KARATINA UNIVERSITY UNIVERSITY EXAMINATIONS
2024/2025 ACADEMIC YEAR
FIRST YEAR FIRST SEMESTER REGULAR EXAMINATION
FOR THE DEGREE OF BACHELOR OF SCIENCE (P100, P101, P102, P103, P105, P106, P107) BACHELOR OF EDUCATION (E100, E101, E111, E112, E103) COURSE CODE: MAT 116
COURSE TITLE: CALCULUS I

DATE: 14th January 2025 TIME: 3.00 pm-5.00pm

INSTRUCTIONS TO CANDIDATES:
See inside

ANSWER QUESTION ONE AND ANY OTHER TWO QUESTIONS

QUESTION ONE: (30 MARKS)

a) Evaluate each of the following limits

i. $\lim_{x \to -2} \frac{x^3+2x^2-1}{5-3x}$ (3marks)

ii. $\lim_{x \to 1} \frac{x^2-1}{x-1}$ (3marks)

b) Determine the derivative of each of the following functions

i. $f(x) = \frac{1-x}{2+x}$ (3marks)

ii. $f(x) = \frac{2}{\sqrt{3-x}}$ (3marks)

iii. $y = (x^2 + 1)\sqrt[3]{x^2 + 2}$ (3marks)

iv. $y = 2x + cos^2x$ (3marks)

c) Find the equation of the tangent line to the curve $y = x^3 - 4x + 1$ at point (1,-2) (3marks)

d) Given the equation of a circle $x^2 + y^2 = 25$ find $\frac{dy}{dx}$ at the point (3,4). (4 marks)

e) The equation of motion of a particle is $s = 2t^3 - 5t^2 + 3t = 4$ where s is measured in centimetres and t in seconds. Find velocity and acceleration as a function of time. (2marks)

f) The cost function of a product x is given by $C(x) = 2500 + 2\sqrt{x}$. Find the marginal cost at the production of 100 units. (3 marks)

QUESTION TWO 20 MARKS

a) Find $f'(x)$ given that $f(x) = \frac{1}{\sqrt[3]{x^2+x+1}}$ (3marks)

b) Show that the derivative of the function $y = arcsinx$ is given by $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$ (5marks)

c) Evaluate $\lim_{t\to 9} \frac{9-t}{3-\sqrt{t}}$. (3marks)

d) Consider the equation $y = 3x^2 + x + 1$. Find the following

   i. gradient at the point (-2,1) (2marks)

   ii. Equation of the tangent (3marks)

   iii. Equation of the normal at point (-2,1) (3marks)

QUESTION THREE 20 MARKS

a) find the derivative of the function $y = \frac{3x}{e^{2x}}$ (3marks)

b) If a tank holds 5000 gallons of water, which drains from the bottom of the tank in 40 min, then Torricelli's Law gives the volume V of water remaining in the tank after t minutes as $V = 5000(1 - \frac{t}{40})^2$ .Find the rate at which water is draining from the tank after 20 min (4marks)

c) Determine and classify the stationery points for the function $f(x) = x^4 - 4x^3 + 3x^2 - 2$. (13marks)

QUESTION FOUR 20 MARKS

a) Given that the cost and demand functions for a product x are $C(x) = 3800 + 5x - \frac{x^2}{1000}$ and $p(x) = 50 - \frac{x}{100}$ respectively. Determine the following

   i. Marginal Revenue. (3marks)

   ii. Marginal cost. (2marks)

   iii. Maximum profit. (4marks)

b) Find the values of constants c and d to make the function

$$h(x) = \begin{cases} 2x, & \text{for} \quad x < 1 \\ cx^2 + d, & \text{for} \quad 1 \leq x \leq 2 \\ 4x, & \text{for} \quad x > 2 \end{cases}$$

continuous (5marks)

c) Given that the distance moved by an object in time t is $10t - 20t^2$ metres, determine

i. The maximum distance travelled. (4mks)

ii. Velocity after 3 seconds. (2mks)

QUESTION FIVE 20 MARKS

a) If $f(x) = \sqrt{x-1}$, find the derivative of f using the first principles method. (2marks)

b) Where does the normal line to the ellipse $x^2 - xy + y^2 = 3$ at the point (-1,1) intersect the ellipse a second time. (6marks)

c) The rate at which photosynthesis takes place for a species of phytoplankton is modeled by the function $P = \frac{100L}{L^2+L+4}$ where L is the light intensity. For what light intensity is P a maximum. (7marks)

d) Find the value of constant a to make the function

$$f(a) = \begin{cases} 7a^2 + a, & \text{for} \qquad x \leq a \\ a+1, & \text{for} \qquad x > a \end{cases}$$

continuous (5marks)