Inspiring Innovation and Leadership

# KARATINA UNIVERSITY

## UNIVERSITY SPECIAL/SUPPLEMENTARY EXAMINATIONS

## 2023/2024 ACADEMIC YEAR

**FOURTH** YEAR **SECOND** SEMESTER EXAMINATION

**FOR THE DEGREE OF:**

**BACHELOR OF SCIENCE WITH EDUCATION (P106)**

**BACHELOR OF EDUCATION science (E101)**

**COURSE CODE: MAT 426**

**COURSE TITLE: METHODS II**

**DATE:** *23RD JULY 2024* **TIME:** *3.00PM TO 5.00PM*

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**INSTRUCTION TO CANDIDATES**

- SEE INSIDE

**INSTRUCTION:** **Answer question ONE and any other TWO QUESTIONS**

**Question One (30 marks)**

a) i) Define an integral equation (1mark)

ii) State two types of integral equations giving examples (3marks)

b) Describe the tensor $B^{p}_{kl}$ (3marks)

c) Define the modified Bessel's differential equation of order n and give the form of its gene ral solution. (2marks)

d) Express $\frac{d\phi}{dt} = \frac{\partial\phi}{\partial x^1}\frac{dx^1}{dt} + \frac{\partial\phi}{\partial x^2}\frac{dx^2}{dt} + - - - + \frac{\partial\phi}{\partial x^n}\frac{dx^n}{dt}$ in summation convention (2marks)

e) Show that the Kronecker delta $\bar{\delta}_i^k$ is a mixed tensor of rank 2 having the same compone nts in every coordinate system. (4marks)

f) Show that the expression $A(i, j, k)$ is a covariant tensor of rank three if $A(i, j, k)B^k$ is cov ariant tensor of rank two and $B^k$ is contravariant vector (4marks)

a) Construct the Green's function for the BVP

$$\frac{\partial^2 y}{\partial x^2} + \frac{1}{4}y = f(x) \text{ with } y(0) = 0 = y(\pi)$$ (4marks)

b) Given that $A_i$ is a covariant tensor, prove that $\frac{\partial A_i}{\partial x^j}$ do not form a tensor. (3marks)

c) Reduce the initial value problem to the Volterra integral equation

$$u' - 3x^2u = 0$$
$$u(0) = 1$$ (4marks)

**Answer any Two questions**

**Question Two (20 marks)**

a) State two properties of Asymptotic sequences (2marks)

b) Differentiate between linear integral equation and homogenous integral equation. (3marks)

c) Find $g$ and $g^{ij}$ corresponding to metric tensor

$$(ds)^2 = 5(dx^1)^2 + 3(dx^2)^2 + 4(dx^3)^2 - 6dx^1dx^2 + 4dx^2dx^3$$ (10marks)

d) Show that any inner product of the tensors $A_r^p$ and $B_t^{qs}$ is a tensor of rank three (5marks)

**Question Three (20marks)**

**a)** If $g_{ij}$ denotes a covariant tensor of rank 2, show that a product $g_{ij}\partial x^i \partial x^j$ is an invariant. (4marks)

**b)** Find the solution of the integral equation by the Neumann series.

$$u(x) = e^x + \frac{1}{e}\int_0^1 u(y)\,dy \qquad \text{(8marks)}$$

**c)** Solve the differential equation using the Green's function

$$y'' + y = 1$$
$$y(0) = y(1) = 0 \qquad \text{(8marks)}$$

**Question Four (20marks)**

a) If $\phi = a_{jk} A^j A^k$ Show that we can always write $\phi = b_{jk} A^j A^k$ where $b_{jk}$ is symmetric. (5marks)

**b)** Given the Bessel function $J_n(x) = \sum_{p=0}^{\infty} \frac{(-1)^n}{p!\Gamma(p+n+1)}\left(\frac{x}{2}\right)^{n+2p}$

show that $J_{\frac{1}{2}}(x) = \left(\frac{2}{\pi x}\right)^{\frac{1}{2}} \sin x$ (6marks)

c) Find the steady-state temperature $u(r,\theta)$ inside a sphere of radius $a$ with $u(a,\theta) = f(\theta)$ (9marks

**Question Five (20marks)**

a) Evaluate the rank of the tensor $A_X^{YZ}$ (3marks)

b) If $g_{ij}$ denotes a covariant tensor of rank 2, show that a product $g_{ij}\partial x^i \partial x^j$ is an invariant. (4marks)

c) Convert the integral equation into an initial value problem

$$u(x) = x^3 + \int_0^x (x-y)^2 u(y)\,dy \qquad \text{(4marks)}$$

d) Find the solution of the integral equation with separable kernel.

$$y(x) = x + \lambda\int_0^1 (xt^2 + x^2 t)\, y(t)\,dt \qquad \text{(9marks)}$$