Inspiring Innovation and Leadership

KARATINA UNIVERSITY

UNIVERSITY EXAMINATIONS

2024/2025 ACADEMIC YEAR

FOURTH YEAR **FIRST** SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF

BACHELOR OF SCIENCE (P106, P103), BACHELOR OF EDUCATION (E100, E101, E111)

COURSE CODE : MAT 418

COURSE TITLE : PARTIAL DIFFERENTIAL EQUATIONS I

DATE: 13th Dec 2024 TIME: 12.00noon-2.00pm

INSTRUCTION TO CANDIDATES

SEE INSIDE

ANSWER **QUESTION ONE (COMPULSORY)** AND ANY OTHER **TWO QUESTIONS**

QUESTION ONE (30 marks)

a) Evaluate the following partial derivative

$$\frac{\partial^2}{\partial x^2}(x^3 e^{x+y})^2$$ (2marks)

b) Consider the transport equation $au_t(x,t)+bu_x(x,t)=0$ where a and b are constants. Show that $u(x,t)=f(bt-ax)$ is a solution where f is an arbitrary differentiable function in one variable. (3marks)

c) Given the following family of hyperbolas $x^2-y^2=k$ where k is arbitrary. Determine the orthogonal trajectories to these curves. (3marks)

d) Using the change of variables technique with $v=y-2x$ and $w=x$. Solve the PDE $u_x+2u_y=1$ (5marks)

e) Solve the nonlinear PDE $u_x+uu_y=0$ using the following transformations $\xi=x$ and $\eta=y-xu$. (5marks)

f) Determine whether the function $u(x,y)=e^{-\lambda^2\alpha^2 t}(\cos\lambda x-2\sin\lambda x)$ is a solution to the PDE $u_t-\alpha^2 u_{xx}=-u_x$ (3marks)

g) Use the definition of the partial derivative to find $\frac{\partial f}{\partial x}$ as a function of x and y, for $f(x,y)=3x^2+xy$ (4marks)

h) Given $r(t)=(t^2,\ln(t),t)$. Determine the following

i. The normal to the level curve at t=2 (2mark)

ii. The directional derivative of $r(t)$ at $t=2$ in the direction of $a=(1,1,1)$ (3marks)

QUESTION TWO (20 marks)

a) Solve the linear PDE $u_x + u_y = u$ using the following transformations $\xi = x + y$ and $\eta = x - y$. (6marks)

b) Show that $u(x, y) = e^{-2y}\sin(x - y)$ is the solution to the initial value problem

$u_x + u_y + 2u = 0$ for x,y>1
u(x,0)=sinx (3marks)

c) Find the solution to $-3u_x + u_y = 0$, $u(x,0) = e^{-x^2}$ (4marks)

d) Solve the initial value problem below using the method of Cauchy. (7marks)

$u_x + u_y = 1$
$u(x,0) = f(x)$

QUESTION THREE (20 marks)

a) Find the general solution of the equation $xu_x + yu_y = xe^{-u}$, $x > 0$ (5marks)

b) Given $f(x, y, z) = e^{x+y+z}$. Determine the following

i. The level surface (1mark)

ii. The normal to the level surface (2mark)

iii. The directional derivative of f at $(0,0,0)$ in the direction of $a = (1,1,1)$ (3marks)

c) Solve the partial differential equation $u_x + u_y = 1$ by introducing the change of variables $s = x + y$ and $t = x - y$. (6marks)

d) Find the general solution of the PDE $yu_x + xu_y = x^2 + y^2$ (7marks)

QUESTION FOUR (20 marks)

a) Solve the problem below using linear change of variables technique (10marks)

$$5\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = x$$

$u(x,0) = \sin 2\pi x$ for $-\infty < x < \infty$

a) Solve the following non-linear PDE using the Charpit method

$(p^2 + q^2)y = qz$ (10marks)

QUESTION FIVE (20 marks)

a) Determine every function u(x,t) that solves the function using the linear change of variable technique (10marks)

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial x} = 0$$

Where v is a fixed constant.

b) Solve the following partial differential equation $p^2 x + q^2 y = z$ using Jacobi's method. (10marks)