

# Multiagent Systems

Course code : INFOMAS      Date : 9 April 2020      Time : 13:30-16:30

## Question 1

Consider the following game with players  $A$  and  $B$ . The outcome  $X \backslash Y$  indicates that the  $A$ 's payoff is  $X$  and the  $B$ 's payoff is  $Y$ .

$A \backslash B$	$\beta_1$	$\beta_2$	$\beta_3$
$\alpha_1$	$5 \backslash 6$	$0 \backslash 8$	$9 \backslash 5$
$\alpha_2$	$8 \backslash 1$	$7 \backslash 7$	$8 \backslash 4$
$\alpha_3$	$10 \backslash 10$	$1 \backslash 8$	$10 \backslash 3$

- (a) What are the maxmin (security level) and minmax values for players  $A$  and  $B$  in this game?
- (b) What are the Pareto efficient outcomes?
- (c) What are the pure and mixed strategy (Nash) equilibria of this game? Provide the calculation of the mixed strategy.
- (d) What is the expected utility of the players for the mixed strategy equilibrium?
- (e) Is the declaration to play  $\beta_3$  by player  $B$  a self-committed utterance? Is it a self-revealing utterance? Motivate your answer.
- (f) Is the declaration to play  $\alpha_3$  by player  $A$  a self-committed utterance? Is it a self-revealing utterance? Motivate your answer.

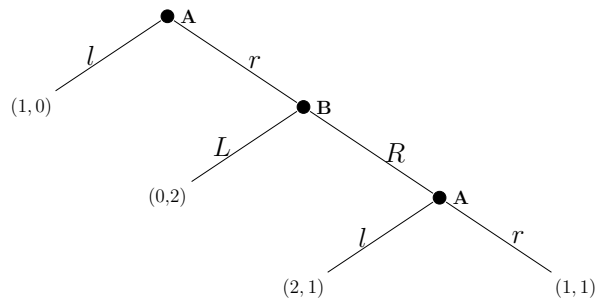
### Question 2

Three players go to an all-you-can-eat restaurant. They can go to the restaurant individually or as a group. Because of their age, if they go individually player 1 has to pay €25, player 2 €30, and player 3 €35. The price for a group of two persons is €50 and for a group of three persons is €70.

- (a) Model this scenario as a cooperative game  $(N, v)$  where  $v$  specifies the payment (not utility).
- (b) Is the core of this game empty? If not, give two outcomes that are in the core.
- (c) Determine the marginal contribution  $u_i(S)$  for each player  $i$  and each coalitions  $S$ . Provide the calculation of the marginal contributions.
- (d) Determine the Shapley value  $sh_i$  for each player  $i$ . Provide the calculation of the Shapley values.

### Question 3

Consider the following extensive game with players  $A$  and  $B$ . The outcome  $(X, Y)$  indicates that the payoff of player  $A$  is  $X$  and the payoff of player  $B$  is  $Y$ .



- (a) Enumerate the strategies of players  $A$  and  $B$ ?
- (b) What are the Nash equilibria of this game?
- (c) What are the dominant strategies of players  $A$  and  $B$ ?
- (d) What are the subgame-perfect Nash equilibria?

#### Question 4

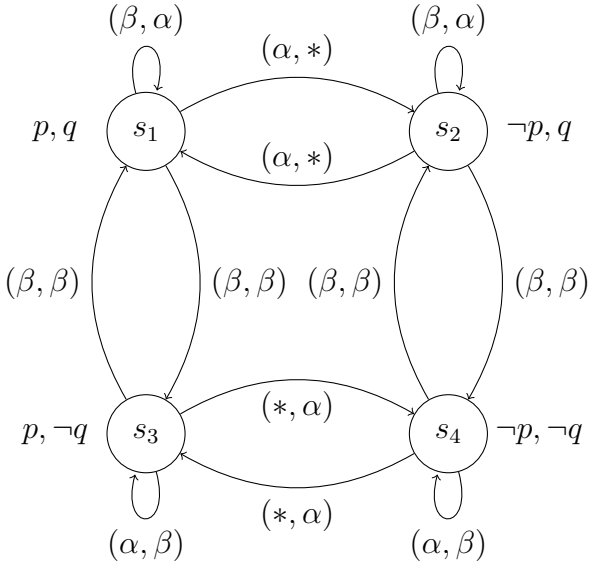
Consider the following voting scenario.

5	3	2	3
$a$	$d$	$b$	$c$
$d$	$a$	$a$	$d$
$c$	$b$	$d$	$a$
$b$	$c$	$c$	$b$

- Give the winners according to the plurality, majority, Condorcet, and Borda voting systems.
- Which candidate is the winner according to the method of Plurality with Elimination? Explain why.
- Are these preferences single-peaked? If yes, show the order of the candidates.
- Which candidate is the winner of the median voting rule? Explain why.

#### Question 5

Consider the concurrent game structure  $M$  shown below on the left side. Considering the memoryless strategies of the two players, indicate for each statement on the right side whether it is true or false. If a statement is true, give the memoryless strategy of the players.



- $M, s_1 \models \langle\langle \{1\} \rangle\rangle \Box q$
- $M, s_2 \models \langle\langle \{1\} \rangle\rangle X \langle\langle \{1, 2\} \rangle\rangle \Box \neg q$
- $M \models \langle\langle \{2\} \rangle\rangle \Diamond \neg q$
- $M, s_4 \models \langle\langle \{1, 2\} \rangle\rangle X \langle\langle \{2\} \rangle\rangle \Diamond (p \wedge q)$
- $M, s_3 \models \langle\langle \{2\} \rangle\rangle \Diamond (\neg p \vee q)$