



# KARATINA UNIVERSITY

## UNIVERSITY EXAMINATIONS 2024/2025 ACADEMIC YEAR

### FOURTH YEAR FIRST SEMESTER REGULAR EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

**COURSE CODE: ACS 411**

**COURSE TITLE: MATHEMATICS FOR  
DEMOGRAPHY AND  
GRADUATION**

**DATE: 18<sup>TH</sup> DECEMBER 2024 TIME: 9.00-11.00 AM**

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### INSTRUCTIONS TO CANDIDATES

- **SEE INSIDE**

**INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS**

**QUESTION ONE (30 MARKS)**

- a) Distinguish between the following terms;
- (i) Total Fertility Rate (TFR) and Gross Reproduction Rate (GRR). (2 marks)
  - (ii) Infant Mortality Rate and Under Five Mortality. (2 marks)
  - (iii) Population estimation and Population projection. (2 marks)
- b) Why is the Infant Mortality Rate (IMR) considered an important indicator of population health? How does it relate to other demographic measures such as life expectancy and fertility? (4 marks)
- c) Explain how the separation factor is used in the adjustment of infant mortality rates. Why is this adjustment necessary? (4 marks)
- d) What is meant by "graduation" in the context of demographic analysis? (2 marks)
- e) The mortality rate at age 30 is 0.002, and at age 50 is 0.008. Assuming mortality follows a Gompertz function, estimate the mortality rate at age 40. (4 marks)
- f) The data below shows the proportion of women in rural and urban areas of a country X and the Age Specific Fertility Rates (ASFR) per woman by age in 2009.

Age	Proportion		ASFR	
	Urban Areas	Rural Areas	Urban Areas	Rural Areas
15-19	9.7	9.4	0.135	0.165
20-24	10.1	7.8	0.268	0.291
25-29	9.0	6.3	0.242	0.273
30-34	6.3	5.3	0.210	0.261
35-39	4.7	4.4	0.149	0.202
40-44	3.0	4.4	0.086	0.123
45-49	1.9	3.1	0.012	0.062

Given that, the total number of urban women in the survey is 1334 and the total number of rural women is 10518, calculate the Total fertility rate for X. (4 marks)

- g) The following data was collected from the out patients records of a county hospital.

Age	Males	Females
0-9	1688	1040
10-19	456	396
20-29	328	312
30-39	360	530
40-49	504	568
50+	528	456

Calculate the age-sex accuracy score (joint score). Explain the results. (6 marks)

### QUESTION TWO (20 MARKS)

- a) Distinguish between cohort fertility and period fertility. (2 marks)
- b) The following data is for a population:

Age Group (years)	Number of Births	Female Population
15-19	120	3,000
20-24	200	2,800
25-29	250	2,600
30-34	180	2,400
35-39	90	2,200
40-44	30	2,000

- (i) Calculate the **Age-Specific Fertility Rates (ASFR)** for each age group. Plot the **ASFR** curve. (6 marks)
- (ii) Calculate the **Total Fertility Rate (TFR)** for this population. Interpret the TFR in terms of average children per woman. (4 marks)
- (iii) Using the **TFR** calculated above, estimate the **Gross Reproduction Rate (GRR)** if 48% of the births are female. What does this value represent in terms of population replacement? (4 marks)
- (iv) Given a survival ratio of 0.95 for females from birth to reproductive age, calculate the **Net Reproduction Rate (NRR)**. Explain the demographic significance of the NRR being greater than, less than, or equal to 1. (4 marks)

### QUESTION THREE (20 MARKS)

- a) Distinguish between direct standardization and indirect standardization. (2 marks)
- b) In what situations would you prefer to use indirect standardization over direct standardization? (2 marks)
- c) Consider the following data on mortality for two populations, A and B, and a standard population. Calculate the age-standardized mortality rate for population A and B using **direct standardization**

Age group	Population A	Mortality rate per 1000	Population B	Mortality rate per 1000	Standard Population
0-19	8000	2.0	10,000	1.8	12,000
20-39	6000	3.0	8,000	2.5	10,000
40-59	4000	5.0	5,000	4.0	8,000
60+	2000	8.0	3,000	7.5	6,000

(6 marks)

- d) The table below shows the births and deaths for a given county for the years 2012 to 2014.

Year	Birth cohort	Births	Deaths	Infant deaths
2012	2012	67349	661	561
2013	2012	-----	-----	98
2013	2013	69304	640	542
2014	2013	-----	-----	94
2014	2014	68515	613	519

- (i) Calculate the crude infant mortality rate for country X for 2013. (2 marks)
- (ii) Calculate the infant mortality rate for 2013 using the numerator separation and the cohort probability methods. (4 marks)
- e) You are provided with the following age distribution of a population. Graduate the population distribution by age using the **Carrier-Farrag method**, and refine the results using **Newton's method** to improve the graduation

Age group	Observed population
0-4	1200
5-9	1100
10-14	950
15-19	850
20-24	700

(4 marks)

#### QUESTION FOUR (20 MARKS)

- a) A city's population is 200,000, and has a carrying capacity of 500,000. If it grows at a rate of 5% annually, project the population in 5 years using the
- (i) Geometric Growth Model. (2 marks)
- (ii) Exponential Growth Model. (2 marks)
- (iii) Logistic Growth Model. (2 marks)
- b) The table below is the abridged life table for a country Y.

Age	${}_nq_x$	$l_x$	${}_nd_x$	${}_nL_x$	$T_x$	$e_x$
0-1	0.005880	100000	588		8010591	80.1
1-4	0.000999	99412	99	397409	7911090	79.6
5-9	0.000650	99313	65	496402	7513681	75.7
10-14	0.000750	-----	74	496054	7017279	-----
15-19	0.001898	99174	-----	-----	6521225	65.8
20-24	0.002247	98985	222	494371	6025827	60.9
25-29	0.002597	98763	257	493174	5531457	56.0
30-34	-----	98506	349	491660	5038283	51.1
35-39	0.005584	98157	548	489417	4546623	46.3
40-44	0.008712	97609	850	485920	4057206	41.6
45-49	0.012719	96759	1231	480718	3571286	36.9
50-54	0.018330	95528	1751	473264	-----	32.4
55-59	0.028488	93777	2672	462207	2617304	27.9

- (i) Complete the life table. (show all your working) (6 marks)

- (ii) What is the probability of dying between ages 15 and 25 given survival to age 15. (2 marks)
- (iii) What is the average age at death for those dying between ages 20 and 55 given survival to age 20. (4 marks)
- (iv) Of 3000 members of this population aged 30 years, how many are likely to survive to exact age 50. (2 marks)

#### **QUESTION FIVE (20 MARKS)**

- a) A country experiences 50,000 immigrants and 30,000 emigrants in a given year. The total population at mid-year is 1,000,000. Calculate the Net Migration Rate (**NMR**). Interpret the result. (3 marks)
- b) In a region, the number of in-migrants is 15,000, and the number of out-migrants is 10,000. Calculate the **Migration Effectiveness Index (MEI)** Interpret the effectiveness of migration in terms of population redistribution. (3 marks)
- c) The following data was collected from the out patients records of a county hospital.

Terminal Digit "a"	Population with Terminal Digit "a" starting at 10 +a	Population with Terminal Digit "a" starting at 20 +a
0	1569	1183
1	526	393
2	788	447
3	583	360
4	478	277
5	1134	836
6	493	295
7	392	226
8	535	280
9	297	165

Calculate the Myer's Blended Index and explain the results. (6 marks)

- a) What do we mean by Parity Progression Ratio. (2 marks)
- b) Suppose a certain hypothetical cohort for women has the parity progression ratios;

$$p_0 = 0.968, p_1 = 0.940, p_2 = 0.929, p_3 = 0.913, p_4 = 0.888$$

. Assuming that no woman in this cohort had more than five children, out of 2000 women calculate the;

- (i) Number of women who remain childless. (2 marks)
- (ii) Number of women who have exactly one child. (2 marks)
- (iii) Complete family size. (2 marks)