Coalition Logic

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Logics of Strategic Ability

- logics for reasoning about what (groups of) agents can and cannot achieve
- models look like state transition systems, but also like games
- CL: Coalition Logic (Pauly 2000-2001), this lecture
- ATL: Alternating-time Temporal Logic (Alur, Henzinger, Kupferman 1997-2002) (next couple of weeks)

can express: coalition *A* has a collective strategy to enforce some property

Strategies

- a *strategy* is a conditional plan (a mapping from states or sequences of states to actions, possibly joint actions by a coalition of agents)
- intuitively, a strategy is intended to work whatever the opponents (other agents, environment) do
- coalition logic looks at one-step strategies (choices of a single action):
- is there an action by a coalition of agents A such that whatever action other agents perform, A 'win' (make sure some property holds)?

Models: Concurrent Game Structures

- agents, actions, transitions between states
- atomic propositions + interpretation
- actions are executed concurrently (as opposed to turn-based games)
- transitions between states are by joint actions

Models: Concurrent Game Structures

Definition (Concurrent Game Structure)

A concurrent game structure is a tuple $M = \langle \mathbb{A}\mathrm{gt}, St, \mathcal{V}, Act, d, o \rangle$, where:

- $\mathbb{A}\mathrm{gt}$: a finite set of all agents
- St: a set of states
- $\mathcal{V}$: a valuation of propositions
- Act: a finite set of (atomic) actions
- $d : \mathbb{A}\mathrm{gt} \times St \to 2^{Act}$ defines actions available to an agent in a state
- o: a deterministic transition function that assigns outcome states $q' = o(q, \alpha_1, \ldots, \alpha_k)$ to states and tuples of actions actions are simultaneous: that's why the games are called concurrent

Some notation

- sometimes write $d_a(q)$ instead of $d(a, q)$ for agent a's available actions in q
- sometimes write $d_A(q)$ for joint actions available to a coalition of agents A
- $d_A(q)$ is a set of tuples of actions $\langle \alpha_1, \ldots, \alpha_m \rangle$, one for each agent in A
- formally, $d_A(q) = \Pi_{i \in A} d(i, q)$

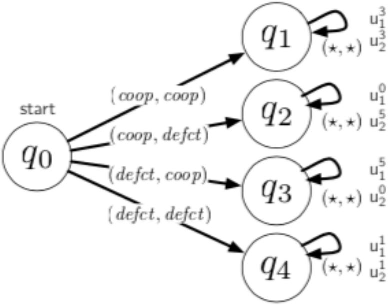
Prisoner's dilemma

Each of the two players can choose to 'cooperate' with the other one (i.e., play action *coop*) or to 'defect' (i.e., play action *defct*).

1/2	*coop*	*defct*
coop	(3,3)	(0,5)
defct	(5,0)	(1,1)

Example: robots and carriage

1/2	*coop*	*defct*
coop	(3,3)	(0,5)
defct	(5,0)	(1,1)



Prisoner's dilemma as a CGS

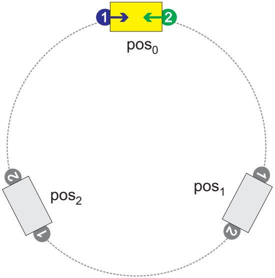
The corresponding CGS is
$M_{pris} = \langle \{1,2\}, \{q_0, q_1, q_2, q_3, q_4\}, \mathcal{V}, \{coop, defct\}, d, o \rangle$, where:

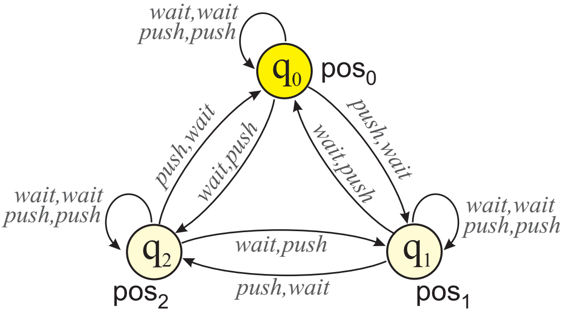
- q_0 is the initial state, $q_1 = (3,3)$, $q_2 = (0,5)$, $q_3 = (5,0)$ and $q_4 = (1,1)$)
- $\mathcal{V}$ assigns sets of states to propositions of the form u_a^j where j is utility and a agent
  $\mathcal{V}(\mathrm{u}_1^3) = \{q_1\}$, $\mathcal{V}(\mathrm{u}_1^0) = \{q_2\}$, $\mathcal{V}(\mathrm{u}_1^5) = \{q_3\}$, $\mathcal{V}(\mathrm{u}_1^1) = \{q_4\}$, similarly for utility variables for agent 2. In addition, $\mathcal{V}(\mathrm{start}) = \{q_0\}$.
- $d(a,q) = \{coop, defct\}$ (d makes all actions available to all agents in all states)
- o is like in the game description: $o(q_0, coop, coop) = q_1$, $o(q_0, coop, defct) = q_2$, $o(q_0, defct, coop) = q_3$, $o(q_0, defct, defct) = q_4$. In addition, $o(q_i, a_1, a_2) = q_i$ for $i \in \{1,2,3,4\}$ and $a_1, a_2 \in \{coop, defct\}$.

What we want to be able to say about one step strategies in prisoner's dilemma

- there is a strategy for both players to ensure utility of 3 for each of them
- player 1 cannot ensure getting utility of 5 on his own (he can defect, but if the other player defects as well, both get utility 1)

Example: robots and carriage





Robots and carriage as a CGS

$M_{carr4} = \langle \{1, 2\}, \{q_0, q_1, q_2\}, \mathcal{V}, \{wait, push\}, d, o \rangle$

- states and $\mathcal{V}$ are as before
- $Act = \{wait, push\}$
- $d(a, q) = \{wait, push\}$ for any q and a
- $o(q, wait, wait) = o(q, push, push) = q$, $o(q_i, push, wait) = q_{i+1}$ and $o(q_i, wait, push) = q_{i-1}$ (addition modulo 2).

Sample properties of one-step strategies for the robots

- in state q_0, there is a strategy for both agents to ensure in the next step the carriage is in pos1
- in state q_0, there is a strategy for both agents to ensure in the next step the carriage is in pos2
- agent 1 cannot on its own ensure that in the next step the carriage is in pos1 (it can push, but if the other agent pushes too, the carriage does not move)

Language of Coalition Logic

Syntax of coalition logic

$\varphi ::= \mathsf{p} \mid \neg\varphi \mid \varphi \wedge \varphi \mid [A]\varphi$

where p $\in$ *PV* and $A \subseteq \mathbb{A}\mathrm{gt}$

Semantics of Coalition Logic

Definition (Semantics of CL)

- $\mathcal{M}, q \models p$ iff $q \in \mathcal{V}(p)$;
- $\mathcal{M}, q \models \neg\varphi$ iff not $\mathcal{M}, q \models \varphi$;
- $\mathcal{M}, q \models \varphi \wedge \psi$ iff $\mathcal{M}, q \models \varphi$ and $\mathcal{M}, q \models \psi$;
- $\mathcal{M}, q \models [A]\varphi$ iff there is a collective action $\alpha_A \in d_A(q)$ such that, for every response $\alpha_{\mathbb{A}\mathrm{gt}\setminus A} \in d_{\mathbb{A}\mathrm{gt}\setminus A}$, we have that $\mathcal{M}, o(q, \alpha_A, \alpha_{\mathbb{A}\mathrm{gt}\setminus A}) \models \varphi$

Example: prisoner's dilemma

- $\mathcal{M}_{pris}, q_0 \models [\{1,2\}](\mathrm{u}_1^3 \wedge \mathrm{u}_2^3)$ iff there is a collective action $\alpha_{\{1,2\}} \in d_{\{1,2\}}(q_0)$ such that, for every response $\alpha_{\mathbb{A}\mathrm{gt}\setminus\{1,2\}} \in d_{\mathbb{A}\mathrm{gt}\setminus\{1,2\}}$, we have that $\mathcal{M}_{pris}, o(q_0, \alpha_{\{1,2\}}, \alpha_{\mathbb{A}\mathrm{gt}\setminus\{1,2\}}) \models \mathrm{u}_1^3 \wedge \mathrm{u}_2^3$
- yes: there is an action by $\{1,2\}$, $(coop, coop)$ (there are no other agents outside $\{1,2\}$) such that $o(q, coop, coop) = q_1$ and $\mathcal{M}_{pris}, q_1 \models \mathrm{u}_1^3 \wedge \mathrm{u}_2^3$.
- $\mathcal{M}_{pris}, q_0 \models [\{1\}]\mathrm{u}_1^5$ iff there is a collective action $\alpha_{\{1\}} \in d_{\{1\}}(q_0)$ such that, for every response $\alpha_{\mathbb{A}\mathrm{gt}\setminus\{1\}} \in d_{\mathbb{A}\mathrm{gt}\setminus\{1\}}(q_0)$, we have that $\mathcal{M}_{pris}, o(q_0, \alpha_{\{1\}}, \alpha_{\mathbb{A}\mathrm{gt}\setminus\{1\}}) \models \mathrm{u}_1^5$
- no: if 1 does *defct*, and 2 does *defct* too, 1 does not get u_1^5.

Example: two robots

- $\mathcal{M}_{carr4}, q_0 \models [\{1,2\}]$pos1 iff there is a collective action $\alpha_{\{1,2\}} \in d_{\{1,2\}}(q_0)$ such that, for every response $\alpha_{\mathbb{A}\mathrm{gt}\setminus\{1,2\}} \in d_{\mathbb{A}\mathrm{gt}\setminus\{1,2\}}(q_0)$, we have that $\mathcal{M}_{carr4}, o(q_0, \alpha_{\{1,2\}}, \alpha_{\mathbb{A}\mathrm{gt}\setminus\{1,2\}}) \models$ pos1
- yes: $\alpha_{\{1,2\}} = \langle push, wait \rangle$
- $\mathcal{M}_{carr4}, q_0 \models [\{1\}]$pos1 iff there is a collective action $\alpha_{\{1\}} \in d_{\{1\}}(q_0)$ such that, for every response $\alpha_{\mathbb{A}\mathrm{gt}\setminus\{1\}} \in d_{\mathbb{A}\mathrm{gt}\setminus\{1\}}$, we have that $\mathcal{M}_{carr4}, o(q_0, \alpha_{\{1\}}, \alpha_{\mathbb{A}\mathrm{gt}\setminus\{1\}}) \models$ pos1
- no: if $\alpha_{\{1\}} = push$, for $\alpha_{\mathbb{A}\mathrm{gt}\setminus\{1\}} = wait$, the resulting state $o(q_0, \alpha_{\{1\}}, \alpha_{\mathbb{A}\mathrm{gt}\setminus\{1\}})$ does not satisfy pos1