CTLK and Interpreted Systems

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CTLK

Combining modalities

- we have seen how to model and reason about knowledge and time using modal logic
- similarly, one can model other dimensions of a MAS: beliefs, desires, obligations, etc.
- how about **combining various dimensions** in one framework?

CTLK

CTLK combines temporal and epistemic logic

CTLK = CTL + Knowledge

- language includes both kinds of operators; for example:

$$AG\,C_{\{1,2\}}(\mathrm{pos0} \vee \mathrm{pos1} \vee \mathrm{pos2})$$

- models include both kinds of modal relations
- semantics: union of semantic clauses

Language of CTLK

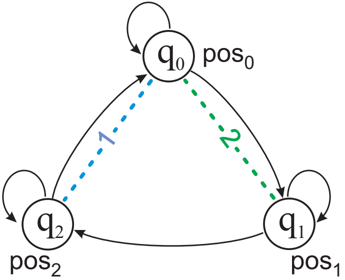
Definition (Syntax of CTLK)

$$p \mid \neg\varphi \mid \varphi \wedge \psi \mid EX\varphi \mid EF\varphi \mid EG\varphi \mid E\varphi\, U\, \psi \mid AX\varphi \mid AF\varphi \mid AG\varphi \mid A\varphi\, U\, \psi \mid$$

$$K_i\varphi \mid E_A\varphi \mid C_A\varphi \mid D_A\varphi$$

- where $p \in \mathcal{PV}$, $i \in Agt$ and $A \subseteq Agt$

Robots and Carriage revisited



$\neg EF(\bigvee_i K_1 \mathrm{pos_i} \wedge \bigvee_i K_2 \mathrm{pos_i})$

Interpreted Systems

Towards More Grounded Models

- Where do the global states come from in a multi-agent system?
- Each agent has its own view of the global state $\leadsto$ local state
- Some features of the environment might be invisible to all agents
- More grounded notions of epistemic state and global state

  $\leadsto$ **interpreted systems** by Halpern, Fagin, Moses, and Vardi

Interpreted Systems

- Global states are **tuples of local states** of individual agents
- St_i: set of local states of agent i
- St_{env}: set of local states of the environment
- Global states: $St \subseteq St_1 \times \cdots \times St_k \times St_{env}$

- Epistemic relations are based on local states:

$$\langle q_1, ..., q_k \rangle \sim_i \langle q'_1, ..., q'_k \rangle \text{ iff } q_i = q'_i$$

- Temporal dimension: **runs** (paths)

Interpreted Systems

Definition (System)

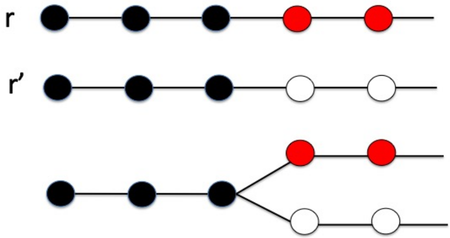
A **system** is a set of runs.

Definition (Interpreted system)

An **interpreted system** $\mathcal{I}$ is a set of runs $\mathcal{R}$ plus valuation of propositions: $\mathcal{V} : PV \rightarrow 2^{St}$.

Runs and Trees

A set of runs with a common initial state can be seen as a branching-time tree:



Applications of interpreted systems

Interpreted systems have been applied to modeling of synchrony and asynchrony, perfect recall, message passing systems, knowledge bases, distributed systems etc.

Semantics of CTLK in Interpreted Systems

- Formulae evaluated wrt time points $\langle r, m\rangle$: a run r plus a time moment m.
- That is, we have an implicit Kripke model with $St = \mathcal{R} \times \mathbb{N}$.
- Epistemic equivalence between points:

$$\langle r, m\rangle \sim_i \langle r', m'\rangle \text{ iff } r_m \sim_i r'_{m'}.$$

Semantics of CTLK in Interpreted Systems

Knowledge is interpreted as before:

- $\mathcal{I}, \langle r, m \rangle \models K_i \varphi$ iff $\mathcal{I}, \langle r', m' \rangle \models \varphi$ for every $\langle r', m' \rangle$ such that $\langle r, m \rangle \sim_i \langle r', m' \rangle$.
- similarly for E_A, C_A, D_A

Interpretation of temporal operators (AX, AU, AG are similar):

$\mathcal{I}, \langle r, m \rangle \models EX\varphi$	iff there is r' such that $r'[0...m] = r[0...m]$ and $\mathcal{I}, \langle r', m+1 \rangle \models \varphi$;
$\mathcal{I}, \langle r, m \rangle \models E\varphi\, U\, \psi$	iff there is r' such that $r'[0...m] = r[0...m]$ and $\mathcal{I}, \langle r', m' \rangle \models \psi$ for some $m' \geq m$ and $\mathcal{I}, \langle r', m'' \rangle \models \varphi$ for all m'' such that $m \leq m'' < m'$.
$\mathcal{I}, \langle r, m' \rangle \models EG\varphi$	iff there is r' such that $r'[0...m] = r[0...m]$ and $\mathcal{I}, \langle r', m' \rangle \models \varphi$ for all $m' \geq m$

Interpreted systems: infinite runs?

arguably we have a more grounded notions of epistemic state and global state, but at the same time an infinite set of infinite runs

however, as before, we assume that the runs come from an unfolding of a finite transition system

Reference

R. Fagin, J. Y. Halpern, Y. Moses, and M. Y. Vardi.
Reasoning about Knowledge.
MIT Press: Cambridge, MA, 1995.