KARATINA UNIVERSITY

UNIVERSITY EXAMINATIONS

2024/2025 ACADEMIC YEAR

THIRD YEAR FIRST SEMESTER

EXAMINATIONS

FOR THE DEGREE OF

BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

COURSE CODE: ACS 312

COURSE TITLE: FINANCIAL MATHEMATICS II

DATE: 20^{TH} DEC, 2024 TIME: 9.00A.M.-11.00A.M.

Instructions: See inside

SECTION A

Answer **all** questions from this section

QUESTION ONE (30 marks)

(a) Calculate the present value of an annuity that pays £150 annually in arrears forever using an annual effective rate of interest of 8%. **[3 mark]**

(b) The present value at 6% pa of the following series of payments is \$245.32.
\$1 at time 1, \$4 at time 2, \$9 at time 3, ... ,\$100 at time 10. What is the present value of the series of payments if, instead of being paid at the end of each year, the payments are made in three equal instalments at the end of each third of a year? **[4 marks]**

(c) Calculate the yield achieved by investing in a 15-year pure discount bond if P_{15}=0.54 . **[3 marks]**

(d) The prices for zero coupon bonds of various terms are as follows:
1 year = £94% 5 years = £70% 10 years = £47% 15 years = £30%
£x% means £x per £100 nominal. Calculate the spot rates for these terms and sketch a graph of these rates as a function of the term. **[5 mark]**

(e) Define a swap. **[1 mark]**

(f) What happens to yields of fixed interest securities if:

(i) bond prices fall **[1 marks]**

(ii) demand for fixed-interest securities falls **[1 marks]**

(iii) the government issues many more stocks **[1 marks]**

(iv) institutional investors suddenly decide to invest less in equities and more in fixed-interest securities **[1 marks]**

(v) bond prices rise **[1 marks]**

(g) Determine an expression for the convexity of the cashflows arising from a lump sum deposited in a bank account that pays simple interest at a constant rate at the end of each year.

Assume that the lump sum is invested for an indefinite term. **[4 marks]**

(h) Evaluate the discounted mean term of a bond redeemable at par in 10 year's time with annual coupons of 8% at interest rates of 5%, 10% and 15%. **[5 marks]**

SECTION B

Answer **any Two** questions from this section

QUESTION TWO (20 marks)

(a) A fund must make payments of £50,000 at the end of the sixth and eighth years. Show that, if interest rates are currently 7% pa at all durations, immunisation to small changes in interest rates can be achieved by holding an appropriately chosen combination of a 5- year zero-coupon bond and a 10-year zero-coupon bond. **[14 marks]**

(b) At 1 July 2004, an investor has a liability of £20,000 to be paid on 1 January 2008 and a liability of £18,000 to be paid on 1 July 2010. The investor currently holds assets with a present value equal to the present value of the liabilities. The investor wishes to immunise its position by investing in two zero coupon bonds with outstanding terms of four years and seven years. Determine whether or not this is possible assuming an effective interest rate of 10% per annum. **[6 marks]**

QUESTION THREE (20 marks)

(a) You are given the following information about the term structure of interest rates:

The 1-year spot rate is 4.1% pa.

The 2-year spot rate is 4.0% pa.

The 4-year spot rate is 4.4% pa.

The 5-year spot rate is 4.5% pa.

(i) Calculate the theoretical price of a 3-year zero-coupon bond that is issued at time 2 and is redeemable at par **[3 marks]**

(ii) A 1-year zero-coupon bond is issued at time 3 and has a theoretical price of £95.50 per £100 nominal. Given that the bond is redeemable at par, calculate the :

(a) 3-year spot rate, **[3 marks]**

(b) 1-year forward rate starting at time 2 **[2 marks]**

(iii) Calculate the gross redemption yield on a bond that pays annual coupons of 6% pa (at times 1, 2, 3, 4 and 5) and is redeemable at 110% in 5 years' time. **[6 marks]**

(b) An individual purchases £100,000 nominal of a bond on 1 January 2003 which is redeemable at 105% in four years time and pays coupons of 4% per annum at the end of each year. The investment manager wishes to invest the coupon payments on deposit until the bond is redeemed. It is assumed that the rate of interest at which the coupon payments can be invested is a random variable and the rate of interest in any one year is independent of that in any other year. Deriving the necessary formulae, calculate the mean value of the total accumulated investment on 31 December 2006 if the annual effective rate of interest has an expected 7alue of $5\frac{1}{2}\%$ in 2004, 6% in 2005 and $4\frac{1}{2}\%$ in 2006. **[6 marks]**

QUESTION FOUR (20 marks)

(a) List any four characteristics that are particular to property investments. **[4 marks]**

(b) Outline the role of margin and the clearing house in reducing counterparty credit risk. **[2 marks]**

(c) Calculate the mean and variance of the accumulated value of an initial investment of £40,000 at the end of 25 years if the annual rates of return are assumed to conform to the varying interest rate model and follow a Gamma (16,200) distribution. **[6 marks]**

(d) A lump sum of \$14,000 will be invested at time 0 for 4 years at an annual rate of interest i . The interest rate, once determined, will be the same in each of the four years. $1 + i$ has a log-normal distribution with mean 1.05 and variance 0.007. Calculate the probability that the investment will accumulate to more than \$20,000 in 4 years' time. **[8 marks]**

QUESTION FIVE (20 marks)

(a) A company wishes to invest £10,000 in 182-day bills from the British government. The bills are currently issued at an annual discount of 10%. Calculate the par value of the bills that could be purchased. **[2 marks]**

(b) What are the main features of:

(i) government bills **[3 marks]**

(ii) fixed interest government bonds **[3 marks]**

(c) An index-linked bond is issued on 28 February 2006 and pays half-yearly coupons and is redeemable at par on 28 February 2024. There is no time lag on the indexation. The coupon paid on 28 February 2008 was £2.10. A non-taxpayer buys £100 nominal of the bond on 29 February 2008. Assuming that past inflation

has been 4% pa and future inflation is 5.25% pa, how much should this investor pay in order to obtain a money rate of return of 10% pa? **[4 marks]**

(d) An investor purchases a bond 3 months after issue. The bond will be redeemed at par ten years after issue and pays coupons of 6% per annum annually in arrears. The investor pays tax of 25% on both income and capital gains (with no relief for indexation). Calculate the purchase price of the bond per £100 nominal to provide the investor with a rate of return of 8% per annum effective. **[8 marks]**