Ramsey Theory: Example Sheet 1

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1. Let $f : [n] \to [m] \cup \{*\}$ such that $f^{-1}(\{*\}) \neq \emptyset$. We can identify f with a combinatorial line by $I = \{i : f(i) = *\}$ and for $i \notin I$, $x_i = f(i)$; similarly, given a combintorial line L there is a unique such f that represents it.

So the number of combinatorial lines in $[m]^n$ is just the number of such fs, which equals $(\#\{f : [n] \to [m] \cup \{*\}\} - \#\{f : [n] \to [m]\}) = (m+1)^n - m^n$.

4. Suppose that this is true, and let $c_n : \mathbb{N}^{(n)} \to [2]$ be the 2-colouring whereby A is red if $n \in A$, and blue otherwise. If $M \subseteq \mathbb{N}$ is infinite and monochromatic under c_n, then it cannot contain n since then its colour must be red, but there of course exists $A \subseteq M \backslash \{n\}$ with $|A| = n$, and then $c_n(A)$ is blue.

So let $c = \bigcup c_n$ be the 2-colouring of $\bigcup \mathbb{N}^{(n)}$. Then there exists an infinite $M \subseteq \mathbb{N}$ such that, for each r, c is constant on $M^{(r)}$. But for each r, $c \restriction M^{(r)} = c_r$, and since c_r is constant on M we conclude that $r \notin M$. But r was arbitrary, so M is empty, contradiction.

10. With choice: false. Wellorder $\mathbb{R}$ $(<_\alpha)$ and colour $\{a, b\}$ red if their order in the wellorder agrees with their natural order in $\mathbb{R}$. A monochromatic subset is then $M = \{a_i : i \in I\}$ with $a_i < a_j$ for $i < j$ such that either