KARATINA UNIVERSITY

UNIVERSITY EXAMINATIONS

2023/2024 ACADEMIC YEAR

<u>SECOND</u> YEAR SPECIAL/ SUPPLIMENTARY EXAMINATION

FOR THE DEGREE OF

BACHELOR OF SCIENCE ACTUARIAL SCIENCE

COURSE CODE: MAT 230

COURSE TITLE: COMPUTATIONAL METHODS AND DATA ANALYSIS

DATE: 22nd JULY 2024   TIME: 12.00noon-2.00pm

INSTRUCTION TO CANDIDATES

SEE INSIDE:

ANSWER QUESTION ONE (COMPULSORY) AND ANY OTHER TWO QUESTIONS

SECTION A

QUESTION ONE (30 marks)

a) Outline any four Features of Matlab   (4marks)

b) Create a valid Matlab code that solves the following system and outputs the value of e only.
   $c = a + b$, $d = c + a\sin b$, $e = 5d$ given that $a = 5$ and $b = 7$ (3marks)

c) State and explain the two forms in which a number in a computer word can be stored.   (2marks)

d) Convert the following numbers

   i.   The hexadecimal number 1EFC into its decimal equivalence   (3marks)
   ii. The binary number 1111011111100 into its decimal equivalence.   (3marks)
   iii. Compare the two answers in (i) and (ii) above   (1mark)

e) A computer has a word length of twelve, determine the form in which the number $(0.96457)_{10}$ is stored by this computer's binary system.   (2marks)

f) Given that the negative root of the equation $2x^3 - 5x^2 - 3x + 6 = 0$ lies between -1 and -2,
   i. use secant method with x_0=-1 and x_1=-2 to find this root, iterate 3 times.   (5 marks)
   ii. Given that the correct answer to four decimal places is -1.1375, determine the percentage error in your answer above. (2 marks)

g) Describe the condition that must be met to guarantee that the Jacobi method for solving a linear system of equations converges.   (3marks)

SECTION B (40marks)

QUESTION TWO (20 marks)

a) Consider the following matrix
   $$A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$$
   i. Determine the spectral radius of matrix A   (4 marks)
   ii. Does the matrix converge?   (1 mark)

b) For the data below calculate the differences and obtain the forward difference polynomial. Interpolate at x=0.25 (4marks)

X	0.1	0.2	0.3	0.4	0.5
F(x)	1.40	1.56	1.76	2.00	2.28

c) Write down a Matlab code to find a root of the equation $x^3 - \frac{1}{2} = 0$ using Muller's method take the three initial approximations as $x_0=0$, $x_1=1$ and $x_2=1/2$. Do not calculate the values (6marks)

d) Define the term Algorithm, hence outline any four important features of an algorithm. (5marks)

QUESTION THREE (20 marks)

a) Convert the decimal number $(0.65)_{10}$ into binary, expressing your answer in eight bits. (2marks)

b) Outline six qualities of a good computer software. (6marks)

c) Given that the largest positive root of the nonlinear equation $x^3 - 4x^2 + 5 = 0$ lies between 3 and 4,

    i. Determine this root using Newton Raphson method with $x_0 = 3.0$, iterate 4 times. (6marks)

    ii. Given that the exact root is 3.6180, determine absolute error in your answer above. (1mark)

d) An exact decimal number must be converted before storage in the computer memory.

    i. Explain how an error can occur during the storage process.

        (2marks)

    ii. State any three types of errors. (3marks)

QUESTION FOUR (20 marks)

a) Outline the five phases that a problem goes through for a computer to produce meaningful results. (5marks)

b) Solve the following system of equations using Jacobi method by first converting it into a matrix

$$4x_1 + x_2 + x_3 = 2$$
$$x_1 + 5x_2 + 2x_3 = -6$$
$$x_1 + 2x_2 + 3x_3 = -4$$

Take $x^{(0)} = [0.5, -0.5, -0.5]^T$ iterate twice   (8marks)

c) Given that a real root of the equation $f(x) = 2x^3 - 6x + 3$ lies in the interval (0,1), use the Regula Falsi method to perform four iterations to find this root.   (7marks)

QUESTION FIVE (20 marks)

a) Using Chebyshev method find the smallest positive root of the equation $f(x) = x^3 - 2x + 1$, take $x_0 = 0.5$ and iterate 2 times.   (5marks)

b) Given the system of nonlinear equations below, use Newton Raphson method with initial approximation $x_0 = 1.5$ and $y_0 = 0.5$ and three iterations to approximate its solution.   (8marks)

$$x^2 + xy + y^2 = 7$$
$$x^3 + y^3 = 9$$

c) Use Gauss-Seidel method to set up an iteration scheme for solving the following system of equations by converting it into matrix.   (7marks)
$$2x_1 - x_2 = 7$$
$$-2x_1 + 3x_2 - x_3 = 5$$
$$-x_2 + 5x_3 = 8$$