KARATINA UNIVERSITY

UNIVERSITY EXAMINATIONS

2023/2024 ACADEMIC YEAR

SECOND YEAR **SECOND** SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF:

BACHELOR OF EDUCATION (SCIENCE)

BACHELOR OF SCIENCE APPLIED STATISTICS WITH COMPUTING

COURSE CODE: MAT 220

COURSE TITLE: CALCULUS III

DATE: 22/04/2024 **TIME: 3:00PM-5:00PM**

INSTRUCTIONS TO CANDIDATES

- SEE INSIDE

INSTRUCTIONS: **Answer QUESTION ONE in SECTION A and any other TWO in SECTION B**

SECTION A (30 MARKS)

QUESTION ONE (30 MARKS)

a) Determine if $\lim_{(x,y)\to(0,0)} \frac{4xy^2}{x^2+y^2}$ exists (3 marks)

b) Explain the three conditions for any function $f(x,y)$ to be continuous at a point (a,b) (3 marks)

c) Find the total differential dz given that $z = x^2y + x^2y^2 + xy^3$ (3 marks)

d) Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$, where $z = f(x,y)$ is defined by $xy + yz^2 + zx = 1$. (4 marks)

e) Evaluate the iterated integral $\int_0^1 \int_x^1 \sin(y^2)dydx$ (3 marks)

f) Given that $z = x^2 + 3xy + 5y^2$, $x = sin\, t$ and $y = cos\, t$. Find $\frac{dz}{dt}$ (3 marks)

g) Show that the series $\sum_{n=1}^{\infty} \frac{n}{n^3 + 2}$ converges using comparison test. (3 marks)

h) Find the tangent plane and normal line of the surface $x^3 + y^4 - z - 9 = 0$ at the point (1,1,0) (4 marks)

i) Obtain the Taylor's series expansion of the following functions about the indicated points. $f(x) = e^{\frac{x}{2}}$ about $x_0 = 2$. (4 marks)

SECTION B (40 MARKS)

QUESTION TWO (20 MARKS)

a) A farmer wishes to build a rectangular storage bin, without a top, with a volume of 500 cubic meters using the least amount of materials possible. If we let x and y be the dimensions of the base of the bin and z the height, all measured in meters, minimize the surface area of the bin, given by

$$S = xy + 2xz + 2yz$$

subject to the constraint on the volume, namely,

$$500 = xyz$$

(10 marks)

b) Evaluate $I = \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{2} (x^2 + y^2)dzdydx$ (6 marks)

c) Given that $Z = f(x, y)$ has continuous second order partial derivatives and that $x = r^2 + s^2, y = 2rs$, find Z_{rr} (4 marks)

QUESTION THREE (20 MARKS)

a) Test for maxima-minima in the function $Z = x^3 + y^3 + 3xy$ (10 marks)

b) A cup-like object is made by rotating the area between $y = 2x^2$ and $y = x + 1$ with $x \geq 0$ around the x-axis. Find the volume of the material needed to make the cup. Units are cm (7 marks)

c) Find the equation of the tangent plane to the elliptic paraboloid $Z = 2x^3 + y$ at (1,1,0) (3 marks)

QUESTION FOUR (20 MARKS)

a) Find absolute extrema of $f(x, y) = 2 + 2x + 2y - x^2 - y^2$ defined on the triangular region bounded by the straight lines $x = 0, y = 0$ and $x + y = 9$. (15 marks)

b) Evaluate $\iint_R (1 - 6x^2y)dA$ where $R = [0,2] \times [-1,1]$ (5 marks)

QUESTION FIVE (13 MARKS)

a) Investigate $f(x, y) = x^3 + y^3 - 3xy + 1$ for extreme values (10 marks)

b) Use an appropriate test to determine if the following series converges or diverges. State the test first.

i. $\sum_{n=1}^{\infty} \frac{2n^2+3n}{\sqrt{5-n^2}}$. (5 marks)

ii. $\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$. (5 marks)