# Multiagent Systems

Course code: INFOMAS Date: 15 April 2021 Time: 19:00-22:00

### Question 1

Consider the following game with players A and B. The outcome  $X \setminus Y$  indicates that the A's payoff is X and the B's payoff is Y.

$A \setminus B$	$\beta_1$	$\beta_2$	$\beta_3$
$\alpha_1$	$1 \setminus 2$	$3 \setminus 2$	$4 \setminus 4$
$\alpha_2$	$1 \setminus 4$	$2 \setminus 2$	$1 \setminus 4$
$\alpha_3$	$3 \setminus 3$	$4 \setminus 1$	$2 \setminus 1$

- (a) What are the maxmin (security level) and minmax values for players A and B in this game?
- (b) What are the Pareto efficient outcomes?
- (c) What are the pure and mixed strategy (Nash) equilibria of this game? Provide the calculation of the mixed strategy.
- (d) What is the expected utility for each player in the mixed strategy equilibrium?
- (e) Is the declaration to play  $\alpha_2$  by player A a self-committed utterance? Is it a self-revealing utterance? Motivate your answer.
- (f) Is the declaration to play  $\beta_1$  by player B a self-committed utterance? Is it a self-revealing utterance? Motivate your answer.

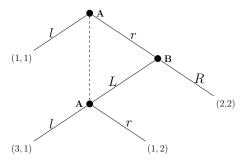
## Question 2

There are three parties a, b, and c with respectively 50, 30, and 20 votes. A 65% vote is required to pass a  $\in$ 100 bill. A coalition of parties with a minimum of 65 votes can thus divide the  $\in$ 100 bill.

- (a) Model this scenario as a cooperative game (N, v).
- (b) Is the core of this game empty? If yes, explain why. If not, give an outcome that is in the core.
- (c) Determine the Shapley value for each party. Provide the calculation of the Shapley values.

# Question 3

Consider the following imperfect-information extensive game with players A and B. The outcome (X,Y) indicates that the payoff of player A is X and the payoff of player B is Y.



- (a) What are the Nash equilibria of this extensive game?
- (b) What are the subgame-perfect Nash equilibria of this extensive game?
- (c) Transform this extensive game to a normal-form strategic game and determine the dominant strategies of players A and B.

# Question 4

Consider the following voting scenario.

- (a) Give the winners according to the plurality, majority, Condorcet, approval and Borda voting systems. For the approval voting assume that each voter gives one vote to his/her first two candidates.
- (b) Which candidate is the winner according to the method of Plurality with Elimination? Explain why.
- (c) Investigate whether different comparison orders of the candidates in this voting scenario result in different winners using the pairwise elimination method.
- (d) Are these preferences single-peaked? If yes, show the order of the candidates.
- (e) Which candidate is the winner of the median voting rule? Explain why.

#### Question 5

An indivisible object O should be assigned to one of the five rational players a, b, c, d, and e. The true independent private values of these players for O are respectively  $210 \in$ ,  $230 \in$ ,  $205 \in$ ,  $235 \in$ , and  $233 \in$ .

- (a) How should these players bid in the Vickrey auction? Who is the winner in this auction and how much should the winner pay?
- (b) What is the Nash equilibrium of the players in the first-price sealed-bid auction?