

Model checking CTL

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Local model checking

Model checking problem

$$M, q \stackrel{?}{=} \varphi$$

state in the model

temporal formula

- this problem is sometimes called **local model checking**: it checks whether a formula is true locally (in a particular state)
- implemented as a function $mcheck(M, q, \varphi)$ such that:

$$mcheck(M, q, \varphi) = \begin{cases} \text{true} & \text{if } M, q \models \varphi \\ \text{false} & \text{else} \end{cases}$$

Global model checking

- however it is more efficient to work with **sets** of states

Global model checking

We want to implement the function

$$mcheck(M, \varphi) = \{q \in St \mid M, q \models \varphi\}$$

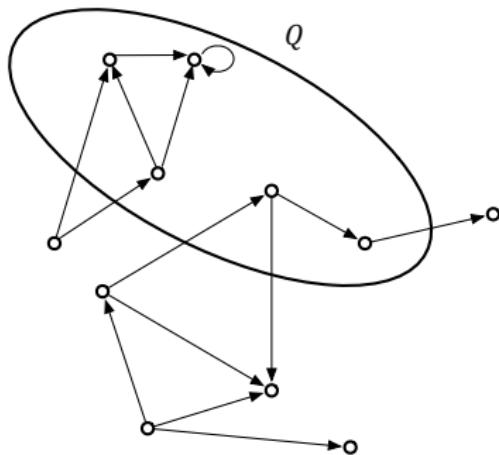
- often no harder than local model checking
- from now on, we will refer to global model checking as simply “model checking”

Model Checking for CTL

- as explained in the previous lecture, paths satisfying CTL formulae can be constructed incrementally, using **fixpoint computations**
- for “**next time**”, we simply take the appropriate **pre-image**
- for “**long-term**” properties, we repeatedly take the pre-image to compute the **fixpoint**
- computing the pre-image is the key building block in computing the fixpoint

Existential pre-image

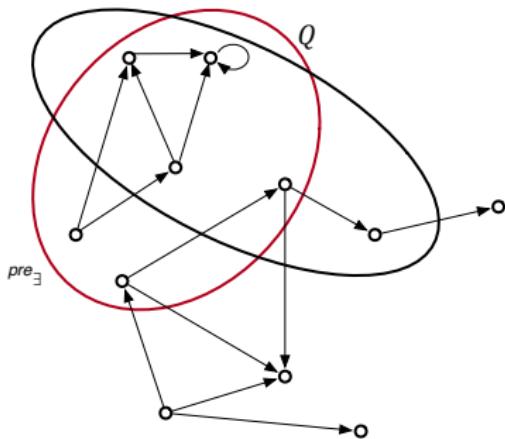
The existential pre-image of a set of states Q , $\text{pre}_{\exists}(Q)$ is all states q which have **some** state $q' \in Q$ as a successor



$$\text{pre}_{\exists}(Q) = \{q \mid \exists q'. q \rightarrow q' \& q' \in Q\}$$

Existential pre-image

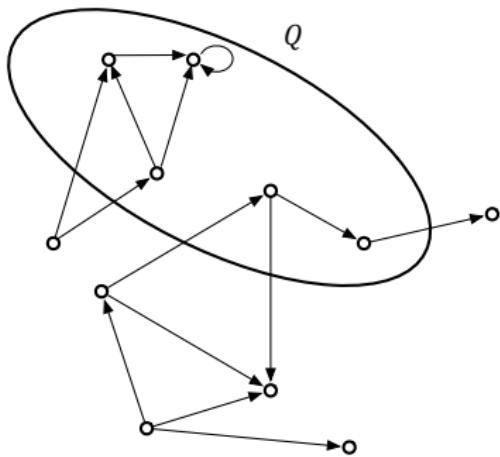
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Universal pre-image

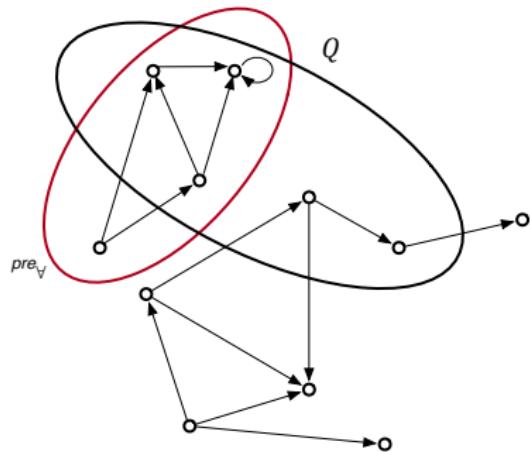
The universal pre-image of a set of states Q , $\text{pre}_{\forall}(Q)$ is all states q where **all** successor states q' are in Q



$$\text{pre}_{\forall}(Q) = \{q \mid \forall q'. q \rightarrow q' \Rightarrow q' \in Q\}$$

Universal pre-image

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CTL model checking algorithm

To implement $mcheck(M, \varphi_0)$, we proceed as follows:

- compute the set of subformulas of φ_0 , $Sub(\varphi_0)$
- the elements of $Sub(\varphi_0)$ are in order of complexity, atomic propositions first, then negations of atomic formulas, and so on, ending with φ_0 itself
- label states of M with each subformula $\varphi' \in Sub(\varphi_0)$ in turn, so that when we reach φ' , states are already labelled with the subformulas of φ'
- labelling of states for each subformula proceeds by cases, one for each logical connective

function MCHECK(M, φ_0)

for $\varphi' \in Sub(\varphi_0)$ **do**

case $\varphi' = p$

$[\varphi']_M \leftarrow \mathcal{V}(p)$

case $\varphi' = \neg\psi$

$[\varphi']_M \leftarrow St \setminus [\psi]_M$

case $\varphi' = \psi_1 \wedge \psi_2$

$[\varphi']_M \leftarrow [\psi_1]_M \cap [\psi_2]_M$

case $\varphi' = \psi_1 \vee \psi_2$

$[\varphi']_M \leftarrow [\psi_1]_M \cup [\psi_2]_M$

case $\varphi' = EX\psi$

$[\varphi']_M \leftarrow pre_{\exists}([\psi]_M)$

case $\varphi' = EG\psi$

$Q_1 \leftarrow St; Q_2 \leftarrow [\psi]_M$

while $Q_1 \not\subseteq Q_2$ **do**

$Q_1 \leftarrow Q_2; Q_2 \leftarrow pre_{\exists}(Q_1) \cap Q_1$

$[\varphi']_M \leftarrow Q_1$

case $\varphi' = E\psi_1 \cup \psi_2$

$Q_1 \leftarrow \emptyset; Q_2 \leftarrow [\psi_2]_M$

while $Q_2 \not\subseteq Q_1$ **do**

$Q_1 \leftarrow Q_1 \cup Q_2; Q_2 \leftarrow pre_{\exists}(Q_1) \cap [\psi_1]_M$

$[\varphi']_M \leftarrow Q_1$

case $\varphi' = AX\psi$

$$[\varphi']_M \leftarrow \text{pre}_\forall([\psi]_M)$$

case $\varphi' = AG\psi$

$$Q_1 \leftarrow St; \quad Q_2 \leftarrow [\psi]_M$$

while $Q_1 \not\subseteq Q_2$ **do**

$$Q_1 \leftarrow Q_2; \quad Q_2 \leftarrow \text{pre}_\forall(Q_1) \cap Q_1$$

$$[\varphi']_M \leftarrow Q_1$$

case $\varphi' = A\psi_1 \cup \psi_2$

$$Q_1 \leftarrow \emptyset; \quad Q_2 \leftarrow [\psi_2]_M$$

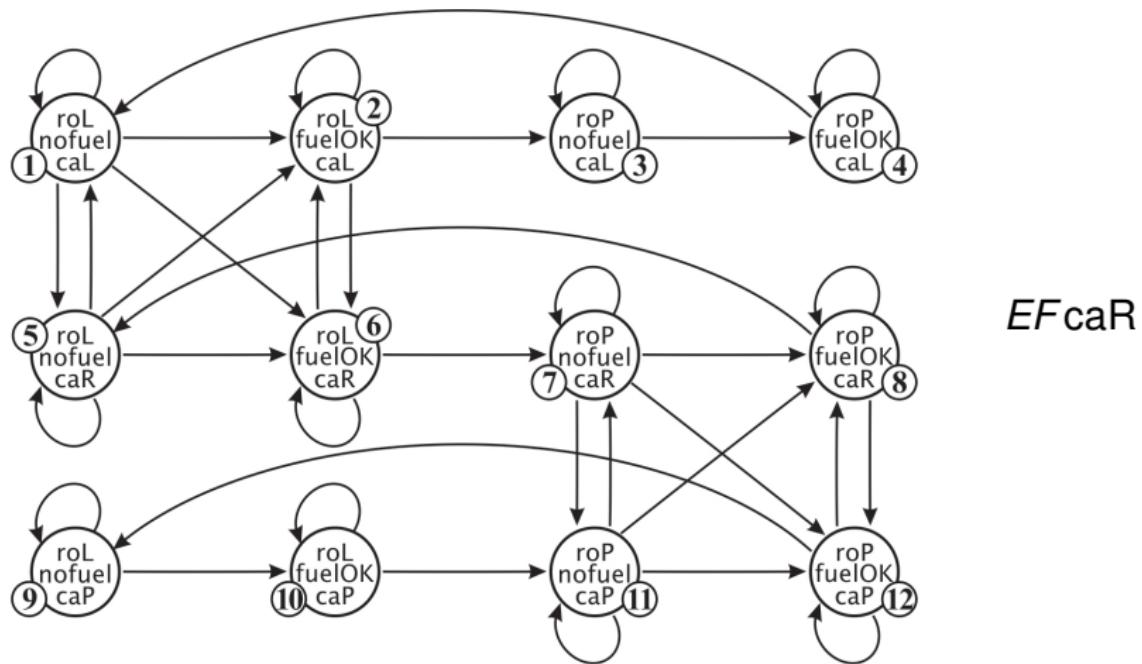
while $Q_2 \not\subseteq Q_1$ **do**

$$Q_1 \leftarrow Q_1 \cup Q_2; \quad Q_2 \leftarrow \text{pre}_\forall(Q_1) \cap [\psi_1]_M$$

$$[\varphi']_M \leftarrow Q_1$$

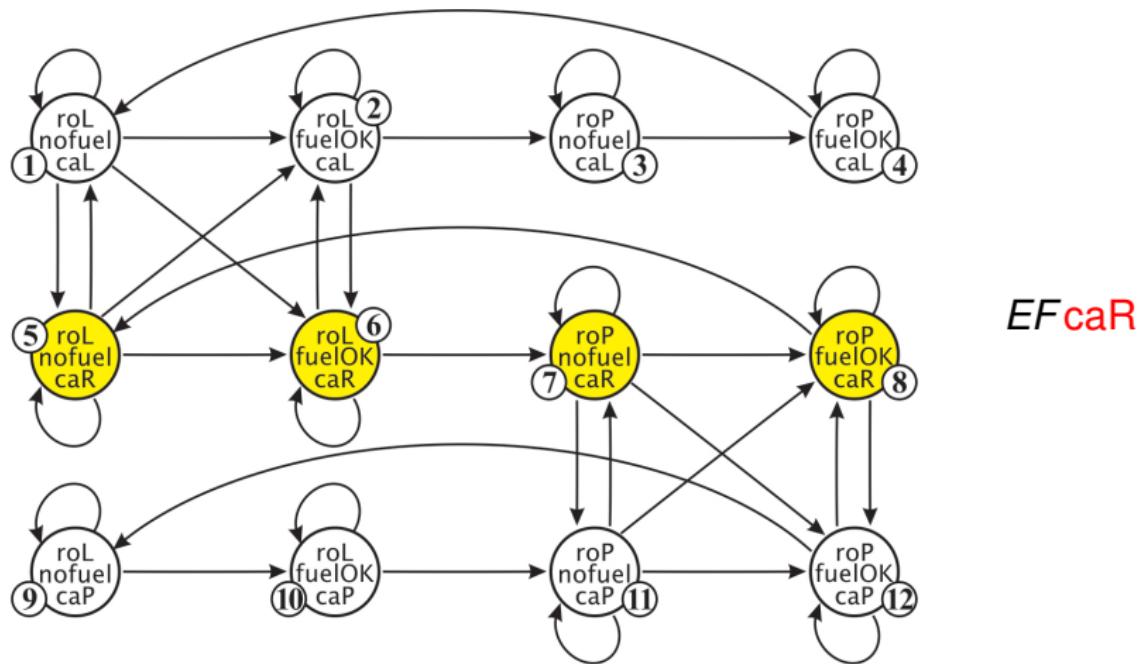
Example: Simple Rocket

Compute $mcheck(M, EF\text{caR})$



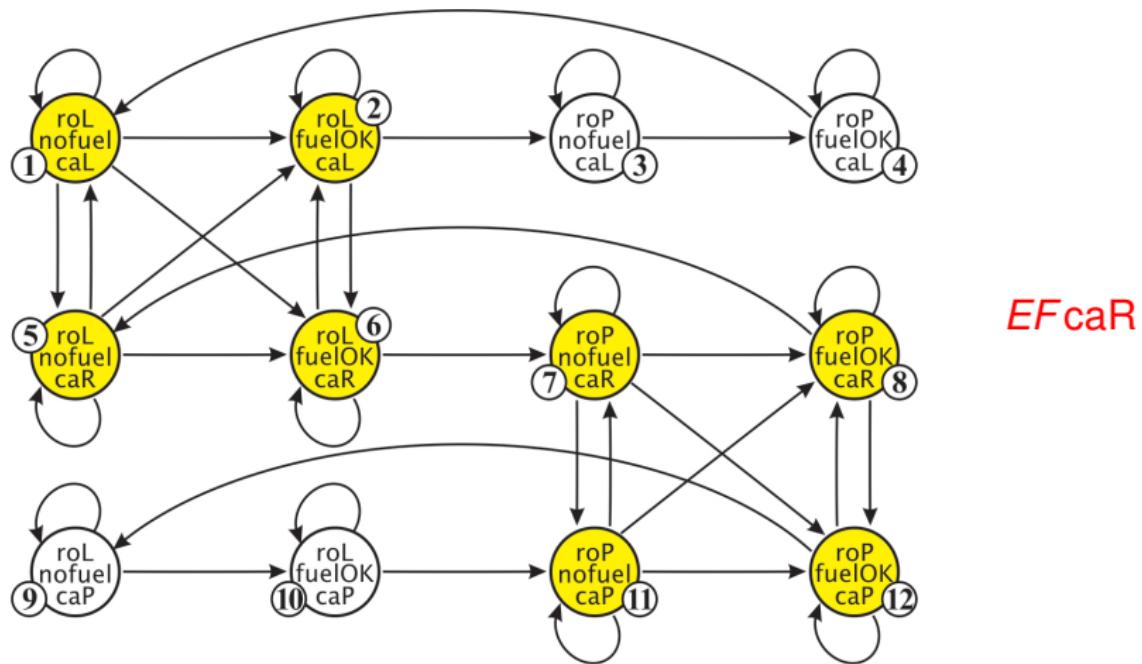
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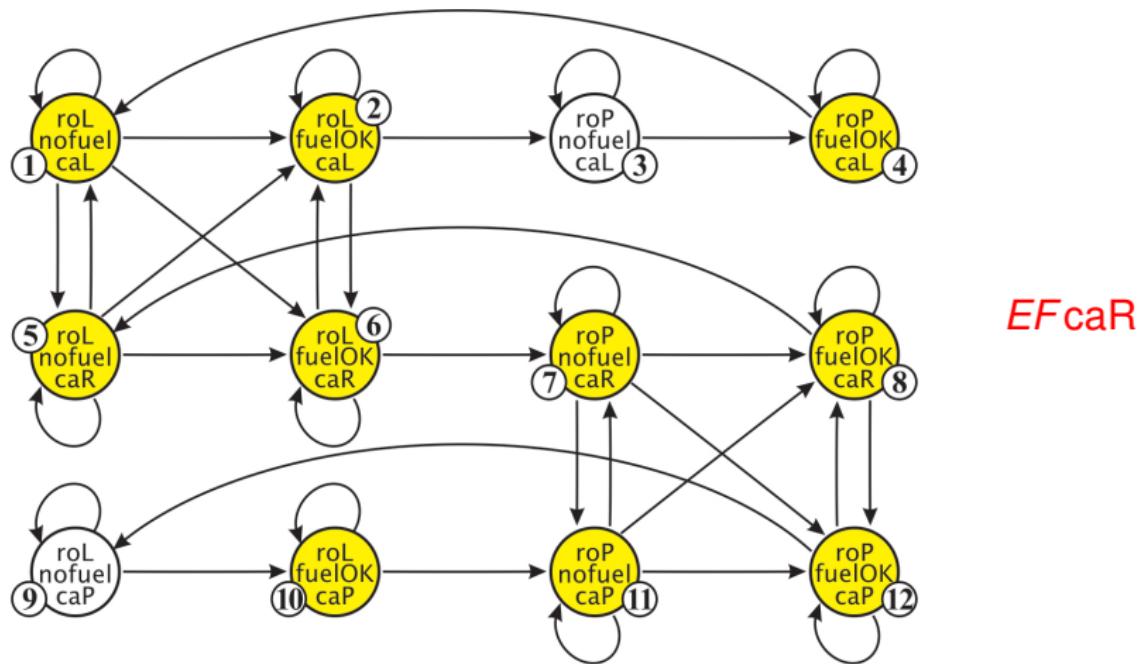
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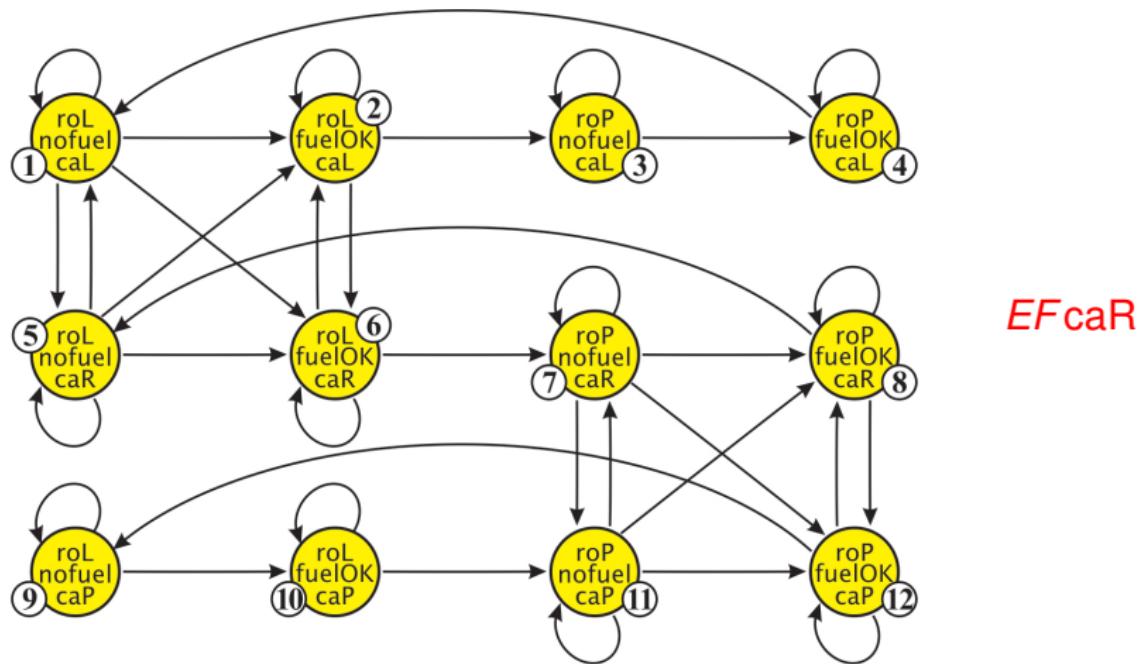
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Compute $mcheck(M, EFcaR)$



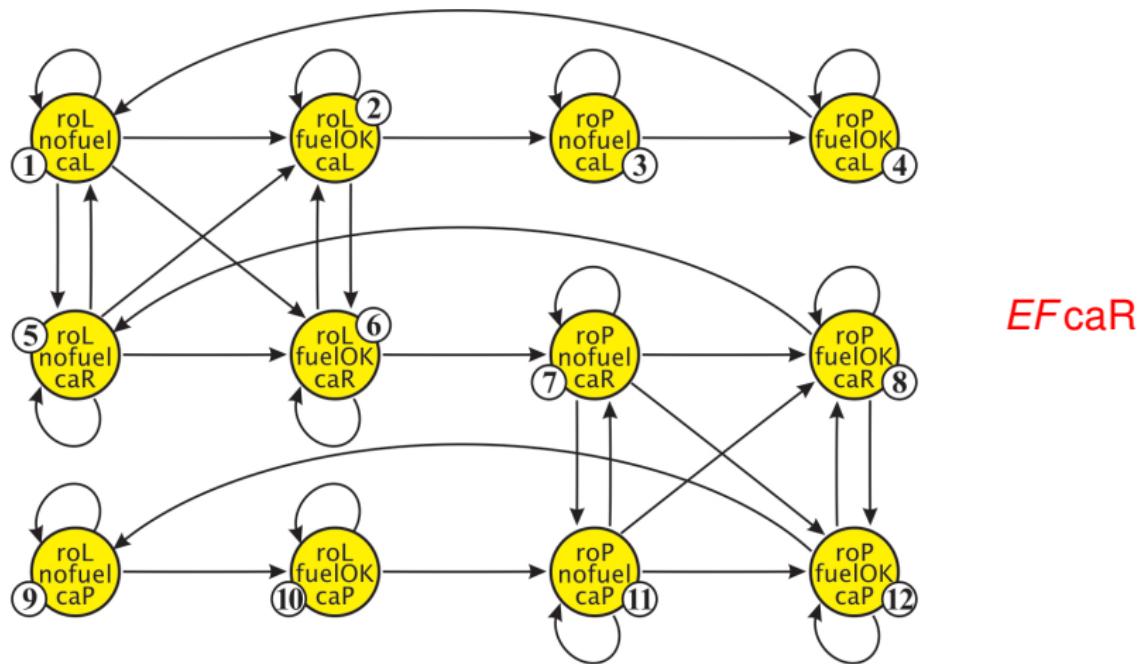
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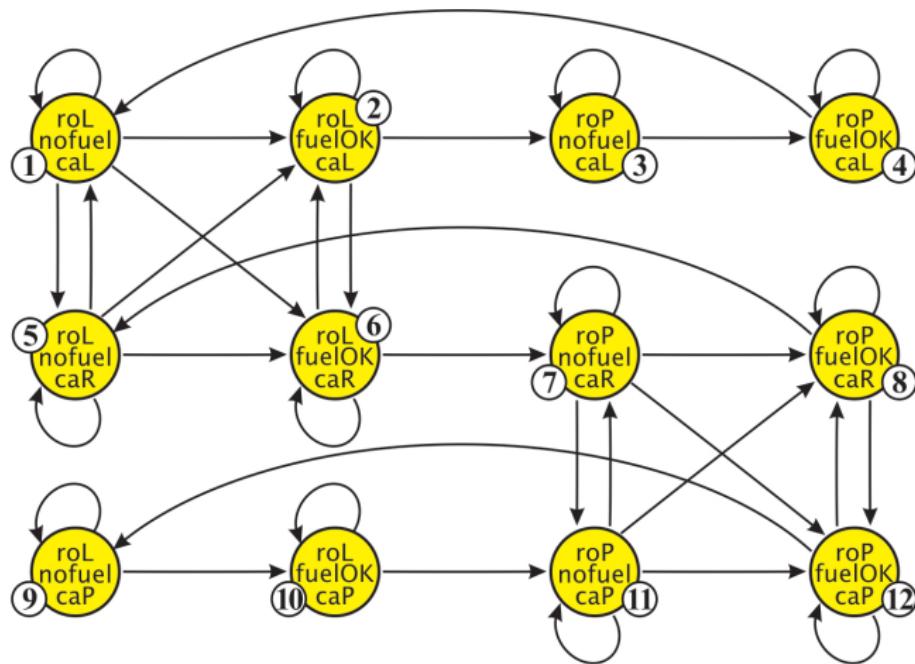
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Compute $mcheck(M, EFcaR)$



Example: Simple Rocket

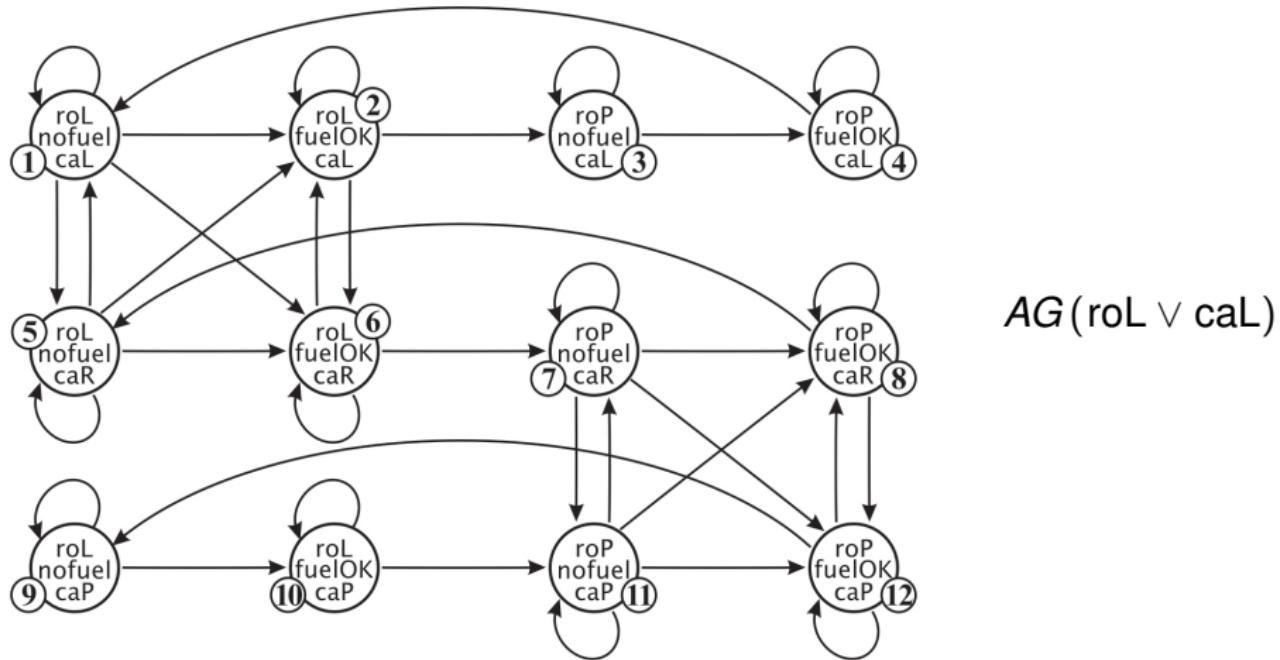
Compute $mcheck(M, EF_{caR})$



EF_{caR}
Done!

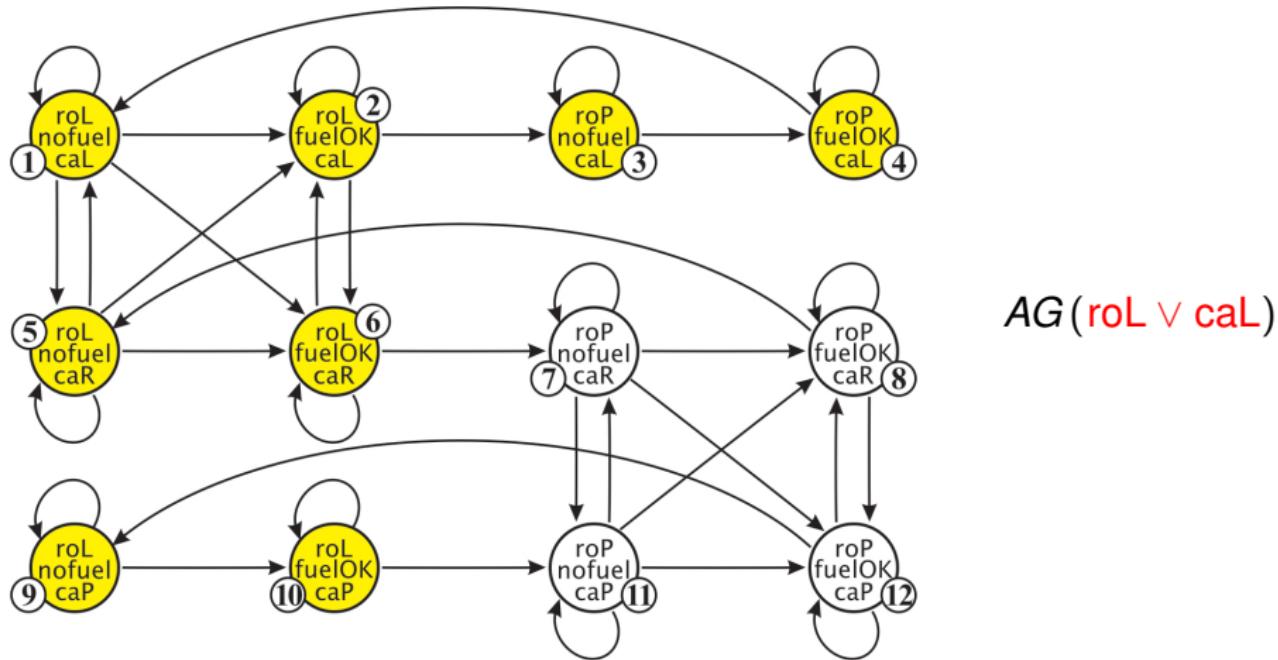
Example: Simple Rocket

Compute $mcheck(M, AG(\text{roL} \vee \text{caL}))$



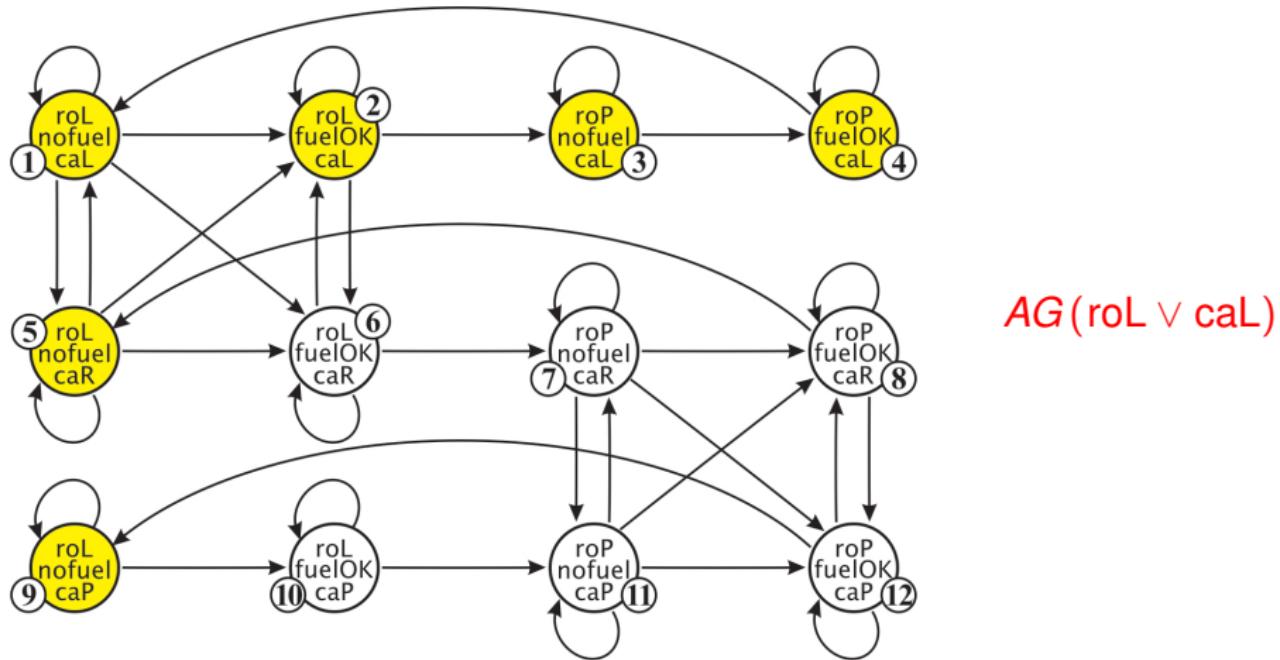
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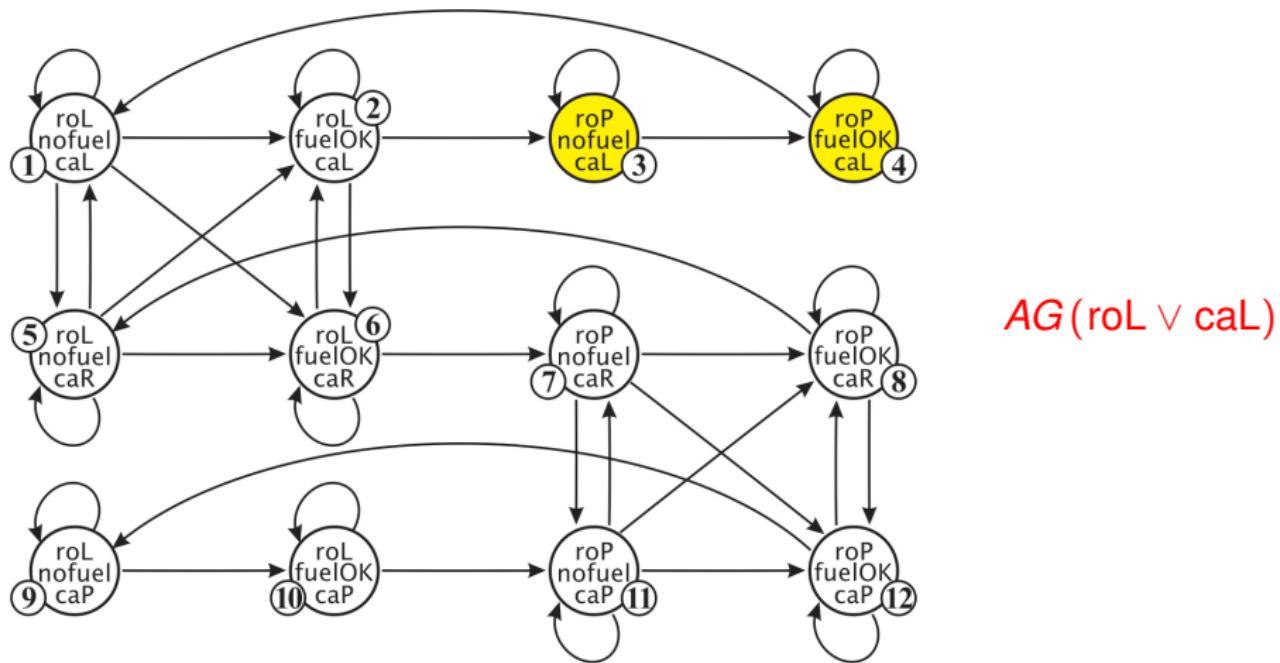
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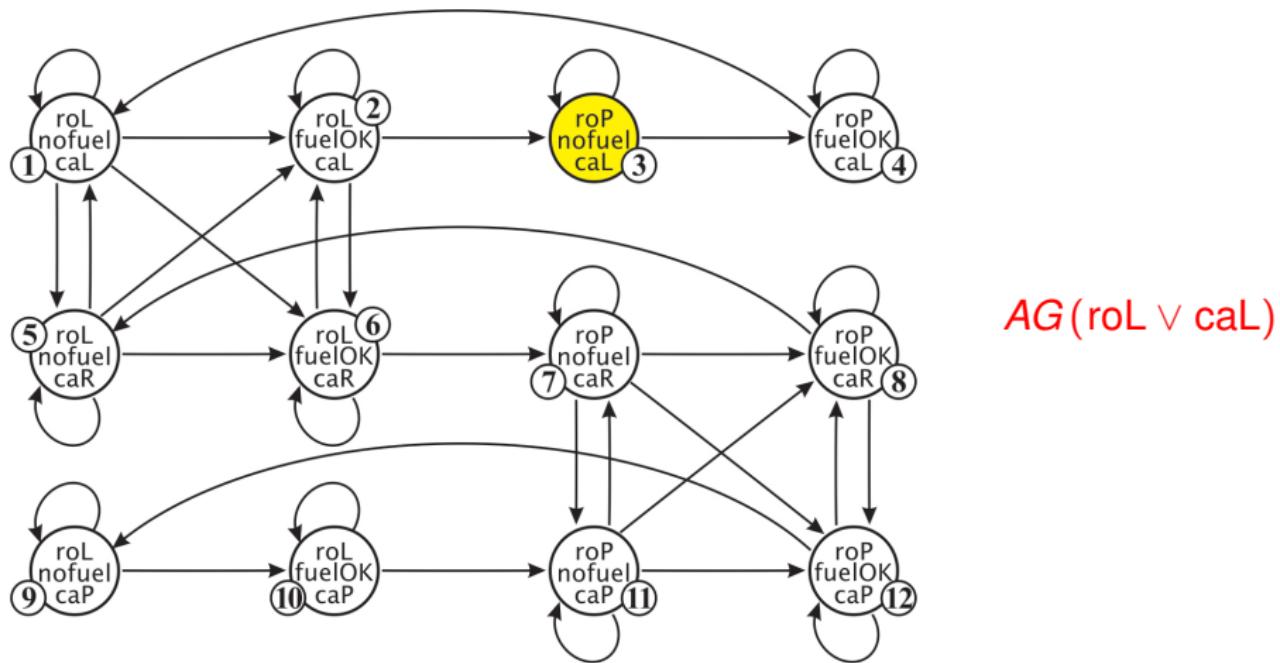
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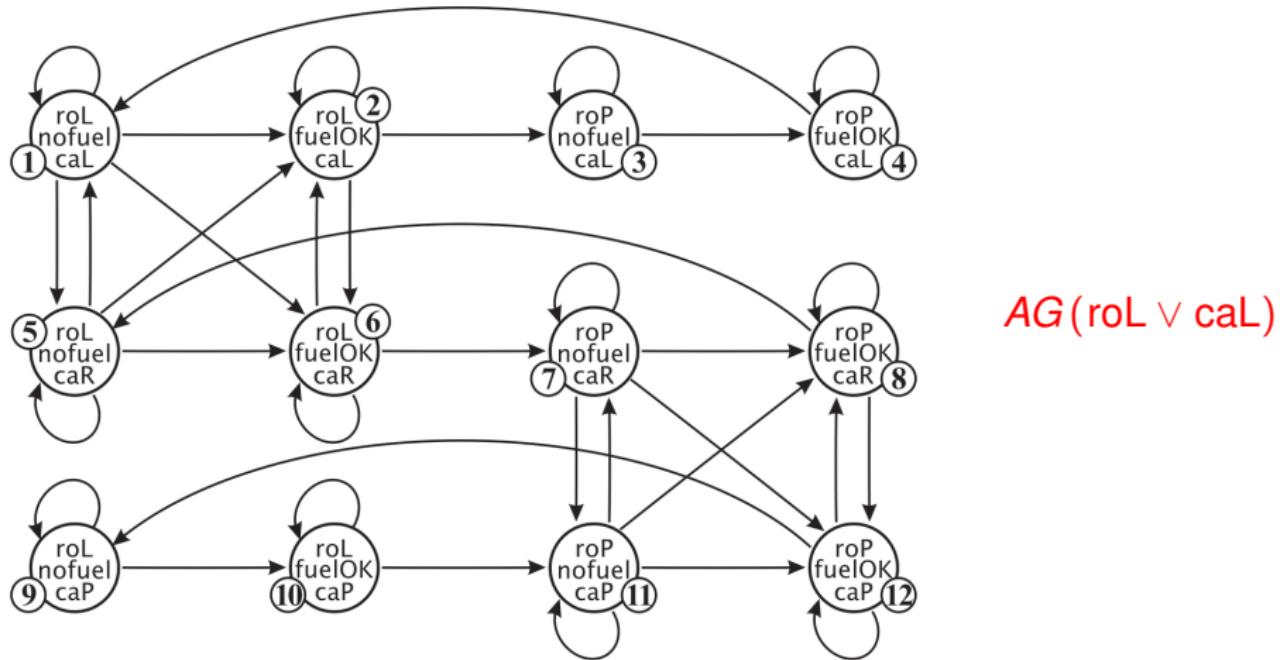
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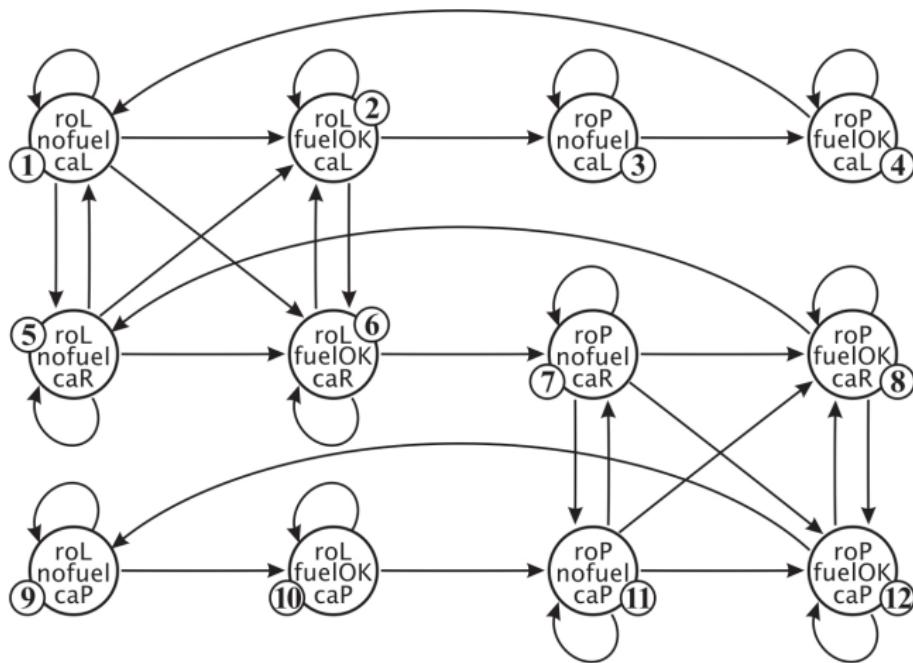
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Example: Simple Rocket

Compute $mcheck(M, AG(\text{roL} \vee \text{caL}))$



$AG(\text{roL} \vee \text{caL})$
Done!

Complexity of CTL Model Checking

Theorem (Complexity of CTL Model Checking)

*Model checking of CTL is **P-complete**, and can be done in time $O(m \cdot l)$ where **m** is the **number of transitions in the model** and the **l** is the number of subformulae in the formula.*