Inspiring Innovation and Leadership

# KARATINA UNIVERSITY

## UNIVERSITY EXAMINATIONS
## 2017/2018 ACADEMIC YEAR

## **FOURTH** YEAR **FIRST** SEMESTER EXAMINATION

## FOR THE DEGREE OF

## BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

**COURSE CODE: ACS 412**

**COURSE TITLE: SURVIVAL MODELS AND ANALYSIS**

**DATE:** **TIME:**

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**INSTRUCTION TO CANDIDATES**

- ANSWER **ALL QUESTIONS** IN SECTION A
- ANSWER **ANY TWO** QUESTIONS FROM SECTION B

## SECTION A

## ANSWER ALL QUESTIONS IN THIS SECTION.

## QUESTION ONE (30 marks)

(a) Explain the concept of data censoring, clearly distinguishing between the three types. [6 marks]

(b) The population of elderly people in a prison is observed during the period of 1st January 1994 to 31st December 1996. The duration of residence (measured to the nearest number of months) is recorded for those who die during the period. The recorded data measured in months are:

| 9, 13, 13+,18, 23, 28+, 31, 34, 45+, 48, 61+ |
| --- |

The plus observations are censored times.

(i) Compute the Kaplan-Meir estimator of the survival function for this data. [10 marks]

(ii) Compute 95% confidence limits on the survival function for the first three survival times, that is, for $\tau = 9,13,13+$ using Greenwood's formula. [7 marks]

(iii) Compute the median survival time. [2 marks]

(c) Given the hazard function $\lambda(t) = c$ derive the survivorship function and the probability density function. [5 marks]

## SECTION B

## ANSWER <u>ANY TWO</u> QUESTIONS FROM THIS SECTION.

## QUESTION TWO (20 marks)

The time (in months) from the start of treatment to relapse or the end of follow-up for 15 children with rhabdomyosarcoma treated with surgery and radiation but no chemotherapy was as follows:

Relapsed: 2, 3, 9, 10, 10, 15, 16, 30

Disease free: 12, 15, 18, 24, 36, 40, 45

Estimate by calculator the disease-free survival as a function of time since treatment using the Product Limit Estimator (method of Kaplan and Meir) by completing the rows of the following table:

| **Time (Months)** | **Number at Risk** | **Number Died** | **Proportion Survived** | **P.L.E of Survival** |
|---|---|---|---|---|
| 2 | 15 | 1 | 0.9333 | 0.9333 |
| 3 | 14 | 1 | 0.9286 | 0.8667 |
| etc. | | | | |

## QUESTION THREE (20 marks)

Consider the following life table data from patients with cancer of the ovary.

| Time from Diagnosis (yr) | Number lost To Follow-up $l_i$ | Number Withdrawn Alive $w_i$ | Number Dying $d_i$ | Number Entering $n_i$ |
|---|---|---|---|---|
| 0 – 10 | 20 | 0 | 731 | 949 |
| 10 – 20 | 18 | 0 | 52 | 200 |
| 20 – 30 | 8 | 67 | 14 | 132 |
| 30 – 40 | 0 | 33 | 10 | 43 |

(a) Compute the life table for these data. [16 marks]

(b) Plot the estimated hazard function $\hat{\lambda}(t)$ versus time for the above data and explain. [4 marks]

## QUESTION FOUR (20 marks)

The data below shows survival times (in months) of patients with Hodgkin's disease who were treated with nitrogen mustards. Group A patients received heavy prior therapy, whereas Group B patients received little or no prior therapy.

**Group A:** 23, 16+, 18+, 20+, 24+

**Group B:** 15, 18, 19, 19, 20

Compare the survival distributions of the two therapy groups at 5% level of significance using:

(a) Gehan's generalized Wilcoxon test and interpret. [10 marks]

(b) Compute the linear rank – type variance and the linear rank type – Statistic for the data set and interpret. [10 marks]

## QUESTION FIVE (20 marks)

(a) Consider the following survival times for a group of patients on a treatment (+ denote right censored) that follow the exponential distribution with parameter $\lambda$.

6+, 6, 6, 6, 7, 9+, 10+, 10, 11+, 13, 16, 17+, 19+, 20+, 22, 23, 25+, 32+, 32+, 34+, 35+.

Calculate the maximum likelihood estimate of the parameter of the distribution. [6 marks]

[Hint: The pdf of an exponential distribution is defined as:

$$f(t) = \begin{cases} \lambda e^{-\lambda t}, & t > 0 \\ 0, & elsewhere \end{cases}$$ ]

(b) The life time of light bulbs follows a Weibull distribution with parameters $\alpha$ and $\beta$, where $\alpha$ is the scale parameter and $\beta$ is the shape parameter.

The survival function of these light bulbs is given by:

$$S(t) = e^{-\alpha t^{\beta}}$$

Find the;

(i) Probability density function $f(t)$. [2 marks]

(ii) Hazard function $\lambda(t)$ [2 marks]

(iii) Cumulative hazard function $\Lambda(t)$. [2 marks]

(c) Explain in detail the following survival analysis techniques demonstrating how these analyses can be carried out in R software:

(i) Cox Proportional Hazards models. [4 marks]

(ii) Accelerated Failure Time models. [4 marks]