



KARATINA UNIVERSITY

UNIVERSITY EXAMINATIONS

2023/2024 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER

SUPPLEMENTARY/SPECIAL EXAMINATIONS

FOR THE DEGREE OF:

**BACHELOR OF SCIENCE (P102,P103,P106,P107) AND
BACHELOR OF EDUCATION (E100,E101,E103,E111,E112)**

COURSE CODE: MAT 221

**COURSE TITLE: ORDINARY DIFFERENTIAL
EQUATIONS I**

DATE: 18th JULY, 2024

TIME: 3:00 PM - 5:00 PM

Instructions: See Inside

Answer **all** questions in section A and any other **two** from section B.

SECTION A: COMPULSORY (30 MARKS)

Answer **all** questions from this section

QUESTION ONE (30 Marks)

- (a) A circuit in series has a constant electromotive force of 40V, a resistance of 10Ω , an inductance of 0.2 henry and an initial current of 0. Solve the basic differential equation to find the current i on the circuit at any time $t > 0$ [5 Marks]

- (b) Use the method of solution by integration to solve the given ordinary differential equations

$$\frac{dy}{dx} = \frac{xy^2 + x}{yx^2 + y}$$

[4 Marks]

- (c) Solve the given differential equation using the method of substitution

$$2xy \frac{dy}{dx} = y^2 - x^2$$

[5 Marks]

- (d) Solve the given differential equation using reduction by order method

$$y'' + 4y = 0, \quad y_1 = \cos 2x$$

[4 Marks]

- (e) Solve the given Bernoulli differential equation

$$x \frac{dy}{dx} + y = x^3 y^6$$

[5 Marks]

- (f) Find the solution to the given Non-Homogeneous Linear Equations of Second Order

$$y'' + 3y' + 2y = 4x^2$$

[7 Marks]

SECTION B: ELECTIVE (40 MARKS)

Answer **any Two** questions from this section

QUESTION TWO (20 Marks)

- (a) Show that

$$3x(xy - 2)dx + (x^3 + 2y)dy = 0$$

is an exact differential, hence solve the equation.

[5 Marks]

- (b) Suppose that a pie is removed from $350^\circ F$ oven and placed in a room with a temperature of $75^\circ F$. In 15 minutes, the pie has a temperature of $150^\circ F$. Determine the time required to cool the pie to a temperature of $80^\circ F$ when one can actually enjoy the eating?

[5 Marks]

- (c) Using the method of variation of parameters evaluate

$$y'' - 4y' + 3y = e^{-x}$$

[10 Marks]

QUESTION THREE (20 Marks)

- (a) Solve the initial value problem

$$x \frac{dy}{dx} = y + \sqrt{x^2 - y^2} \quad y(x_0) = 0 \quad x_0 > 0$$

[6 Marks]

- (b) Using the method of variation of parameters compute the solution to

$$y'' + y = \sec x$$

[8 Marks]

- (c) Solve the given Cauchy-Euler Differential Equation

$$4x^2y'' + 8xy' + y = 0$$

[6 Marks]

QUESTION FOUR (20 Marks)

- (a) Find the general solution to

$$\frac{1}{x} \frac{dy}{dx} - \frac{2y}{x^2} = x \cos x \quad x > 0$$

[4 Marks]

- (b) Suppose a population has **39** members at time $t = 8$ and **60** members at time $t = 12$. Find the original population

[5 Marks]

- (c) Find the particular integral of

$$\frac{d^2y}{dx^2} + y = \csc x$$

[6 marks]

- (d) Solve the differential equation using the method of variable separable

$$\frac{dy}{dx} = (9x + y + 1)^2$$

[5 marks]

QUESTION FIVE (20 Marks)

- (a) Solve the exact differential equation given

$$2xydx + (x^2 - 1)dy = 0$$

[5 Marks]

- (b) A colony of bacteria increases according to the law of exponential growth. If the number of bacteria doubles in 3 hours. How long will it take for the size of the colony to triple.

[5 Marks]

- (c) Use substitution method to solve the special case of homogeneous differential equation given as

$$\frac{dy}{dx} = \frac{x + y + 4}{x - y - 6}$$

[10 Marks]