Multi-Agent Systems

Question 1

a- Give three 2×2 strategic normal-form games which have zero, one, and two Nash equilibria, respectively. The Nash equilibria should not be the product of dominant strategies.

a\b	c_1	c_2
r_1	1 , 0	0 , 1
r_2	0 , 1	1 ,0

a\b	c_1	c_2
r_1	1 , 0	0 , 1
r_2	0 , 1	2 , 2

a\b	c_1	c_2
r_1	0 , 0	1 , 1
r_2	1 , 1	0 , 0

b- Determine Pareto efficient outcomes of the three games from part 1-a.
Game 1: all outcomes are Pareto efficient.
Game 2: outcome (2,2) is Pareto efficient.
Game 3: outcome (1,1) is Pareto efficient.

c- Consider the following game in which two players (a and b) wish to go to either a Bach or a Stravinsky concert.

a\b	Bach	Stravinsky
Bach	2 \ 1	0 \ 0
Stravinsky	0 \ 0	1 \ 2

Determine the mixed Nash strategy equilibrium of this game? What is the expected utility of players a and b for this mixed Nash Equilibrium?

Suppose player b plays Bach with probability p. The expected utility of the pure strategies of player a should then be the same, i.e.,
$2p + 0(1 - p) = 0p + 1(1 - p)$
$2p = 1 - p$
$3p = 1$
$p = 1/3$
So, the mixed strategy in equilibrium for player b is $(1/3, 2/3)$, i.e., player b plays Bach with $1/3$ and plays Stravinsky with $2/3$.

We do now the same for player a. Suppose player a plays Bach with probability q. The expected utility of the pure strategies of player b should then be the same, i.e.,
$1q + 0(1 - q) = 0q + 2(1 - q)$
$1q = 2 - 2q$
$3q = 2$
$q = 2/3$
So, the mixed strategy in equilibrium for player a is $(2/3, 1/3)$, i.e., player a plays Bach with $2/3$ and plays Stravinsky with $1/3$.

The expected utility of this mixed strategy in equilibrium for both players is the same, i.e.,
$(1/3 * 2/3 * 1) + (2/3 * 1/3 * 2) = 2/3$.

Question 2

a- Describe the Japanese auction in terms of bidding, clearing, and information rules?

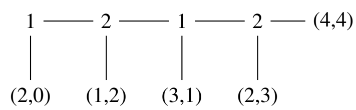
* Bidding rule: Ascending bid auction with in/out bids.

* Clearing rule: when one (or zero) agent is left. In the case, one agent is left, the winner pays the current price announced by the auctioneer. In the case that zero agent is left, one agent from previous round is chose based on a tie breaking rule. The selected agent pays the price announced by the auctioneer at the last round.

* Information rule: all valuations except the valuation of the winner in case there was only one agent is left. Otherwise, all valuations.

b- Do the bidders in this auction have a dominant strategy? If yes, which one? If no, explain why the bidders have no dominant strategy. Yes, under the independent private values model the dominant strategy of the bidders is to bid up to (but not beyond) their true valuations.

Question 3 Consider the following two players (players 1 and 2) extensive game.



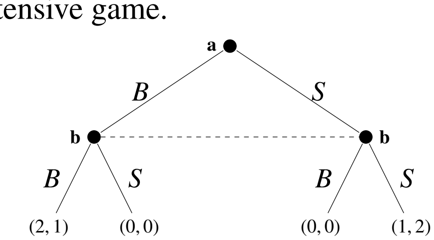
a- Transform this extensive game to a normal-form strategic game.

1\2	LL	LS	SL	SS
LL	2,0	2,0	2,0	2,0
LS	2,0	2,0	2,0	2,0
SL	1,2	1,2	3,1	3,1
SS	1,2	1,2	2,3	4,4

b- Determine the Subgame-perfect Nash equilibria of this game.

The strategy profile (SS, SS) is the only Subgame-perfect Nash equilibrium of the game. This strategy is in equilibrium in all 4 subtrees.

c- Transform the Bach-Stravinsky normal-form game from question 1 to an equivalent extensive game.



d- Let players in the Bach-Stravinsky game declare to play Bach. Would these utterances be self-commitment and self-revealing? Explain why.

The declaration is both self-committed and self-revealing for both players. Consider the row player declaring to play Bach. The declaration is self-committing because assuming that the column player believes this declaration, the column player will play Bach such that the row player can do no better than playing Bach too.

The declaration is self-revealing because there is no other reason for this choice of the row player other than the fact that the row player has intention to play Bach, i.e., declaring Bach by the row player would not increase the expected utility of other strategies of the row player.

Question 4

Three players share a taxi because their destinations are along one and the same route. It is assumed that the taxi does not charge for additional stops. The costs of each individual journey is as follows: 6€ for player 1, 12€ for player 2, and 42€ for player 3.

a- Model this situation as a cooperative game (N, v).$N = \{1, 2, 3\}$
$v(\emptyset) = 0,\ v(\{1\}) = 6,\ v(\{2\}) = 12,\ v(\{3\}) = 42,\ v(\{1, 2\}) = 12,\ v(\{1, 3\}) = 42\ ,\ v(\{2, 3\}) = 42\ ,\ v(\{1, 2, 3\}) = 42$

b- Is the core of this game empty? If not, give an outcome that is in the core.The core is not empty. An outcome is $(1, 1, 40)$.

c- Determine the Shapley value for each player.
$\mu_1(C_1(123)) = \mu_1(\emptyset) = v(\{1\}) - v(\emptyset) = 6 - 0 = 6$
$\mu_1(C_1(132)) = \mu_1(\emptyset) = v(\{1\}) - v(\emptyset) = 6 - 0 = 6$
$\mu_1(C_1(213)) = \mu_1(\{2\}) = v(\{1, 2\}) - v(\{2\}) = 12 - 12 = 0$
$\mu_1(C_1(231)) = \mu_1(\{2, 3\}) = v(\{1, 2, 3\}) - v(\{2, 3\}) = 42 - 42 = 0$
$\mu_1(C_1(312)) = \mu_1(\{3\}) = v(\{1, 3\}) - v(\{3\}) = 42 - 42 = 0$
$\mu_1(C_1(321)) = \mu_1(\{2, 3\}) = v(\{1, 2, 3\}) - v(\{2, 3\}) = 42 - 42 = 0$

$Sh(1) = 12/6 = 2.$

$\mu_2(C_2(123)) = \mu_2(\{1\}) = v(\{1, 2\}) - v(\{1\}) = 12 - 6 = 6$
$\mu_2(C_2(132)) = \mu_2(\{1, 3\}) = v(\{1, 2, 3\}) - v(\{1, 3\}) = 42 - 42 = 0$
$\mu_2(C_2(213)) = \mu_2(\emptyset) = v(\{2\}) - v(\emptyset) = 12 - 0 = 12$
$\mu_2(C_2(231)) = \mu_2(\emptyset) = v(\{2\}) - v(\emptyset) = 12 - 0 = 12$
$\mu_2(C_2(312)) = \mu_2(\{1, 3\}) = v(\{1, 2, 3\}) - v(\{1, 3\}) = 42 - 42 = 0$
$\mu_2(C_2(321)) = \mu_2(\{3\}) = v(\{2, 3\}) - v(\{3\}) = 42 - 42 = 0$

$Sh(2) = 30/6 = 5.$

$\mu_3(C_3(123)) = \mu_3(\{1, 2\}) = v(\{1, 2, 3\}) - v(\{1, 2\}) = 42 - 12 = 30$
$\mu_3(C_3(132)) = \mu_3(\{1\}) = v(\{1, 3\}) - v(\{1\}) = 42 - 6 = 36$
$\mu_3(C_3(213)) = \mu_3(\{1, 2\}) = v(\{1, 2, 3\}) - v(\{1, 2\}) = 42 - 12 = 30$
$\mu_3(C_3(231)) = \mu_3(\{2\}) = v(\{2, 3\}) - v(\{2\}) = 42 - 12 = 30$
$\mu_3(C_3(312)) = \mu_3(\emptyset) = v(\{3\}) - v(\emptyset) = 42 - 0 = 42$
$\mu_3(C_3(321)) = \mu_3(\emptyset) = v(\{3\}) - v(\emptyset) = 42 - 0 = 42$

$Sh(3) = 210/6 = 35.$

Question 5 Consider the following voting scenario.

3	4	3	5
c	b	b	a
b	c	a	b
a	a	d	c
d	d	c	d

a- Give the winners according to the plurality, majority, Condorcet, and Borda voting systems.
plurality = b
majority = none
Condorcet = b
Borda = b (with 37 points)

b- Show if these preferences are single-peaked? Which candidate is the winner of the median voting rule?

The preferences are single peaked as all preferences can be plotted as single peaked using the linear order *cbad* or *dabc*. The winner of the median voting rule is the median voter's ideal point given the order of the outcomes, in this case it is the b candidate.

Question 6

Design a mechanism with two alternatives and two players that implements the following social choice function in dominant strategy.

$$f(\succ_1, \succ_2) = \begin{cases} a & \text{if} \quad \succ_1 = a \succ b \quad \& \quad \succ_2 = a \succ b \\ b & \text{otherwise} \end{cases}$$

$1\backslash 2$	$a \succ_2 b$	$b \succ_2 a$
$a \succ_1 b$	a	b
$b \succ_1 a$	b	b