Lecture 13: Non-monotonic reasoning

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Methods in AI Research

Non-monotonic reasoning

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  - Therefore, I will attend the concert on March 12th
  - Pandemic $\Rightarrow$ I will not attend the concert on March 12th
- *Non-monotonicity – some conclusions are retracted after receiving new information*

What is non-monotonicity?

Monotonicity

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Two restricted variants of monotonicity:

Cautious monotonicity

CM If $KB \mid\!\sim \alpha$ *and* $KB \mid\!\sim \beta$, then $KB \cup \{\beta\} \mid\!\sim \alpha$

Rational monotonicity

RM If $KB \mid\!\sim \alpha$ *and* $KB \not\mid\!\sim \neg\beta$, then $KB \cup \{\beta\} \mid\!\sim \alpha$

Example 1

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α: "Tweety flies"

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β: “Tweety is a penguin”

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Example 2

Nixon Diamond

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- Republicans are not pacifists

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- Is Nixon a pacifist?

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  - false for every object *except* those for which they are known to be true
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  - We assume $\neg Abnormal_1(Tweety)$, if we do not already know that $Abnormal_1(Tweety)$ holds
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- an example of *model preference* logic
  - one model is preferred to another if it has *fewer* abnormal objects

Multiple inheritance

Nixon Diamond

$Republican(Nixon) \wedge Quaker(Nixon)$
$Republican(x) \wedge \neg Abnormal_2(x) \rightarrow \neg Pacifist(x)$
$Quaker(x) \wedge \neg Abnormal_3(x) \rightarrow Pacifist(x)$

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  - m_1: $Abnormal_2(Nixon)$ and $Pacifist(Nixon)$ hold and
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- a variant – *prioritized circumscription*
  - giving priority to religious beliefs $\Rightarrow$ $Abnormal_3$ minimized
  - conclusion: $Pacifist(Nixon)$

Default logic

Default logic

- Default rules:

$$P : J_1, \ldots, J_n / C$$

  - P – prerequisite, C – conclusion, J_i – justifications
  - if P, then infer C, unless some J_i can be proven false
- Example:

$$Bird(x) : Flies(x) / Flies(x)$$

  - $Bird(x)$ is true and $Flies(x)$ is consistent with what is inferred, then $Flies(x)$ may be concluded by default

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- *extension* – a maximal set of consequences of the theory
  - if we apply one rule first, we extend KB in a way that another rule can be blocked
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What follows from a default theory:

- *Sceptical* consequences – formulas that are contained in every extension
- *Credulous* consequences – formulas that are contained in some extension

Argumentation-based approach

Argumentation-based approach to NMR

1. Formalize the problem: build a knowledge base KB and a rule base R
2. Construct arguments and attacks (ASPIC+)
3. Calculate extensions (choose a semantics S)
4. Derive conclusions of *justified* arguments

Argumentation-based approach to NMR

1. Formalize the problem: build a knowledge base KB and a rule base R
2. Construct arguments and attacks (ASPIC+)
3. Calculate extensions (choose a semantics S)
4. Derive conclusions of *justified* arguments
   - α is sceptically $S-$acceptable if and only if all $S-$extensions contain an argument with conclusion α
   - α is credulously $S-$acceptable if and only if at least one $S-$extension contains an argument with conclusion α

Tweety & argumentation

Rules:

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- Penguins don't fly

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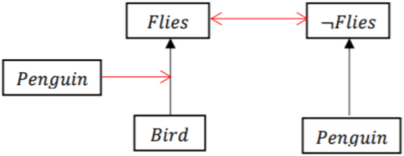

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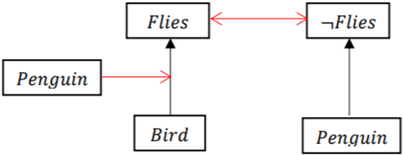

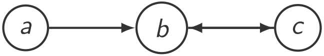

- One extension: $\{a, c\}$
- Conclusion of c accepted $\Rightarrow$ Tweety does not fly!

System P

Idea: preferences over models

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For example:

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Then:

- $p \mid\sim r$
- $p \wedge q \not\mid\sim r$

System P – The core properties of non-monotonic reasoning

REF $\alpha \mid\!\sim \alpha$ [reflexivity]

LLE $$\frac{\models \alpha \leftrightarrow \beta \quad \alpha \mid\!\sim \gamma}{\beta \mid\!\sim \gamma}$$ [left logical equivalence]

RW $$\frac{\models \alpha \rightarrow \beta \quad \gamma \mid\!\sim \alpha}{\gamma \mid\!\sim \beta}$$ [right weakening]

CUT $$\frac{\alpha \land \beta \mid\!\sim \gamma \quad \alpha \mid\!\sim \beta}{\alpha \mid\!\sim \gamma}$$

OR $$\frac{\alpha \mid\!\sim \gamma \quad \beta \mid\!\sim \gamma}{\alpha \lor \beta \mid\!\sim \gamma}$$

CM $$\frac{\alpha \mid\!\sim \beta \quad \alpha \mid\!\sim \gamma}{\alpha \land \beta \mid\!\sim \gamma}$$ [cautious monotonicity]

Characterization result

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Theorem (KLM '90)

A consequence relation $\mid\sim$ satisfies System P iff $\mid\sim = \mid\sim_S$ *for some preferential structure S.*

Rational consequence relations

- *Rational relation* = System P + *RM*

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- Characterized by *ranked structures*= preferential structures + modularity