Ramsey Theory revision questions

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1. What is Ramsey Theory all about?
2. State Ramsey's Theorem.
3. Prove Ramsey's Theorem.
4. What type of proof is this called?
5. Explain how the above generalises to k-colourings.
6. Show that any sequence in a totally ordered set has a monotone subsequence.
7. State Ramsey's Theorem for r-sets.
8. Prove Ramsey's Theorem for r-sets.
9. Prove that given a collection of points (i, x_i) in the plane, the induced function on some subset of them is either concave or convex.
10. State Finite Ramsey.
11. Prove Finite RAmsey.
12. What type of argument is this called?
13. Let c be a colouring of $\mathbb{N}^{(2)}$ with an arbitrary set of colours. Then there exists an infinite $M \subset \mathbb{N}$ such that one of which four conditions must hold?
14. Prove this.
15. What are the canonical colourings fo the r-sets?
16. Define focused/colour-focused APs.
17. Define $W(m, k)$.
18. State Van der Waerden's Theorem.
19. Prove Van der Waerden's Theorem for $m = 3$.
20. Prove Van der Waerden's Theorem.
21. Define the Ackermann/Grzegorcyzk Hierarchy.
22. Define what it means for a function $f : \mathbb{N} \to \mathbb{N}$ to be of type n.

23. What is the type of our bound for $W(m, k)$ for fixed m?
24. What is the type of our bound for $W(m) = W(m, 2)$?
25. What did Shelah improve this bound to, and when?
26. Who offered how much for an $f_3(m)$ bound?
27. Who found an 'almost' type 2 bound, what was it, and when?
28. State a lower bound on $W(m)$.
29. Give two arguments to show that we cannot guarantee infinitely long mono APs.
30. State the Strengthened Van der Waerden Theorem.
31. Prove the Strengthened Van der Waerden Theorem.
32. What kind of bounds do we get here?
33. What is the case $m = 2$ called?
34. Prove this directly from Ramsey.
35. Define the n-dimensional cube on alphabet X.
36. Define a combinatorial line.
37. Define active coordinates.
38. State the Hales-Jewett Theorem.
39. Define $\mathrm{HJ}(m, k)$.
40. Show that HJ $\implies$ VdW.
41. Define what it means for lines to be focused/colour-focused.
42. Prove the Hales-Jewett Theorem.
43. Define a d-parameter set/d-dimensional subspace of X^n.
44. State the Extended Hales-Jewett Theorem.
45. Prove the Extended Hales-Jewett Theorem.
46. For $S \subset \mathbb{N}^d$ finite, define a homothetic copy of S.
47. State Gallai's Theorem.
48. Prove Gallai's Theorem.
49. Why didnt' we use extended HJ for this, when it might seem so natural?
50. Define a partition regular matrix A.
51. Express Schur's Theorem as a monochromatic solution to a linear equation.

52. Express Strengthened VdW similarly.
53. Define what it means for A to have the columns property.
54. State Rado's Theorem.
55. State a handy $(p-1)$-colouring of $\mathbb{N}$, for p prime.
56. Prove that if $(a_1 \ a_2 \ \ldots \ a_n)$ is PR, then $\sum_{i \in I} a_i = 0$ for some $\emptyset \neq I \subset [n]$.
57. Prove that, for $\lambda \in \mathbb{Q}$, W$\mathbb{N}$FC there exists mono x, y, z with $x + \lambda y = z$.
58. Prove Rado's Theorem for a single equation.
59. State Rado's Boundedness Conjecture.
60. How many colours suffice for a 1×3? Who proved this, and when?
61. Show that if A ($m \times n$ rational matrix) is PR, then it has the columns property.
62. Define an (m, p, c)-set
63. Define a row of an (m, p, c)-set.
64. What is a $(2, p, 1)$-set?
65. What is a $(2, p, 3)$-set?
66. Let $m, p, c \in \mathbb{N}$. Prove that W$\mathbb{N}$FC there exists a monochromatic (m, p, c)-set.
67. What does the $(m, 1, 1)$ case mean?
68. State the Finite Sums Theorem, and all of its other names.
69. What does this mean for $m = 2$?
70. Deduce the 'Finite Products Theorem'.
71. What about FS *and* FP?
72. Show that if matrix A has CP, then there exists m, p, c such that every (m, p, c)-set contains a solution of $Ax = 0$.
73. Prove Rado's Theorem, assuming any earlier results.
74. State and prove the Consistency Theorem.
75. Is this obvious?
76. Can this be proven directly?
77. Show that W$\mathbb{N}$FC there exists a colour class containing a solution to every PR system of equations.
78. State Rado's Conjecture.
79. Who proved it, and when?

80. What, roughly speaking, was their method?
81. State Hindman's Theorem.
82. Define a filter on $\mathbb{N}$.
83. Give some examples.
84. Define an ultrafilter on $\mathbb{N}$.
85. Give an equivalent characterisation of ultrafilters.
86. Show that every filter can be extended to an ultrafilter.
87. An ultrafilter is non-principal iff it extends which filter?
88. What ingredient do we need in order to actually get an ultrafilter?
89. Define the topological space $\beta\mathbb{N}$.
90. How can we view $\mathbb{N}$ as a subset of $\beta\mathbb{N}$?
91. Show that $\mathbb{N}$ is dense in $\beta\mathbb{N}$.
92. Prove that $\beta\mathbb{N}$ is Hausdorff.
93. Prove that $\beta\mathbb{N}$ is compact.
94. Give an alternative proof of compactness, via Tychonoff.
95. Why is $\beta\mathbb{N}$ interesting?
96. Define $\forall_{\mathcal{U}} x \; p(x)$.
97. Show that ultrafilter quantifiers play nicely with logical connectives.
98. Which logical property do ultrafilter quantifiers fail to uphold? Demonstrate this.
99. Define $\mathcal{U} + \mathcal{V}$, and show it is an ultrafilter.
100. Show that $+$ on $\beta\mathbb{N}$ is associative.
101. Show that $+$ is left-continuous.
102. Show that $+$ is not right-continuous.
103. Show that $+$ is not commutative.
104. State the Idempotent Lemma.
105. Prove the Idempotent Lemma.
106. Does $\beta\mathbb{N}$ contain any non-trivial finite subgroups?
107. Prove Hindman's Theorem.
108. Give a 2-colouring of $\mathbb{N}^{(\omega)}$ for which no $M \subset \mathbb{N}^{(\omega)}$ is monochromatic.
109. Define what it means for $Y \in \mathbb{N}^{(\omega)}$ to be Ramsey.

110. Give a base of open sets for the product topology on $\mathbb{N}^{(\omega)}$.
111. Give two other names for this topology on $\mathbb{N}^{(\omega)}$.
112. Define $(A, M)^{(\omega)}$.
113. Define M accepts A.
114. Define M rejects A.
115. If M accepts/rejects A, what can we say about any $L \in M^{(\omega)}$?
116. If M accepts A, then M accepts any $A \cup B$ where...?
117. Need M either accept or reject A?
118. State the Galvin-Prikry Lemma.
119. Prove the Galvin-Prikry Lemma.
120. Prove that if $Y \subset \mathbb{N}^{(\omega)}$ is open, then it is Ramsey.
121. Show also that closed sets are Ramsey.
122. Define the $*$-topology on $\mathbb{N}^{(\omega)}$, and give two other names for it.
123. Demonstrate that this really is a base for a topology.
124. Show that if Y is $*$-open, then it is Ramsey.
125. Hence show also that $*$-closed sets are Ramsey.
126. Define a completely Ramsey set.
127. Demonstrate that this is strictly stronger than being Ramsey.
128. Show that $*$-open/closed sets are completely Ramsey.
129. Define a nowhere dense subset of a general topological space.
130. Show that $*$-ND sets are CR.
131. Define a meagre set.
132. Show that $*$-meagre sets are ND.
133. Define a Baire set.
134. Show the Baire sets form a σ-algebra.
135. Show that Y is $*$-Baire iff it is completely Ramsey.
136. Why do we bother with meagre sets, since they are just the same as nohwere dense here?
137. Show that all τ-Borel sets are Ramsey.
138. Give a neat example of an applicaiton of this result.