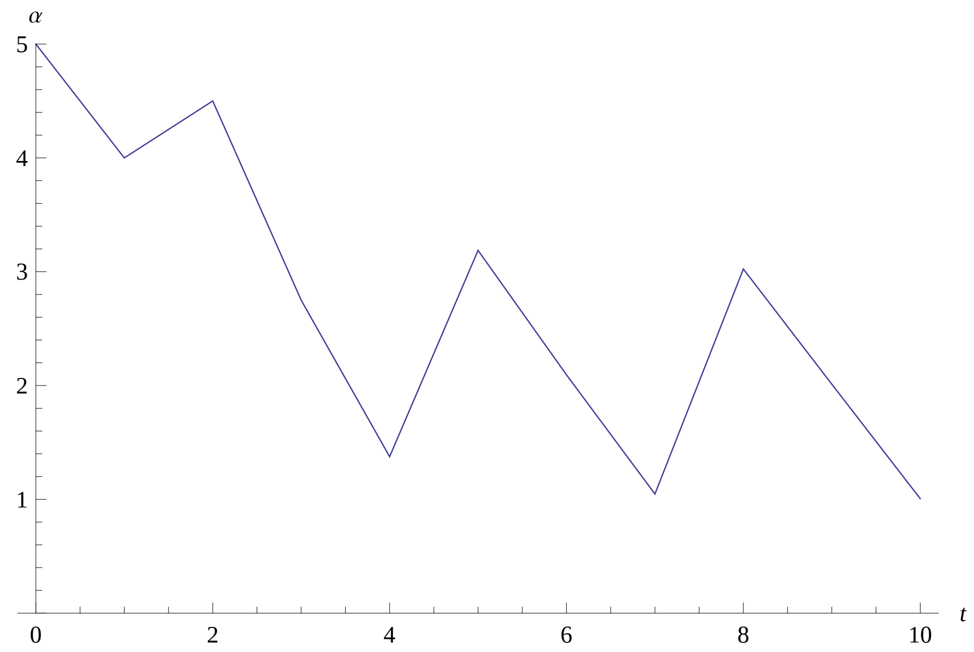
t	TFT	A_t	π_t	α_t
0	C	C	3	5
1	C	**D**	5	4
2				



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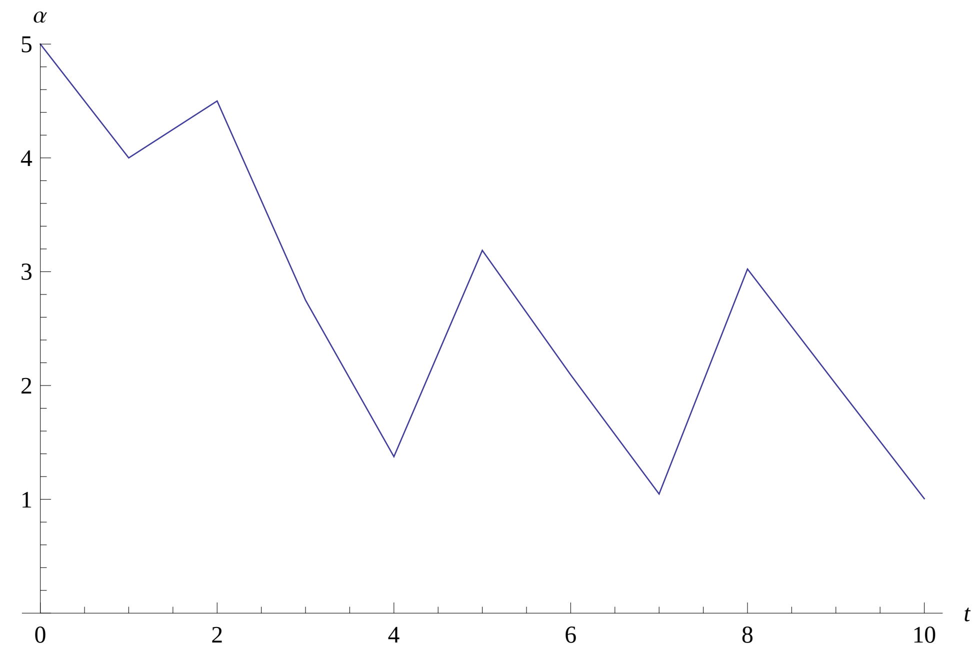
2	D			



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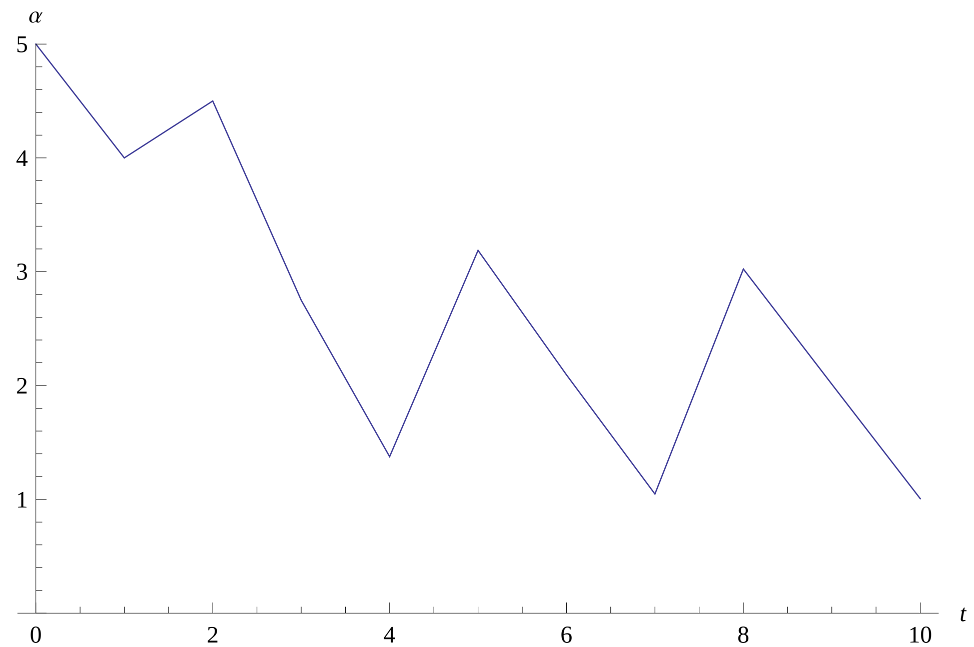
2	D	D		



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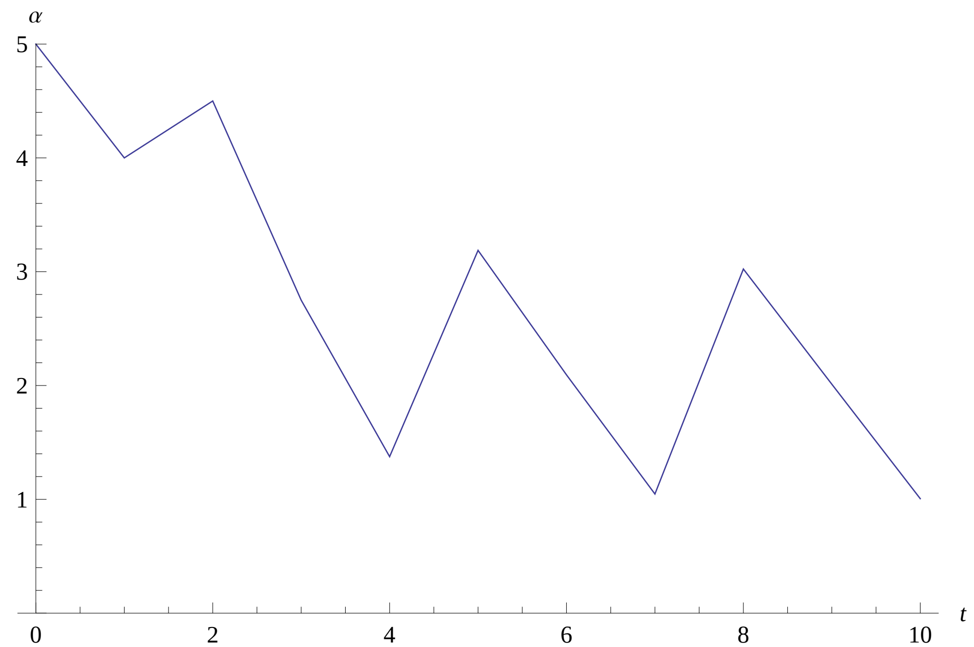
0	C	C	3	5
1	C	**D**	5	4
2	D	D	1	



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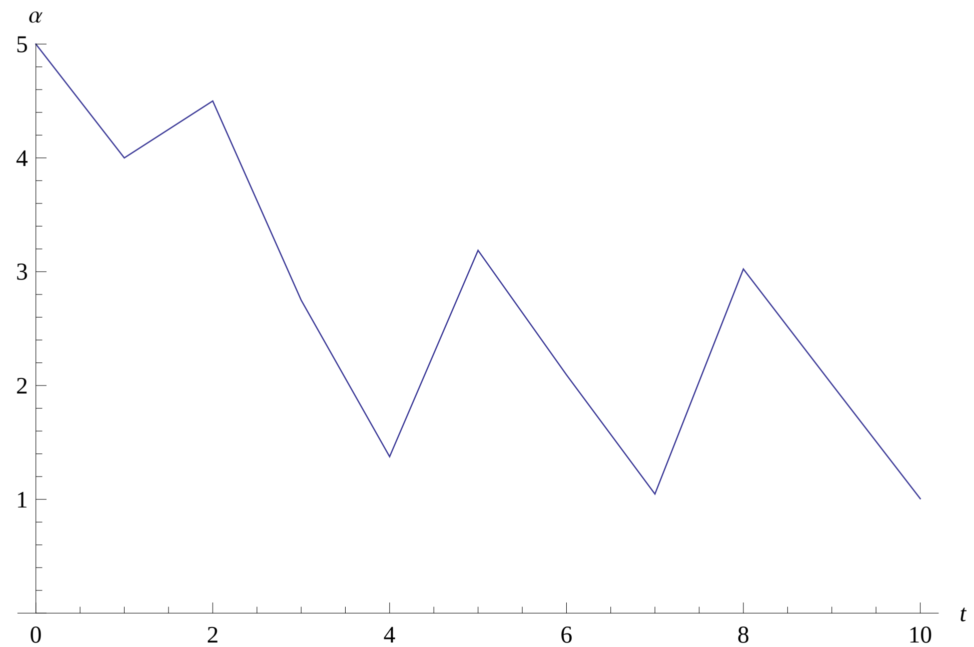
0	C	C	3	5
1	C	**D**	5	4
2	D	D	1	4.5



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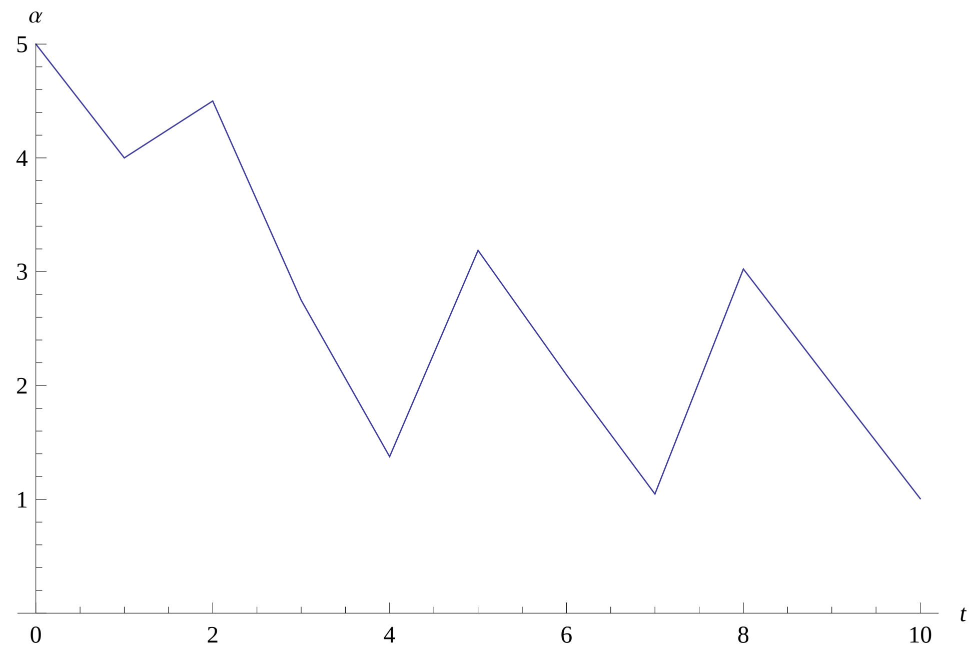
3				



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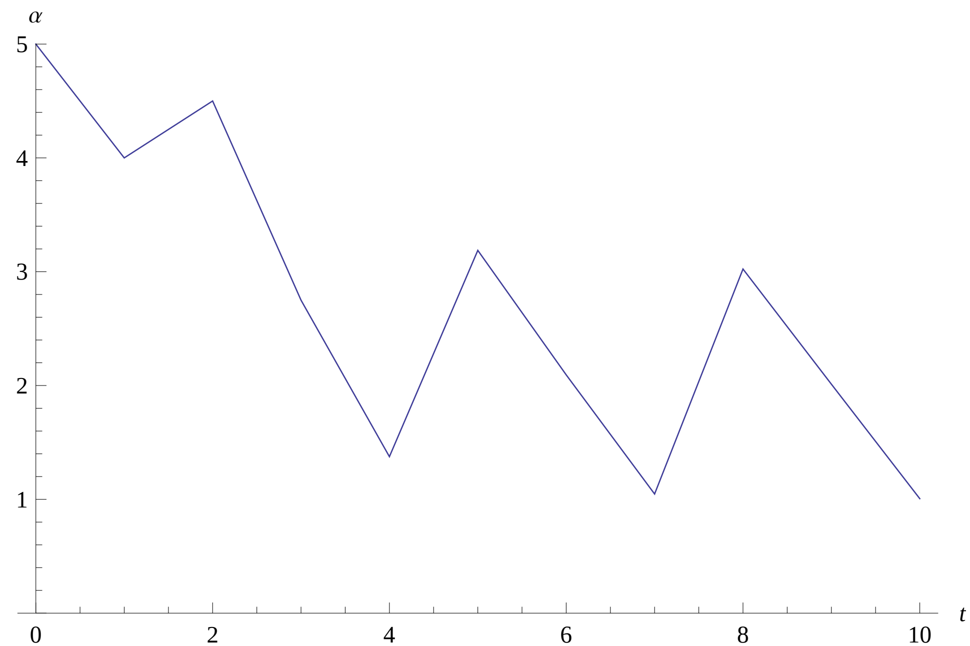
3	D			



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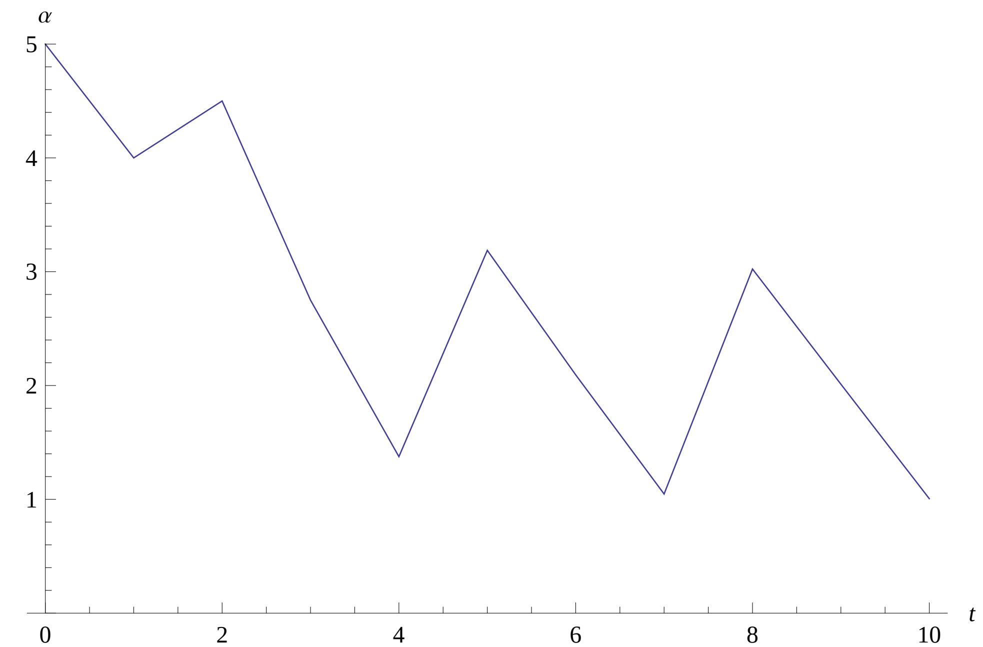
Game: prisoner's dilemma. Strategy player 1: tit-for-tat. Strategy player 2: satisficing with initial state $(A_0, \alpha_0) = (C, 5)$. Persistence rate: $\lambda = 0.5$.

t	TFT	A_t	π_t	α_t
0	C	C	3	5
1	C	**D**	5	4
2	D	D	1	4.5
3	D	**C**		



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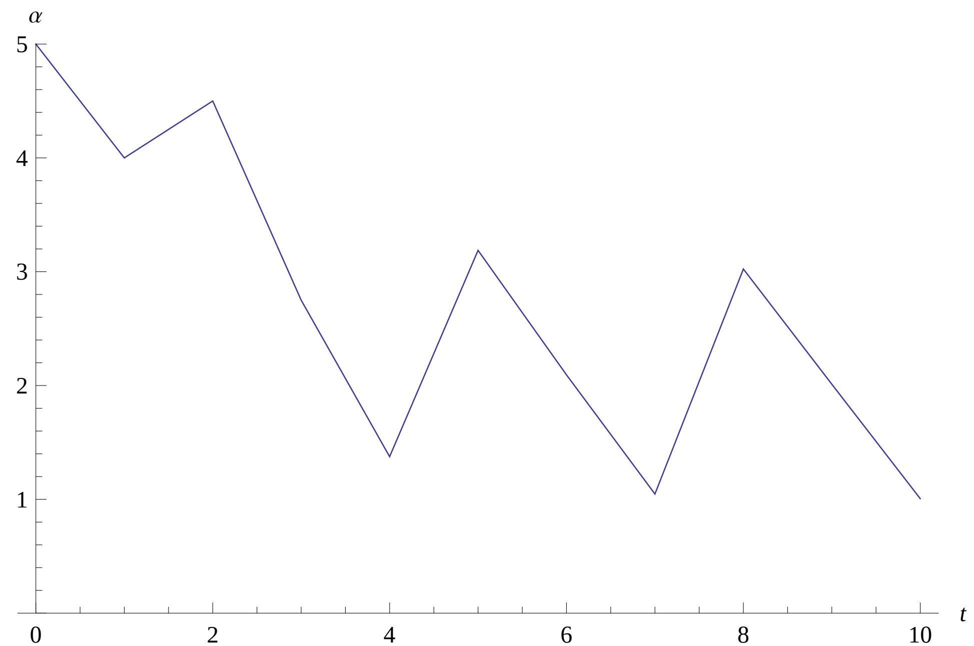
0	C	C	3	5
1	C	**D**	5	4
2	D	D	1	4.5
3	D	**C**	0	



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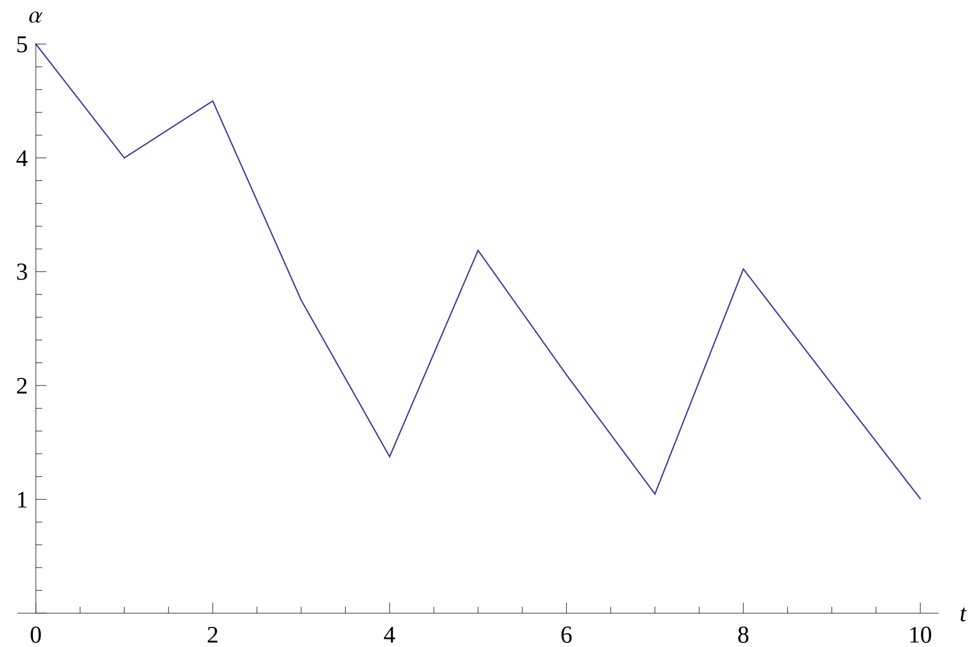
0	C	C	3	5
1	C	**D**	5	4
2	D	D	1	4.5
3	D	**C**	0	2.75



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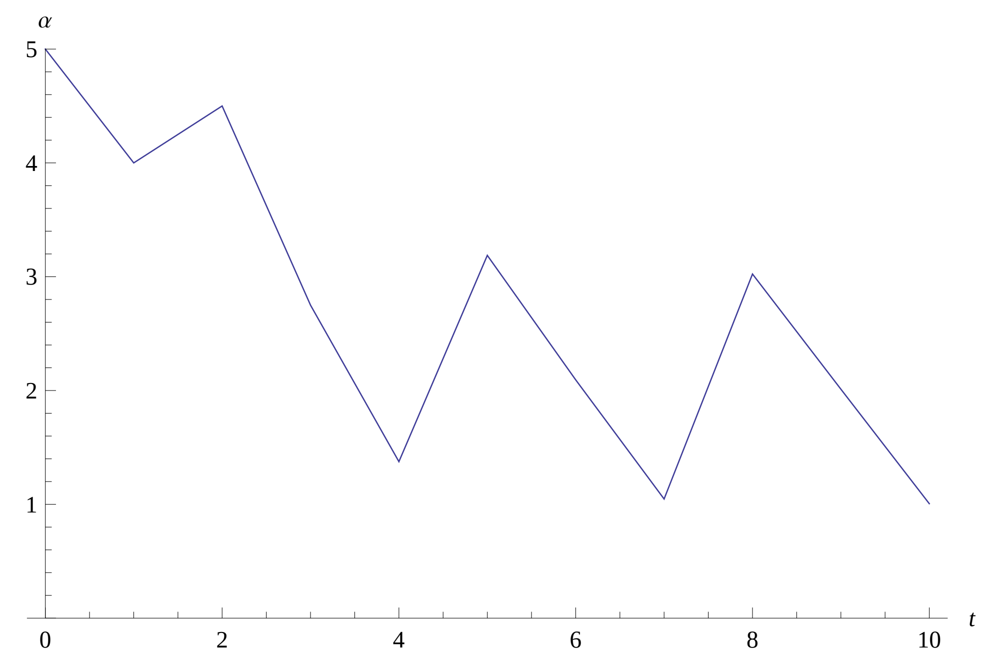
4				



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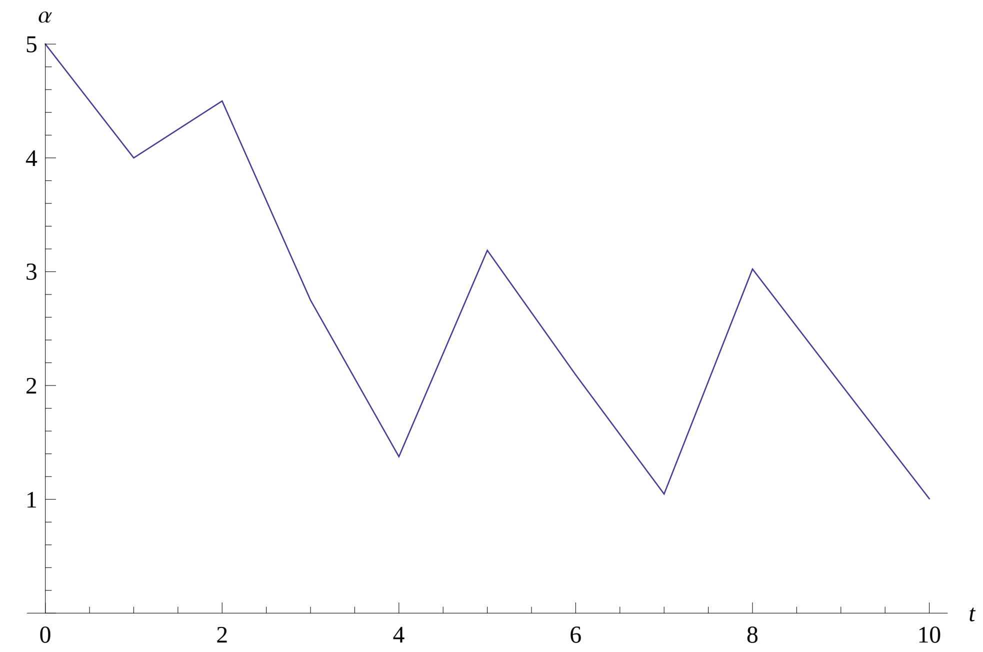
4	C			



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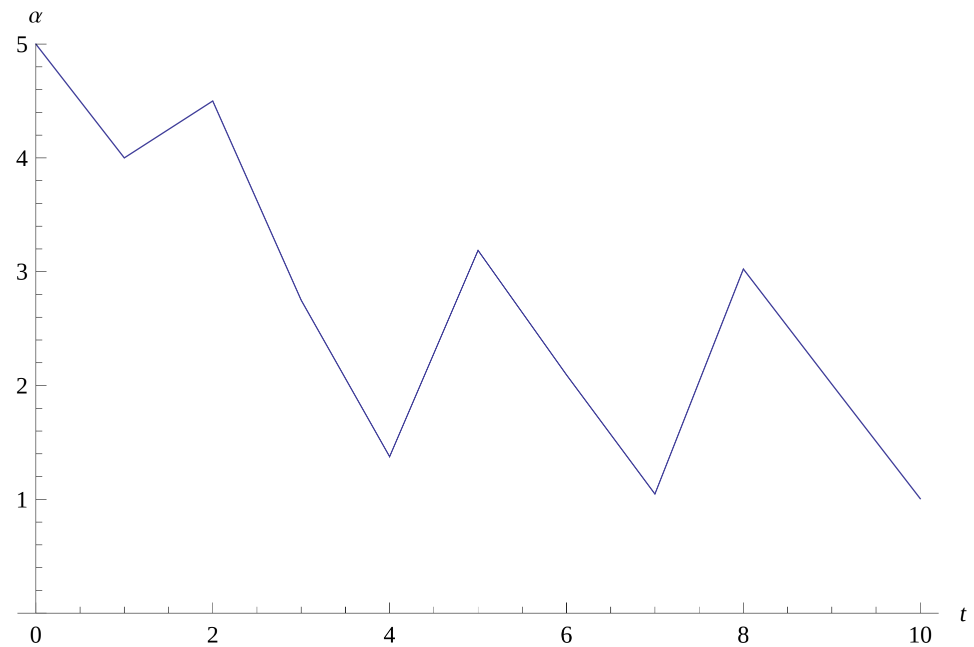

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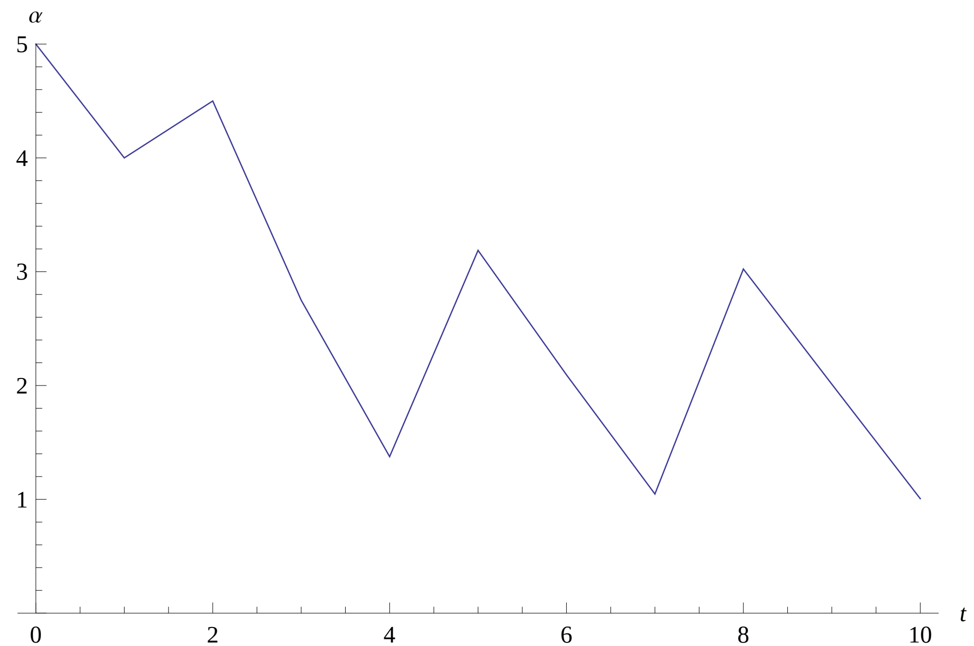

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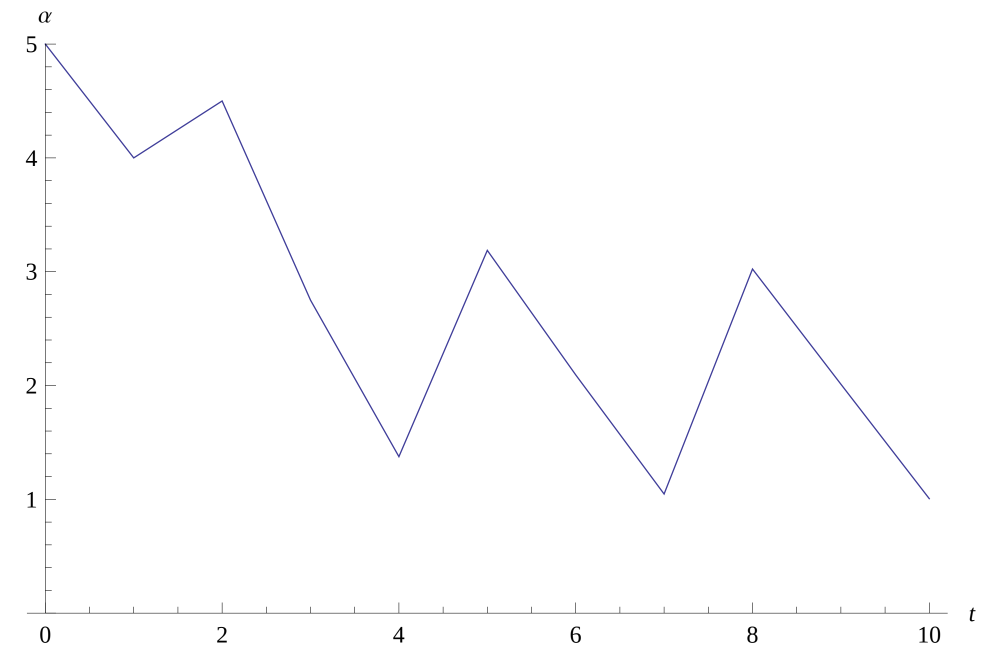
3	D	**C**	0	2.75
4	C	**D**	5	1.375



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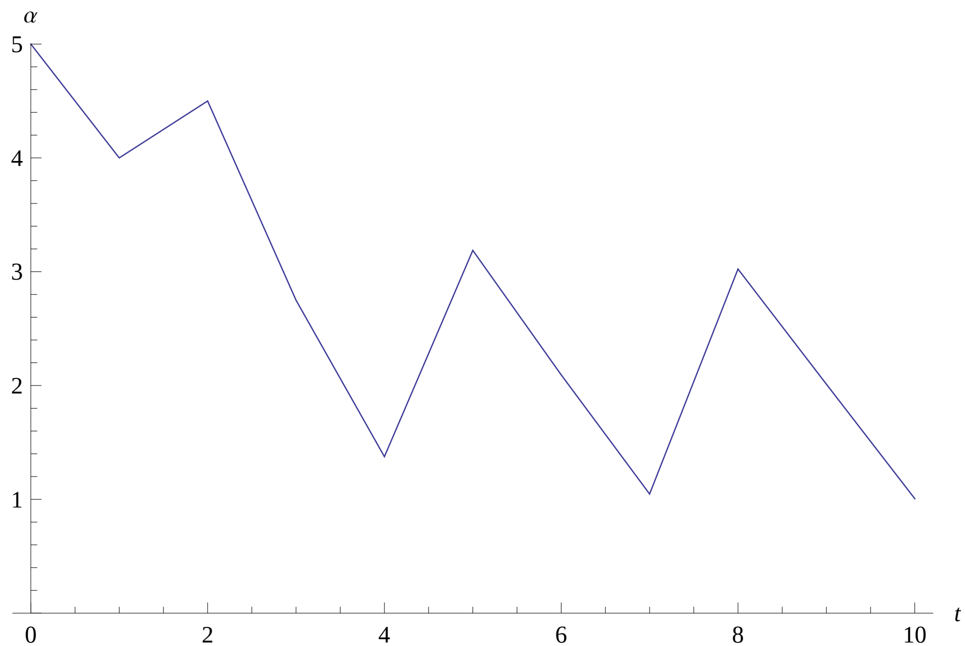
3	D	**C**	0	2.75
4	C	**D**	5	1.375
5				



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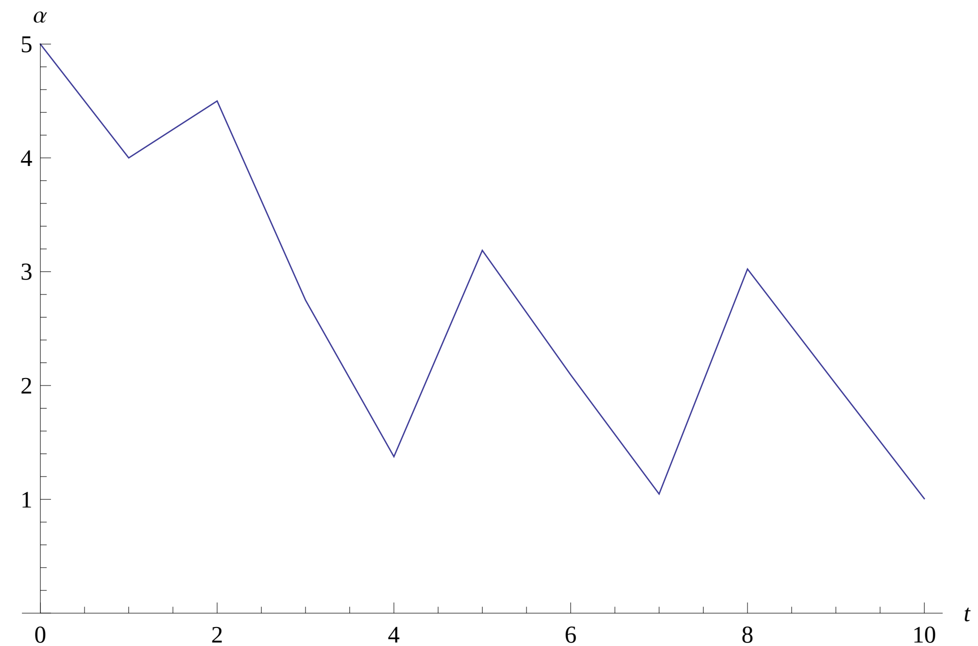
5	D			



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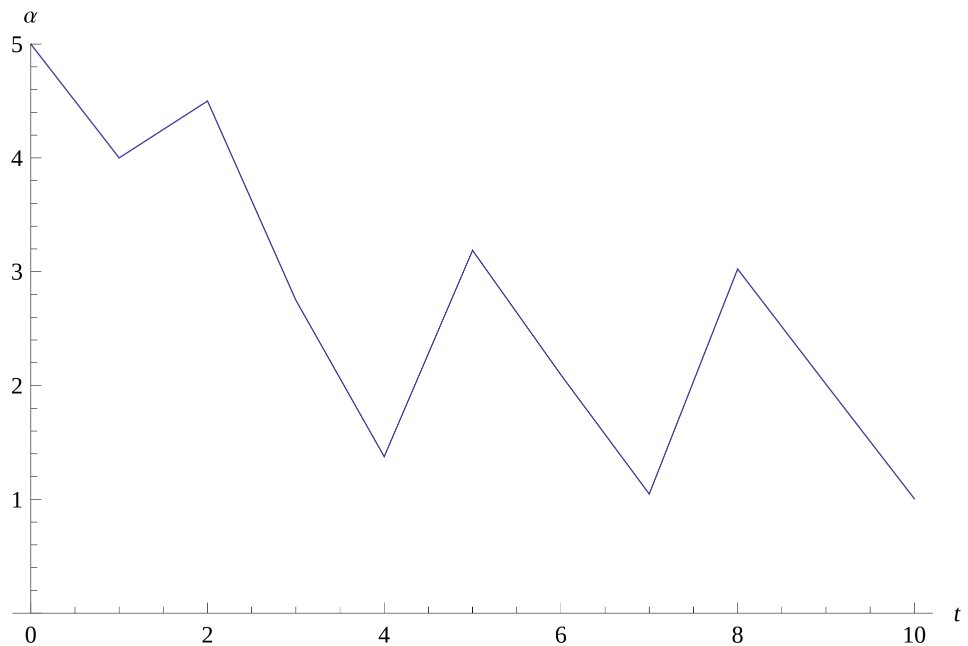
5	D	D		



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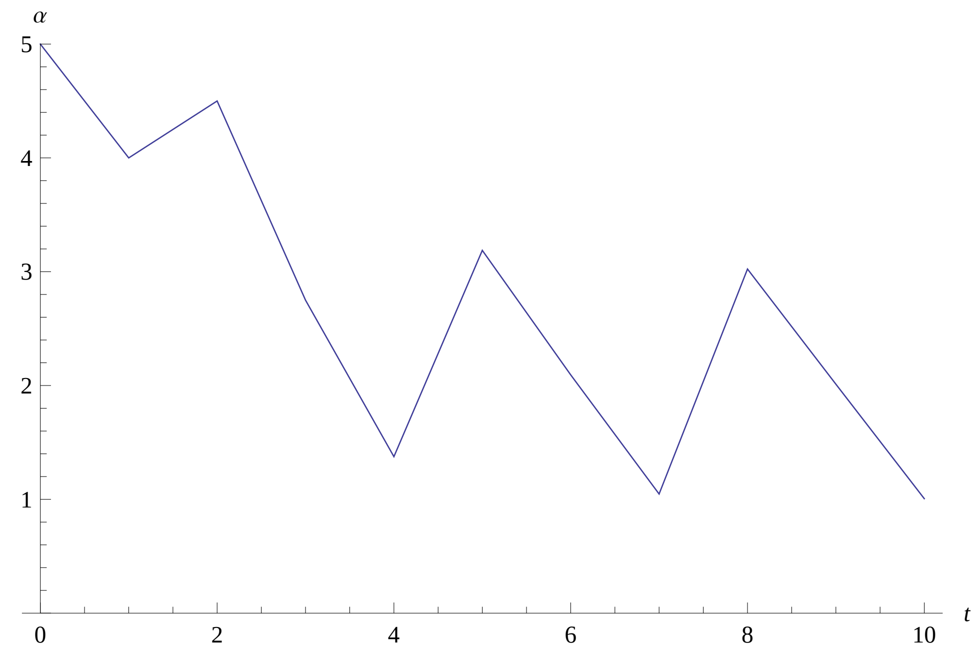
5	D	D	1	



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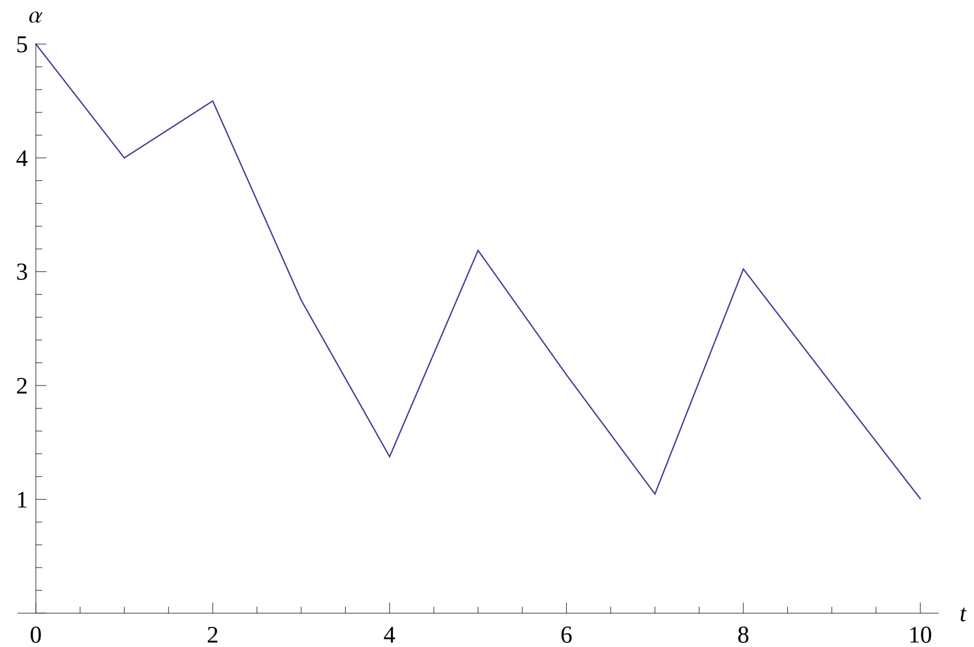

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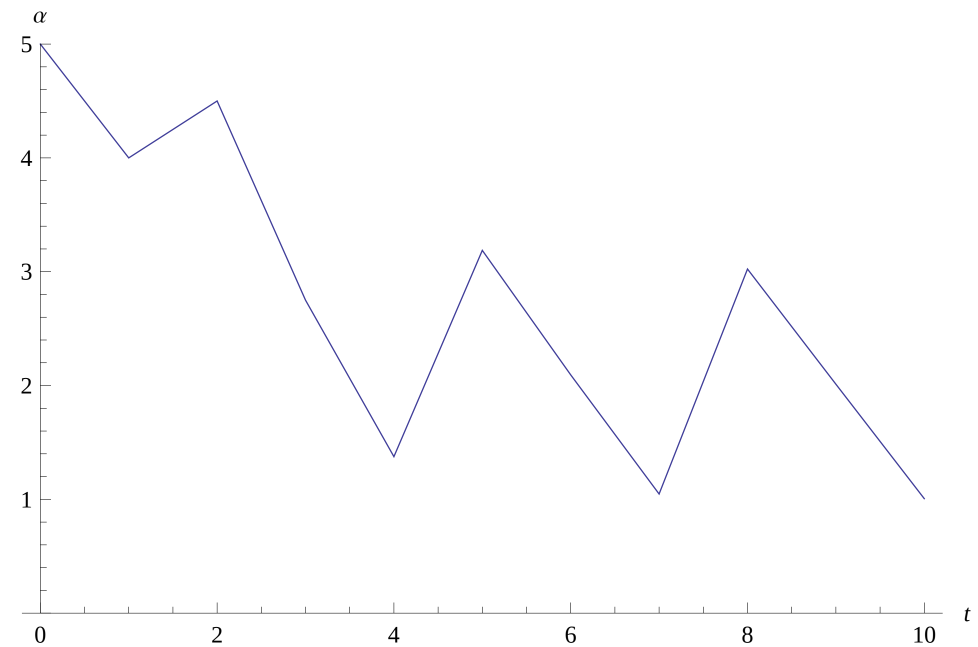
6				



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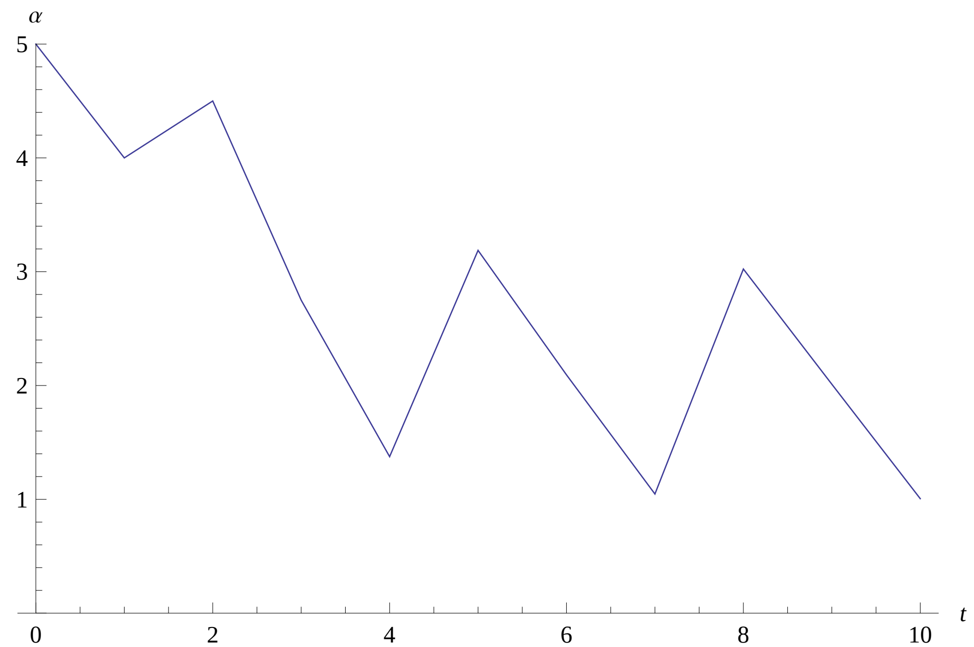
6	D			



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5	D	D	1	3.1875
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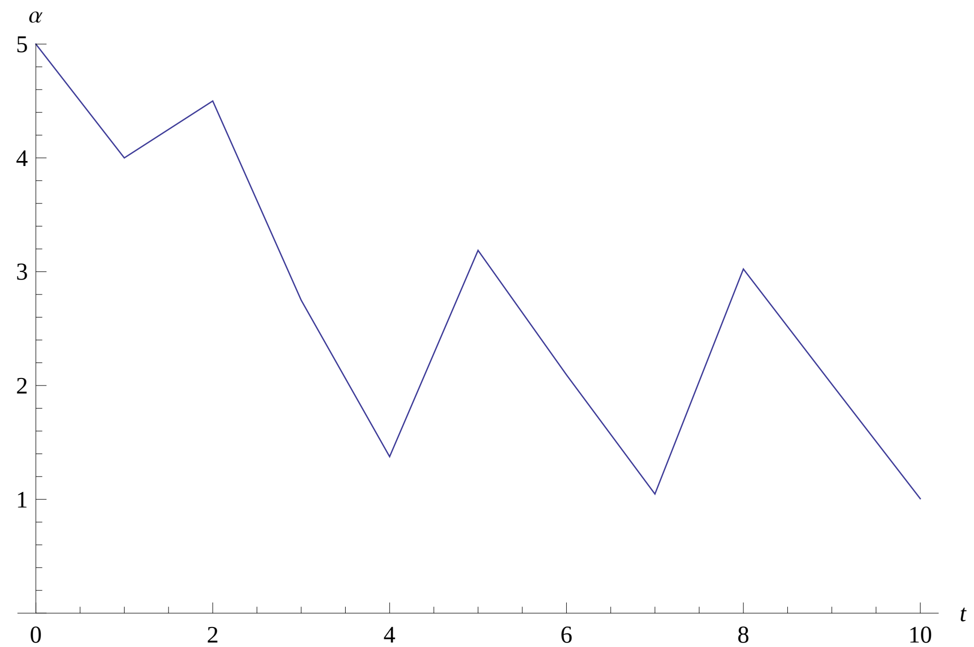

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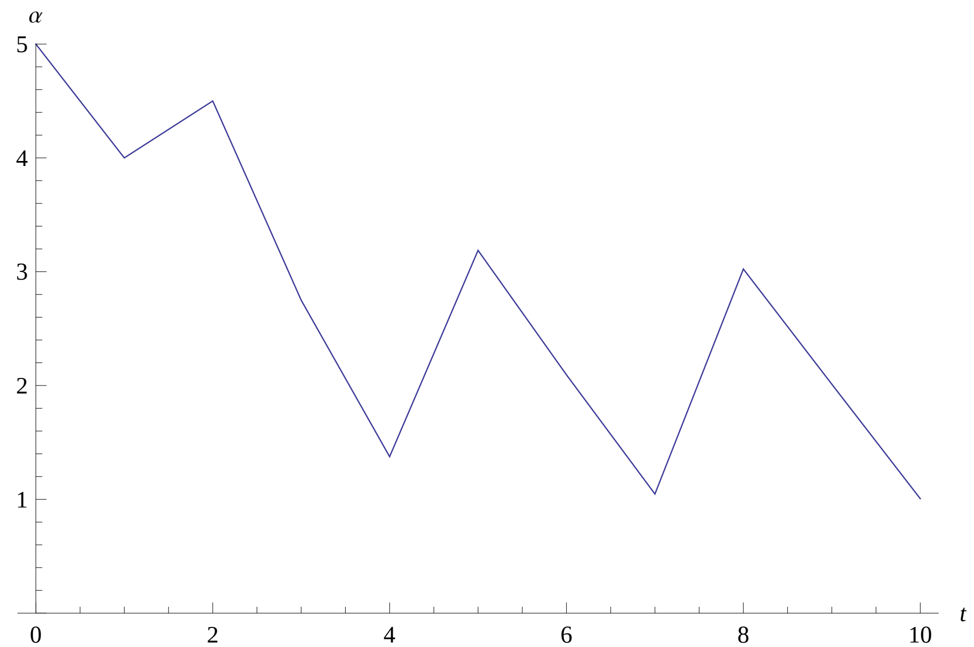
4	C	**D**	5	1.375
5	D	D	1	3.1875
6	D	**C**	0	



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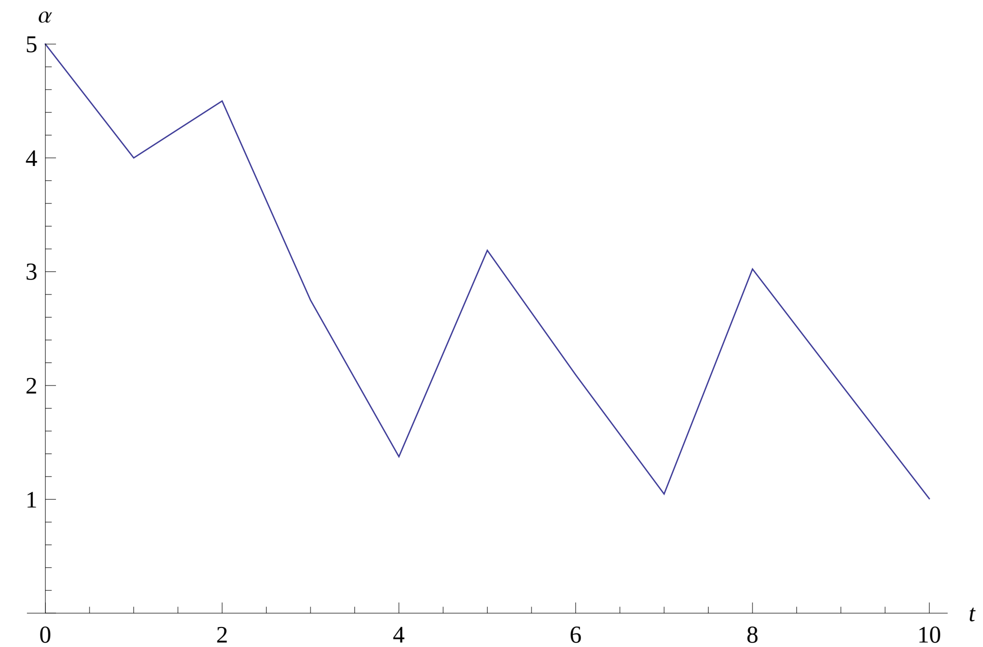
4	C	**D**	5	1.375
5	D	D	1	3.1875
6	D	**C**	0	2.09375



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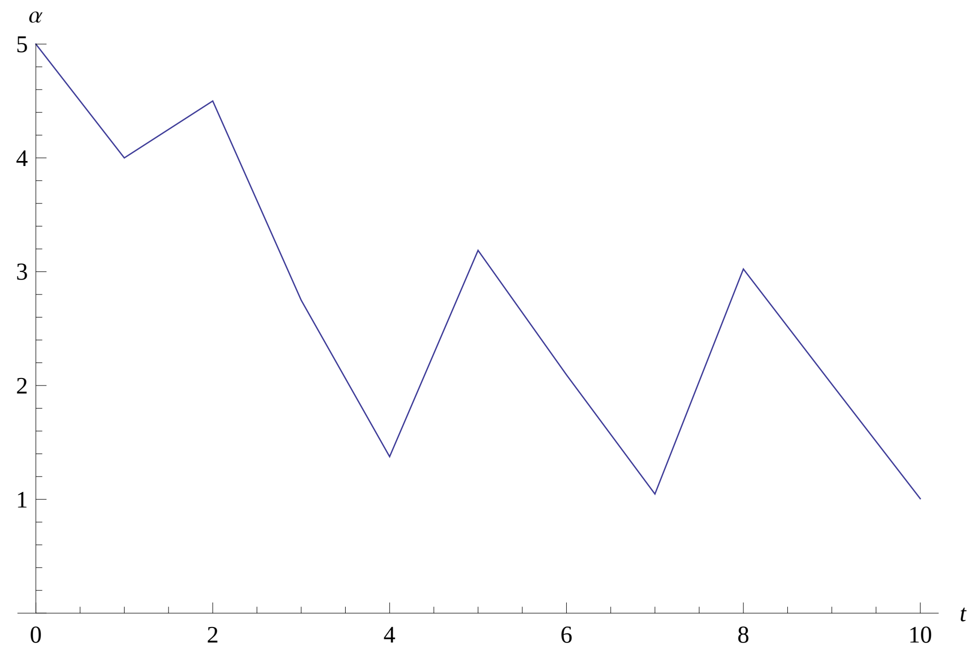
4	C	**D**	5	1.375
5	D	D	1	3.1875
6	D	**C**	0	2.09375
7				



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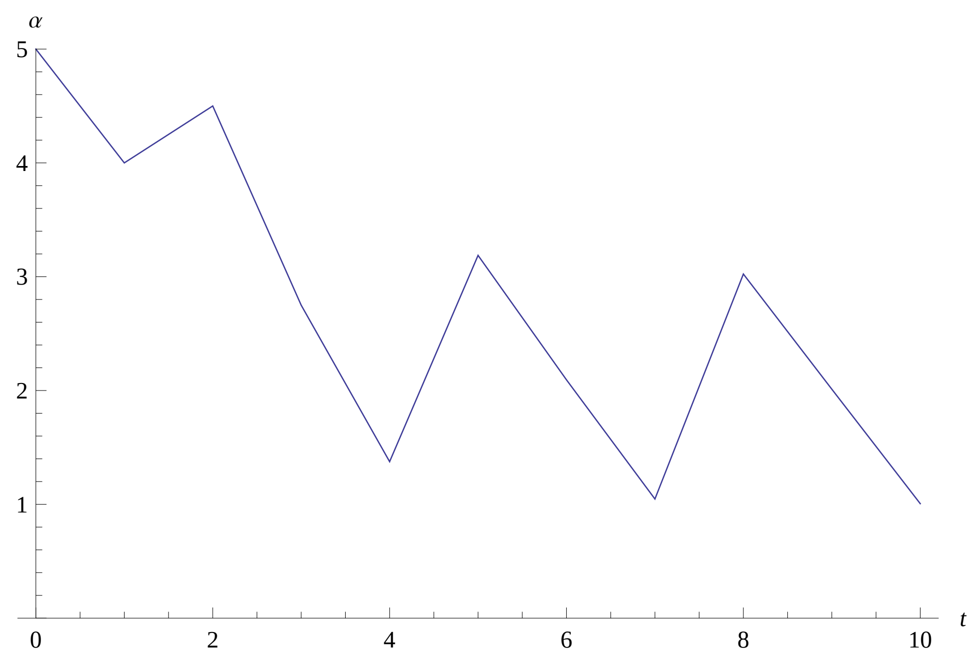
4	C	**D**	5	1.375
5	D	D	1	3.1875
6	D	**C**	0	2.09375
7	C			



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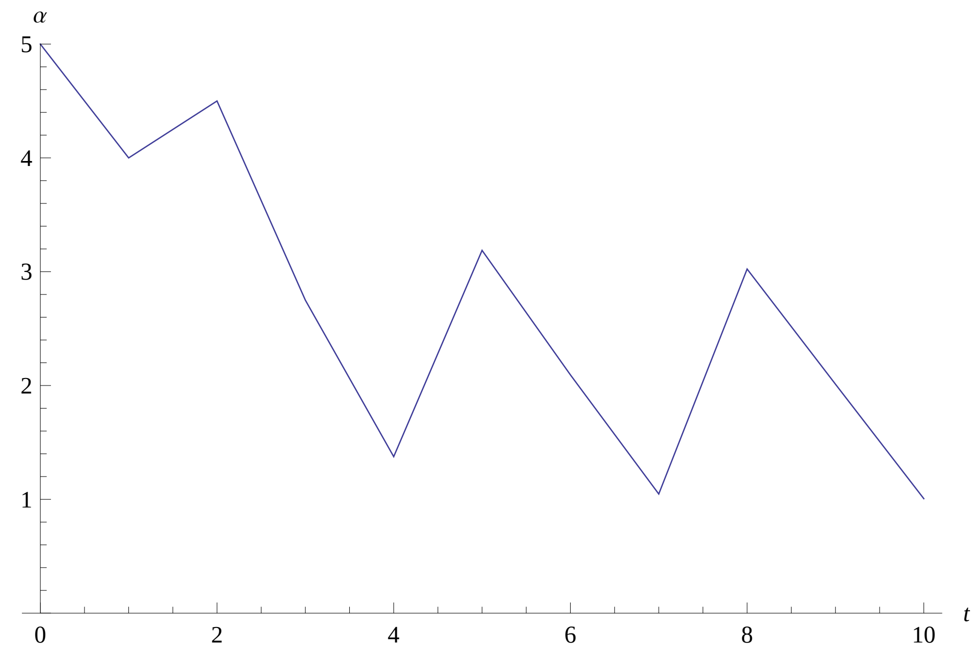
5	D	D	1	3.1875
6	D	**C**	0	2.09375
7	C	**D**		



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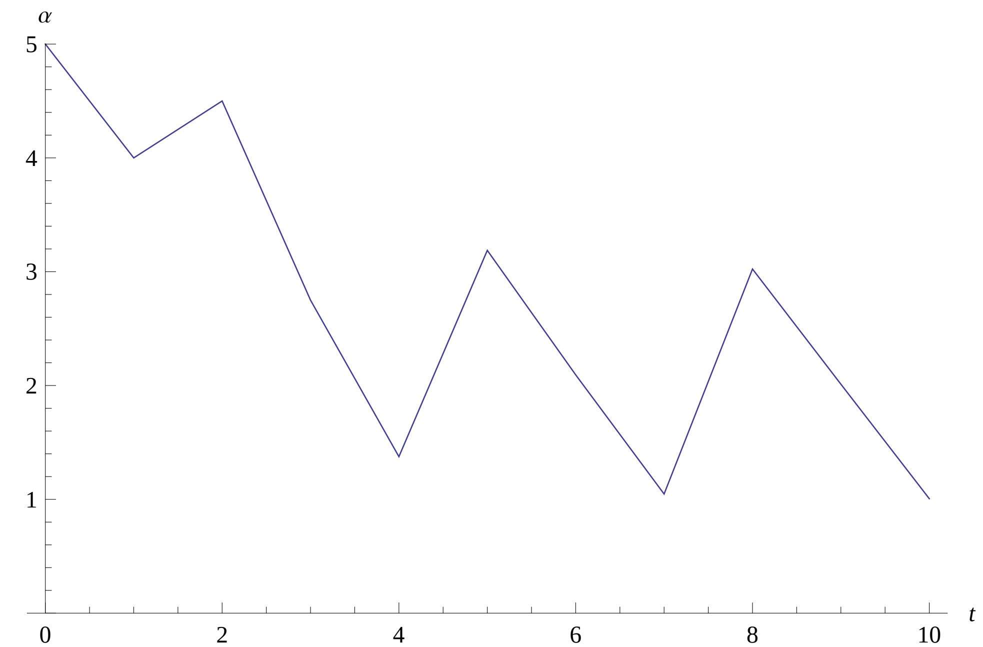

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5	D	D	1	3.1875
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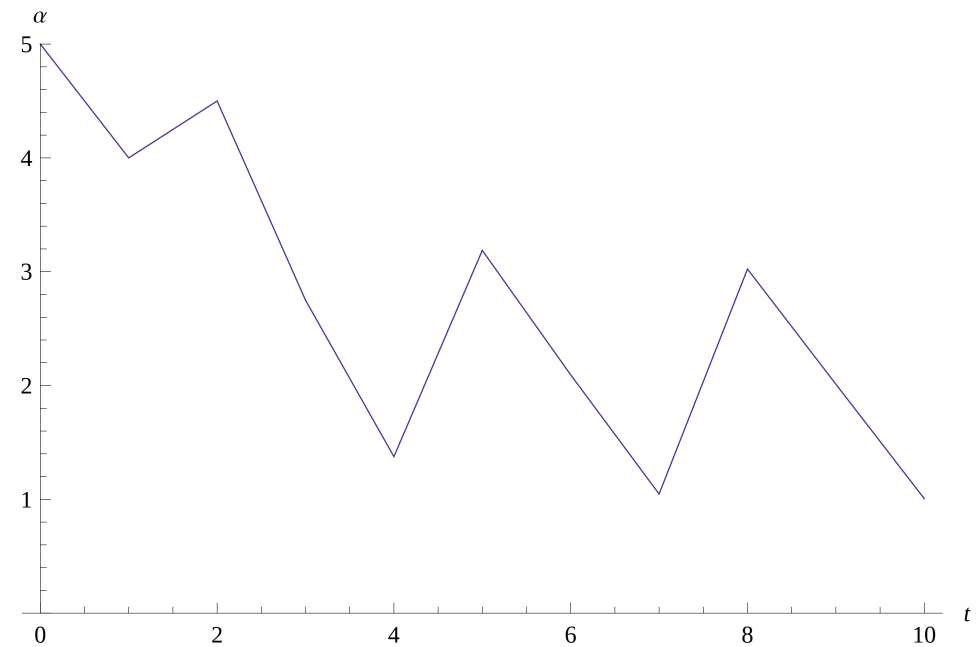
7	C	**D**	5	1.046875



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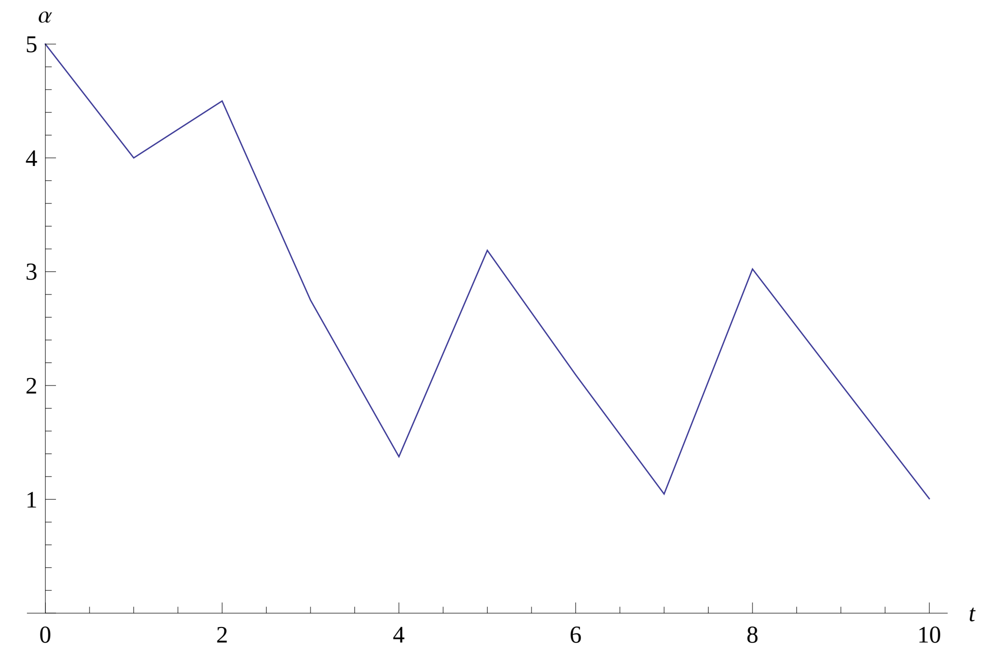
7	C	**D**	5	1.046875
8				



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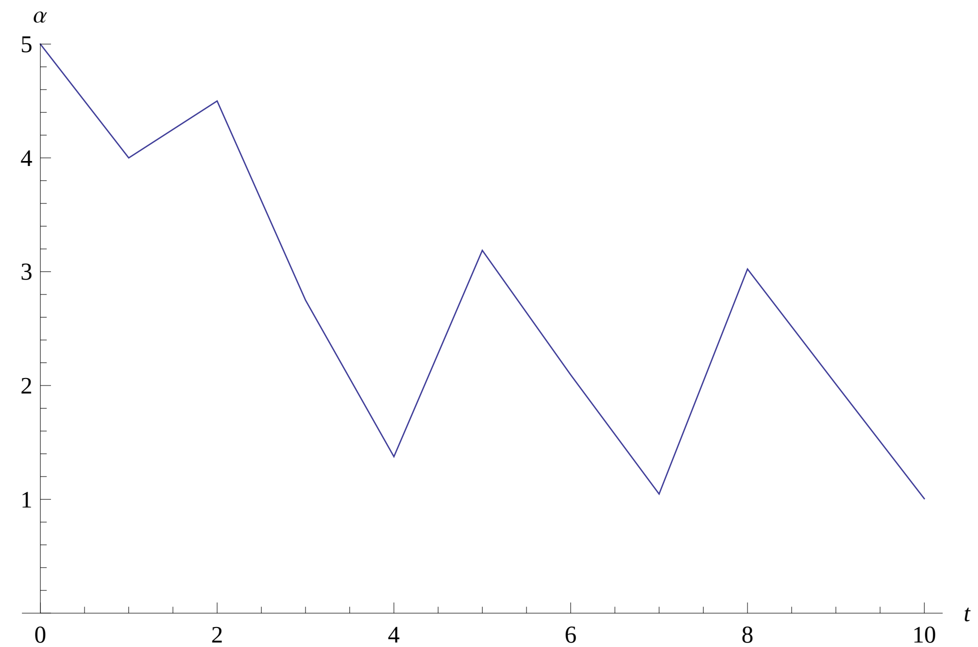

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6	D	**C**	0	2.09375
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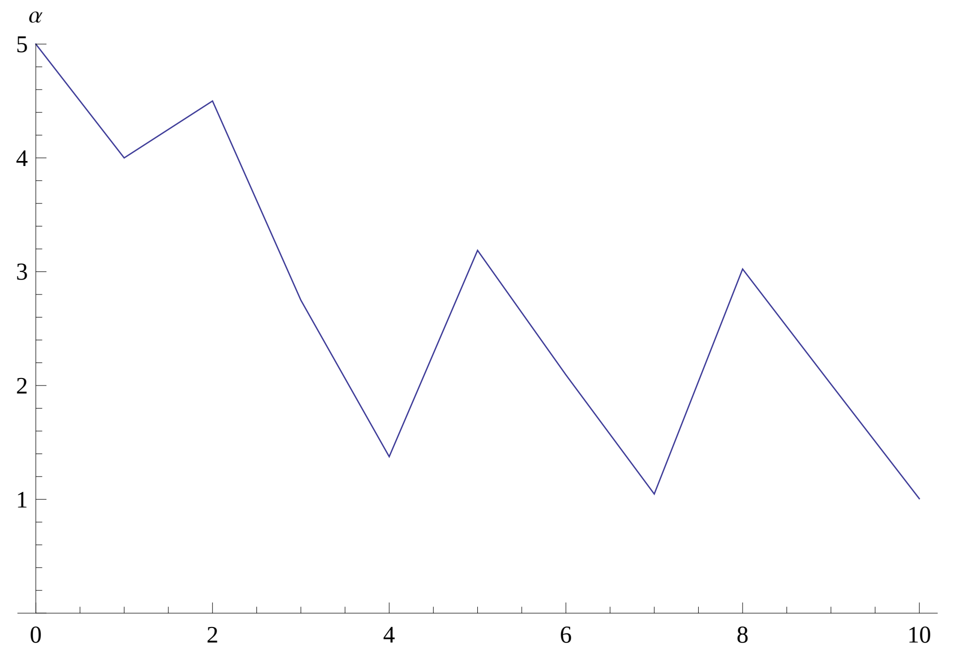
8	D	D		



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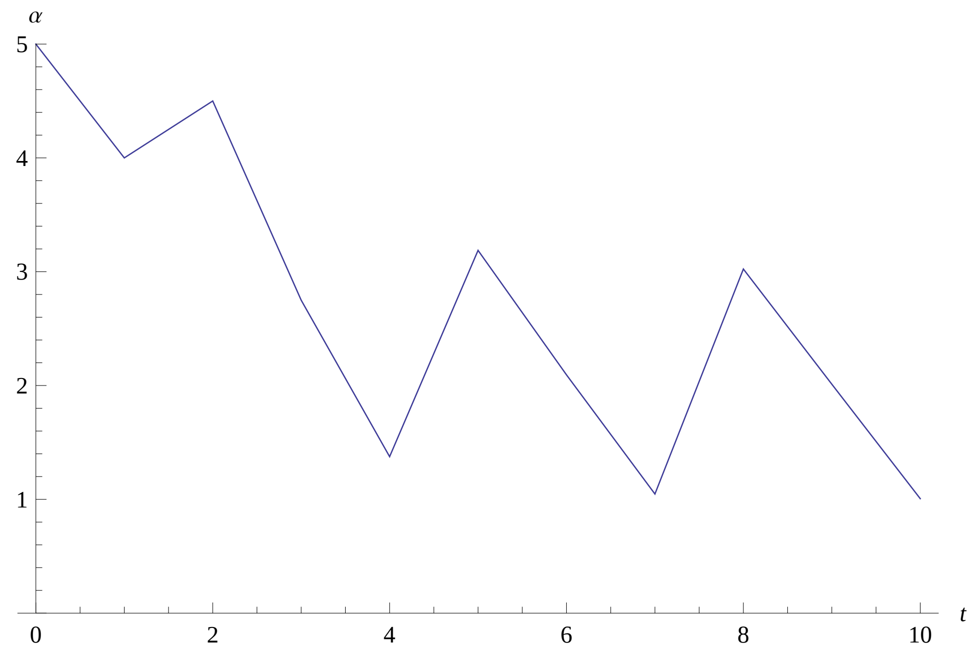

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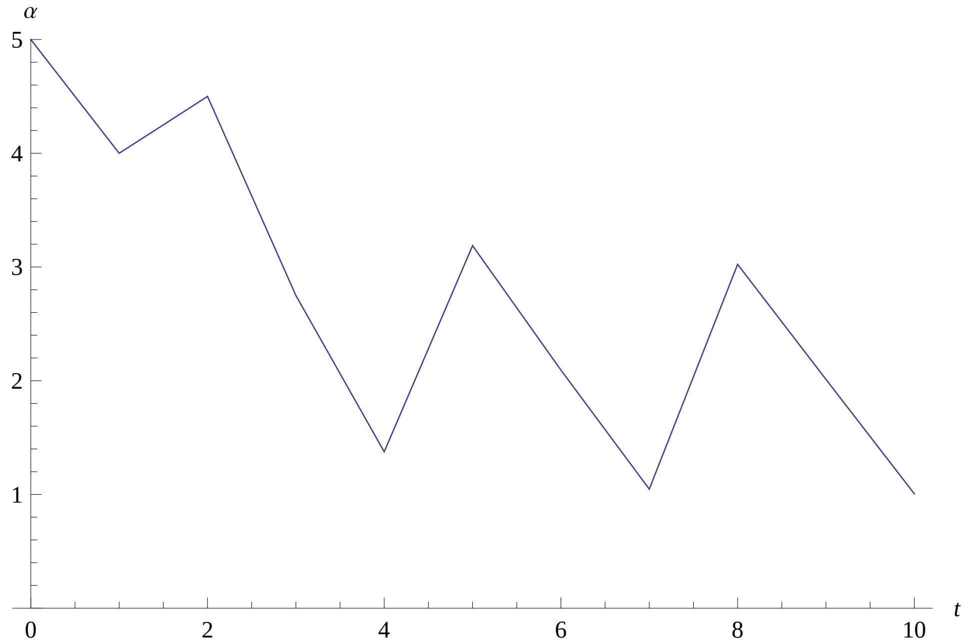

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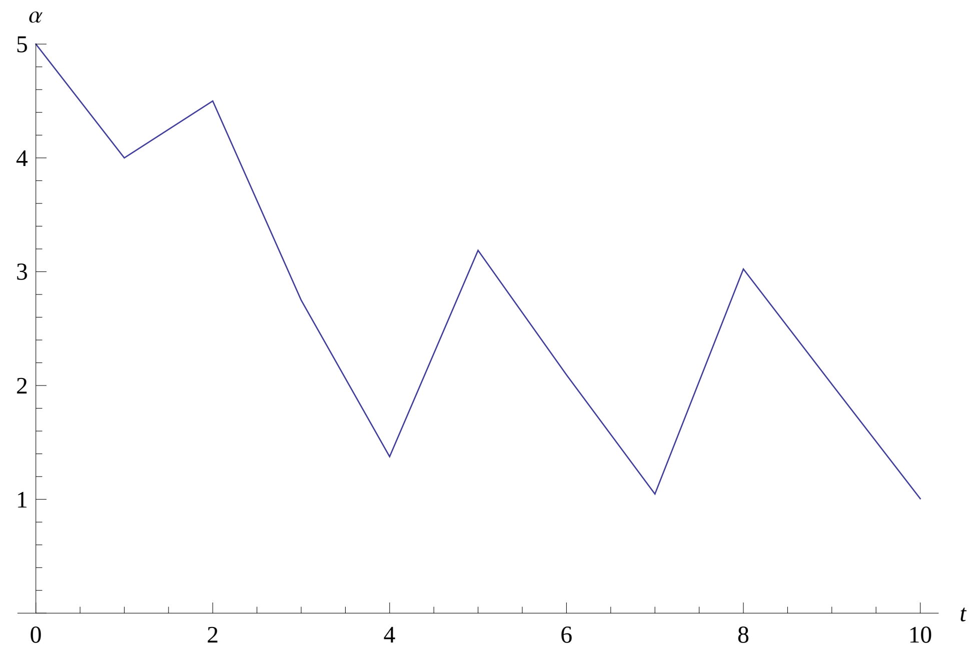
9				



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Game: prisoner's dilemma. Strategy player 1: tit-for-tat. Strategy player 2: satisficing with initial state $(A_0, \alpha_0) = (C, 5)$. Persistence rate: $\lambda = 0.5$.

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0	C	C	3	5
1	C	**D**	5	4
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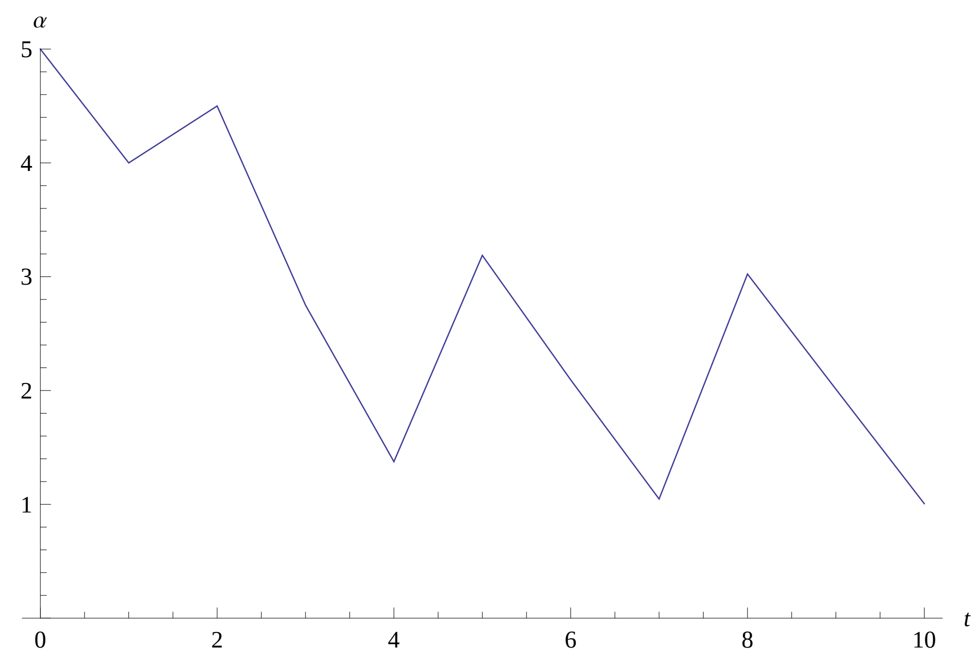
3	D	**C**	0	2.75
4	C	**D**	5	1.375
5	D	D	1	3.1875
6	D	**C**	0	2.09375
7	C	**D**	5	1.046875
8	D	D	1	3.0234375
9	D			



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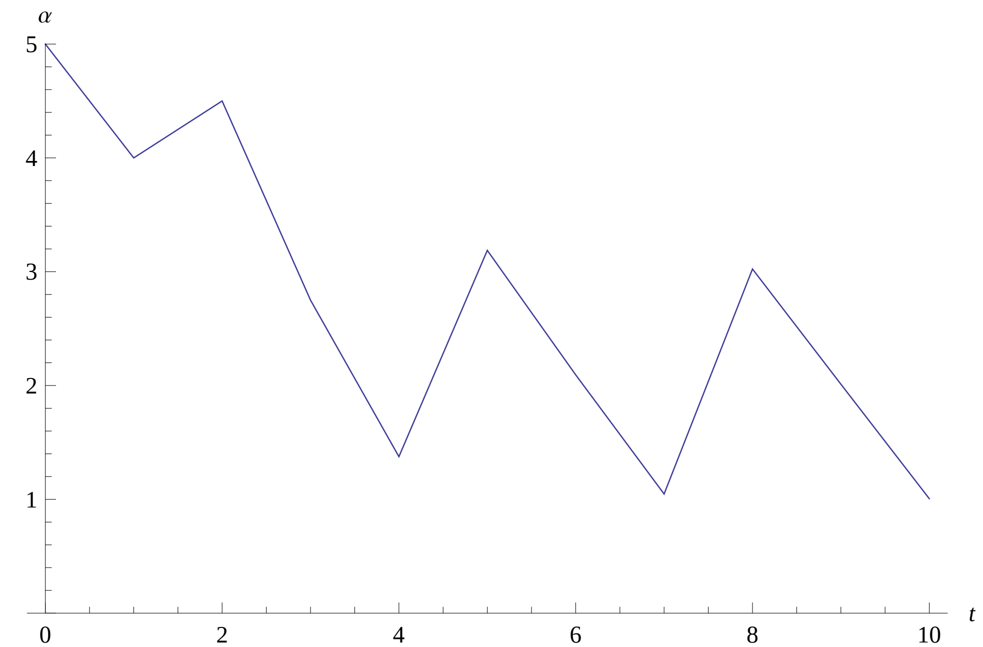

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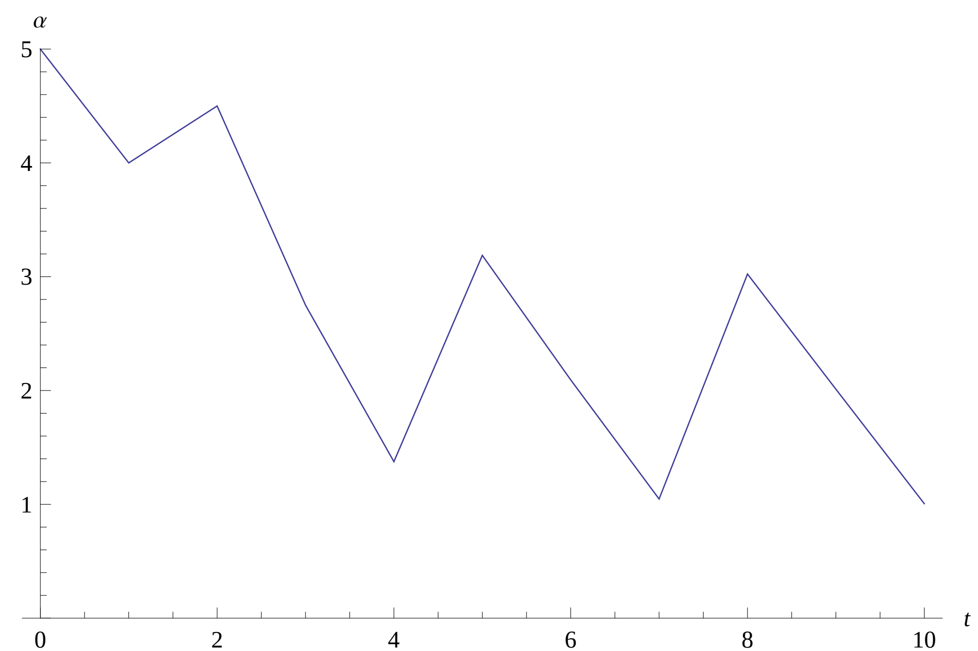

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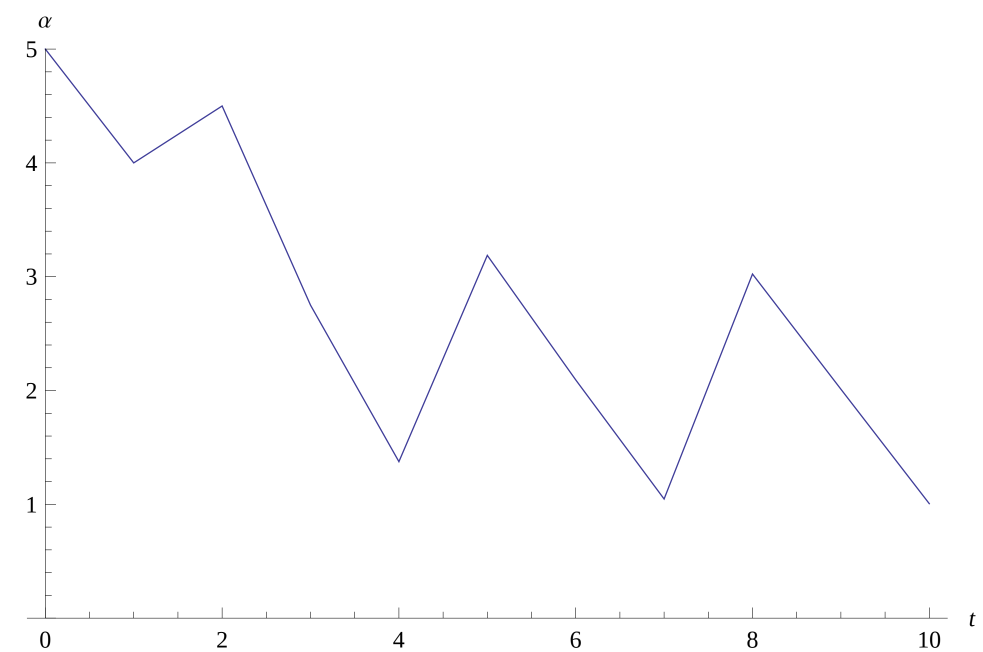

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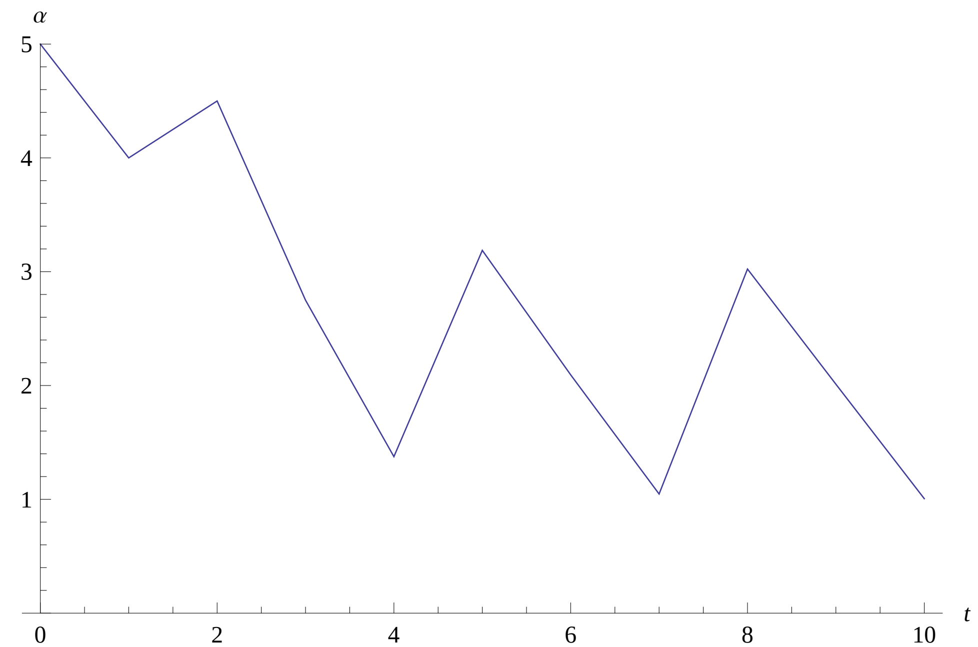

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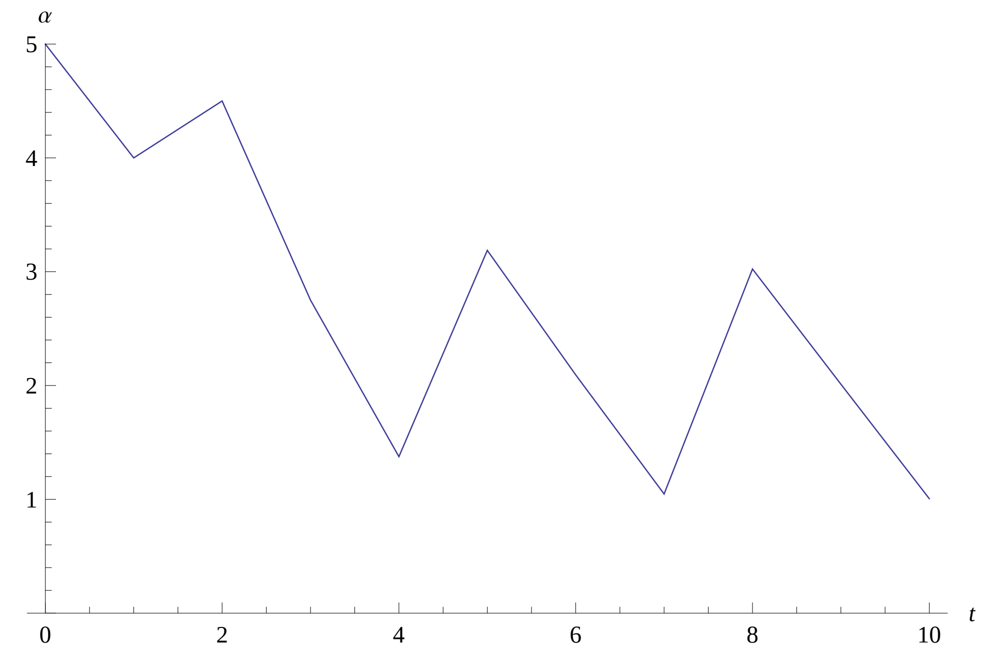

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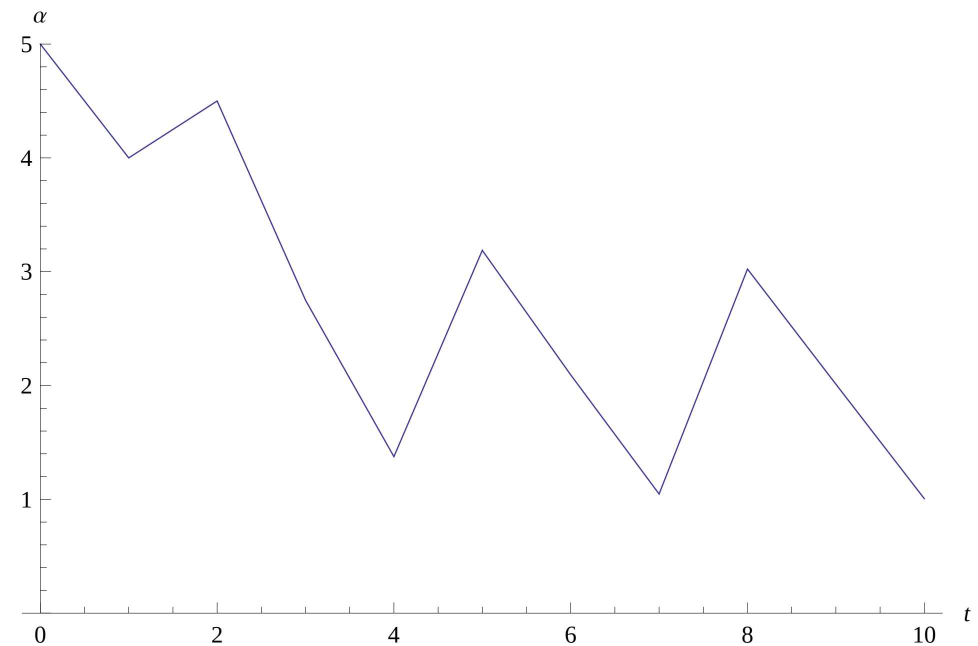

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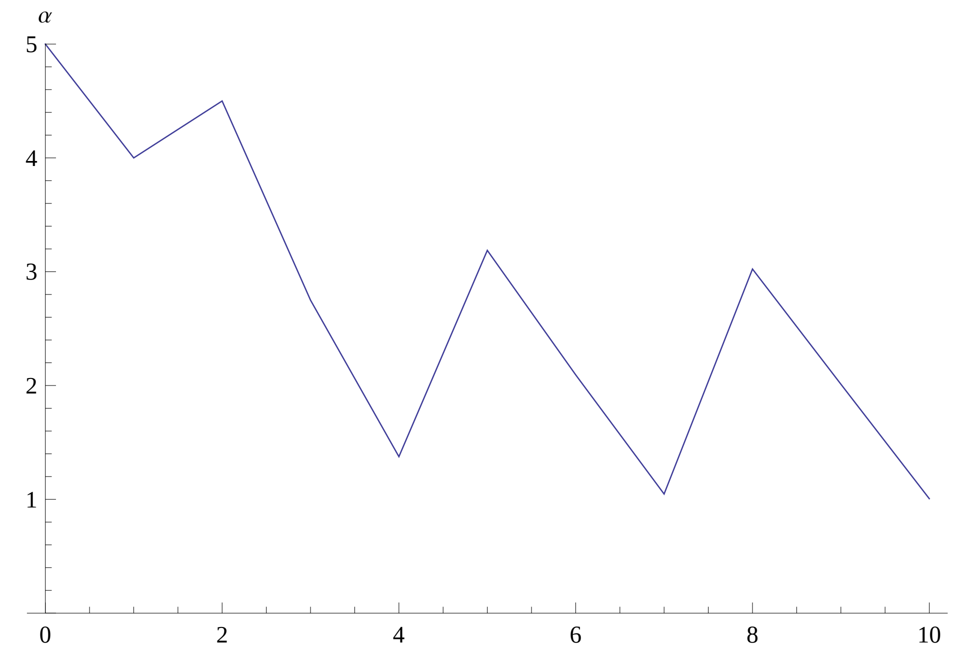

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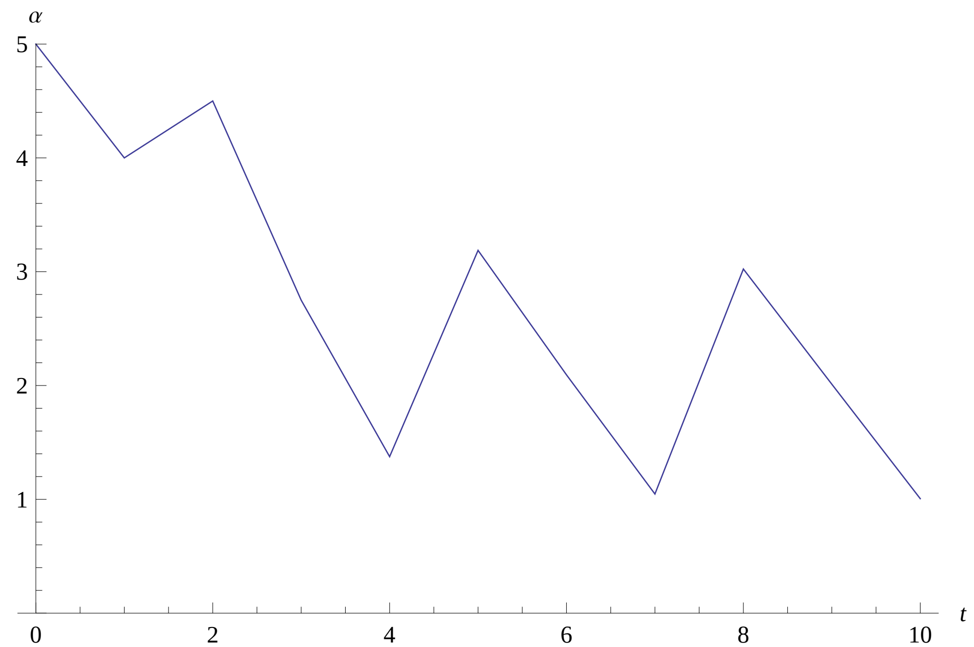
9	D	**C**	0	2.01171875
10	C	D	5	1.005859375



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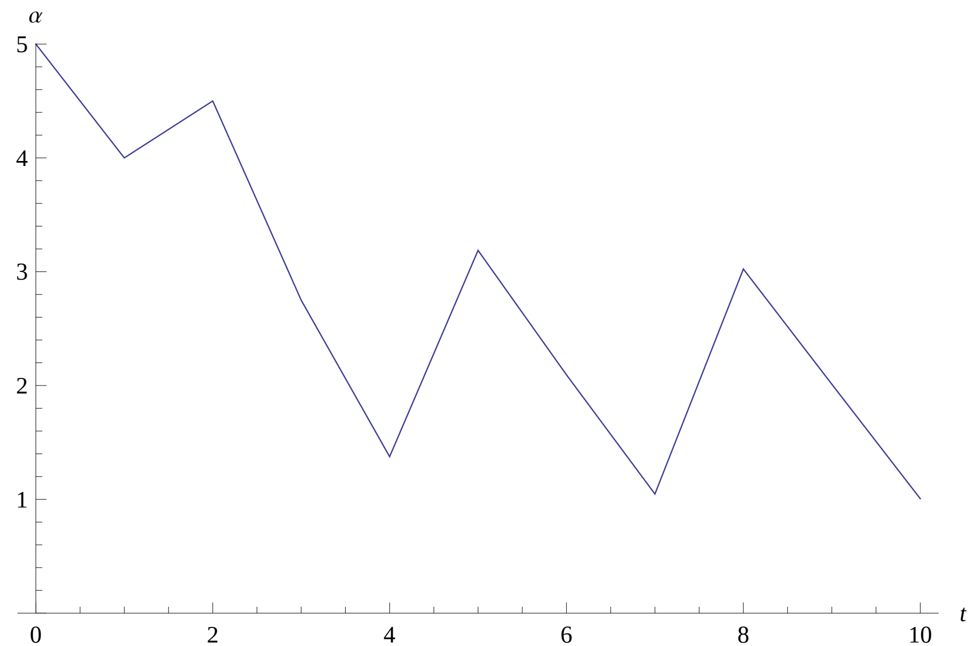
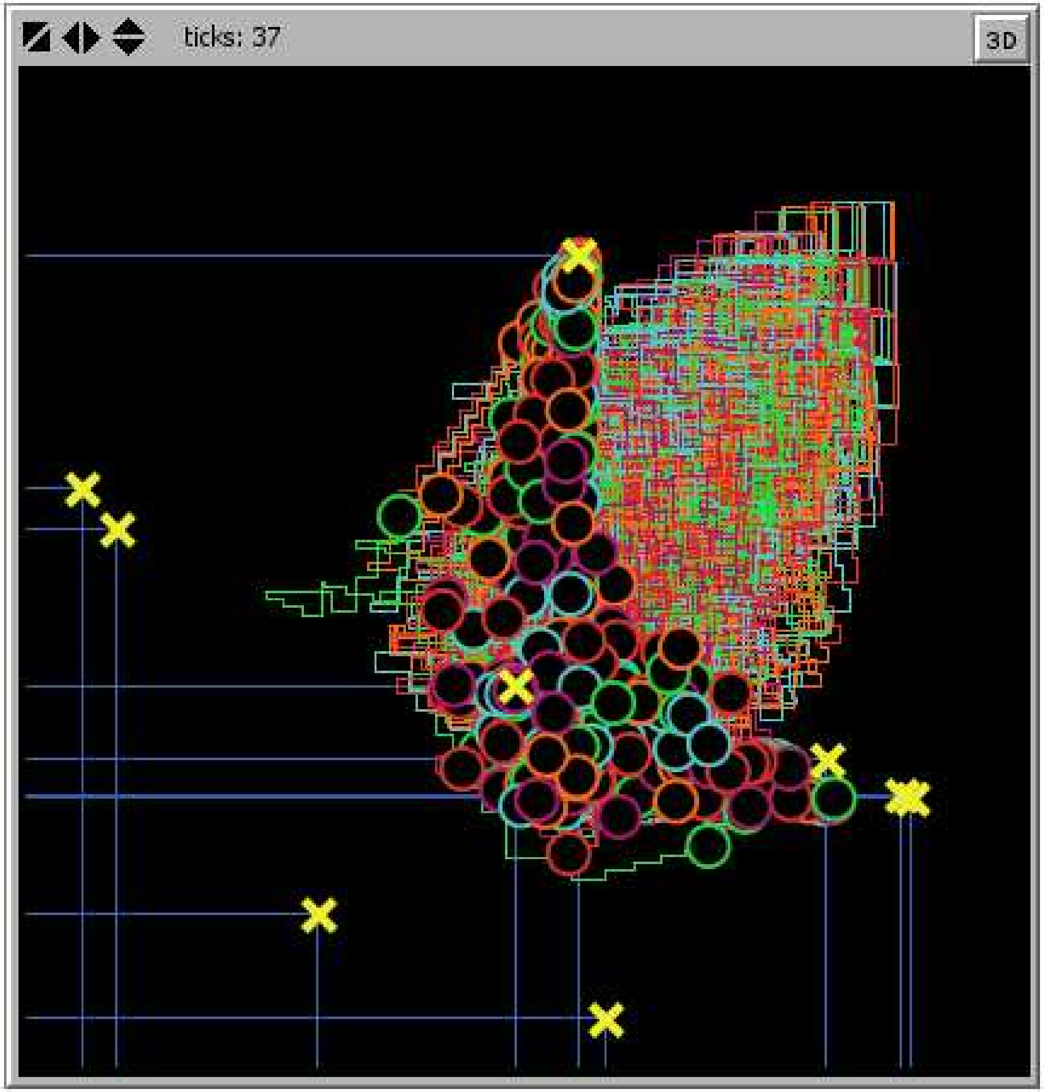


Progress of aspirations.

Demo:

Satisficing play in general 2-player 3x3 matrix games

Approach



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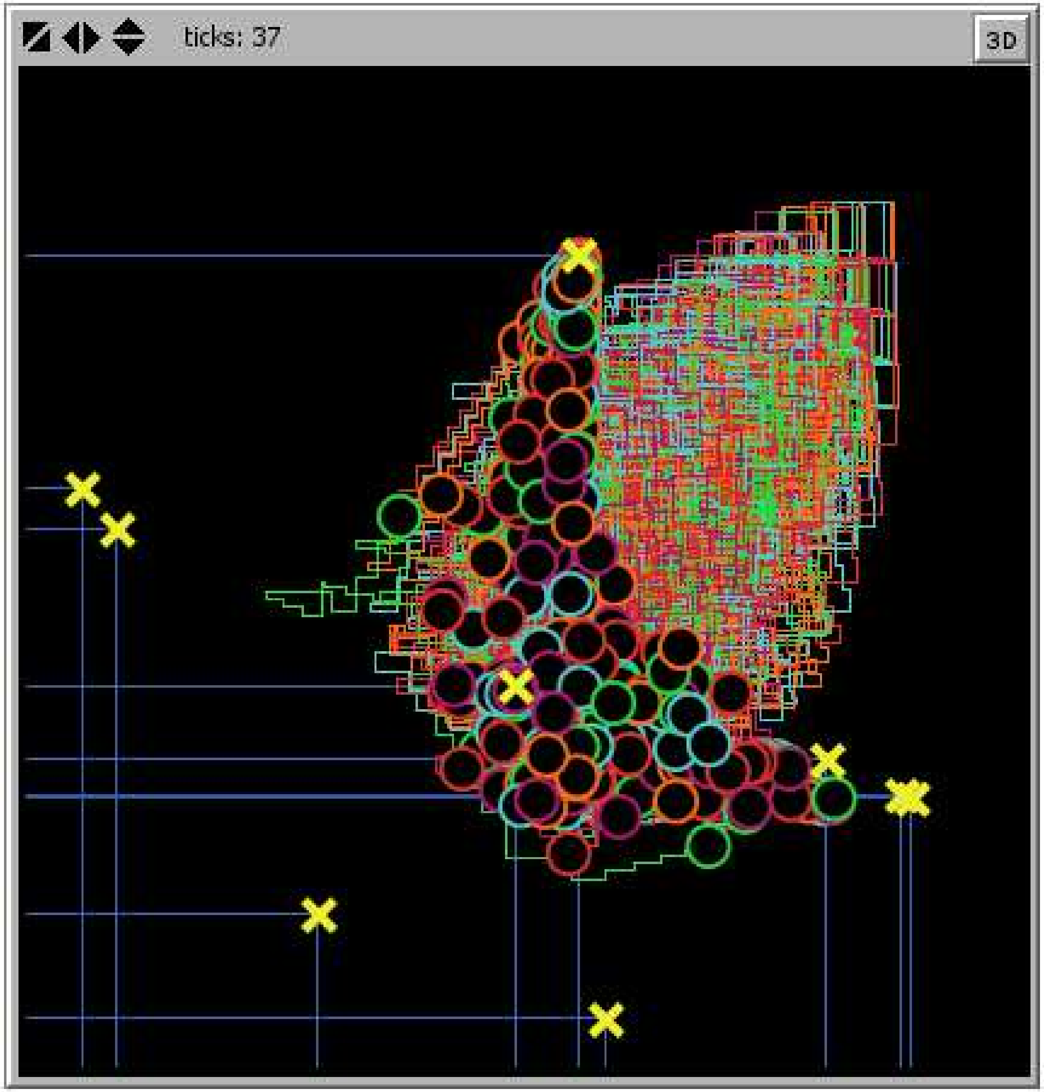
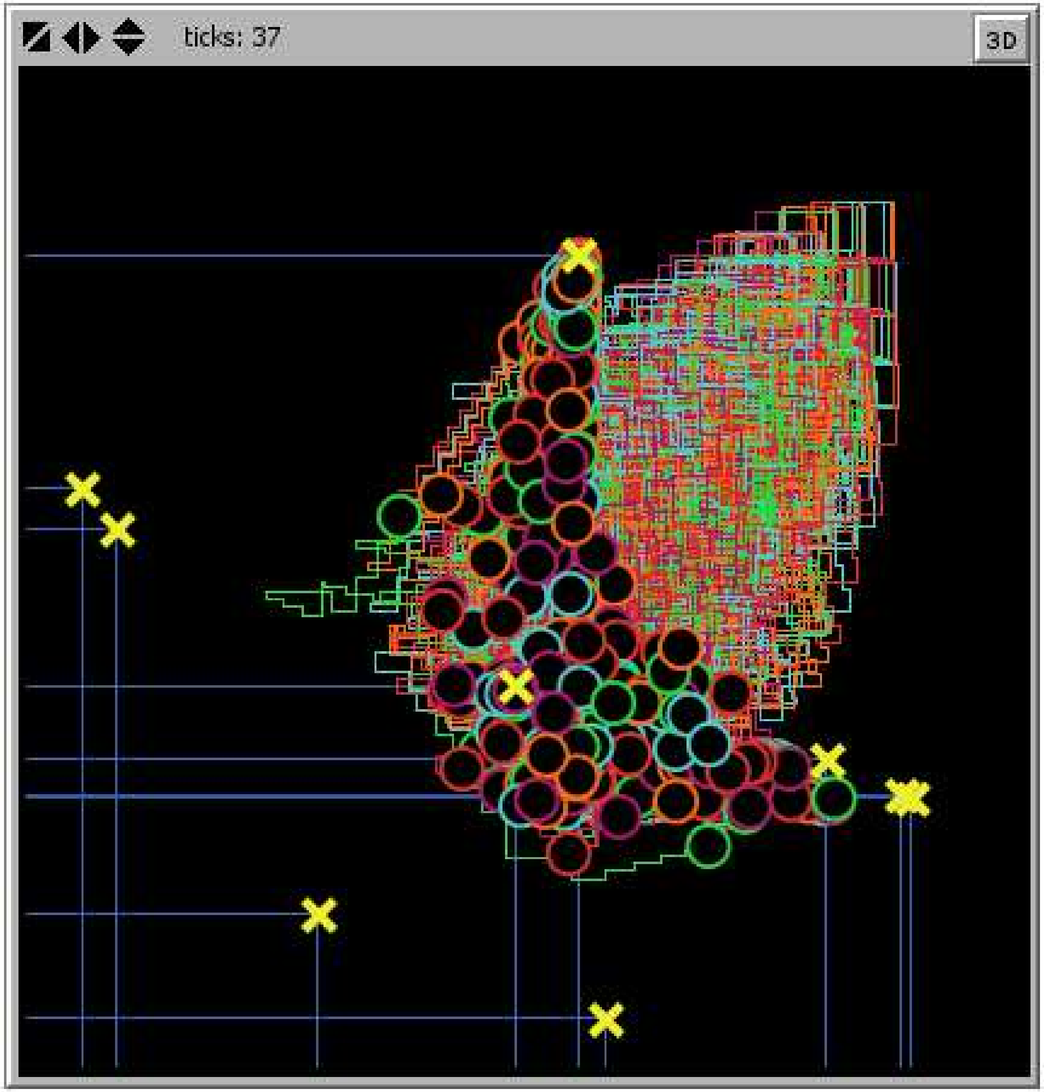


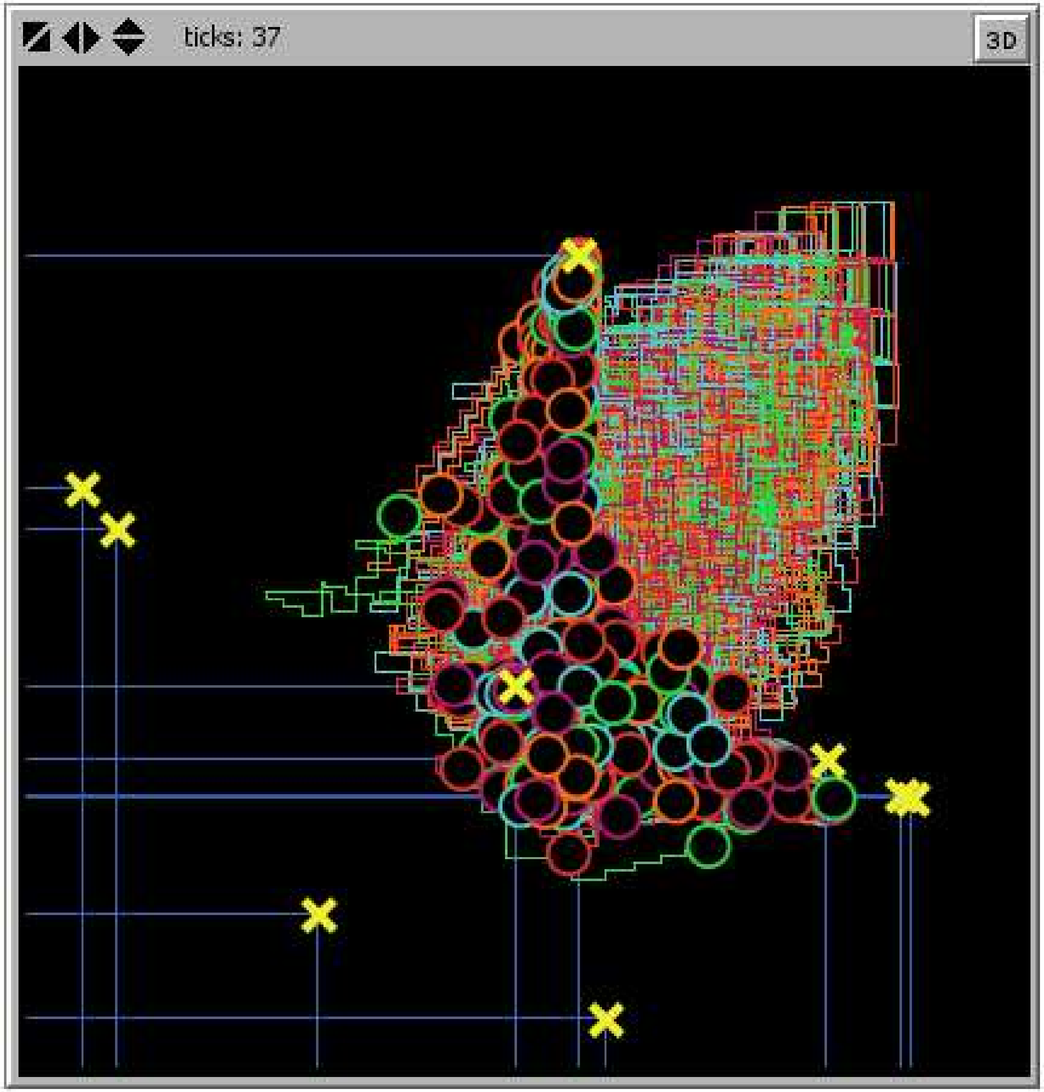
- Take a 2-player 3×3 game in normal form.

Approach



- Take a 2-player 3×3 game in normal form.
- Plot all 9 pure payoff profiles in 2D.

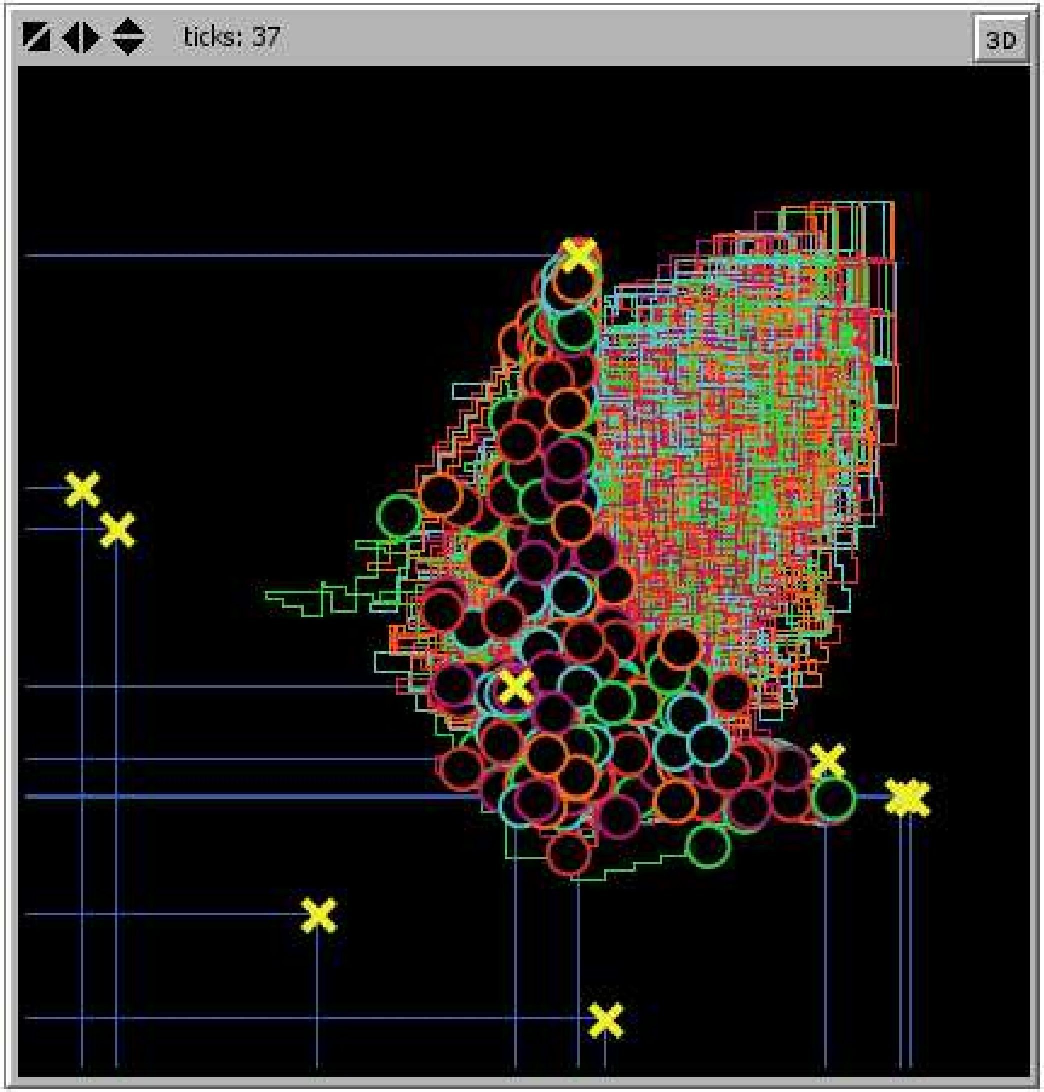
Approach



- Take a 2-player 3×3 game in normal form.
- Plot all 9 pure payoff profiles in 2D.
- Initialize, say, 100 profiles. One profile looks like:

$$(\ (A_t, \alpha_t)\ ,\ (B_t, \beta_t)\ ).$$

Approach



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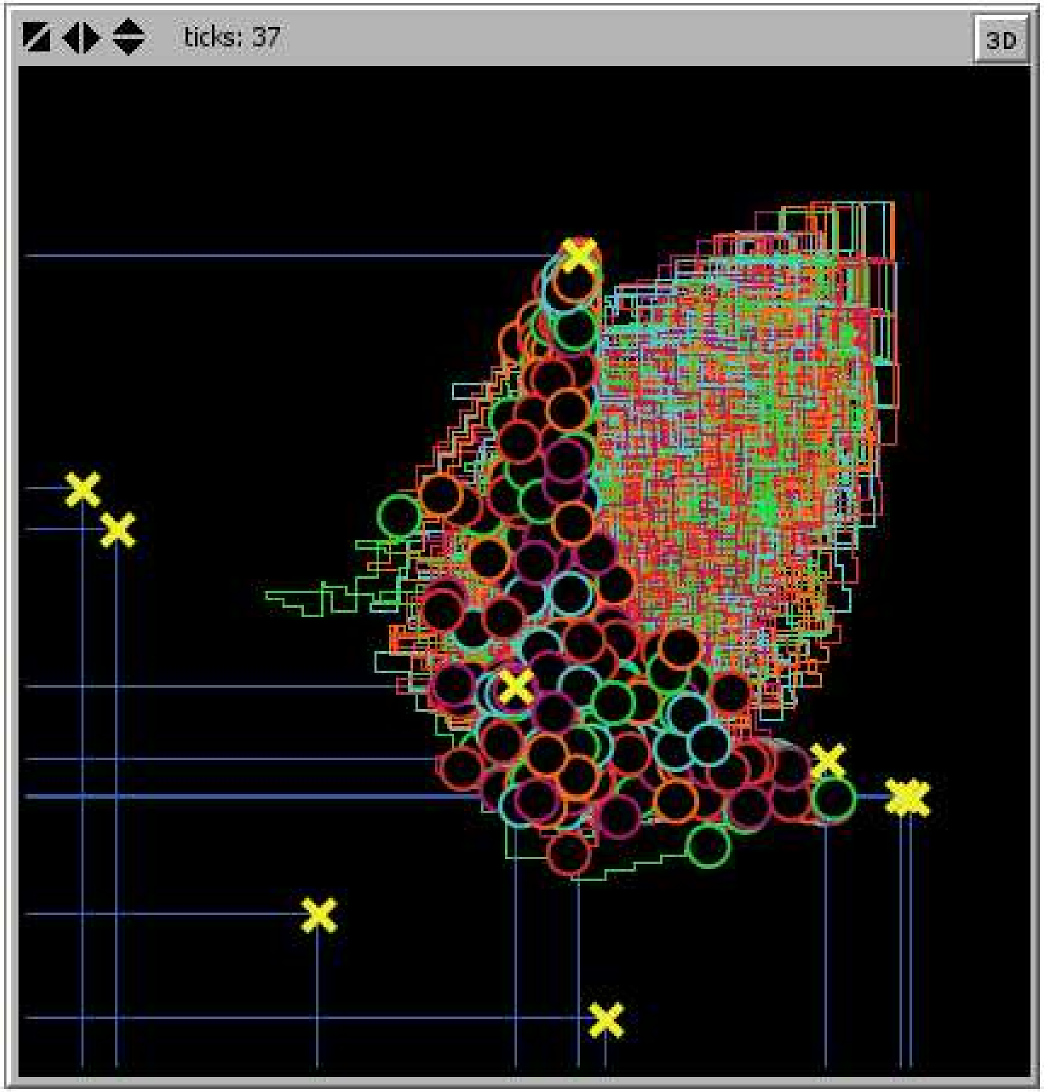
Plot the corresponding 100 aspiration profiles (α_t, β_t) in the same canvas.

Approach



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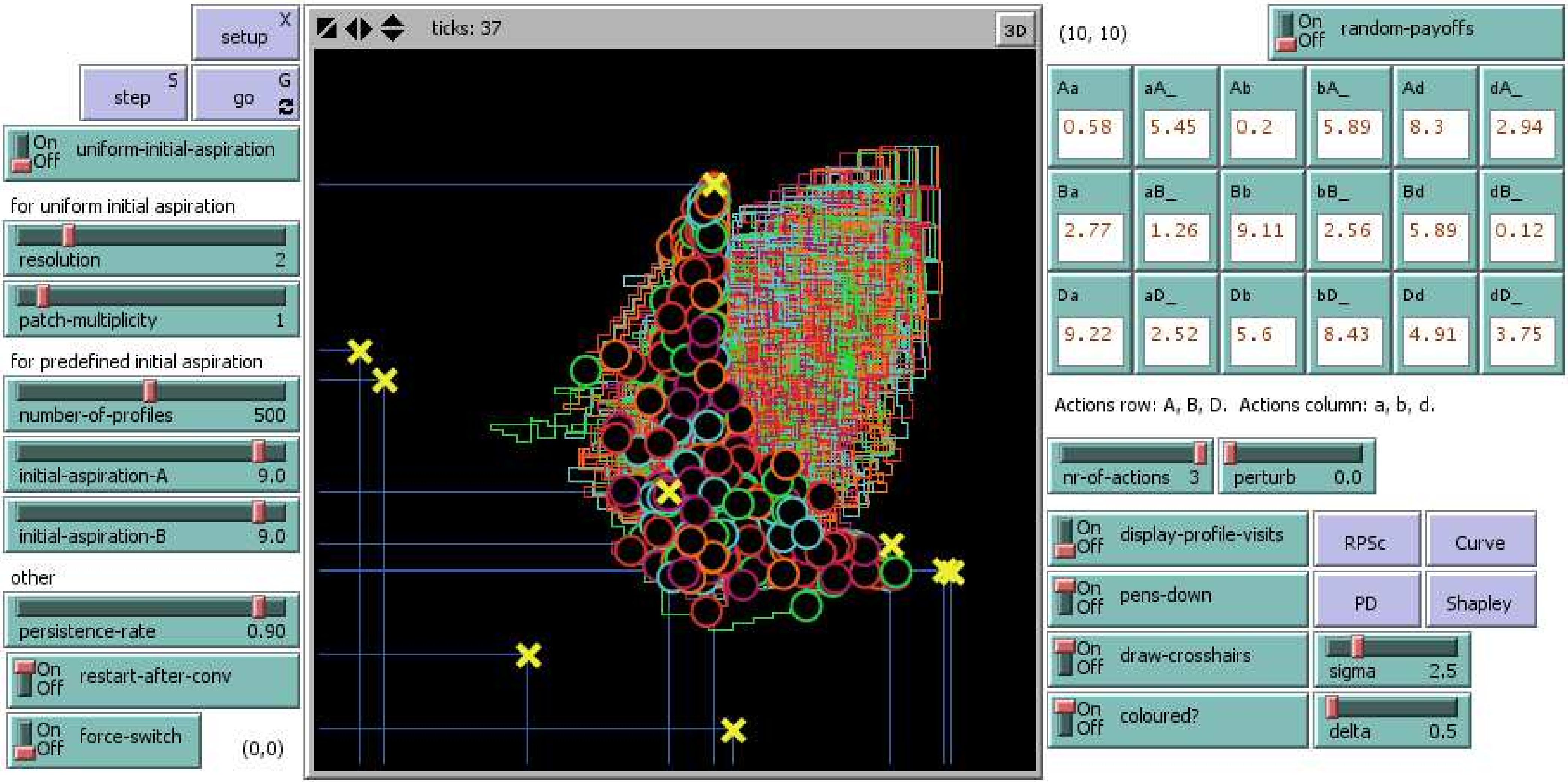
  Plot the corresponding 100 aspiration profiles (α_t, β_t) in the same canvas.

- Execute satisficing play for all player profiles simultaneously.

Satisficing play in a 2-player matrix game



Satisficing play in a generalised prisoner's dilemma with self-play (Stimpson *et al.*, 2001)

The generalised prisoner's dilemma (GPD)

The generalised prisoner's dilemma (GPD)

- Generalised payoff matrix

	C	D
C	σ, σ	$0, 1$
D	$1, 0$	δ, δ

Reward payoff:	σ	Sucker payoff:	0
Temptation payoff:	1	Punishment payoff:	δ

The generalised prisoner's dilemma (GPD)

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Constraints: $0 < \delta < \sigma < 1$ and $1/2 < \sigma$.

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  - States for satisficing play:
    - (A_t, α_t) for the row player.
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  - The initial states are denoted by (A_0, α_0) and (B_0, β_0), respectively.

Self-play: possible dynamics

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1. **Stability**. Convergence to a fixed action profile.

Self-play: possible dynamics

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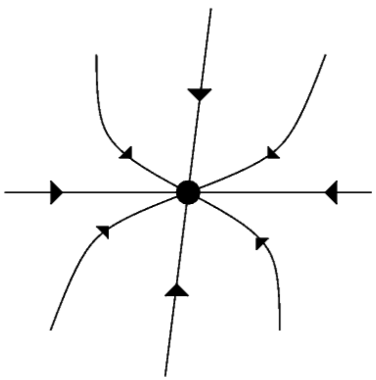

$$\alpha_t^A \leq \pi_t^A \quad \text{and} \quad \alpha_t^B \leq \pi_t^B.$$

for all $t \geq T$, for some $T \geq 0$.

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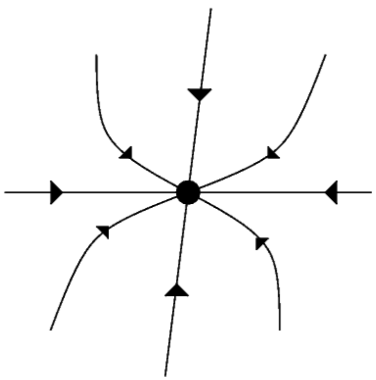

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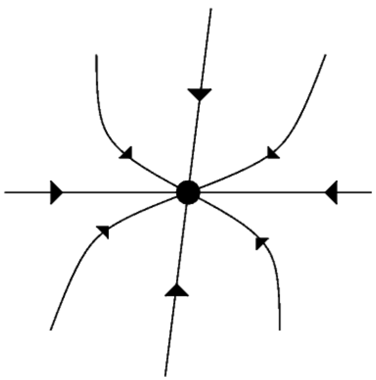

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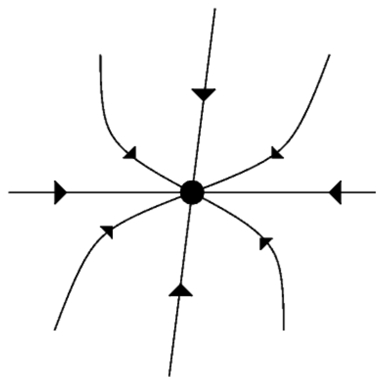

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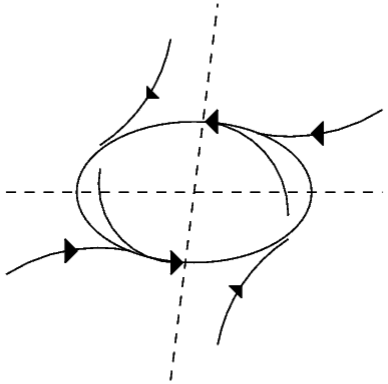
$$(D,D),\ (D,C),\ (C,D),\ (D,D),\ (D,C),\ (C,D),\ \dots$$



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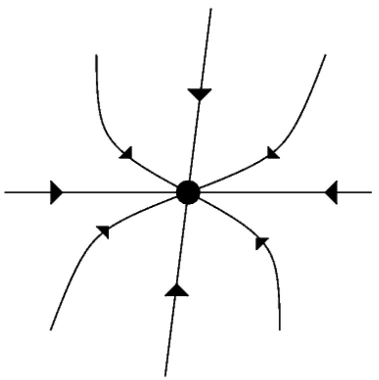

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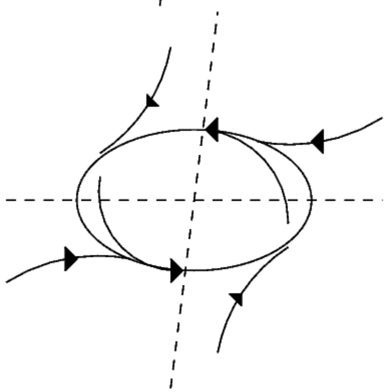

3. **Chaos**.

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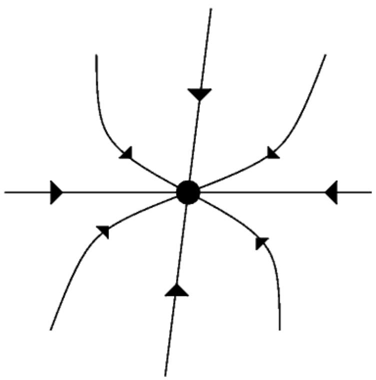

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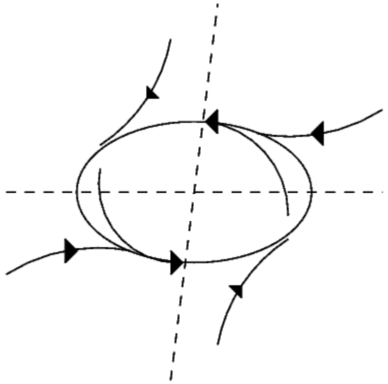
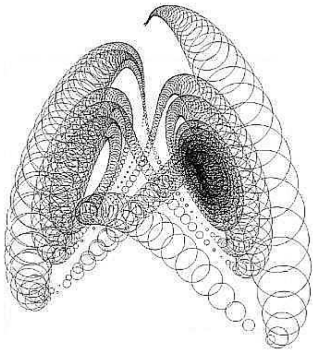


3. **Chaos**. Deterministic but non-periodic behaviour.



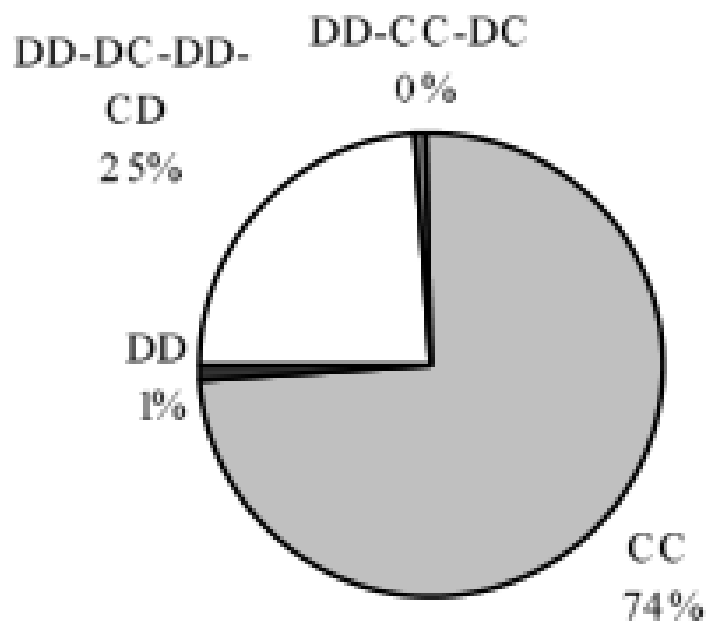
Experiments throughout the parameter space

Parameter space

	Symbol	Min	Max
Reward payoff	σ	0.51	1.0
Punishment payoff	δ	0.1	σ
Initial aspirations	α_0, β_0	0.5	2.0
Initial actions	A_0, B_0	50% C, 50% D	
Persistence rate	λ	0.1	0.9

Table 1: Distribution of parameters for simulations.

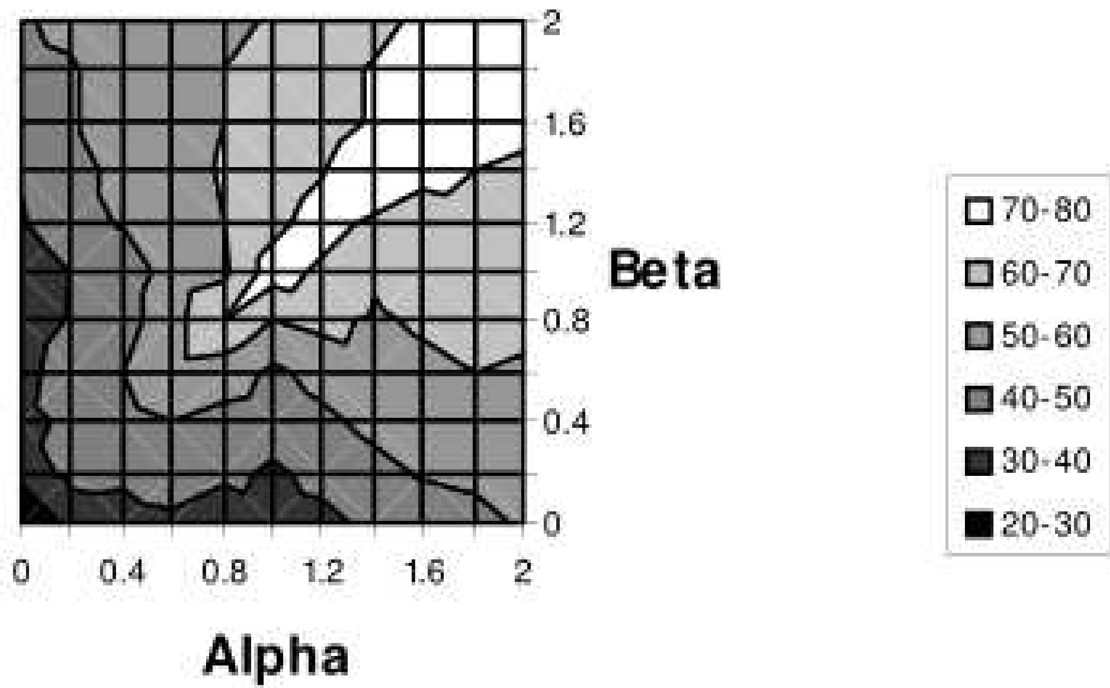
Frequencies of each of the possible outcomes



Frequencies of each of the possible outcomes from 5,000 trials. Parameters were randomly selected as described in Table 1.

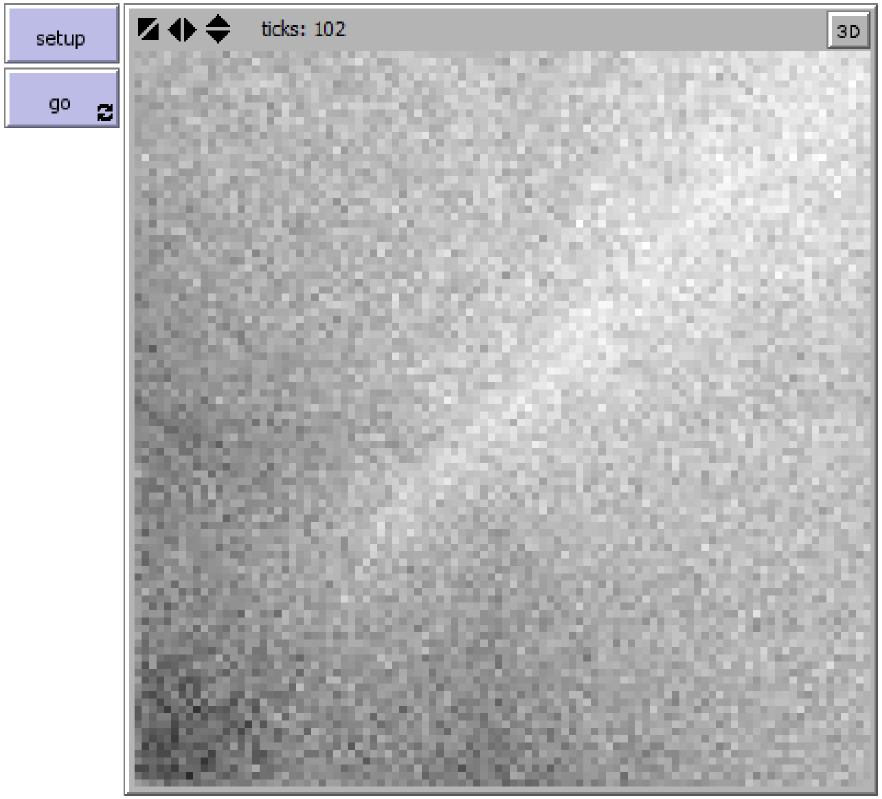
(From: “Satisficing and Learning Cooperation in the Prisoner’s Dilemma”, Stimpson *et al.*, 2001.)

Mutual cooperation as a result of initial aspirations



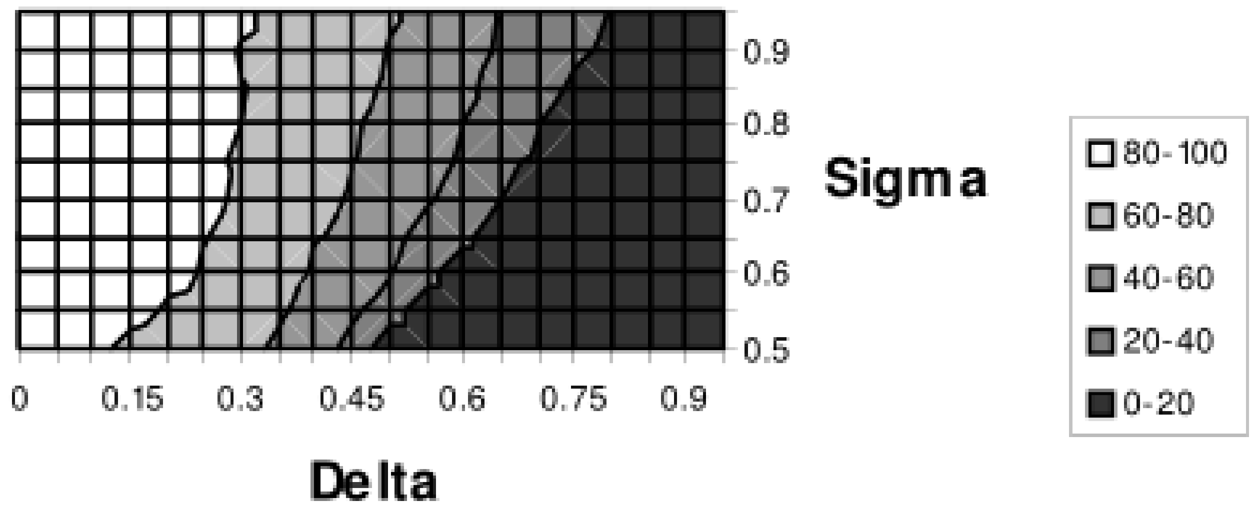
A contour plot of the percentage of trials out of 1,000 that converged to mutual cooperation as a function of initial aspirations. Light colors indicate that in most of the trials with the given initial aspirations, the agents learned to cooperate. Parameters other than α_0 and β_0 were randomly selected from Table 1. (From: Stimpson *et al.*, 2001.)

Same experiment with Netlogo



A Netlogo plot of the percentage of trials out of 100 that converged to mutual cooperation as a function of initial aspirations. Light colors indicate that in most of the trials the agents learned to cooperate. Parameters other than α_0 and β_0 were randomly selected from Table 1.

Mutual cooperation as a result of reward and punishmen



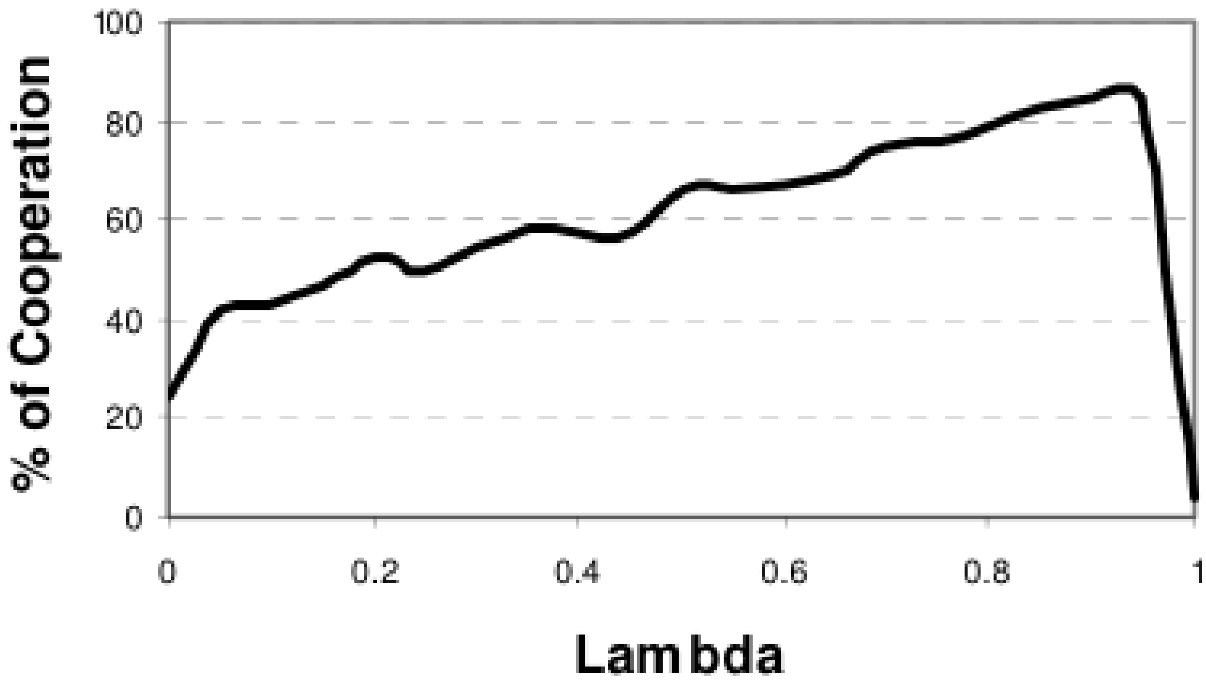
A contour plot of the percentage of trials out of 1,000 that converged to mutual cooperation as a function of each (δ, σ) pair. Light colors indicate that most of the trials converged to mutual cooperation. Parameters other than δ and σ were randomly selected from Table 1. (From: Stimpson *et al.*, 2001.)

Effects of the initial actions

Initial actions	**Cooperation**
Random	73.7%
CC	81.6%
DD	81.6%
CD or DC	66.7%

Table 2: Percentage of cooperation out of 1,000 trials as a function of initial actions. Parameters other than A_0 and B_0 were randomly selected from Table 1. (From: "Satisficing and Learning Cooperation . . . ", Stimpson *et al.*, 2001.)

Effect of the persistence rate

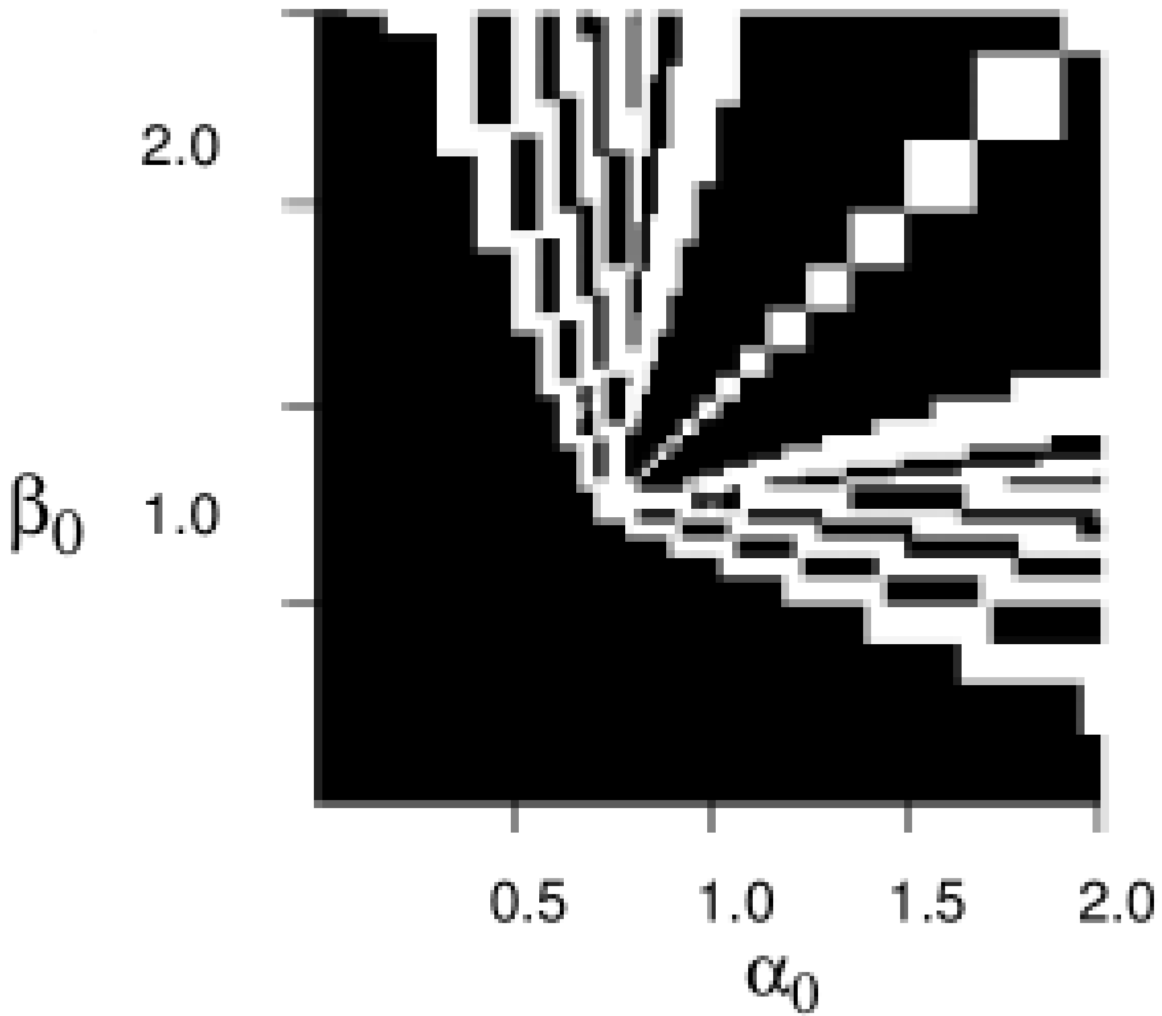


Percentage of trials out of 1,000 that converged to mutual cooperation as a function of the persistence rate, λ. Parameters other than λ were selected randomly as described in Table 1.

(From: "Satisficing and Learning Cooperation …", Stimpson *et al.*, 2001.)

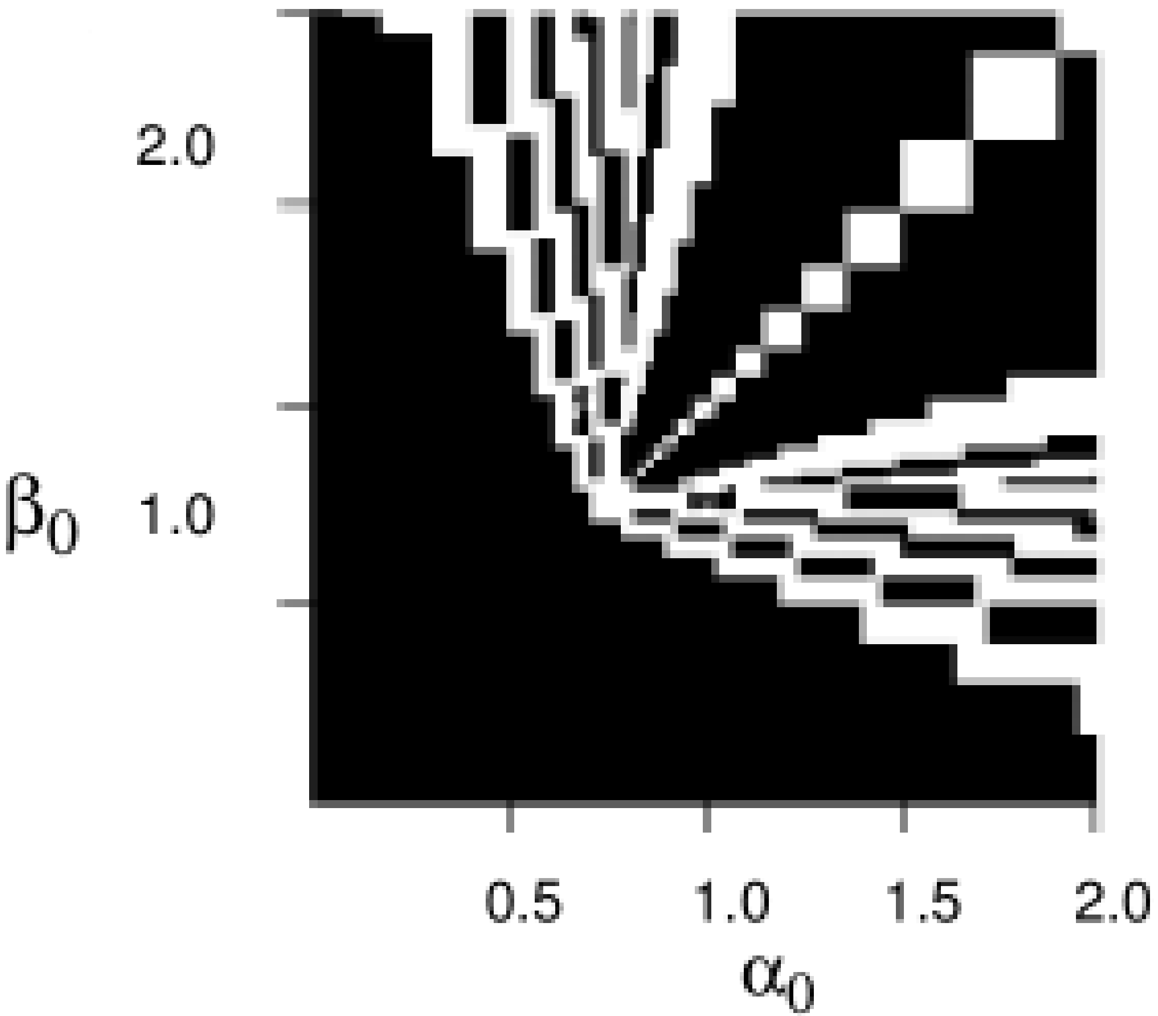
Experiments with specific parameters

Final outcome as a result of initial aspirations



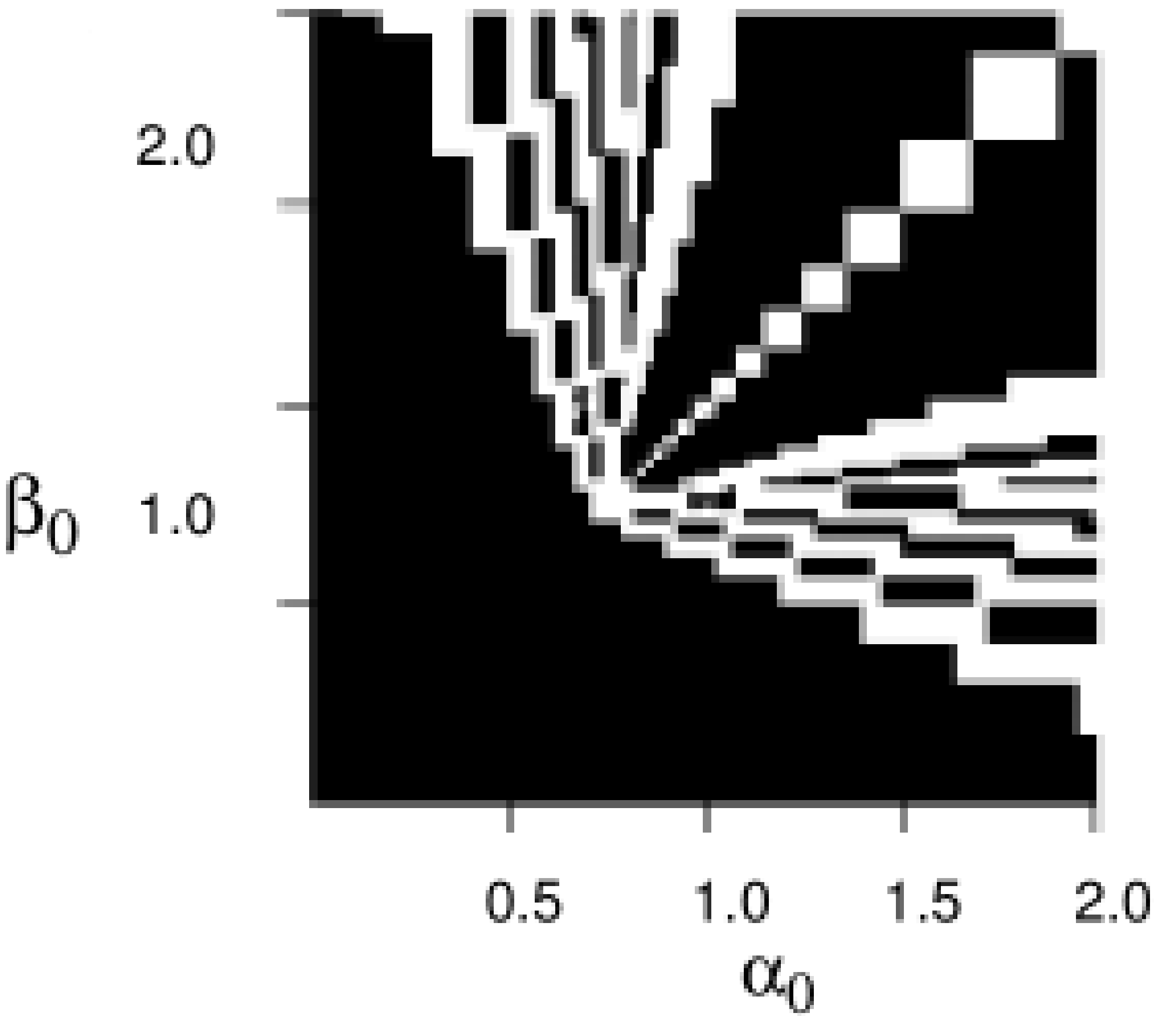
Final outcome as a result of initial aspirations

- Initial aspiration of player A on *x*-axis; Initial aspiration of player B on *y*-axis.



Final outcome as a result of initial aspirations

- Initial aspiration of player A on x-axis; Initial aspiration of player B on y-axis.
- White: convergence to (C, C); black: convergence to (D, D); grey: periodic or chaotic behaviour.



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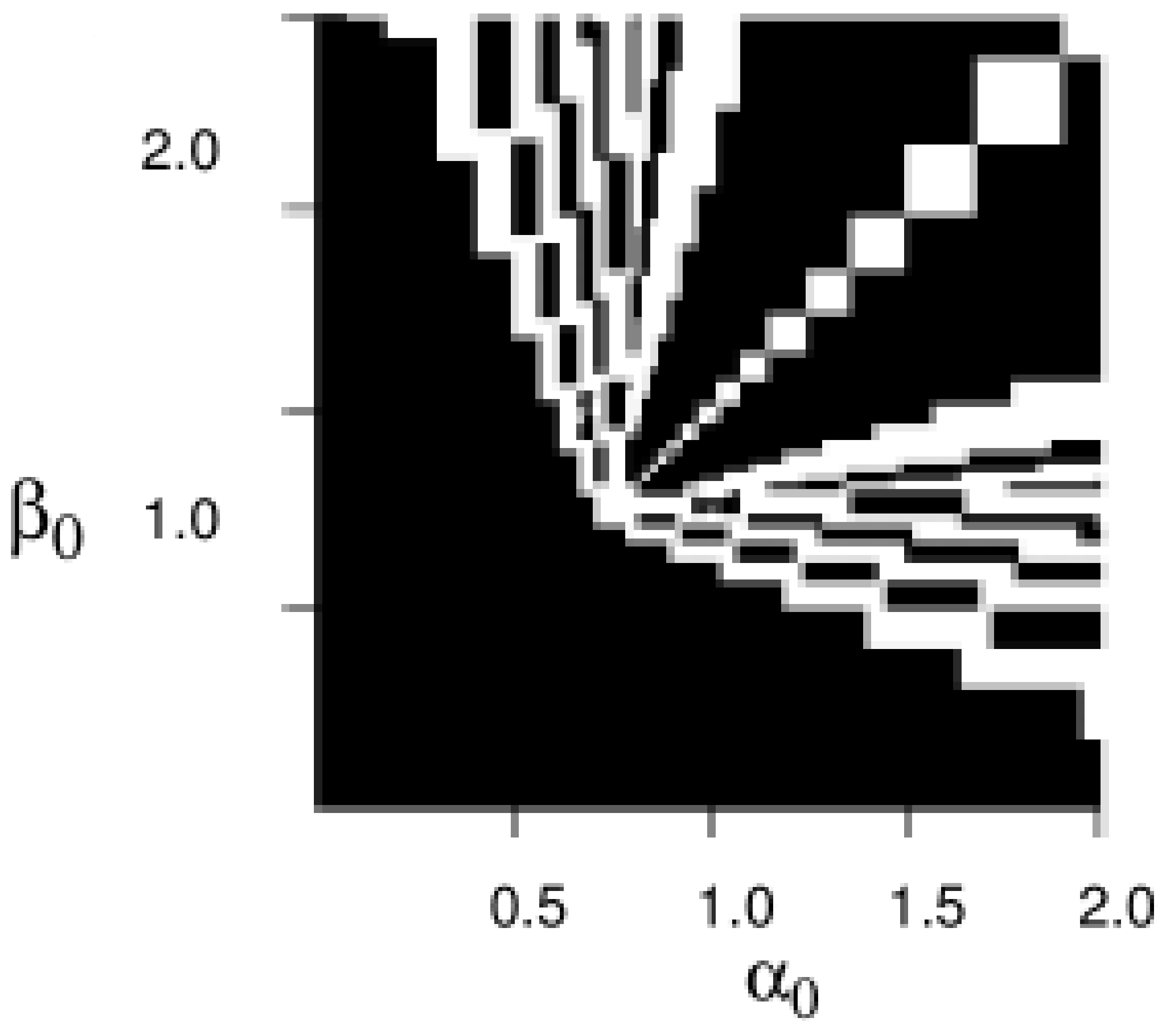
- Initial aspiration of player A on x-axis; Initial aspiration of player B on y-axis.
- White: convergence to (C, C); black: convergence to (D, D); grey: periodic or chaotic behaviour.
- $(A_0, B_0) = (D, D)$, $\sigma = 0.8$, $\delta = 0.7$, $\lambda = 0.9$.



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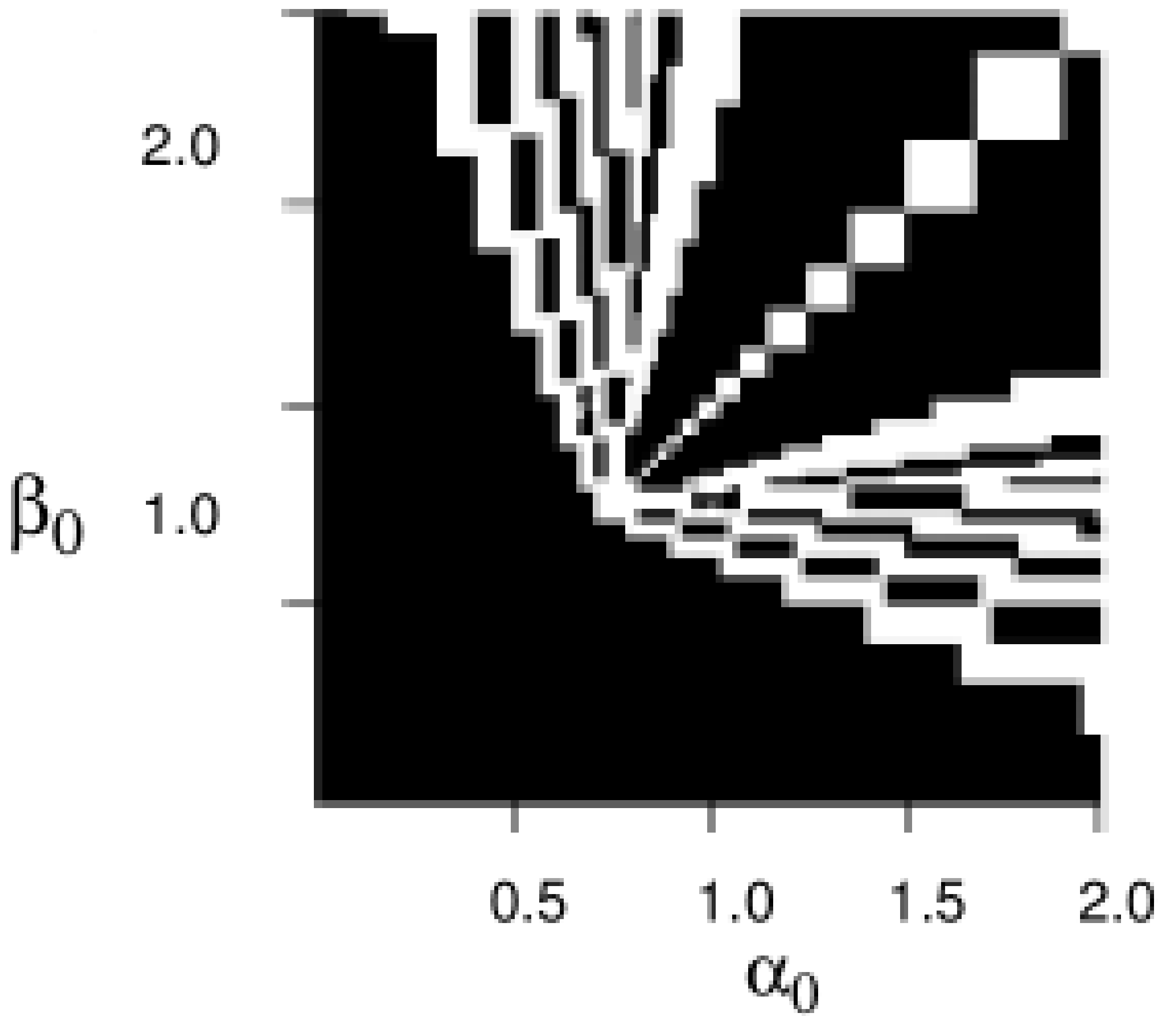
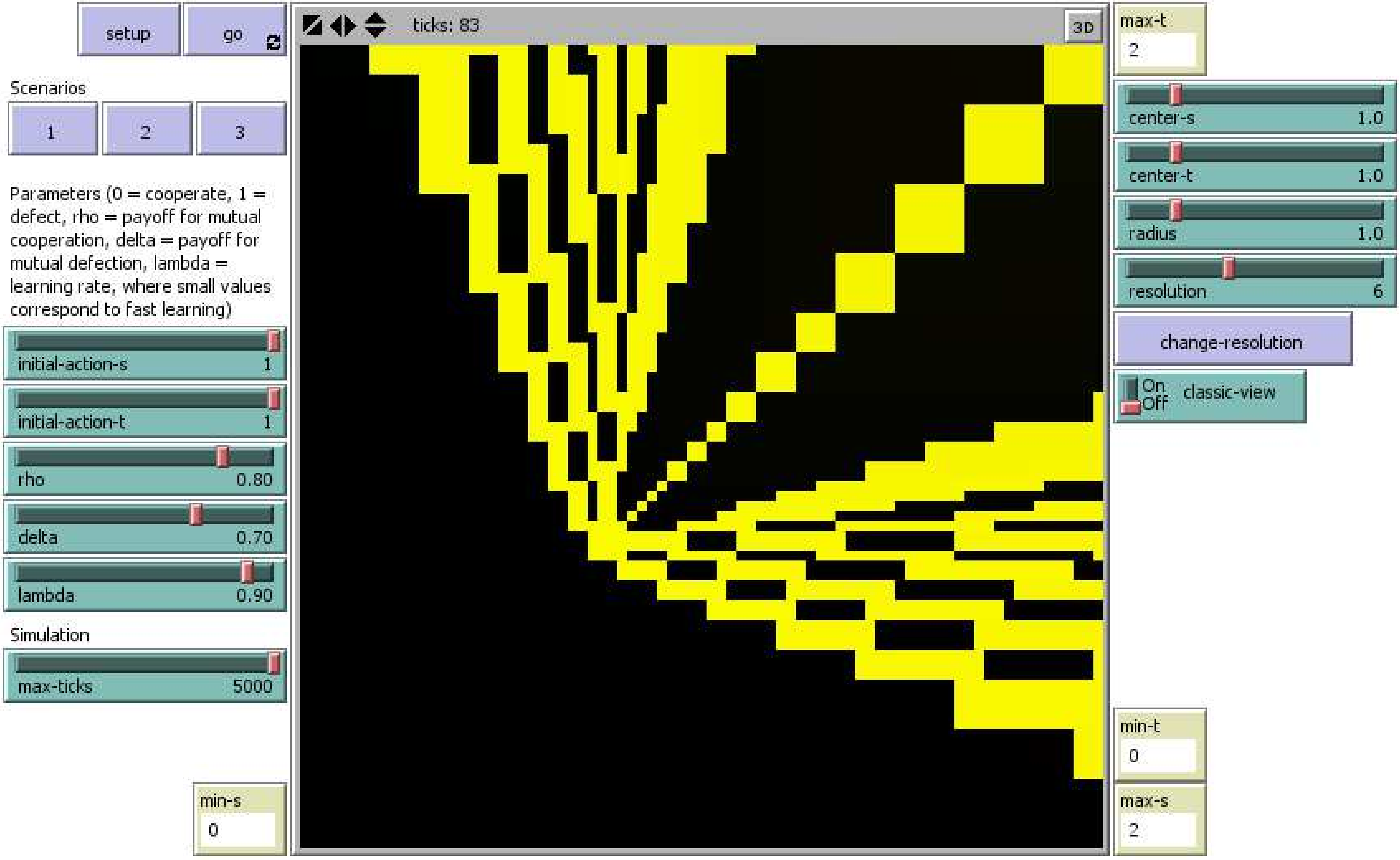
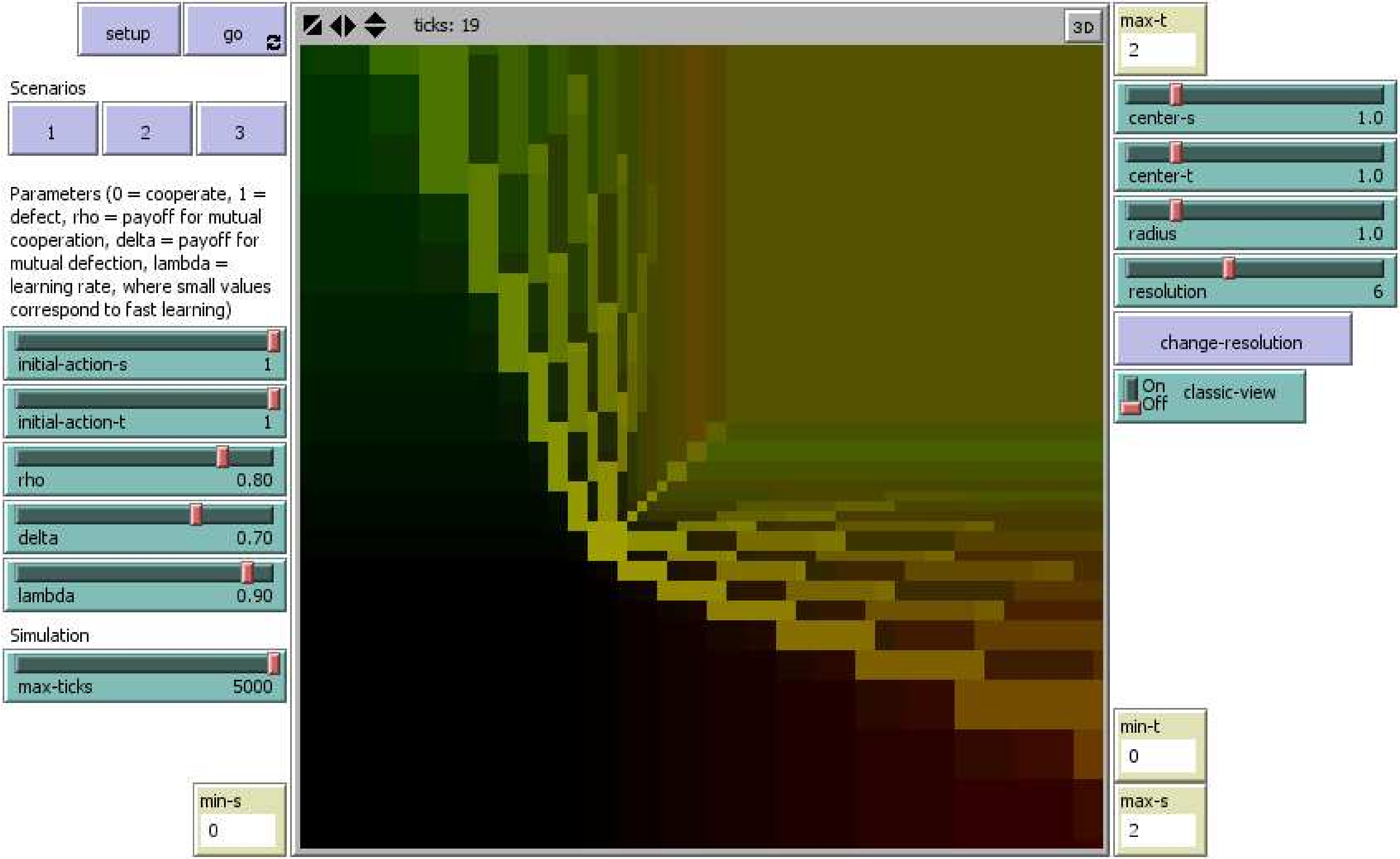


Final outcome as a result of initial aspirations (demo)



Final outcome as a result of initial aspirations (buildup)



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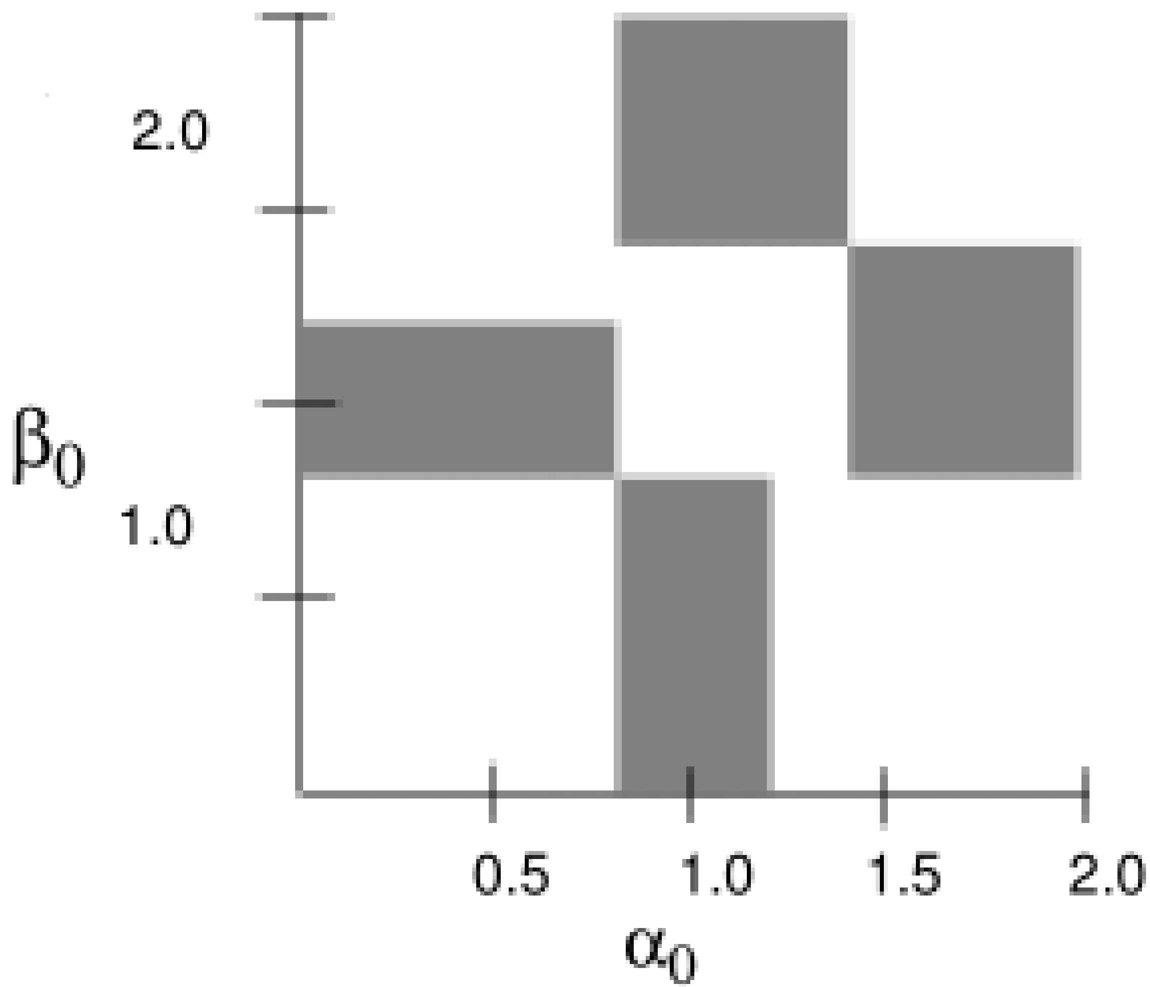
- $(A_0, B_0) = (C, C)$, $\sigma = 0.8$, $\delta = 0.5$, $\lambda = 0.5$.

(From: "Satisficing and Learning Cooperation in the Prisoner's Dilemma", Stimpson *et al.*, 2001.)



Final outcome as a result of initial aspirations

- Initial aspiration of player A on x-axis; Initial aspiration of player B on y-axis.
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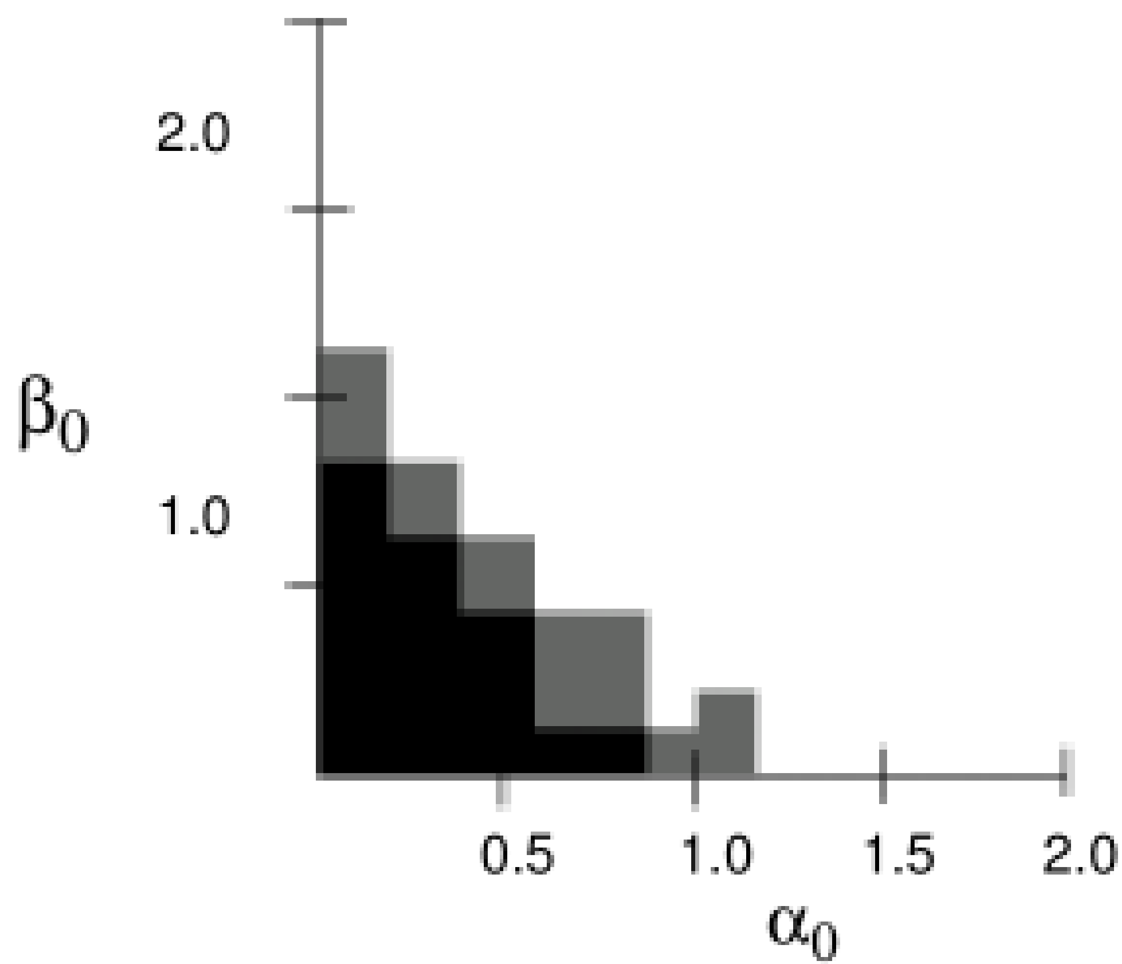
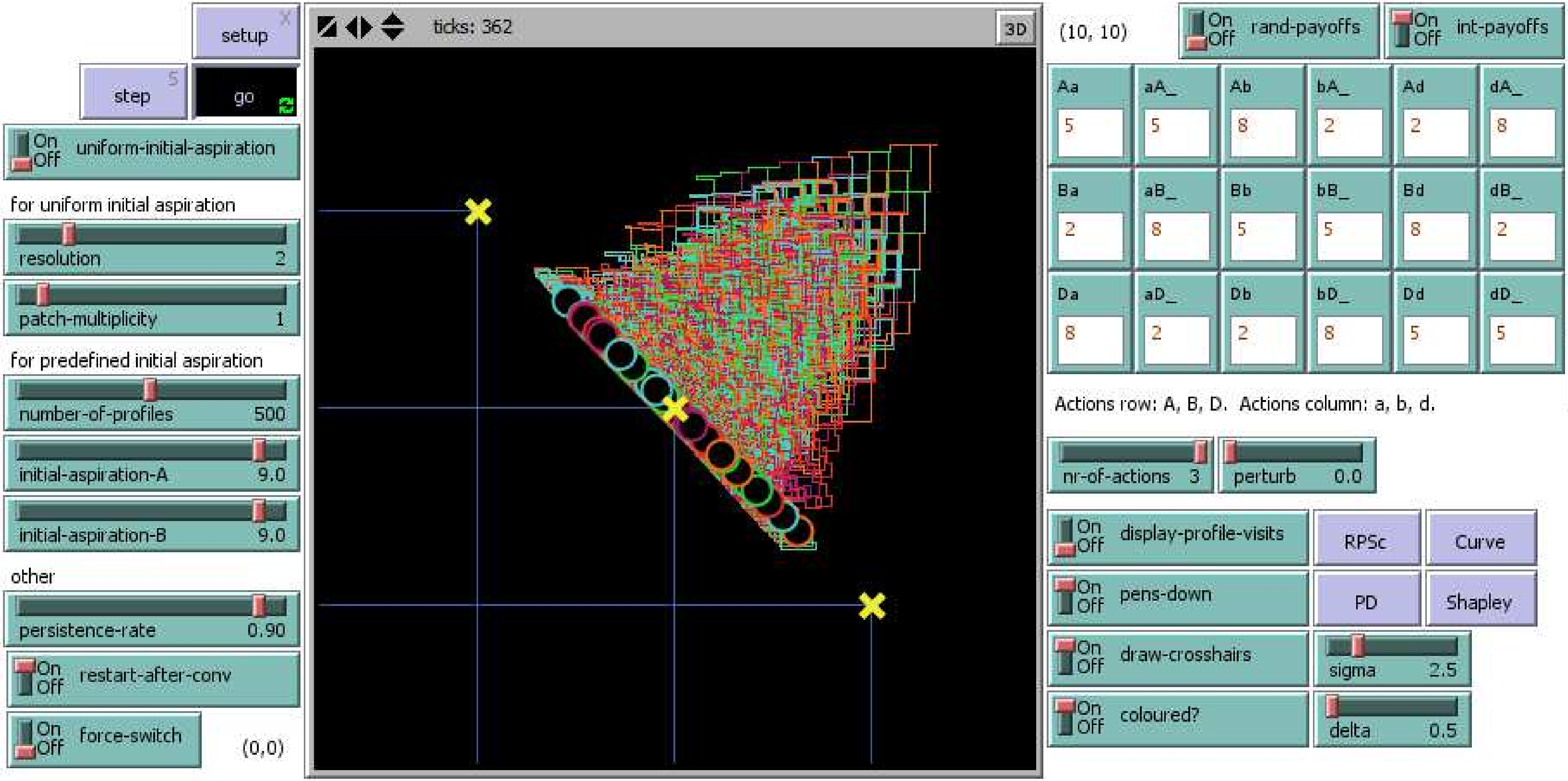
- $(A_0, B_0) = (D, C)$, $\sigma = 0.6$, $\delta = 0.5$, $\lambda = 0.8$.

(From: "Satisficing and Learning Cooperation in the Prisoner's Dilemma", Stimpson *et al.*, 2001.)

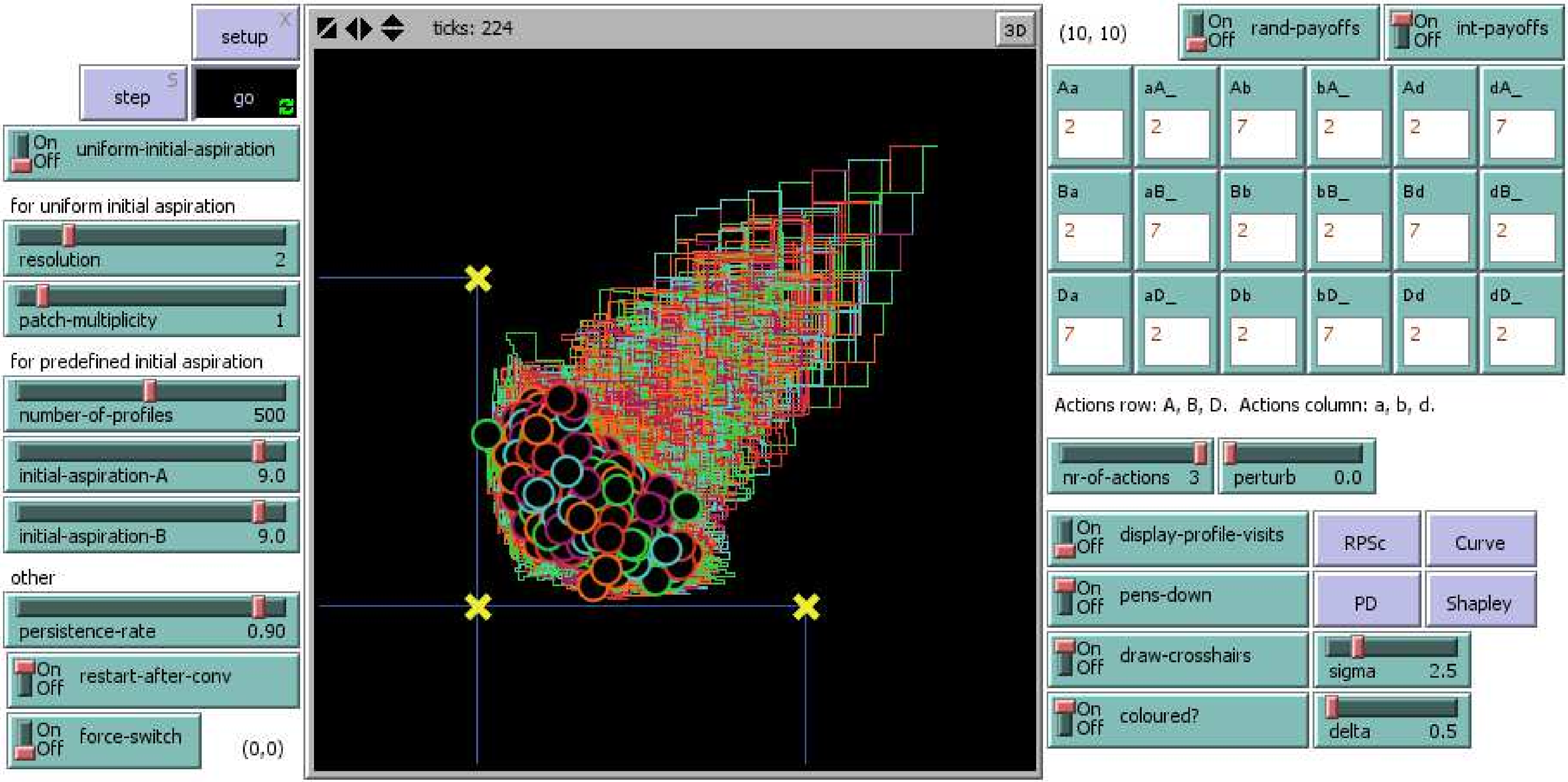


Difficult games for satisficing play

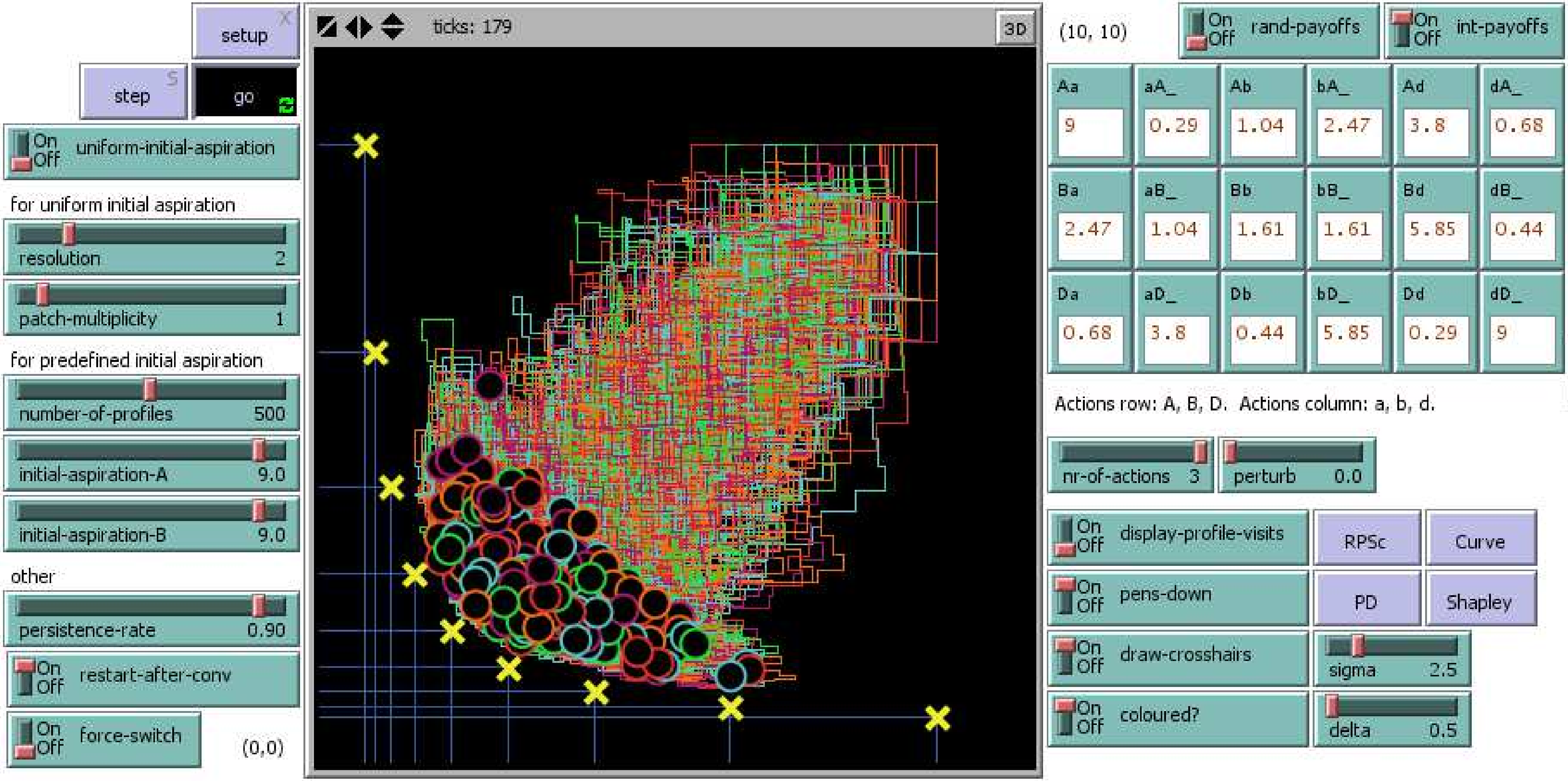
Difficult games for satisficing play (RPSc)



Difficult games for satisficing play (Shapley)



Difficult games for satisficing play (Curve)



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  The vector Δr^t is a vector of virtual reinforcements—gains or losses relative to the current average that that *would have* materialised if a given action x had been played at time t.

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Result: 100% mutual cooperation.

What next?

Gradient dynamics:

- Like fictitious play, players model (or assess) each other through mixed strategies.
- Strategies are not played, only maintained.
- Due to CKR (common knowledge of rationality, cf. Hargreaves Heap & Varoufakis, 2004), all models of mixed strategies are correct. (I.e., $q^{-i} = s^{-i}$, for all i.)
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