Model checking ATL with Imperfect Information

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ATL_{ir}

- models of ATL_{ir} are called Concurrent Epistemic Game Structures (CEGS)
- a CEGS is a CGS with epistemic indistinguishability relations added, one for each agent
- $M = \langle Agt, St, \{\sim_i \mid i \in Agt\}, \mathcal{V}, Act, d, o\rangle$
- ATL_{ir} semantics:
- $\langle\!\langle a \rangle\!\rangle_{ir}\gamma$: agent a has a memoryless uniform strategy to enforce γ, and this strategy works from all the states a considers possible (the 2nd condition is called strong uniformity)
- $\langle\!\langle A \rangle\!\rangle_{ir}\gamma$: agents in A have a joint memoryless uniform strategy to enforce γ, and it works from all the states indistinguishable by $\sim_A^E = \bigcup_{a \in A} \sim_a$

Notation

- in lecture 8/1, coalition modalities $\langle\langle A \rangle\rangle_{ir}$ were subscripted with $_{ir}$ to stress they have a different meaning from $\langle\langle A \rangle\rangle$ in the lectures in weeks 6 & 7
- however, it is more common to use the same $\langle\langle A \rangle\rangle$ syntax whatever the semantics, and instead subscript the semantic relation: $\models_{ir}$

Semantics of ATL_{ir}

Definition (Semantics of ATL_{ir})

$M, q \models_{ir} \langle\langle A \rangle\rangle X \varphi$	iff there is a collective uniform strategy s_A such that, for every path $\lambda \in \bigcup_{q' \sim_A^E q} out(q', s_A)$, we have $M, \lambda[1] \models_{ir} \varphi$
$M, q \models_{ir} \langle\langle A \rangle\rangle G \varphi$	iff there is a collective uniform strategy s_A such that, for every path $\lambda \in \bigcup_{q' \sim_A^E q} out(q', s_A)$, we have $M, \lambda[i] \models_{ir} \varphi$ for all $i \geq 0$
$M, q \models_{ir} \langle\langle A \rangle\rangle \varphi_1 \, U \, \varphi_2$	iff there is a collective uniform strategy s_A such that, for every path $\lambda \in \bigcup_{q' \sim_A^E q} out(q', s_A)$, we have $M, \lambda[i] \models_{ir} \varphi_2$ for some $i \geq 0$, and $M, \lambda[j] \models_{ir} \varphi_1$ forall $0 \leq j < i$;

How to model check ATL$_{ir}$?

- fixpoint equivalences no longer hold
- we therefore need a different model checking algorithm
- basic idea: given a model M, generate all possible memoryless uniform strategies for the coalition A
- each strategy induces a new model M_i (where A just have a single joint action in each state, assigned by the strategy), and we check whether the property $\langle\langle A \rangle\rangle \varphi$ holds in at least one such model

Deterministic algorithm for checking $M, q \models_{ir} \langle\langle A \rangle\rangle \varphi$

In the initial model M

- $d : Agt \times St \to 2^{Act}$ defines actions available to an agent in a state
- e.g., $d(a_1, q_1) = \{\alpha^1_1, \ldots, \alpha^1_{m_1}\}$, $d(a_2, q_1) = \{\alpha^2_1, \ldots, \alpha^2_{m_2}\}$, etc.

We generate new models $M_1, \ldots, M_N$, where $N = O(|Act|^{|St|\,k})$

- $|Act|$ is the number of actions available
- $|St|$ is the number of states in M, and
- k is the number of agents in A

Induced models

- in each M_i, each agent in A is restricted to a single action
- e.g., $d_1(a_1, q_1) = \{\alpha_1^1\}$, $d_1(a_2, q_1) = \{\alpha_1^2\}$, $\ldots d_1(a_k, q_1) = \{\alpha_1^k\}$
- the d_i are such that, if two states q, q' are indistinguishable for an agent a_j, $q \sim_j q'$, then $d_i(a_j, q) = d_i(a_j, q')$, i.e., the agents' strategies are uniform
- the models $M_1, \ldots, M_N$ therefore encode all possible uniform strategies for the coalition A

Model checking $M, q \models_{ir} \langle\langle A \rangle\rangle \varphi$

- for each induced model, M_i, we run the standard ATL model checking algorithm $\textsc{mcheck}(M_i, \varphi)$ to label the states $[\langle\langle A \rangle\rangle \varphi]_{M_i}$ in which the uniform strategy encoded by M_i makes $\langle\langle A \rangle\rangle \varphi$ true
- we check whether $q \in [\langle\langle A \rangle\rangle \varphi]_{M_i}$
- for strong uniformity, we check if $\forall q' \in St$ such that $q \sim_A^E q'$ it holds that $q' \in [\langle\langle A \rangle\rangle \varphi]_{M_i}$
- i.e., that the uniform strategy encoded by M_i enforces $\langle\langle A \rangle\rangle \varphi$ in all states indistinguishable from q
- if such M_i is found, $M, q \models_{ir} \langle\langle A \rangle\rangle \varphi$ holds

Non-deterministic algorithm for checking $M, q \models_{ir} \langle\langle A \rangle\rangle \varphi$

- idea: when dealing with a subformula $\langle\langle A \rangle\rangle \varphi$, *guess* a uniform strategy s_A for A
- remove from M all A's actions not conforming to s_A
- check whether in the resulting model, all states $q' \sim_A^E q$ satisfy $\langle\langle A \rangle\rangle \varphi$ (in fact, can even use CTL model checking of $A\varphi$, since we are checking all paths generated by s_A)
- $\Delta_2^P = P^{NP}$ complexity: we need an NP oracle for guessing uniform strategies,the rest of the computation is polynomial
- ATL$_{ir}$ model checking is Δ_2^P-complete, so there is no polynomial algorithm (unless $P = \Delta_2^P$ which is even more unlikely than P = NP).

MCMAS

- unfortunately mcmas 1.1.0 does not yet have uniform strategies implemented
- from 1.2.1, there is a -uniform option
- it is implemented using the deterministic algorithm above, without the check for strong uniformity (but if the set of initial states includes all states connected by $\sim_A^E$, then strong uniformity holds too)