KARATINA UNIVERSITY

UNIVERSITY EXAMINATIONS

2023/2024 ACADEMIC YEAR

FOURTH YEAR SPECIAL/SUPPLIMENTARY EXAMINATION

FOR THE DEGREE OF

BACHELOR OF SCIENCE WITH EDUCATION, BACHELOR OF EDUCATION SCIENCE, BACHELOR OF EDUCATION ARTS and BACHELOR OF SCIENCE

COURSE CODE: MAT 417

COURSE TITLE: FLUID MECHANICS II

DATE: 22nd JULY 2024 TIME: 3.00pm-5.00pm

INSTRUCTION TO CANDIDATES

SEE INSIDE

ANSWER **QUESTION ONE (COMPULSORY)** AND ANY OTHER **TWO QUESTIONS**

QUESTION ONE (30 marks)

a) Define the following terms
   i. Potential flow (2marks)
   ii. Circulation (2marks)

b) For a two-dimensional Irrotational flow define the complex velocity potential. (2marks)

c) Given the complex velocity potential $w(z) = (3 + 2z)^2$ find the potential function and the stream function (4marks)

d) State any two fictitious forces that characterize rotating reference frames.(2marks)

e) The stream function for a flow is given by $\Psi = xy$. Determine

   i. Velocity components in the x and y directions (2marks)

   ii. The condition for which the flow is irrotational (3marks)

f) Highlight two properties of the equipotential function. (2marks)

g) Given that $u = x^2 - y^2$ and $v = -xy$, determine the stream function. (3marks)

h) The velocity distribution for the flow of a fluid given by $u = 5x + \frac{x^3}{3}$, $v = -xy - \frac{3y^2}{2}$. Show that this represents a possible case of rotational flow at the point (-1,2). (4marks)

i) From the continuity equation for an incompressible steady flow show that the equipotential function satisfies the Laplace equation. (4marks)

QUESTION TWO (20**marks**)

a) A stream function for a two-dimensional flow is given by $\psi = x + 2x^2 - 2y^2$. Determine
   i. Whether the flow is rotational (4marks)
   ii. Velocity potential of the flow (4marks)

b) Consider a uniform line doublet of strength μ per unit length with its axis along $x-axis$. Determine the equipotential lines and stream lines at $z_1 = 0$. (5marks)

c) Consider a line source of strength m at $z = z_1$ where $R_e z \geq 0$ and the plane $x = 0$ is a rigid boundary. Show that the image of a line source in a rigid infinite plane is a line source of equal strength. (7marks)

QUESTION THREE (20 **marks**)

a) The velocity potential function is given by an expression $\phi = \frac{-xy^3}{3} - x^2 + \frac{x^3 y}{3} + y^2$

i. Find the velocity components in x and y directions (4marks)

ii. Show that ϕ represents a possible case of flow (3marks)

b) Make a sketch of the image of a line doublet at points A and B along the X- axis. (2marks)

c) Consider a line source of strength $-m$ at point A and another of strength m at point B, show that the complex velocity potential at a point P due to the two line sources is given by $w = \mu \frac{e^{i\alpha}}{z - z_1}$ (6marks)

d) Proof that the curl of velocity is zero for irrotational flow $\nabla \times \vec{v} = 0$ (5marks)

QUESTION FOUR (20**marks**)

a) State the three main assumptions in the derivation of D'Alembert's paradox . (3marks)

b) A flow field is described by the equation $\psi = y - x^2$, calculate its vorticity and explain whether the flow is irrotational. (5marks)

c) Derive the continuity equation for an incompressible steady flow in three dimensions by considering a cuboid control element. (12marks)

QUESTION FIVE (20**marks**)

a) Consider the flow whose velocity complex potential is $w = z^2$. Determine

i. Velocity components (2marks)

ii. Speed of the flow at point (-1,2) (2marks)

b) A two dimensional doublet of strength μi is at the point $z = ia$ in a stream of velocity $-ui$ in a semi-infinite liquid of constant density occupying the half plane $y > 0$ and having $y = 0$ as a rigid boundary (i is a unit vector in the positive $x-axis$). Show that the complex potential of the motion is $w = uz + 2\mu \frac{z}{z^2 + a^2}$

(7marks)

c) In a two-dimensional incompressible flow the fluid velovity components are given by $u = x - 4y$ and $v = -y - 4x$, show that

i. The flow is potential (3marks)

ii. The flow satisfies the continuity equation (2marks)

iii. Obtain an expression for the stream function (4marks)