

# Multiagent Systems

Course code : INFOMAS      Date : 15 April 2021      Time : 19:00-22:00

## Question 1

Consider the following game with players  $A$  and  $B$ . The outcome  $X \backslash Y$  indicates that the  $A$ 's payoff is  $X$  and the  $B$ 's payoff is  $Y$ .

$A \backslash B$	$\beta_1$	$\beta_2$	$\beta_3$
$\alpha_1$	$5 \backslash 5$	$1 \backslash 4$	$3 \backslash 3$
$\alpha_2$	$4 \backslash 1$	$3 \backslash 3$	$5 \backslash 1$
$\alpha_3$	$2 \backslash 7$	$1 \backslash 7$	$2 \backslash 6$

- (a) What are the maxmin (security level) and minmax values for players  $A$  and  $B$  in this game?
- (b) What are the Pareto efficient outcomes?
- (c) What are the pure and mixed strategy (Nash) equilibria of this game? Provide the calculation of the mixed strategy.
- (d) What is the expected utility for each player in the mixed strategy equilibrium?
- (e) Is the declaration to play  $\alpha_1$  by player  $A$  a self-committed utterance? Is it a self-revealing utterance? Motivate your answer.
- (f) Is the declaration to play  $\beta_3$  by player  $B$  a self-committed utterance? Is it a self-revealing utterance? Motivate your answer.

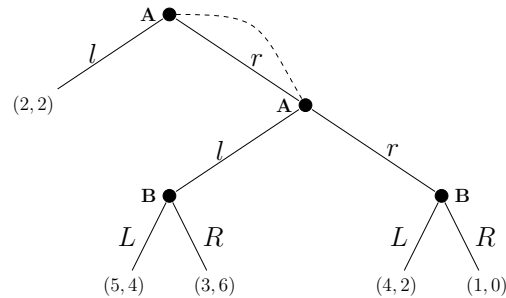
### Question 2

There are three parties  $a$ ,  $b$ , and  $c$  with respectively 40, 25, and 35 votes. A majority vote is required to pass a €100 bill. A coalition of parties with a majority vote divides the €100 bill.

- (a) Model this scenario as a cooperative game  $(N, v)$ .
- (b) Is the core of this game empty? If yes, explain why. If not, give an outcome that is in the core.
- (c) Determine the Shapley value for each party. Provide the calculation of the Shapley values.

### Question 3

Consider the following imperfect-information extensive game with players  $A$  and  $B$ . The outcome  $(X, Y)$  indicates that the payoff of player  $A$  is  $X$  and the payoff of player  $B$  is  $Y$ .



- (a) What are the Nash equilibria of this extensive game?
- (b) What are the subgame-perfect Nash equilibria of this extensive game?
- (c) Transform this extensive game to a normal-form strategic game and determine the dominant strategies of players  $A$  and  $B$ .

#### Question 4

Consider the following voting scenario.

2	2	5	6
<i>d</i>	<i>d</i>	<i>b</i>	<i>c</i>
<i>a</i>	<i>c</i>	<i>a</i>	<i>d</i>
<i>b</i>	<i>a</i>	<i>d</i>	<i>a</i>
<i>c</i>	<i>b</i>	<i>c</i>	<i>b</i>

- (a) Give the winners according to the plurality, majority, Condorcet, approval and Borda voting systems. For the approval voting assume that each voter gives one vote to his/her first two candidates.
- (b) Which candidate is the winner according to the method of Plurality with Elimination? Explain why.
- (c) Investigate whether different comparison orders of the candidates in this voting scenario result in different winners using the pairwise elimination method.
- (d) Are these preferences single-peaked? If yes, show the order of the candidates.
- (e) Which candidate is the winner of the median voting rule? Explain why.

#### Question 5

An indivisible object  $O$  should be assigned to one of the five rational players  $a$ ,  $b$ ,  $c$ ,  $d$ , and  $e$ . The true independent private values of these players for  $O$  are respectively 20€, 30€, 15€, 35€, and 25€.

- (a) How should these players bid in the Vickrey auction? Who is the winner in this auction and how much should the winner pay?
- (b) What is the Nash equilibrium of the players in the first-price sealed-bid auction?