Solution to study question 3.5.1

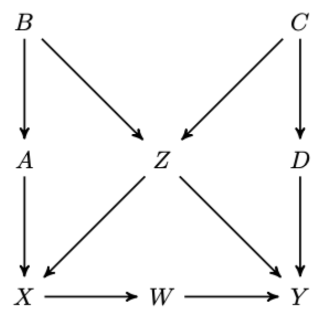
November 28, 2019

(Source: Pearl et al, 2016, solution manual)

The graph of this exercise is available at dagitty.net/m331. Students can solve parts (a) and (b) interactively in class by forcing adjustment for single covariates (move mouse pointer over the variable and press "a" key).

An *R* solution of this exercise is provided at dagitty.net/primer/3.5.1

Repeating Figure 3.8 for ease of reference:



Part (a)

By Rule 2, we must adjust for a set of variables that satisfies the backdoor criterion, conditional on C. We observe that when we condition on C, there is still an open backdoor from $X \leftarrow Z \rightarrow Y$, which we can block by conditioning on Z. So, we may claim that:

$$P(Y = y|do(X = x), C = c) = \sum_z P(Y = y|X = x, Z = z, C = c)P(Z = z)$$

Above, Rule 2 is applicable because the set $\{Z, C\}$ satisfies the backdoor criterion to assess the c-specific effect of X on Y.

Part (b)

Again using Rule 2, we see that $\{X, Y, Z, C\}$ is such a set since $\{Z, C\}$ satisfies the backdoor criterion. We can then write:

$$P(Y = y|do(X = x), Z = z) = \sum_c P(y|x, z, c)P(c)$$

Advanced students may be challenged to show that $\{X, Y, Z, W\}$ is also such a set, since W satisfies the front-door criterion when Z is specified.

Part (c)

Since our choice of X relies upon the value of Z, we need to adopt the conditional policy $do(X = g(Z))$, where:

$$g(Z) = \begin{cases} 0 & Z \leq 2 \\ 1 & Z > 2 \end{cases}$$

We then assume that $Z \in \{1, 2, 3, 4, 5\}$, so by Eq. (3.17) we have:

$$\begin{aligned} P(Y = y|do(X = g(Z))) &= \sum_z P(Y = y|do(X = g(z)), Z = z)P(Z = z) \\ &= P(Y = y|do(X = 0), Z = 1)P(Z = 1) \\ &\quad + P(Y = y|do(X = 0), Z = 2)P(Z = 2) \\ &\quad + P(Y = y|do(X = 1), Z = 3)P(Z = 3) \\ &\quad + P(Y = y|do(X = 1), Z = 4)P(Z = 4) \\ &\quad + P(Y = y|do(X = 1), Z = 5)P(Z = 5) \end{aligned}$$

So, for each term above in the format $P(Y = y|do(X = x), Z = z)$, we can substitute our findings from (b) to find an expression free of the *do*-operator.