KARATINA UNIVERSITY

UNIVERSITY EXAMINATIONS

2023/2024 ACADEMIC YEAR

SECOND YEAR SECOND SEMESTER REGULAR EXAMINATIONS

FOR THE DEGREE OF:

BACHELOR OF SCIENCE (P102,P103,P106,P107) AND BACHELOR OF EDUCATION (E100,E101,E103,E111,E112)

COURSE CODE: MAT 221

COURSE TITLE: ORDINARY DIFFERENTIAL EQUATIONS I

DATE: 25th APRIL, 2024 **TIME: 3:00 PM - 5:00 PM**

Instructions: See Inside

Answer **all** questions in section A and any other **two** from section B.

SECTION A: COMPULSORY (30 MARKS)

Answer **all** questions from this section

QUESTION ONE (30 Marks)

(a) Use the method of solution by integration to solve the given ordinary differential equations

$$\frac{dy}{dx} = \frac{xy^2 + x}{yx^2 + y}$$

[4 Marks]

(b) Solve the given differential equation using the method of substitution

$$2xy\frac{dy}{dx} = y^2 - x^2$$

[5Marks]

(c) A circuit in series has a constant electromotive force of 40V, a resistance of $\mathbf{10\Omega}$, an inductance of 0.2 henry and an initial current of 0. Solve the basic differential equation to find the current i on the circuit at any time $t > 0$ **[5 Marks]**

(d) Solve the given differential equation using reduction by order method

$$y'' + 4y = 0, \qquad y_1 = \cos 2x$$

[4 Marks]

(e) Solve the given Bernoulli differential equation

$$x\frac{dy}{dx} + y = x^3y^6$$

[5 Marks]

(f) Find the solution to the given Non-Homogeneous Linear Equations of Second Order

$$y'' + 3y' + 2y = 4x^2$$

[7 Marks]

SECTION B: ELECTIVE (40 MARKS)

Answer **any Two** questions from this section

QUESTION TWO (20 Marks)

(a) Show that

$$3x(xy - 2)dx + (x^3 + 2y)dy = 0$$

is an exact differential,hence solve the equation. **[5 Marks]**

(b) Suppose that a pie is removed from 350^oF oven and placed in a room with a temperature of 75^oF. In 15 minutes, the pie has a temperature of 150^oF Determine the time required to cool the pie to a temperature of 80^oF when one can actually enjoy the eating? **[5 Marks]**

(c) Using the method of variation of parameters evaluate

$$y'' - 4y' + 3y = e^{-x}$$

[10 Marks]

QUESTION THREE (20 Marks)

(a) Use substitution method to solve the special case of homogeneous differential equation given as

$$\frac{dy}{dx} = \frac{x + y + 4}{x - y - 6}$$

[10 Marks]

(b) Find solution to the given Non-Homogeneous Linear Equations of Second Order with Constant Coefficients expressed as

$$y'' - 5y' + 6y = -3\sin 2x$$

[10 Marks]

QUESTION FOUR (20 Marks)

(a) Solve the initial value problem

$$x\frac{dy}{dx} = y + \sqrt{x^2 - y^2} \qquad y(x_0) = 0 \qquad x_0 > 0$$

[6 Marks]

(b) Using the method of variation of parameters compute the solution to

$$\mathrm{y}'' + \mathrm{y} = \sec \mathrm{x}$$

[8 Marks]

(c) Solve the given Cauchy-Euler Differential Equation

$$4x^2y'' + 8xy' + y = 0$$

[6 marks]

QUESTION FIVE (20 Marks)

(a) Find the general solution to

$$\frac{1}{x}\frac{dy}{dx} - \frac{2y}{x^2} = x\cos x \quad x > 0$$

[4 Marks]

(b) Suppose a population has **39** members at time $t = 8$ and **60** members at time $t = 12$. Find the original population **[5 Marks]**

(c) Find the particular integral of

$$\frac{d^2y}{dx^2} + y = \csc x$$

[6 Marks]

(d) Solve the differential equation using the method of variable separable

$$\frac{dy}{dx} = (9x + y + 1)^2$$

[5 Marks]