Inspiring Innovation and Leadership

KARATINA UNIVERSITY

UNIVERSITY EXAMINATIONS

2024/2025 ACADEMIC YEAR

FIRST YEAR SECOND SEMESTER REGULAR EXAMINATIONS

FOR THE DEGREE OF MSC IN PURE MATHEMATICS

COURSE CODE: MAT 823

COURSE TITLE: ALGEBRAIC CODING THEORY

DATE: th ., 2025 **TIME:**

Instructions: See Inside

Answer question **ONE** in section A and any other **Two** from section B.

SECTION A

Question ONE is Compulsory

QUESTION ONE (20 marks)

(a) What is the information rate of the following code:

$C = \{0000, 1110, 1111, 0101, 1010\}$? **[3 marks]**

(b) Determine whether $C = \{1101, 1110, 1011, 1111\}$ is a linear code over $\mathbb{F}_2$. **[3 marks]**

(c) Let $q = 2$, $S = \{0001, 0010, 0100\}$ and the vector space $V = \langle S \rangle$, then

$V = \{0000, 0001, 0010, 0100, 0011, 0101, 0110, 0111\}$. The set S is linearly independent.

Determine the number of different bases for the vector space V. **[3 marks]**

(d) Let $q = 2$ and let $S = \{(0, 1, 0, 0), (0, 1, 0, 1)\}$.Find $S^{\perp}$. **[3 marks]**

(e) Consider the binary linear code C with the following generator matrix:

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}.$$

Determine whether C is a cyclic code. **[3 marks]**

(f) Assign messages to the words in $\mathbb{F}_2$ as follows:

000	100	010	001	110	101	011	111
A	C	D	E	G	I	N	O

Let C be the binary linear code with generator matrix

$$G = \begin{pmatrix} 10101 \\ 01010 \\ 00011 \end{pmatrix}.$$

Use G to encode the message "CAGE". **[5 marks]**

SECTION B

Answer **any Two** questions from this section

QUESTION TWO (20 marks)

(a) Let $C = \{001, 011\}$ be a binary code.

i) Suppose we have a memoryless binary channel with the following probabilities:

$$P(0 \text{ received} \mid 0 \text{ sent}) = 0.1,$$

$$P(1 \text{ received} \mid 1 \text{ sent}) = 0.5.$$

Use the maximum likelihood decoding rule to decode

the received word 000. **[5 marks]**

ii) Use the nearest neighbour decoding rule to decode 000. **[3 marks]**

(b) Consider the Hamming Code with the following generator matrix

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

Suppose we have the received word $r = (1\ \ 1\ 1\ 0\ 1\ 1\ 1\ 1)$.

Decode the word. **[4 marks]**

(c) $x^7 - 1 = (x-1)(x^3+x+1)(x^3+x^2+1)$ over $\mathbb{Z}_2$.

Construct the generator matrix and the parity-check matrix of the cyclic code C generated by $g(x) = 1 + x + x^3$. **[8 marks]**

QUESTION THREE (20 marks)

(a) List the cosets of the binary linear code $C = \{0000, 1011, 0101, 1110\}$. **[5 marks]**

(b) Find all binary cyclic codes of length 3. **[6 marks]**

(c) Suppose that codewords from the binary code $C = \{000, 100, 111\}$ are being sent over a BSC (binary symmetric channel) with crossover probability $p = 0.03$. Use the maximum likelihood decoding rule to decode the following received words: $010, 011001$. **[9 marks]**

QUESTION FOUR (20 marks)

(a) Given the binary linear code: $C = \{0000, 1011, 0101, 1110\}$, and the parity-check matrix:

$$H = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix},$$

construct the syndrome look-up table assuming complete nearest neighbor decoding. Use the coset leaders: $0000, 0001, 0010, 1000$. **[6 marks]**

(b) Use the syndrome look-up table constructed above to decode

(i) $w = 1101$ **[3 marks]**

(ii) $w = 1111$. **[3 marks]**

(c) Find a generator matrix. a parity-check matrix and the parameters $[n, k, d]$ for the binary linear code $C = \langle S \rangle$, where $S = \{11101, 10110, 01011, 11010\}$. **[8 marks]**