Exercises Reasoning Part MAIR

1. Consider the following statement.

   (a) For all sentences α and β: if $\models \alpha \vee \beta$, then $\models \alpha$ or $\models \beta$.

   (b) For all sentences α and β: if $\neg\alpha \models \beta$, then $\models \alpha \vee \beta$.

   Are these statements true? (Justify your answer.)

2. Write a sentence of FOL whose models are:

   (a) all first-order models with *at most* two elements in the domain

   (b) all first-order models with *at least* two elements in the domain

   (c) all first-order models with *exactly* two elements in the domain.

3. Using unary predicate symbols $Student$, $Green$ and $Bicycle$ ($Student(x)$ stands for "x is a student", $Green(x)$ for "x is green" and $Bicycle(x)$ stands for "x is a bicycle"), and a binary predicate symbol Has ($Has(x,y)$ stands for "x has y" - i.e., "x owns y"), translate the following sentences from English into first order logic.

   (a) Every bicycle is green.

   (b) Every student has a green bicycle.

4. Express the following sentences in first order logic using predicate symbols $Student$ (unary, $Student(a)$ means a is a student), $Tutor$ (binary, $Tutor(b,a)$ means b is a's tutor), $Lazy$ (unary), $Happy$ (unary):

   (a) Every student has a tutor.

   (b) There are no lazy students.

   (c) No student has two different tutors.

   (d) If a student is lazy, then the student's tutor is not happy.

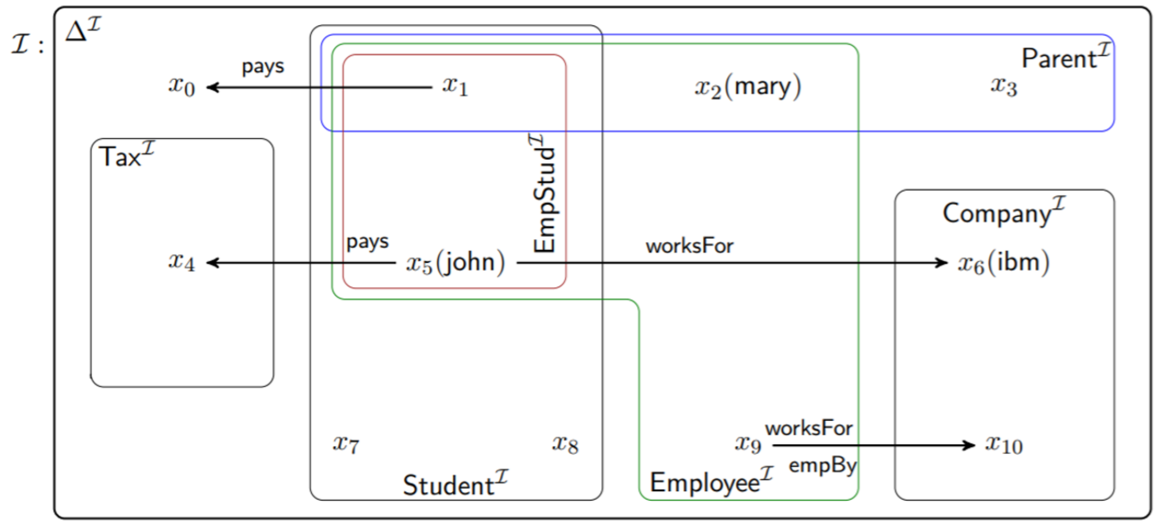
   (e) There is a tutor all of whose tutees are lazy.

5. Consider an interpretation where the domain consists of 4 suitcases a, b, c, d where a and b are large and c and d are small. In other words, the predicate symbol $Large$ is interpreted as the set $\{a, b\}$ and $Small$ is interpreted as the set $\{c, d\}$. There is also a predicate symbol $FitsIn$ that is interpreted as the set of pairs $\{(c,a),(c,b),(d,a),(d,b)\}$ (small suitcases fit inside large ones). Are the following first order sentences true or false in this interpretation (and why):

   (a) $\forall x \forall y (Large(x) \wedge Small(y) \rightarrow FitsIn(x,y))$

   (b) $\forall x \forall y (Large(x) \wedge Small(y) \rightarrow FitsIn(y,x))$

   (c) $\exists x \forall y FitsIn(x,y)$



(d) $\forall x \exists y \neg FitsIn(x, y)$

(e) $\forall x \forall y (\neg FitsIn(x, y) \vee \neg FitsIn(y, x))$

6. Which class of human beings is described by the following concept?

$$\neg \forall child.Male \sqcap \neg \forall child.Female$$

7. Translate the concept in the previous exercise in FOL.

8. Translate the following FOL sentence in DL and describe the concept in natural language.

$$\exists y(neighbour(x, y) \wedge Old(y)) \wedge \forall y(neighbour(x, y) \rightarrow Friendly(y)).$$

9. Translate the following FOL sentence in DL and describe the concept in natural language.

$$\exists y\big(neighbour(x, y) \wedge \forall z(neighbour(y, z) \rightarrow \neg Friendly(z))\big).$$

10. For the interpretation $\mathcal{I}$ from the picture, determine:

(a) $(\neg\mathsf{Employee})^{\mathcal{I}}$

(b) $(\neg\mathsf{EmpStud} \sqcap \forall \mathsf{empBy}.\mathsf{Company})^{\mathcal{I}}$

(c) $(\mathsf{Student} \sqcap \forall \mathsf{pays}.\bot)^{\mathcal{I}}$

11. For the interpretation $\mathcal{I}$ from the picture, determine whether:

    (a) $\mathcal{I} \Vdash \exists \mathsf{worksFor}.\top \sqsubseteq \mathsf{Employee}$

    (b) $\mathcal{I} \Vdash \mathsf{Employee} \sqsubseteq \exists \mathsf{worksFor}.\top$

    (c) $\mathcal{I} \Vdash \exists \mathsf{empBy}.\top \sqsubseteq \exists \mathsf{worksFor}.\mathsf{Company}$

12. For the interpretation $\mathcal{I}$ from the picture, determine whether:

    (a) $\mathcal{I} \Vdash (\mathsf{mary}, \mathsf{ibm}) : \mathsf{empBy}$

    (b) $\mathcal{I} \Vdash (\mathsf{ibm}, \mathsf{john}) : \mathsf{worksFor}$

    (c) $\mathcal{I} \Vdash \mathsf{john} : \forall \mathsf{empBy}.\mathsf{Company}$

13. Explain for each of the following FOL sentences why they are not expressible in DL (at least not in the standard way).

    (a) $\exists xyz P(x, y, z)$, where P is a 3-ary predicate.

    (b) $\forall x \forall y (P(x, y) \rightarrow P(y, x))$.

    (c) $\forall x P(x, x)$.

14. What does it mean for a logic to be decidable?

15. Is propositional logic decidable? And FOL?

16. Is any extension of a decidable logic decidable?