KARATINA UNIVERSITY

UNIVERSITY EXAMINATIONS

2024/2025 ACADEMIC YEAR

FOURTH YEAR **FIRST** SEMESTER REGULAR EXAMINATION

FOR THE DEGREE OF

BACHELOR OF SCIENCE WITH EDUCATION, BACHELOR OF EDUCATION SCIENCE, BACHELOR OF EDUCATION ARTS and BACHELOR OF SCIENCE

COURSE CODE: MAT 417

COURSE TITLE: FLUID MECHANICS II

DATE: 17th Dec 2024 TIME: 12.00noon-2.00pm

INSTRUCTION TO CANDIDATES

SEE INSIDE

ANSWER **QUESTION ONE (COMPULSORY)** AND ANY OTHER **TWO QUESTIONS FROM SECTION B**

SECTION A

QUESTION ONE (30 marks)

a) Define the meaning of the following terms

i. Irrotational flow (2marks)

ii. Equipotential lines (2marks)

b) Given the velocity complex potential $w(z)$ explain how the velocity components (u,v) in the (x,y) directions are obtained. (1mark)

c) Write down the continuity equation for an incompressible steady flow in three dimensions. (2marks)

d) State the three main assumptions in the derivation of the D'Alembert's paradox . (3marks)

e) The stream function for a flow is given by $\psi = xy$. Determine

i. Velocity components in the x and y directions (2marks)

ii. The condition for which the flow is irrotational (3marks)

f) From the continuity equation for an incompressible steady flow show that the stream function satisfies the Laplace equation. (4marks)

g) Explain the fluid flow dynamics through a Tee- Junction. Make a sketch to support your explanation. (4marks)

h) Consider a line source of strength m at $z = z_1$ where $R_e z \geq 0$ and the plane $x = 0$ is a rigid boundary. Show that the image of a line source in a rigid infinite plane is a line source of equal strength. (7marks)

SECTION B

QUESTION TWO (20 marks)

a) For a two dimensional irrotational flow, show that the stream function ψ is harmonic. (5marks)

b) The velocity distribution for the flow of an incompressible fluid is given by $u = 3 - x, v = 4 + 2y, w = 2 - z$. Show that this satisfy the requirements of the continuity equation. (3marks)

c) The velocity distribution for the flow of a fluid given by $u = 5x + \frac{x^3}{3}$, $v =- xy - \frac{3y^2}{2}$. Show that this represents a possible case of rotational flow at the point (-1,2). (4marks)

d) Given that $u = x^2 - y^2$ and $v =- xy$, determine the stream function. (3marks)

e) Proof that the curl of velocity is zero for irrotational flow $\nabla \times \vec{v} = 0$ (5marks)

<u>QUESTION THREE</u> (20**<u>marks</u>**)

a) A stream function is given by $\psi = 5x - 6y$. determine the following

i. Velocity components (2marks)

ii. Velocity magnitude (2marks)

iii. Direction of the resultant velocity any point (3marks)

b) Consider the flow whose velocity complex potential is $w = z^2$. Determine

i. Equipotential and stream functions (2marks)

ii. Streamline and equipotential lines (2marks)

iii. Velocity components (2marks)

c) A stream function for a two-dimensional flow is given by $\psi = x + 2x^2 - 2y^2$. Determine

i. Whether the flow is rotational (4marks)

ii. Velocity potential of the flow (3marks)

<u>QUESTION FOUR</u> (20**<u>marks</u>**)

a) Derive the complex velocity potential function for a uniform flow whose velocity is $-ui$. (6marks)

b) The velocity potential function is given by an expression $\phi = \frac{-xy^3}{3} - x^2 + \frac{x^3 y}{3} + y^2$

i. Find the velocity components in x and y directions (4marks)

ii. Show that ϕ represents a possible case of flow (4marks)

c) show that in a potential flow the circulation around any closed curve is zero. (6marks)

QUESTION FIVE (20**marks**)

a) Determine the complex velocity potential of a uniform flow that is inclined at an angle $\propto$ to the positive x-axis. (10marks)

b) Discuss the flow whose complex velocity potential is given by $w = uze^{-i\alpha}$ (10marks)