



Tree Search



Reinforcement Learning

An Introduction
second edition

Richard S. Sutton and Andrew G. Barto

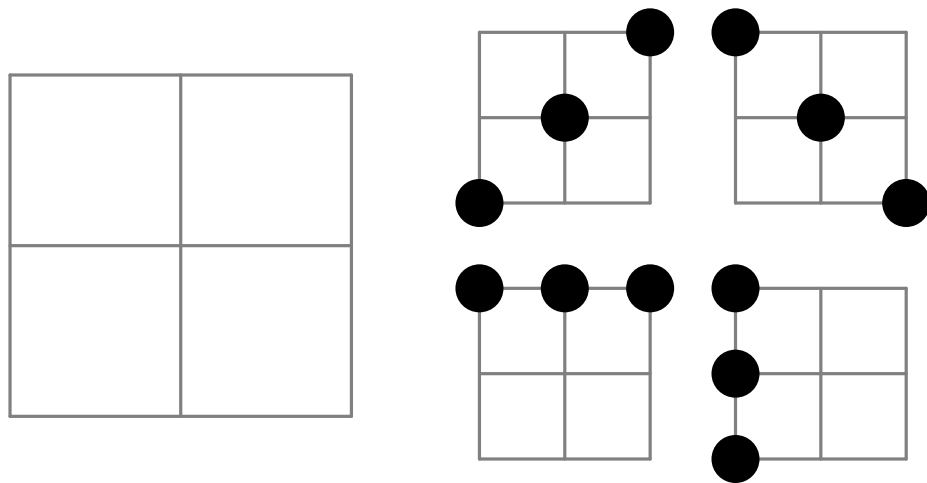
partially Section 8.11

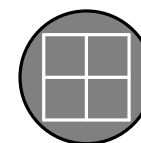


Game Tree

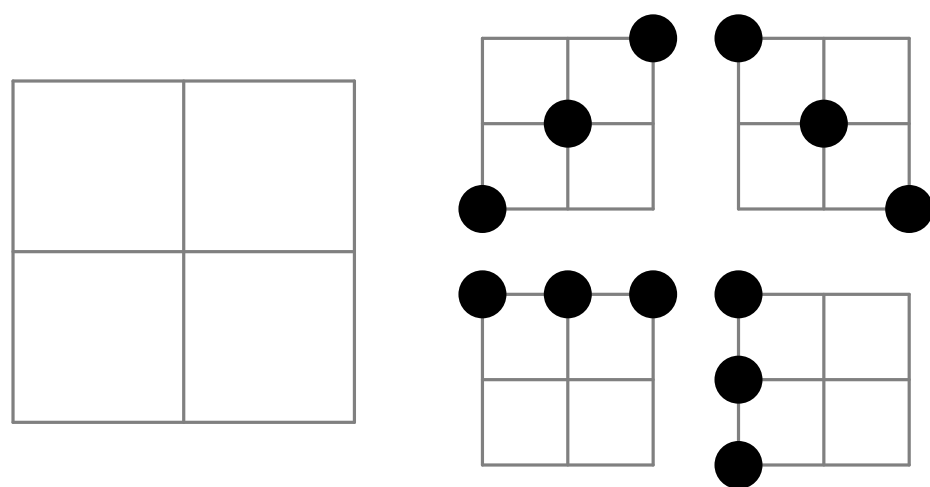


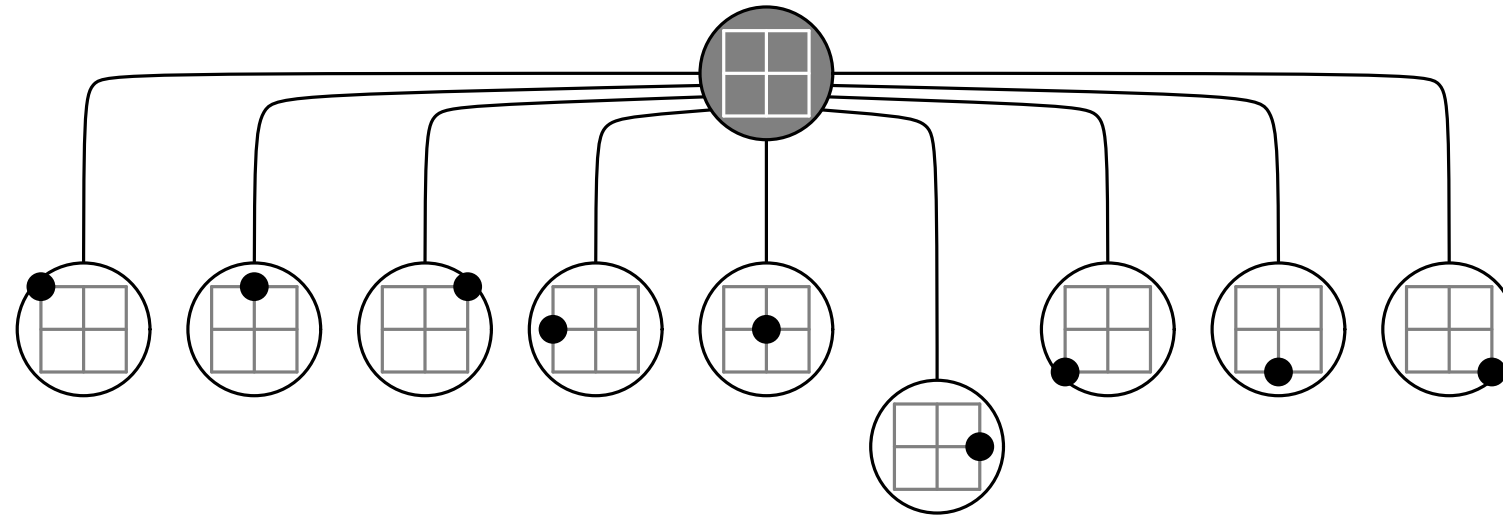
Game Tree



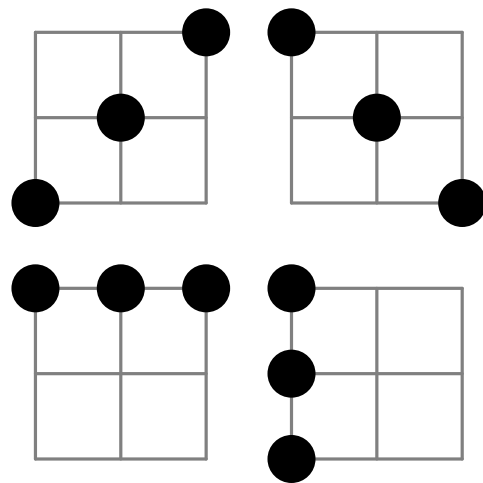
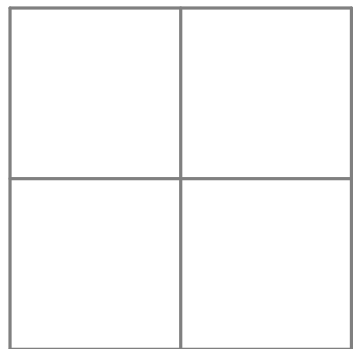


Game Tree



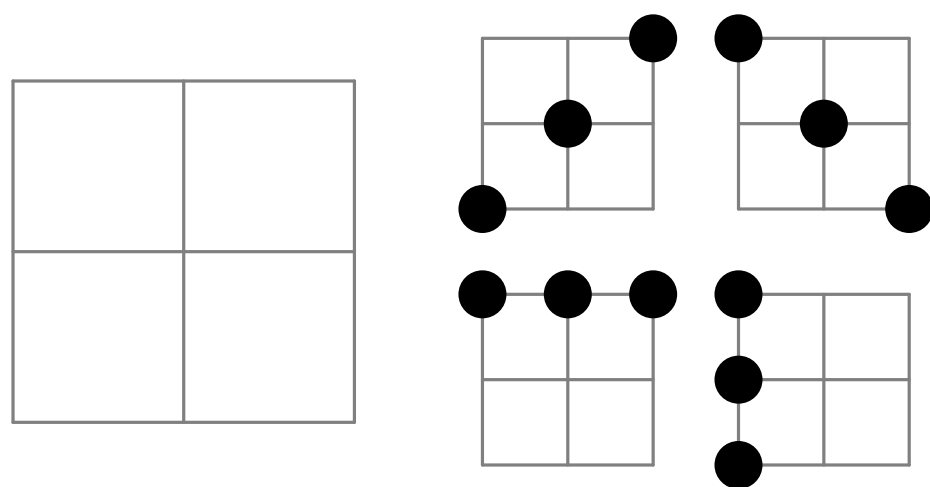
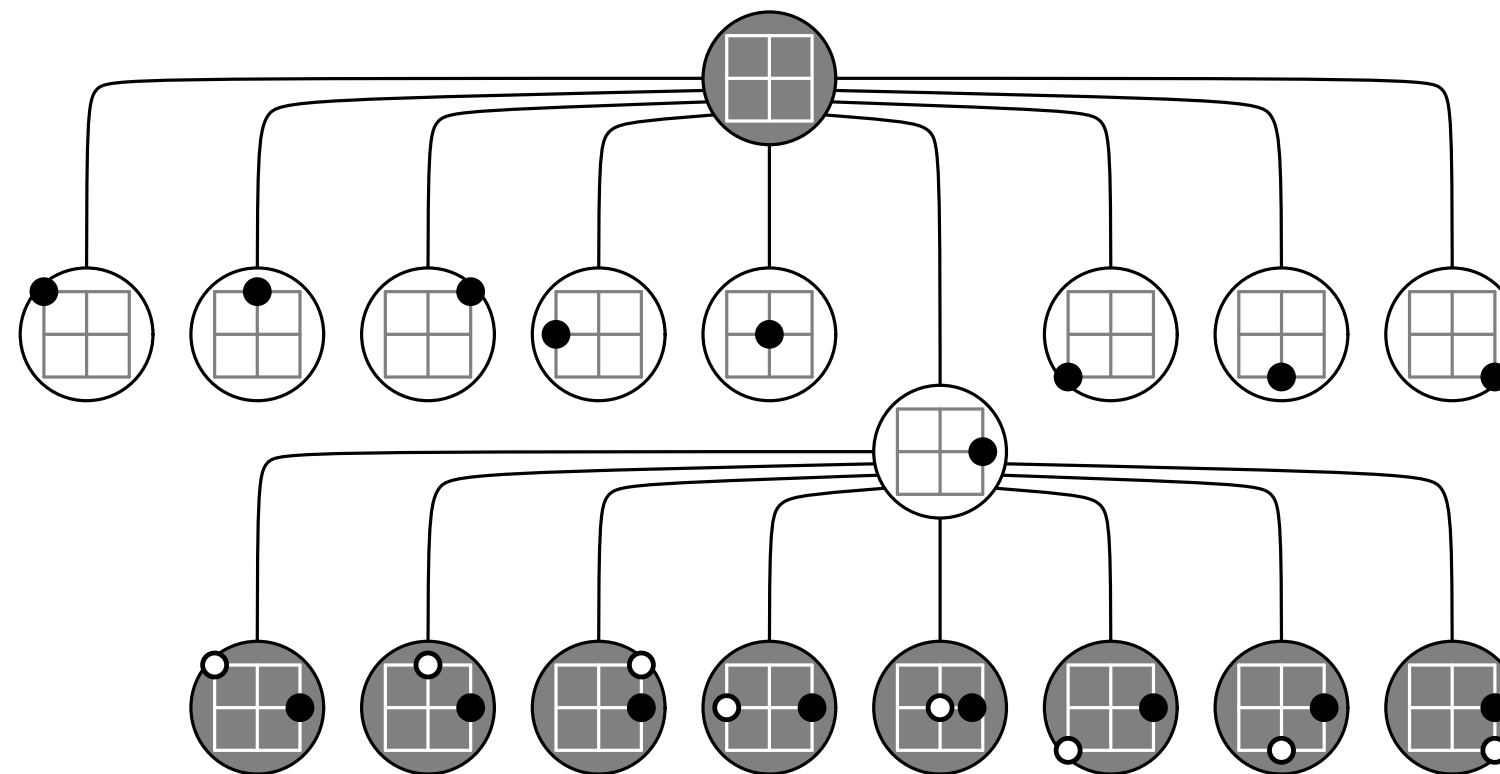


Game Tree



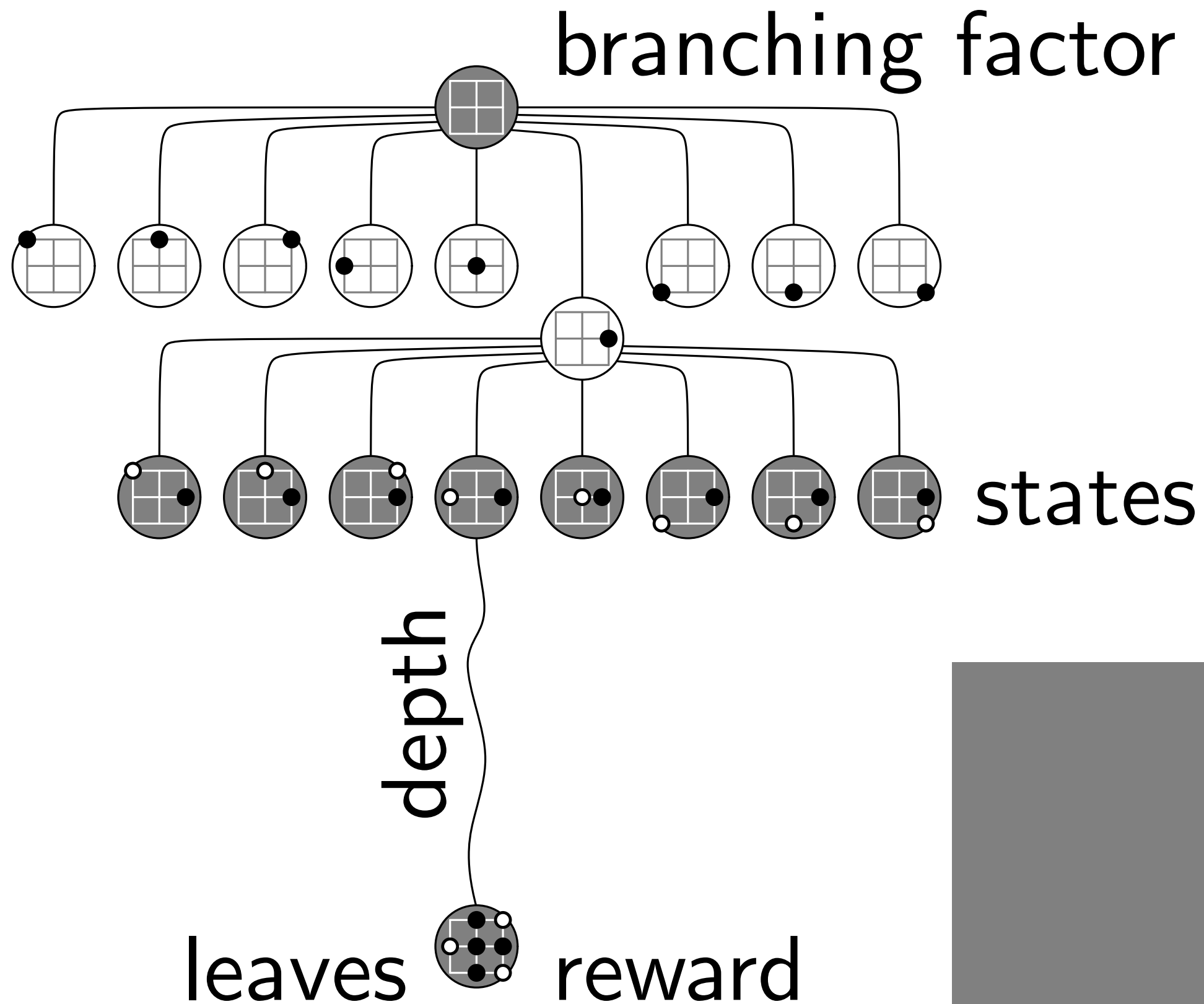
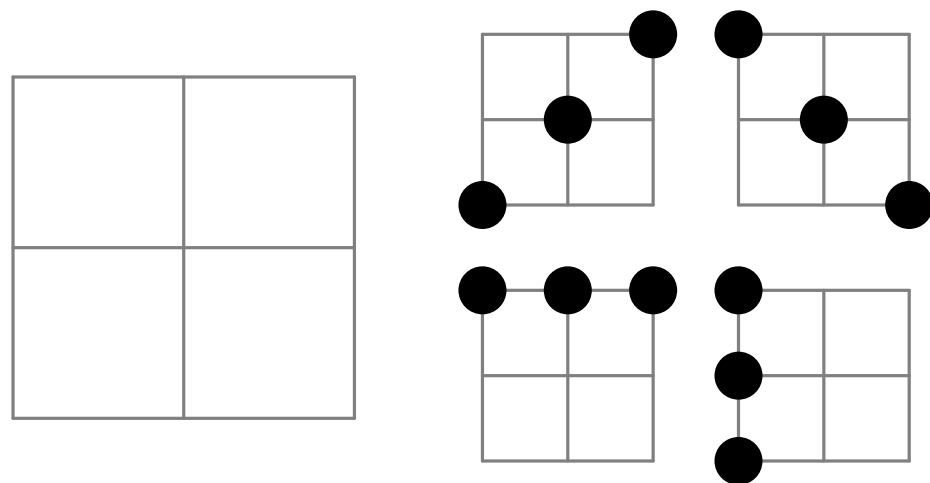


Game Tree



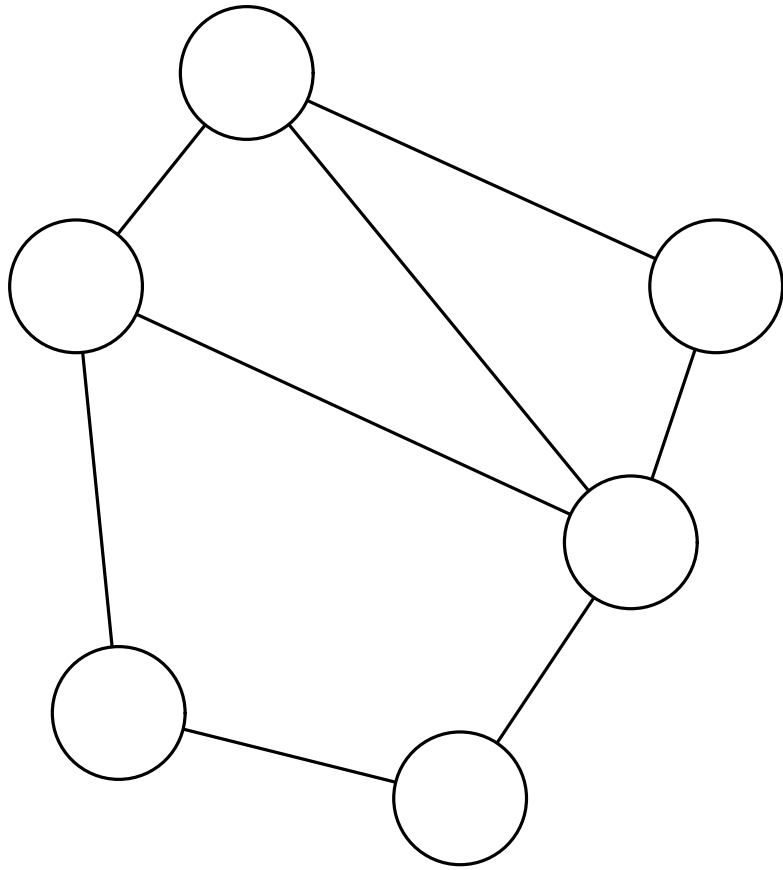


Game Tree

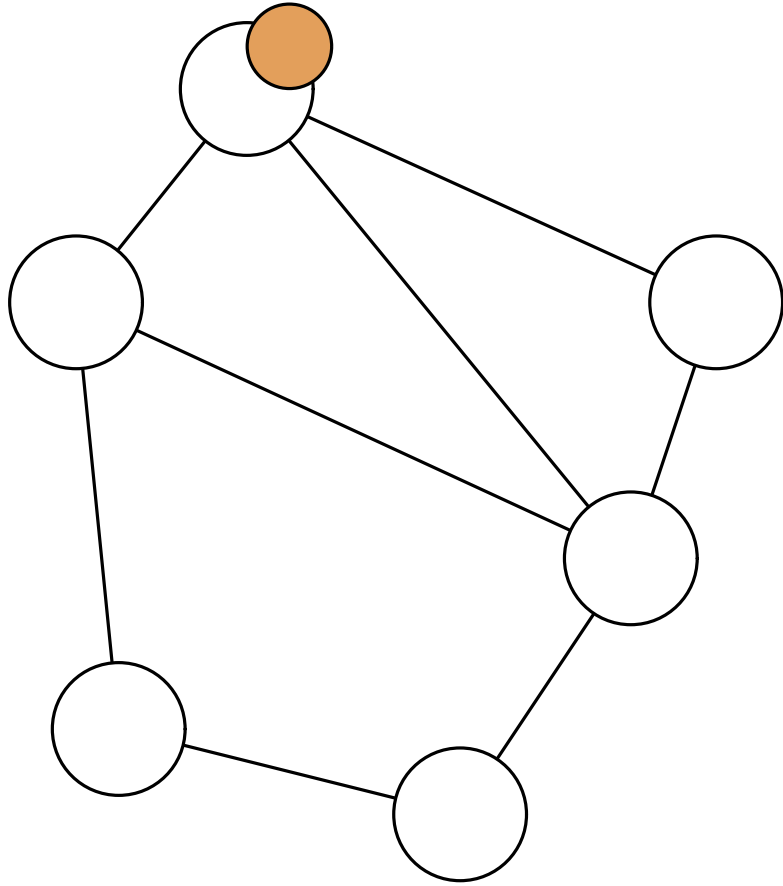


Example

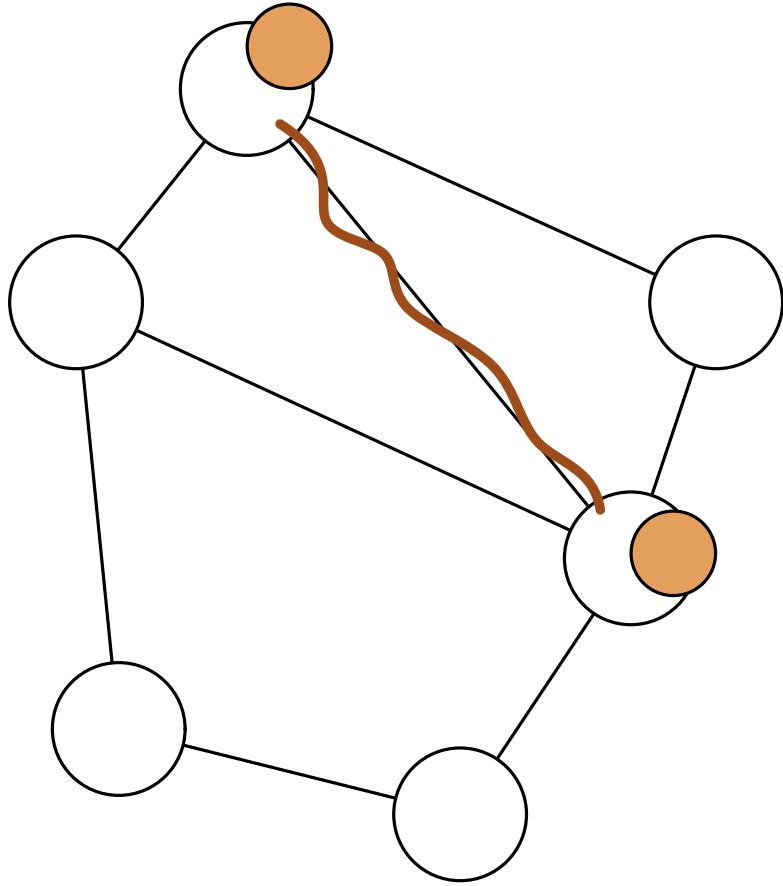
Geography



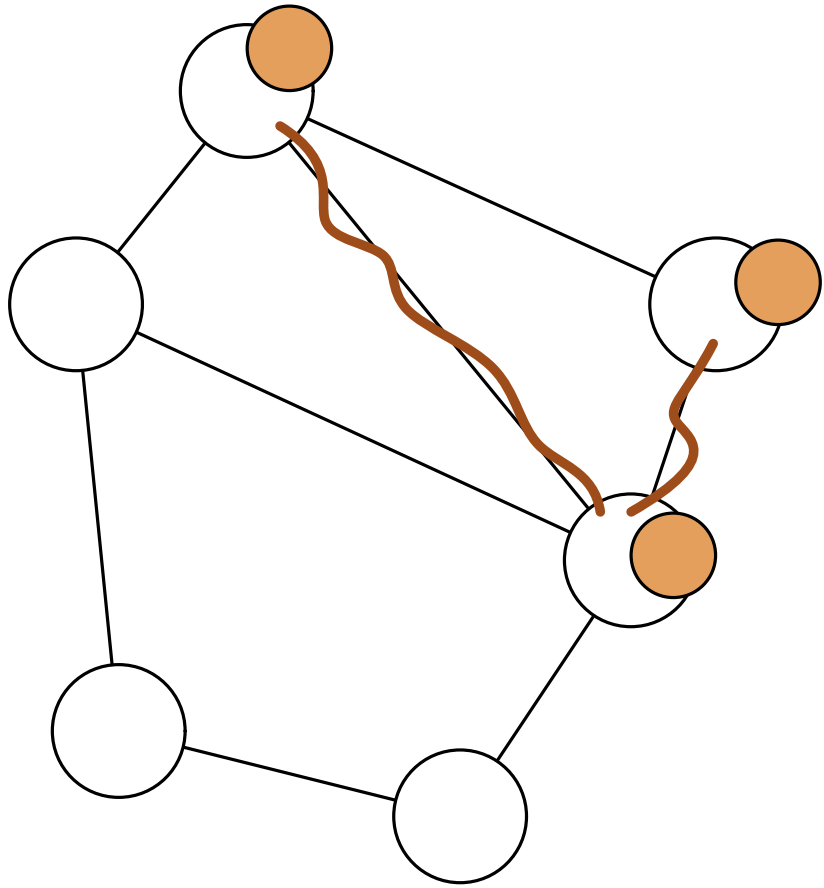
Geography



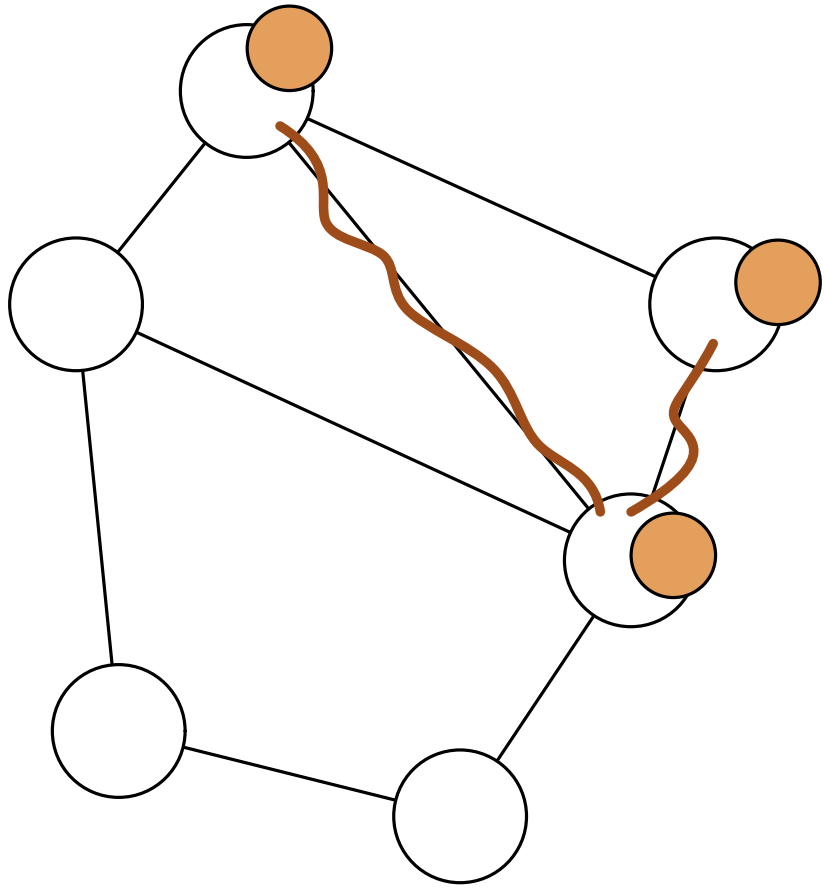
Geography



Geography

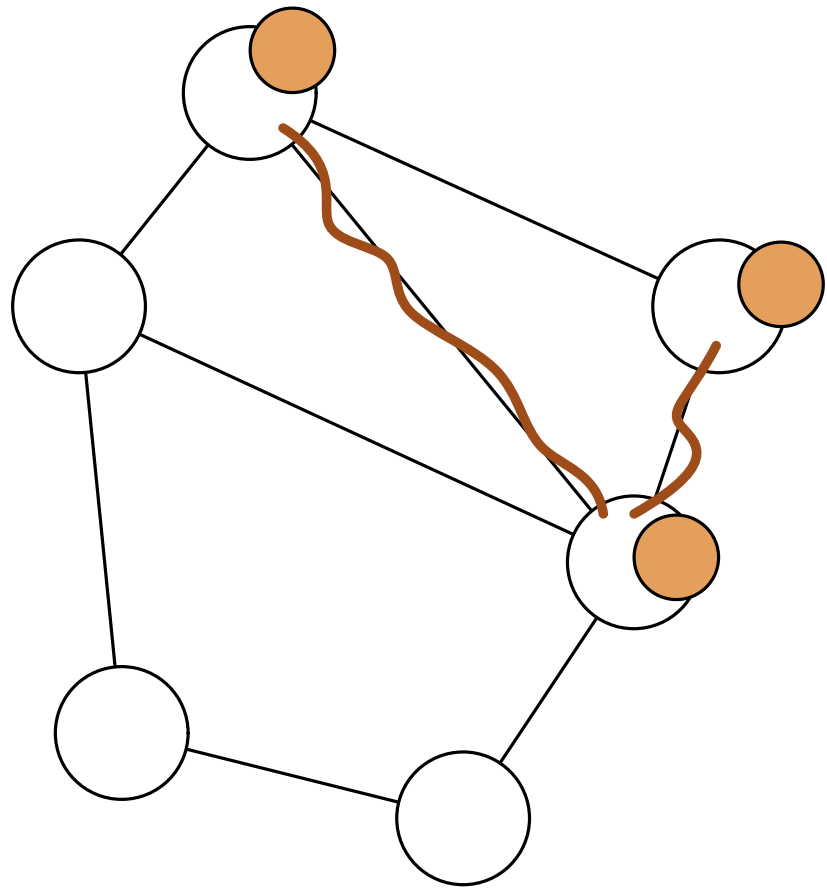


Geography



Lost

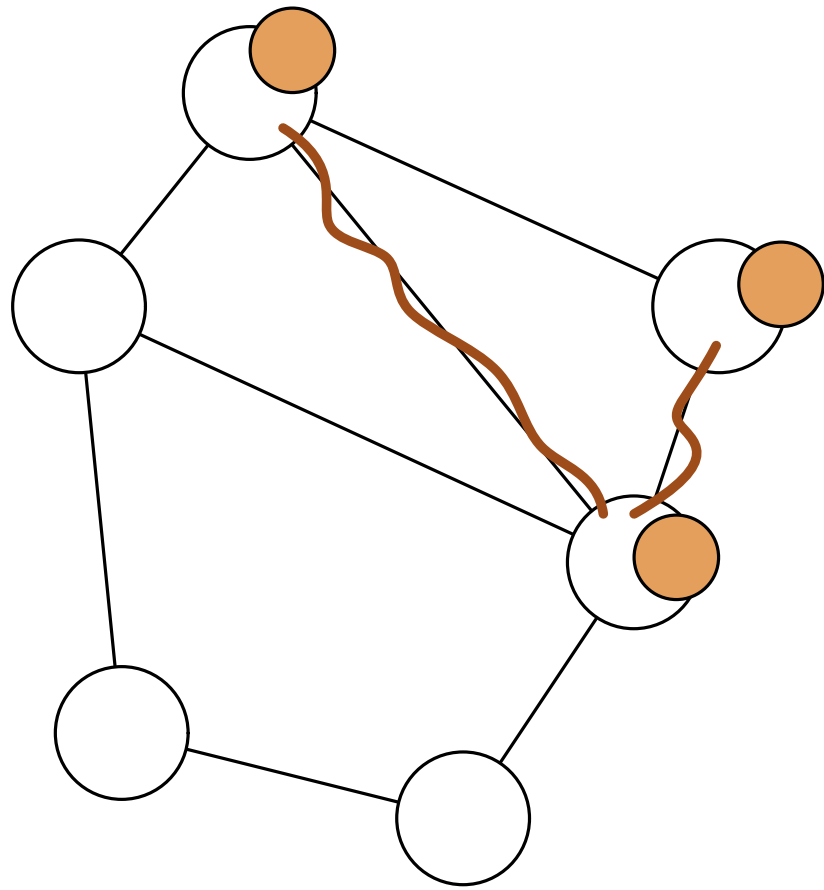
Geography



Lost

Graph with n nodes
max degree d

Geography



Lost

Graph with n nodes
max degree d

states $\leq 2^n \cdot n$
branching factor $\leq d$
depth $= n$
leaves = “dead-end”
reward $= -1/1$



Assignment 1

Checkers

Nim (5,5,5)

branching factor

depth

leaves

states

reward



Post on
Teams

Checkers

Nim (5,5,5)

branching factor

depth

leaves

states

reward



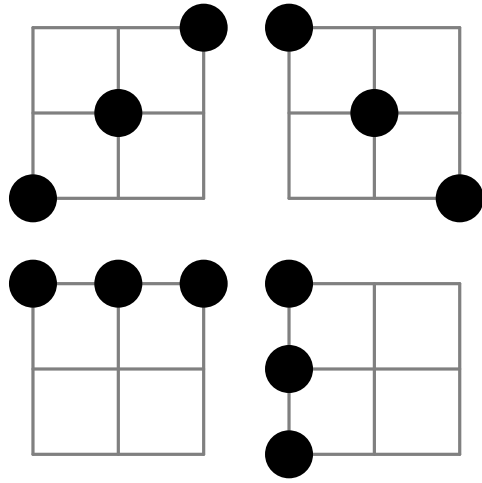
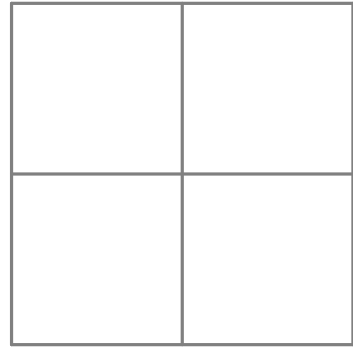
Post on
Teams

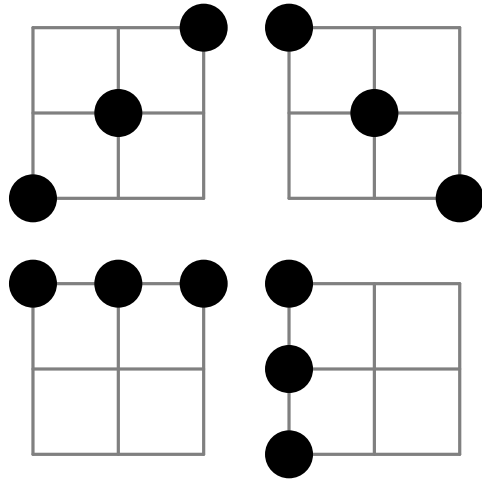
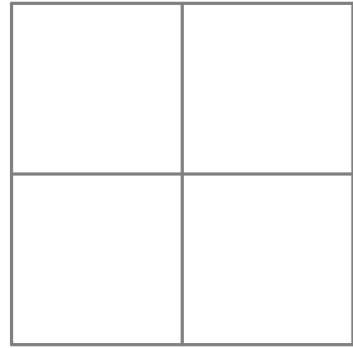


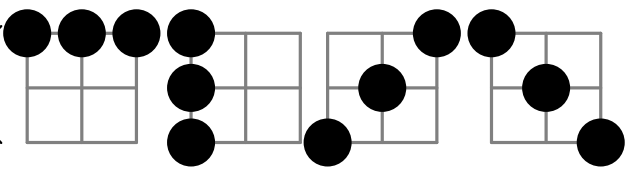
State $\longrightarrow \mathbb{R}$

Static Evaluation
Score Function
(Approximate) Value v

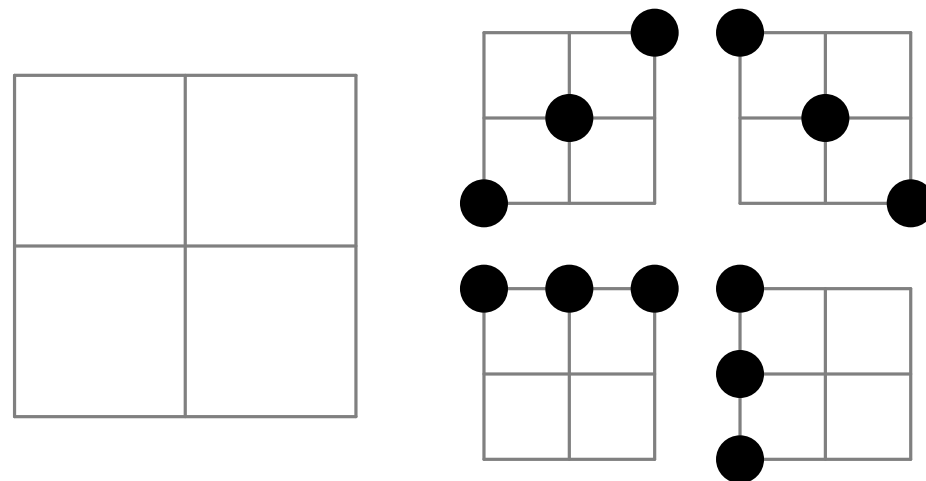
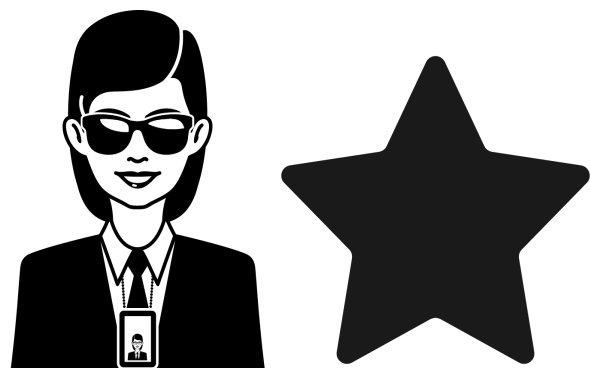
Example





32 (# ())

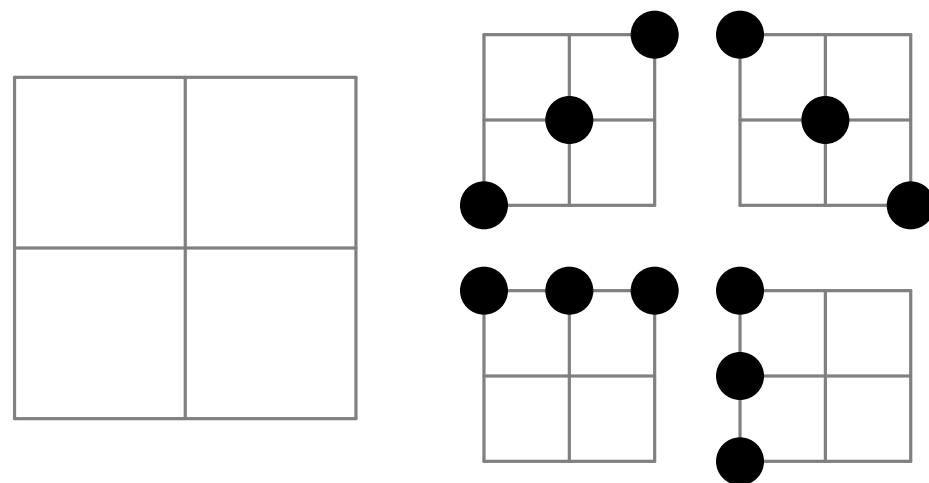
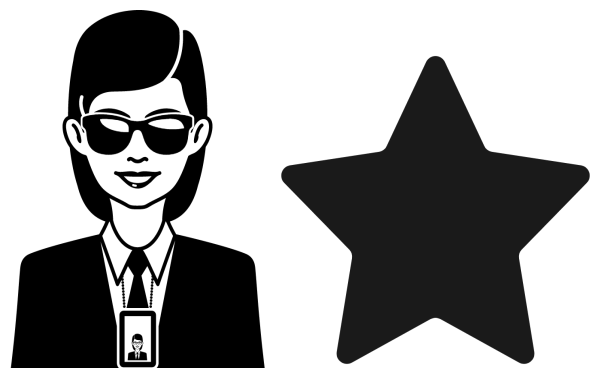




$$32 \textcircled{\#} \left(\begin{array}{cc|cc|cc|cc} \bullet & \bullet & \bullet & \bullet & & & & \\ \bullet & & & & \bullet & & \bullet & \\ \hline & & & & & & & \\ & & & & & & & \end{array} \right)$$

$$+ \textcircled{\#} \left(\begin{array}{cc|cc|cc|cc} \bullet & \bullet & \square & \bullet & \bullet & & & \\ \bullet & & & & & & \square & \\ \hline & & & & \square & & & \\ & & & & \bullet & & \bullet & \end{array} \right) \dots \right)$$



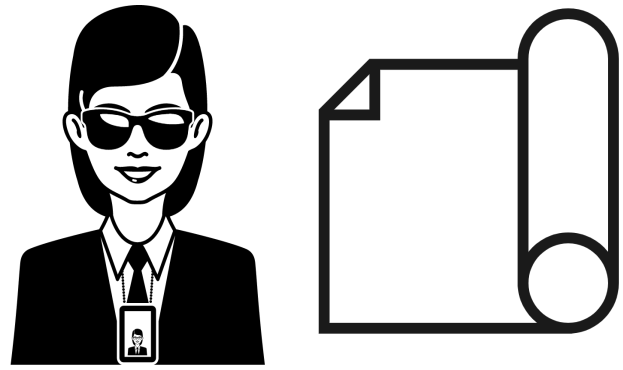


$$32 \textcircled{\#} \left(\begin{array}{c} \bullet \bullet \bullet \bullet \\ \square \square \square \square \end{array} \right)$$

$$+ \textcircled{\#} \left(\begin{array}{c} \bullet \bullet \square \bullet \\ \square \square \square \square \end{array} \right) \dots$$

$$- 32 \textcircled{\#} \left(\begin{array}{c} \circ \circ \circ \circ \\ \square \square \square \square \end{array} \right)$$

$$- \textcircled{\#} \left(\begin{array}{c} \circ \circ \square \circ \\ \square \square \square \square \end{array} \right) \dots$$



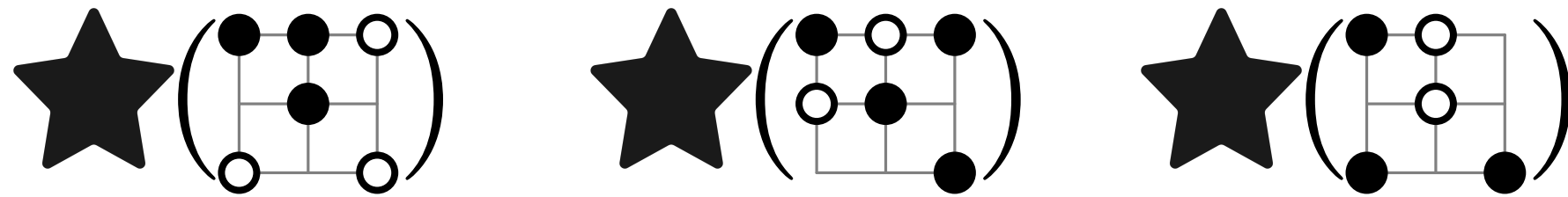
100 roll-outs with random agent.

value = average winrate



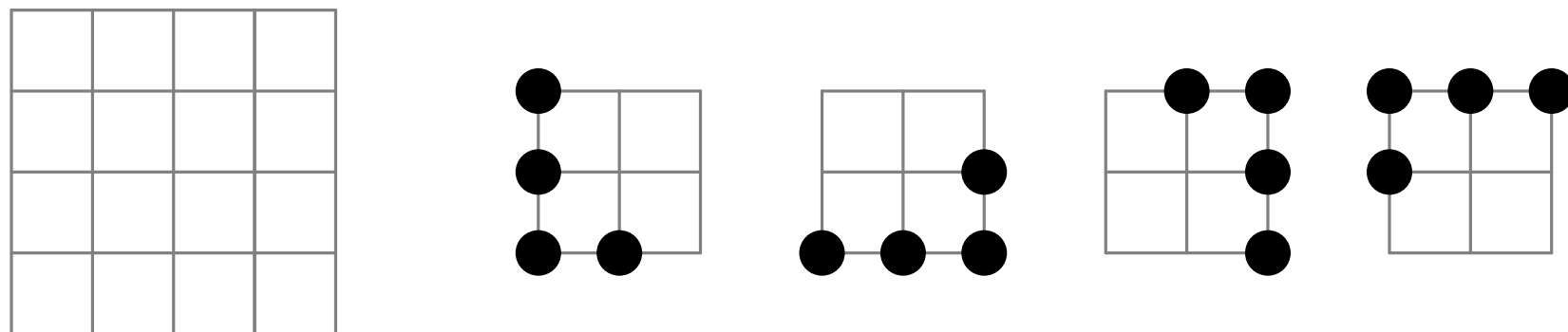
Assignment 2

Compute

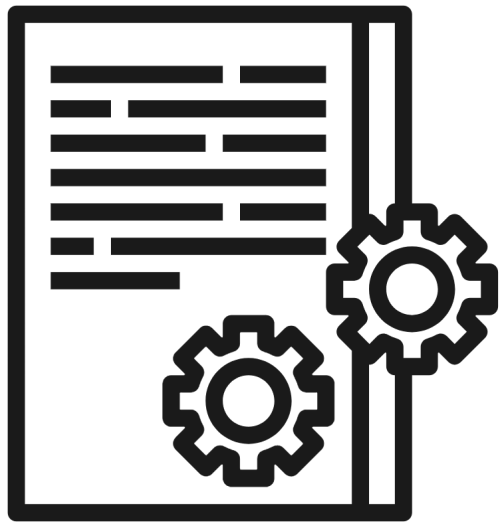


Post on
Teams

Describe a score function for the following setup.



(several possible solutions)



Policy

deterministic

State \longrightarrow Action

probabilistic

State \longrightarrow Probability Distribution over Actions



Assignment 3

How does a value function determine a policy?
deterministic / random ?

How does a policy define a value function?

Why might a **random** policy be better?
(for learning)



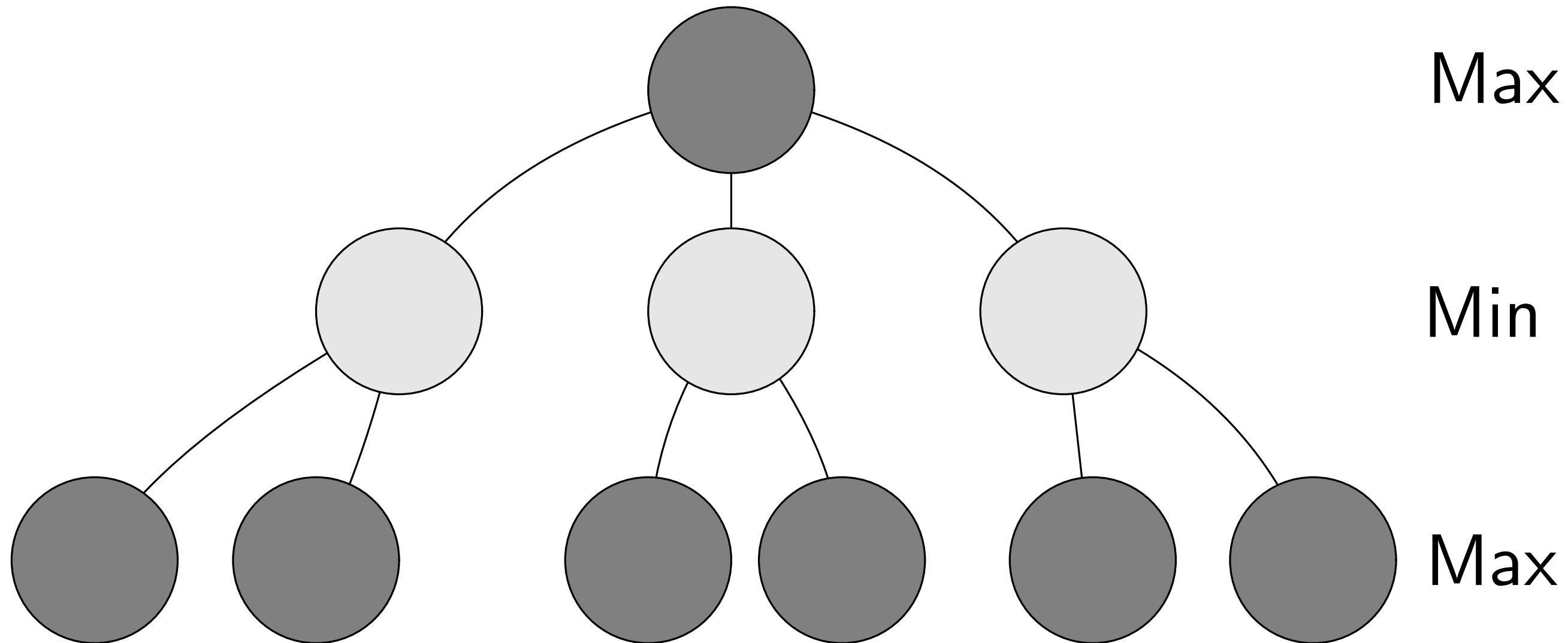
Post on
Teams



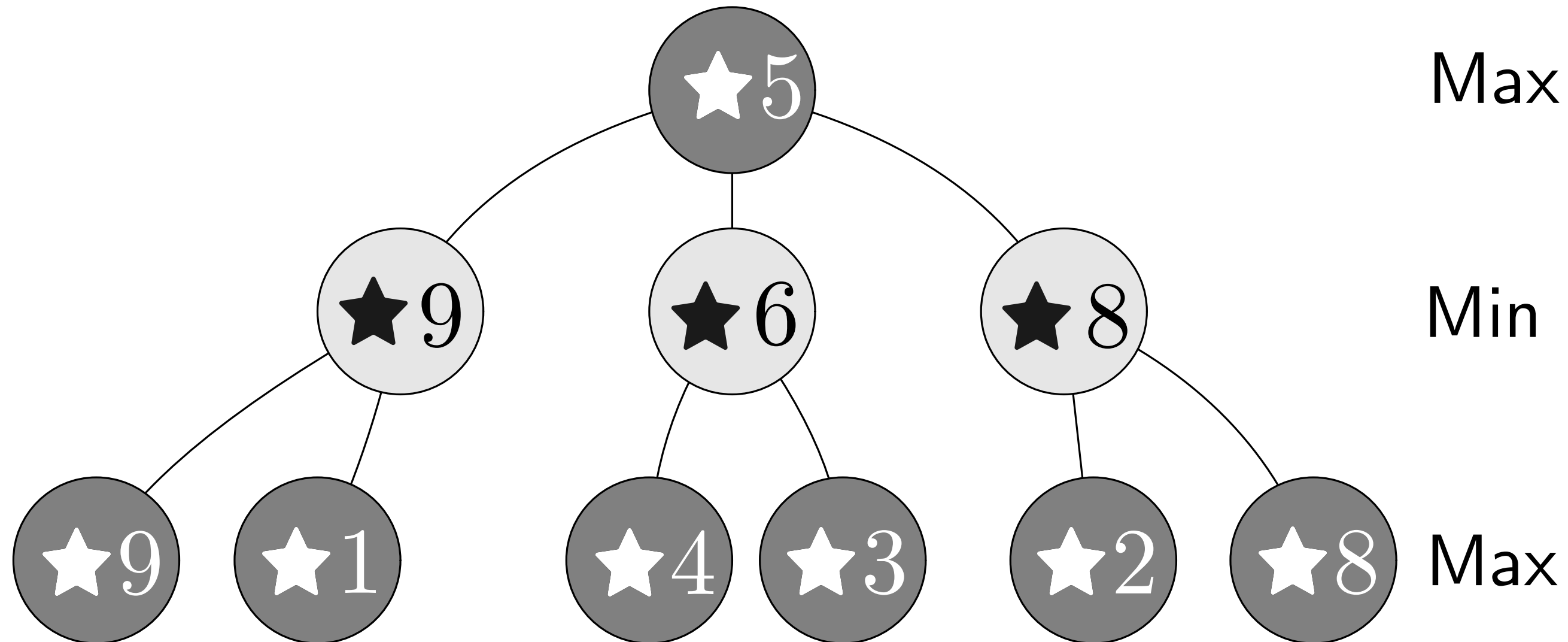
Min Max



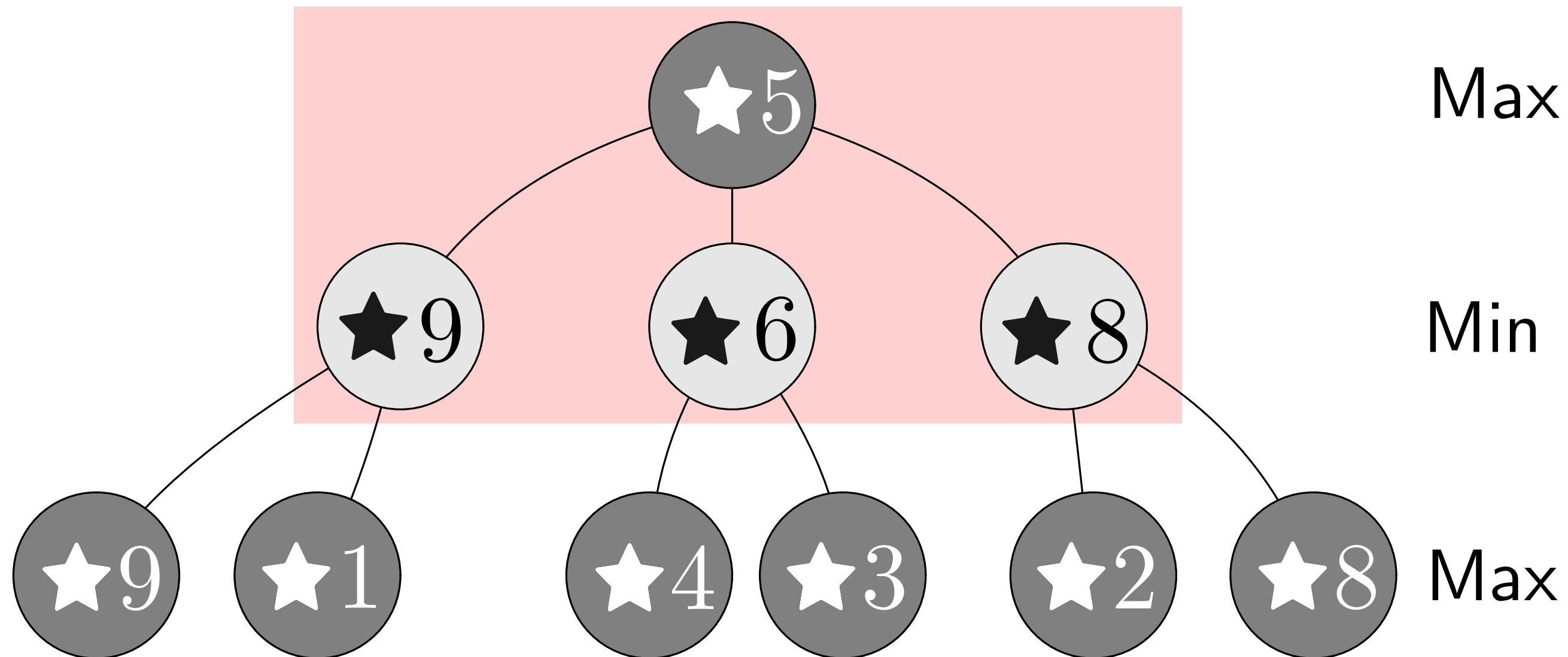
Min Max



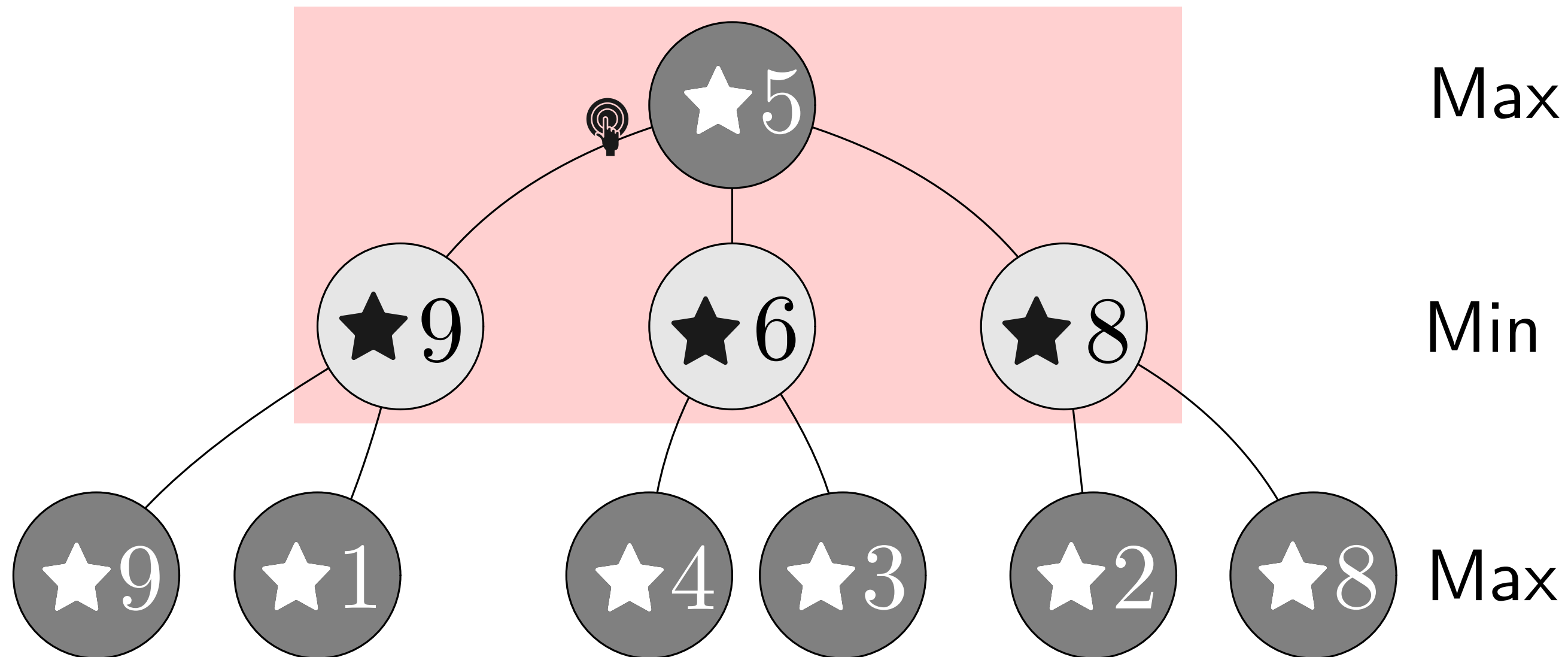
Min Max



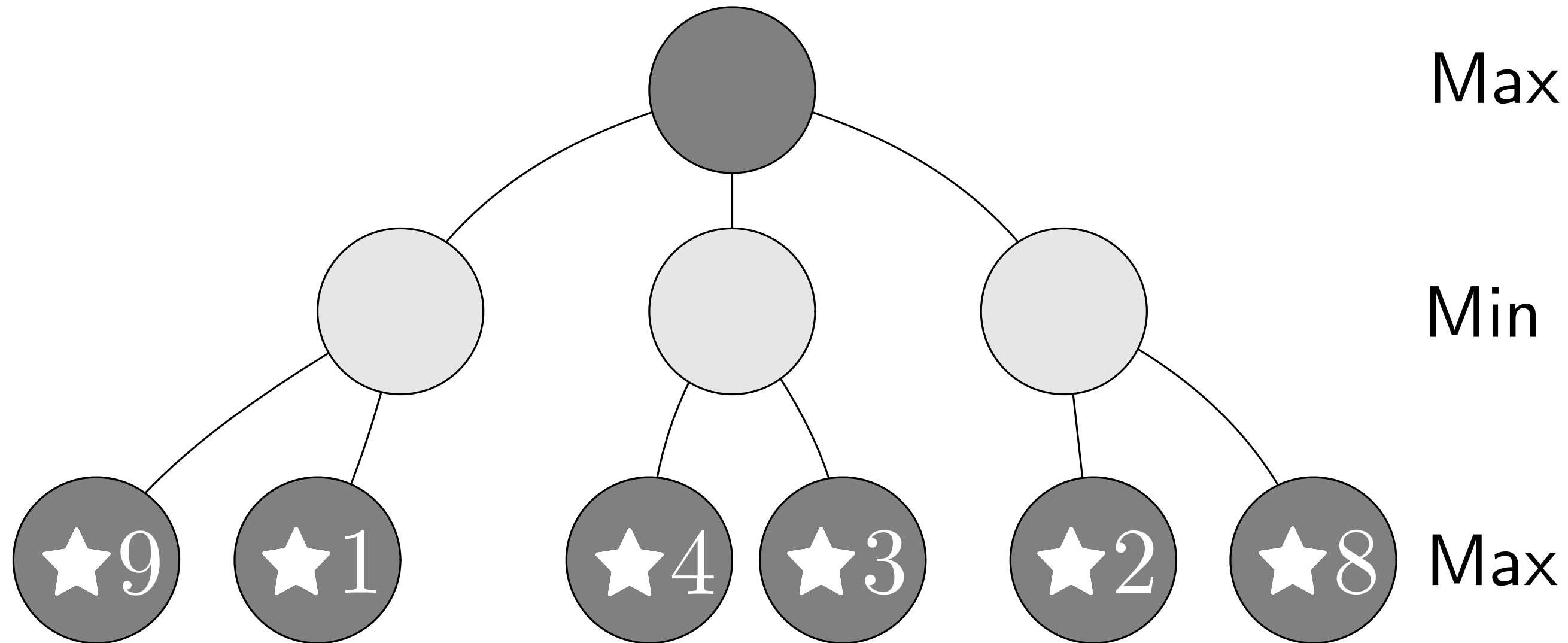
Min Max



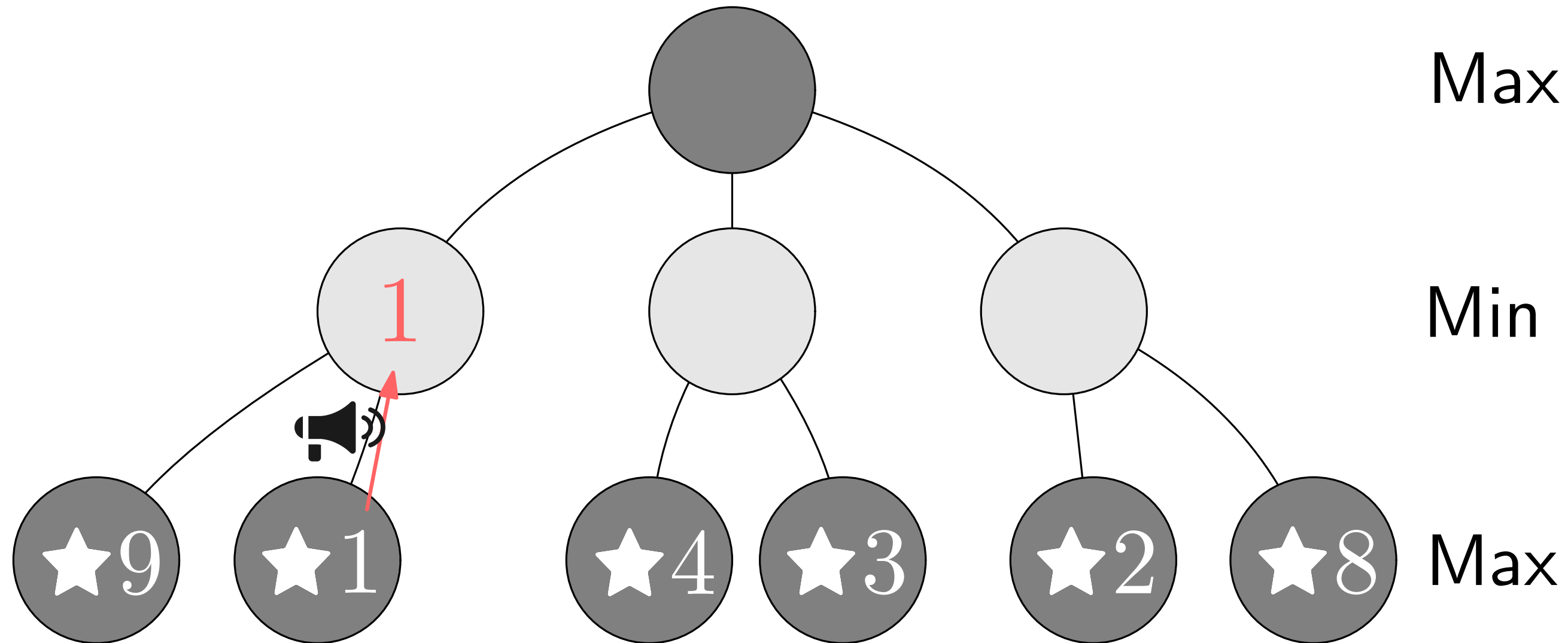
Min Max



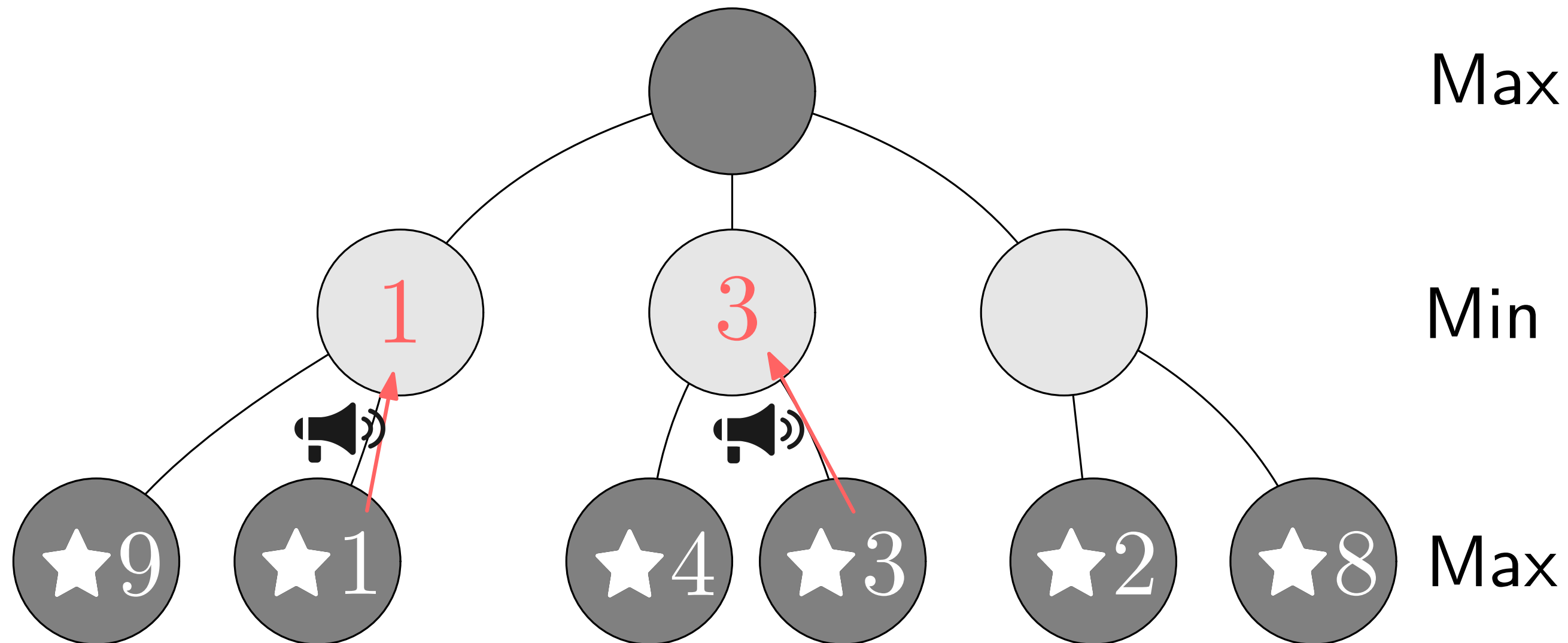
Min Max



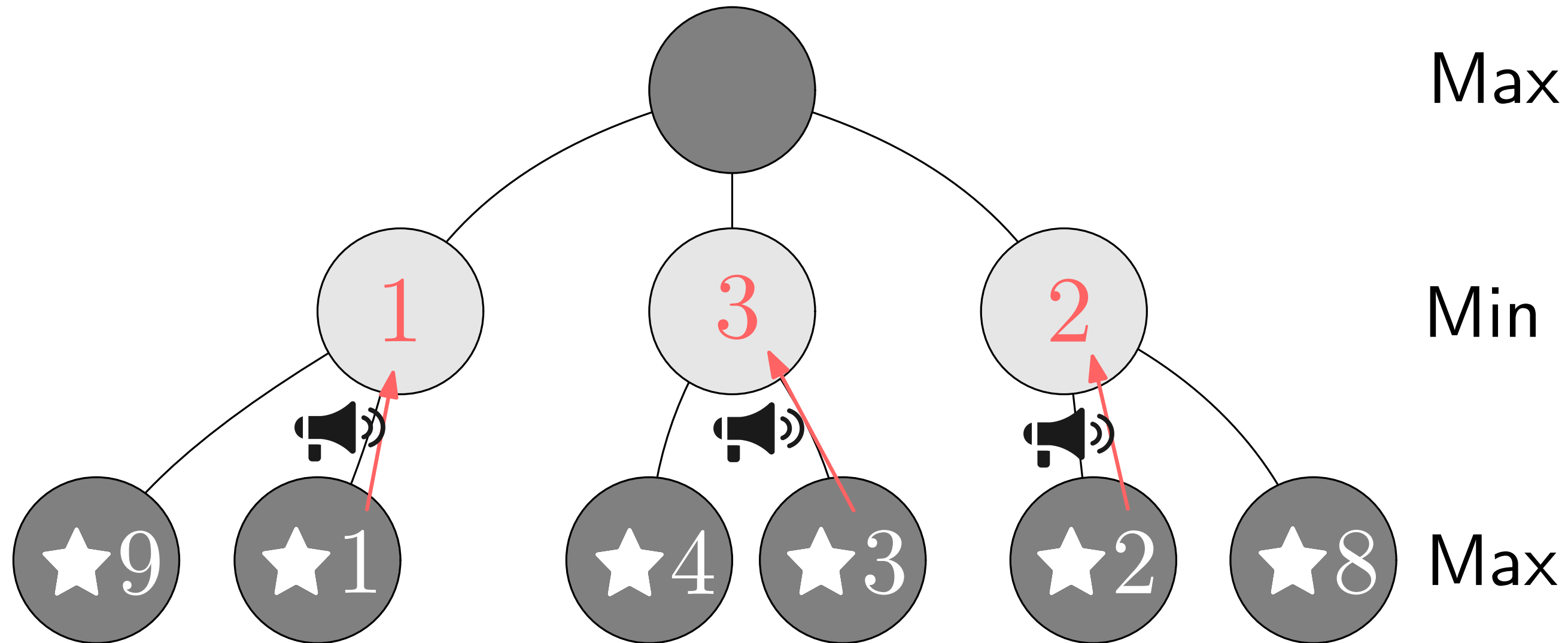
Min Max



Min Max

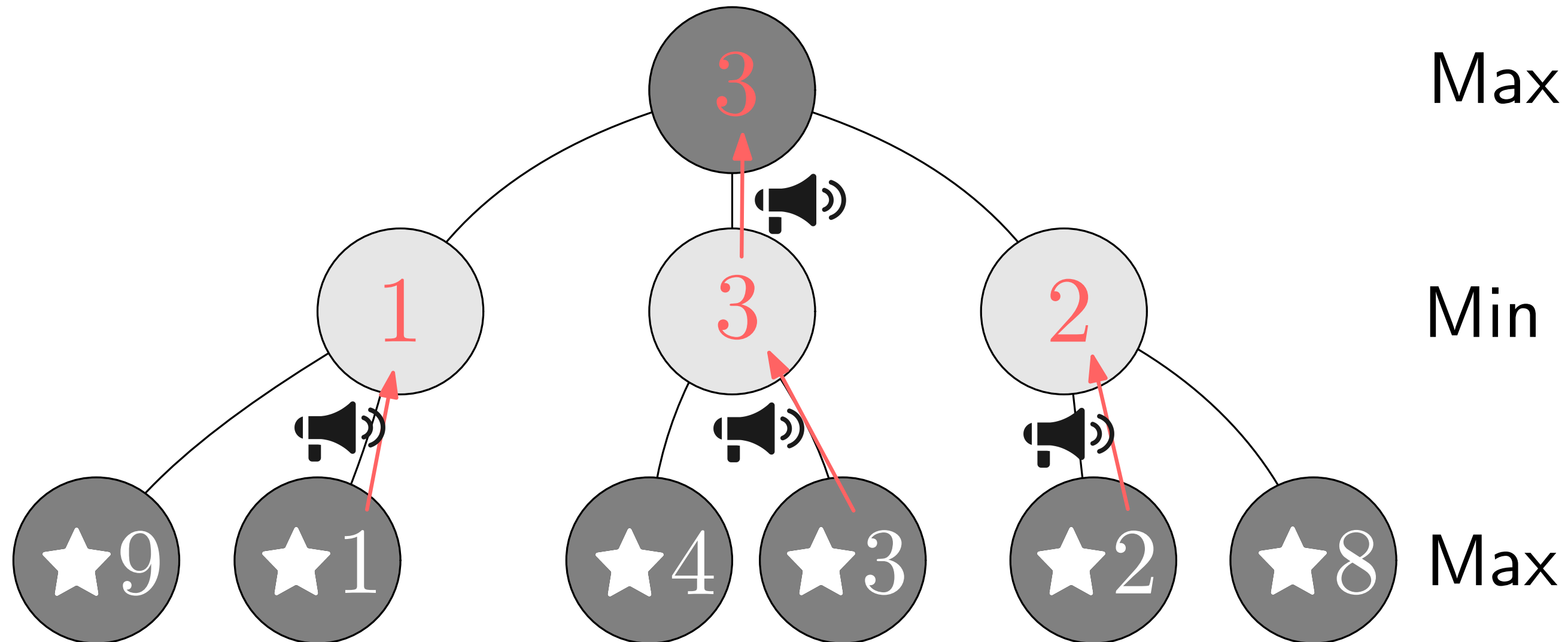


Min Max



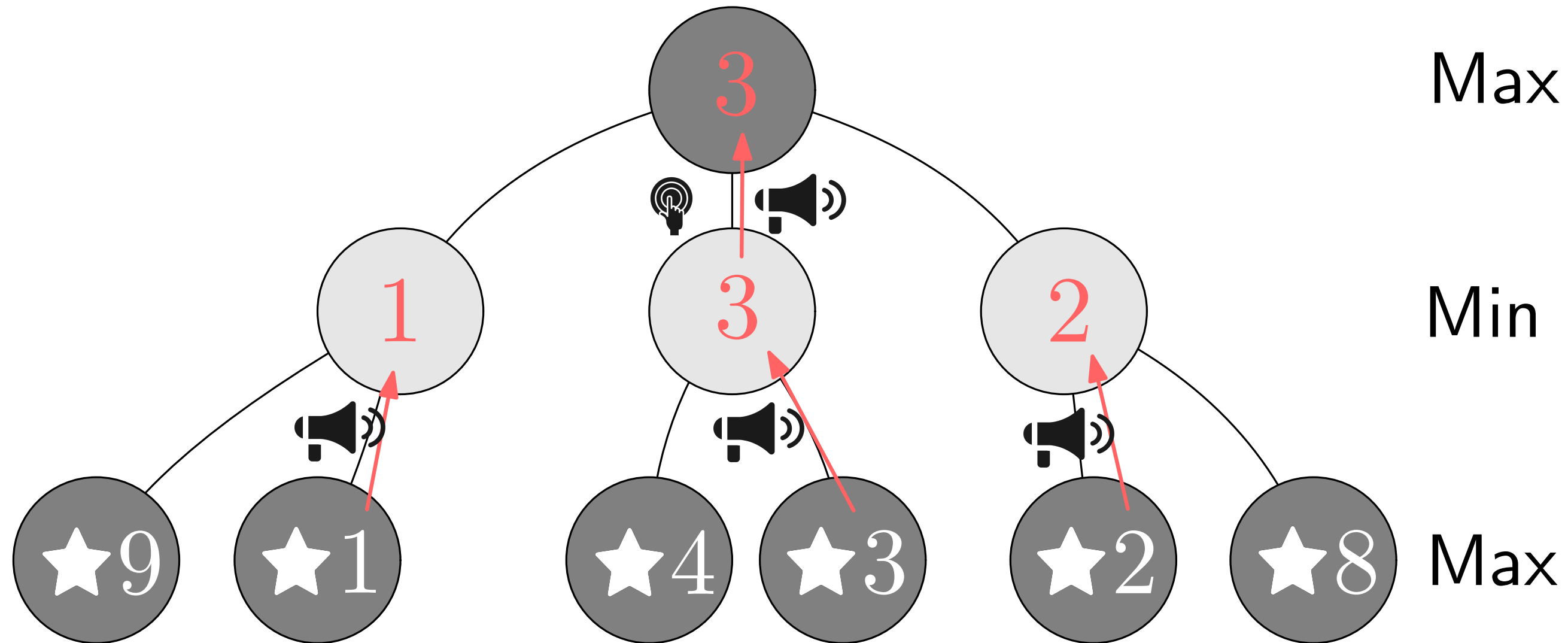
Min Max

 Backpropagation



Min Max

 Backpropagation





Assignment 4

Write Mini Max recursive Pseudocode

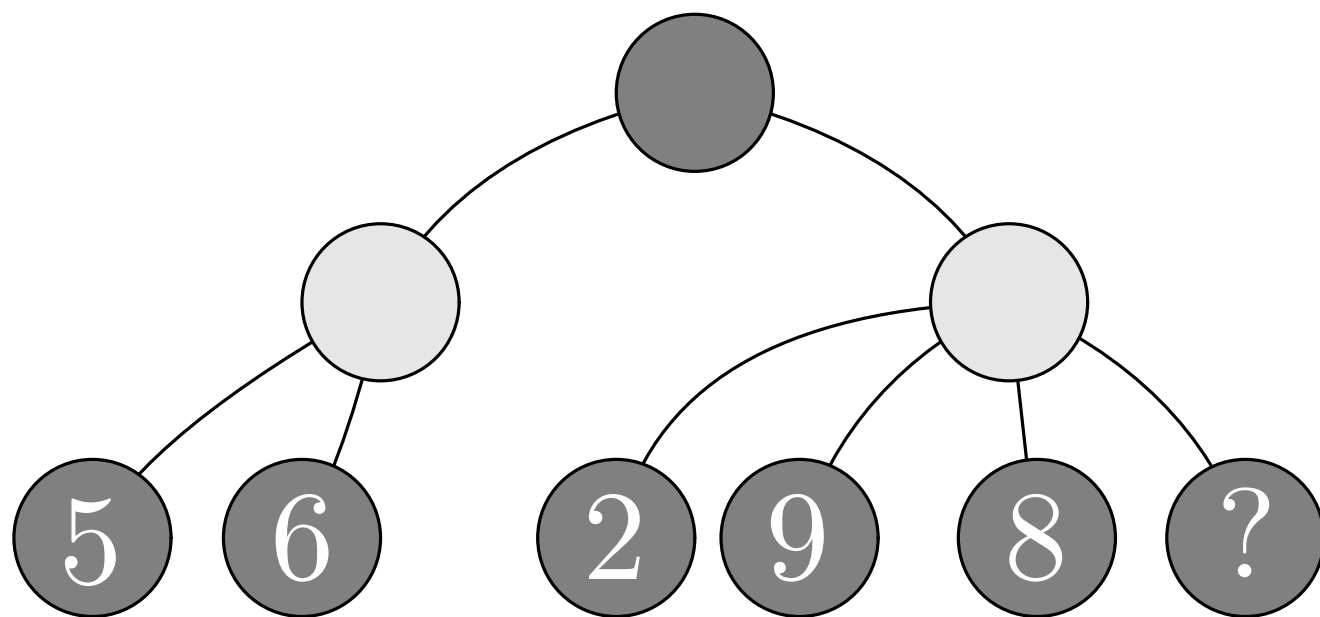
```
function minimax(state depth player)
```

depth means how deep you want to go.

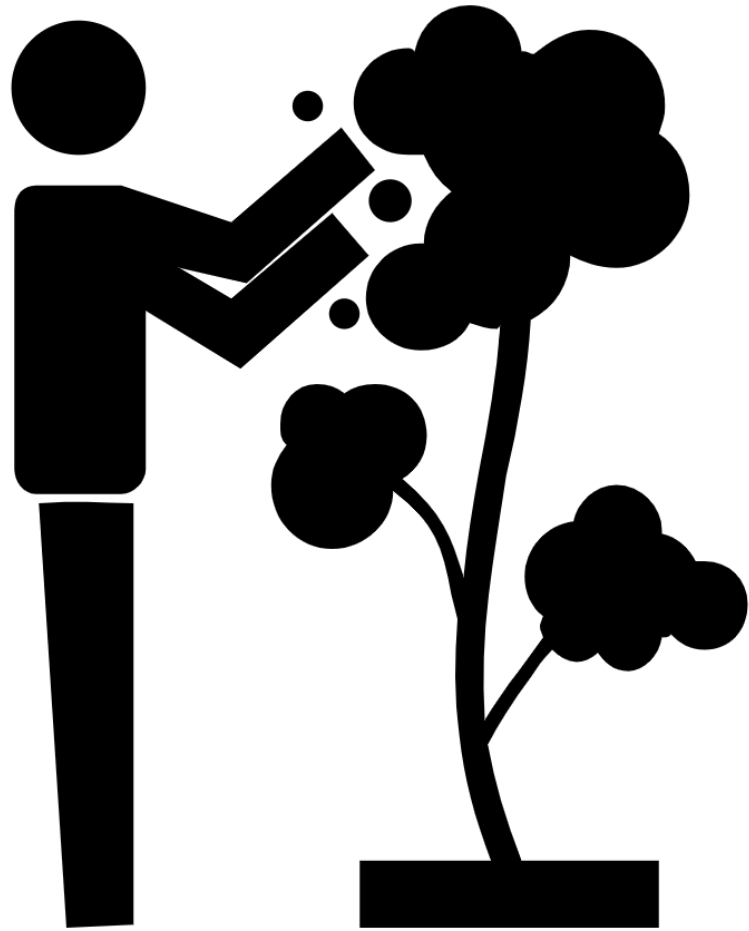
You can access all children



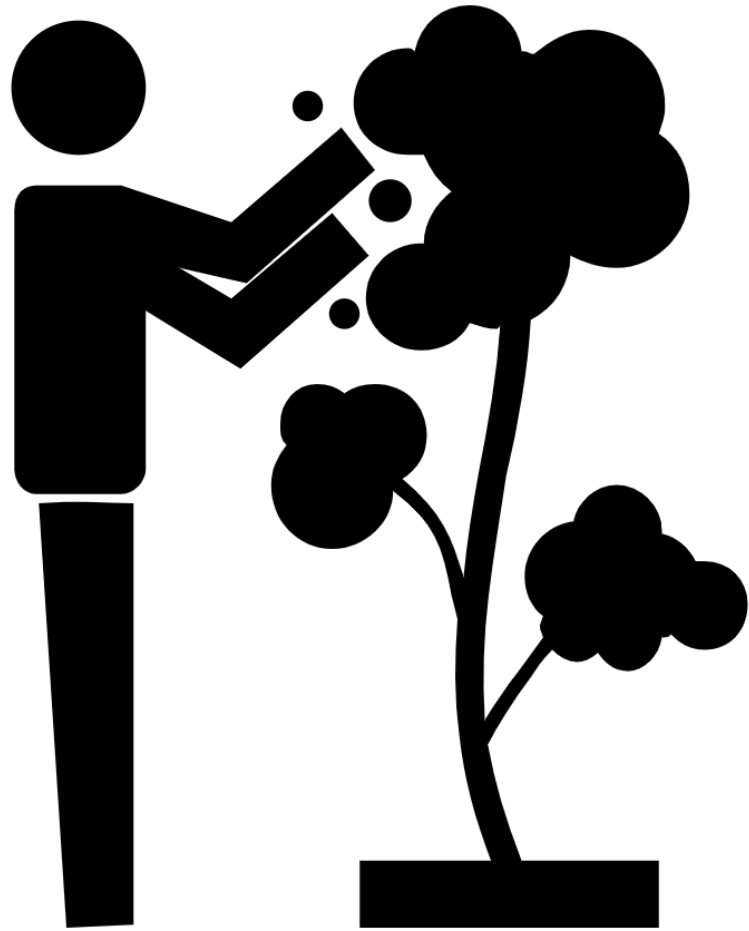
Post on
Teams



Evaluate
Observations?



Alpha-Beta Pruning



Don't look, if you don't have to!

Alpha-Beta Pruning



Monte Carlo

Tree

Search





Selection

Expansion

Simulation

Backpropagation

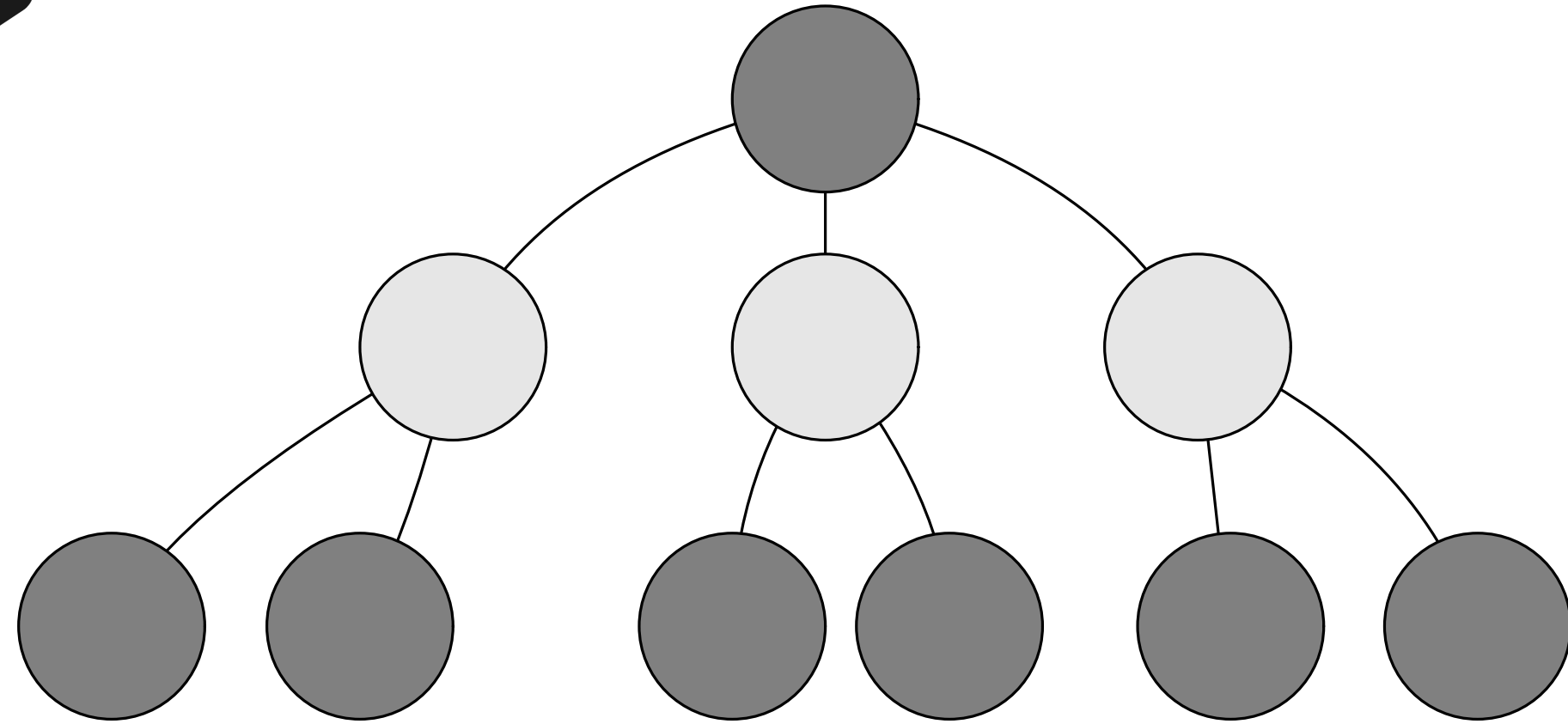


Selection

Expansion

Simulation

Backpropagation



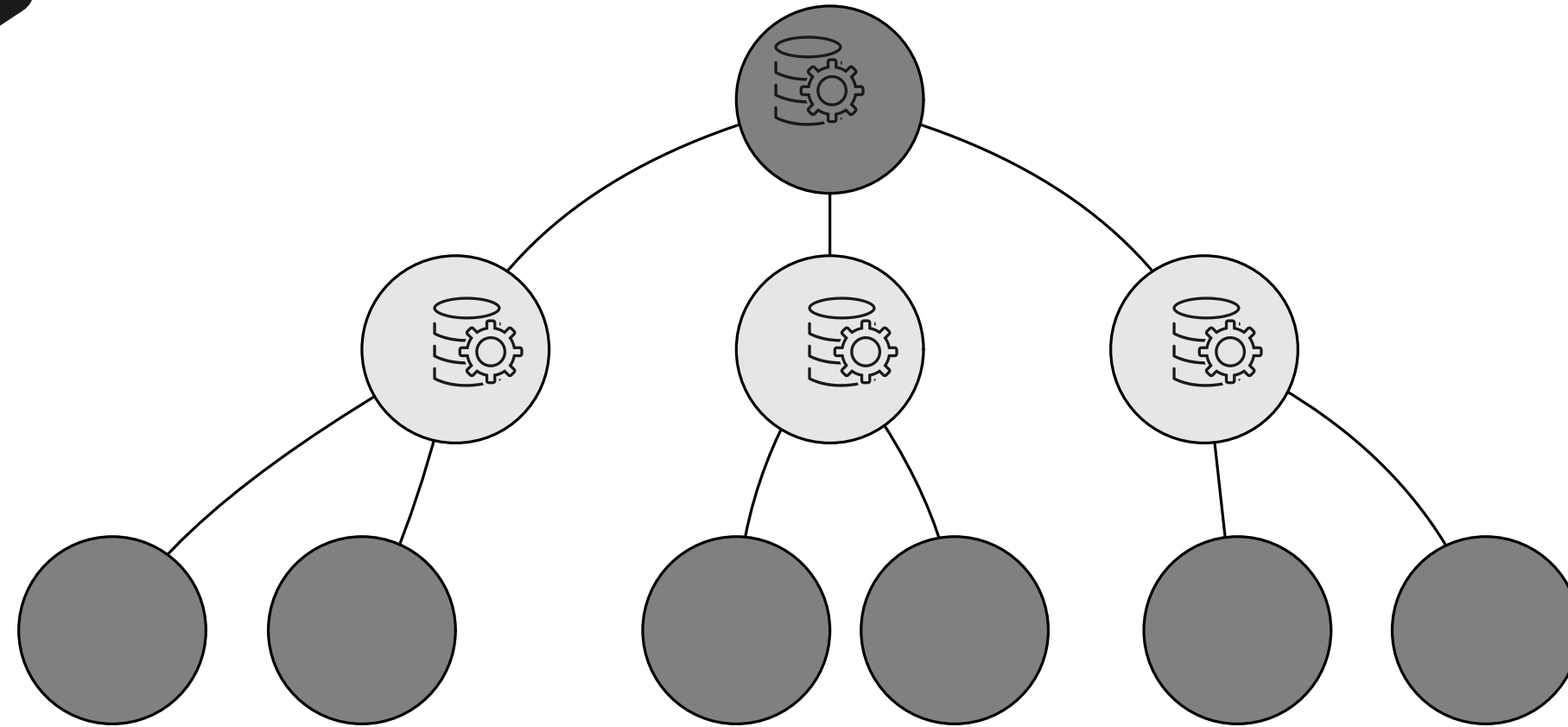


Selection

Expansion

Simulation

Backpropagation



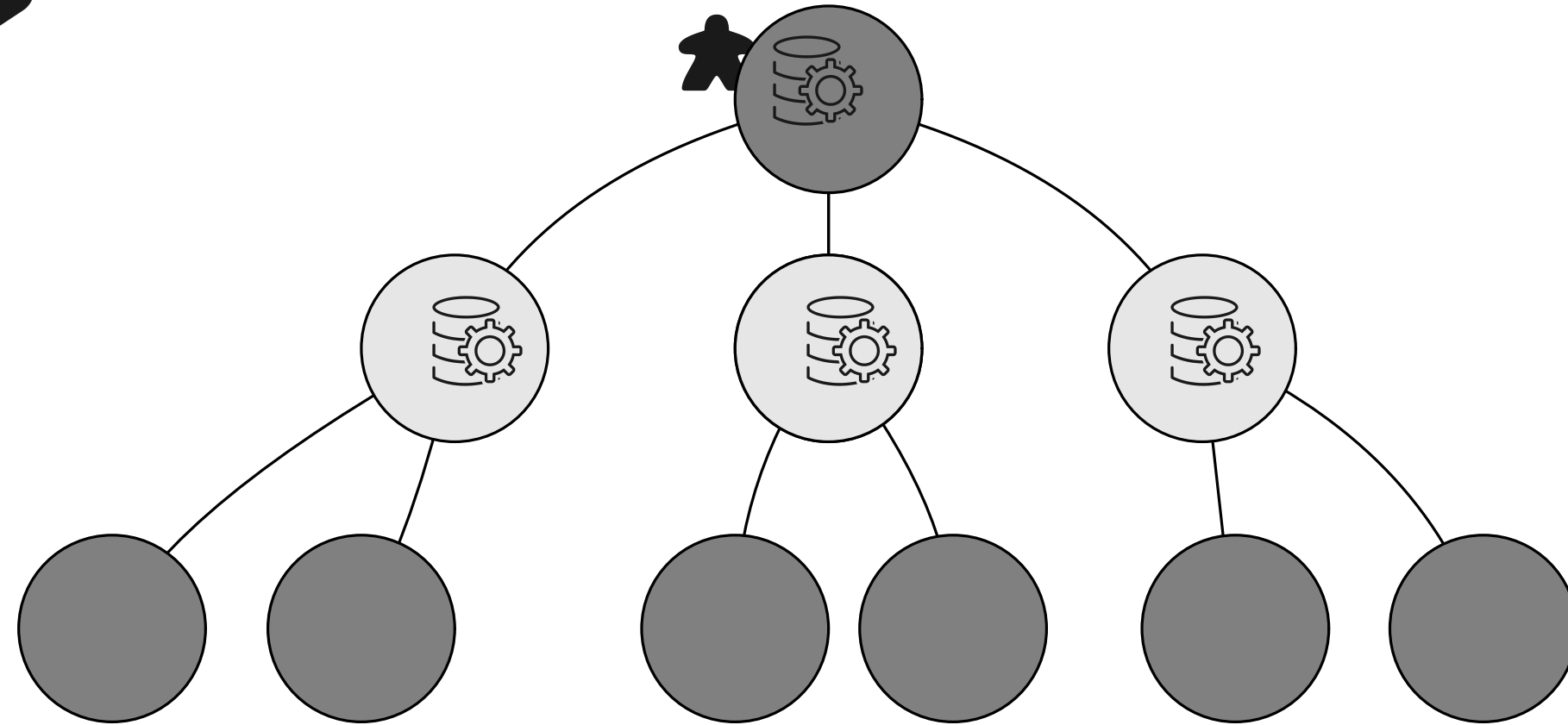


Selection

Expansion

Simulation

Backpropagation



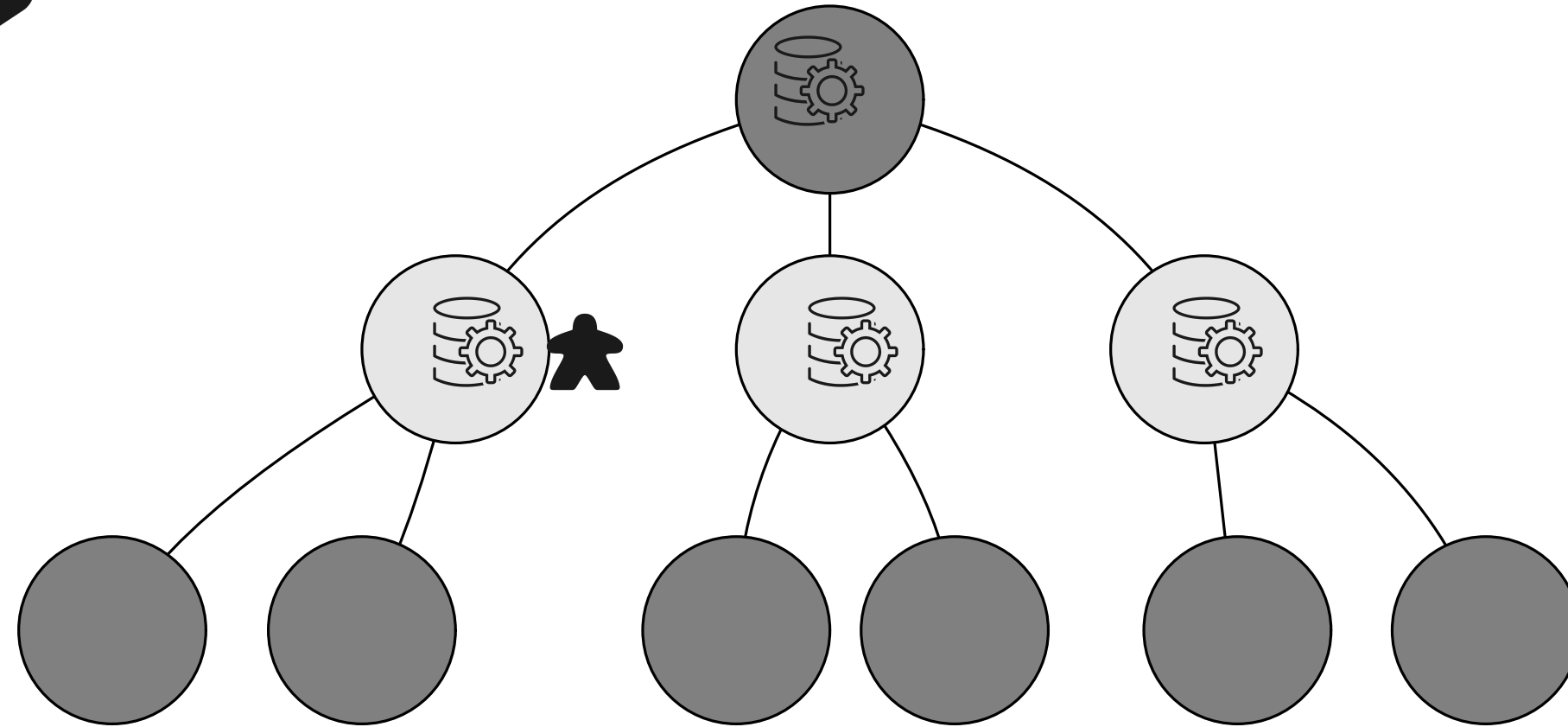


Selection

Expansion

Simulation

Backpropagation



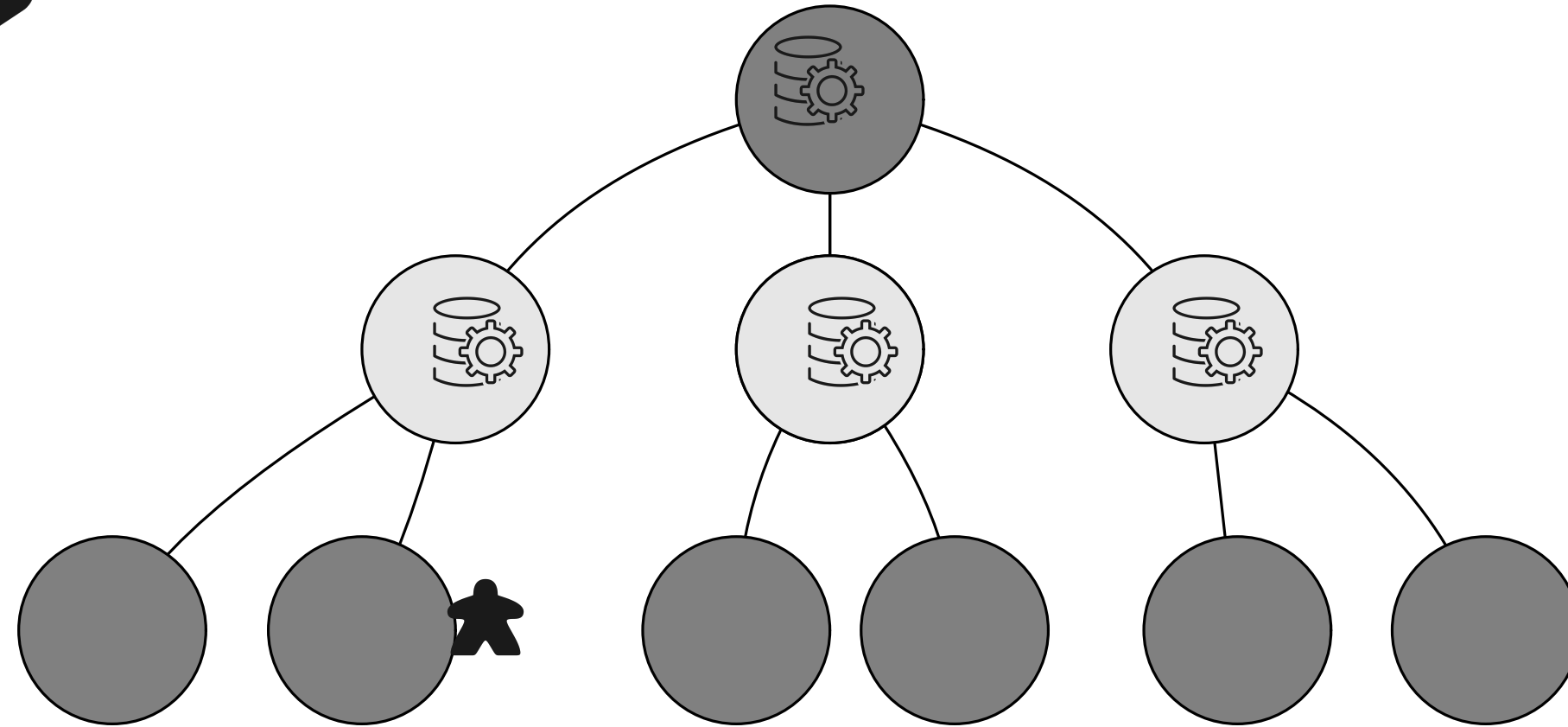


Selection

Expansion

Simulation

Backpropagation



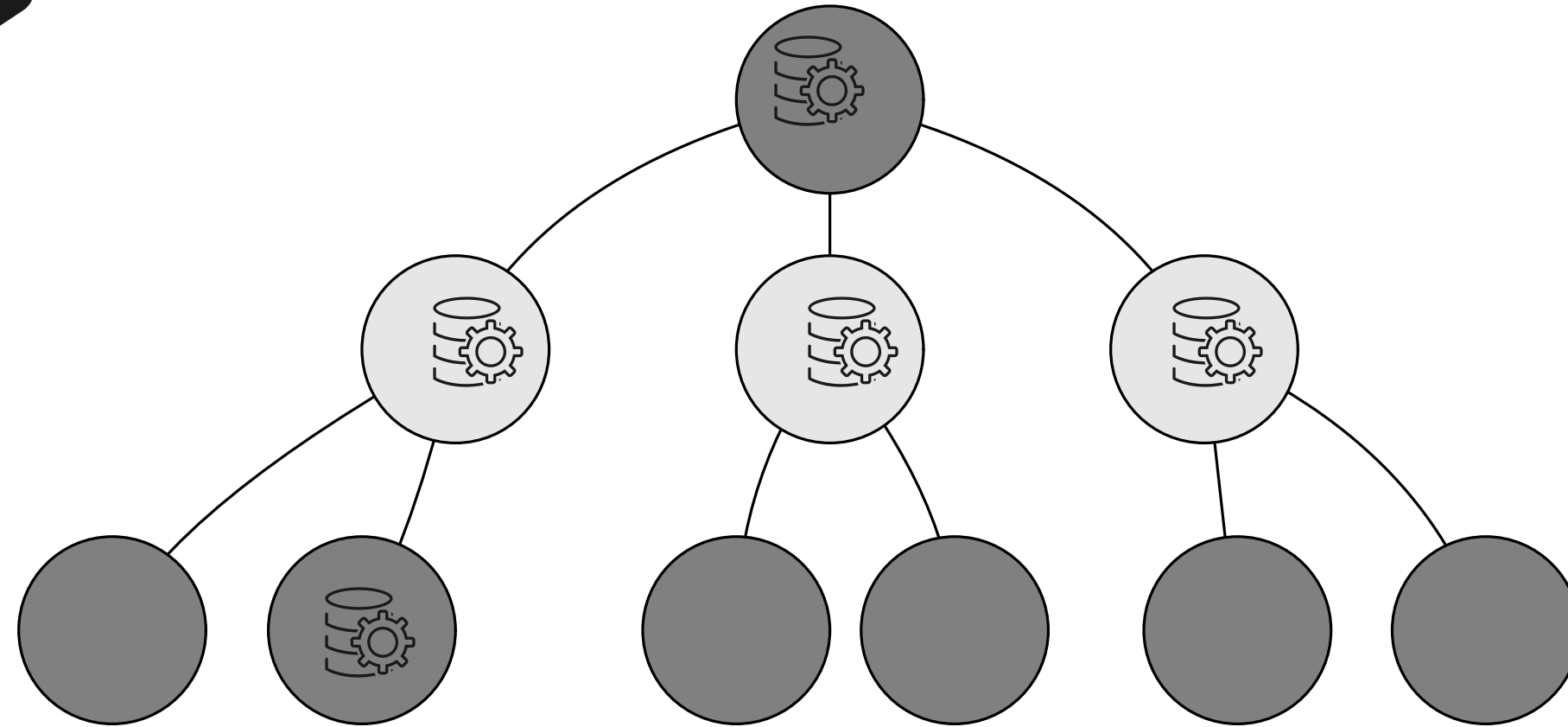


Selection

Expansion

Simulation

Backpropagation



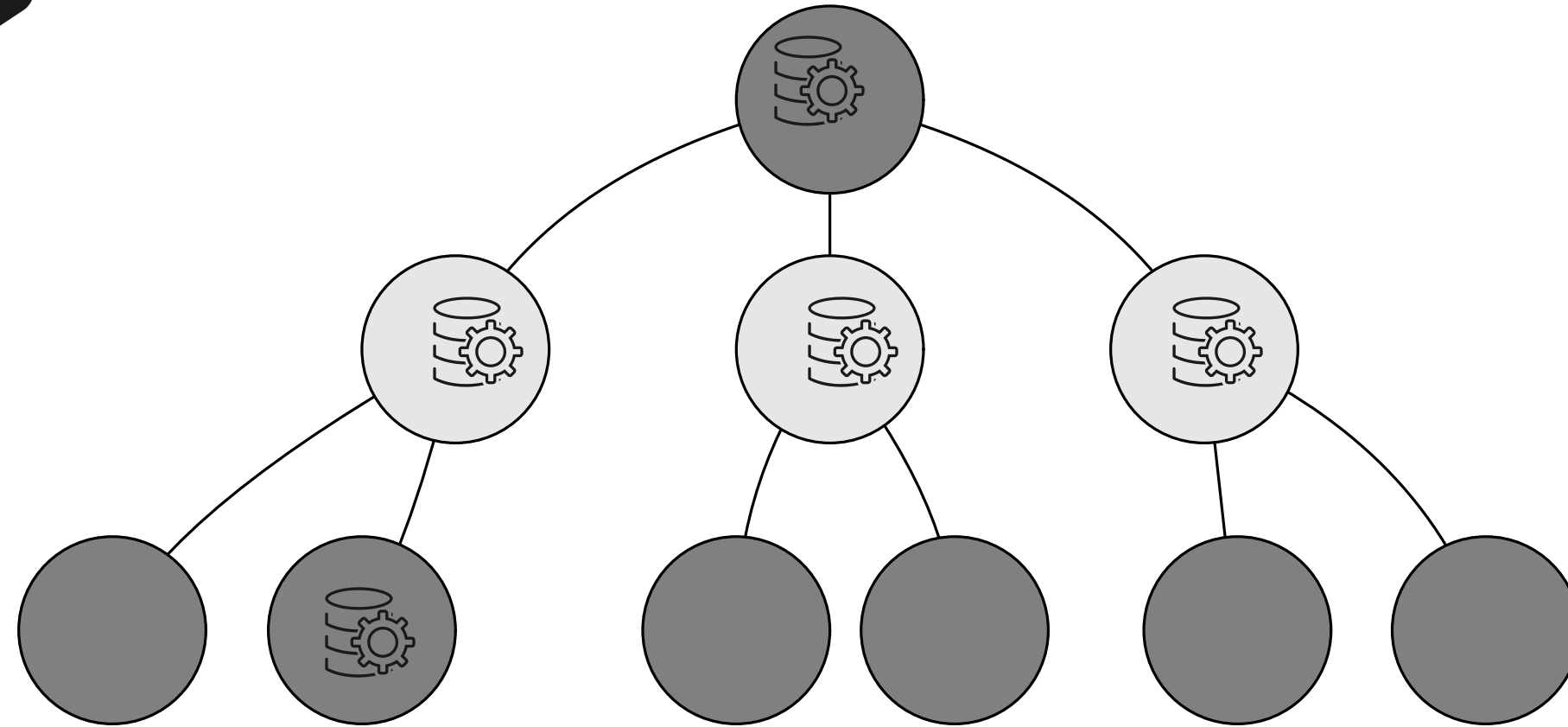


Selection

Expansion

Simulation

Backpropagation



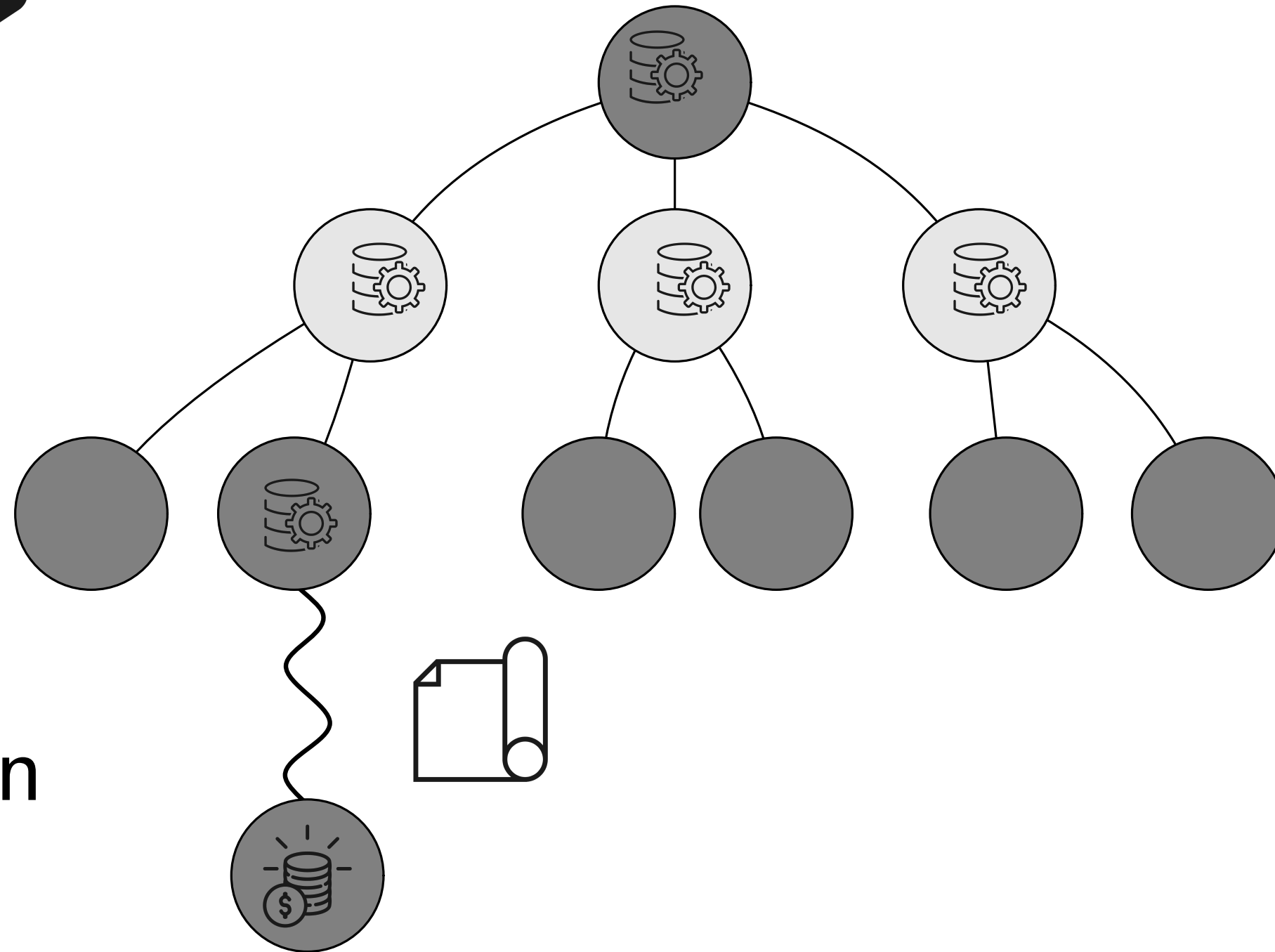


Selection

Expansion

Simulation

Backpropagation



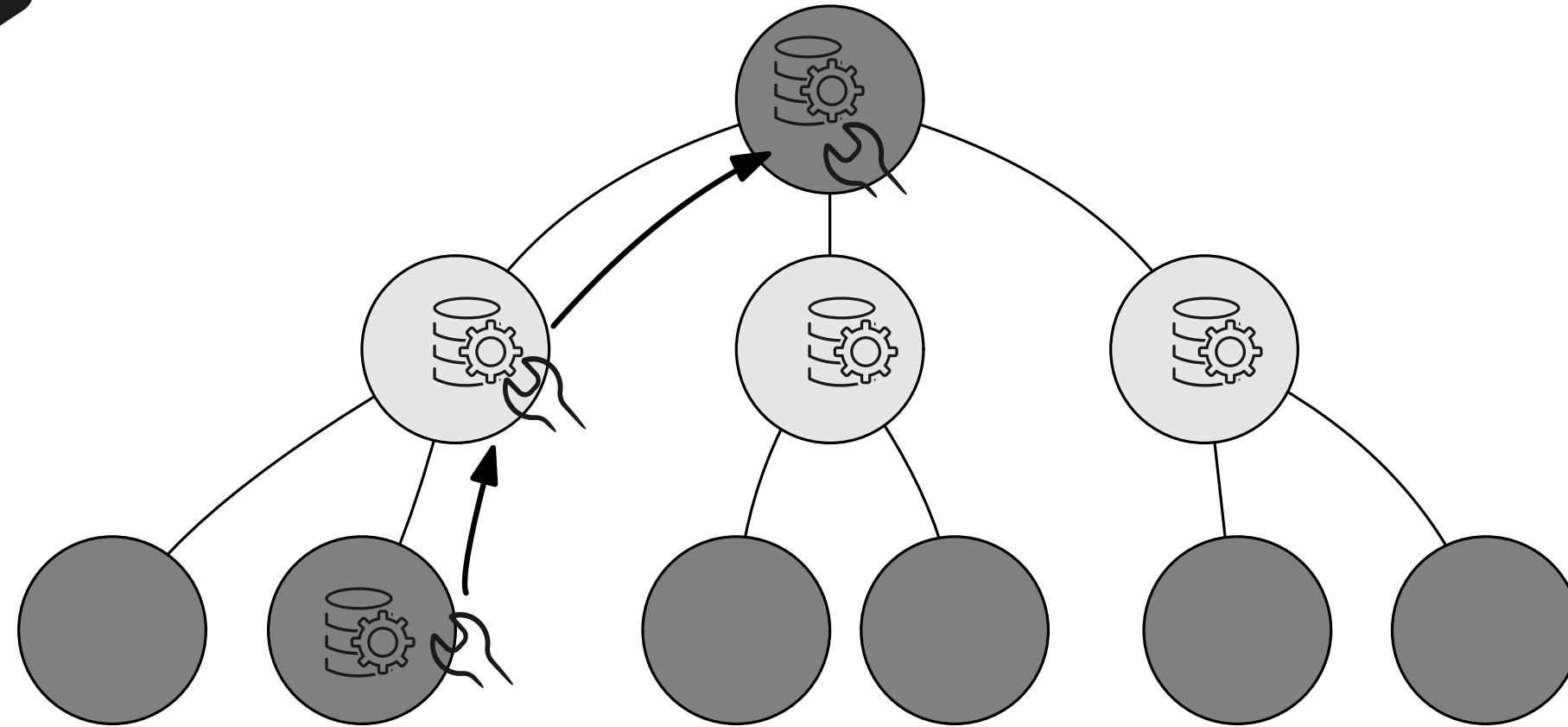


Selection

Expansion

Simulation

Backpropagation



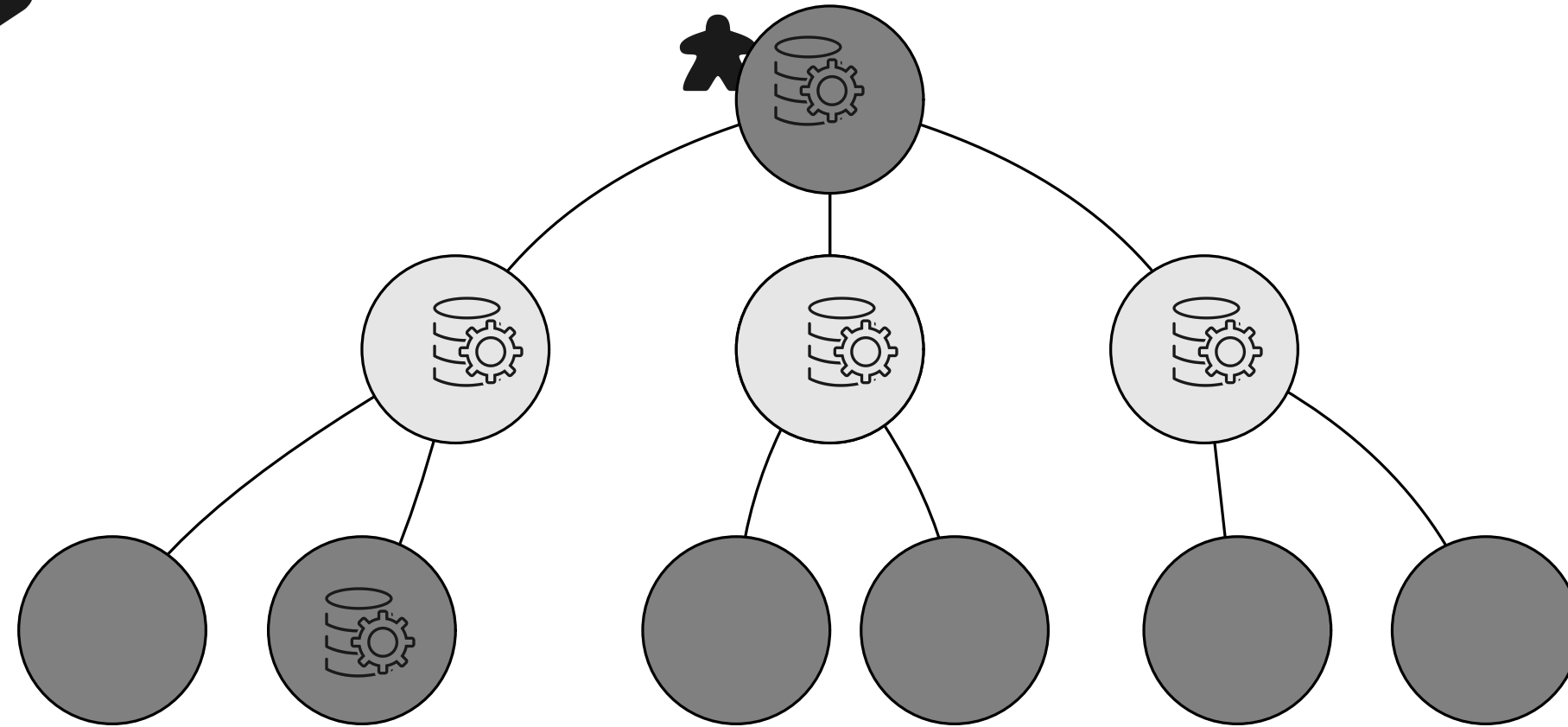


Selection

Expansion

Simulation

Backpropagation



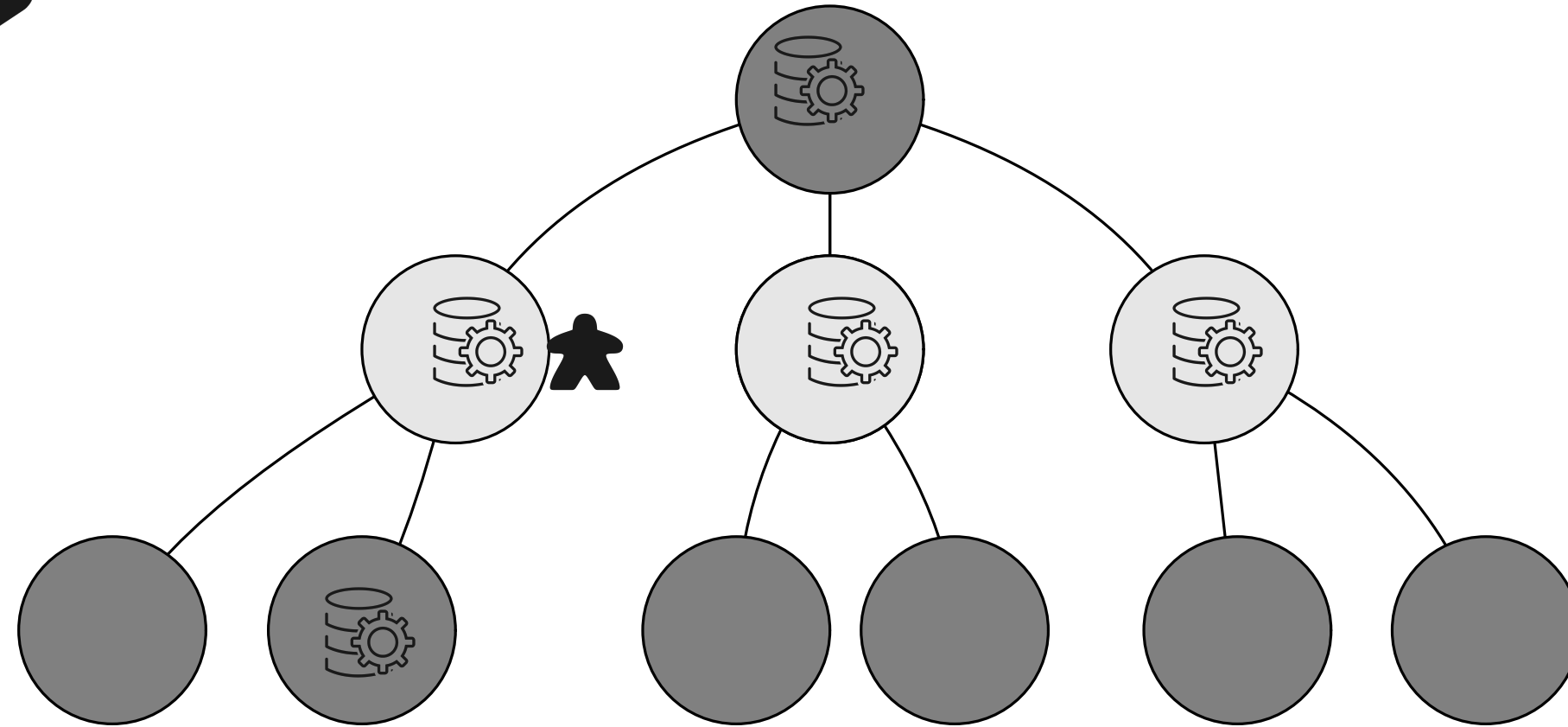


Selection

Expansion

Simulation

Backpropagation



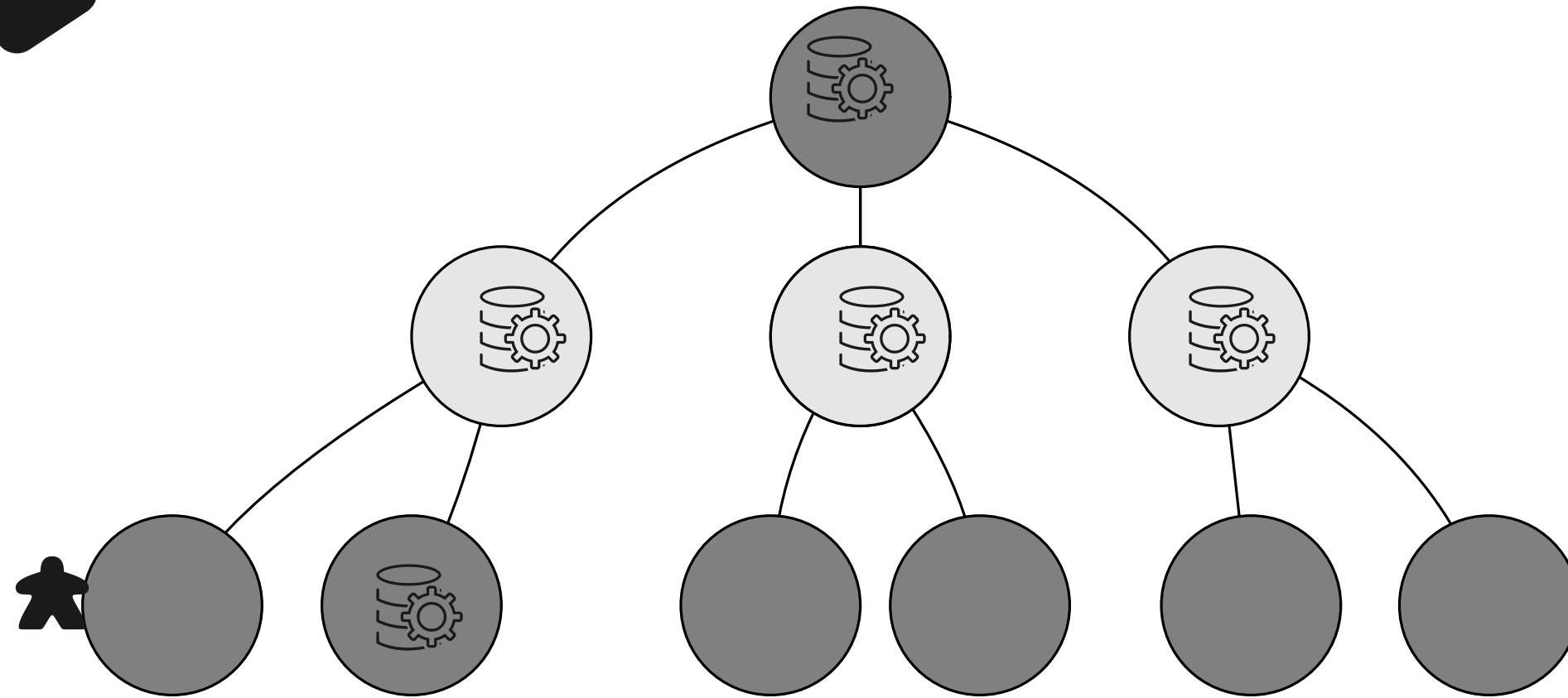


Selection

Expansion

Simulation

Backpropagation



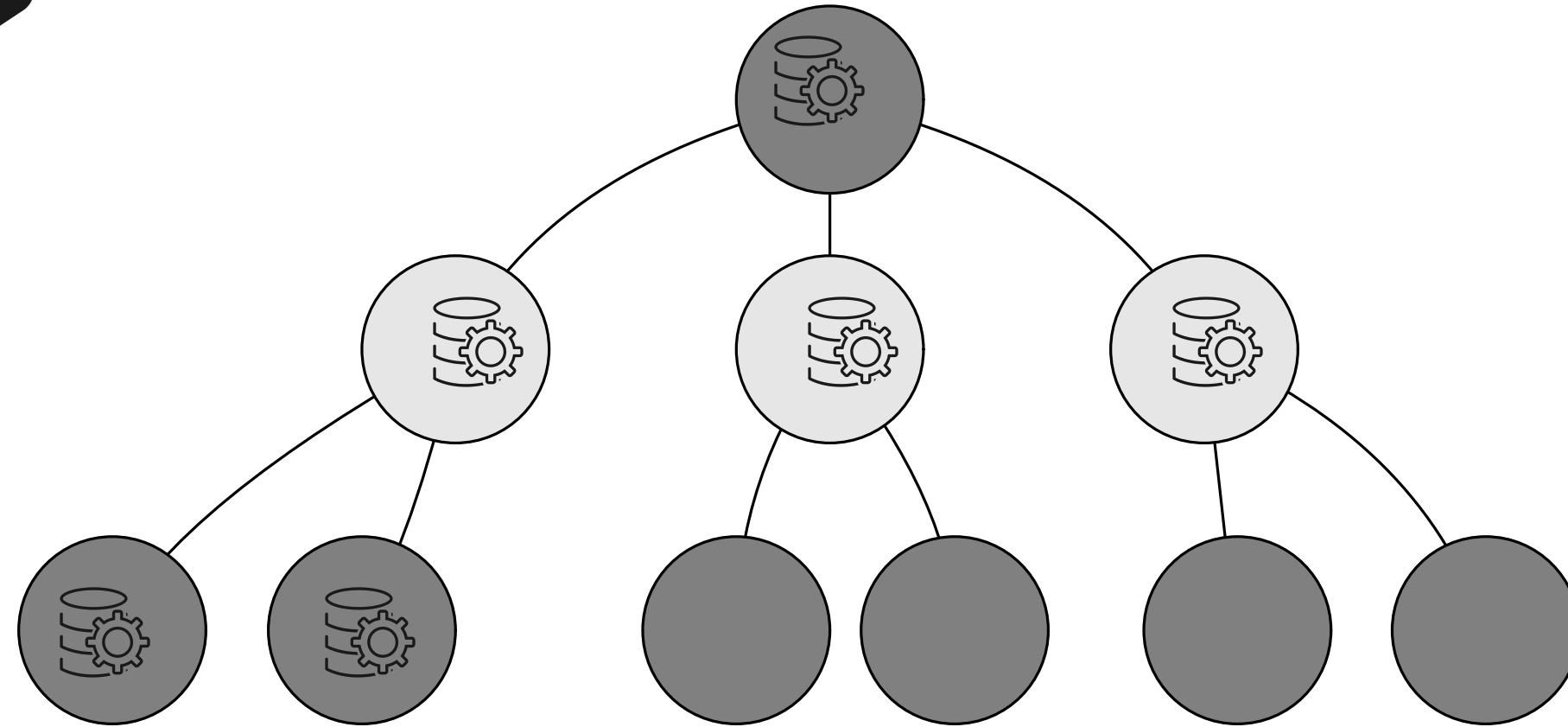


Selection

Expansion

Simulation

Backpropagation



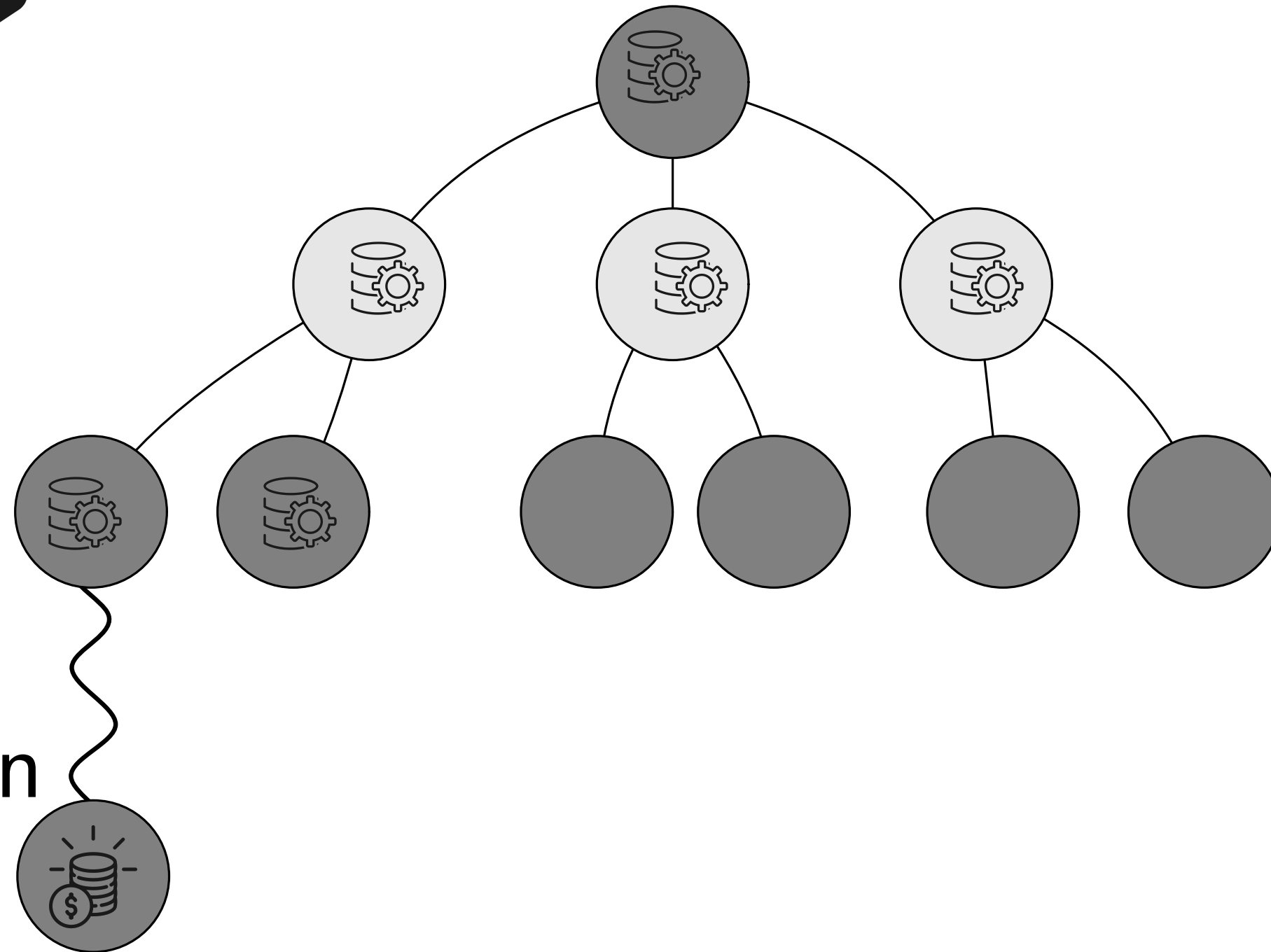


Selection

Expansion

Simulation

Backpropagation



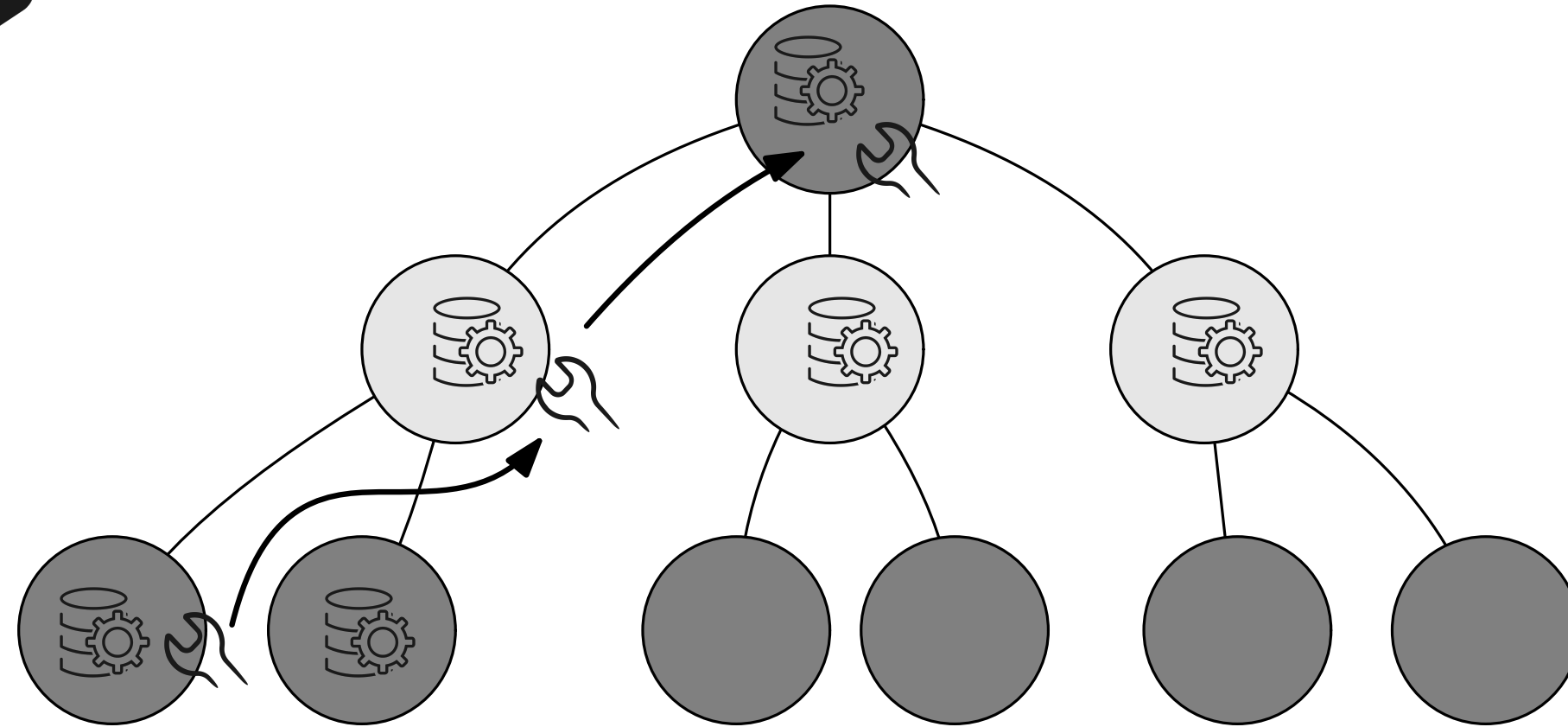


Selection

Expansion

Simulation

Backpropagation



Selection



Selection

UCB

ϵ -greedy



Selection

UCB

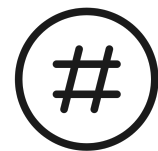
ϵ -greedy



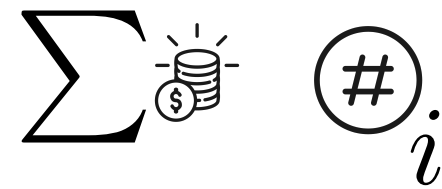
Selection



UCB



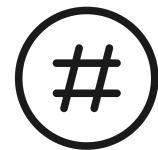
ϵ -greedy



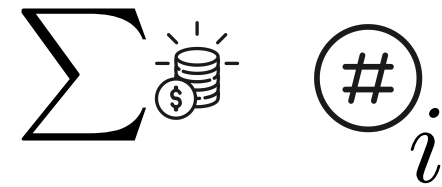
Selection



UCB



ϵ -greedy



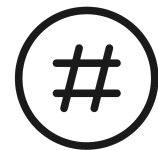
Simulation



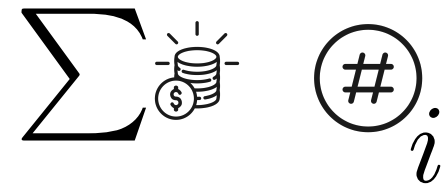
Selection



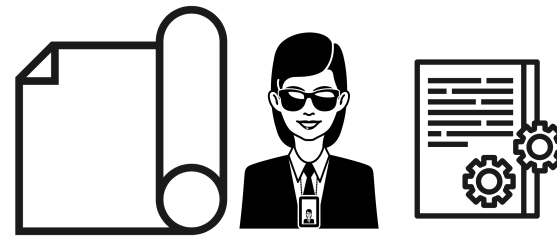
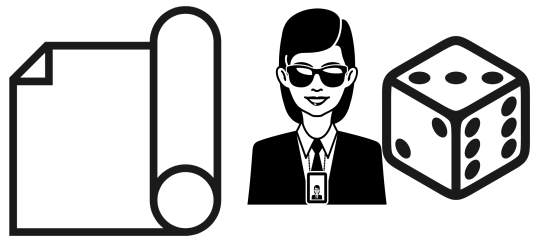
UCB



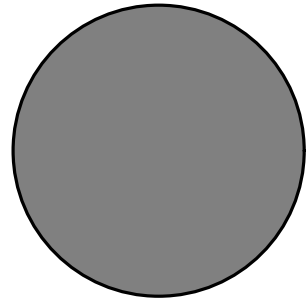
ϵ -greedy



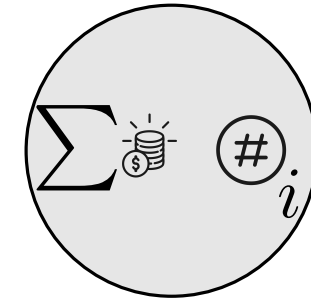
Simulation



Selection
Expansion
Simulation
Backpropagation



$$\# = 0$$

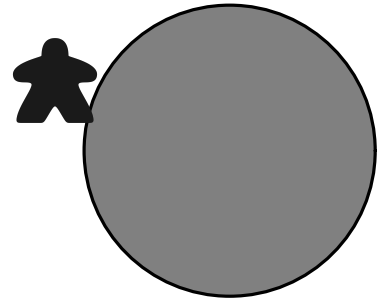


Selection

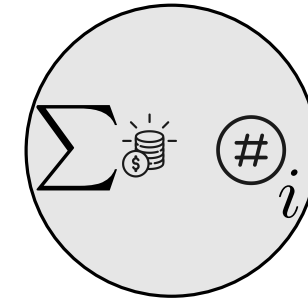
Expansion

Simulation

Backpropagation



$$\textcircled{\#} = 0$$

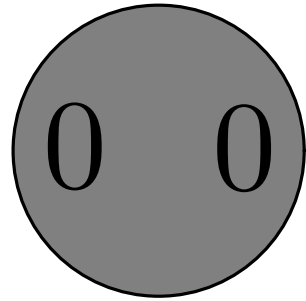


Selection

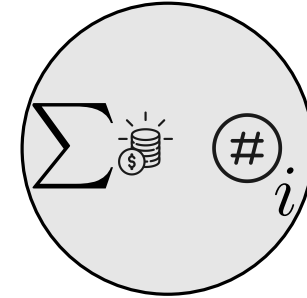
Expansion

Simulation

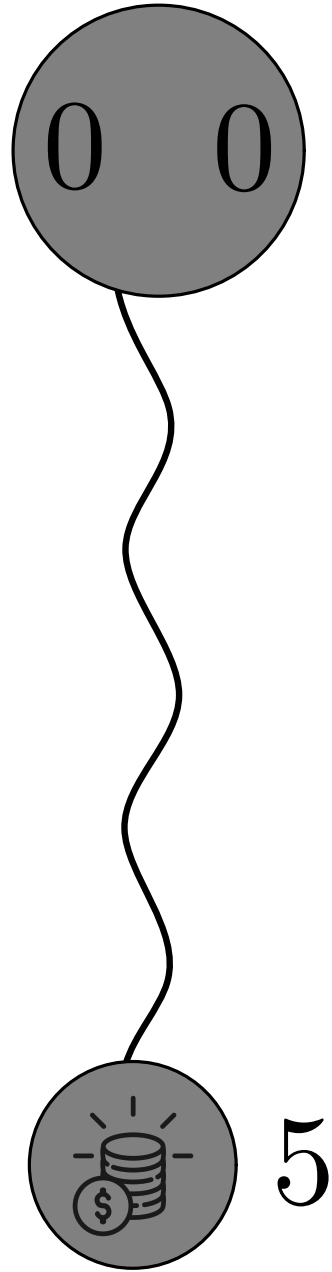
Backpropagation



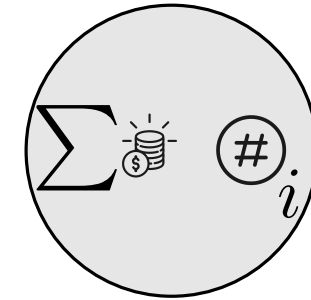
$$\# = 0$$



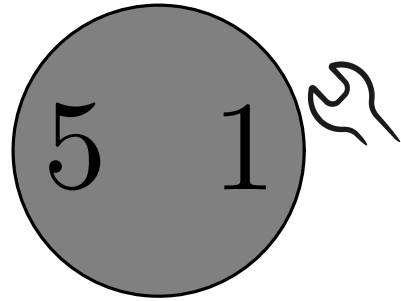
Selection
Expansion
Simulation
Backpropagation



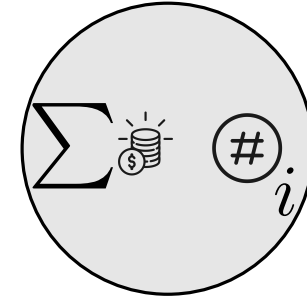
$$\# = 0$$



Selection
Expansion
Simulation
Backpropagation



$$\textcircled{\#} = 1$$

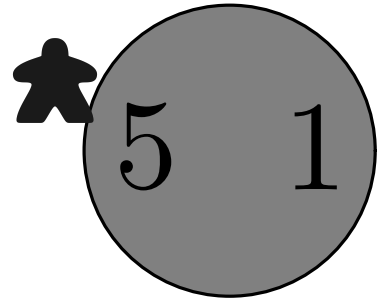


Selection

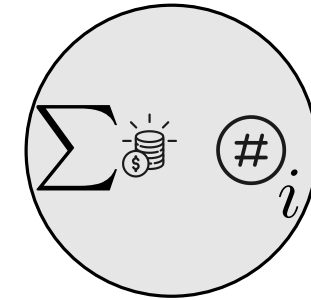
Expansion

Simulation

Backpropagation



$$\textcircled{\#} = 1$$

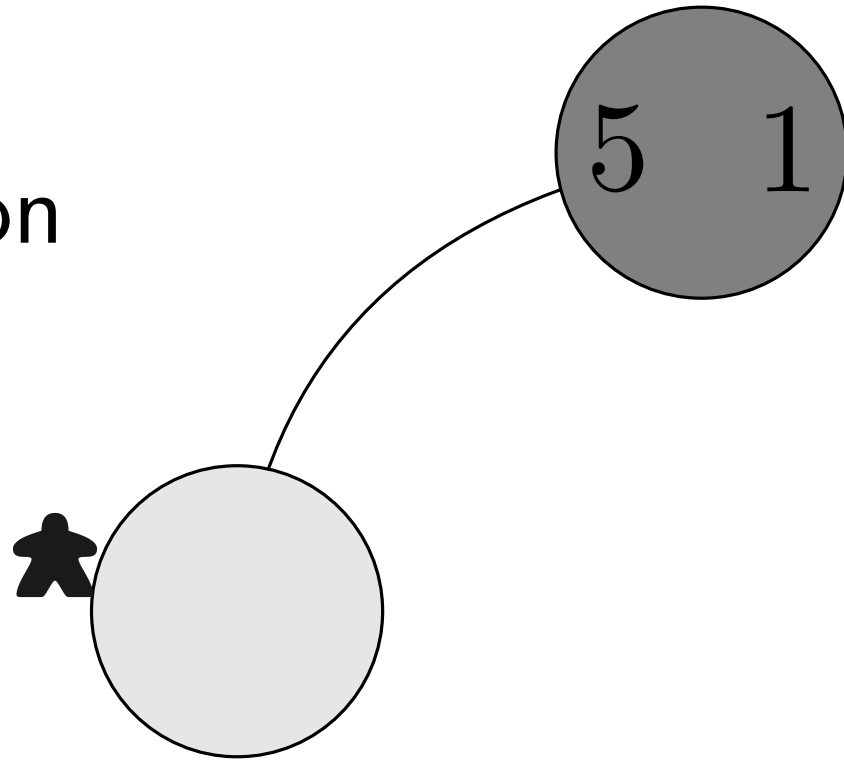


Selection

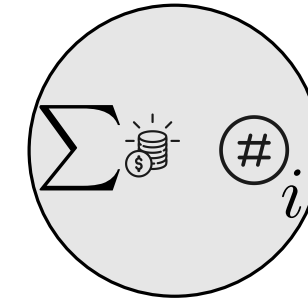
Expansion

Simulation

Backpropagation



$$\textcircled{\#} = 1$$

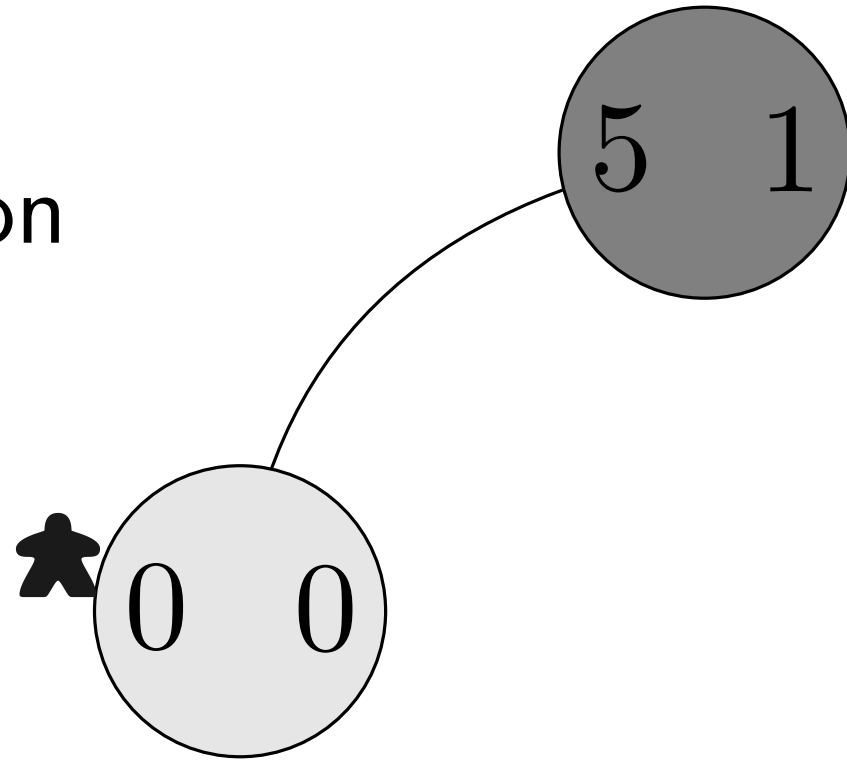


Selection

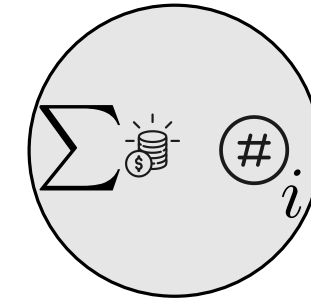
Expansion

Simulation

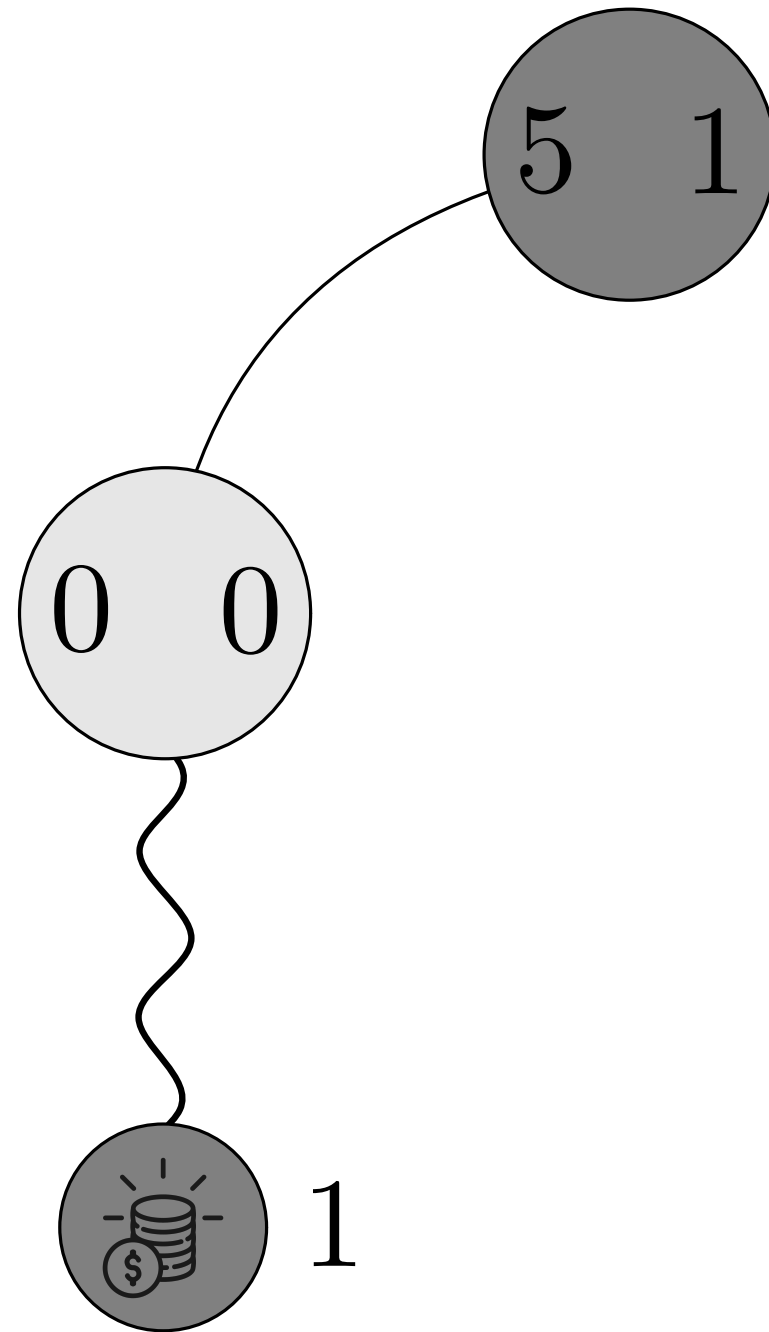
Backpropagation



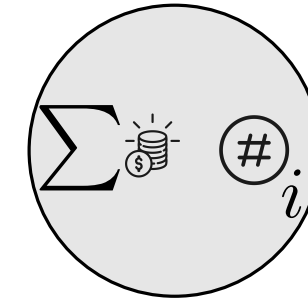
$$\textcircled{\#} = 1$$



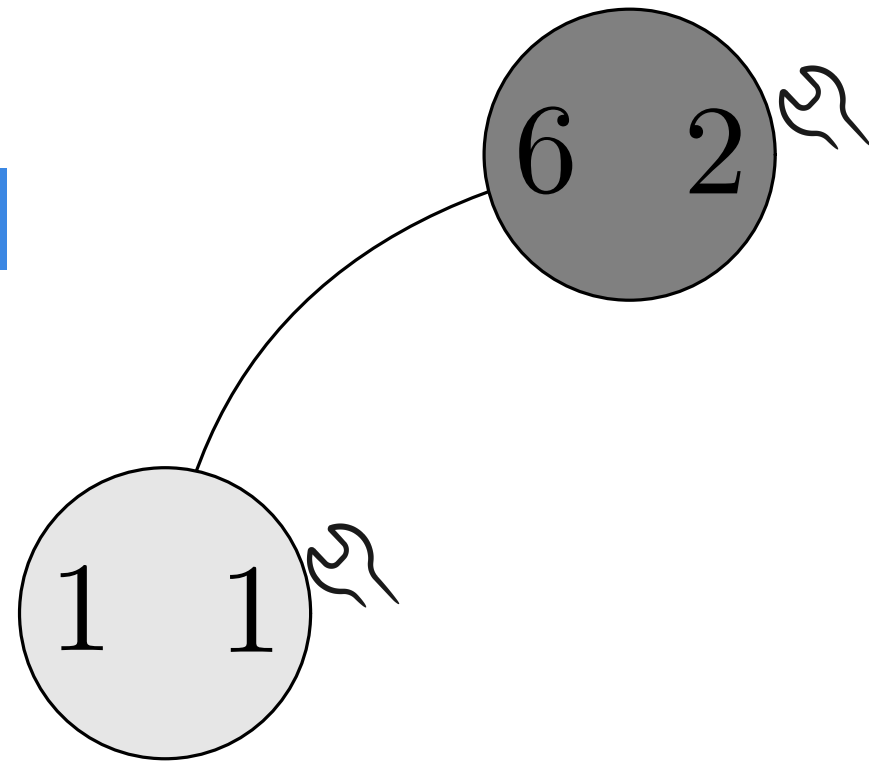
Selection
Expansion
Simulation
Backpropagation



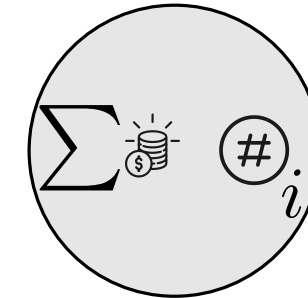
$$\textcircled{\#} = 1$$



Selection
Expansion
Simulation
Backpropagation



$$\textcircled{\#} = 2$$

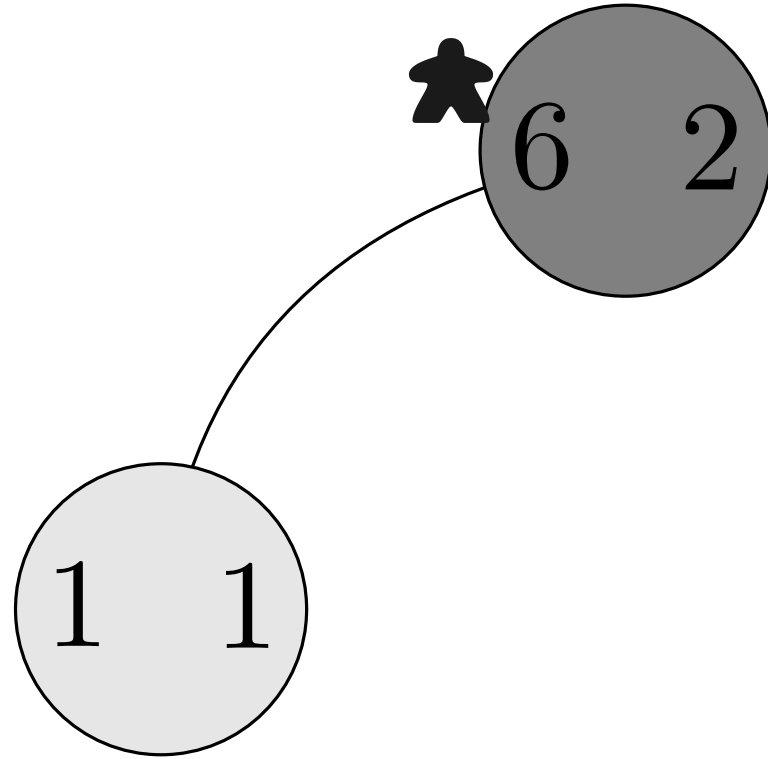


Selection

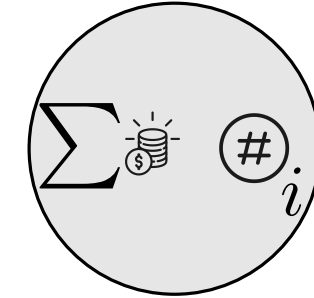
Expansion

Simulation

Backpropagation



$$\textcircled{\#} = 2$$

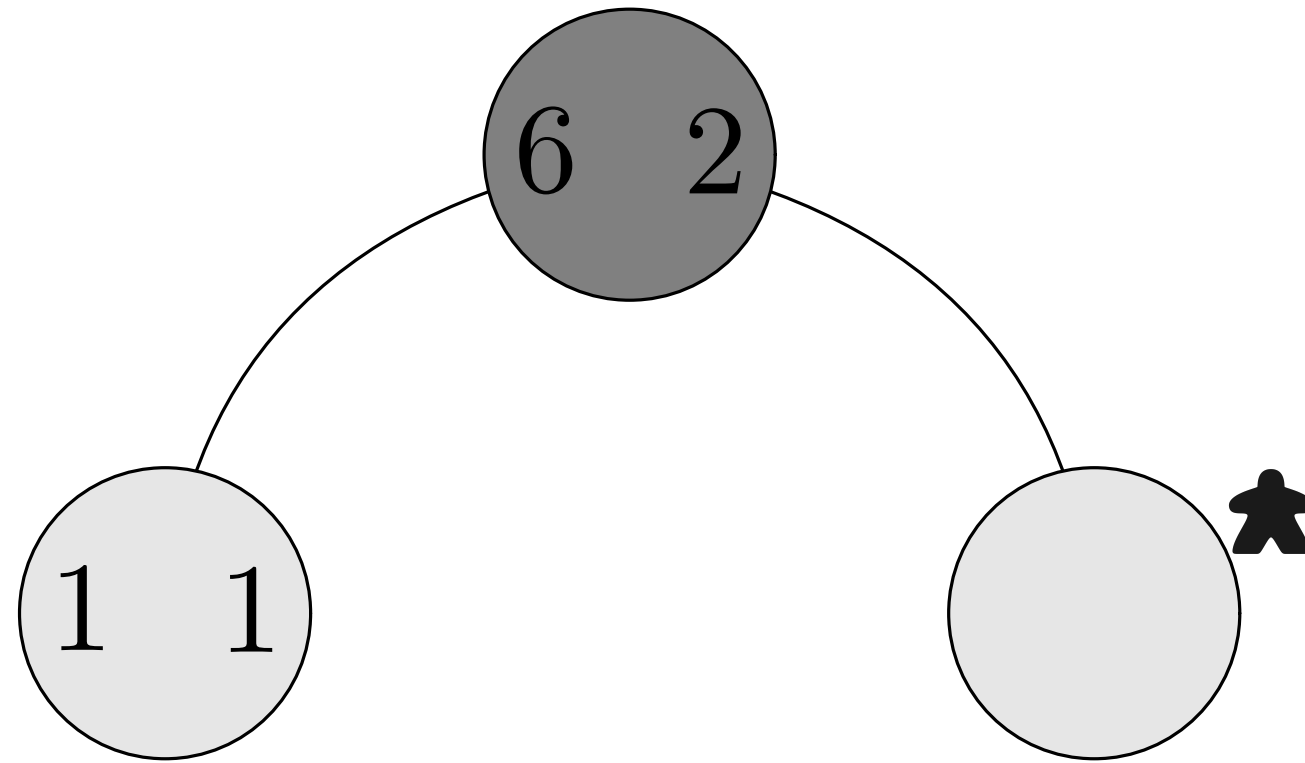


Selection

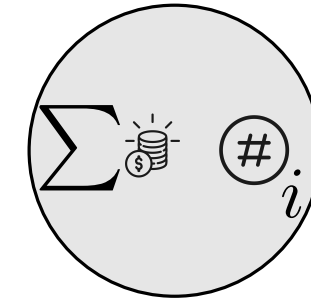
Expansion

Simulation

Backpropagation



$$\textcircled{\#} = 2$$

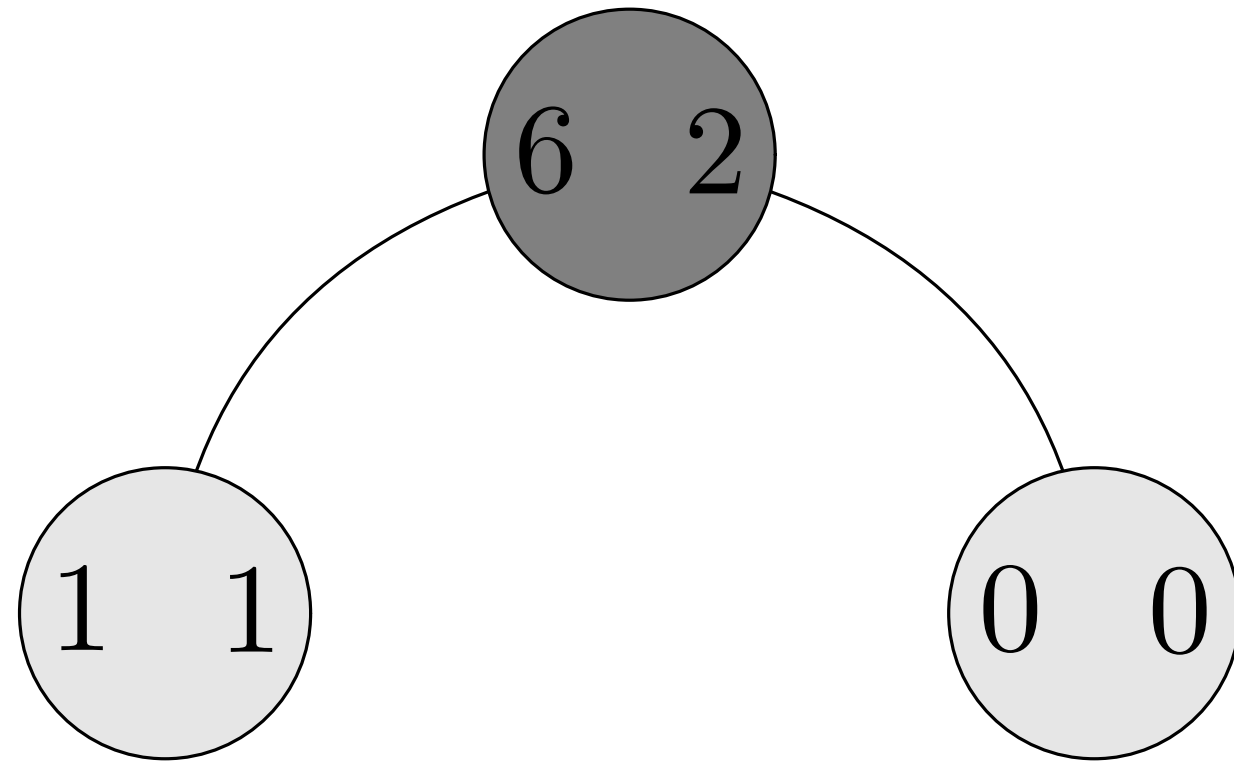


Selection

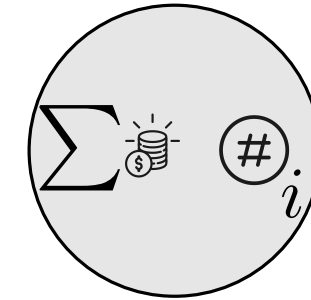
Expansion

Simulation

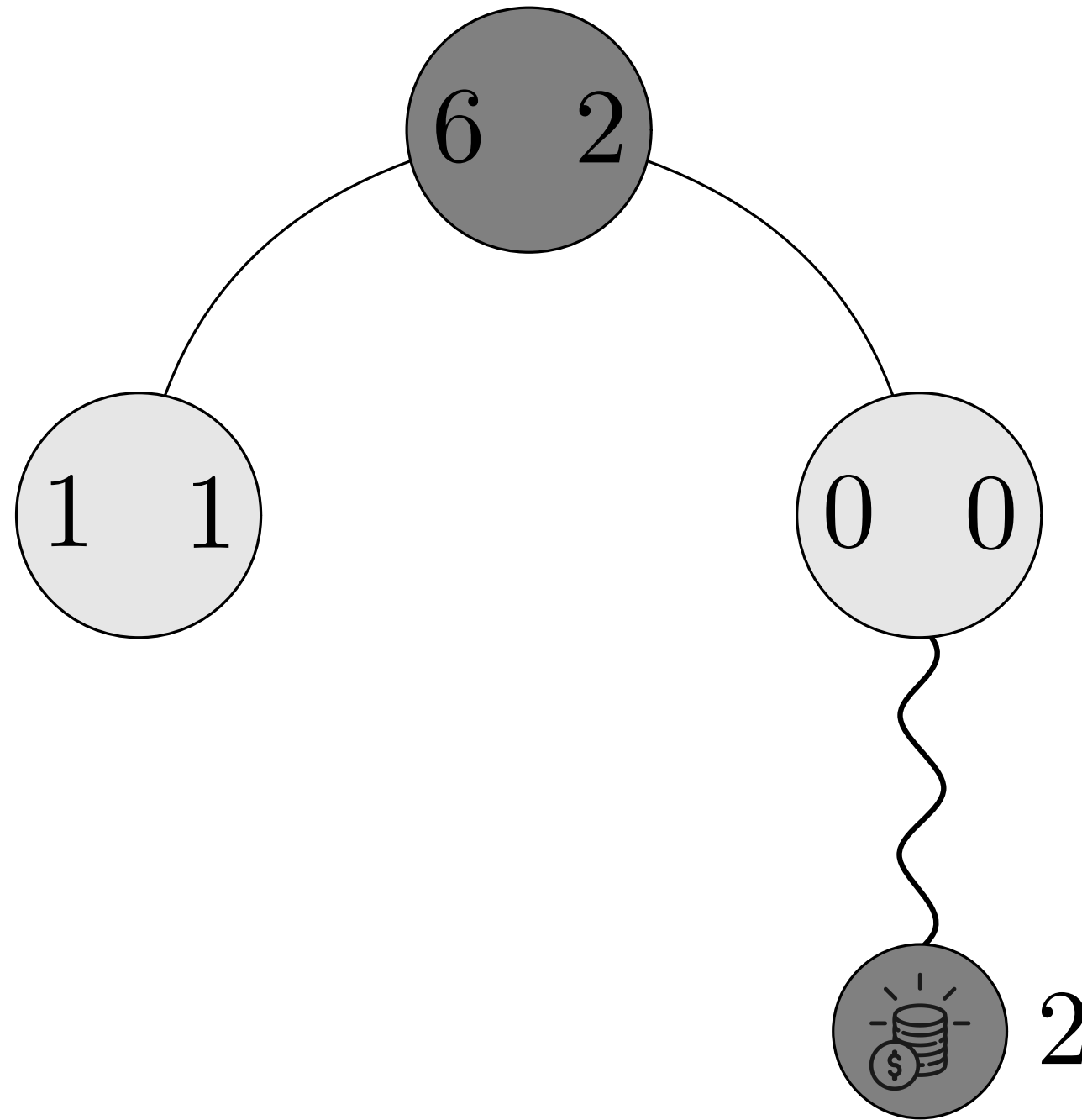
Backpropagation



$$\textcircled{\#} = 2$$



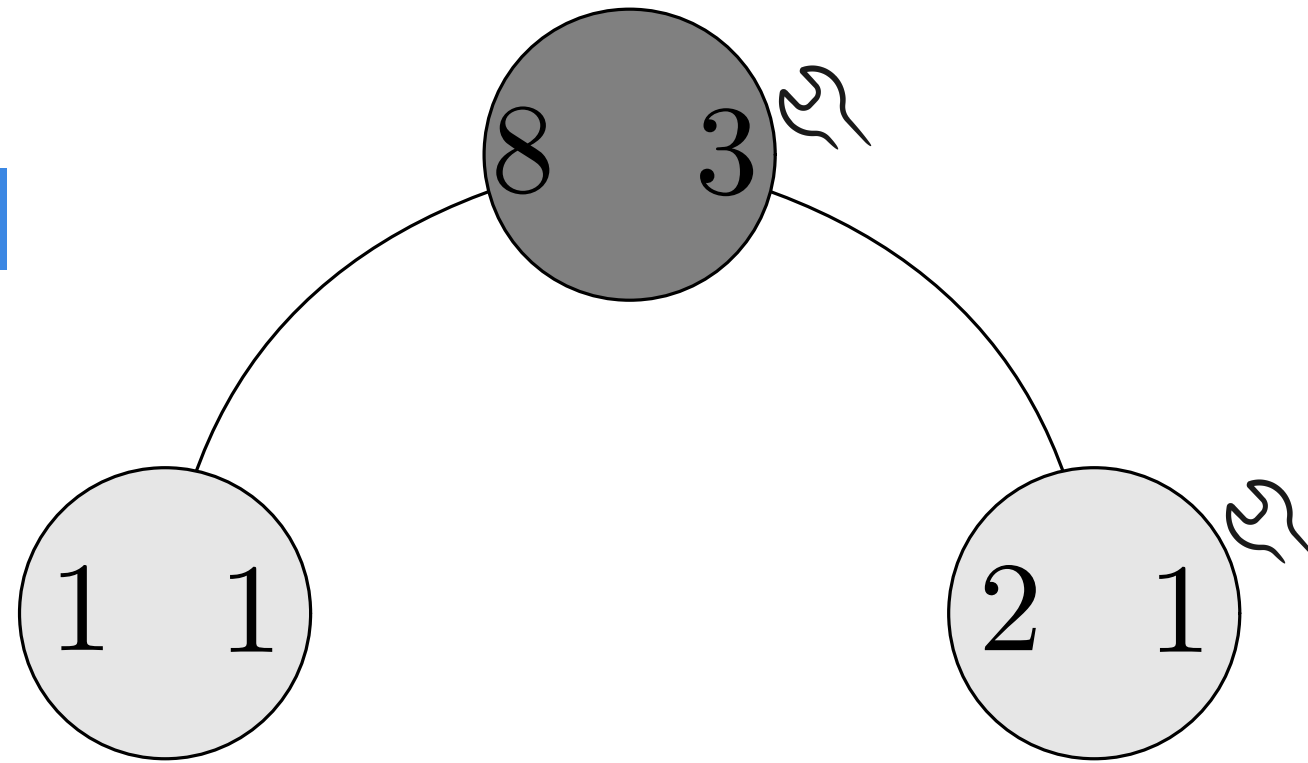
Selection
Expansion
Simulation
Backpropagation



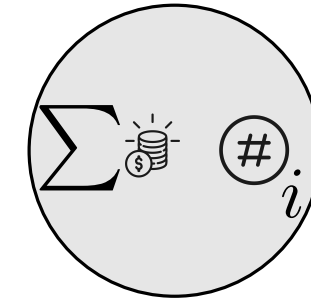
$$\textcircled{\#} = 2$$

A light gray circle containing a summation symbol Σ , a stack of coins icon, and a circled hash symbol $\#_i$.

Selection
Expansion
Simulation
Backpropagation



$$\textcircled{\#} = 3$$

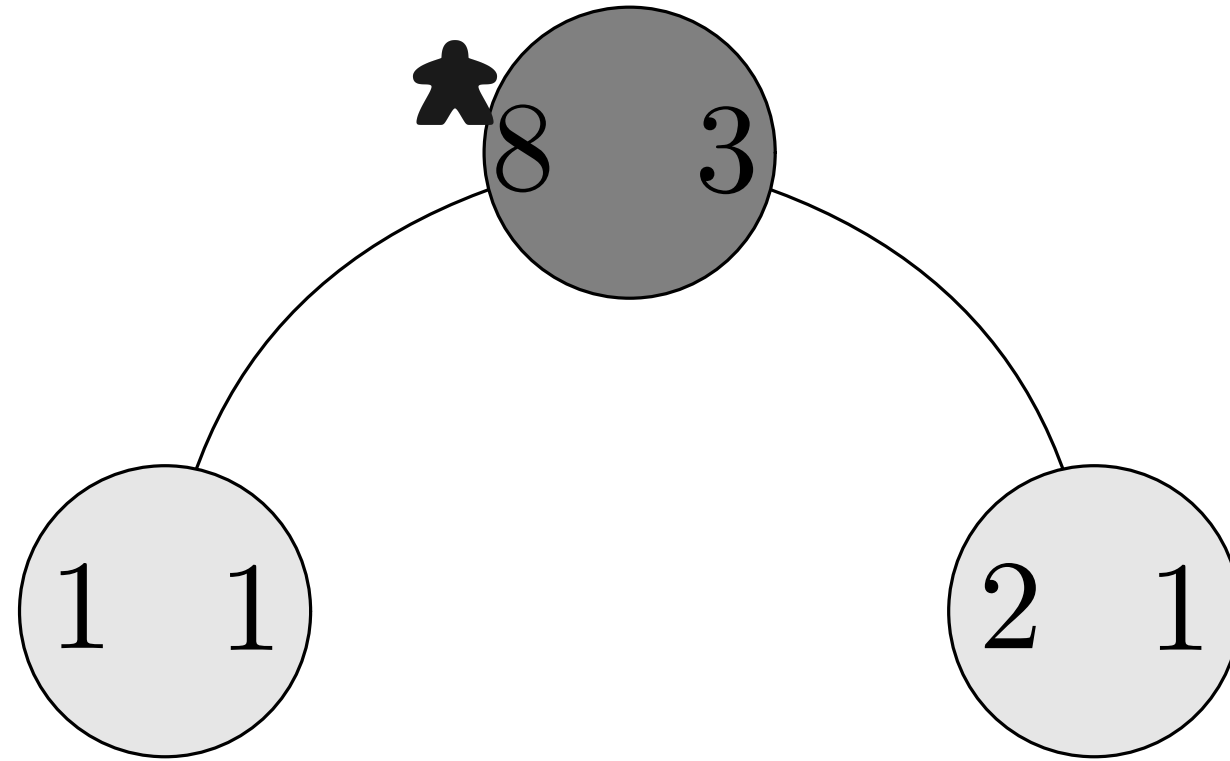


Selection

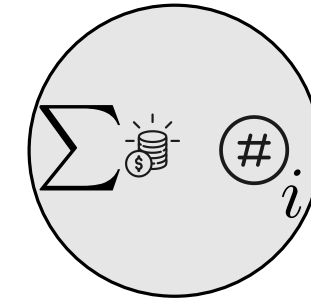
Expansion

Simulation

Backpropagation



$$\textcircled{\#} = 2$$

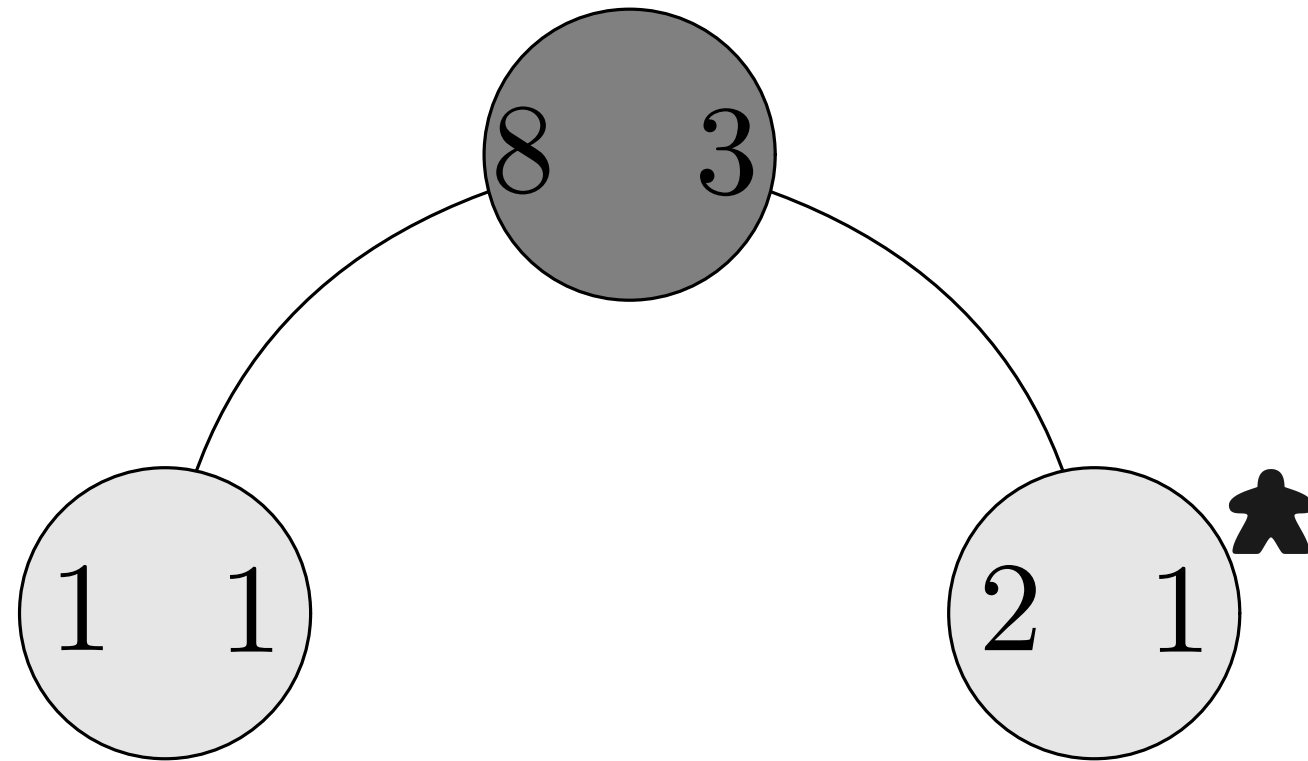


Selection

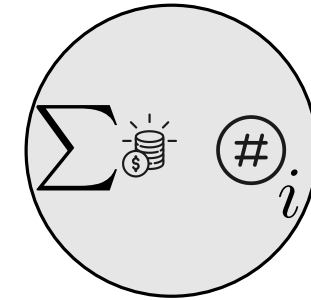
Expansion

Simulation

Backpropagation



$$\textcircled{\#} = 2$$

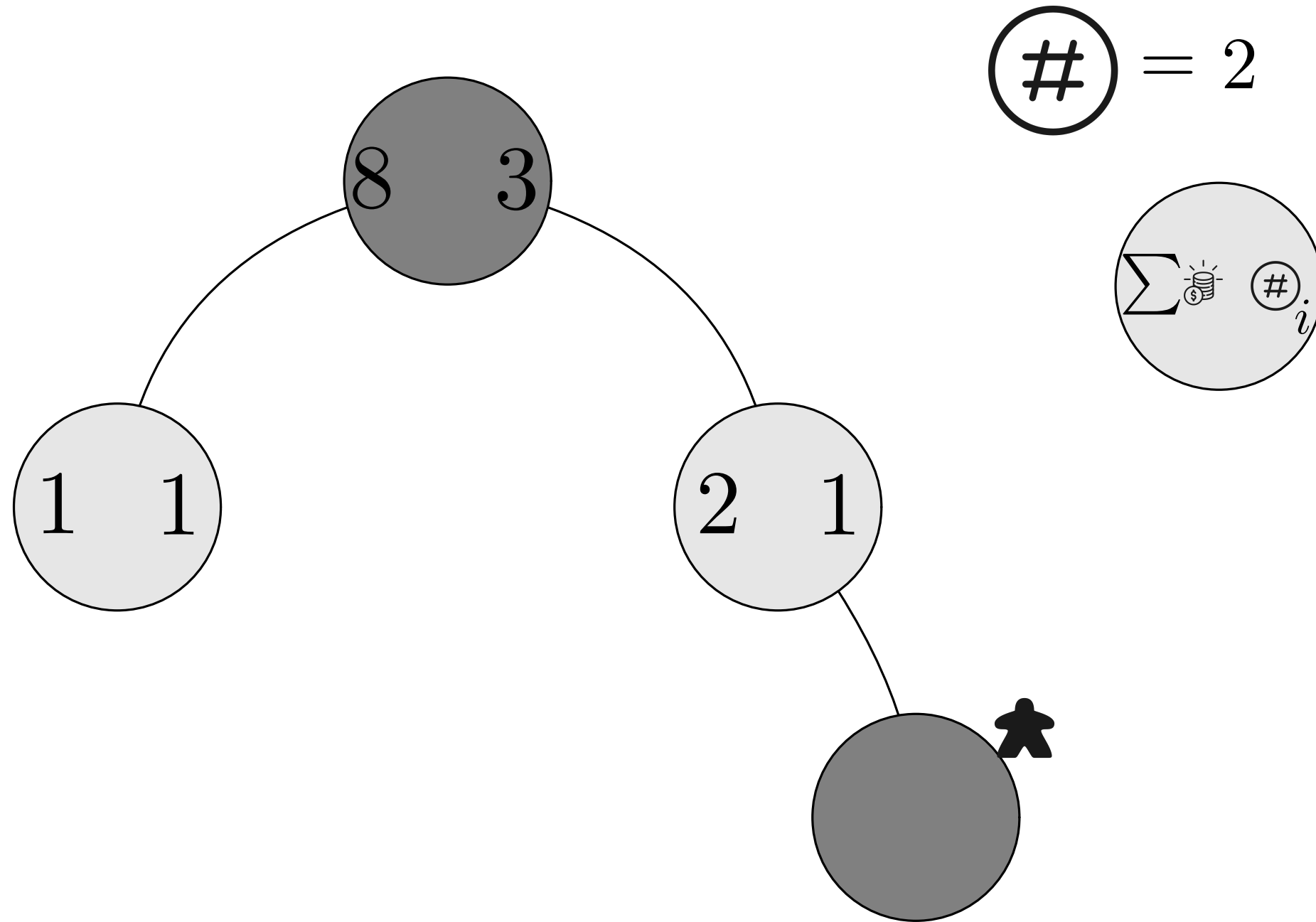


Selection

Expansion

Simulation

Backpropagation

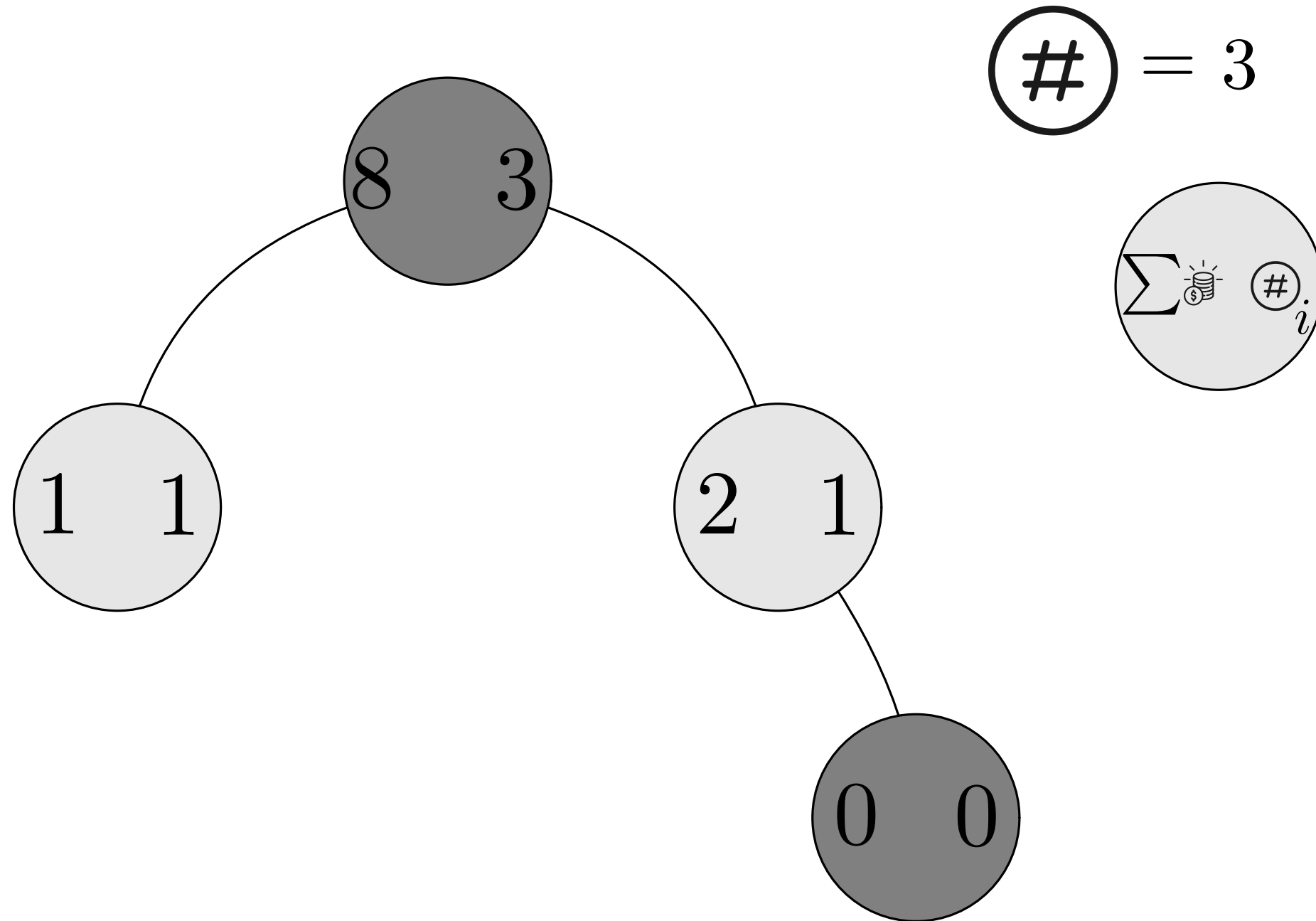


Selection

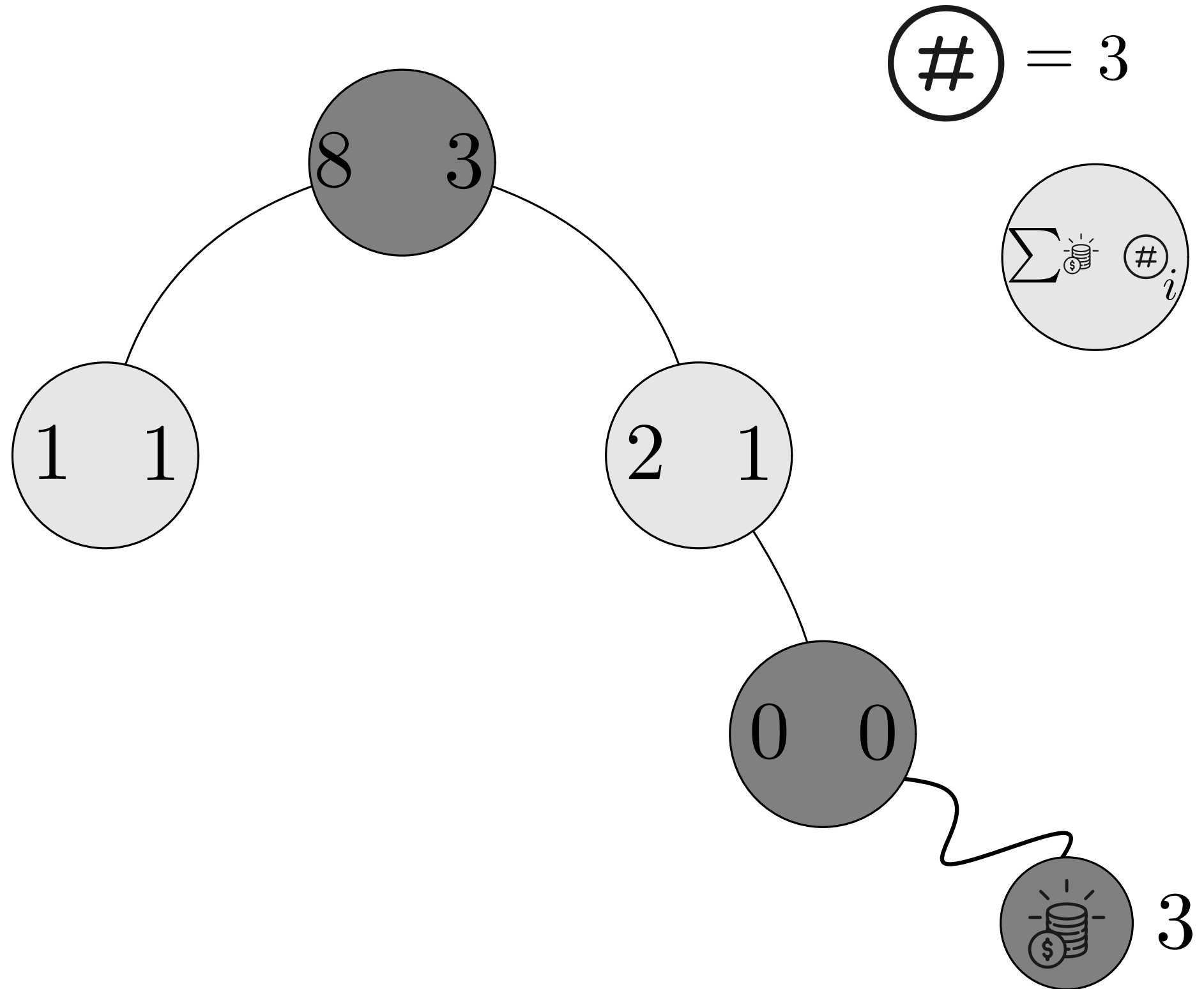
Expansion

Simulation

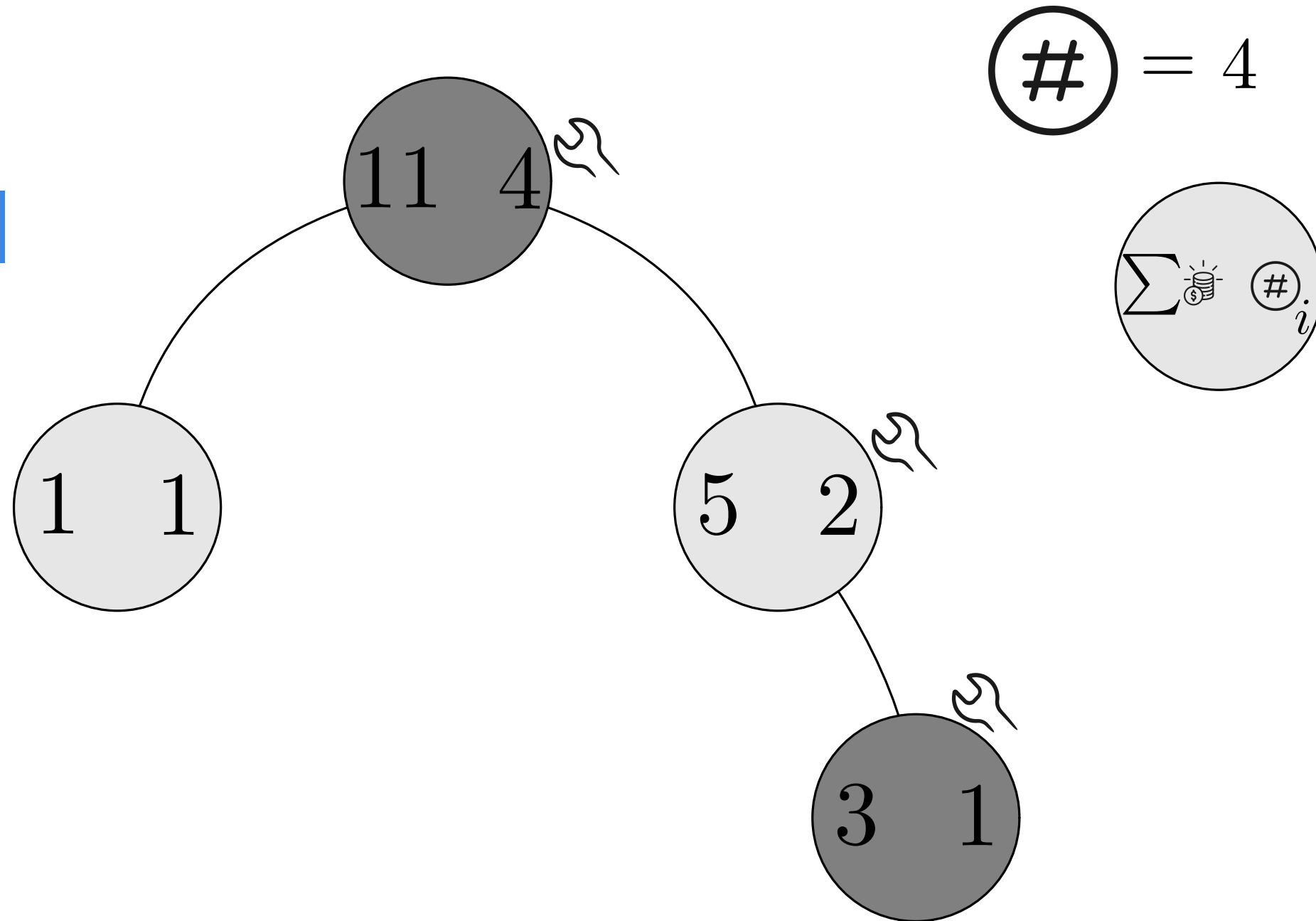
Backpropagation



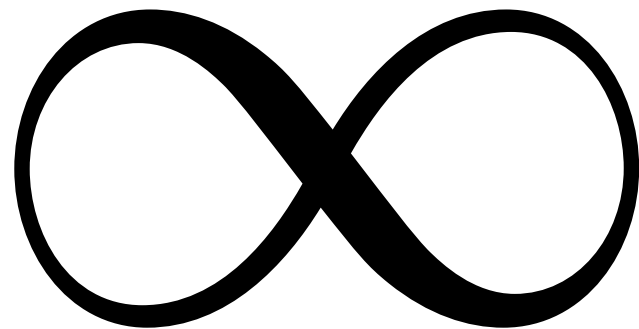
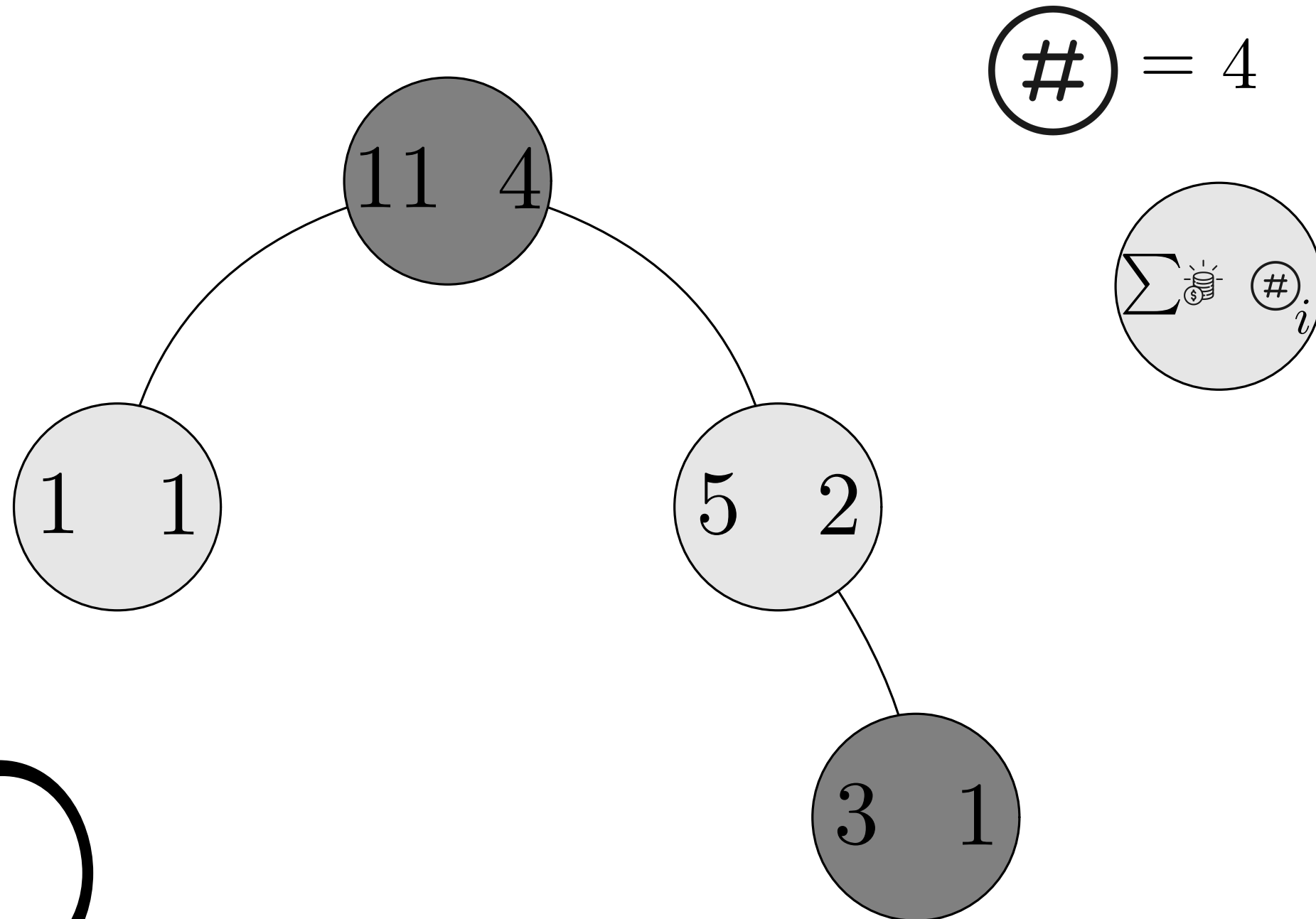
Selection
Expansion
Simulation
Backpropagation

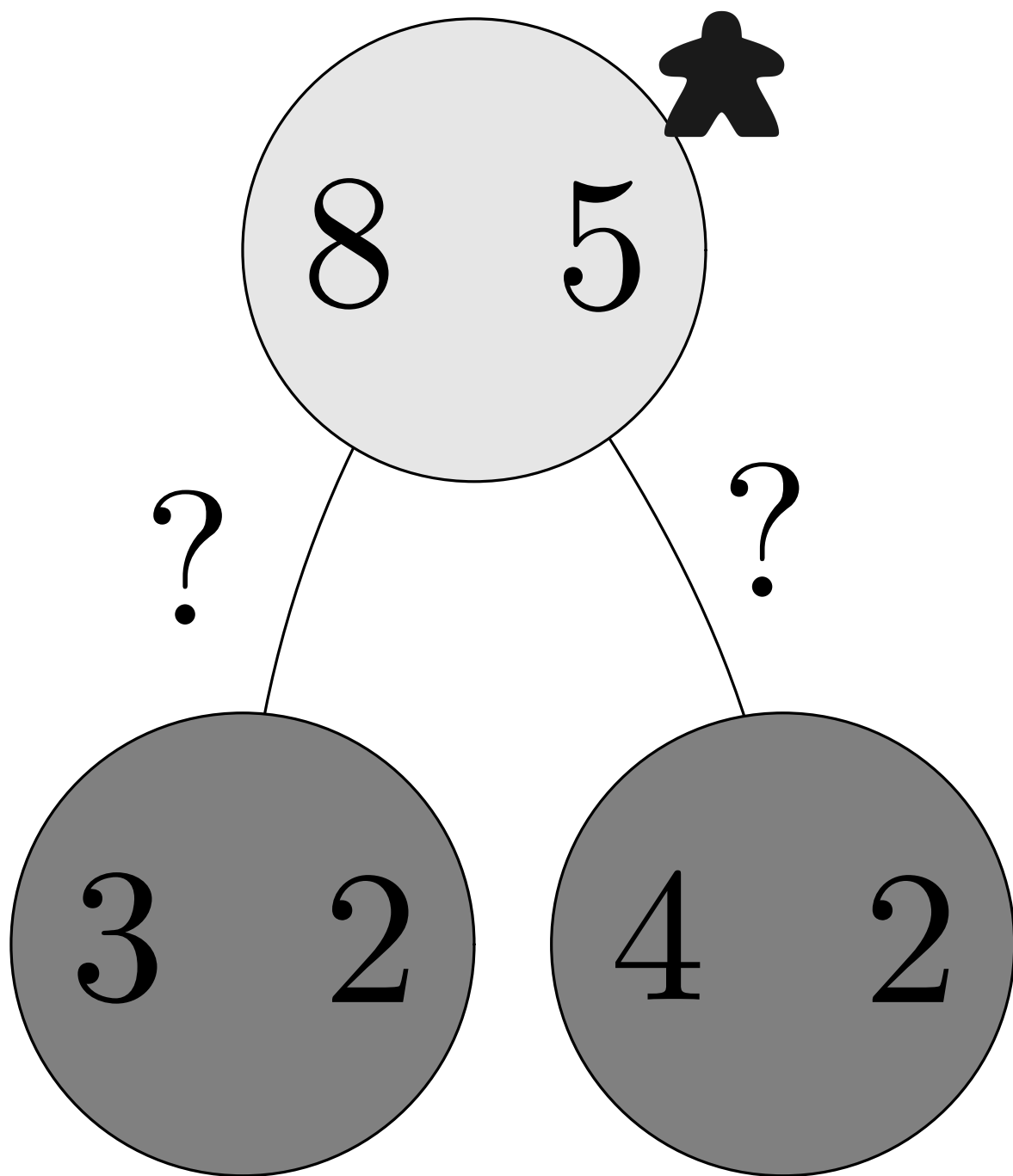


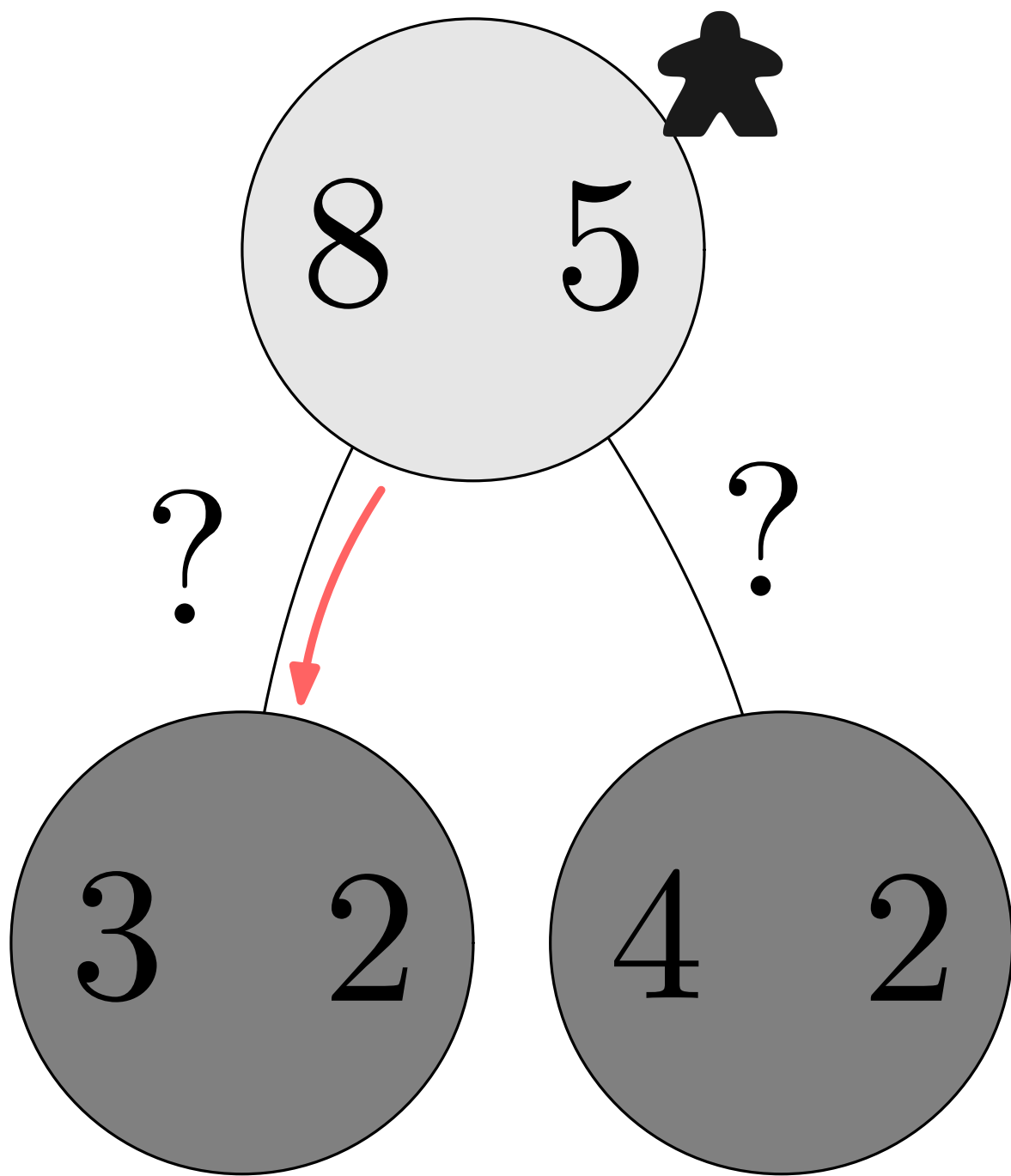
Selection
Expansion
Simulation
Backpropagation

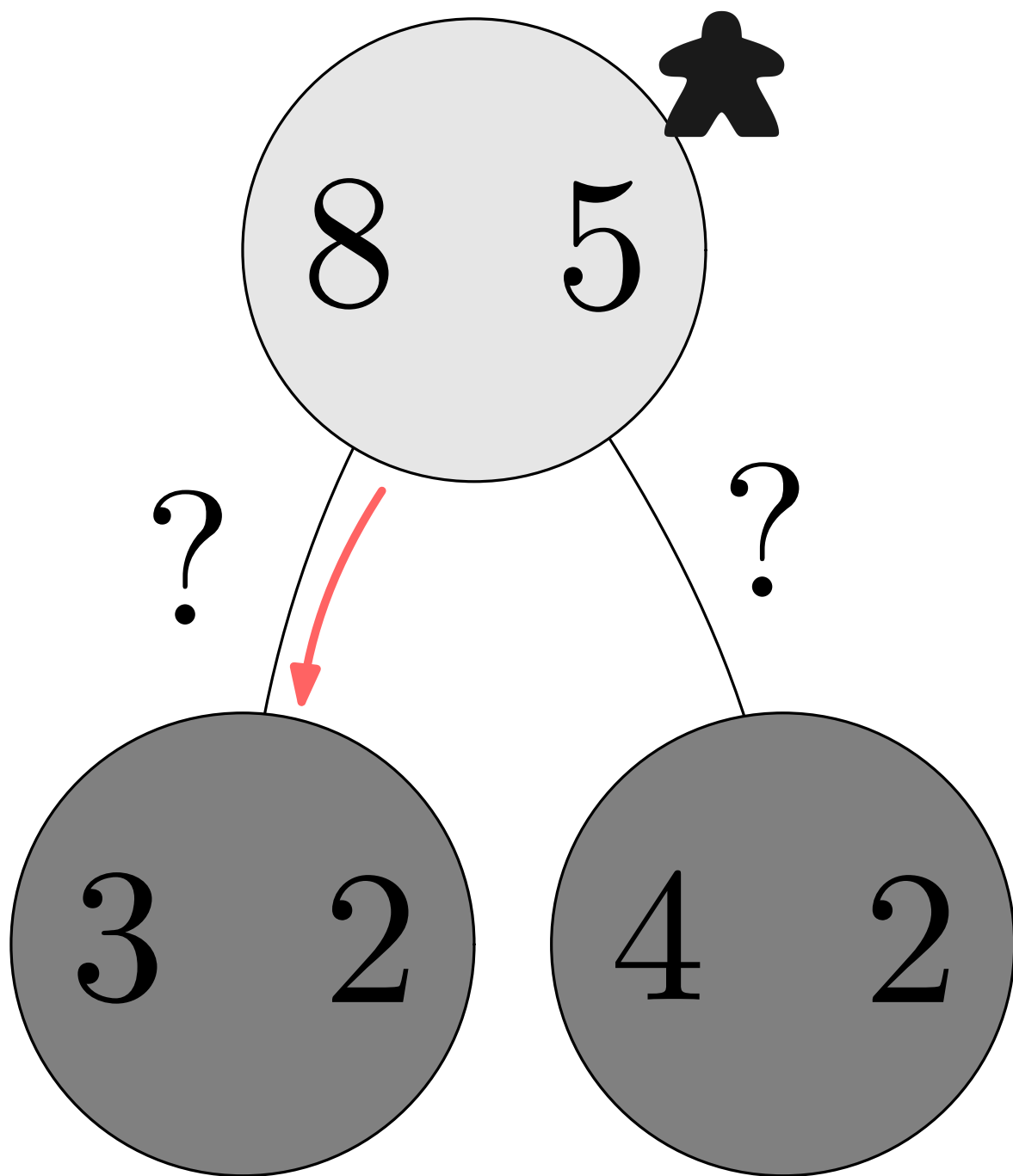


Selection
Expansion
Simulation
Backpropagation



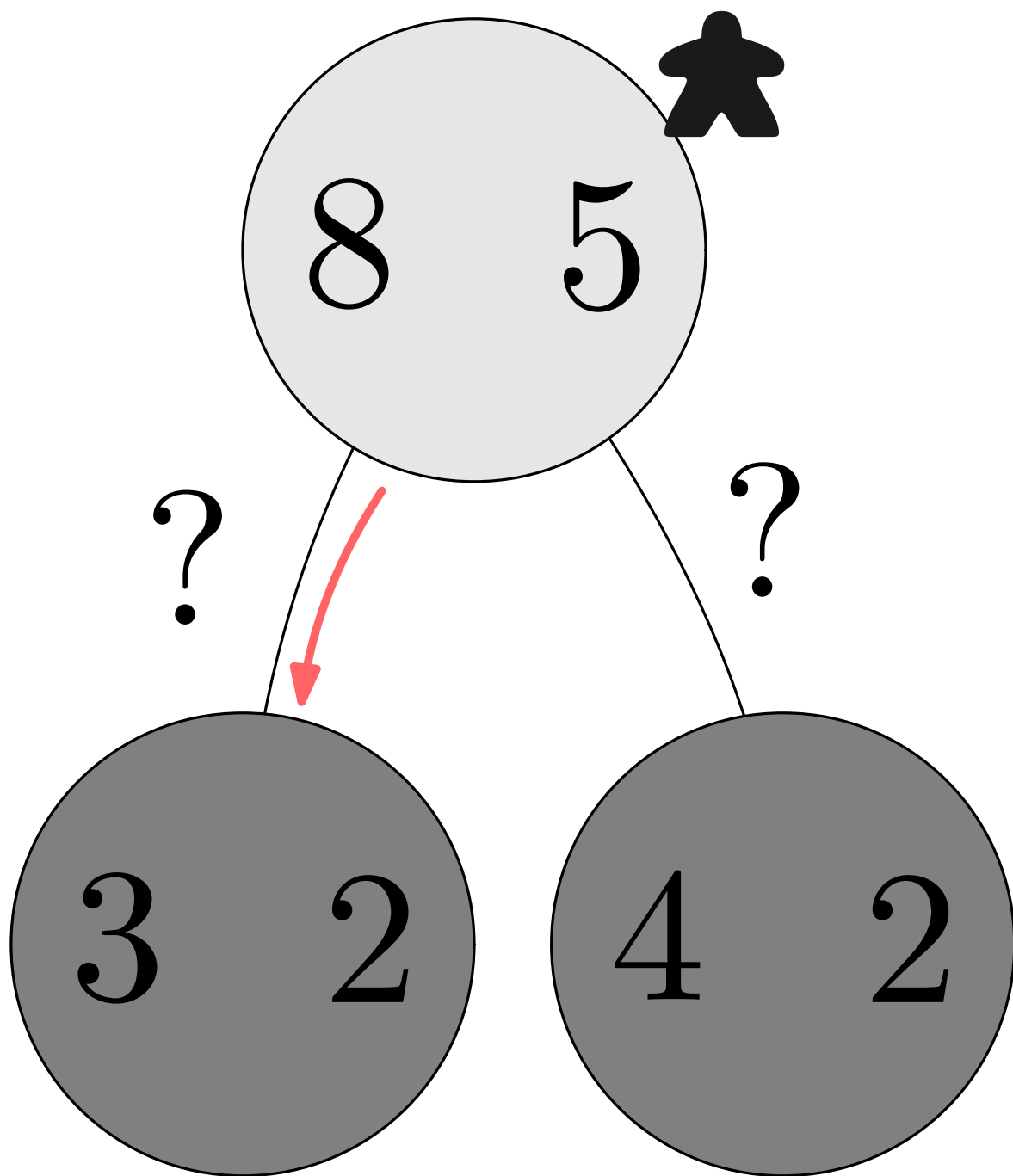






$$\max \frac{\sum \text{💡}_i}{\text{Ⓢ}_i} + c \sqrt{\frac{\log \text{Ⓢ}}{\text{Ⓢ}_i}}$$



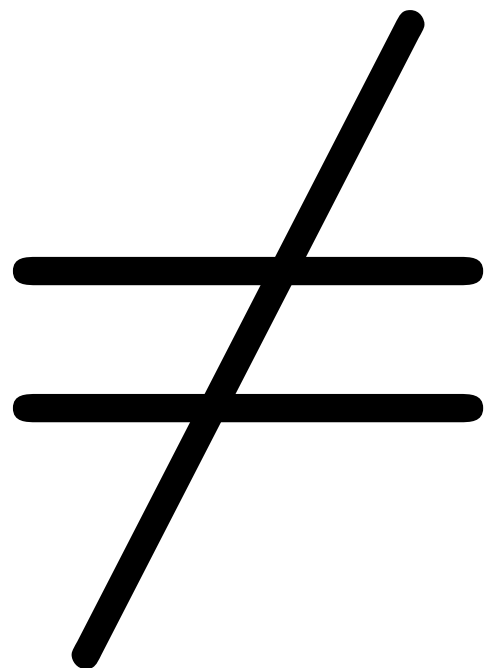


$$\max \frac{\sum \text{💰}_i}{\textcircled{\#}_i} + c \sqrt{\frac{\log \textcircled{\#}}{\textcircled{\#}_i}}$$

↓

$$\max \frac{-\sum \text{💰}_i}{\textcircled{\#}_i} + c \sqrt{\frac{\log \textcircled{\#}}{\textcircled{\#}_i}}$$

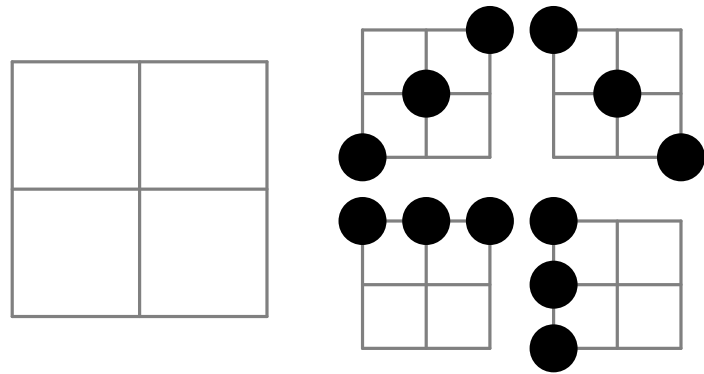






Assignment 5

Tic Tac Toe

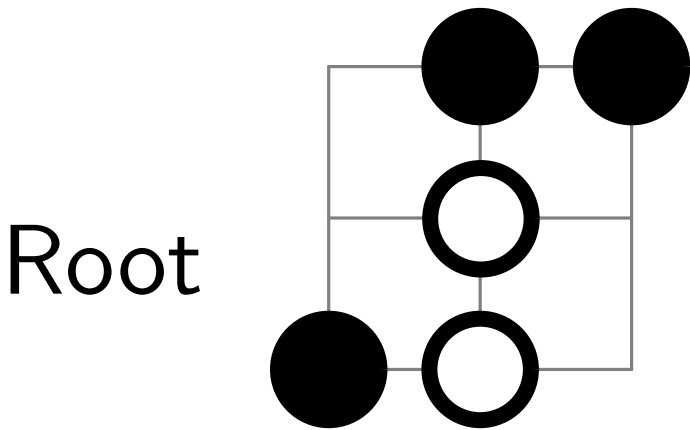


Selection Upper Confidence Bound ($c = 1$)



Simulation ★ as earlier

Post on
Teams



10 steps
final tree

tip: use a computer :p

