

Lecture 11: Knowledge Based Systems

Dragan Doder

Methods in AI Research

Before we start

Next four lectures: Reasoning

- ▶ Knowledge based reasoning
- ▶ Description logic
- ▶ Non-monotonic reasoning – *bonus lecture*
- ▶ Formal argumentation

Outline of this lecture

Knowledge representation and reasoning

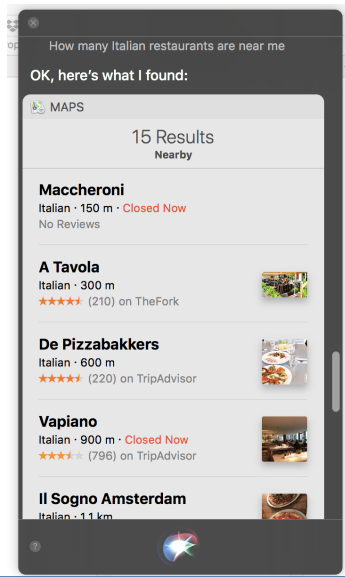
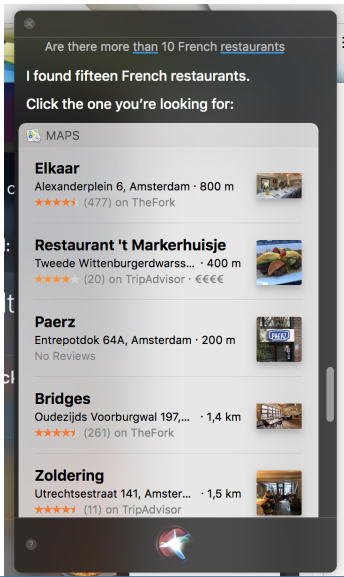
Recap – Propositional logic

Some fundamental logical notions

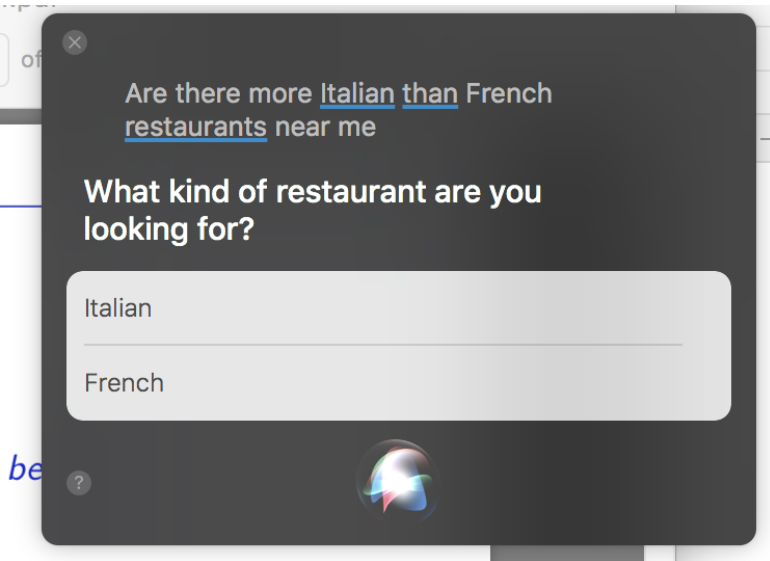
First-Order logic

Knowledge representation and reasoning

Conversational agents - where are we now?



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User: *My father likes Football.*

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User: *Marc's favorite football team is Ajax.*

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(On May 22nd)

CA: Hey, Marc's birthday is in a month and you wanted to buy him football tickets.

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(On May 22nd)

CA: Hey, Marc's birthday is in a month and you wanted to buy him football tickets.

CA: How about I order today the tickets for Ajax–Utrecht happening on July 3rd?

Conversational agents - What do we need for that?

Various types of logical and commonsense reasoning:

- ▶ Coreference resolution
- ▶ Reasoning about superlatives
- ▶ Reasoning about family relations
- ▶ Time reasoning
- ▶

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Major components of AI:

knowledge, reasoning, language understanding, learning.

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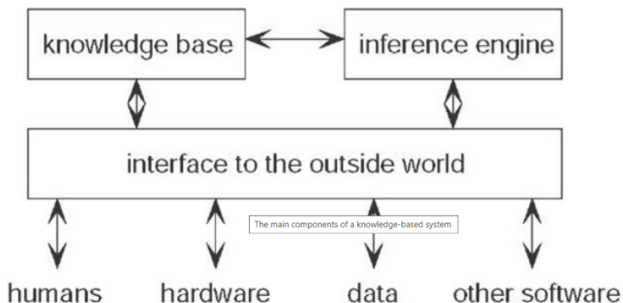
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Lectures 11–14

Knowledge based systems



Knowledge base: represents facts about the world

Inference engine: applies logical rules to KB to deduce new facts

Knowledge Representation and Reasoning

Knowledge representation

- ▶ use **formal symbols** to represent knowledge of an agent
- ▶ not all the knowledge is explicitly represented

Reasoning

- ▶ bridges the gap between what is represented and what is known
- ▶ **formal manipulation** of the symbols

The role of logic

- ▶ Logic is study of **entailment** relations
 - ▶ languages, truth conditions, inference rules

MYCIN - sample rule

MYCIN ('70s, Stanford U.): Reasoning about bacterial infections

IF the stain of the organism is gram negative
AND the morphology of the organism is rod
AND the aerobiocity of the organism is gram anaerobic
THEN the there is strongly suggestive evidence (0.8) that the class of
the organism is enterobacteriaceae

Propositional logic



Propositions

Which sentences are propositions?

Any statement which can be evaluated as *true* or *false*.

Examples of propositions:

- ▶ Chris is a lecturer of MAIR.
- ▶ The Earth is flat.
- ▶ $15 + 3 \leq 20$
- ▶ “P=NP”

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- ▶ Chris is a lecturer of MAIR.
- ▶ The Earth is flat.
- ▶ $15 + 3 \leq 20$
- ▶ “P=NP”

The following sentences are *not* propositions:

- ▶ Are you a lecturer of MAIR?
- ▶ Don't cheat at the exam!
- ▶ $2x + 3 = 10$

Composite propositions

Examples:

- ▶ It is **not** the case that the Earth is flat.
- ▶ Chris is a lecturer of MAIR **or** the Earth is flat.
- ▶ Chris is a lecturer of MAIR **and** the Earth is flat.
- ▶ **If** the Earth is flat, **then** “P=NP”
- ▶ $15 + 3 \leq 20$ **if and only if** $2(15 + 3) \leq 2 \cdot 20$

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Course manual:

If all three individual grades are at least 5.0 **and** the weighted final grade is at least 5.5, **then** you passed the course.

Propositional logic – Syntax

Alphabet: the symbols of propositional logic are:

- ▶ **propositional letters:** p, q, r etc.
- ▶ **(Boolean) connectives:** $\neg, \wedge, \vee, \rightarrow$ and \leftrightarrow
- ▶ **brackets** “(” and “)”

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Formulas: the well defined propositional formulas are:

- ▶ propositional letters: p, q, r, \dots
- ▶ If α and β are formulas, then the following are also formulas.

$\neg\alpha$	negation	“not ...”
$(\alpha \wedge \beta)$	conjunction	“... and ...”
$(\alpha \vee \beta)$	disjunction	“... or ...”
$(\alpha \rightarrow \beta)$	implication	“if ..., then ...”
$(\alpha \leftrightarrow \beta)$	equivalence	“... iff ...”

Propositional logic – Semantics

A *model* (of a logic) is a precisely described situation in which the truth of each formula is determined.

A model *m* is a *model of a formula* α , if α is true in *m*.

$$m \models \alpha$$

- ▶ *Models* of propositional logic – *valuations*.
- ▶ A *valuation* assigns a truth value to every propositional variable

Example:

	<i>p</i>	<i>q</i>	<i>r</i>
model <i>m</i> :	<i>true</i>	<i>true</i>	<i>false</i>

Given *m*, for every formula we can determine its truth value:

- ▶ Propositional variables have their values directly assigned by *m*
- ▶ *[Truth functionality:]* The truth value of a compound formula is a function of the truth values of its subformulas.

Truth table

The following *truth table* characterizes the Boolean connectives.

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \rightarrow Q$	$P \leftrightarrow Q$
false	false	true	false	false	true	true
false	true	true	false	true	true	false
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model m : $\begin{matrix} p & q & r \\ \text{true} & \text{true} & \text{false} \end{matrix}$

$$\neg p \wedge (q \vee r)$$

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$$\neg p \wedge (q \vee r) = \neg \text{true} \wedge (\text{true} \vee \text{false})$$

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p q r
 model m : *true* *true* *false*

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Some fundamental logical notions

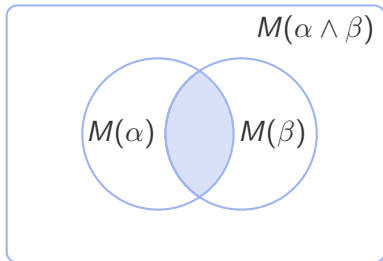
Model of a KB

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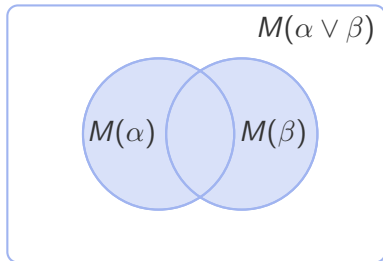
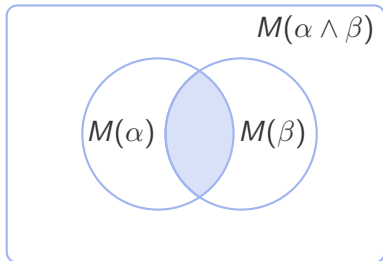


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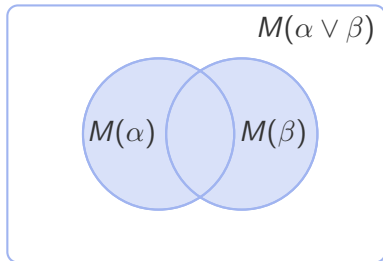
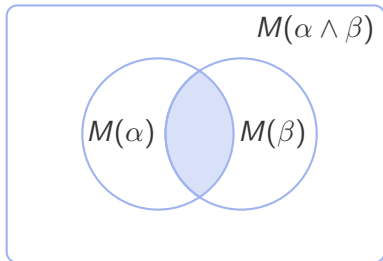


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$M(KB)$ – the set of all models of *every* formula from KB

If $KB = \{\alpha, \beta, \gamma\}$, then $M(KB) = M(\alpha) \cap M(\beta) \cap M(\gamma)$

Some semantic concepts: satisfiability and validity

Satisfiability

A formula α is **satisfiable** iff there is a model m such that $m \models \alpha$ holds (e.g. $p \wedge q$)

α is **satisfiable** iff $Mod(\alpha) \neq \emptyset$

Validity

A formula α is **valid** (notation $\models \alpha$) iff $m \models \alpha$ holds for every model m (e.g. $p \vee \neg p$)

Some semantic concepts: Entailment

Entailment

A formula α is a **logical consequence** of a set of formulas KB ($KB \models \alpha$) iff α is true in every model in which KB is true.
(e.g. $\{p, p \rightarrow q\} \models q$)

Entailment is a relationship between sentences (i.e., **syntax**) that is based on **semantics**.

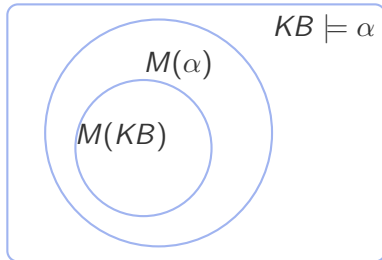
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- ▶ $KB \models \alpha$ iff $M(KB) \subseteq M(\alpha)$
- ▶ $\beta \models \alpha$ iff $\models \beta \rightarrow \alpha$
- ▶ An unsatisfiable formula entails every formula



Inference

$KB \vdash \alpha$ – “ α is *inferred* from KB ” (by. . .)

Two main groups of inference algorithms:

- ▶ **Model-based**: (e.g., truth table enumeration)
- ▶ **Proof-based**: apply inference rules to infer new knowledge (e.g., modus ponens:

$$\frac{\alpha \quad \alpha \rightarrow \beta}{\beta})$$

Inference in logic

Logical language, formulas

- ▶ inference system
(inference rules)
- ▶ semantics
(models, satisfiability)
- ▶ derivations (\vdash)
- ▶ entailment (\models)

Soundness: $KB \vdash \alpha \implies KB \models \alpha$

Completeness: $KB \models \alpha \implies KB \vdash \alpha$

First-Order logic

On expressiveness of Propositional logic

Question: Can we formalize this kind of reasoning in propositional logic?

- ▶ All humans are mortal
- ▶ Socrates is a human
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An attempt:

p : All humans are mortal

q : Socrates is a human

r : Socrates is mortal

$$\{p, q\} \not\models r$$

First-Order Logic

Whereas propositional logic assumes world contains *facts*, first-order logic (like natural language) assumes the world contains:

- ▶ *Objects*: people, houses, numbers, theories, MAIR lecturers, colors, centuries . . .
- ▶ *Relations*: red, round, prime, boring. . . , sister, bigger than, inside, part of, has color, occurred after, comes between . . .
- ▶ *Functions*: father of, one more than, plus . . .

Syntax of FOL: Basic elements

Constant symbols	<i>George, 2, UU, ...</i>
Predicate symbols (relation symbols)	<i>Yellow, Sister, \geq, ...</i>
Function symbols	<i>Sqrt, FatherOf, ...</i>
Variables	<i>x, y, a, b, ...</i>
Connectives	$\wedge \vee \neg \rightarrow \leftrightarrow$
Equality	$=$
Quantifiers	$\forall \exists$

Atomic formulas

Atomic formula = *predicate_symbol*(*term*₁, ..., *term*_{*n*})
or *term*₁ = *term*₂

Term = *function_symbol*(*term*₁, ..., *term*_{*n*})
or *constant_symbol* or *variable*

E.g.

- ▶ *Spouses*(*FatherOf*(*George*), *MotherOf*(*George*))
- ▶ \geq (*Height*(*George*), *Height*(*FatherOf*(*George*)))

Complex formulas

Complex formulas are made from atomic formulas using connectives and quantifiers

$$\neg\alpha, \quad \alpha \wedge \beta, \quad \alpha \vee \beta, \quad \alpha \rightarrow \beta, \quad \alpha \leftrightarrow \beta \quad \exists x \alpha, \quad \forall x \alpha$$

E.g.:

- ▶ $Spouses(FatherOf(George), MotherOf(George)) \rightarrow Spouses(MotherOf(George), FatherOf(George))$
- ▶ $\geq(1, 2) \vee \neg(\geq(1, 2))$
- ▶ $\forall x At(x, UU) \rightarrow Smart(x)$
“Everyone at UU is smart”
- ▶ $\exists x At(x, UU) \wedge Tall(x)$
“Someone at UU is Tall”

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“Someone at UU is Tall”
- ▶ $\exists x \exists y Lec(x, MAIR) \wedge Lec(y, MAIR) \wedge \neg(x = y)$
“There are at least two lecturers og MAIR”

Truth in first-order logic

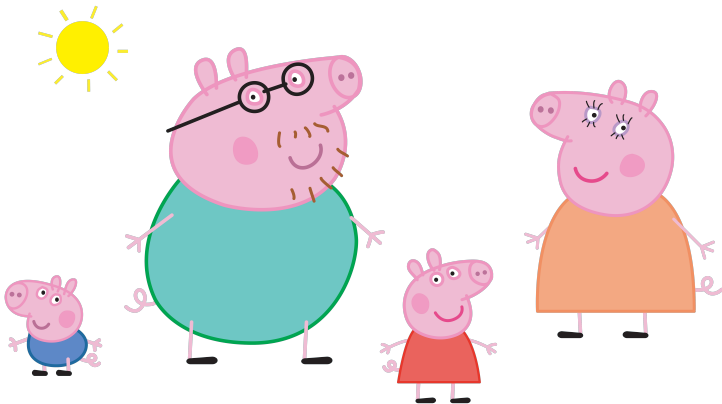
Formulas are true with respect to a *model/interpretation/structure*, which consists of

- ▶ A *non empty domain D* of objects
- ▶ An *interpretation function I* linking the syntax with the domain:
 - ▶ constant symbols \rightarrow objects
 - ▶ predicate symbols \rightarrow relations
 - ▶ function symbols \rightarrow functions

An atomic formula *predicate_symbol*(*term*₁, ..., *term*_{*n*}) is true iff the *objects* referred to by *term*₁, ..., *term*_{*n*} are in the *relation* referred to by *predicate_symbol*

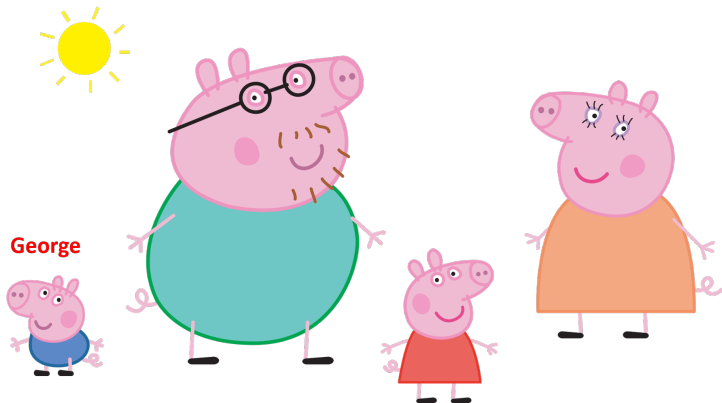
$\forall x \alpha$ is true in a model if α is true with x being each possible *object* in the model.

Models for FOL: Example



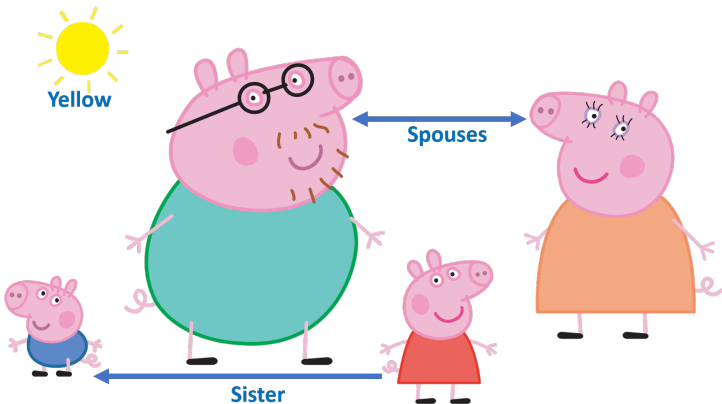
Objects: Peppa Pig, George Pig, Mammy Pig, Daddy Pig, Sun

Models for FOL: Example



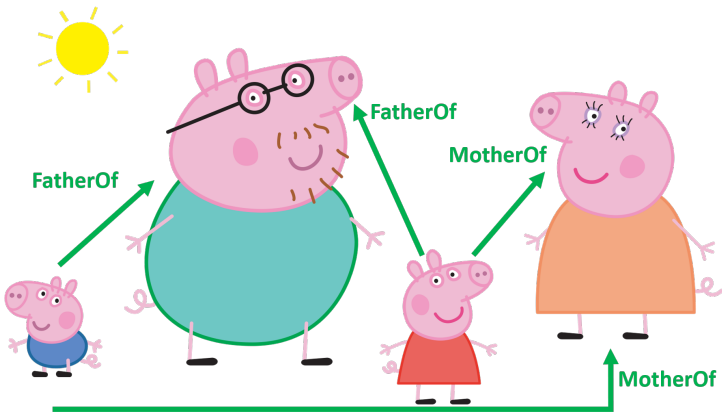
Constants: George

Models for FOL: Example



Relations: Yellow, Sister, Spouses

Models for FOL: Example



Functions: MotherOf, FatherOf

Semantics, more formally

An interpretation function I maps elements of the syntax to the domain as follows (superscripts – **arity**):

- ▶ For each constant symbol c , $I(c) = o \in D$
- ▶ For each n -ary function symbol f^n , $I(f^n) = \phi^n: D^n \rightarrow D$
- ▶ For each n -ary predicate symbol P^n , $I(P^n) = R^n \subseteq D^n$

So, the interpretation function maps all elements of the syntax to the domain except for variables.

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- ▶ $FatherOf(x) \quad ?$

Denotation – Interpreting terms

Let $\mathcal{M} = (D, I)$ be a model.

A *variable assignment* μ maps every variable to an object in D :

$$\mu : \text{Variables} \rightarrow D$$

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Denotation maps terms to objects of D (given \mathcal{M} and μ):

- ▶ $\|x\|_{\mathcal{M}, \mu} = \mu(x)$
- ▶ $\|c\|_{\mathcal{M}, \mu} = I(c)$
- ▶ $\|f(t_1, \dots, t_n)\|_{\mathcal{M}, \mu} = I(f)(\|t_1\|_{\mathcal{M}, \mu}, \dots, \|t_n\|_{\mathcal{M}, \mu})$

Satisfiability relation \models

Let $\mathcal{M} = (D, I)$ be a model, and let μ be a variable assignment.

$$\begin{array}{ll}
 \mathcal{M}, \mu \models P_i^n(t_1, \dots, t_n) & \Leftrightarrow I(P_i^n)(\|t_1\|_{\mathcal{M}, \mu}, \dots, \|t_n\|_{\mathcal{M}, \mu}) \\
 \mathcal{M}, \mu \models \neg \alpha & \Leftrightarrow \text{not } \mathcal{M}, \mu \models \alpha \\
 \mathcal{M}, \mu \models \alpha \wedge \beta & \Leftrightarrow \mathcal{M}, \mu \models \alpha \text{ and } \mathcal{M}, \mu \models \beta \\
 \vdots & \vdots \\
 \mathcal{M}, \mu \models \forall x \alpha & \Leftrightarrow \text{for all } \mu' =_x \mu: \mathcal{M}, \mu' \models \alpha \\
 \mathcal{M}, \mu \models \exists x \alpha & \Leftrightarrow \text{for some } \mu' =_x \mu: \mathcal{M}, \mu' \models \alpha
 \end{array}$$

μ is an *x-variant* of μ' (written as $\mu =_x \mu'$) if they map all variables to the same object in D except for x .

Satisfiability relation \models

Let $\mathcal{M} = (D, I)$ be a model, and let μ be a variable assignment.

$$\begin{array}{ll}
 \mathcal{M}, \mu \models P_i^n(t_1, \dots, t_n) & \Leftrightarrow I(P_i^n)(\|t_1\|_{\mathcal{M}, \mu}, \dots, \|t_n\|_{\mathcal{M}, \mu}) \\
 \mathcal{M}, \mu \models \neg \alpha & \Leftrightarrow \text{not } \mathcal{M}, \mu \models \alpha \\
 \mathcal{M}, \mu \models \alpha \wedge \beta & \Leftrightarrow \mathcal{M}, \mu \models \alpha \text{ and } \mathcal{M}, \mu \models \beta \\
 \vdots & \vdots \\
 \mathcal{M}, \mu \models \forall x \alpha & \Leftrightarrow \text{for all } \mu' =_x \mu: \mathcal{M}, \mu' \models \alpha \\
 \mathcal{M}, \mu \models \exists x \alpha & \Leftrightarrow \text{for some } \mu' =_x \mu: \mathcal{M}, \mu' \models \alpha
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Sentence: a formula in which all variables are bounded by a quantifier

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Sentence: a formula in which all variables are bounded by a quantifier (evaluated in \mathcal{M} , assignments irrelevant)

Beyond FOL

Jon Barwise (1942–2000) an American logician

To claim that logic is first-order logic is like to claim that astronomy is the study of the telescope.

Reasoning about:

▶ knowledge	epistemic logic
▶ time	temporal logic
▶ norms	deontic logic
▶ inconsistent knowledge	paraconsistent logic
▶ defeasible inferences	non-monotonic logic
▶ uncertainty	probabilistic logic, ...
▶ vague statements	fuzzy logic
⋮	⋮

Summary

This lecture:

- ▶ Knowledge-based systems apply **logical inference** to a **knowledge base** to derive new information.
- ▶ Logics:
 - ▶ Propositional logic
 - ▶ First order logic

Next lecture:

- ▶ Description logics