A Reading Course in Computability and Logic

Thomas Forster

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This is a reading course for 20/21 Part III students in Mathematics. The choice of material is guided by a desire to show how ideas of Computability can be useful in Mathematics, rather than by a desire to develop these ideas for their own sake. Thus, for example, there is essentially no degree theory, beautiful tho' that stuff is. Its point of departure is the material, [8], which I have accumulated over the years for Part III courses with titles like the above, and which I would be lecturing in person were it not for COVID-19. It is linked (along with other resources) from my webpage at DPMMS: `www.dpmms.cam.ac.uk/~tf/cam_only/partiiimaterials.html`. This material is useful to students because it is (mostly) at the correct level and is also a useful source of exercises, many of which have worked answers in the appendix.

Students should read the first seven chapters. They should also read [5].

Students will be examined on the following topics:

Wellfounded relations; wellfounded induction and recursion; DC; Rank functions. Read chapter 2 of [8].

Primitive Recursion, Multiple Recursion and Ackermann's Function. Read Chapter 3 of [8].

Justification of recursive definitions. Gödel's β function or equivalent. See [8] section 3.1.2.

Knaster-Tarski. Read [9] ch 3.

Craig's theorem about recursive axiomatisability. See Remark 8, ch 4 of [8].

Myhill-Nerode. Read [8] section 2.1.2.

Lambda calculus. Representability of recursive functions by lambda terms; Church-Rosser. Read Paulson [11]; Pitts [12].

Gödel's incompleteness Theorem. Read [8] ch 7.

Trakhtenbrot's Theorem. Read [8] 7.4. Also ch 5 of [3].

Recursive and automatic Structures. Tennenbaum's theorem: read [10] and the appendix to [4].

The Rudiments of Automatic Groups: [2]; first two chapters of [6].

Defining the Naturals without quantifying over infinite Sets. Read section 2.7 of [8].

Baker-Gill-Solovay. Read [1].

Friedberg-Muchnik. Read chapter 7 of Soare [13].

References

[1] T. P. Baker, J. Gill, R. Solovay. "Relativizations of the P =? NP? Question". SIAM Journal on Computing, 4(4): 431-442 (1975).

[2] Baumslag. Review of [6] Bull Am Maths Soc **31** 1994 pp 86–91.

[3] Max Block. Undecidability of finite satisfiability and characterization of NP in finite model theory

`https://uu.diva-portal.org/smash/get/diva2:818862/FULLTEXT01.pdf`

[4] George S Boolos and Richard C Jeffrey "Computability and Logic", various editions. CUP The appendix relevant to Tennenbaum is also at `www.dpmms.cam.ac.uk/~tf/cam_only/tennenbaumstheorem.pdf`.

[5] N.J. Cutland "Computability, an Introduction to Recursive Function Theory", CUP (A pirate version seems to be online at `http://index-of.co.uk/Theory-of-Computation/An%20Introduction%20To%20Recursive%20Function%20Theory%20-Nigel%20Cutland.pdf`)

[6] David B. A. Epstein, J. W. Cannon, D. F. Holt, S. V. F. Levy, M. S. Paterson and W. P. Thurston. "Word Processing in Groups". Jones and Bartlett 1992.

see `www.dpmms.cam.ac.uk/~tf/cam_only/Word-Processing-in-Groups.pdf`

[7] Benson Farb "Automatic Groups a guided Tour". Enseignement Mathematique **38** (1992) pp 291–313. Also at `http://retro.seals.ch/digbib/view?rid=ensmat-001:1992:38::528&id=&id2=&id3=`

[8] Thomas Forster. Lecture Notes on Computability and Logic `www.dpmms.cam.ac.uk/~tf/cam_only/partiiicomputability2020.pdf`

[9] Thomas Forster. Logic, Induction and Sets, CUP ppback.

[10] R.W. Kaye. "Tennenbaum's theorem for models of arithmetic". `http://web.mat.bham.ac.uk/R.W.Kaye/papers/tennenbaum/tennenbaum.pdf`

[11] Larry Paulson's Computer Science 1b functional programming notes: `http://www.cl.cam.ac.uk/~lp15/papers/Notes/Founds-FP.pdf`

[12] Andrew Pitts. CS Lecture Notes for 1a RLFA:

`http://www.cl.cam.ac.uk/teaching/1112/RLFA/materials.html`

and IB Computation Theory:

`https://www.cl.cam.ac.uk/teaching/1112/CompTheory/comt-notes.pdf`

[13] Robert I Soare. Recursively enumerable sets and degree A study of Computable functions and Computably generated Sets. Springer 1980

Additional Support

I am available for zoom consultations every evening until the exams. I welcome queries concerning both the mathematical substance and the availability and suitability of further literature.

If you wish to pursue this course please make yourself known to me.