KARATINA UNIVERSITY

UNIVERSITY SPECIAL/SUPPLEMENTARY EXAMINATIONS

2023/2024 ACADEMIC YEAR

THIRD YEAR **SECOND** SEMESTER EXAMINATION

FOR THE DEGREE OF:

BACHELOR OF SCIENCE WITH EDUCATION

BACHELOR OF EDUCATION SCIENCE

COURSE CODE: MAT 323

COURSE TITLE: METHODS 1

DATE: *23rd July 2024* **TIME:** *3.00pm to 5.00pm*

INSTRUCTION TO CANDIDATES

- SEE INSIDE

<u>INSTRUCTION:</u> Answer ALL questions in section A and any other TWO in Section B

SECTION A: Answer ALL questions

QUESTION ONE (30marks)

a) Given that $\Gamma(x)\Gamma(1-x) = \dfrac{\pi}{\sin \pi x}$

Prove that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$ (3marks)

b) Given the diferrential equation $2x^2y''+3xy'+(2x-3)y = 0$

i) classify the singular points of the differential equation (4marks)

ii) determine the indicial equation (4marks)

c) Find $\mathcal{L}^{-1}\left[\dfrac{2}{(s+5)^4}\right]$ (3mark

s)

d) Show that

$\int_0^\infty x^{1/2} e^{-x^2} dx = \dfrac{\sqrt{\pi}}{3}$ (4marks)

e) classify the partial differential equation

$\dfrac{\partial u}{\partial t} = \alpha^2 \dfrac{\partial^2 u}{\partial x^2}$ (3marks)

f) Separate the partial differential equation into two ordinary differential equations

$\dfrac{\partial^2 u}{\partial r^2} + \dfrac{1}{r}\dfrac{\partial u}{\partial r} + \dfrac{1}{r^2}\dfrac{\partial^2 u}{\partial^2 \theta} = 0$ (4marks)

g) Find the recurrence relation for the series solution of the differential equation

$y'' + y = 0$, about $x = 0$ (5marks)

SECTION B: Answer any TW0 questions

Question Two (20 marks)

a) Show that the equation

$$\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial y^2} + (x^2 + y^2)^2 u = 0$$

is not separable in x and y. (4marks)

b) Prove that $\mathcal{L}\left[f'(t)\right] = s\mathcal{L}[f(t)] - f(0)$ (4marks)

c) Given the differential equation

$$5x^2 y'' + x(1+x) - y = 0$$

i) Find its indicial equation (4marks)

ii) Find the power series solutions for $x > 0$ corresponding to the larger root (8marks)

Question Three (20 marks)

a) A periodic function f with period 4 is defined on one period by

$$f(x) = \begin{cases} 0, & if -2 < x < 0 \\ 1, & if\, 0 < x < 2 \end{cases}$$

i) Sketch the graph of f. (2marks)

ii) Obtain the Fourier series for f. (4marks)

b) Determine the first two terms of the series solution for the differential equation

$y'' - xy = 0$, about $x = 0$ (7marks)

c) Apply Laplace transform to find the solution of this initial-value problem.

$y'' - 3y' - 10y = 2$, $y(0) = 1, y'(0) = 0$ (7marks)

Question Four (20 marks)

a) Use the gamma function to evaluate

$$I = \int_0^\infty y^3 e^{-2y} dy$$ (4marks)

b) Find the Laplace transform of the function

$$f(x) = e^{ax}$$ (4marks)

c) solve the heat equation using the method of separation of variables

$$\frac{\partial u}{\partial t} = k\frac{\partial^2 u}{\partial x^2} \qquad u(0,t) = u(1,t) = 0, t > 0$$

$$u(x,0) = f(x) = 100, 0 < x < 1$$ (12marks)

Question Five (20 marks)

a) Using the Beta function to evaluate

$$I = \int_0^2 x(2-x)^{\frac{-1}{2}} dx$$ (4marks)

b) Find the Laplace transform of $f(t) = \begin{cases} 1, & 0 \le t < 2 \\ t-2, & 2 \le t \end{cases}$ (4marks)

c) Determine the half-range Fourier cosine series for $f(x) = e^x$ on $(0,1)$. (4marks)

d) Given the equation:

$$9x^2 y'' + (x+2) y = 0$$

i) Test for singularity. (4marks)

ii) Determine the roots of its indicial equation. (4marks)