Inspiring Innovation and Leadership

KARATINA UNIVERSITY

UNIVERSITY EXAMINATIONS

2024/2025 ACADEMIC YEAR

FOURTH YEAR REGULAR EXAMINATIONS

FOR THE DEGREE OF:

BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

COURSE CODE: ACS 414

COURSE TITLE: LOSS MODEL I

DATE: 19th **December, 2024 TIME:12.00PM-2.00PM**

Instructions: See inside.

SECTION A

Answer **all** questions from this section

QUESTION ONE (30 marks)

(a) If X has a Pareto distribution with parameters $\lambda = 400$ and $\alpha = 3$, and N has a Poisson(50) distribution, find the expected value of S . **[5 marks]**

(b) The probability of a claim arising on any given policy in a portfolio of 1,000 one year term assurance policies is 0.004. Claim amounts have a $Gamma(5, 0.002)$ distribution.

Find the mean and variance of the aggregate claim amount. **[4 marks]**

(c) The loss severity distribution for a portfolio of household insurance policies is assumed to be Pareto with parameters α= 3.5 , λ= 1,000 . Next year, losses are expected to increase by 5%, and the insurer has decided to introduce a policyholder excess of £100. Calculate the probability that a loss next year is borne entirely by the policyholder. **[3 marks]**

(d) The distribution of the number of claims from a motor portfolio is negative binomial with parameters $k = 4,000$ and $p = 0.9$. The claim size distribution is Pareto with parameters $\alpha = 5$ and $\lambda = 1200$. Find the ;

(i) mean and **[2 mark]**

(ii) variance of the aggregate claim distribution. **[3 mark]**

(e) Find;

(i) an expression for the MGF of the aggregate claim amount if the number of claims has a bin(100, 0.01) distribution, and individual claim sizes are gamma(10, 0.2). **[2 marks]**

(ii) the mean and variance of the aggregate claim amount. **[3 marks]**

(f) Suppose that claim amounts are uniformly distributed over the interval (0,500). The insurer effects individual excess of loss reinsurance with a retention limit of 375. Calculate the expected amounts paid by the insurer and the reinsurer in respect of a single claim. **[4 marks]**

(g) Claim numbers from individual policies in a portfolio have a $Poisson(\lambda)$ distribution. The parameter λ varies over the risks in the portfolio and is assumed to have a $Gamma(\alpha, \theta)$ distribution. Show that, if α is a positive integer, then the mixture distribution of the claim numbers is negative binomial with parameters $k = \alpha$ and $p = \frac{\theta}{\theta+1}$ **[4 marks]**

SECTION B

Answer **any Two** questions from this section

QUESTION TWO (20 marks)

Let N be the number of claims on a risk in one year. Suppose claims $[X_1, X_2, ..]$are independent, identically distributed random variables, independent of N . Let S be the total amount claimed in one year.

(a) Derive $E(S)$ and $var(S)$ in terms of the mean and variance of N and X_1. **[6 marks]**

(b) Derive an expression for the moment generating function $MS(t)$ of S in terms of the moment generating functions $M_X(t)$ and $M_N(t)$ of X_1 and N respectively. **[6 marks]**

(c) If N has a Poisson distribution with mean l , show that:

$M_S(t) = \exp(\lambda(M_X(t) - 1))$ **[3 marks]**

(d) If N has a binomial distribution with parameters m and q , determine the moment generating function of S in terms of m, q and $M_X(t)$. **[5 marks]**

QUESTION THREE (20 marks)

A portfolio consists of 500 independent risks. For the ith risk, with probability $1-q_i$ there are no claims in one year, and with probabilityq_i there is exactly one claim $(0 < q_i < 1)$. For all risks, if there is a claim, it has mean m , variance sˆ2 and moment generating function M(t). Let T be the total amount claimed on the whole portfolio in one year.

(a) Determine the mean and variance of T. The amount claimed in one year on risk i is approximated by a compound Poisson random variable with Poisson parameter q_i and claims with the same mean m , the same variance σ^2 and the same moment generating function M(t)as above. **[8 marks]**

(b) Determine the mean and variance of T, and compare your answers to those in part (a). Assume that $q_i = 0.02$ for all i , and if a claim occurs, it is of size μ with probability one. **[6 marks]**

(c) Derive the moment generating function of T and show that T has a compound binomial distribution **[3 marks]**

(d) Determine the moment generating function of the approximating T and show that T has a compound Poisson distribution. **[3 marks]**

QUESTION FOUR (20 marks)

(a) Claims in a portfolio are believed to arise as an $Exp(\lambda)$ distribution. There is a retention limit of 1,000 in force and claims in excess of 1,000 are paid by the reinsurer. The insurer, wishing to estimate λ, observes a random sample of 100 claims, and finds that the average amount of the 90 claims that do not exceed 1,000 is 82.9. There are 10 claims that do exceed the retention limit. Find the MLE for λ. **[7 marks]**

(b) Claims from a portfolio are believed to have a Pareto(α, λ) distribution. In Year 0, and λ=1 000. An excess of loss reinsurance arrangement is in force, with a retention limit of 500. Inflation is a constant 10% pa. Find the ;

(i) distribution of the insurer's claim payments in Years 1 and 2 before reinsurance. **[5 marks]**

(ii) percentage increase in the insurer's mean net claims payout in each year. **[8 marks]**

QUESTION FIVE (20 marks)

(a) The annual number of claims from an individual policy in a portfolio has a $Poisson(\theta)$ distribution. The variability in θ among policies is modelled by assuming that over the portfolio, individual values of θ have a $Gamma(\alpha, \delta)$distribution.

Derive the mixture distribution for the annual number of claims from each policy in the portfolio. **[7 marks]**

(b) A random sample of 100 claim amounts $x_1, x_2, ...x_{100}$ is observed from a Weibull distribution with parameter $\gamma = 2$ where c is unknown. For these data:

$\sum x_i = 487,926$, $\sum x_i^2 = 976,444,000$ median 4,500

(i) Show that the maximum likelihood estimator for c is given by:

$\bar{c} = \frac{n}{\sum x_i^2}$ **[6 marks]**

(ii) Hence estimate the value of c . **[4 marks]**

(c) The loss severity distribution for a portfolio of household insurance policies is assumed to be Pareto with parameters $\alpha = 3.5$, $\lambda = 1,000$. Next year, losses are expected to increase by 5%, and the insurer has decided to introduce a policyholder excess of £100. Calculate the probability that a loss next year is borne entirely by the policyholder. **[3 marks]**