Multi-Agent Systems

Question 1

a- Give three 2×2 strategic normal-form games which have zero, one, and two Nash equilibria, respectively. The Nash equilibria should not be the product of dominant strategies.

b- Determine Pareto efficient outcomes of the three games from part 1-a.

c- Consider the following game in which two players (*a* and *b*) wish to go to either a Bach or a Stravinsky concert.
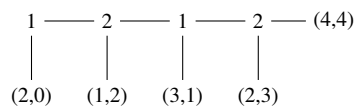
a\b	Bach	Stravinsky
Bach	2 \ 1	0 \ 0
Stravinsky	0 \ 0	1 \ 2

Determine the mixed Nash strategy equilibrium of this game? What is the expected utility of players *a* and *b* for this mixed Nash Equilibrium?

Question 2

a- Describe the Japanese auction in terms of bidding, clearing, and information rules?

b- Do the bidders in this auction have a dominant strategy? If yes, which one? If no, explain why the bidders have no dominant strategy.

Question 3 Consider the following two players (players 1 and 2) extensive game.

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1 —— 2 —— 1 —— 2 — (4,4)
|      |      |      |
(2,0)  (1,2)  (3,1)  (2,3)
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a- Transform this extensive game to a normal-form strategic game.

b- Determine the Subgame-perfect Nash equilibria of this game.

c- Transform the Bach-Stravinsky normal-form game from question 1 to an equivalent extensive game.

d- Let players in the Bach-Stravinsky game declare to play Bach. Would these utterances be self-commitment and self-revealing? Explain why.

Question 4

Three players share a taxi because their destinations are along one and the same route. It is assumed that the taxi does not charge for additional stops. The costs of each individual journey is as follows: 6€ for player 1, 12€ for player 2, and 42€ for player 3.

a- Model this situation as a cooperative game (N, v).

b- Is the core of this game empty? If not, give an outcome that is in the core.

c- Determine the Shapley value for each player.

Question 5 Consider the following voting scenario.

3	4	3	5
c	b	b	a
b	c	a	b
a	a	d	c
d	d	c	d

a- Give the winners according to the plurality, majority, Condorcet, and Borda voting systems.

b- Show if these preferences are single-peaked? Which candidate is the winner of the median voting rule?

Question 6

Design a mechanism with two alternatives and two players that implements the following social choice function in dominant strategy and show that the mechanism is indeed a correct implementation.

$$f(\succ_1, \succ_2) = \begin{cases} a & \text{if} \quad \succ_1 = a \succ b \quad \& \quad \succ_2 = a \succ b \\ b & \text{otherwise} \end{cases}$$