ATL with Imperfect Information

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Strategies under Uncertainty

- ATL includes no notion of knowledge (or, dually uncertainty)
- ...which makes reasoning in ATL rather unrealistic for MAS
- in this lecture, we show how to introduce knowledge and uncertainty into reasoning about strategic abilities

Motivating Example: Rescue Robots

Properties to express

- the robots can rescue person *i*
- the robots can rescue person *i*, and they know that they can
- the robots can rescue person *i*, and they know how to do it

Motivating Example: Rescue Robots

Different notions of knowledge (Moore)

- the robots know that they can rescue i: *knowledge de dicto*
- know that they have an action, but they may not know which action
- the robots know how to rescue i: *knowledge de re*
- know which action to perform
- the second notion is much more useful

Strategies and Knowledge

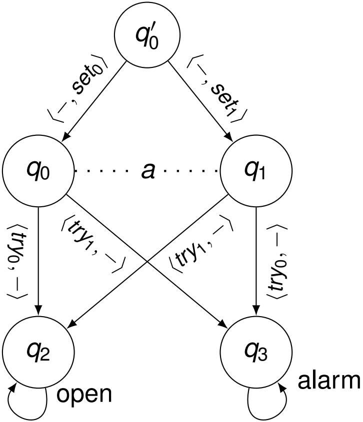
How can we reason about **multi-step games with imperfect information**?

Let's put **ATL** and **epistemic logic** together:

- we extend CGS with indistinguishability relations $\sim_a$, one per agent
- we add epistemic operators to ATL
- independent combination: the semantics is given by the union of semantic clauses of epistemic logic and ATL

$\rightsquigarrow$ Problems!

Schobbens' Robber



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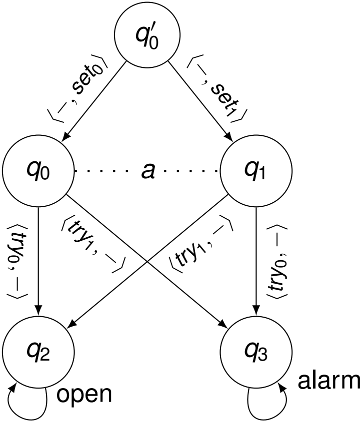

Robber (a) does not know what the code is but:
in q_0',
$\langle\langle a \rangle\rangle F$open is true!
strategy:
$q_0 \mapsto try_0$,
$q_1 \mapsto try_1$,
even worse: in q_0 and q_1,
$K_a \langle\langle a \rangle\rangle F$open is true
this does not make sense!

Strategies and Knowledge

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In many cases, we also mean that A are able to identify the strategy...

In order to identify a strategy as successful, the agents must check its outcome paths from indistinguishable states

Uniform Strategies

Definition (Uniform strategy)

Strategy s_a is **uniform** iff it specifies the same choices for indistinguishable situations:

- (no recall:) if $q \sim_a q'$ then $s_a(q) = s_a(q')$
- (perfect recall:) if $h \approx_a h'$ then $s_a(h) = s_a(h')$
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A collective strategy is uniform iff it consists only of uniform individual strategies

Strategies and Knowledge

Note:

Having a successful strategy does not imply knowing that we have it!

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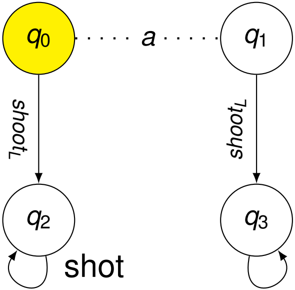
Note:

Having a successful strategy does not imply knowing that we have it!

Knowing that a successful strategy exists does not imply knowing the strategy itself!

Example: Poor Duck with Fixed Gun



There is a uniform strategy (same action in q_0 and q_1), but it only *works* from q_0, and it is not known to the agent that it works

Levels of Strategic Ability

Our cases for $\langle\langle A \rangle\rangle\gamma$ under imperfect information:

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Knowing How to Play

Case **[4]**: **knowing how to play**

- single agent case: we take into account the paths starting from indistinguishable states (i.e., $\bigcup_{q' \sim_a q} out(q', s_A)$)

- what about coalitions?
- question: in what sense should the coalition know the strategy?
- common knowledge (C_A), mutual knowledge (E_A), distributed knowledge (D_A)?

Four versions of ATL (Pierre-Yves Schobbens)

- ATL_{IR}: perfect Information and perfect Recall
- ATL_{Ir}: perfect Information and imperfect recall
- ATL_{iR}: imperfect information and perfect Recall
- ATL_{ir}: imperfect information and imperfect recall

Schobbens' ATL_{ir}

- **$\mathbf{ATL_{ir}}$**: Alternating-time logic with **imperfect information** and **imperfect recall** (Schobbens 2004)

- $\langle\langle a \rangle\rangle_{ir}\gamma$: agent *a* knows how to play to enforce γ from all the states she considers possible

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- what about coalitions?

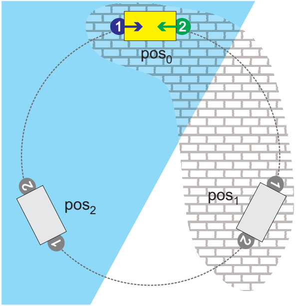
- $\langle\langle A\rangle\rangle_{ir}\gamma$: agents A know how to play in the sense of "everybody knows" (E_A)

Semantics of ATL_{ir}

Definition (Semantics of ATL_{ir})

$M, q \models \langle\langle A \rangle\rangle_{ir} X \varphi$	iff there is a collective uniform strategy s_A such that, for every path $\lambda \in \bigcup_{q' \sim_A^E q} out(q', s_A)$, we have $M, \lambda[1] \models \varphi$
$M, q \models \langle\langle A \rangle\rangle_{ir} G \varphi$	iff there is a collective uniform strategy s_A such that, for every path $\lambda \in \bigcup_{q' \sim_A^E q} out(q', s_A)$, we have $M, \lambda[i] \models \varphi$ for all $i \geq 0$
$M, q \models \langle\langle A \rangle\rangle_{ir} \varphi_1 \, U \, \varphi_2$	iff there is a collective uniform strategy s_A such that, for every path $\lambda \in \bigcup_{q' \sim_A^E q} out(q', s_A)$, we have $M, \lambda[i] \models \varphi_2$ for some $i \geq 0$, and $M, \lambda[j] \models \varphi_1$ forall $0 \leq j < i$;

Example: Robots and Carriage



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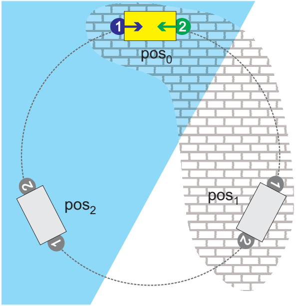

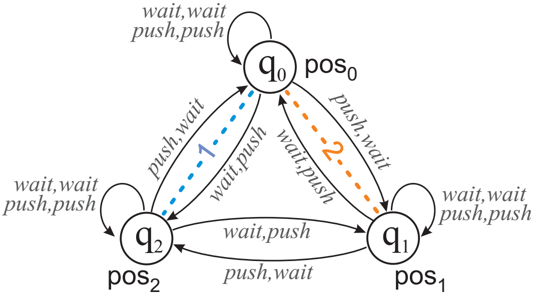

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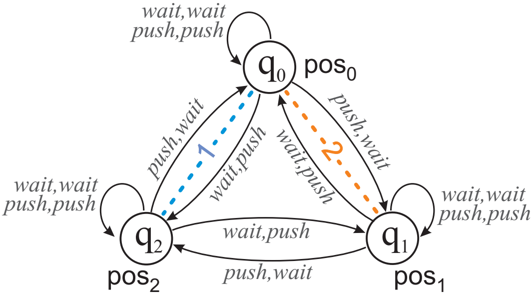

$\mathsf{pos}_0 \rightarrow \neg\langle\langle 1\rangle\rangle_{ir} G\neg\mathsf{pos}_1$

$\mathsf{pos}_0 \rightarrow \neg\langle\langle 1,2\rangle\rangle_{ir} G\neg\mathsf{pos}_1$

$\mathsf{pos}_0 \rightarrow \langle\langle 1,2\rangle\rangle_{ir} F\mathsf{pos}_1$

Fixpoint (Non-)Equivalences

Interesting: $\langle\langle A \rangle\rangle_{ir}$ **are not fixpoint operators** any more!

Theorem

*The following formulae are **not** valid for ATL_{ir}:*

- $\langle\langle A \rangle\rangle_{ir} G \varphi \quad \leftrightarrow \quad \varphi \wedge \langle\langle A \rangle\rangle_{ir} X \langle\langle A \rangle\rangle_{ir} G \varphi$
- $\langle\langle A \rangle\rangle_{ir} \varphi_1 \, U \, \varphi_2 \quad \leftrightarrow \quad \varphi_2 \vee \varphi_1 \wedge \langle\langle A \rangle\rangle_{ir} X \langle\langle A \rangle\rangle_{ir} \varphi_1 \, U \, \varphi_2.$

Fixpoint (Non-)Equivalences

Conjecture

Strategy for A cannot be synthesized incrementally.

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Conjecture

Strategy for *A* cannot be synthesized incrementally.

Indeed...

Theorem (Schobbens 2004; Jamroga & Dix 2006)

Model checking ATL_{ir} is Δ_2-complete in the number of transitions in the model and the length of the formula.