ATL with Imperfect Information

Natasha Alechina Brian Logan

Utrecht University

n.a.alechina@uu.nl b.s.logan@uu.nl

Strategies under Uncertainty

- ATL includes no notion of knowledge (or, dually uncertainty)
- ...which makes reasoning in ATL rather unrealistic for MAS
- in this lecture, we show how to introduce knowledge and uncertainty into reasoning about strategic abilities

Motivating Example: Rescue Robots

Properties to express

- the robots can rescue person *i*
- the robots can rescue person *i*, and they know that they can
- the robots can rescue person *i*, and they know how to do it

Motivating Example: Rescue Robots

Different notions of knowledge (Moore)

- the robots know that they can rescue i: *knowledge de dicto*
- know that they have an action, but they may not know which action
- the robots know how to rescue i: *knowledge de re*
- know which action to perform
- the second notion is much more useful

Strategies and Knowledge

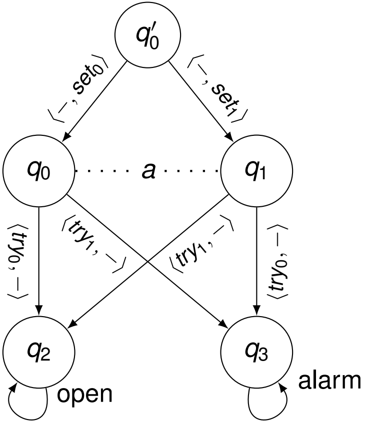
How can we reason about **multi-step games with imperfect information**?

Let's put **ATL** and **epistemic logic** together:

- we extend CGS with indistinguishability relations $\sim_a$, one per agent
- we add epistemic operators to ATL
- independent combination: the semantics is given by the union of semantic clauses of epistemic logic and ATL

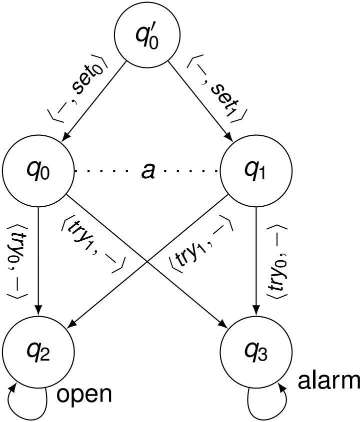
$\rightsquigarrow$ Problems!

Schobbens' Robber



Schobbens' Robber

We have a vault,
it can be open(q1) or closed (q0),
robber does not know which it is (q0 or q1).
Robber can have two actions (try0 or try1) which enter the right password and open the vault if chosen correctly, otherwise trigger the alarm.



Robber (a) does not know what the code is but:
in q_0',
$\langle\langle a \rangle\rangle F$open is true!
strategy:
$q_0 \mapsto try_0$,
$q_1 \mapsto try_1$,
even worse: in q_0 and q_1,
$K_a \langle\langle a \rangle\rangle F$open is true
this does not make sense!

The problem is, that standard ATL can say through a, that we can reach both q0 and q1.
In ATL, we can just say q0 > try0 and q1>try1 but unfortunately, robber does not know which state it is in, so this stops making sense in this way.

Strategies and Knowledge

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In many cases, we also mean that *A* are able to identify the strategy...

Strategies and Knowledge

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Strategic and epistemic abilities are *not* independent!

Coalition of agents A

$\langle\langle A \rangle\rangle \gamma$ = A can **enforce** γ

It should at least mean that A are able to execute the right strategy!

Executable strategies = uniform strategies

In many cases, we also mean that A are able to identify the strategy...

In order to identify a strategy as successful, the agents must check its outcome paths from indistinguishable states

Uniform Strategies

Definition (Uniform strategy)

Strategy s_a is **uniform** iff it specifies the same choices for indistinguishable situations:

- (no recall:) if $q \sim_a q'$ then $s_a(q) = s_a(q')$
- (perfect recall:) if $h \approx_a h'$ then $s_a(h) = s_a(h')$
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- (perfect recall:) if $h \approx_a h'$ then $s_a(h) = s_a(h')$

  where $h \approx_a h'$ iff $h[i] \sim_a h'[i]$ for every i If two outcome histories (h, h') are the same, same action should be chosen for both states

It is indistinguishable, when for both histories (h,h'), each point i in h corresponds to each point i in h'.

A collective strategy is uniform iff it consists only of uniform individual strategies

Uniform if every agent is executing a uniform strategy

Strategies and Knowledge

Note:

Having a successful strategy does not imply knowing that we have it!

Strategies and Knowledge

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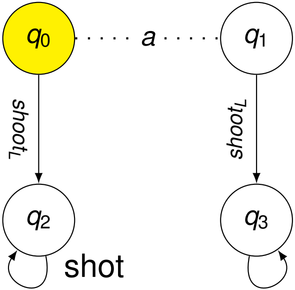
Having a successful strategy does not imply knowing that we have it!

Knowing that a successful strategy exists does not imply knowing the strategy itself!

Example: Poor Duck with Fixed Gun



There is a uniform strategy (same action in q_0 and q_1), but it only *works* from q_0, and it is not known to the agent that it works

Levels of Strategic Ability

Our cases for $\langle\langle A \rangle\rangle\gamma$ under imperfect information:

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Knowing How to Play

Case **[4]**: **knowing how to play**

- single agent case: we take into account the paths starting from indistinguishable states (i.e., $\bigcup_{q' \sim_a q} out(q', s_A)$)

- what about coalitions?
- question: in what sense should the coalition know the strategy?
- common knowledge (C_A), mutual knowledge (E_A), distributed knowledge (D_A)?

Four versions of ATL (Pierre-Yves Schobbens)

- ATL_{IR}: perfect Information and perfect Recall
- ATL_{Ir}: perfect Information and imperfect recall
- ATL_{iR}: imperfect information and perfect Recall
- ATL_{ir}: imperfect information and imperfect recall

Schobbens' ATL$_{ir}$

- **ATL$_{\mathbf{ir}}$**: Alternating-time logic with **imperfect information** and **imperfect recall** (Schobbens 2004)

- $\langle\langle a \rangle\rangle_{ir}\gamma$: agent *a* knows how to play to enforce γ from all the states she considers possible

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- what about coalitions?

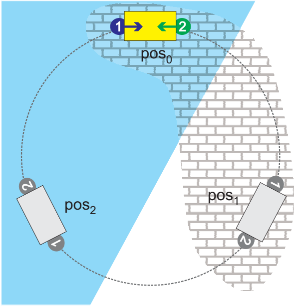
- $\langle\langle A \rangle\rangle_{ir} \gamma$: agents A know how to play in the sense of "everybody knows" (E_A)

Semantics of ATL$_{\text{ir}}$

Definition (Semantics of ATL$_{\text{ir}}$)

$M, q \models \langle\langle A \rangle\rangle_{ir} X \varphi$	iff there is a collective uniform strategy s_A such that, for every path $\lambda \in \bigcup_{q' \sim_A^E q} out(q', s_A)$, we have $M, \lambda[1] \models \varphi$
$M, q \models \langle\langle A \rangle\rangle_{ir} G \varphi$	iff there is a collective uniform strategy s_A such that, for every path $\lambda \in \bigcup_{q' \sim_A^E q} out(q', s_A)$, we have $M, \lambda[i] \models \varphi$ for all $i \geq 0$
$M, q \models \langle\langle A \rangle\rangle_{ir} \varphi_1 \, U \, \varphi_2$	iff there is a collective uniform strategy s_A such that, for every path $\lambda \in \bigcup_{q' \sim_A^E q} out(q', s_A)$, we have $M, \lambda[i] \models \varphi_2$ for some $i \geq 0$, and $M, \lambda[j] \models \varphi_1$ forall $0 \leq j < i$;

Example: Robots and Carriage



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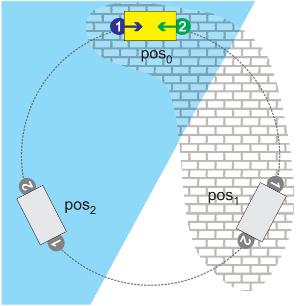

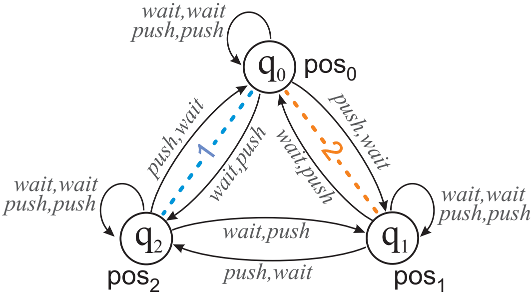

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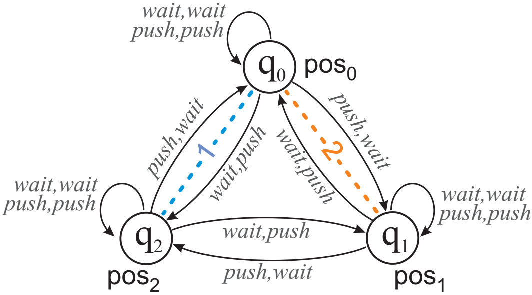

$\mathsf{pos}_0 \rightarrow \neg\langle\langle 1 \rangle\rangle_{ir}\, G \neg \mathsf{pos}_1$

$\mathsf{pos}_0 \rightarrow \neg\langle\langle 1, 2 \rangle\rangle_{ir}\, G \neg \mathsf{pos}_1$

$\mathsf{pos}_0 \rightarrow \langle\langle 1, 2 \rangle\rangle_{ir}\, F \mathsf{pos}_1$

Fixpoint (Non-)Equivalences

Interesting: $\langle\langle A \rangle\rangle_{ir}$ **are not fixpoint operators** any more!

Theorem

The following formulae are ***not*** *valid for* ATL_{ir}*:*

- $\langle\langle A \rangle\rangle_{ir} G \varphi \quad \leftrightarrow \quad \varphi \wedge \langle\langle A \rangle\rangle_{ir} X \langle\langle A \rangle\rangle_{ir} G \varphi$
- $\langle\langle A \rangle\rangle_{ir} \varphi_1 \, U \, \varphi_2 \quad \leftrightarrow \quad \varphi_2 \vee \varphi_1 \wedge \langle\langle A \rangle\rangle_{ir} X \langle\langle A \rangle\rangle_{ir} \varphi_1 \, U \, \varphi_2.$

Fixpoint (Non-)Equivalences

Conjecture

Strategy for A cannot be synthesized incrementally.

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Strategy for A cannot be synthesized incrementally.

Indeed...

Theorem (Schobbens 2004; Jamroga & Dix 2006)

Model checking ATL_{ir} is Δ_2-complete in the number of transitions in the model and the length of the formula.