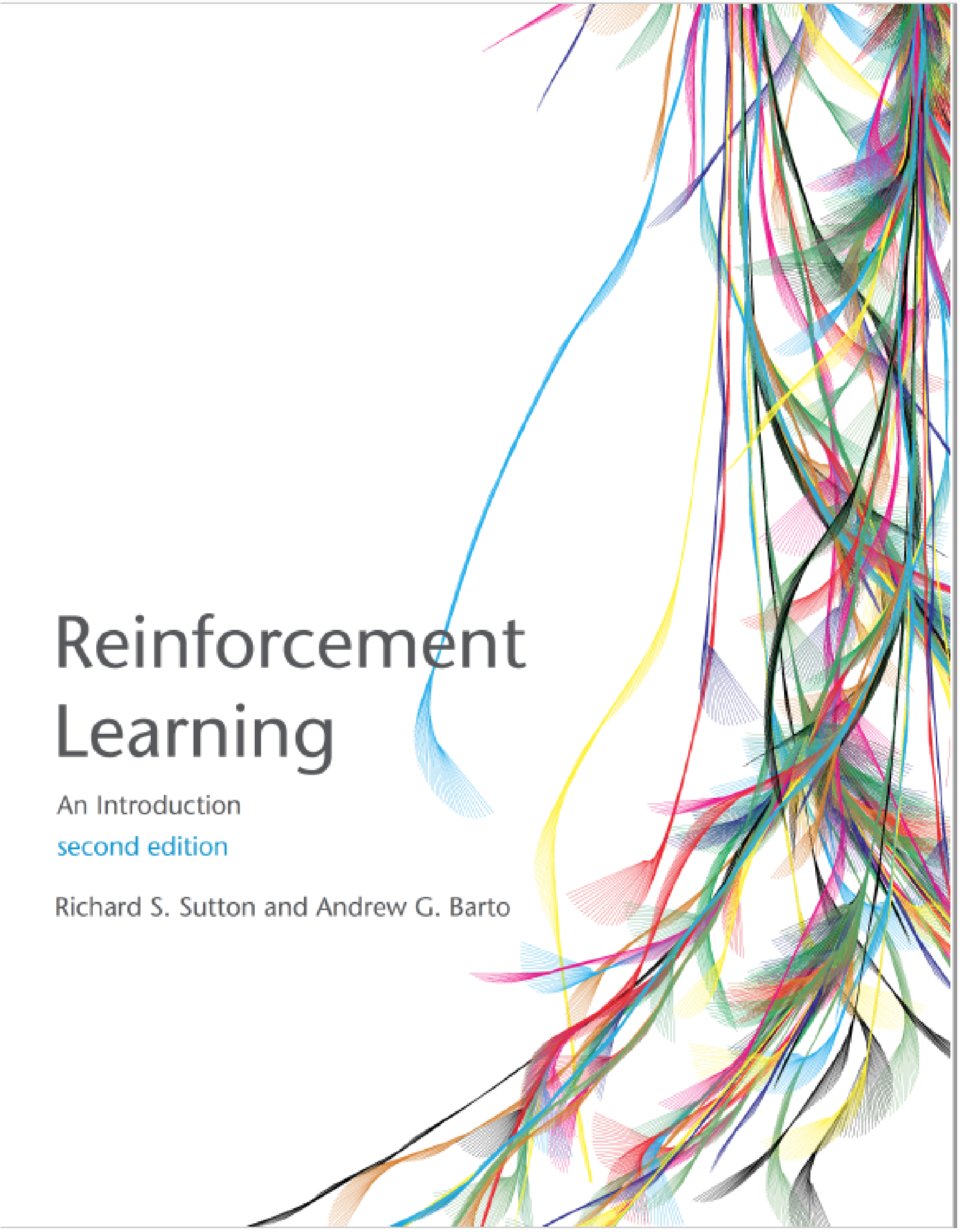
Markov Decision Process

OR: confusion via math!


Reinforcement
Learning
An Introduction
second edition
Richard S. Sutton and Andrew G. Barto

Chapter 3

Markov Chain

no actions, no rewards

Markov Reward Porcess

rewards

Markov Decision Process

actions + rewards

Markov Decision Process

actions + rewards + observations

Markov Chain

Markov Property:
next state only dependent on previous state

Example

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state $=$ order of cards

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dynamics = riffle shuffle

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distributions $= d_0, d_1, d_2, d_3, \ldots$

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$d =$ stationary distribution

THM $\log n$ steps to d.

Markov Chain

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trajectory: $s_1, s_2, s_3, \ldots$

$$\boxed{s_1 \longrightarrow s_2 \longrightarrow s_3 \longrightarrow s_4}$$

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transition matrix: $M \qquad M_{ij} = p(i|j)$

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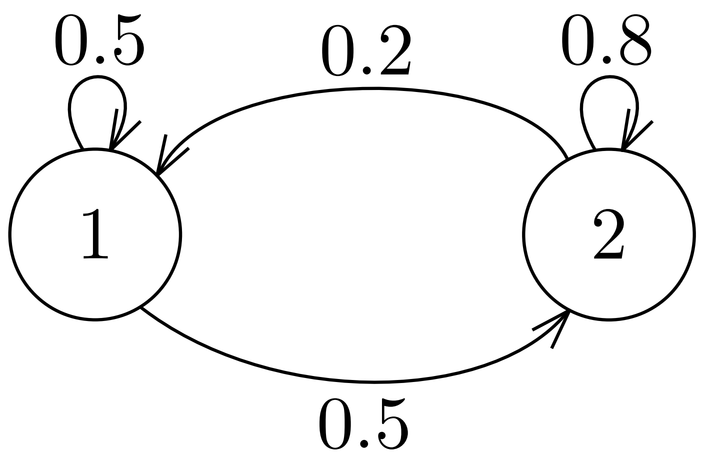
transition: $d_{i+1} = M d_i$

Assignment 3

1 

Use a random number generator and produce a short trajectory.

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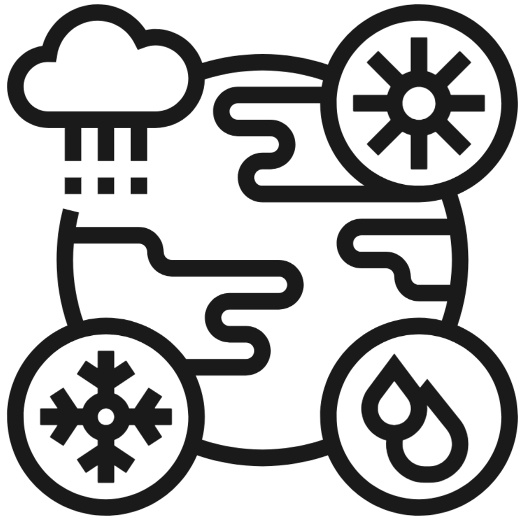
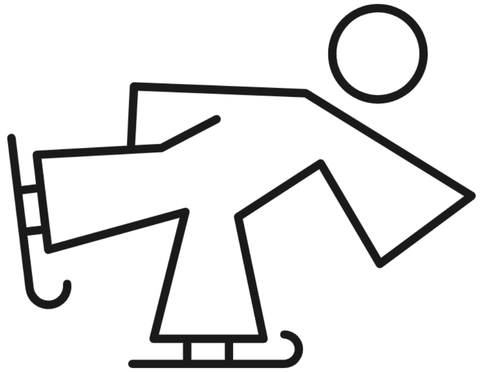
2 Write down the transition matrix.
Compute the distributions d_1, d_2, d_3.
$d_0 = (1, 0)$

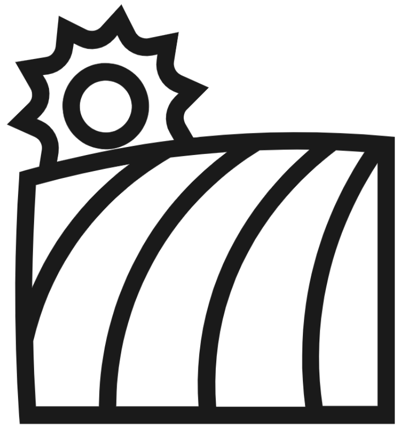
3 Find one example related to nature.

Example Markov Reward Process

The weather in the entire world. The weather tomorrow is a probabilistic function of the weather today. Weather gives rewards in many ways plants grow, freezing temperatures allow us to go skating. :)







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$$
\begin{tikzcd}
s_1 \arrow[r] \arrow[d] & s_2 \arrow[r] \arrow[d] & s_3 \arrow[r] \arrow[d] & s_4 \arrow[d] & \cdots \\
r_1 & r_2 & r_3 & r_4 &
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horizon $=$ steps till terminal state

episodic vs continuing

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return $G(s_t) = r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots$

discount factor $0 < \gamma \leq 1$.

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discount factor $0 < \gamma \leq 1$.

value: $v(s) = \mathbb{E}\, G(s)$

Assignment 4

Give an example of a **continuing** Markov reward process related to the real world.

- Describe the statespace.
- Describe the rewards.
- Give a short example trajectory.
- Describe the dynamics.
- Argue that you satisfy the Markov property.
- What discount factor seems useful?
- Compute the return of your example trajectory.

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Example Markov Decision Process

Quiz Game, 10 levels, random question, win and go to the next level or loose everything, actions: continue or quit.

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states S, rewards R, actions A

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policy $\pi : S \to A \qquad \pi : S \to d(A)$

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value: $v_\pi(s) = \mathbb{E}_\pi \; G(s)$