# Question 1

(a)

maxmin (A) = 2

maxmin (B) = 2

minmax (A) = 2

minmax (B) = 2

(b)

{α1,β3}

(c)

Pure strategy Nash equilibria - {α1,β3}, {α3,β1}

Mixed strategy Nash equilibria

- Player A
  u(β1) = 2*p + 4*q + 3*(1-p-q) = 2p + 4q + 3 - 3p - 3q = -p + q + 3
  u(β2) = 2*p + 2*q + 1*(1-p-q) = 2p + 2q + 1 - p - q = p + q + 1
  u(β3) = 4*p + 4*q + 3*(1-p-q) = 4p + 4q + 3 - 3p - 3q = p + q + 3
- Player A is indifferent between their pure strategies:
  - {α1,α2} if
    u(β1) = u(β2)
    -p + q + 3 = p + q + 1
    q = -2
  - {α2,α3} if
    u(β2) = u(β3)
    p + q + 1 = p + q + 3
    2p = 2q + 2
    p = q + 1
  - Substitutions with q = -2
    p = -2 + 1
    p = -1

- Player B
  $u(\alpha 1) = 1*p + 3*q + 4*(1-p-q) = p + 3q + 4 - 4p - 4q = -3p - q + 4$
  $u(\alpha 2) = 1*p + 2*q + 1*(1-p-q) = p + 2q + 1 - p - q = q + 1$
  $u(\alpha 3) = 3*p + 4*q + 2*(1-p-q) = 3p + 4q + 2 - 2p - 2q = p + 2q + 2$
- Player B is indifferent between their pure strategies:
  - {β1,β2} if
    $u(\alpha 1) = u(\alpha 2)$
    $-3p - q + 4 = q + 1$
    $-2q = 4p - 3$
    $q = -2p + 1{,}5$
  - {β2,β3} if
    $u(\alpha 2) = u(\alpha 3)$
    $q + 1 = p + 2q + 2$
    $-p = q + 1$
    $p = -q - 1$
  - Substitutions with $q = -2p + 1{,}5$
    $p = 2p - 1{,}5 - 1$
    $-p = -2{,}5$
    $p = 2{,}5$

(d)

(e)

α2 is not self-revealing as player A does not wish to play that row anyway. It is also not self-committing as player A can not expect player B to believe A's desire to play this row.

(f)

β1 is self-committing and self-revealing. As player B expects player A to believe him so that he can declare the optimal action.

# Question 2

(a)

- $N = \{a, b, c\}$
- $v(\varnothing) = 0$
  $v(\{a\}) = 50$
  $v(\{b\}) = 30$
  $v(\{c\}) = 20$
  $v(\{a, b\}) = 80$
  $v(\{a, c\}) = 70$
  $v(\{b, c\}) = 50$
  $v(\{a, b, c\}) = 100$

(b)

Core is not empty as the grand coalition is feasible, efficient, individually rational and stable. An outcome is {50,40,20}.

(c)

- $\mu_a(C_a(abc)) = \mu_a(\varnothing) = v(\{a\}) - v(\varnothing) = 50 - 0 = 50$
  $\mu_a(C_a(acb)) = \mu_a(\varnothing) = v(\{a\}) - v(\varnothing) = 50 - 0 = 50$
  $\mu_a(C_a(bac)) = \mu_a(\{b\}) = v(\{a, b\}) - v(\{b\}) = 80 - 30 = 50$
  $\mu_a(C_a(bca)) = \mu_a(\{b, c\}) = v(\{a, b, c\}) - v(\{b, c\}) = 100 - 50 = 50$
  $\mu_a(C_a(bac)) = \mu_a(\{c\}) = v(\{a, c\}) - v(\{c\}) = 70 - 20 = 50$
  $\mu_a(C_a(cba)) = \mu_a(\{b, c\}) = v(\{a, b, c\}) - v(\{b, c\}) = 100 - 50 = 50$
  ***S h*(a) = 300/6 = 50**.
- $\mu_b(C_b(abc)) = \mu_b(\{a\}) = v(\{a, b\}) - v(\{a\}) = 80 - 50 = 30$
  $\mu_b(C_b(acb)) = \mu_b(\{a, c\}) = v(\{a, b, c\}) - v(\{a, c\}) = 100 - 70 = 30$
  $\mu_b(C_b(bac)) = \mu_b(\varnothing) = v(\{b\}) - v(\varnothing) = 30 - 0 = 30$
  $\mu_b(C_b(bca)) = \mu_b(\varnothing) = v(\{b\}) - v(\varnothing) = 30 - 0 = 30$
  $\mu_b(C_b(cab)) = \mu_b(\{a, c\}) = v(\{a, b, c\}) - v(\{a, c\}) = 100 - 70 = 30$
  $\mu_b(C_b(cba)) = \mu_b(\{c\}) = v(\{b, c\}) - v(\{c\}) = 50 - 20 = 30$
  ***S h*(b) = 180/6 = 30**.
- $\mu_c(C_c(abc)) = \mu_c(\{a, b\}) = v(\{a, b, c\}) - v(\{a, b\}) = 100 - 80 = 20$
  $\mu_c(C_c(acb)) = \mu_c(\{a\}) = v(\{a, c\}) - v(\{a\}) = 70 - 50 = 20$
  $\mu_c(C_c(bac)) = \mu_c(\{a, b\}) = v(\{a, b, c\}) - v(\{a, b\}) = 100 - 80 = 20$
  $\mu_c(C_c(bca)) = \mu_c(\{a\}) = v(\{b, c\}) - v(\{b\}) = 50 - 30 = 20$
  $\mu_c(C_c(cab)) = \mu_c(\varnothing) = v(\{c\}) - v(\varnothing) = 20 - 0 = 20$
  $\mu_c(C_c(cba)) = \mu_c(\varnothing) = v(\{c\}) - v(\varnothing) = 20 - 0 = 20$
  ***S h*(c) = 120/6 = 20**.

# Question 3

(a)

The strategy profiles **(rl,R),(rr,R)** are Nash equilibria.

(b)

The only subgame-perfect Nash equilibria is **(rr,R).**

(c)

| | **L** | **R** |
|---|---|---|
| **ll** | 1,1 | 1,1 |
| **lr** | 1,1 | 1,1 |
| **rl** | 3,1 | 2,2 |
| **rr** | 1,2 | 2,2 |

Player A - dominant strategy is **(rl).**

Player B - dominant strategy is **(R).**

# Question 4

(a)

- Plurality - **c**
- Majority - **none**
- Condorcet - **b**
- Approval - **d**
- Borda - **d**

(b)

- Plurality with Elimination - **a**
  This is because of how voters decide in the rounds after elimination. After the first elimination round (b), a gets two additional votes (4+2=6). After the second elimination round (d), a gets four additional votes (6+4=10). At this elimination round, only two candidates (a,c) remain with c being the candidate with lowest amount of votes (5). After the final elimination, a is standing.

(c)

Not really, no.

(d)

The preferences are single-peaked as all preferences can be plotted as single peaked using the linear order ***cdab*** or ***badc***.

(e)

The winner of the median voting rule is the median voter's ideal point given the order of the outcomes, in this case it is the ***d*** candidate.

# Question 5

(a)

The auction format incentivises players to bid their **true valuation** of the object to maximise their expected utility. So, the players should bid what the object is actually worth to them.

This is a sealed-bid auction where the highest bidder wins and pays the second-highest bid. Player ***d*** wins, and should pay **233€**.

(b)

As there are five players, each player should bid **4/5 of their true valuation** of the product.

Bidder $i$ bids $\dfrac{n-1}{n} v_i$

- a - 168€
- b - 183€
- c - 164€
- d - **188€**
- e - 186,4€