# Implied Volatility Under Heavy-Tailed Returns: A Monte Carlo Exploration with Student–$t$ Shocks

Otto Montgomery

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**Abstract**

The Black–Scholes model assumes Gaussian log-returns, which implies a flat implied-volatility curve across strikes. Equity options do not behave this way: implied volatility varies with moneyness and displays a pronounced skew. I replace the normal shock in geometric Brownian motion with a standardized Student–$t$ shock and use Monte Carlo simulation to isolate the effects of fat tails on option prices. The Gaussian Monte Carlo benchmark reproduces a flat surface over most strikes but shows an artificial uplift for very deep in-the-money calls due to numerical inversion. The Student–$t$ specification preserves this numerical effect on the left but adds a genuine right-hand skew: heavier positive tails increase the value of high-strike calls and raise their implied volatilities. Symmetric fat tails therefore account for the right wing of the volatility smile but cannot generate the strong downside skew observed in equity markets.

# 1 Introduction

The Black–Scholes model prices options under the assumption that log-returns are normally distributed with constant variance. Under this Gaussian structure all higher moments are pinned down by a single volatility parameter, and the model predicts a flat implied-volatility surface in strike and maturity.

Actual equity options look very different. When implied volatility is plotted against moneyness for a fixed maturity, the curve bends sharply in the wings: out-of-the-money puts and, to a lesser extent, out-of-the-money calls trade at higher implied volatilities than at-the-money contracts. This "smile" or skew is standard evidence that the Black–Scholes assumptions are too simple [1].

This project uses Monte Carlo simulation to isolate how fat tails alone deform the implied-volatility surface. I replace the Gaussian return shock with a standardized Student–$t$ shock and compare the implied volatilities recovered from simulated option prices. The aim is to identify which features of the volatility smile arise purely from heavier tails and which features require additional structure such as asymmetry or stochastic volatility. I am not proposing a new model; the goal is to measure the specific contribution of tail thickness within an otherwise standard geometric Brownian motion framework.

# 2 Black–Scholes Benchmark

The Black–Scholes framework assumes the underlying price follows geometric Brownian motion:

$$dS_t = \mu S_t \, dt + \sigma S_t \, dW_t, \tag{1}$$

where $\mu$ is the drift, $\sigma$ is constant volatility, and $W_t$ is a standard Brownian motion. Under the risk-neutral measure, the drift becomes $\mu = r$.

Solving this stochastic differential equation yields

$$S_t = S_0 \exp\!\left(\left(r - \tfrac{1}{2}\sigma^2\right) t + \sigma W_t\right), \tag{2}$$

so that

$$\ln\!\left(\frac{S_t}{S_0}\right) \sim N\!\left((r - \tfrac{1}{2}\sigma^2)t,\ \sigma^2 t\right).$$

The call price has the familiar closed form

$$C = S_0 \, \Phi(d_1) - K e^{-rT} \, \Phi(d_2), \tag{3}$$

with

$$d_1 = \frac{\ln(S_0/K) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}, \qquad d_2 = d_1 - \sigma\sqrt{T}.$$

Because $\sigma$ is the only volatility parameter, Black–Scholes implies a flat implied-volatility surface. Figure 1 shows this behavior for a 45-day maturity.

Real option data look very different. Figure 2 shows the implied-volatility curve for AAPL options with roughly 45 days to expiry: implied volatility is highest for low strikes, dips near the money, and rises again for high strikes.

The contrast between Figures 1 and 2 shows that the Gaussian log-return assumption does not adequately describe equity return dynamics [2].

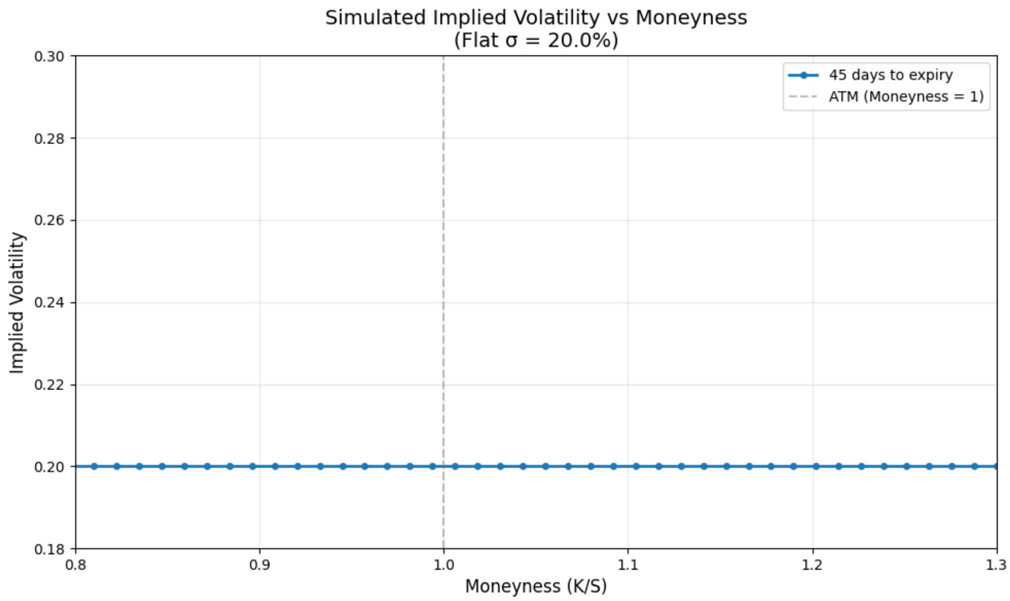


Figure 1: Black–Scholes implied volatility for a 45-day maturity under constant-$\sigma$ lognormal returns. The curve is flat across moneyness.

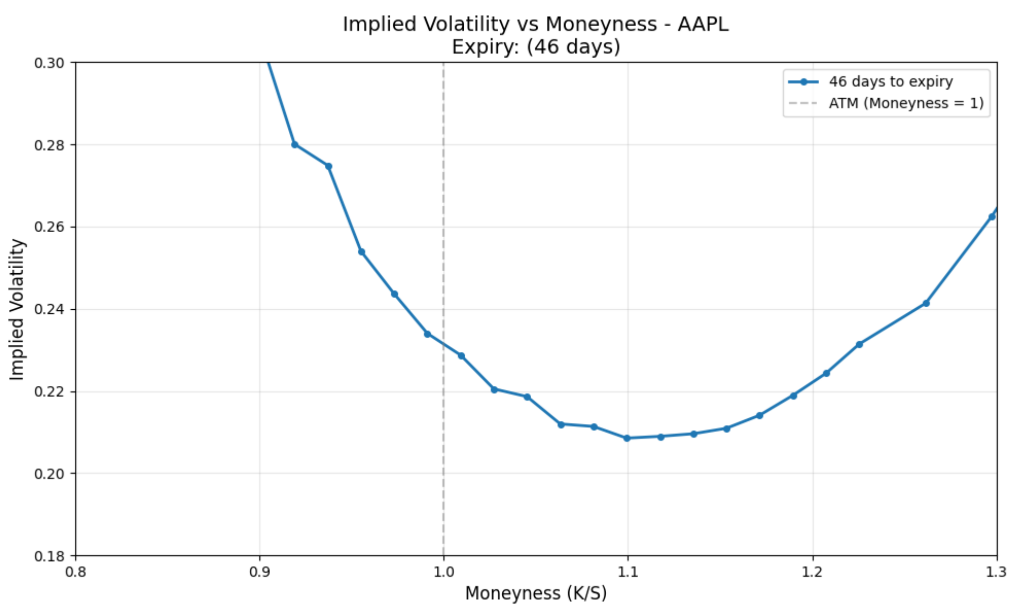


Figure 2: Implied-volatility smile for AAPL options at a 45-day maturity, plotted against moneyness $K/S_0$.

## 3 Heavy-Tailed Return Model

To introduce heavier tails while changing as little as possible, I keep the exponential price update but replace the normal increment with a standardized Student–$t$ shock. A raw $t_\nu$ variable has variance

$$\mathrm{Var}(t_\nu) = \frac{\nu}{\nu - 2}, \qquad \nu > 2.$$

I rescale it to unit variance:

$$\varepsilon_t = \frac{t_\nu}{\sqrt{\nu/(\nu-2)}}, \qquad \mathrm{Var}(\varepsilon_t) = 1.$$

Standardization ensures that differences in option prices arise from tail behavior rather than scale.

The resulting price update is

$$S_{t+\Delta t} = S_t \exp\Big[(r - \tfrac{1}{2}\sigma^2)\Delta t + \sigma\sqrt{\Delta t}\,\varepsilon_t\Big]. \tag{4}$$

I focus on $\nu \in [2.0, 4.0]$: values near 2 generate very heavy tails; values near 4 are close to Gaussian.

## 4 Monte Carlo Setup

All code used to generate simulations, figures, and implied-volatility curves is available at

`https://github.com/ottomontgomery/Heavy-Tail-Options-Simulation`.

I fix the baseline parameters

$$S_0 = 100, \qquad r = 0.02, \qquad \sigma = 0.20, \qquad T = 45/365.$$

I use moneyness levels $K/S_0 \in [0.80, 1.30]$ and simulate daily steps over $[0, T]$. For each model I simulate 100,000 paths and compute discounted call payoffs

$$\Pi^{(i)}(K) = e^{-rT}\,(S_T^{(i)} - K)^+.$$

The Monte Carlo price at strike $K$ is

$$\hat{C}(K) = \frac{1}{N_{\mathrm{paths}}} \sum_{i=1}^{N_{\mathrm{paths}}} \Pi^{(i)}(K).$$

I then invert the Black–Scholes formula to compute implied volatilities $\hat{\sigma}(K)$.

Because call payoffs are convex, rare large positive shocks have disproportionate impact on expected payoffs, so fat-tailed return distributions can influence option prices even when extreme events remain unlikely.

## 5 Results

Figure 3 shows the implied-volatility curve obtained from the Monte Carlo simulation under Gaussian innovations. For strikes from roughly $K/S_0 \approx 0.9$ up through 1.3, the recovered implied volatility is flat at 20%, matching the Black–Scholes benchmark. For very deep in-the-money calls ($K/S_0 \lesssim 0.9$), the curve bends upward: the implied volatility inferred from the simulated prices rises to about 28% at $K/S_0 = 0.8$.

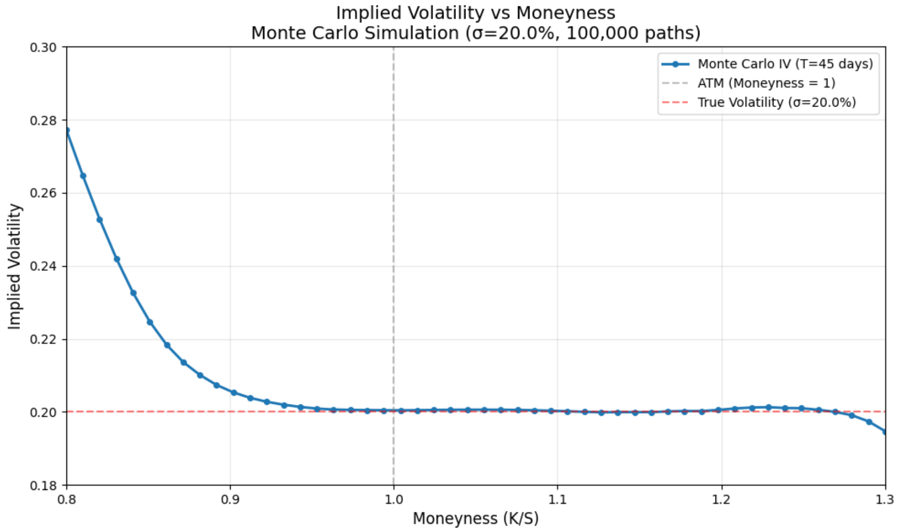


Figure 3: Monte Carlo implied volatility under Gaussian innovations. The curve is flat at 20% over most strikes; the uplift for very deep in-the-money calls is a numerical artifact of Monte Carlo pricing and Black–Scholes inversion.

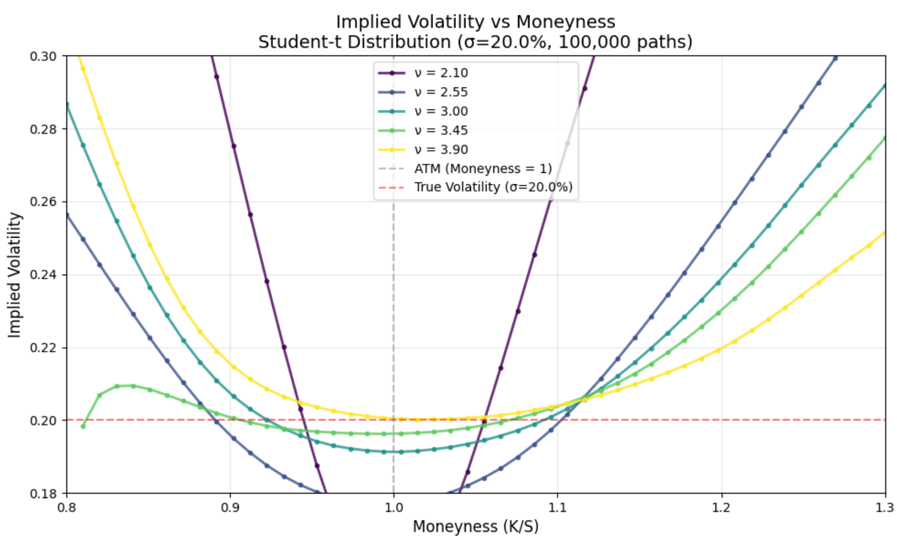


Figure 4: Monte Carlo implied volatilities under Student–$t$ innovations for several $\nu$. All curves share the deep in-the-money uplift from numerical effects; heavier tails generate additional right-hand curvature for high strikes.

This left-hand wing is a numerical effect rather than a feature of the model. Deep in-the-money call prices are dominated by intrinsic value, so the option's time value is small. Monte Carlo noise and discretization error therefore represent a large fraction of the time value, and the Black–Scholes inversion amplifies these small pricing errors into visible changes in implied volatility.[1]

Figure 4 shows the implied-volatility curves when the normal shocks are replaced by standardized Student–$t$ shocks for several values of $\nu$. All curves inherit the same deep in-the-money uplift on the left, for the same numerical reasons as in the Gaussian case. In addition, as $\nu$ decreases and the tails become heavier, the right wing bends upward: far out-of-the-money calls require higher implied volatilities than Black–Scholes predicts. Around and slightly below the money, the Student–$t$ curves remain close to the Gaussian Monte Carlo curve.

Taken together, these plots show that symmetric heavy tails primarily affect the right side of the volatility surface by increasing the value of high-strike, low-probability payoff events. The left-hand uplift visible in the Monte Carlo figures is mostly a byproduct of numerical inversion for very deep in-the-money calls, not a fundamental change in the pricing model.

## 6 Conclusion & Extensions

This project examines how implied volatility changes when Gaussian log-return shocks are replaced by heavier-tailed Student–$t$ shocks. The Gaussian benchmark produces the expected flat implied-volatility curve, aside from a numerical uplift for deep in-the-money strikes. The Student–$t$ specification inherits this left-hand artifact but adds a genuine right-hand skew: heavier positive tails raise the value of high-strike calls and increase their implied volatilities.

Comparing these curves to the example volatility smile derived from a true (AAPL) options chain shows that symmetric fat tails explain only part of the surface. The simulated left wing is far too shallow, because negative tail events do not increase call payoffs, while actual equity smiles reflect strong downside risk premia. The curves are also centered at the money, whereas equity smiles typically bottom at slightly higher strikes, indicating that symmetry in the return distribution is itself a limitation.

More broadly, this experiment does not identify a single degree-of-freedom parameter $\nu$ that matches market data, nor does it establish that Student–$t$ is the appropriate heavy-tailed family. Matching curvature does not guarantee accurate pricing in levels. These shortcomings point to the need for asymmetric return dynamics—such as downward jumps or stochastic volatility with leverage—if one aims to reproduce the full shape of the equity implied-volatility surface.

## References


[1] Roger W. Lee. "Implied Volatility: Statics, Dynamics, and Probabilistic Interpretation". In: *Recent Advances in Applied Probability* (2004).

[2] Bernd Walter. "The Equity Volatility Smile and Default Risk". 2007.


[1] If I price the same contracts directly from the Black–Scholes formula instead of from simulated payoffs, the implied-volatility curve is exactly flat at $\sigma = 20\%$ over the same strike range.