KARATINA UNIVERSITY

UNIVERSITY EXAMINATIONS
2024/2025 ACADEMIC YEAR

THIRD YEAR **FIRST** SEMESTER REGULAR EXAMINATION
FOR THE DEGREE OF
BACHELOR OF SCIENCE ACTUARIAL SCIENCE
COURSE CODE: ACS 311

COURSE TITLE: ACTUARIAL MATHEMATICS I

DATE:19/12/2024 **TIME: 9.00AM-11.00AM**

INSTRUCTION TO CANDIDATES

- SEE INSIDE

INSTRUCTIONS: Answer All Questions in Section A and Any other Two Questions from Section B.

SECTION A (30 Marks)

QUESTION ONE

a) A life insurance company sells level premium conventional whole life assurance contracts. Explain two reasons why the company will need to set up reserves for the contracts. [2 marks]

b) Using AM92 mortality and 4% p.a. interest, calculate the values of:

i. ${}_{10|}a_{56}$ [2 marks]

ii. $A_{64:\overline{10|}}$ [2 marks]

c) Explain, including formulae, the following expressions assuming that the sum assured is payable at the end of the year of death:

i. death strain at risk [2 marks]

ii. expected death strain [2 marks]

iii. actual death strain [2 marks]

d) A life insurance company issues whole of life assurance policies to lives aged 35 exact for a sum assured of 85,000 payable at the end of year of death. Premiums are payable annually in advance. Calculate the annual office premium for each policy using the basis below.

The office holds reserves using the following basis:

Mortality PFA92C20

Interest 4% per annum

[4 marks]

e) A life insurance company is considering selling with-profit endowment policies with a term of twenty years and initial sum assured of £100,000. Death benefits are payable at the end of the policy year of death. Bonuses will vest at the end of each policy year.

The company is considering three different bonus structures as outlined below. Calculate the amount payable at maturity under each of the three structures.

i. Simple reversionary bonuses of 4.5% per annum. [1 mark]

ii. Compound reversionary bonuses of 3.84615% per annum.[1 mark]

iii. Super compound bonuses where the original sum assured receives a bonus of 3% each year and all previous bonuses receive an additional bonus of 6% each year. [2 marks]

f) A life insurance company sells 1,000 whole life annuities on 1 January 2007 to policyholders aged 65 exact. Each annuity is for £25,000 payable annually arrear. 5 annuitants die during 2007.

The office holds reserves using the following basis:

Mortality PFA92C20

Interest 4% per annum

Calculate the profit or loss from mortality for this group for the year ending 31 December 2007. [3 marks]

g)

Given the following select and ultimate q_x values, calculate the values of $l_{[x]}$ and $l_{[x]+1}$ for $x = 45$ and $x = 46$, assuming that $l_{47} = 1{,}000$.

Age	Duration 0	Duration 1	Duration 2+ (*ie* Ultimate)
45	0.000838		
46	0.000924	0.001158	
47	0.001018	0.001284	0.001415
48		0.001423	0.001564
49			0.001729

[3 marks]

h) A life insurance company sells a special immediate annuity policy to a life aged 65 exact. The policy provides an annuity of 30,000 a year payable monthly in

advance. Payment is guaranteed for the first five years and thereafter ceases immediately on the death of the policyholder.

Calculate the expected present value of this annuity. [4 marks]

Basis:

- Mortality PFA92C20
- Rate of Interest 4% per annum
- Expenses Ignore

SECTION B (40 Marks)

QUESTION TWO (20 Marks)

a) For each of the following expected present values, calculate the value assuming AM92 mortality and 4% p.a. interest.

i. $\overline{A}_{47:\overline{11}|}$ [5 marks]

ii. $\ddot{a}^{(4)}_{[53]:\overline{13}|}$ [5 marks]

b) Consider an endowment assurance of 1 payable at the end of the year of death for a life aged 40 exact, with a term of 15 years.

On the following basis:

- Mortality AM92 Select
- Rate of interest 4% per annum
- Expenses Nil

Calculate the:

i. expected present value of the assurance. [4 marks]

ii. variance of the present value of the assurance. [6 marks]

QUESTION THREE (20 Marks)

A life insurance company issued a with profits whole life policy to a life aged 40 exact on 1 January 2000. Under the policy, the basic sum assured of £50,000 and

attaching bonuses are payable immediately on death. Level premiums are payable annually in advance under the policy until age 65 or earlier death.

The company declares simple reversionary bonuses at the start of each year including the first year and the bonus entitlement on the policy is earned immediately the bonus is declared.

a) Give an expression for the gross future loss random variable under the policy at the outset, defining symbols where necessary. [6 marks]

b) Calculate the annual premium using the following assumptions: [7 marks]
   - Mortality: AM92 Select
   - Interest: 6% per annum
   - Bonus loading: 2.5% per annum simple
   - Initial expenses: £300
   - Renewal expenses: £25 at the start of the second and subsequent policy years while the policy is in force
   - Claim expenses: £250

c) On 31 December 2009, the policy is still in force. Bonuses declared to date total £13,750. Calculate the gross premium prospective reserve for the policy as at 31 December 2009 using the following assumptions: [7 marks]
   - Mortality: AM92 Ultimate
   - Interest: 4% per annum
   - Bonus loading: 3% per annum simple
   - Renewal expenses: £35 at the start of each policy year while the policy is in force
   - Claim expenses: £250

QUESTION FOUR (20 Marks)

A life insurance company issues with-profits whole of life policies to lives aged 35 exact with the sum assured of £100,000 together with any attaching bonuses

payable immediately on death of the life assured. Level premiums are payable monthly in advance to age 65 or until earlier death.

The company markets two types of this policy as follows:

Policy A – assumed to provide compound bonuses of 4% of the sum assured vesting at the end of each policy year.

Policy B – assumed to provide simple bonuses of b% per annum of the sum assured, again vesting at the end of each policy year.

The death benefit under each version does not include any bonus relating to the policy year of death.

The following basis is used to price these contracts:

- Mortality: AM92 Select
- Rate of Interest: 4% per annum
- Initial expenses: £275
- Renewal expenses: 2.5% of the second and subsequent monthly premiums
- Initial commission: 40% of the total premiums payable in the first policy year, all incurred at the policy commencement date
- Renewal commission: 2.5% of the second and subsequent monthly premiums
- Claim expenses: £225 at payment of death claim

a) Show that the monthly premium under Policy A of this policy is approximately £511. [13marks]

b) Calculate the level simple bonus rate b% that can be supported each year under Policy B of this policy if the monthly premium calculated in part (a) is charged. [7 marks]

QUESTION FIVE (20 Marks)

On 1 January 2005, a life insurance company issued 1,000 10-year term assurance policies to lives aged 55 exact. For each policy, the sum assured is £50,000 for the

first five years and £25,000 thereafter. The sum assured is payable immediately on death and level annual premiums are payable in advance throughout the term of this policy or until earlier death.

The company uses the following basis for calculating premiums and reserves:

- Mortality AM92 Select
- Interest 4% per annum
- Expenses Nil

a) Calculate the annual premium for the policy. [7 marks]

b) Calculate the net premium retrospective reserve per policy as at 31 December 2009. [6 marks]

c) State two conditions under which the retrospective reserve is equal to the prospective reserve. [2 marks]

d) There were, in total, 20 deaths during the years 2005 to 2008 inclusive and a further 8 deaths in 2009. Calculate the total mortality profit or loss to the company during 2009. [5 marks]

- Interest: 8% per annum
- Mortality: AM92 Select
- Expenses ignore
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