KARATINA UNIVERSITY

UNIVERSITY EXAMINATIONS

2024/2025 ACADEMIC YEAR

FIRST YEAR **FIRST** SEMESTER **REGULAR**

EXAMINATIONS FOR THE DEGREE OF:

BACHELOR OF SCIENCE: (P100, P101, P103, P106, P107);

BACHELOR OF EDUCATION (E100, E101, E103, E111, E112, E113).

COURSE CODE: MAT 115

COURSE TITLE: DISCRETE MATHEMATICS I

DATE: 23rd JANUARY, 2025 **TIME:** 9:00AM - 11:00PM

INSTRUCTIONS: ***See Inside***

Answer question **ONE** in section A and any other **Two** from section B.

SECTION A

Question ONE is Compulsory

QUESTION ONE (30 marks)

a) Define *'Symmetric Difference'* of two nonempty subsets **A** and **B** of a universal set $\boldsymbol{U}$. **(2 marks)**

b) List all the elements of the set

i) $\mathbf{A} = \{x \in \mathbb{Z} : x^2 - 5x - 36 = 0\}$ **(3 marks)**

ii) $\mathbf{B} = \{x \in \mathbb{N} : x^2 - 5x - 36 = 0\}$ **(1 mark)**

c) Let R be a relation from a set $\mathbf{A} = \{-6, 7, 9\}$ to a set $\mathbf{B} = \{p, q, r, s\}$ represented by the matrix $M_R = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

i) List down all the ordered pairs in the relation R. **(2 marks)**

ii) Represent the relation R using an arrow diagram. **(2 marks)**

d) A drug company is considering manufacturing a new toothpaste. They are considering two flavors, ***regular*** and ***mint***. In a sample of 450 people, it was found that:

200 liked the regular flavor;

198 liked the mint flavor;

72 did not like either of the flavors.

i) Create a Venn diagram to model the information. **(4 marks)**

ii) How many liked both flavours? **(1 mark)**

iii) How many liked only the regular flavor? **(1 mark)**

e) Compute:

i) $\lceil -12.56 \rceil$. **(1 mark)**

ii) $\dfrac{13!}{8!\,2!}$ **(2 marks)**

f) Determine if:

i) $28 \equiv 4 \bmod 8$. **(1 mark)**

ii) $-45 \equiv 17 \bmod 11$. **(1 mark)**

g) Find $g(-3)$ given $g(x) = \sqrt{5x^3 - 9}$. **(2 marks)**

h) Find the domain of the function $g(x) = \dfrac{5x+8}{x^2+4x-21}$. **(3 marks)**

i) How many permutations can one have for the word ***"MISSISSIPPI"*** if the *"I's"* do not come together? **(4 marks)**

SECTION B

Answer ***ANY TWO*** questions from this section.

QUESTION TWO (20 marks)

a) Find $\mathbf{B} \times \mathbf{C}$ given that $\mathbf{B} = \{-5, 7\}$ and $\mathbf{C} = \{\theta, \pi, \alpha\}$. **(3 marks)**

b) A committee of 5 people is to be chosen from a group of 6 men and 4 women. How many committees are possible in the following cases?

i) No restrictions. **(2 marks)**

ii) The committee must have 3 men and 2 women? **(3 marks)**

c) Let $\mathbf{A} = \{-3, 7, 9\}$ and define the relation S on $\mathbf{A}$ as:

$S = \{\, (-3,-3),\ (7,9),\ (9,9),\ (7,-3),\ (9,7),\ (-3,7),\ (7,7)\,\}$.

Determine if the relation S is an equivalence relation. **(4 marks)**

d) Describe the set $\mathbf{C} = \{\, \cdots,\ \frac{1}{27},\ \frac{1}{9},\ \frac{1}{3},\ 1,\ 3,\ 9,\ 27,\ \cdots \}$ in words. **(2 marks)**

e) Define the term "partition" of a set $\mathbf{S}$. **(3 marks)**

f) For the function $g(x) = \begin{cases} -3 & x < -12 \\ 2x^3 - 7x & -12 \leq x \leq 45 \\ \ln(x) & 45 < x \leq 160 \end{cases}$,

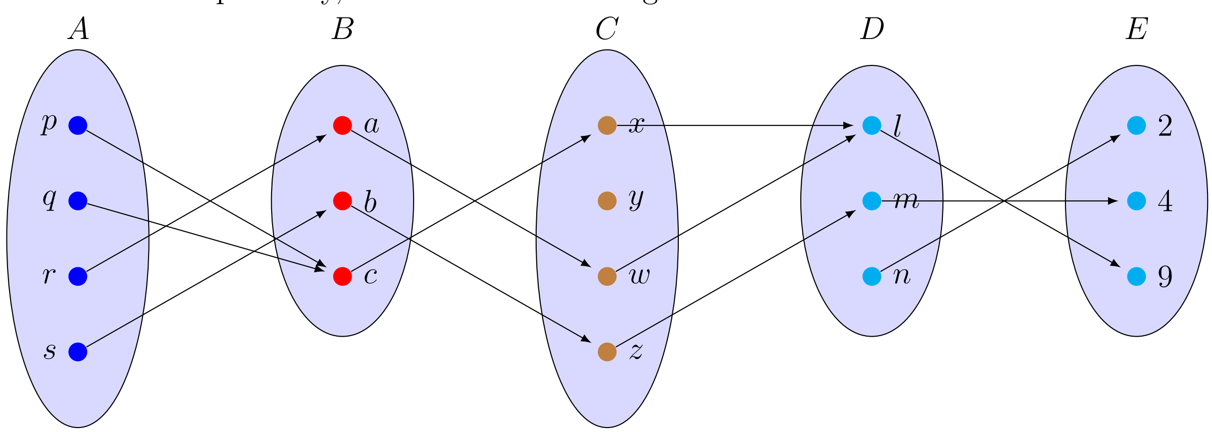
find $g(-17) + g(9)$. **(3 marks)**

QUESTION THREE (20 marks)

(a) Compute $\dfrac{5+2i}{4-3i}$. **(3 marks)**

(b) Let $g : \mathbb{R} \to \mathbb{R}$ be defined by $g(x) = 8x - 11$. Find a formula for the inverse function $g^{-1} : \mathbb{R} \to \mathbb{R}$. **(3 marks)**

(c) Let $f_1,\ f_2,\ f_3$ and f_4 be relations defined from sets **A** to **B** , **B** to **C** , **C** to **D** and **D** to **E** respectively, and whose arrowndiagrams are shown below.



Identify with reasons a relation that is:

i) Not a function. **(2 marks)**

ii) A one to one function. **(2 marks)**

iii) An onto function. **(2 marks)**

iv) An invertible function. **(2 marks)**

(d) Let $\mathbf{A} = \{3, 4, 6, 7\}$ and $\mathbf{B} = \{2, 4, 6, 8, 9, 10, 13, 15\}$. Also let S be a relation from **A** to **B** defined by: $S = \{\ (a, b)\ :\ a \text{ divides } b\ \}$

List the ordered pairs of relations this relation. **(3 marks)**

(e) Let $\mathbf{U} = \{\ -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7\ \}$, $\mathbf{A} = \{\ -3, -1, 0, 1, 3, 5, 6\ \}$, $\mathbf{B} = \{\ -2, -1, 3, 4, 5\ \}$ and $\mathbf{C} = \{\ -4, -3, -1, 0, 3, 4, 5\ \}$. Find all members of the set:

i) $(\mathbf{C} \cup \mathbf{A}) \cap (\mathbf{B} \cup \mathbf{A})$. **(2 marks)**

ii) $(\overline{\mathbf{A}} \cap \overline{\mathbf{B}} \cap \overline{\mathbf{C}})$. **(1 mark)**

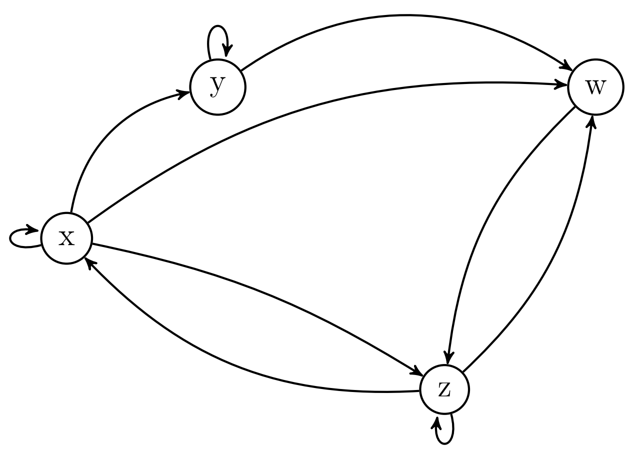
QUESTION FOUR (20 marks)

a) The relation S defined on the set $\mathbf{A} = \{1, 2, 3, 4, 5, 6\}$ is known to be an equivalence relation.

$S = \{(1, 1), (1, 5), (2, 2), (2, 4), (2, 6), (3, 3), (4, 2), (4, 4), (4, 6), (5, 1), (5, 5), (6, 2), (6, 4), (6, 6)\}$.

Determine the contents of its equivalence classes. **(3 marks)**

b) Below is a digraph for a relation S defined on a set $\mathbf{C} = \{\ x,\ y,\ z,\ w\ \}$.



List the elements of the relation S. **(3 marks)**

c) What is the cardinality of the set $\mathbf{B}$ given by

$\{x \in \mathbb{Z} : |x| \geq 11, -16 \leq x < 18\}$. **(3 marks)**

d) Let $\mathbf{A} = \{1, 2, 3, 4\}$. Further let R and S be the following relations on $\mathbf{A}$:

$R = \{(1, 2), (1, 4), (2, 1), (3, 2), (3, 4), (4, 1), (4, 4)\}$ and

$S = \{(1, 1), (1, 3), (2, 1), (2, 2), (2, 4), (3, 3), (4, 1), (4, 2)\}$.

Compute:

i) $R \cap S^C$. **(2 marks)**

ii) The reflexive closure of relation R. **(2 marks)**

iii) The symmetric closure of relation S **(2 marks)**

e) Draw a Venn diagram that represents the set $(A \cup B) \cap C$ **(3 marks)**

f) Find $\overline{z_1 - z_2}$ given $z_1 = 9 + 4i$ and $z_2 = -11 + 8i$. **(3 marks)**

QUESTION FIVE (20 marks)

a) A survey of 550 university students on the games they played revealed the following information:

- 300 play Soccer,
- 240 play Basketball,
- 265 play Volleyball,
- 110 play Soccer and Basketball,
- 140 play Soccer and Volleyball,
- 105 play Basketball and Volleyball,
- 55 play all the three games.

i) Create a Venn diagram to model the information. **(4 marks)**

ii) How many students did not play any of the three games? **(1 mark)**

iii) How many play table Volleyball but not Basketball? **(1 mark)**

iv) How many students play Soccer and Basketball but not Volleyball? **(1 mark1)**

v) How many played only one game? **(1 mark)**

vi) How many play at least two games? **(2 marks)**

b) Let $\mathbf{A} = \{p, q, r, s\}$ A = 1, 2, 3, $\mathbf{B} = \{a, b, c\}$ and $\mathbf{A} = \{x, y, w, z\}$.

Consider the relations R and S from **A** to **B** and from **B** to **C**, respectively given by:

$R = \{(p, 3), (p, 1), (q, 3), (r, 2)\}$ and

$S = \{(1, z), (2, y), (3, z), (3, w)\}$.

i) Draw the arrow diagram for each of the relations. **(3 marks)**

ii) Find the composition relation $R \circ S$. **(2 marks)**

c) Let $\mathbf{A} = \{\, 1,\ 2,\ 3 \,\}$ and $\mathbf{B} = \{\, a,\ b \,\}$.

Determine:

i) The possible number of relations from set $\mathbf{A}$ to set $\mathbf{B}$. **(2 marks)**

ii) The possible number of relations on $\mathbf{A} \times \mathbf{B}$. **(3 marks)**