KARATINA UNIVERSITY

UNIVERSITY EXAMINATIONS

2024/2025 ACADEMIC YEAR

SECOND YEAR **FIRST** SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF

BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

COURSE CODE: ACS 212

COURSE TITLE: FUNDAMENTALS OF ACTUARIAL MATHEMATICS II

DATE: 10^{th} DEC, 2024 TIME: 3:00 PM - 5:00 PM

Instructions: See Inside

Answer question **ONE** in section A and any other **TWO** from section B.

SECTION A

Question **ONE** is compulsory

QUESTION ONE (30 Marks)

(a) Calculate using AM92 mortality at 4% , showing all your workings:

i. $10|5\,q_{65}$ **[2 marks]**

ii. $5P_{40}$ **[2 marks]**

iii. $10q_{20}$ **[2 marks]**

(b) Describe force of mortality **[2 marks]**

(c) In a certain population, $l_x = 50000(130 - x)^2$. Find the probability that;

(i) A newborn will not celebrate his 20th birthday. **[3 marks]**

(ii) A life aged 20 will die within ages 45 and 55. **[3 marks]**

(d) Why do insurance companies set reserves when operating in the market? **[4 marks]**

(e) Using AM92 mortality at 4% calculate the present value of an ordinary endowment assurance of 20 years issued to a life aged 35 with a sum assured of ksh. 1.5m.

[3 marks]

(f) A life aged 35 purchases a whole-life assurance cover with a sum assured of ksh 2m. Annual premiums are payable up to age 60. Calculate the policy reserves after 18 years. **[5 marks]**

Basis:

Mortality = AM92

Interest = 4%

Expenses = 5% of each premium

(g) Using AM92 Mortality at 4% , Evaluate:

(i) $10|\,\ddot{a}_{20:\overline{6}|}$ **[2 marks]**

(ii) $5|\,\ddot{a}_{22}$ **[2 marks]**

SECTION B

Answer **any Two** questions from this section

QUESTION TWO (20 Marks)

(a) A life aged 38 purchases a whole-life assurance policy with a sum assured of 2M and annual indefinite premiums. The office modifies the policy to include a return of the premium paid if death occurs within the first 12 years.

Calculate using AM92 mortality at interest, 4% and expenses are 5% of all premiums:

(i.) The Net premiums **[4 marks]**

(ii.) The Gross premiums **[4 marks]**

(b) (i) Name any two categories of expenses insured by life insurance companies.

[2 marks]

(ii) Give one example of each category in part (i) and indicate how it is usually allowed for, in the calculation of premiums. **[4 marks]**

(c) You are given that ${}_5p_{40} = 0.8$, ${}_{10}p_{45} = 0.6$, ${}_{10}p_{55} = 0.4$. Find the probability that a life aged 40 will die between the ages of 55 and 65. **[6 marks]**.

QUESTION THREE (20 Marks)

(a) Prove that

$$A_{x:\overline{n}|} = \frac{M_x - M_{x+n} + D_{x+n}}{D_x}$$

[4 marks]

(b) For a certain population $l_x = 70000(1+x)^2$. calculate the force of mortality at age 5 years. **[3 marks]**

(c) A life aged 35 effects a 25-year without profit endowment assurance policy with a sum assured of Kshs. 400,000 payable at the end of the year of death or on survival

to the end of the term. Level premiums are payable annually in advance throughout the term of the policy or until earlier death of the life assured. Calculate the level premium P, using the following basis

Mortality : AM92

Interest : 4% p.a

Expenses: None

[5 marks]

(d) Discuss the four types of life insurance products sold in the Kenyan market.

[8 marks]

QUESTION FOUR (20 marks)

(a) Prove that $N_x = D_x - dN_x$ **[4 marks]**

(b) The following are values taken from a life table

$$q_{85} = 0.1256, \quad q_{86} = 0.1375, \quad q_{87} = 0.1498, \quad q_{88} = 0.1616, \quad q_{89} = 0.1729$$

What is the probability of dying before reaching exactly age 90 for those who survive to exactly age 85? **[4 marks]**

(c) Assuming AM92 ultimate mortality, find the present value at 4% interest of a sum of ksh. 450000 due to a life aged 23 payable on:

i) survival of 30 years. **[3 marks]**

ii) Either death within 30 years or survival of 30 years. **[3 marks]**

(d) A life aged 40 purchases a 30-year with profit ordinary endowment assurance with sum assured of 2M. Annual premiums are payable for at most 20 years. The office approximates an initial expense of 2% of the sum assured and a renewal expense of 5% of all premiums and bonuses at a rate of 7% of the sum assured. Calculate

the gross premium payable (Assume the office uses a simple bonus accumulation method) **[6 marks]**

QUESTION FIVE (20 Marks)

(a) Twenty years ago a life office issued a large number of annual premium 20-year pure endowment policies with a sum assured of 1M to lives then aged 40, assuming mortality according to the A1967-70 ultimate table at 4% p.a interest with expenses of 2% of each premium. If the assumption on interest was correct but expenses were 3% of each premium and mortality experienced was A1967-70 rated down 10 years, find the average loss to the life office on each policy. **[10 marks]**

(b) A 25-year term assurance with a sum assured of ksh. 1M for a life aged 25 on the A1967-70 select table at 4 % interest has initial expenses of 2% of the sum assured and continuing administration cost of ksh. 1000 each year. Find the single premium payable given that the premium paid is:

(i) To be returned with the sum assured in case of a claim. **[5 marks]**

(ii) Not to be returned with the sum assured in case of a claim. **[5 marks]**