# KARATINA UNIVERSITY

## UNIVERSITY REGULAR EXAMINATIONS

## 2024/2025 ACADEMIC YEAR

**SECOND** YEAR **FIRST** SEMESTER

REGULAR EXAMINATION

**FOR THE DEGREE OF:**

**BACHELOR OF SCIENCE AND EDUCATION**

**(P102/P106/P107 & E100/E101/E103/E111/E112)**

**COURSE CODE: MAT 214**

**COURSE TITLE: VECTOR ANALYSIS**

**DATE: 20/12/24** **TIME: 9:00-11:00AM**

**INSTRUCTION TO CANDIDATES**

- SEE INSIDE

**<u>INSTRUCTION:</u> Answer questions ONE in section A and any other TWO in section B**

## <u>SECTION A</u>

### <u>Question One (30 marks)</u>

a) Find the directional derivative of $\varphi(x, y, z) = x^2y^2z + 4x^2$ at $(1,2,-1)$ in the direction of the vector $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$. (4 marks)

b) If $\varphi = 3x^2y - y^3z^2$, find grad $\phi$ at the point $(1, -2, -1)$. (4 marks)

c) Given that the three vectors $\vec{A} = \hat{\imath} + \hat{\jmath} + a\hat{k}$, $\vec{B} = 7\hat{\imath} + 2\hat{\jmath} + 0\hat{k}$ and $\vec{C} = 0\hat{\imath} + \hat{\jmath} + 7\hat{k}$ are coplanar, determine the value of $a$ . (4 marks)

d) Determine the curl of the vector $\vec{F} = y\hat{\imath} + 2xz\hat{\jmath} + ze^x\hat{k}$ (4 marks)

e) Given the parametric equation $x = sint$, $y = cost$, $z = 45t$, $0 \leq t \leq 2\pi$. Find;

   i. The tangent vector $\vec{F}'(t)$. (3 marks)

   ii. The arc length. (4 marks)

f) Find the volume of a parallelopiped whose edges are represented by

$\vec{F} = 3\hat{\imath} + \hat{\jmath} + \hat{k}$, $\vec{G} = 2\hat{\imath} - 3\hat{\jmath} + \hat{k}$, $\vec{H} = \hat{\imath} - 3\hat{\jmath} - 4\hat{k}$ (4 marks)

g) Determine the value of 'd' such that the vectors of $\vec{A}$ and $\vec{B}$ are orthogonal.

$\vec{A} = 6\hat{\imath} - d\hat{\jmath} + 2\hat{k}$ and $\vec{B} = 3\hat{\imath} + \hat{\jmath} + 4\hat{k}$ (3 marks)

# SECTION B

## Question Two (20 marks)

a) For the space curve $x = 3\,cos(\,t), y = 3\,sin(\,t), z = 4t$.Find the;

i) Unit tangent vector $\hat{T}$. (4 marks)

ii) Curvature k. (3marks)

iii) Principal normal vector $\widehat{N}$. (2 marks)

iv) Binormal vector $\hat{B}$ and torsion $\tau$. (5 marks)

b) Evaluate the triple integral $\int_0^1 \int_0^{\sqrt{1-y^2}} \int_{x^2+y^2}^{1-x^2-y^2} xdzdxdy$ (6 marks)

## Question Three (20 marks)

a) A projectile is fired from the origin over a horizontal ground with an initial speed of 50m/s and a launch angle of 30 degrees where will the projectile be 10 seconds later? (take $g = 9.8m/s^2$) (6 marks)

b) Given the vector $\vec{A} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$ and $\vec{B} = -\hat{\imath} + 2\hat{\jmath} - 2\hat{k}$ determine;

i. $\left|\vec{A}\right|$, (1 mark)

ii. $\left|\vec{B}\right|$, (1 mark)

iii. $\vec{A} \cdot \vec{B}$ (1 mark)

iv. hence determine the angle between the two vectors (1 mark)

c) Given $\emptyset(x, y, z) = xyz^2$ and $\vec{A} = xz\hat{\imath} - xy^2\hat{\jmath} + yz^2\hat{k}$. Compute $\frac{\partial^3}{\partial z \partial x \partial y}(\emptyset\vec{A})$ at the point $(2, -2,2)$. (5 marks)

d) What is the area of a triangle determined by the points $P(-1,2,0)$, $Q(2,3, -2)$ and $R(0, -3,4)$. (5 marks)

**Question Four (20 marks)**

a) Show that $\overline{F} = (y^2 \cos x + z^3)\hat{i} + (2y\sin x - 4)j + (3xz^2 + 2)k$ is a conservative force field. Determine the scalar potential and work done in moving a particle in this force field from $(0, 1, -1)$ to $(\frac{\pi}{2}, -1, 2)$. (10 marks)

b) Find an equation for the plane determined by the points $(1,2,1), (-1,1,3)$ and $(-2,-2,-2)$. (6 marks)

c) The formula for the orthogonal projection of u onto v is $proj_v u = \left(\frac{u\cdot v}{v\cdot v}\right) v$.

i. Find $proj_v u$ if $u = \langle 2,1,2\rangle$ and $v = \langle 0,-1,1\rangle$. (3 marks)

ii. Find the projection of u ORTHOGONAL to v. (1 mark)

**Question Five (20 marks)**

a) Show that $\vec{F} = \frac{\vec{r}}{r^2}$ is an irrotational vector for $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$. (6 marks)

b) A particle moves along a curve whose parametric equations are $x = \sin(t)$, $y = 2e^{-t},\ z = t^2$ where t is the time. Then

i. Determine its velocity at any time. (2 marks)

ii. acceleration at any time. (1 mark)

iii. determine the speed of the particle at any time. (1 mark)

iv. Find the magnitudes of the velocity and acceleration at $t = 0$. (2 marks)

c) Use divergence theorem to evaluate: $\int\int_s \vec{F} \cdot \hat{n}\, ds$ for the vector field $\vec{F} = \sin(\pi x)\,\hat{\imath} + zy^3\hat{\jmath} + (z^2 + 4x)\hat{k}$ where $s$ is the surface of the of the box with $-1 \le x \le 2$, $0 \le y \le 1$ and $1 \le z \le 4$. (8 marks)