KARATINA UNIVERSITY

UNIVERSITY EXAMINATIONS
2024/2025 ACADEMIC YEAR

FIRST YEAR **FIRST** SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF

BACHELOR OF SCIENCE WITH EDUCATION (P106)

COURSE CODE: MAT 122

COURSE TITLE: ANALYTICAL GEOMETRY 1

DATE: 23RD JANUARY 2025   **TIME:** 3:00PM- 5:00PM

INSTRUCTION TO CANDIDATES

- SEE INSIDE

INSTRUCTIONS: Answer _ALL_ questions in section A and any other **TWO** questions in section **B.**

SECTION A (30 marks)

QUESTION ONE (30 marks)

a) A point moves such that its distance from the point (4,0) is half that of its distance from the line $x = 16$. Find the locus of the point. (4 marks)

b) Find the equation of the line passing through the intersection of the lines $3x + y = 2$ and $x - 3y = 4$ and is perpendicular to the line $3x + 4y = 0$. (6 marks)

c) The parametric equations of a circle are $x = 3 + \sqrt{2}\ cos\theta$ and $y = -5 + \sqrt{2}\ sin\theta$. Find the Cartesian equation of the circle. (4 marks)

d) Classify the graph of the following equation as a circle, a parabola, an ellipse or a hyperbola $4y^2 - 2x^2 - 4y - 8x - 15 = 0$. (3 marks)

e) The polar coordinates of a point are $(-5, 0.23)$. Determine the Cartesian coordinates for the point. (3 marks)

f) Determine the equation of the tangent line to the parabola $y^2 - 2y - 12x - 23$ at the point $(-\frac{7}{4}, 10)$. (5 marks)

g) Find the length of the latus rectum of an ellipse $4x^2 + 9y^2 - 24x + 36y - 72 = 0$. (5 marks)

SECTION B (40 marks)

QUESTION TWO (20 marks)

a) Find the angle between the lines whose equations are: $y = 2x - 3$ and $y = 5x + 2$. (3 marks)

b) Determine the centre and the radius of a circle whose equation is $x^2 - 10x + y^2 - 12y - 39 = 0$ (3 marks)

c) Find the distance from point $P(-2, -3)$ to the line $8x + 15y - 24 = 0$. (3 marks)

d) For a given hyperbola $\frac{(x-3)^2}{9} - \frac{(y+2)^2}{16} = 1$. Find the following:

    (i)   Length of the axes. (3 marks)
    (ii)   coordinates of vertices and foci. (4 marks)
    (iii) the eccentricity. (2 marks)
    (iv) length of the latus rectum. (2 marks)

QUESTION THREE (20 marks)

a) Find the equation of a point's locus so that the sum of its distances from (0, -1) and (0,1) is 3. What curve does the equation represent? (6 marks)

b) Determine the equation of a hyperbola, whose eccentricity is 3/2 and focus are $F'(-2,0)$ and $F(2,0)$. (3 marks)

c) A parabolic antenna has a cross-section of width 12m and a depth of 2m.
   (i) Determine the point the receiver should be placed for best reception. (3 marks)
   (ii) Find the equation of the parabolic antenna. (2 marks)

d) Discuss the triangle. (6 marks)
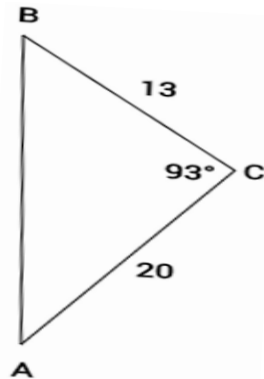
QUESTION FOUR (20 marks)

a) The slope of the line joining the points $(3 + 2t, 5t)$ and $(1, 2t - 1)$ is -3. Find the value of t. (3 marks)

b) Determine the point of intersection of the circles $x^2 + y^2 - 4x - 6y - 12 = 0$ and $x^2 + y^2 - 8x - 2y - 19 = 0$. (8 marks)

c) Given an ellipse whose foci are at $(\pm 4, 0)$ and the eccentricity is ⅓. Find the equation of the ellipse. (4 marks)

d) Discuss the applications of analytical geometry in Machine Learning, Artificial Intelligence and robotic systems. (5 marks)

QUESTION FIVE (20 marks)

a) Determine whether the point (4,5) lies inside, outside or within the circle $x^2 + y^2 - 12x - 10y + 36 = 0$ (3 marks)

b) Write a set of parametric equations for $y^2 = 4x - 4$ (4 marks)

c) The equation of a parabola is given by: $(y + 1)^2 = -4(x - 8)$. Find;
   (i) The focus and the vertex of this parabola. (3 marks)
   (ii) The axis of symmetry and the directrix. (3 marks)
   (iii) The endpoints of the focal diameter. (3 marks)
   (iv) Sketch the parabola. (4 marks)