Inspiring Innovation and Leadership

KARATINA UNIVERSITY

UNIVERSITY EXAMINATIONS

2023/2024 ACADEMIC YEAR

SECOND YEAR **SECOND** SEMESTER SUPPLEMENTARY/ SPECIAL EXAMINATION

FOR THE DEGREE OF:

BACHELOR OF SCIENCE WITH EDUCATION

COURSE CODE: MAT 223

COURSE TITLE: DYNAMICS I

DATE: WED 17TH JULY 2024 **TIME:** 9 – 11 AM

INSTRUCTION TO CANDIDATES

1. ANSWER QUESTIONS IN **SECTION A** AND ANY OTHER TWO QUESTIONS FROM **SECTION B**
2. DO ALL YOUR WORKINGS ON THE ANSWER BOOKLET PROVIDED
3. STRICTLY NO USE OF MOBILE PHONES IN THE EXAMINATION ROOM

SECTION A

QUESTION ONE (30 Marks)

(a) (i) Define torque and state its mathematical expression. (2 marks)

(ii) Considering the Newton's second law of motion, $\vec{F} = \frac{d\vec{p}}{dt}$where $\vec{P}$ is linear momentum and $\vec{F}$ is the resultant force, derive the expression of the law of conservation of angular momentum $\vec{L}$. (6 marks)

(b) A ball kicked from ground level at an initial velocity of 60 m/s and an angle θ with ground reaches a horizontal distance of 200 meters.

(i) What is the size of angle θ? (3 marks)

(ii) What is time of flight of the ball? (2 marks)

c) Distinguish between conservative and non-conservative forces. Give an example of each. (4 marks)

d) Show that the force field F defined by, $F = x^2 i + 2yzj + y^2 k$ is a conservative force field. (4 marks)

e) A particle of mass m moves along a space given by $\vec{\boldsymbol{r}} = a\cos\omega t\,\hat{\boldsymbol{\imath}} + b\sin\omega t\,\hat{\boldsymbol{\jmath}}$. Find

(i) The torque about the origin of the force acting upon the particle. (5 marks)

(ii) the angular momentum of the particle about the origin. (4 marks)

SECTION B

QUESTION TWO (20 Marks)

a) Define the moment of a force. (2 marks)

b) (i) Describe what is meant by damped oscillation. (3 marks)

(ii) A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. Determine the time it will take to drop 1/100 of the original amplitude. (6 marks)

c) A baseball is popped straight up into the air and has a hang-time of 6.25s. Determine the height to which the ball rises before it reaches its peak. (5 marks)

d) Find the gravitational force of attraction between the moon and the earth if the mass of the moon is 1/81 the mass of the earth. ($G = 6.67 \times 10^{-11} Nm^2/kg^2$, radius of the moon's orbit is 3.58 x 10^5 km and mass of the earth is 6 x 10^{24} Kg). (4 marks)

QUESTION THREE (20 Marks)

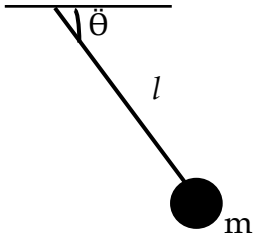
a) Determine the torque about the origin of a particle at a point (5, 2, 7) due to a force $6i - 4j + k$. (3 marks)

b) A particle mass 2 units moves in a force field depending on time t given by $F = 24t^2 i + (36t - 16)j - 12tk$.Assuming that at t=0,the particle is located at

$r_0 = 3i + j + 4k$ and a velocity $v_0 = 6i + 15j - 8k$. Find the velocity and position at any time t. (6 marks)

c) Show that for a body executing circular motion the centripetal acceleration is given by $a = \frac{v^2}{r}$. (3 marks)

d) Consider the following system



(i) If the period of the pendulum is 5 seconds, what is the length of the string? (3 marks)

(ii) If the mass reaches a height of 0.75 and the minimum angle is 30^0, what is the period of the pendulum? (5 marks)

QUESTION FOUR (13 marks)

a) If the amplitude of a simple harmonic oscillator is doubled, how does this affect,

i) its periodic time, (2 marks)

ii) its total energy, (2 marks)

iii) the maximum velocity of the oscillator. (2 marks)

b) A circular disc of mass *m* and radius *r* rolls on a table. If ω is its angular speed. Show that its total energy is given by $E = \frac{3}{4}r^2\omega^2$ = d) (5 marks)

c) A constant force of magnitude **P** newtons accelerates a particle of mass 8 kg in a straight line from a speed of 4 ms^{-1} to a speed of 20 ms^{-1} over a distance of 15 m. Determine the value of **P**. (4 marks)

d) The amplitude of a damped oscillator becomes one third in ten minutes. The amplitude after 30 minutes will be $\mathbf{1/n}$ times the original. Find the value of $\boldsymbol{n}$. (5 marks)

QUESTION FIVE (20 Marks)

a) Find the unit tangent and principal unit normal vectors at the given points.
$r(t) = ti + (t^2 - 1)j + tk\rangle$ at $t = 0$ and $t = 1$ **(**7 marks)

b) Derive the equation of motion $s = ut + \frac{1}{2}at^2$ where *s*, *u*, *a* and *t* have their usual meaning in classical mechanics. (3 marks)

c) Show that impulse is equal to the change in momentum. (3 marks)

d) The motion of an object is described by the equation; $x = 0.3\cos\left(\frac{\pi}{3}t\right)$. Find;

(i) The position of the object at t = 0 and t = 0.6 s (3 marks)

(ii) The amplitude of the motion (2 marks)

(iii) The frequency of the motion, and (1 mark)

(iv) The period of the motion. (1 mark)