KARATINA UNIVERSITY

UNIVERSITY REGULAR EXAMINATIONS

2024/2025 ACADEMIC YEAR

SECOND YEAR **FIRST** SEMESTER

SUPPLEMENTARY EXAMINATION

FOR THE DEGREE OF:

BACHELOR OF SCIENCE AND EDUCATION

(P102/6/7 & E100/101/103/111/112)

COURSE CODE: MAT 214

COURSE TITLE: VECTOR ANALYSIS

DATE: **TIME:**

INSTRUCTION TO CANDIDATES

- SEE INSIDE

<u>INSTRUCTION:</u> Answer questions ONE in section A and any other TWO in section B

<u>SECTION A</u>

<u>Question One (30 marks)</u>

a. Given $r_1 = 3i - 2j + k,\ r_2 = 2i - 4j - 3k,\ r_3 = -i + 2j + 2k,$ find the magnitude of:

i. $r_1 + r_2 + r_3$ (2 marks)

ii. $2r_1 - 3r_2 - 5r_3$ (3 marks)

b. If $r_1 = 2i - j + k, r_2 = i + 3j - 2k, r_3 = -2i + j - 3k,$ and $r_4 = 3i + 2j + 5k,$ find scalars a,b,c such that $r_4 = ar_1 + br_2 +, cr_3.$ (4 marks)

c. If $A = A_1 i + A_2 j + A_3 k$ and $B = B_1 i + B_2 j + B_3 k,$ prove that

$$A \cdot B = A_1 B_1 + A_2 B_2 + A_3 B_3.$$ (3 marks)

d. Determine a unit vector perpendicular to the plane of $A = 2i - 6j - 3k$ and $B = 4i + 3j - k.$ (4 marks)

e. Prove that $\nabla^2(\frac{1}{r}) = 0.$ (5 marks)

a. For the space curve $x = \cos(t) + t\sin(t), y = \sin(t) - t\cos(t), z = t^2, t > 0.$ Find the;

i. Unit tangent vector (2 marks)

ii. Curvature (3 marks)

iii. Unit normal vector (2 marks)

iv. Binormal vector (2 marks)

SECTION B

Question Two (20 marks)

a. Verify Green's theorem in the plane for (7 marks)

$$\oint (2xy - x^2)dx + (x + y^2)dy$$

where C is a closed curve bounded by $C: y = x^2 \ and \ y^2 = x$

b. Evaluate $\iint_S F \cdot ndS$, where $F = 4xzi - y^2j + yzk$ and S is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1.$ (6 marks)

c. A particle moves along a curve whose parametric equations are $x = e^{-t},\ y = 2cos3t,\ z = 2sin3t$ where t is the time. Then

i. Determine its velocity and acceleration at any time. (4 marks)

ii. Find the magnitudes of the velocity and acceleration at $t = 0$. (3 marks)

Question Three (20 marks)

a. If $A = (3x^2 + 6y)i - 14yzj + 20xz^2k$, evaluate $\int_C A \cdot dr$ from (0,0,0) to (1,1,1) along the following paths C :

i. $x = t, y = t^2, z = t^3$. (3 marks)

ii. The straight lines from (0,0,0) to (1,1,1), then to (1,1,0), and then to (1,1,1). (5 marks)

b. Find the total work done in moving a particle in a force field given by $F = 3xyi - 5zj + 10xk$ along the curve $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1 \ to \ t = 2.$ (5 marks)

c. If $A = A_1i + A_2j + A_3k, B = B_1i + B_2j + B_3k,\ C = C_1i + C_2j + C_3k$, show that (7 marks)

$$A \cdot (BXC) = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$

Question Four (20 marks)

a. A vector V is called irrotational if curl V = 0. Find constants a,b,c so that $V = (x + 2y + az)i + (bx - 3y - z)j + (4x + cy + 2z)k.$ (5 marks)

b. Evaluate $\int AX\frac{d^2A}{dt^2}dt$. (4 marks)

c. Given that the vector field $F = 2xyi + [xyz^3 - \sin(yz)]j + ze^{x+y}k$. Determine the divergence of F. (4 marks)

d. Find the area cut from the bottom of a paraboloid $z = x^2 + y^2$ by the plane z=1. (7 marks)

Question Five (20 marks)

a. Find the unit tangent vector to any point on the curve $x = t^2 + 1, y = 4t - 3, z = 2t^2 - 6t$. Determine also the unit tangent at the point where t = 2.

(5 marks)

b. If C_1 and C_2 are constant vectors and λ is a constant scalar, show that $H = e^{-\lambda x}(C_1\, sin\lambda y + C_2\, cos\lambda y)$ satisfies the partial differential equation. (5 marks)

$$\frac{\partial^2 H}{\partial x^2} = \frac{\partial^2 H}{\partial y^2} = 0$$

c. If $\emptyset(x, y, z) = 3x^2y - y^3z^2$, find $\nabla\emptyset$(or grad $\emptyset$) at the point $(1, -2, -1)$.

(3 marks)

d. Show that $\vec{F} = \frac{\vec{r}}{r^2}$ is an irrotational vector for $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$. (7 marks)