KARATINA UNIVERSITY

UNIVERSITY EXAMINATIONS

2024/2025 ACADEMIC YEAR

SECOND YEAR FIRST SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF

BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

COURSE CODE: ACS 211

COURSE TITLE: FINANCIAL MATHEMATICS I

DATE: 19th DEC, 2024 **TIME:** 09.00A.M.-11.00A.M.

Instructions: See inside.

SECTION A

Answer **all** questions from this section

QUESTION ONE (30 marks)

(a) State whether each of the following annuities is paid in arrears or in advance, immediate or deferred, level, simple increasing or simple decreasing.

   (i) Payments of £2 paid at the start and halfway through each of the next five years   **[1 marks]**

  (ii) A payment of £5 in five years, £10 in ten years, ..., £40 in forty years.   **[1 marks]**

(b) Find an approximate value for the present value of a series of payments of £1 each, payable for 1 year at the beginning of each week, assuming an effective rate of interest of 8% per annum.

(Assume that there are 52.18 weeks in a year.)   **[4 marks]**

(c) Define the accumulation factor $A(t_1, t_2)$ in terms of the force of interest

$\delta(t)$ **[1 mark]**

(d) An annuity is payable annually in arrears for 20 years. The first payment is of amount £8,000, and the amount of each subsequent payment decreases by £300 each year. Find the present value of the annuity on the basis of an interest rate of 5% per annum using;

   i) First principles,   **[4 marks]**

  ii) Increasing annuity functions.   **[4 marks]**

(e) A speculator borrows £50,000 at an effective interest rate of 8% per annum to finance a project that is expected to generate £7,500 at the end of each year for the next 15 years.

Find the discounted payback period for this investment.   **[4 marks]**

(f) Define the following terms with respect to project appraisal;

    (i) net present value (NPV)   **[2 marks]**

    (ii) internal rate of return (IRR)   **[1 mark]**

    (iii) payback period   **[1 mark]**

    (iv) discounted payback period   **[1 mark]**

    (v) linked internal rate of return   **[1 mark]**

(g) Find the present value as at 1 June 2004 of payments of £1,000 payable on the first day of each month from July 2004 to December 2004 inclusive, assuming a rate of interest of 8% per annum convertible quarterly.   **[5 marks]**

SECTION B

Answer **any Two** questions from this section

QUESTION TWO (20 marks)

(a)   (i) Calculate the present value of a continuously payable annuity that is initially paid at a rate of £200 pa but decreases linearly to £100 pa after 10 years, assuming a force of interest of $\delta(t) = 0.2 - 0.01t$, where t is the time from commencement (measured in years).   **[7 marks]**

    (ii) Calculate the accumulated value of the annuity after 10 years at the same force of interest.   **[4 marks]**

(b) An investor is to receive a special annual annuity for a term of 10 years in which payments are increased by 5% compounded each year to allow for inflation. The first payment is to be £1,000 on 1 November 2009. Calculate the accumulated value of the annuity payments as at 31 October 2026 if the investor achieves an effective rate of return of 4% per half year   **[6 marks]**

(c) Differentiate between n-year spot rate and forward rate.   **[3 marks]**

QUESTION THREE (20 marks)

An investor is considering making an investment in one or both of two projects. The cashflows associated with the projects are as follows. The unit of time is years.

Project A: Initial payments of £2 million at time zero and £4 million at time 2 are made. In return a sum of £900,000 per annum is paid continuously from time 5 to time 25.

Project B: Regular payments of £100,000 are made at the start of each year for 10 years. In return, amounts of X , 2X , 3X and so on are made annually for 10 years, the first payment being made at time 11.

(i) Find the net present value of Project A at an effective annual interest rate of 10%. **[2 marks]**

(ii) Show that the internal rate of return for Project A is 9.38% pa. **[2 marks]**

(iii) Find the value of X if the internal rate of return for Project B is the same as that for Project A. **[4 marks]**

(iv) Find the value of X if both projects are to have the same net present value at 10% pa. **[3 marks]**

(v) The investor proposes to borrow all the money needed for the project. Funds are available at an interest rate of 7% per annum effective. Repayments can be made at any time, and positive cash balances can be invested to yield 3% per annum. If X=£45,000, find the accumulated value of each project at the end of the 25 year period. **[9 marks]**

QUESTION FOUR (20 marks)

(a) A fund had a value of £21,000 on 1 July 2003. A net cash flow of £5,000 was received on 1 July 2004, and a further net cash flow of £8,000 was received on 1 July 2005. Immediately before receipt of the first net cash flow, the fund had a value of £24,000, and immediately before receipt of the second net cash flow, the fund had a value of £32,000. The value of the fund on 1 July 2006 was £38,000.

   (i) Calculate the annual effective money-weighted rate of return earned on the fund over the period 1 July 2003 to 1 July 2006. **[4 marks]**

  (ii) Calculate the annual effective time-weighted rate of return earned on the fund over the period 1 July 2003 to 1 July 2006. **[4 marks]**

 (iii) Explain why the values in (i) and (ii) differ. **[3 marks]**

(b) Explain why banks and other organisations lending money to people buying homes with mortgages often impose restrictions and/or penalties if all or part of the mortgage is repaid during the first 3 years. **[4 marks]**

(c) In respect of an investment project, define the:

   (i) discounted payback period **[2 marks]**

  (ii) payback period **[1 marks]**

(d) Discuss why both the discounted payback period and the payback period are inferior measures compared with the net present value for determining whether to proceed with an investment project. **[2 marks]**

QUESTION FIVE (20 marks)

On 1 September 2005, a company placed part of its assets with two fund managers. Manager P was given £80,000 and Manager Q was given £140,000. Both managers received a net cashflow of £15,000 on 1 September 2006, bringing their total fund values to £103,000 and £183,000, respectively.

A further net cashflow of £20,000 was received by each manager on 1 September 2007. This brought their total fund values to £143,600 and £239,600, respectively.

On 31 August 2008, the value of Manager P's fund was £172,320 and the value of Manager Q's fund was £263,560.

(a) For the period from 1 September 2005 to 31 August 2008, calculate for each fund manager the:

   (i) time weighted rate of return   **[4 marks]**

   (ii) money weighted rate of return.   **[5 marks]**

(b) By examining the growth factors between cashflows, describe the performance of each manager over the three-year period. Hence, explain why the money weighted rate of return for   **[3 marks]**

(c) An investor borrows £120,000 at an effective interest rate of 7% per annum. The investor uses the money to purchase an annuity of £14,000 per annum payable half yearly in arrears for 25 years. Once the loan is paid off, the investor can earn interest at an effective rate of 5% per annum on money invested from the annuity payments, determine the:

   (i) discounted payback period for this investment.   **[3 marks]**

   (ii) profit the investor will have made at the end of the term of the annuity.   **[5 marks]**