

*Methods in AI Research*  
*Fragments and Subsymbolic vs symbolic AI*

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*Lecture 13*  
*Tuesday, 17 October 2019*

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*Four lectures on logic and reasoning in AI:*

*Lecture 12. Knowledge-based reasoning*

*Lecture 13. Fragments and Subsymbolic vs symbolic AI*

*Lecture 14. Nonmonotonic reasoning*

*Lecture 15. Common sense reasoning*

1. *Last Tuesday*
2. *Fragments of FOL*
3. *Description Logic*
4. *Symbolic versus Subsymbolic AI*

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1. *Last Tuesday*

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Ex. Let the language contain a binary predicate  $S$ , constant (0-ary function)  $c$ , and a unary function  $f$ .  $M = (D, I)$ , where

- $D = \mathbb{N} = \{0, 1, 2, \dots\}$  and  $I(c) = 0$
- $I(S) = \{(m, n) \mid m < n\}$  ( $I(S)$  is the smaller-than relation)
- $I(f)(n) = 2n$ .
- $M \models \neg S(c, f(c))$
- $M \models \forall x (x \neq c \rightarrow S(x, f(x)))$
- $M \not\models \exists x S(x, f(f(c)))$
- $M \not\models \forall x \forall y (S(x, y) \rightarrow S(y, x))$
- $M \models \forall x (f(x) = x \rightarrow x = c)$ .

*Dfn. A **sentence** is a formula without free variables.*

*A sentence  $\varphi$  **follows** from a set of sentences  $\Gamma$  (written  $\Gamma \models \varphi$ ) if  $\varphi$  is valid in all models in which all formulas in  $\Gamma$  are valid.*

*For finite sets  $\Gamma = \{\varphi_1, \dots, \varphi_n\}$  we write  $\varphi_1, \dots, \varphi_n \models \varphi$  for  $\Gamma \models \varphi$ .*

*Ex. For any formulas  $\varphi$  and  $\psi$ :*

- $\varphi \models \varphi$
- $\varphi \models \varphi \vee \psi$
- $\forall x \varphi(x) \models \neg \exists y \neg \varphi(y)$

*Ex. There are formulas  $\varphi$  and  $\psi$  such that*

- $\varphi \not\models \neg \varphi$
- $\varphi \vee \psi \not\models \varphi \wedge \psi$
- $\exists x \varphi(x) \not\models \exists x \neg \varphi(x)$

*Ex. Let the language contain a binary predicate  $R(x, y)$  and a unary function  $f$ .*

- $\forall x \forall y (R(x, y) \rightarrow R(y, x)) \not\models \forall z R(z, z)$
- $\forall x \exists y R(x, y) \not\models \forall z R(z, z)$
- $\forall x R(x, f(x)) \wedge \forall y (y \neq f(y)) \not\models \forall z R(z, z)$
- $\forall x \forall y (R(x, y) \leftrightarrow y = f(x)) \wedge \forall y (y \neq f(y)) \models \forall z \neg R(z, z).$

*Why not express a KB in predicate logic as a set of formulas  $\Gamma$  and reason using the valid inferences of predicate logic: deduce  $\varphi$  for those formulas  $\varphi$  such that  $\Gamma \models \varphi$ ?*

*Because . . .*

*Thm. (Alan Turing 1936–1937)*

*There exists no Turing Machine (algorithm) that decides for any finite set of formulas  $\Gamma$  and any formula  $\varphi$  whether  $\Gamma \models \varphi$ .*

*David Hilbert's Entscheidungsproblem (decision problem) 1900*

*Find an algorithm that decides whether  $\Gamma \models \varphi$ .*

*Alonzo Church and Alan Turing proved that there exists no such algorithm.*



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*FOL is undecidable.*

*Dfn. A logic is **decidable** if there is an algorithm that for any formula  $\varphi$  decides whether it is valid in the logic or not.*

*FOL is undecidable. What to do?*

*Consider decidable fragments of FOL.*

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## 2. *Fragments of FOL*

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*Consider decidable fragments of FOL, i.e. sets of first-order formulas for which there exists an algorithm that decides whether an arbitrary formula belongs to the set or not.*

*In theorie*

*Focus on decidable fragments of FOL:*

- *the quantifier free fragment;*
- *the fragment without functions and only 1-ary predicates;*
- *⋮*

*In AI practice*

*Focus on logics that correspond to a decidable fragment of FOL:*

- *modal logics;*
- *epistemic logics;*
- *agent logics;*
- *description logics;*
- *⋮*

*Tension:*

*expressive power (what can be represented)  
versus  
efficiency (good computational properties)*

*Extremes:*

*FOL (strong expressive power, bad computational properties)  
versus  
propositional logic (weak expressive power, good computational  
properties, decidable)*

*Description Logics as a well-known example between these extremes. They correspond to a fragment of FOL, although they are formulated in a different language.*

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### 3. *Description Logic*

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*(An Introduction to Description Logic – 2017)*

*(Baader & Horrocks & Lutz & Sattler)*

*Description Logics (DLs) are a family of knowledge representation languages that can be used to represent knowledge of an application domain in a structured and well-understood way.*

*Many applications, for example*

*Ontology languages (OWL),*

*DLs introduced in the 1980s.*

*Knowledge Representation before DLs:*

*Frames, semantic networks, ...*

*Important: DLs are variants of the multi-modal logic  $K$ .*



Main notion: *Concept descriptions*, i.e., expressions that are built from atomic concepts (unary predicates) and atomic roles (binary predicates) using the concept and role constructors provided by the particular DL.

Notation: atomic concepts uppercase; atomic roles lowercase.

Ex. *The concept a student that has a sloppy roommate and only happy friends:*

$\text{Student} \sqcap \exists \text{roommate.Sloppy} \sqcap \forall \text{friends.Happy}$

Translation into FOL:

$\text{Student}(x) \wedge \exists y (\text{roommate}(x,y) \wedge \text{Sloppy}(y)) \wedge \forall z (\text{friends}(x,z) \rightarrow \text{Happy}(z))$

*Dfn. The language of the weak Description Logic  $FL^w$ : Concepts are defined via the following rule*

$$C, D \rightarrow A \mid \neg C \mid C \sqcap D \mid C \sqcup D \mid \forall r.C$$

*$C, D$  are concept names,  $r$  a role name.*

*In terms of FOL:  $\wedge = \sqcap$ ,  $\vee = \sqcup$ ,  $C, D$  range over unary predicates,  $r$  ranges over binary predicates.*

$$\forall r.C \text{ in FOL: } \forall y(r(x, y) \rightarrow C(y))$$

$$\exists r.C \text{ in FOL: } \exists y(r(x, y) \wedge C(y))$$

*An interpretation/model for  $FL^w$  is defined as for FOL under the above translation.*

*New concepts can be defined in terms of previously given ones:*

$$\text{Man} \equiv \text{Person} \sqcap \text{Male} \quad \text{Mother} \equiv \neg \text{Male} \sqcap \exists \text{child}.\text{Person}$$

*Ex. Language has atomic concepts  $C_1, C_2, \dots, C_7$  and a role  $r$ .*

*Model  $M = (\mathcal{D}, \mathcal{I})$  has domain  $\mathcal{D} = \{R_1, R_2, \dots, R_7\}$  consisting of seven restaurants, and  $\mathcal{I}(r)(x, y)$  holds when  $x, y$  are in the same street.*

$$C(x) \equiv \forall r. \neg R_2$$

*$\mathcal{I}(C)$  is the concept consisting of all restaurants in  $\mathcal{D}$  that are not in the same street as  $R_2$ .*

*In FOL:*

$$\varphi(x) = \forall y (r(x, y) \rightarrow \neg R_2(y))$$

*$M, \mu \models \varphi(x)$  iff  $\mu(x)$  is not in the same street as  $R_2$ .*

$FL^w$  equals  $FL^-$  in Chapter 16 (16.1.1) without  $[EXISTS\ 1\ r]$  and

$$[ALL\ r\ d] = \forall r.D \quad [AND\ d_1 \dots d_n] = D_1 \sqcap D_2 \sqcap \dots \sqcap D_n.$$

*At exam:*

*Please use the standard notation, which is the one on the slides.*

*The main purpose of reasoning in DLs is to explicate knowledge that is contained only implicitly in a given concept description.*

*Key ingredient: Subsumption*

*Concept  $D$  **subsumes**  $C$  in model  $M = (\mathcal{D}, \mathcal{I})$  if  $\mathcal{I}(C) \subseteq \mathcal{I}(D)$  holds in  $M$ .*

*$C \sqsubseteq D$  denotes that  $D$  subsumes  $C$ .*

*Inferences are of the form:*

$$\text{if } C_1 \equiv D_1, \dots, C_n \equiv D_n, \text{ then } C \sqsubseteq D,$$

*Meaning that if the left  $n$  expressions hold in a model, so does  $C \sqsubseteq D$ .*

*Ex.*

*if  $\text{Disease} \equiv \text{Symptom}X \sqcap \text{Symptom}Y$ ,  $\text{Symptom}X \equiv \text{Symptom}Y$ ,  
then  $\text{Symptom}Y \sqsubseteq \text{Disease}$ .*

*Like many DLs,  $FL^w$  (and  $FL^-$  in book) have good computational properties: subsumption between concepts is not only decidable, it can be decided in time polynomial in the length of the concept expressions.*

*In Chapter 16 an example is given, description logic FL, for which subsumption is decidable, but deciding it is as hard as deciding whether a given propositional formula is unsatisfiable.*

*Since it is assumed that there is no polynomial algorithm for the latter, it is assumed that there is no polynomial algorithm that decides subsumption in FL.*

*Conclusion: For many applications, DLs have sufficient expressive power and are reasonably efficient.*

*Tension:*

*expressive power (what can be represented)*  
*versus*  
*efficiency (good computational properties)*

*Extremes:*

*FOL (strong expressive power, bad computational properties)*  
*versus*  
*propositional logic (weak expressive power, good computational properties, decidable)*

*DLs as a well-known example between these extremes.*

*FOL is normative/absolute.*

*Not all reasoning is normative, especially not reasoning exhibited by humans. (next two lectures)*

*Other logic paradigms than FOL:*

- *Nonmonotonic logic (next lecture)*
- *Fuzzy logic*
- *Many valued logic*
- *⋮*

*The issue of tension remains:*

*expressive power (what can be represented)*  
*versus*  
*efficiency (good computational properties)*

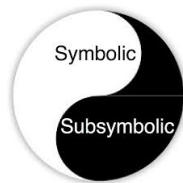


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#### 4. *Symbolic versus Subsymbolic AI*

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*Symbolic AI: concerned with describing and manipulating our knowledge of the world via explicit symbols.  
reasoning, logic, knowledge,...*



*Subsymbolic AI: concerned with obtaining correct responses to input stimuli without looking inside the “black box”.  
networks, learning, probability,...*

# Timeline

Subsymbolic

Symbolic

1940 1950 1960 1970 1980 1990 2000 2010

Henry Lieberman • MIT

*Recent years have seen a strong increase in interest in combining Machine Learning methods with Knowledge Representation methods.*

*(A Boxology of Design Patterns for Hybrid Learning and Reasoning Systems – van Harmelen & ten Teije, 2019)*

*Examples from recent literature:*

*<https://www.theverge.com/2016/10/10/13224930/ai-deep-learning-limitations-drawbacks>*

*<https://www.sciencedirect.com/science/article/pii/S2352154618301943>*

## *Chatbot-based Tourist Recommendations Using Model-based Reasoning*

*(Nica, Tazl & Wotowa 2018)*

<https://www.semanticscholar.org/paper/Chatbot-based-Tourist-Recommendations-Using-Nica-Tazl/a59d177989e33a079348401df726264b731be8ef>

*The reasoning part: find inconsistencies in user requirements and possible solutions.*

*Homework:*

*Exercises 3 & 4 in Section 2.7 of Chapter 2.*

*Exercise 1 in Section 16.7 of Chapter 16.*

*File with additional exercises in Blackboard by Sunday, October 20.*

*Finis*