Inspiring Innovation and Leadership

KARATINA UNIVERSITY

UNIVERSITY EXAMINATIONS

2024/2025 ACADEMIC YEAR

FOURTH YEAR **FIRST** SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF

BACHELOR OF SCIENCE ACTUARIAL SCIENCE

COURSE CODE: ACS 413

COURSE TITLE: RISK MATHEMATICS

DATE: 20/12/2024 **TIME: 12.00PM-2.00PM**

INSTRUCTION TO CANDIDATES

- SEE INSIDE

INSTRUCTION: Answer all questions in section A and any other THREE in section B

SECTION A: 30MKS

question 1

a) Discuss whether time diversification is a fact or a fallacy (3mks)

b) Each year, Mr A is offered the opportunity to invest £1,000 in a risk fund. If successful, at the end of the year he will be given back £2,000. If unsuccessful, he will be given back only £500. There is a 50% chance of either outcome. Calculate the expected rate of return per annum on the investment. (4mks)

c) State an expression for the expectation of the next-period utility of Investor X, again assuming that he invests a proportion a in Equity A and the rest in a non-interest-bearing bank account. He has the utility function $U(w) = \log(w)$. (4mks)

d) Calculate the 97.5% VaR over one year for a portfolio consisting of £200m invested in shares. You should assume that the return on the portfolio of shares is normally distributed with mean 8% pa and standard deviation 8% pa. (3mks)

e) State Lundberg's inequality (1mks)

f) Explain how motor insurance claims could be represented by a Poisson process. (3mks)

g) Calculate the value of e-15R. (3mks)

h) The claims arising during each year from a particular type of annual insurance policy are assumed to follow a normal distribution with mean 0.7P and standard deviation 2.0P , where P is the annual premium. Claims are assumed to arise independently. Insurers assess their solvency position at the end of each year. A small insurer with an initial surplus of £0.1m expects to sell 100 policies at the beginning of the coming year in respect of identical risks for an annual premium of £5,000. The insurer incurs expenses of 0.2P at the time of writing each policy. Calculate the probability that the insurer will prove to be insolvent at the end of the coming year. Ignore interest. (5mks

i) The number of claims from a portfolio of policies has a Poisson distribution with parameter 30 per year. The individual claim amount distribution is lognormal with parameters $\mu = 3$ and $\sigma 2 = 1.1$. The rate of premium income from the portfolio is 1,200 per year. If the insurer has an initial surplus of 1,000, estimate the probability that the insurer's surplus at time 2 will be negative, by assuming that the aggregate claims distribution is approximately normal. (4mks)

SECTION B

Question Two (20 marks)

Investor A has an initial wealth of $100, which is currently invested in a non-interest-bearing account, and a utility function of the form:

$U(w) = \log(w)$

Where w is the investor's wealth at any time.

Investment Z offers a return of –18% or +20% with equal probability.

(i) What is Investor A's expected utility if nothing is invested in Investment Z? (5mks)

(ii) What is Investor A's expected utility if they're entirely invested in Investment Z? (5mks)

(iii) What proportion a of wealth should be invested in Investment Z to maximise expected utility? What is Investor A's expected utility if they invest this proportion in Investment Z? (10mks)

Question Three (20 marks)

An investor is contemplating an investment with a return of £ R , where:

R = 300,000 – 500000U

where U is a uniform [0,1] random variable.

Calculate each of the following four measures of risk:

(a) variance of return (5mks)

(b) downside semi-variance of return (5mks)

(c) shortfall probability, where the shortfall level is £100,000 (4mks)

(d) Value at Risk at the 5% level. (6mks)

Question Four (20 marks)

a) An insurer knows from past experience that the number of claims received per month has a Poisson distribution with mean 15, and that claim amounts have an exponential distribution with mean 500. The insurer uses a security loading of 30%. Calculate the insurer's adjustment coefficient and give an upper bound for the insurer's probability of ruin, if the insurer sets aside an initial surplus of 1,000. (10mks)

b) Write down the equation for the adjustment coefficient for personal accident claims if 90% of claims are for £10,000 and 10% of claims are for £25,000, assuming a proportional security loading of 20%. Show that this equation has a solution in the range

0.00002599 <R<0.00002601 (10mks)

Question Five (20 marks)

a) Claims occur as a Poisson process with rate λ and individual claim sizes X follow an Exp( ) β distribution. The office premium includes a security loading $\theta 1$. An individual excess of loss arrangement operates under which the reinsurer pays the excess of individual claims above an amount M in return for a premium equal to the reinsurer's risk premium increased by a proportionate security loading $\theta 2$. Derive and simplify as far as possible an equation satisfied by the adjustment coefficient for the direct insurer. (11mks

b) Suppose Investor A has a power utility function with $\gamma = 1$, whilst Investor B has a power utility function with $\gamma = 0.5$.

i) Which investor is more risk-averse (assuming that $w > 0$)? (4mks)

ii) Suppose that Investor B has an initial wealth of 100 and is offered the opportunity to buy Investment X for 100, which offers an equal chance of a payout of 110 or 92. Will the Investor B choose to buy Investment X? (5mks)