Firm Dynamics and Innovation: Evidence from Decomposing Top Sales Shares

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Abstract

What do changes in top sales shares signal about changes in firm dynamics? I use an accounting decomposition to identify two sources of top sales shares growth: (i) incumbent top firms grow bigger; (ii) new top firms replace old top firms. Over the 1950-2019 period, incumbent top firms contribute about 3.5 times as much as new top firms to the growth of the sales shares accrued to the top 0.01% firms in the US economy. Using the results from this empirical decomposition, I then build a continuous-time random growth model to estimate a firm dynamics process in which firms grow in response to own innovation shocks and shrink at the impact of creative destruction shocks. The existence of a channel through which own innovation can lead to higher top sales shares growth is supported by the data. My estimation reveals a surge in the rate of own innovation since 1980 and a decline in the rate of creative destruction over time.

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1 Introduction

Recently there has been a revival of interest in the role of large firms in the economy. Studies have documented a rise of "superstar firms", i.e., an increase in the sales shares accrued to the leading firms of the US industries over the 1990-2010 period (e.g., Autor et al. (2020)). As top firms amass a larger share of total sales, what does it signal about the underlying firm dynamics process that drives the rise and fall of large firms? What do changes in the top sales shares imply about aggregate productivity growth?

In this paper, I answer these questions by separately accounting for two potential sources of the growth of top sales shares: (i) incumbent top firms grow bigger; (ii) new top firms replace old top firms. These two sources of top sales shares growth connect naturally to the underlying forces that make firms grow and shrink. Intuitively, forces that make incumbent top firms grow (shrink) increase (decrease) the first component of top sales shares growth. While making incumbent top firms shrink, destructive forces also removes dwindling incumbents from the top percentile. This contributes to the growth of top sales shares via the second component.

Taking advantage of the mechanism that forces which make firms grow or shrink have differential impact on the two components of top sales shares growth, I infer the underlying process that drives firm dynamics based on the decomposition of top sales shares growth. Depending on whether the rise in top sales shares comes from growth of incumbent top firms or new top firms displacing old top firms, different underlying processes that drive the rise and fall of large firms may be at work.

I start by applying the accounting framework developed in Gomez (2020) to decompose the growth of sales shares accrued to the top 0.01% firms (henceforth top sales shares) of the US economy over the 1950-2019 period. I focus on two key components of the growth of top sales shares from this decomposition. The first term, the *within* term, is the contribution to top sales shares growth by firms that belong to the top percentile at the beginning of period, whether or not they remain in the top by the end of period (i.e., holding constant the composition of top firms). The second term, the *displacement* term, measures the effect of compositional changes of top firms on top sales shares growth.

During the whole sample period (1950-2019), the within term is about 3.5 fold the magnitude of the displacement term. That is, the growth of top sales shares is to a large extent due to incumbent top firms growing bigger. However, the displacement of old top firms by new top firms also makes important positive contributions to the growth of top sales shares.

Equipped with the empirical decomposition results, I estimate the underlying firm dynamics process which features a positive shock that makes a firm grow and a negative shock that makes a firm shrink. For instance, these shocks can be interpreted as innovation shocks that take place in different forms. The positive shock can be viewed as an "own innovation" shock in response to which firms improve on their own products and grow larger. In contrast, the negative shock can be viewed as a "creative destruction" shock which occurs to a firm when its product is improved upon by its competitor.

In a continuous-time firm random growth model, I show that when the dynamics of individual firms are subject to these two shocks modeled as a compound poisson process, there exists a unique stationary firm size distribution that is Pareto. In terms of transition dynamics, I obtain closed-form formulas to approximate the asymptotic behavior of the two decomposition components that drive the growth of top sales shares. This mapping between the theoretical and empirical components of the top sales shares growth allows me to estimate the underlying process that drives the dynamics of large firms.

The model predicts that different forms of innovation exert different impact on the total growth of top sales shares: own innovation drives an increase in the top sales shares growth and creative destruction leads to a fall in the growth of the top sales shares. A further dissection disentangle the mechanisms through which these two forms of innovation shape the displacement term. Own innovation increases displacement via an "inflow" effect by promoting previously lower-ranked firms into the top. Creative destruction raises displacement via an "outflow" effect by removing dwindling incumbent firms from the top. When measured in the data, both the inflow and outflow effects contribute positively to the displacement term. Therefore, the data depicts that both forms of innovations are at work. As own innovation leads to top sales shares growth via the within term and the displacement term, my finding highlights a channel through with an increase in innovation (in the form of own innovation) can result in higher top sales shares growth.

With regard to the dynamic process that drives the rise and fall of large firms, I find that prior to 1980, the rate at which large firms are hit by own innovation shocks is low. However, when the shocks occur, they make large firms grow at a large step size. In contrast, in the post 1980s, the rate of own innovation is much higher with a smaller step size. The rate of creative destruction has been declining from 16% prior to the 1980s to 10% in more recent years. However, the step size at which firms shrink at the impact of creative destruction shocks has become larger over the years. Across periods, the top sales shares rise primarily due to increases in own innovation. This rise in the top sales shares prevails despite the offsetting effect of creative destruction.

After estimating the intensities of own innovation shocks and creative destruction shocks, I nest the estimated firm dynamics process in a growth model to study its implications on aggregate productivity growth. I find that the implied aggregate productivity growth is highest over the 1980-1995 period when both own innovation and creative destruction rates are fairly high. It slows down afterwards because of the decline in both own innovation and creative destruction rates over the past twenty years.

Related literature. Existing studies have documented that from the late 1990s to the early 2010s, top firms have captured an increasing proportion of industry sales. For instance, Dorn et al. (2017) and Autor et al. (2020) report an upward trend in the sales shares accrued to top 4 and top 20 firms across four-digit industries by major sector.

I extend the analysis to a longer time horizon covering the 1950-2019 period and document the evolution of top sales shares at the aggregate level by measuring the sales shares accrued to the top 0.01% firms in the entire economy. A contemporaneous paper by Kwon et al. (2021) measures the top 1% and the top 0.1% firms' sales shares from 1960 onwards. We share similar findings that the top sales shares have been on an upward historical trend prior to the 1980-2010 horizon which is the period of focus by most existing studies on the industrial concentration. My study differs in that, beyond the static viewpoint, I add a dynamic perspective to the top sales shares: I take into account the entry and exit of firms in and out of the top percentile by tracking a panel of top firms.

My study contributes to a growing literature that seeks to understand the forces behind the rise of large firms. One prevalent view in this literature is that star firms arise from the reallocation of market share to more productive firms. This reallocation can operate via several channels: (i) network effect or platform competition foster a "winner-takes-most" market structure (Van Reenen (2018); Autor et al. (2020)); (ii) the rising importance of intangible capital benefits large firms disproportionately (Crouzet and Eberly (2018); Haskel and Westlake (2017)); (iii) technological changes favor large firms more either because large firms can better absorb fixed costs or because they can exploit economies of scale (e.g.: Information and Communication Technology (ICT) (Bessen (2017); Aghion et al. (2019); Hsieh and Rossi-Hansberg (2019)), digital capital (Tambe et al. (2020)) and Artificial Intelligence (AI) (Babina et al. (2020))); (iv) increased levels of product differentiation that can result from better search technology and the pervasive power of data analytics (Calligaris et al. (2018)). In addition, Fernández-Villaverde et al. (2021) reveals that a combination of search complementarities in the formation of vendor contracts and monopsony power endogenously generates superstar firms.

The literature remains unclear on whether the scale-biased economic forces that favor large firms disproportionately will make existing large firms more entrenched or launch new superstar firms. I contribute to these work by examining the forces that drive the rise and fall of large firms in different periods when different technological environments are in place.

My paper is also related to the growth literature that emphasizes different forms of innovation. Own innovation and creative destruction are two major innovation forms that appear in growth models either separately or jointly. [1] To quantify the relative importance of own innovation and creative destruction to growth, previous studies have directly sought to observe the substitution of existing products by new products (Christensen (2013), Broda and Weinstein (2010) and Hottman et al. (2016)) or looked at patent citation patterns by same firm or other firm and breakthrough patents (Akcigit and Kerr (2018) and Kelly et al. (2021)). From an indirect perspective, through the lens of an exogenous growth model in which compared to own innovation, creative destruction leaves a more polarized job creation/destruction rate in the data, Garcia-Macia et al. (2019) and Klenow and Li (2020) infer which forms of innovation play a dominant role on aggregate growth.

I also take the indirect approach to infer on the innovation process that drives firm dynamics. Relative to the existing studies, rather than using employment moments which focus more on smaller firms and new firms into the economy, my inference differs by focusing on moments of the right tail of firm size distribution such as components of changes in the top sales shares resulting from compositional changes of top firms and turnover rate due to entry and exit of firms into the top percentile. A significant amount of innovation are conducted by large firms and their products are also subject to challenges from innovative competitors. As a result, disciplining the firm innovation process with moments of top sales shares can provide a new angle in comparison to focusing on moments generated by firms at the lower end of firm size distribution.

I study the evolution and compositional changes of firms that belong to the top 0.01% percentile of the US economy. This connects to studies on the macroeconomic outcomes

[1] Classic examples of creative destruction include Stokey (1988); Grossman and Helpman (1991); Aghion and Howitt (1990); Klette and Kortum (2004). Theories focusing on own innovation include Krusell (1998); Lucas Jr and Moll (2014). Some models combine creative destruction and own innovation (e.g., Akcigit and Kerr (2018)).

due to the presence and growth of a small group of "star" or "dominant" firms (i.e., firms in the upper tail of firm size distribution measured by sales or assets). The dynamics of these star firms have important macroeconomic consequence not only from the exercise of market power, but also from shaping the landscape of employment opportunities and funding of new ideas (White (2002)). Relately, shocks to large firms can drive aggregate fluctuations (Carvalho and Grassi (2019); Gabaix (2011)). Gutiérrez and Philippon (2019) has examined the contribution to aggregate productivity growth by the economy-wide superstar (top 20 by market value) firms.

Given the importance of star firms, whether the giant firms maintain entrenched leadership or become contestable from lower-ranked firms is a subject of continuing interest (Collins and Preston (1961); Stonebraker (1979); Manyika et al. (2018)). Large firms may escape the Schumpeterian process of destruction and remains large via exploiting their political connections to stifle entry (Steffens (1906); Faccio and McConnell (2020)). There is also recent evidence that large incumbents impede innovation of their rival startups through acquisition (Kamepalli et al. (2020); Cunningham et al. (2021)). A separate strand of literature approach the question of leadership turnover from an industrial organization (IO) perspective. Papers in this category feature game theoretical elements on the strategic behavior of one or two market leaders and examine how their gap in market share evolves (e.g., Sutton (2007); Dou et al. (2020); Liu et al. (2019)). My work complements these two strands of literature by examining entry/exit rate into a group of top firms of the whole economy (rather than change of leadership for a few firms at the industry level) and comparing among firms. Additionally, I distinguish between the entry rate of new top firms via organic growth and the exit rate of dwindling old top firm, after purging out the entry and exit induced by mergers and acquisitions.

By bridging changes in the top sales shares to changes in aggregate productivity via the underlying innovation processes, my paper is related to previous work on innovation and productivity growth. The literature on the dynamics of U.S. productivity seeks to explain its surge in the 1990s to early 2000s and the subsequent slowdown (Sparque (2021); Decker et al. (2020); Grullon et al. (2019); Andrews et al. (2015); Bloom et al. (2020)). Innovation can prompt a surge in entry of new firms which first increases productivity dispersion and then leads to productivity growth (Foster et al. (2018)). However, the connection between innovation and productivity growth maybe more subtle since it may lead to a burst in productivity growth at first but with a subsequent productivity slowdown (Klenow et al. (2019)). Relative to this literature, I explore on whether changes in different forms of innovation account for changes in aggregate productivity. Existing studies have shown that, aside from own innovation forces which make existing firms grow, creative destruction also contribute to aggregate productivity growth via resource reallocation (Fujita et al. (2008); Foster et al. (2006) and Petrin et al. (2011)). Equipped with rates of own innovation and creative destruction calibrated from components of changes in the top sales shares, through a standard growth model, I infer on the relative contribution to productivity growth by own innovation as opposed to creative destruction.

Roadmap. The rest of the paper is organized as follows. Section 2 presents the data used for my empirical analysis. In section 3, I present empirical facts on the growth of the sales shares accrued to the top 0.01% firms in the US economy via an accounting decomposition. Section 4 uses these empirical facts to estimate the underlying firm dynamics process. Section 5 discuss the implications of my findings on aggregate productivity growth. Section 6 compares the characteristics of firms that enter, exit and stay in the top 0.01% percentile. Section 7 concludes.

2 Data

Top sales shares. Sales data of firms with headquarter in the US are from *Compustat*. Total sales are proxied by the gross output of all industries from Bureau of Economic

Analysis (BEA). [2] Gross output has not been released by the BEA yet, therefore I impute the gross output in 2020 from the gross output in 2019 and the net GDP growth rate from 2019 to 2020 [3]. The total number of firms in 1978-2018 are from Census Business Dynamics Statistics (BDS) (TableID: BDSNAICS). Number of firms in 1950-1977 are imputed by assuming that the number of firms is a constant fraction of population size [4], where the firm number-to-population ratio is calculated as the average firm number-to-population ratio between 1978 to 1982. Number of firms in 2019 is imputed from the number of firms in 2018 and the growth rate of firm number from 2017 to 2018. With the pandemic outbreak in 2020, I assume the firm number does not grow and remains the same as 2019. Sales and gross output are deflated using US GDP deflator line 1 from NIPA Table 1.1.9.

Merger and acquisition. The merger and acquisition (M&A) data comes from the SDC platinum database. I keep the M&A deals whose status have been completed, remove the deals in which the target and the acquirer have the same name or CUSIP to avoid counting in the share buybacks by the same firm, and keep the deals after which the acquirer owns 100% share of the target. I merge this M&A dataset with the Compustat firm sales data to label firms in the top 0.01% which have been involved in M&A transactions each year, that is, a top firm that is either an acquirer or a target in an M&A deal effective in that year. Since there is no common firm identifier to directly merge between the SDC platinum database and the Compustat firms, I generate the matching in a number of ways: (1) use deal number (in SDC platinum) to acquirer and target GVKEY (in Compustat) crosswalk constructed from Phillips and Zhdanov (2013) and Ewens et al. (2019) [5]; (2) Combine the match with CUSIP (6-digit in SDC platinum and 9-digit in Compustat) and the fuzzy name match to crosswalk the deals without deal number in SDC platinum into Compustat.

[2] Source: https://www.bea.gov/industry/io-histannual and https://apps.bea.gov/iTable/iTable.cfm?reqid=150&step=2&isuri=1&categories=ugdpxind

[3] source: https://fred.stlouisfed.org/series/GDPA

[4] Source: http://www.demographia.com/db-uspop1900.htm

[5] Source: https://github.com/michaelewens/SDC-to-Compustat-Mapping

Firm-level data. Firm-level patent value is from Kogan et al. (2017) [6]. The firm patent value data from Kogan et al. (2017) is matched with firm accounting data using the "PERMNO" firm identifier in CRSP-Compustat Merged (CCM) database.

3 Three facts about top sales shares

In this section, I document three empirical facts on concentration in the upper tail of firm size distribution measured by the share of sales accrued to the top 0.01% firms (henceforth "top sales share") in the US economy over the 1950-2020 period. Starting from the year-by-year static snapshots of top sales shares, these facts further examine the evolution of top sales shares from the perspective of large firm dynamics by taking into account the entry and exit of firms into the top 0.01% percentile.

Fact 1: The sales share accrued to the top 0.01% firms of the US economy has doubled from 17% in 1950 to around 35% in 2020. Over the 1950-1980 period, the top sales share has been on a steady upward trend. In the post-1980s, the trend has become flatter and exhibits more fluctuations, with two declining phases (1980-1995 and 2010-2020) and an increasing phase (1995-2010).

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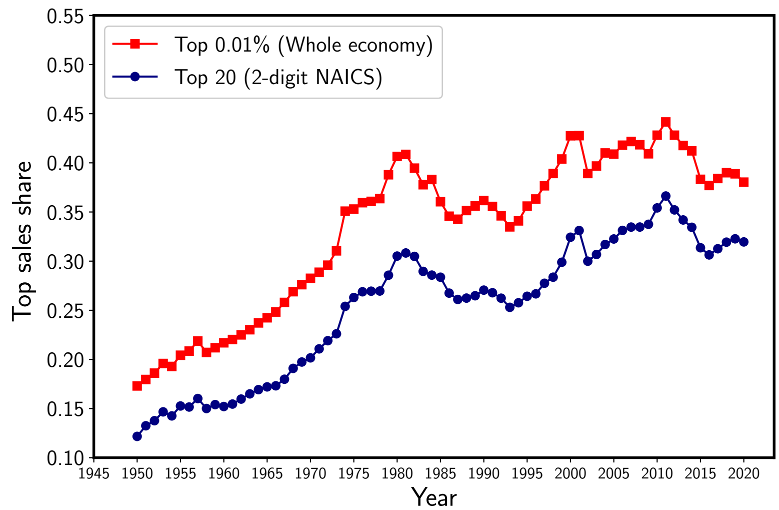
[6] Source: https://github.com/KPSS2017/Technological%2DInnovation%2DResource%2DAllocation%2Dand%2DGrowth%2DExtended%2DData

Figure 1: Aggregate and within-industry top sales share



Note: "Top 0.01% (Whole economy)" plots the sales share accrued to the top 0.01% firms of the US economy. "Top 20 (2-digit NAICS)" plots the average sales share of the top 20 firms of 2-digit NAICS industries (weighted by industry sales). Data are from *Compustat* (sales of top firms), BEA (gross output of all industries) and Census BDS (TableID: BDSNAICS).

To construct the top sales shares [7], I first obtain the total number of firms in the U.S. from the Census Business Dynamics Statistics (BDS). Depending on the total number of firms in the US economy, the top 0.01% group include from a range of 246 firms in 1950 to 532 firms in 2020. I obtain the sales of firms in this top 0.01% group from *Compustat*. Since *Compustat* only contains sales data of firms that are publicly traded, in doing so, I make the assumption that all firms that are ranked in the top 0.01% by sales are public firms. The total sales of all firms in the US economy are proxied by the gross output of all industries from the Bureau of Economic Analysis (BEA). By assembling the above pieces of data, I can calculate the top sales share as the ratio of the sum of sales of the top 0.01% firms to the total sales of all firms in the US.

During the entire sample period (1950-2020), the top sales share has doubled from 17% in 1970 to 37% in 2020. A closer examination of Figure 1 shows that the top sales share

[7] Section 2 provides more details on the construction of top sales shares.

starts low at 17% in 1950, has risen steadily to 40% in 1980, then declines and remains fairly low around 35% over the 1985-1995 period. Starting from the late 1990s, the top sales share rises from 35% to a historical peak at 44% in 2011, then drops to 37% in 2020.

Fact 2: The aggregate trend of top sales share reflects within industry changes.

One question that arises from Fact 1 is whether the aggregate trend reflects changes in the top sales share that have happened within industries or it stems from across-industry effects where top firms in some industries are capturing more sales relative to those in other industries. Figure 1 shows that the sales-weighted average of the within-industry top 20 firms sales share tracks well the aggregate trend of the sales share accrued to the top 0.01% firms of the US economy. This indicates that the aggregate trend in the top sales shares mainly reflects within-industry top sales share changes rather than across-industry effects.

The broad-based rise of top firm sales share at the industry level from the late 1990s to the early 2010s echos the findings from the existing studies that the average sales share accrued to top 4 and top 20 firms across four-digit industries by major sector has risen during the same period (Dorn et al. (2017); Autor et al. (2020)).

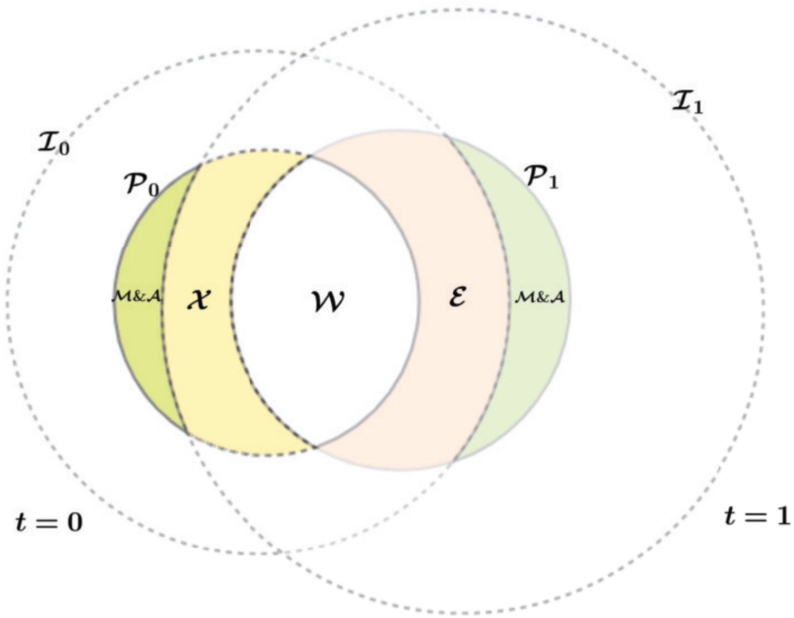
Fact 3: The growth of the top 0.01% firms sales shares can come from: (i) incumbent top firms grow bigger; (ii) new top firms replace old top firms.

The top 0.01% percentile does not comprise the same firms over time. Only 10% of the top 0.01% percentile firms in 2020 were among the top 0.01% firms in 1950 (Appendix Figure 1). The rise and fall of large firms can lead to compositional changes of firms that belong to the top 0.01% group. As a result, both changes in the share of firms that remain inside the top and changes due to the entry/exit of firms into the top can contribute to changes in the top sales share. To understand what is the major driving force behind the aggregate trend of top sales shares and how that evolves over time, I apply an accounting decomposition framework which is developed in Gomez (2020) to decompose the changes in sales share of top firms.

Accounting Framework. To illustrate the accounting framework, one can start by considering the different sets of firms that comprise the top percentile firms in two periods. As demonstrated in the Venn diagram (Figure 2), consider a given top sales percentile $p \in (0,1]$ and denote $\mathcal{P}_t \subset \mathcal{I}_t$ the subset of firms who are in the top percentile at time t ($t \in \{0,1\}$). Denote $\mathcal{W}$ the set of firms who remain in the top percentile in both periods. Let $\mathcal{X}$ be the set of firms who exit the top percentile but remains in the economy at $t = 1$ and $\mathcal{E}$ the set of firms who are in the economy at $t = 0$ and enter the top percentile at $t = 1$. Denote $\mathcal{D}$ the set of firms who are in the top percentile at $t = 0$ and exit the economy at $t = 1$. And denote $\mathcal{B}$ the set of firms who are not in the economy at $t = 0$ and appear in the top percentile at $t = 1$. Firms that belong to the sets $\mathcal{D}$ and $\mathcal{B}$ are targets or new firms that emerge from merger and acquisition (M&A) transactions. They also lead to compositional changes in the top firms and I group them as the $\mathcal{M}\&\mathcal{A}$ sets. Let q_t be the sales of the firm at the percentile threshold (i.e., the top p quantile).

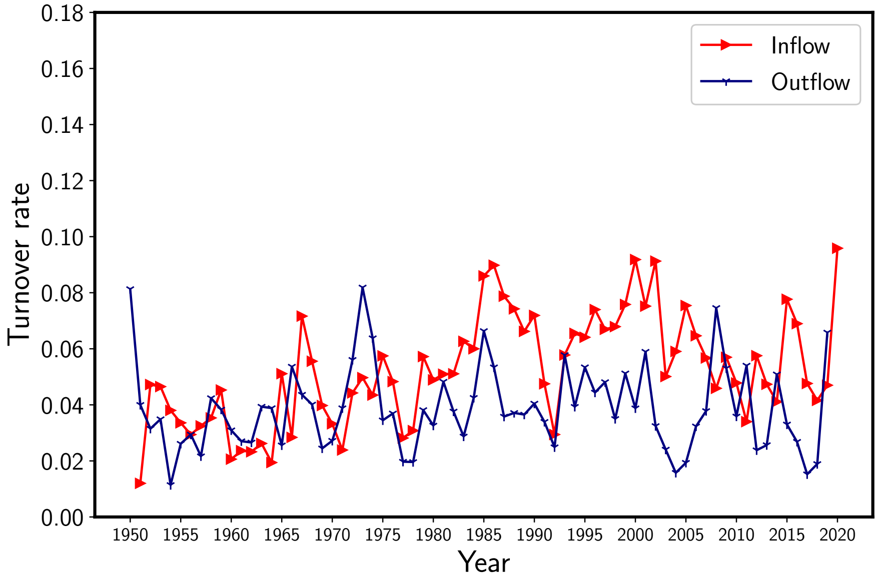
Firms in the sets $\mathcal{X}$ and $\mathcal{E}$ contribute to the compositional change effect that new top firms displace old top firms. In particular, this effect represents that lower-ranked firms surpass old top firms via organic growth rather than mechanical forces such as M&A. Figure 3 plots the fraction of top firms that belongs to the set $\mathcal{E}$ (Inflow) and the set $\mathcal{X}$ (Outflow) and show that around 2-8% of top firms each year enter and exit the top and contribute to this displacement effect.

Figure 2: Venn Diagram Representing Composition Changes in the Top Percentile



Note: This figure plots $\mathcal{I}_t$, the set of all firms in the economy, $\mathcal{P}_t \subset \mathcal{I}_t$ the subset of firms who are in the top percentile at time t ($t \in \{0, 1\}$). The following subsets corresponds to the set of firms used in the accounting decomposition framework from Gomez (2020): $\mathcal{W}$ the set of firms who remain in the top percentile in both periods; $\mathcal{X}$ is the set of firms who exit the top percentile but remains in the economy at $t = 1$; $\mathcal{E}$ is the set of firms who are in the economy at $t = 0$ and enter the top percentile at $t = 1$; the $\mathcal{M}\&\mathcal{A}$ sets contain firms that either directly emerge in the top percentile at $t = 1$ or firms that were in the top at $t = 0$ but disappear from the economy at $t = 1$. Firms in the $\mathcal{M}\&\mathcal{A}$ sets are involved in merger and acquisition transactions.

Figure 3: Turnover rate (Inflow and Outflow)



Note: Turnover rate (Inflow) is the fraction of top 0.01% firms that belongs to the set $\mathcal{E}$, i.e., the set of firms that are new entrants into the top 0.01% firms each year. Turnover rate (Outflow) is the fraction of top 0.01% firms that belongs to the set $\mathcal{X}$, i.e., the set of firms that exit the top 0.01% firms each year.

For a set of firms $\mathcal{G}$, let $|\mathcal{G}|$ be the number of firms in $\mathcal{G}$ and $\overline{y}_{\mathcal{G},t}$ be the average sales of firms in $\mathcal{G}$ at time t. Then the top percentile's sales share $\mathcal{S}_t$ can be expressed as:

$$\mathcal{S}_t = \frac{\sum\limits_{i \in \mathcal{P}_t} y_{it}}{\sum\limits_{i \in \mathcal{I}_t} y_{it}} = \frac{\sum\limits_{i \in \mathcal{P}_t} \frac{y_{it}}{\overline{y}_{\mathcal{I}_t}}}{\sum\limits_{i \in \mathcal{I}_t} \frac{y_{it}}{\overline{y}_{\mathcal{I}_t}}} = \frac{|\mathcal{P}_t| \overline{u}_{\mathcal{P}_t}}{|\mathcal{I}_t|} = p \overline{u}_{\mathcal{P}_t} \tag{1}$$

where $\overline{u}_{\mathcal{P}_t} = \frac{\sum_{i \in \mathcal{P}_t} \frac{y_{it}}{\overline{y}_{\mathcal{I}_t}}}{|\mathcal{P}_t|}$. Therefore, $\frac{\mathcal{S}_t}{\mathcal{S}_{t-1}} = \frac{\overline{u}_{\mathcal{P}_t}}{\overline{u}_{\mathcal{P}_{t-1}}}$, that is, when firm sales is normalized by the average sales of all firms, then the growth of top sales share is equal to the growth of the average of *normalized* sales of the top percentile p.

Proposition 1. *The growth of the average sales in a top percentile between time t = 0 and t = 1 can be decomposed as follows:*

$$\underbrace{\frac{\overline{y}_{\mathcal{P}_{1,1}} - \overline{y}_{\mathcal{P}_{0,0}}}{\overline{y}_{\mathcal{P}_{0,0}}}}_{\text{Change in top share}} = \underbrace{\left(\frac{\overline{y}_{\mathcal{P}_0 \setminus \mathcal{D},1} - \overline{y}_{\mathcal{P}_0 \setminus \mathcal{D},0}}{\overline{y}_{\mathcal{P}_0 \setminus \mathcal{D},0}} \right)}_{\text{Within}}$$

$$+ \underbrace{\underbrace{\frac{|\mathcal{E}|}{|\mathcal{P}_1|} \left(\frac{\overline{y}_{\mathcal{E},1} - q_1}{\overline{y}_{\mathcal{P}_0,0}} \right)}_{\text{Inflow}} + \underbrace{\frac{|\mathcal{X}|}{|\mathcal{P}_1|} \left(\frac{q_1 - \overline{y}_{\mathcal{X},1}}{\overline{y}_{\mathcal{P}_0,0}} \right)}_{\text{Outflow}}}_{\text{Displacement}} \tag{2}$$

$$+ \text{M\&A term} + \Delta \text{ Firm No. term.}$$

The M&A term and the Firm number change term can be further expanded into:

$$\text{M\&A term} + \Delta \text{ Firm No. term} = \underbrace{\underbrace{\left(\frac{\overline{y}^{MA}_{\mathcal{P}_0 \setminus \mathcal{D},1}}{\overline{y}_{\mathcal{P}_0 \setminus \mathcal{D},0}} \right)}_{\text{Within M\&A}} + \underbrace{\frac{|\mathcal{E}|}{|\mathcal{P}_1|} \left(\frac{\overline{y}^{MA}_{\mathcal{E},1}}{\overline{y}_{\mathcal{P}_0,0}} \right)}_{\text{Inflow M\&A}} + \underbrace{\frac{|\mathcal{B}|}{|\mathcal{P}_1|} \left(\frac{\overline{y}_{\mathcal{B},1} - q_1}{\overline{y}_{\mathcal{P}_0,0}} \right)}_{\text{Birth M\&A}}}_{\text{M\&A growth}}$$

$$\underbrace{- \frac{|\mathcal{D}|}{|\mathcal{P}_1|} \left(\frac{\frac{\overline{y}_{\overline{\mathcal{P}}_0 \setminus \mathcal{D},1}}{\overline{y}_{\overline{\mathcal{P}}_0 \setminus \mathcal{D},0}} \overline{y}_{\mathcal{D},0} - q_1}{\overline{y}_{\mathcal{P}_0,0}} \right)}_{\text{M\&A shrink}} \tag{3}$$

$$\underbrace{- \frac{|\mathcal{P}_1| - |\mathcal{P}_0|}{|\mathcal{P}_1|} \left(\frac{\frac{\overline{y}_{\overline{\mathcal{P}}_0 \setminus \mathcal{D},1}}{\overline{y}_{\overline{\mathcal{P}}_0 \setminus \mathcal{D},0}} \overline{y}_{\mathcal{P}_0,0} - q_1}{\overline{y}_{\mathcal{P}_0,0}} \right)}_{\text{Firm number change}}$$

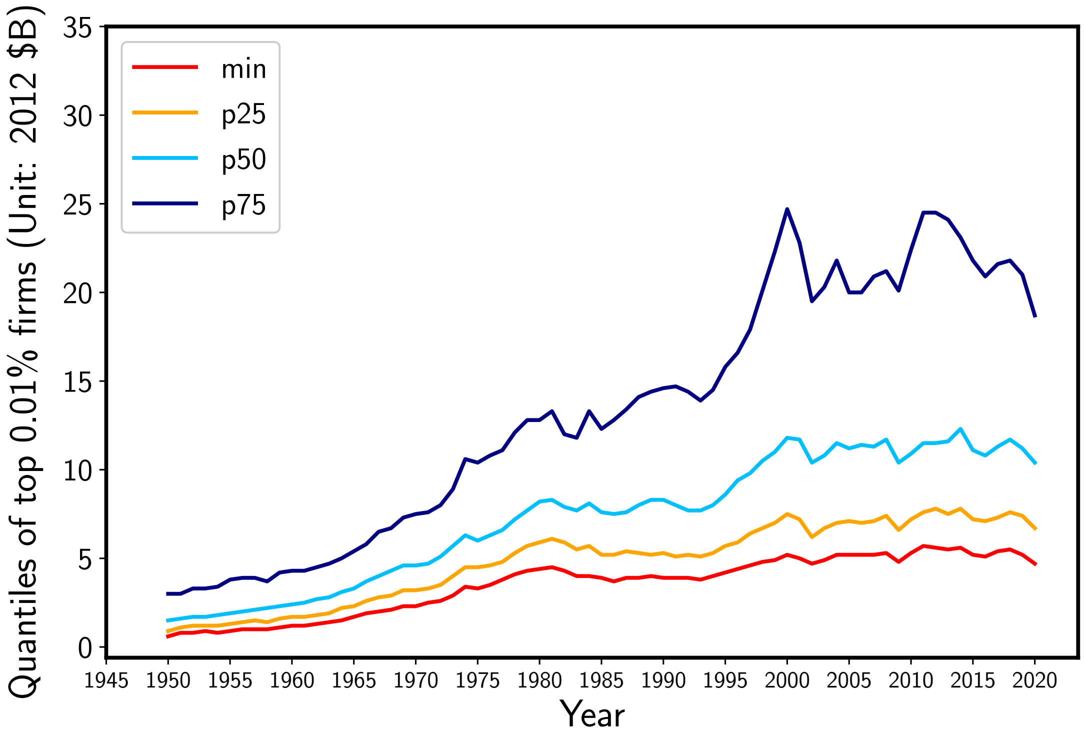
Discuss each term. The within term measures the average growth of firms from $t = 0$ to $t = 1$ in the set $\mathcal{P}_0 \setminus \mathcal{D}$ which are the firms that are in the top percentile at $t = 0$ and do not exit the economy at $t = 1$. This term can be either positive or negative. The displacement term accounts for the change in total top share growth induced by the compositional change of the top percentile between $t = 0$ and $t = 1$. It can be further separated into an inflow term and an outflow term: the inflow term captures the effect of new entrants into the top percentile displacing firms that are close to the percentile threshold and the outflow term reflects that firms exiting the top percentile are replaced by firms just below the percentile threshold. The displacement term is always positive.

Firms who are not in the economy at $t = 0$ may emerge at $t = 1$ as top firms as a consequence of breakups, spinoffs or M&As of existing top firms. This positive effect is accounted for in the M&A growth term. Firms that are at the top percentile at $t = 0$ may exit the economy at $t = 1$, potentially due to merger and acquisition, public firms going private, etc. This negative effect is summarized in the M&A shrink term. Lastly, the total number of firms in the economy changes over time. An increase in the number of firms will add more firms to the top percentile. Since the firms that enter the top due to an expansion in the number of firms have less sales than the existing firms at the top, the firm number change term is negative when there is an increase in the total firm number. Except for the 2008 financial crisis period, firm number has always been on the rise. Therefore, the firm number change term is negative most of the time.

Decompose top shares in data. I apply the accounting framework (Equation (2)) to decompose the annual growth of the average normalized sales of top 0.01% firms in the US economy over the 1950-2019 period. By Equation (1), this is equivalent to decomposing the growth of top sales shares.

As the economy grows, the sales threshold for firms to qualify as the top 0.01% firms has increased steadily from 0.6 billion dollars in 1950 to around 4.7 billion dollars in 2020 (a dollar here is deflated to one 2012 dollar to make comparison over time). Over the past seventy years, there has been an increase in the sales dispersion of firms belonging to the top 0.01% firms. Figure 4 shows that the median and the 75th percentile of top firms have pulled sharply ahead from the rest of top firms that are ranked lower.

Figure 4: Firm size distribution within the top 0.01% percentile



Note: This figure plots the minimum (threshold for top percentile), median, 25th-, 50th- and 75th-percentile of firms that belong to the top 0.01% percentile over the 1950-2020 period. Data are from Compustat (sales of top firms), BEA (gross output of all industries) and Census BDS (total firm number).

Table 1 presents each term geometrically averaged over each period. During the whole sample period, the total 1.13% annual growth is the sum of a within term equal to 1.22%, a displacement term equal to 0.35%, a M&A term equal to 0.36% and a firm number change term equal to -0.81%. During the whole sample period (1950-2019), the within term is about 3.5 fold the magnitude of the displacement term. That is, the growth of top sales shares is to a large extent due to incumbent top firms growing bigger. However, the displacement of old top firms by new top firms also makes important positive contribution to the growth of top sales shares.

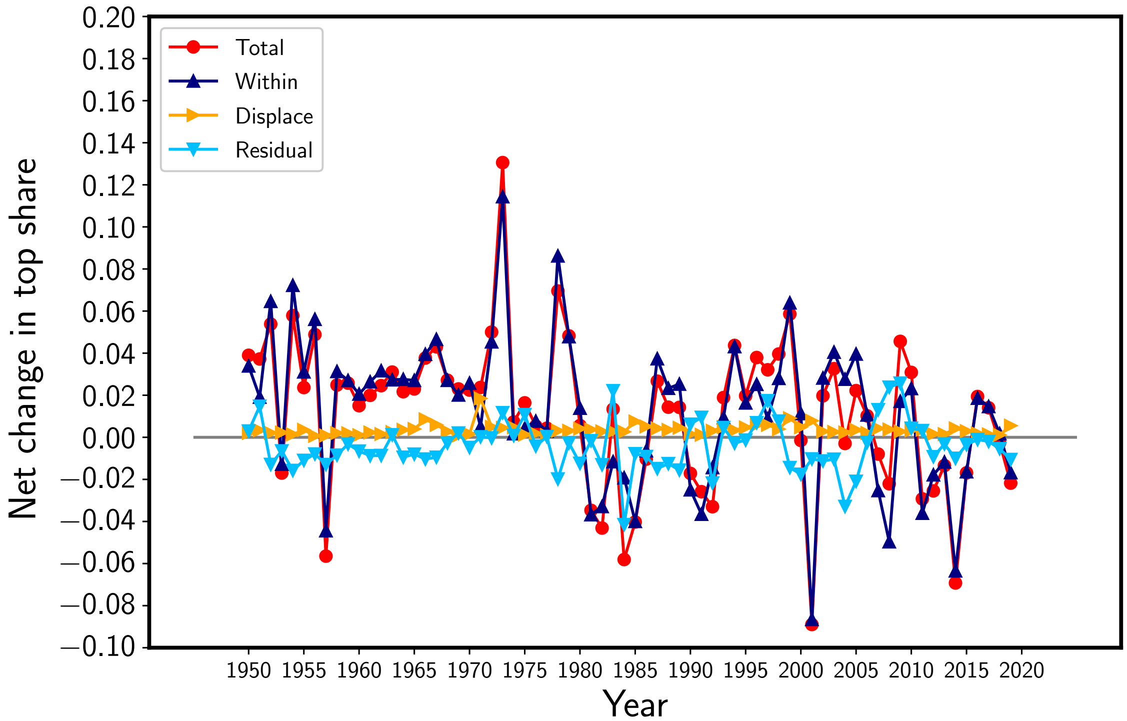
Table 1: Decompose annual growth of top 0.01% firms sales share

Period	Total(%)	Within(%)	Displacement(%)			Residual (%)			
			Total	Inflow	Outflow	Total	M&A Growth	M&A Shrink	Δ Firm No. (%)
1950-2019	1.13	1.22	0.35	0.22	0.13	-0.44	1.32	-0.96	-0.81
1950-1980	2.82	2.95	0.33	0.23	0.1	-0.46	0.67	-0.12	-1
1980-1995	-0.71	-0.37	0.37	0.21	0.17	-0.71	1.56	-1.21	-1.07
1995-2010	1.35	1.05	0.43	0.25	0.19	-0.13	2.34	-2.17	-0.33

Note: Table shows the geometric average of the growth rate of top 0.01% firms sales share, the within, displacement, M&A and firm number change terms. Data are from Census BDS, BEA, *Compustat* and *SDC Platinum* databases.

The contribution to the total top share growth by the within term and the displacement term vary across periods. The within term is high at 2.95% over the 1950-1980 period. Turning into the 1980-1995 period, the within term is negative (-0.37%) and the positive change in top sales shares is mainly driven by the displacement term. Since this positive contribution (0.37%) is less than the negative contributions from the residual term (-0.71%), the total top sales shares are declining during this period. Over the 1995-2010 period, the displacement term peaks at 0.43% and the within term has climbed to 1.05%. The rise in the top sales shares during this period comes from increases in both the within term and the displacement term.

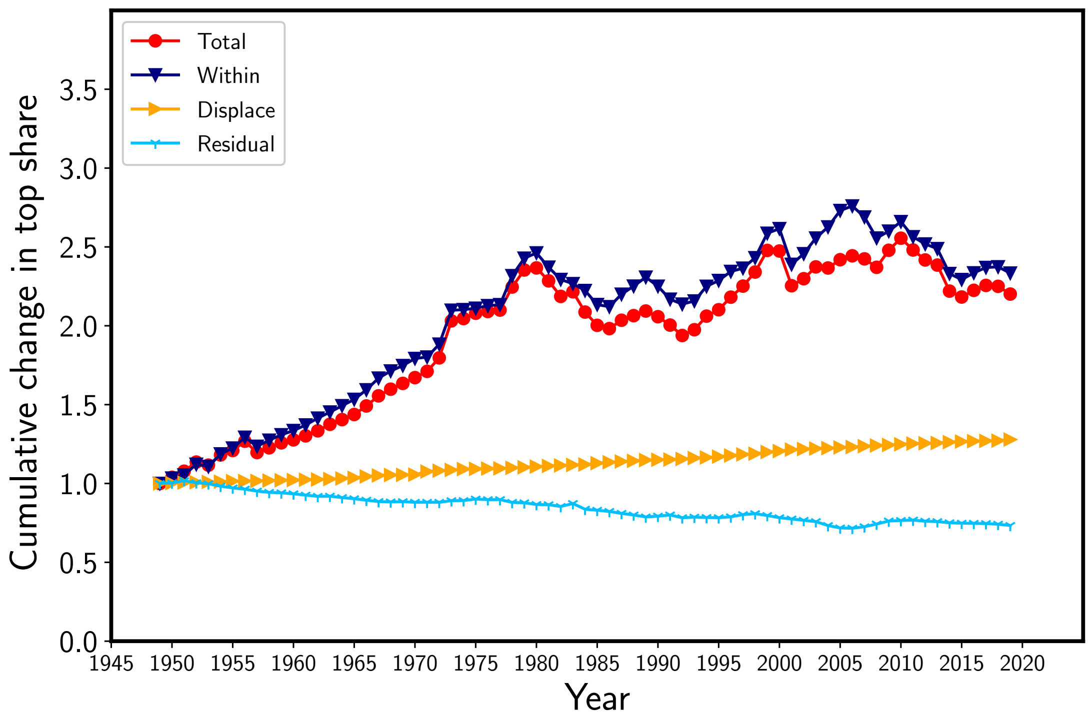
Figure 5: Decomposing Annual Growth of top sales shares



Note: This figure plots the annual growth of the sales shares accrued to the top 0.01% firms over the 1950-2019 period, along with the within, displacement and residual (combining the M&A term and the firm number change term) terms defined in accounting framework (2). Data are from Compustat (sales of top firms), BEA (gross output of all industries) and Census BDS (total firm number), and SDC Platinum (mergers and acquisitions).

Figure 5 plots the decomposition term each year over the 1950-2019 period. In a broad sense, the decompositions terms exhibit different features prior to and after 1980. Prior to 1980, the within term is consistently positive and dominates the displacement term which is much smaller in magnitude. Hence, the growth of top sales shares is mainly driven by the within term and this corresponds to the steady rise in the top sales shares during the period. After 1980, the decomposition time series become more volatile. The within term is mostly negative during 1980-1985, 1990-1993, 2010-2020 and around the 2000 and 2008 crisis periods. The displacement term is small in magnitude but remains fairly stable over time and it becomes the dominant force driving up top sales shares growth when the within term is negative. Figure 6 plots the cumulative sum of each term over time and shows that the cumulative contribution by the within term first rises steadily till 1980, remains roughly stable during 1980-1995, then rise steeply from 1995 to 2005 and declines afterwards. Although smaller in magnitude, the displacement term has consistently made positive contributions to the cumulative growth of top sales shares.

Figure 6: Decomposing Cumulative Growth



Note: This figure plots the cumulated growth of the sales shares accrued to the top 0.01% firms over the 1950-2019 period, along with the within, displacement and residual (combining the M&A term and the firm number change term) terms defined in accounting framework (2). Data are from Compustat (sales of top firms), BEA (gross output of all industries) and Census BDS (total firm number), and SDC Platinum (mergers and acquisitions).

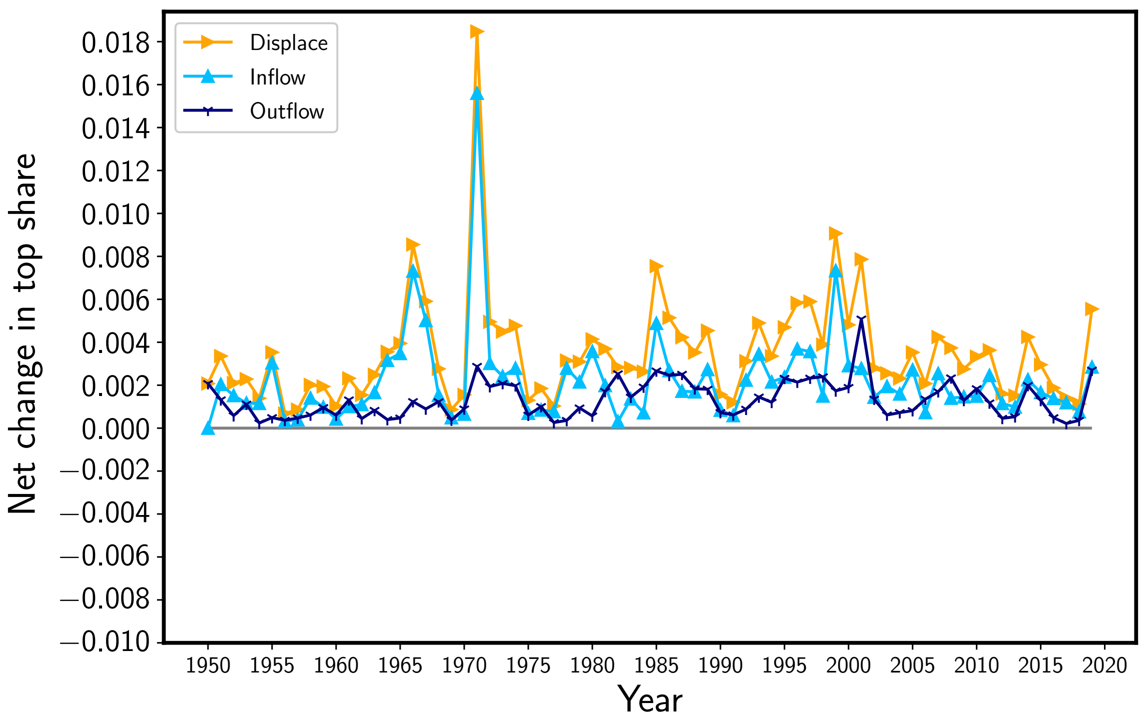
To further understand when each decomposition terms exhibit significant changes, I test for structural breaks in each yearly time series of decomposition terms. Table 2 shows that the within term mirrors the total change in top sales share and has a break point at 1975. The displacement term undergoes structural break around 1963 and this is reflected in the time series of the displacement term plotted in Figure 7 where it remains low prior to 1963 and becomes high and more volatile after 1963.

Table 2: Test for structural break of decomposition terms

Decomposition term	χ^2	Break point
Total	24.950***	1975
Within	19.325***	1975
Displacement	16.969***	1963

Note: The table reports the Wald statistic from structural break test. Significance levels: * (p<0.1), ** (p<0.05), *** (p<0.01).

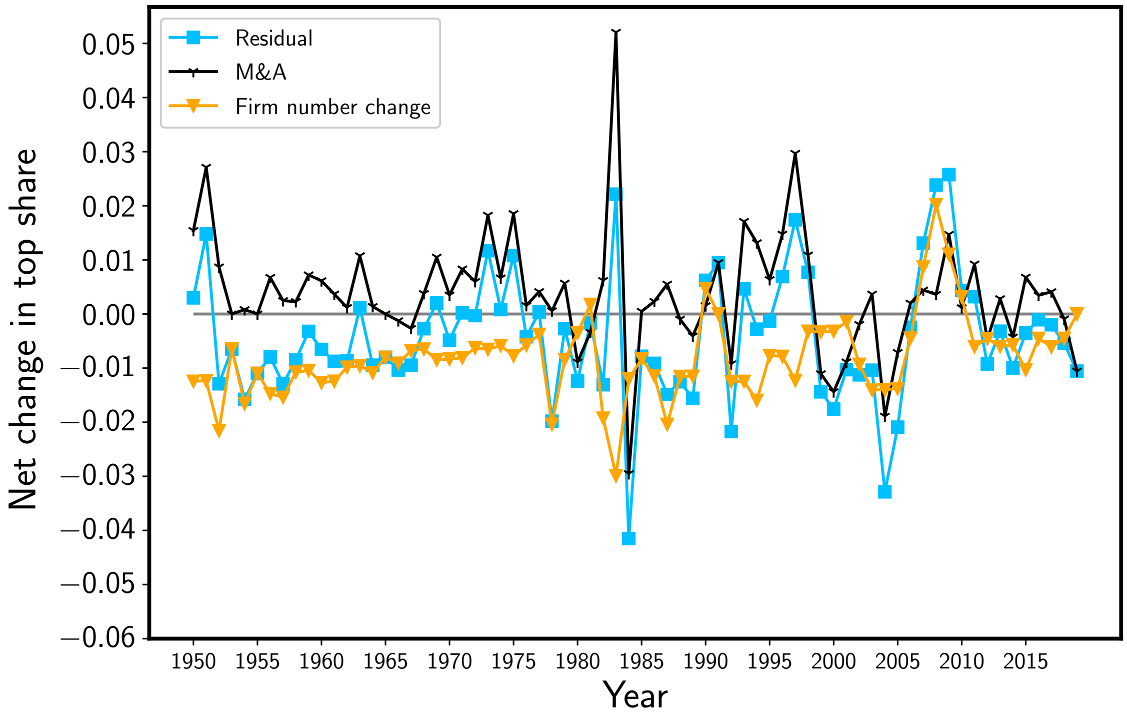
Figure 7: Decomposing Displacement Term into Inflow v.s. Outflow



Note: This figure plots the inflow term and the outflow term, which comprise the displacement term that contributes to the growth of the sales shares accrued to the top 0.01% firms over the 1950-2019 period defined in accounting framework (2). Data are from Compustat (sales of top firms), BEA (gross output of all industries) and Census BDS (total firm number).

Figure 7 shows that aside from two spikes around 1970, displacement terms are high during the mid-1980s and the mid-1990s and stay at a lower level after 2000 for the recent two decades. The accounting framework breaks down the displacement term further into an inflow term and an outflow term. The inflow term reflects the positive contribution from new entrants displacing firms that are close to the percentile threshold. The outflow term reflects the positive effect on the growth of top sales shares due to the replacement of exiting firms by firms just below the percentile threshold. The inflow term is larger than the outflow term. The decline in the displacement term since 2000 reflects declines in both the inflow term and the outflow term. Figure 8 separates the residual term into the M&A term and the firm number change term. The M&A term is high during the early-1980s and the mid-1990s. The firm number change term has been negative for most of the time except during the years around the 2008 Financial Crisis period when the total number of firms in the economy has reduced.

Figure 8: Decomposing Residual Term into M&A vs Firm Population Growth



Note: This figure plots the M&A term and the firm number change term, which comprise the residual term that contributes to the growth of the sales shares accrued to the top 0.01% firms over the 1950-2019 period defined in accounting framework (3). Data are from Compustat (sales of top firms), BEA (gross output of all industries) and Census BDS (total firm number), and SDC Platinum (mergers and acquisitions).

4 Implications for Firm Dynamics

4.1 Firm dynamics and top sales shares.

Equipped with the empirical decomposition result which accounts for different sources of top sales shares growth, I now discuss its implication for the dynamics of large firms. To do so, I build an analytical framework to map individual firm dynamics into the two components of the dynamics of the changes in top sales shares, namely the growth of the incumbent top firms and the growth from entry/exit of firms into and out of the top percentile. This mapping between the theoretical and empirical components of the top sales share growth allows me to estimate the underlying process that drives the dynamics of large firms.

To be more specific, I model firm dynamics as subject to a positive shock which makes a firm grow and a negative shock which makes a firm shrink. For instance, these shocks can be interpreted as innovation shocks that take place in different forms. The positive shock can be viewed as an "own innovation" shock in response to which firms improve on their own products and grow larger. In contrast, the negative shock can be viewed as a "creative destruction" shock which occurs to a firm when its product is improved upon by its competitor. When the dynamics of individual firms follow a random growth process subject to these two shocks, I obtain closed-form formulas for the components of top sales share growth that can be mapped to the within term and the displacement term obtained from the empirical decomposition of top sales share growth.

A model of firm random growth. Consider a firm dynamics process in which a firm i's sales y_{it} follows the random growth process:

$$\frac{dy_{it}}{y_{it}} = \mu dt + G_{\lambda} dN_{it}^{\lambda} - G_{\delta} dN_{it}^{\delta}, \tag{4}$$

where μ is the growth rate of firm i relative to the average growth rate of all firms in the economy in absence of any shocks, dN_{it}^{λ} is the poisson process for own innovation shocks which occur at rate λ, G_{λ} is the net percentage increase in firm sales when own innovation occurs for firm i, dN_{it}^{δ} is the poisson process for creative destruction which occurs at rate δ, G_{δ} is the (absolute) net percentage decrease in firm sales when creative destruction occurs for firm i.

In the following propositions, I first derive the stationary firm size distribution when individual firms follow the dynamic process specified in Equation (4). More specifically, I show that the stationary distribution is a Pareto distribution with exponent ζ. To suit my purpose of studying how the top sales shares evolve over time, I then derive an approximate to the asymptotic behavior of the transition dynamics of the top sales shares. This approximate is arbitrarily close to the actual transition dynamics as time goes to infinity.

Proposition 2. (Stationary firm size distribution) *Given the initial density function* $g_0(y) = a_0 y^{-\zeta-1} + \varepsilon\hat{\zeta}y^{-\hat{\zeta}-1} - \varepsilon\check{\zeta}y^{-\check{\zeta}-1}$, *where* a_0, ε, $\hat{\zeta}$ *and* $\check{\zeta}$ *are constants* [8], *when the sales of individual firms follows the jump process specified by the compound poisson process depicted in Equation* (4), *the stationary firm size distribution is a Pareto distribution with exponent* ζ. ζ *is the unique positive root to the function* $\Psi(s) := \mu s + \lambda[e^{U_1 s} - 1] + \delta[e^{U_2 s} - 1] = 0$.

Proposition 3. (Transition dynamics: decompose top sales shares) *Given the initial density function* $g_0(y) = a_0 y^{-\zeta-1} + \varepsilon\hat{\zeta}y^{-\hat{\zeta}-1} - \varepsilon\check{\zeta}y^{-\check{\zeta}-1}$, *where* a_0, ε, $\hat{\zeta}$ *and* $\check{\zeta}$ *are constants, when the sales of individual firms follows the jump process in Equation* (4), *the average firm size in the top percentile* $\overline{y}$ *follows the law of motion:*

$$
\begin{aligned}
\frac{d\overline{y}_t}{\overline{y}_t} = {} & \underbrace{(\mu + \lambda G_\lambda - \delta G_\delta)\, dt}_{\text{within}} \\
& \underbrace{+ \underbrace{\lambda \left(\frac{(1+G_\lambda)^\zeta - 1 - \zeta G_\lambda}{\zeta} \right) dt}_{\text{inflow}} + \underbrace{\delta \left(\frac{(1-G_\delta)^\zeta - 1 + \zeta G_\delta}{\zeta} \right) dt}_{\text{outflow}}}_{\text{displacement}} \\
& + \underbrace{O(e^{\Psi(\hat{\zeta})t})}_{\text{error term}}\, dt,
\end{aligned}
\tag{5}
$$

where the last term $O(e^{\Psi(\hat{\zeta})t})$ denotes a function that is bounded by $Ce^{\Psi(\hat{\zeta})t}$ for some constant $C > 0$. By the choice $\Psi(\hat{\zeta}) < 0$, this term decays to 0 exponentially fast as $t \to \infty$. Therefore, it becomes as small as possible when t is large enough. Equation (5)

[8] see Equation (46) for further details on the choice of these constants

approximates the asymptotic behavior of the transition dynamics of the average firm size in the top percentile, up to an error term that is arbitrarily small as $t \to \infty$.

As mentioned previously in Equation (1), when firm size is normalized by the average firm size, this is equivalent to the law of motion of top share S_t since $\frac{dS_t}{S_t} = \frac{d\bar{y}_t}{\bar{y}_t}$.

Proposition 4. (Decompose turnover) *When individual firm size follows the jump process (4), the turnover rate in the top percentile is:*

$$\begin{aligned} \text{Turnover Rate (Inflow)} &= \lambda \left((1+G_\lambda)^\zeta - 1 \right) + O(e^{\Psi(\hat{\zeta})t}) \\ \text{Turnover Rate (Outflow)} &= \delta \left(1 - (1-G_\delta)^\zeta \right) + O(e^{\Psi(\hat{\zeta})t}) \end{aligned} \tag{6}$$

where the last term $O(e^{\Psi(\hat{\zeta})t})$ denotes a function that is bounded by a positive constant times $e^{\Psi(\hat{\zeta})t}$. By the choice $\Psi(\hat{\zeta}) < 0$, this term decays to 0 exponentially fast as $t \to \infty$. Therefore, it becomes as small as possible when t is large enough. Equation (6) approximates the asymptotic behavior of the turnover rate due to the inflow and outflow effects, up to an error term that is arbitrarily small as $t \to \infty$.

The model predicts that a rise in the top sales share could be driven by a change in any of these two forces: either an increase in own innovation or a decrease in creative destruction. The within term increases with own innovation and decreases with creative destruction. The displacement term can be further separated into an inflow term and an outflow term. When own innovation increases, the inflow term increases. The outflow term increases as creative destruction increases. Therefore, creative destruction has a negative effect on the growth contribution by the incumbent firms (the within term) and has a positive effect on the growth contribution through exit of firms whose sales go below the percentile threshold (the outflow term). Taking advantage of the fact that creative destruction has opposite effects on the decomposition terms, one can tease out how much a change in top share growth can be attributed to a change in own innovation versus a change in creative destruction.

4.2 Estimating the firm dynamics process

The above model provides closed-form solutions to the key components of changes in the top sales shares including the growth of incumbent firms (the within term), the growth through entry/exit of firms into the top percentile (the displacement term), the turnover rates which measure the entry/exit rate into and out of the top percentile. The displacement term can be separately characterized by an inflow term that results from own innovation shocks and an outflow term that results from creative destruction shocks.

The within term increases with own innovation probability λ and step size G_λ and decreases with rate of creative destruction δ and step size G_δ. This corresponds to the fact that own innovation drives the growth of incumbent top firms whereas creative destruction makes top firms shrink and undermines incumbent growth. The inflow component of the displacement term is driven by own innovation parameters and increases with λ and G_λ, reflecting the effect that lower-ranked firms can displace incumbent top firms after experiencing positive own innovation shocks. On the other hand, displacement also increases through the outflow term triggered by creative destruction shocks. The turnover rate due to entry of firms into the top percentile increases with own innovation rate λ and step size G_λ, whereas the turnover rate due to exit of firms out of the top percentile increases with creative destruction rate δ and (absolute) step size G_δ.

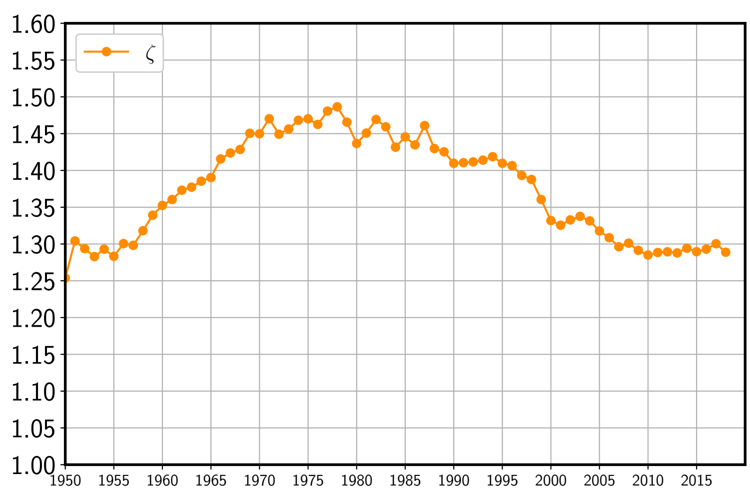
Effect of inequality on displacement. It is also noteworthy that displacement is mediated by the top tail inequality of firm size distribution. When firm size follows Pareto distribution, this inequality is characterized by Pareto exponent ζ, which comes from the density function $g(y) = y^{-\zeta-1}$. From the equations that give top percentile $p = \int_q^\infty y^{-\zeta-1} dy$

and top share $S = \int_q^\infty y \cdot y^{-\zeta-1} dy$. Eliminating q, we get

$$\frac{S/p}{q} = \frac{\zeta}{\zeta - 1}, \tag{7}$$

that is, $\frac{\zeta}{\zeta-1}$ is equal to the ratio of the average sales of top percentile p and the $1-p$ percentile quantile q. ζ reflects the degree of inequality at the upper tail of top distribution: when ζ is low, $\frac{S/p}{q}$ is high, which implies that the average sales of top percentile is pulling far ahead from the sales of the firm that just reaches the top percentile. When ζ is low, both the inflow and outflow terms are low, which means that the displacement term decreases as inequality at the top tail of the firm size distribution increases. Intuitively, when inequality is high as measured by a more dispersed upper tail distribution, the effect of displacement on top share growth is small and turnover is lower because larger jump size is required to get into the top percentile.

Figure 9: Pareto tail index ζ



Note: This figure plots the estimated Pareto exponent ζ of firm size distribution using the ratio of the average sales of the firms in the top 0.01% percentile and the sales of the firm at the threshold of the top 0.01% percentile according to Equation (7).

Estimate Pareto exponent ζ. I estimate the ratio of the average sales of top percentile p and the $1-p$ percentile quantile q from the top 0.01% percentile firms and calculate ζ from equation (7). Figure 9 shows that the estimated ζ has declined from its peak close to 1.5 in the pre-1980s to around 1.3 in recent years. This suggests that the average sales of firms above the threshold at the top 0.01% percentile is roughly three times the sales of the firm at the threshold around 1980. This ratio has increased to 4.3 due to the increased inequality among top firms over the past three decades.

Estimate innovation parameters. After estimating the Pareto exponent ζ, I use the model-implied formulas for the five moments (Within, Inflow, Outflow, Turnover (Inflow) and Turnover (Outflow)) in equation (8) to calibrate the five parameters $(\mu, \lambda, G_\lambda, \delta, G_\delta)$ underlying the firm dynamics process.

$$\begin{cases} \text{Within} = \mu + \lambda G_\lambda - \delta G_\delta \\ \text{Inflow} = \lambda \left(\frac{(1+G_\lambda)^\zeta - 1 - \zeta G_\lambda}{\zeta} \right) \\ \text{Outflow} = \delta \left(\frac{(1-G_\delta)^\zeta - 1 + \zeta G_\delta}{\zeta} \right) \\ \text{Turnover Rate (Inflow)} = \lambda \left((1+G_\lambda)^\zeta - 1 \right) \\ \text{Turnover Rate (Outflow)} = \delta \left(1 - (1-G_\delta)^\zeta \right) \end{cases} \tag{8}$$

Based on Figure 1, the aggregate trend in the sales share of top 0.01% firms can be roughly delineated into three periods: (1) the 1950-1980 period when top sales share has steadily increased; (2) the 1980-1995 period when it has declined; (3) the 1995-2010 period when the top sales share has risen again. For each period, I estimate the innovation parameters that govern the firm dynamics process. Table 3 reports the moments used in the estimation.

Table 3: Moments used in estimation

Period	Within(%)	Inflow(%)	Outflow(%)	Turnover (Inflow)(%)	Turnover (Outflow)(%)
1950-2019	1.22	0.22	0.13	5.16	3.87
1950-1980	2.95	0.23	0.1	3.82	3.73
1980-1995	-0.37	0.21	0.17	6.28	4.18
1995-2010	1.05	0.25	0.19	6.65	4.09

Note: Table shows the moments used in estimating innovation parameters. The within, inflow and outflow terms are obtained from the accounting decomposition in Equation (2). Turnover (Inflow) and Turnover (Outflow) are the average turnover rate due to entry and exit of firms in and out of the top 0.01% percentile within relevant periods. Data are from Census BDS, BEA, *Compustat* and *SDC Platinum* databases.

Discuss estimated innovation process. Table 4 reports the estimated innovation parameters λ, G_λ, δ and G_δ. From the estimation results, we learn that changes in the growth rate of top share across different periods reflect changes in the underlying innovation processes. During the whole sample period, the rate of own innovation is similar to the rate of creative destruction with own innovation occurring at a larger step size than creative destruction. The 1950-1980 period is featured by a low rate of own innovation is around 4%, but when own innovation shock occurs to a firm, it makes the firm grow at a large step size ($\sim$ 50% net percentage change). Turning to the 1980-1995 and the 1995-2010 periods, the rate of own innovation becomes higher ($\sim$ 15%), while the step size has dropped to around $20-30\%$. Across the three periods, the rate of creative destruction has been declining from 16% prior to the 1980s to 10% in more recent years. However, the step size at which firms shrink at the impact of creative destruction shock has become larger over the years.

Although not an innovation parameter, the relative growth rate of top firms to average growth rate of the economy μ also plays an important role in the growth of top sales shares. The period prior to 1980 is a time when large firms is growing at a much higher speed in absence of shocks relative to the rest of the economy. And this trend is reversed after 1980 when μ becomes negative.

Table 4: Estimated innovation parameters

Period	μ(%)	λ(%)	G_λ(%)	δ(%)	G_δ(%)
1950-2019	0.63	9.97	35.5	12.96	22.75
1950-1980	3.22	4.8	52.33	16.39	16.96
1980-1995	-1.47	17.41	23.97	12.98	23.73
1995-2010	-0.42	14.32	32.94	10.31	31.48

Note: μ, λ, δ, G_λ, G_δ are estimated using the five moments: "Within", "Inflow", "Outflow", "Turnover rate (Inflow)", "Turnover rate (Outflow)".

Innovation and top sales shares growth. After estimating the innovation parameters, I examine how each of the three components that drives the firm dynamics process, namely drift, own innovation and creative destruction, contribute to top sales shares growth. Rewriting Equation (9) gives:

$$\begin{aligned}\frac{d\overline{y}_t}{\overline{y}_t} &= \underbrace{\mu dt}_{\text{Drift}} + \underbrace{\left(\lambda G_\lambda + \lambda \underbrace{\frac{(1+G_\lambda)^\zeta - 1 - \zeta G_\lambda}{\zeta}}_{\text{Inflow}}\right) dt}_{\text{Own Innovation}} \\ &\quad + \underbrace{\left(-\delta G_\delta + \delta \underbrace{\frac{(1-G_\delta)^\zeta - 1 + \zeta G_\delta}{\zeta}}_{\text{Outflow}}\right) dt}_{\text{Creative Destruction}} \\ &= \underbrace{\mu dt}_{\text{Drift}} + \underbrace{\lambda \frac{(1+G_\lambda)^\zeta - 1}{\zeta} dt}_{\text{Own Innovation}} + \underbrace{\delta \frac{(1-G_\delta)^\zeta - 1}{\zeta} dt}_{\text{Creative Destruction}}\end{aligned} \tag{9}$$

Own innovation increases the growth of top sales shares via the two components: the within term and the inflow term. As a shock that leads to firm growth, own innovation increases the within term by driving the growth of incumbent top firms. It can also promote previously lower-ranked firms to the top percentile, displacing some old top firms. Creative destruction affects the top sales shares growth via the within term and the outflow term, with opposite effects. It is a destructive force that makes incumbent top firms shrink thereby reducing the within term. On the other hand, by removing firms whose sales are no longer above the percentile threshold to qualify as being a top firm, creative destruction increases the outflow component of the displacement term. The negative effect of creative destruction on the within term is larger than the positive effect on the displacement term. Therefore, creative destruction contributes negatively to the growth of top sales shares.

Table 5 reports the contribution to the growth of top sales shares by drift, own innovation and creative destruction calculated as the respective terms in Equation (9). During the entire sample period (1950-2019), the drift component is positive, and own innovation also makes positive contribution to top sales shares growth, whereas creative destruction offsets top sales shares growth. The drift component fluctuates dramatically across subperiods: it is very large prior to 1980 and turns negative in the post-1980 period. Over the 1950-1980 period, own innovation makes a smaller contribution to top sales shares growth relative to the two periods 1980-1995 and 1995-2010. Creative destruction has also increased in magnitude in the post-1980 era, but the increase is not enough to offset the positive contribution from own innovation. To sum up, prior to 1980, a high drift component is the major driving force for the growth of top sales shares, while in the post-1980 era, own innovation makes a large contribution to the growth of top sales shares. Creative destruction offsets a fraction of the top sales shares growth due to own innovation.

Bringing the model to the data reveals that larger innovation can lead to either an increase or a decrease in the top sales shares growth, depending on which form of innovation is at play. Through the lens of the model, as shown in Equation (9), own innovation drives an increase in the top sales shares growth and creative destruction leads to a fall in the growth of the top sales shares. A further dissection disentangles the effects of the two forms of innovation on the displacement term. Own innovation increases displacement via the inflow term by promoting previously lower-ranked firms into the top. Creative destruction raises displacement via the outflow term by removing dwindling incumbent firms from the top.

When measured in the data, as shown in Table 3, both the inflow and outflow terms are positive. The inflow term is only driven by own innovation and the outflow term is only driven by creative destruction. This rules out alternative models in which only one form of innovation takes place. With both types of innovations at work, coupled with their differential impact on the growth of top sales shares, a higher level of innovation can contribute to either a rise or a fall in top sales shares growth. In particular, my finding highlights a channel through with an increase in innovation (in the form of own innovation) can result in higher top sales shares growth. In addition, this channel has a larger impact than the offsetting effect from creative destruction that dampens the growth of top sales shares.

Table 5: Contribution to top sales shares growth by:
Drift, Own Innovation and Creative Destruction

Period	Total (%)	Drift (%)	Own Innovation (%)	Creative Destruction (%)
1950-2019	1.13	0.63	3.76	-2.82
1950-1980	2.82	3.22	2.75	-2.68
1980-1995	-0.71	-1.47	4.38	-2.91
1995-2010	1.35	-0.42	4.97	-3.06

Note: Table shows the contribution to top sales shares growth with the corresponding periods from the three components: drift, own innovation and creative destruction. Based on Equation 9, Drift $= \mu$, Own innovation $= \lambda \frac{(1+G_\lambda)^\zeta - 1}{\zeta}$, Creative Destruction $= \delta \frac{(1-G_\delta)^\zeta - 1}{\zeta}$.

Changes in innovation and top sales shares. I then use the model-implied innovation parameters to quantify how much changes in the top sales shares growth from period to period comes from changes in own innovation as opposed to changes in creative destruction. The following equations quantify how much change in the growth rate of top share can be attributed to changes in own innovation as opposed to changes in creative destruction:

1. The decrease in top share growth rate from 2.82% during the 1950-1980 period to -0.71% during the 1980-1995 period can be decomposed to:

$$\underbrace{-3.53\%}_{\text{Change in top share growth}} \approx \underbrace{-4.69\%}_{\text{drift}} \underbrace{+1.63\%}_{\text{own innovation}} \underbrace{-0.23\%}_{\text{creative destruction}} \tag{10}$$

2. The increase in top share growth rate from -0.71% during the 1980-1995 period to 1.35% during the 1995-2010 period can be decomposed

$$\underbrace{+2.06\%}_{\text{Change in top share growth}} \approx \underbrace{+1.05\%}_{\text{drift}} \underbrace{+0.59\%}_{\text{own innovation}} \underbrace{-0.15\%}_{\text{creative destruction}} \tag{11}$$

The above quantification reveals a prominent role by increases in own innovation in leading to the rise of top sales shares growth. Changes in creative destruction can offset a small amount of top sales shares growth across periods.

5 Implications for Aggregate Growth

To study the implications of the estimated innovation process that drives firm dynamics on aggregate productivity growth, I now nest this firm dynamics process in a growth model by Jones and Kim (2018). Jones and Kim (2018) uses a growth model with quality ladders in the traditions of Grossman and Helpman (1991) to study how entrepreneurial effort and creative destruction jointly shape income inequality. For my purpose, I abstract from how optimal innovation rate is determined by endogenous entrepreneurial effort and consider the case where own innovation and creative destruction shocks arrive at exogenous poisson rates.

Production. There is a unit measure of varieties in the economy and varieties combine to produce a single final output good:

$$Y = \left(\int_0^1 Y_i^\theta di \right)^{1/\theta}, 0 < \theta < 1. \tag{12}$$

Each variety i is produced by a firm i with constant return to scale in labor L_i, and n_t is the step on quality ladder, γ is the step size of quality ladder:

$$Y_{it} = \underbrace{(1+\gamma)^{n_t}}_{\text{aggregate}} \underbrace{x_{it}^{1-\theta}}_{\text{idiosyncratic}} L_{it}, \tag{13}$$

where firm productivity x_{it} follows a jump process:

$$\frac{dx_{it}}{x_{it}} = \mu dt + G_\lambda dN_{it}^\lambda - G_\delta dN_{it}^\delta, \tag{14}$$

Assume when innovation occurs for one variety, it generates spillovers that move all other varieties up the quality ladder by one step:

$$\dot{n}_t = \lambda + \delta \tag{15}$$

Proposition 5. Firm sales and total output: *Let Y_{it} denote the amount of sales for firm i. Sales of each firm are given by:*

$$Y_{it} = (1+\gamma)^{n_t} X_t^{-\theta} L_t x_{it}, \tag{16}$$

Aggregate output is given by:

$$Y_t = (1+\gamma)^{n_t} X_t^{1-\theta} L_t, \tag{17}$$

where $X_t \equiv (\int_0^1 x_{it}^\theta di)^{\frac{1}{\theta}}$ is the CES aggregate of the productivity distribution across firms and

Table 6: Contribution to aggregate growth by each form of innovation

Period	Own innovation (%)	Creative Destruction (%)
1950-2019	0.72	0.94
1950-1980	0.35	1.19
1980-1995	1.26	0.94
1995-2010	1.04	0.75

$L_t \equiv \int_0^1 L_{it} di$ is total labor supply.

On balanced growth path, aggregate growth (output per person) is

$$\begin{aligned}\Delta log(\frac{Y}{L}) &= \dot{n}_t log(1+\gamma) \\ &= \underbrace{\lambda log(1+\gamma)}_{\text{own innovation}} + \underbrace{\delta \log(1+\gamma)}_{\text{creative destruction}}\end{aligned}$$

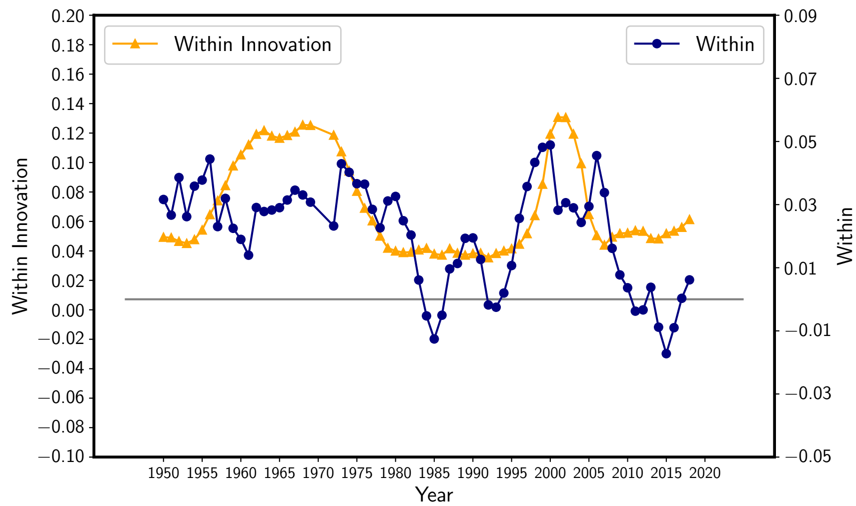
I take $\gamma = 7.5\%$ following Garcia-Macia et al. (2019) and report the estimated contribution to aggregate productivity growth by own innovation and creative destruction in Table 6.

I find that the implied aggregate productivity growth is highest over the 1980-1995 period. Prior to 1980, the contribution to aggregate productivity growth from own innovation is much smaller than that by creative destruction due to a low own innovation rate. Over the 1995-2010 period, the aggregate productivity growth is lower than that during the 1980-1995 period because both own innovation and creative destructions rates have declined.

6 Characteristics of firms that enter, exit and stay in the top

I have estimated a firm dynamics process in which the growth of firms are driven by innovation shocks in section 4. In reality, the growth of firms could be driven by many factors, and innovation is one of them. Now I provide some empirical evidence showing that the growth of incumbent top firms are correlated with their innovating activities.

Figure 10: 5-year Rolling Average of Innovation and Within term



Note: This figure plots the 5-year rolling average of the within term defined in accounting framework (2) and the total innovation by incumbent top 0.01% firms. The total innovation by the group of incumbent top firms is measured as the sum of the patent values of all firms in this group divided by their total assets. Data are from Compustat and Kogan et al. (2017).

Within term and innovation. I show that at low-frequency, the growth from incumbent top firms is positively correlation with the total innovation by the group of incumbent top firms. I measure the total innovation by the group of incumbent top firms as the sum of the patent values of all firms in this group divided by their total assets. And I obtain patent values at the firm level from Kogan et al. (2017). Figure 10 shows that the 5-year rolling average of the within term and the total innovation by incumbent top firms are positively correlated (with correlation coefficient +0.436). This suggests that at low-frequency, the fluctuations of the within term is correlated with the innovation activities by the incumbent top firms.

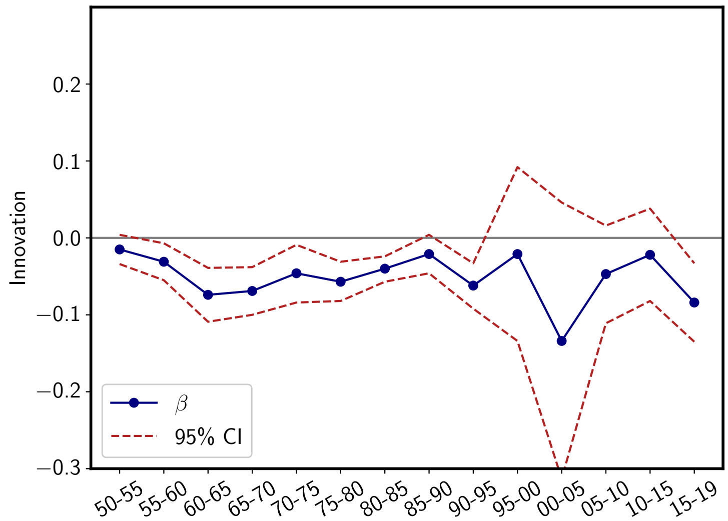
Entry, exit vs stayer: innovation. By comparing the innovation of firms that enter, exit and stay in the top, I find that firms that stay in the top have higher innovation than firms that exit the top, suggesting that innovation is an important factor for firms to stay in the top percentile.

I compare the characteristics of firms that enter, exit and stay in the top 0.01% percentile by running Regression 18. The innovation of a firm i in year t, Innovation$_{it}$, is measured as the ratio of a firm's patent value to its total asset. Since a firm may get approval on several patents in a certain year while in other years there are no patents accredited to the firm, I smooth out firm-level innovation by using the 5-year rolling average of the patent value-to-asset ratio. Entry vs Stayer$_{it}$ is a dummy variable that takes the value 1 if a firm is a new entrant into the top in year t and that takes the value 0 if a firm remains in the top from year $t-1$ to year t. Exit vs Stayer$_{it}$ is a dummy variable that takes the value 1 if a firm is in the top in year $t-1$ but exit the top in year t and that takes the value 0 if a firm remains in the top from year $t-1$ to year t. λ_t is year fixed effect to absorb the effect of time-varying factors. T is the relevant 5-year window within the whole 1950-2019 sample period.

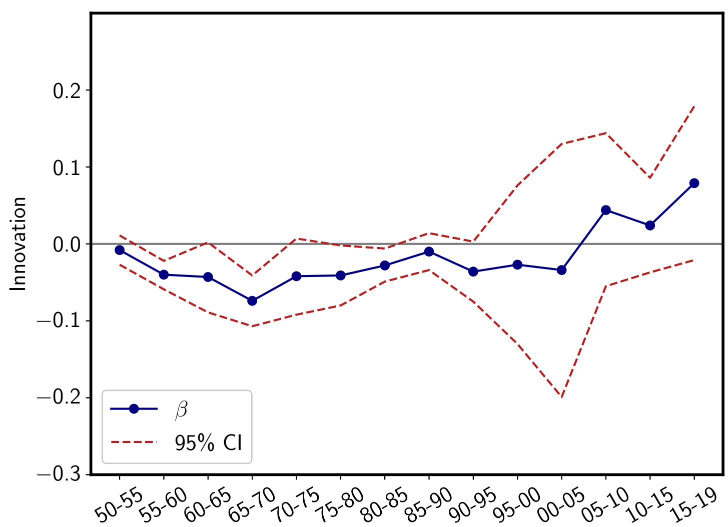
$$\text{Innovation}_{it}^{T} = \alpha + \beta \text{Entry/Exit vs Stayer}_{it} + \lambda_t + \epsilon_{it} \tag{18}$$

Figure 11a shows that over each 5-year window throughout the whole 1950-2019 period, firms that exit from the top have lower innovation than firms that remain in the top. This suggests that firms that remain in the top are more innovative than the old top firms that are being displaced from the top by new firms. Figure 11b shows that firms that remain in the top have innovated more than the firms that are new entrants to the top in the early years prior to 1980. However, this innovation gap has been closing over time and during recent years (2005-2019), new entrants have higher innovation than incumbent top firms.

Figure 11: Compare the innovation of firms that enter, exit and stay in the top 0.01% percentile



(a) Innovation: Exit vs Stayer



(b) Innovation: Entry vs Stayer

Note: Figure 11 plots the coefficient β from Regression (18) for each consecutive 5-year window from1950 to 2019.

Taken together, innovation is an important driving force for firms to stay in the top. Firms that do not innovate enough may get displaced out of the top percentile. Compared with old top firms, new top firms have become more innovative over time.

7 Conclusion

The presence and growth of large firms are themes of continuing interests for researchers, policy makers and the public. The dynamics at the upper tail of firm size distribution are key factors in understanding macroeconomic outcomes. In this paper, I infer the dynamic process that drives the rise and fall of large firms from changes in the top sales shares. To do so, I apply the accounting framework developed in Gomez (2020) to decompose the growth of sales shares accrued to top 0.01% firms in a longer time horizon over the 1950-2019 period.

I obtain two key components of the growth of top sales shares from this decomposition. The first term, the *within* term, is the contribution to top sales shares growth by existing top firms. The second term, the *displacement* term, measures the effect of compositional change of top firms on top sales shares growth. I find that during the whole sample period (1950-2019), the within term is larger in magnitude than the displacement term. That is, the growth of top sales shares is to a large extent due to incumbent top firms growing bigger. However, the displacement of old top firms by new top firms also makes important positive contributions to the growth of top sales shares.

Using continuous-time methods, I derive the dynamics of individual firms to the dynamics of the share of sales accrued to firms in a top percentile. In doing so, I obtain a mapping between the theoretical and empirical components of the top sales share growth. This allows me to estimate the underlying innovation process that drives the dynamics of large firms.

Bringing the model to the data reveals that the rise in top sales shares growth can be driven by an increase or a decrease in innovation, depending on which form of innovation is at work. The data supports that own innovation and creative destruction jointly shape the top sales shares growth. Own innovation leads to top sales shares growth by making incumbent top firms grow larger and promoting lower-ranked firms to surpass existing top firms. Creative destruction decreases top sales shares growth since its negative impact on existing top firms dominates its positive effect via removing dwindling old top firms from the top. My finding highlights a channel through with an increase in innovation (in the form of own innovation) can result in higher top sales shares growth.

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Appendices

A Robustness

I now examine the robustness of my decomposition exercise. First, I explore how the decomposition terms would change if I decompose over longer windows: a 5-year and a 10-year horizon. Second, I check whether the decomposition terms are robust to removing financial firms from the sample.

Decompose over 5-year horizon. In the baseline decomposition exercise, I decompose the one-year growth of the top 0.01% firms top sales into the within term and the displacement term. In doing so, I count firms that are new entrants into the top percentile over the one-year window into the group of firms that generate the displacement effect. However, it may take a longer time for new top firms to grow bigger and contribute to the displacement effect. Therefore, I explore a scenario in which I decompose the growth of the top sales shares over a 5-year window. This allows me to classify all new entries and exits over the 5-year horizon as contributions to the displacement term.

Table 1 reports the decomposition terms of the growth of the top 0.01% firms sales shares over 5-year horizons. To facilitate comparison with the yearly decomposition in the baseline scenario, I report in this table the annualized terms that result from the decomposition over 5-year windows.

Table 1: Decompose the growth of the top 0.01% firms sales shares over 5-year horizon

Period	Total(%)	Within(%)	Displacement(%)			Residual (%)			
			Total	Inflow	Outflow	Total	M&A Growth	M&A Shrink	Δ Firm No. (%)
1950-2019	1.15	1.63	0.44	0.33	0.11	-0.93	0.83	-0.94	-0.88
1950-1980	2.89	2.95	0.32	0.22	0.1	-0.37	0.74	-0.07	-1.08
1980-1995	-0.88	-0.07	0.53	0.41	0.12	-1.35	0.74	-1	-1.13
1995-2010	1.24	2.03	0.62	0.5	0.12	-1.42	1.25	-2.42	-0.44

Note: Table shows the annualized 5-year growth rate of the top 0.01% firms sales shares, the within, displacement, M&A and firm number change terms. Data are from Census BDS, BEA, *Compustat* and *SDC Platinum* databases.

Over the 1950-2019 period, decomposition over the 5-year window yields a similar set of the within term and the displacement term as in the yearly decomposition. Compared with the decomposition over a one-year window, both the within term and the displacement term become larger during the two periods: 1980-1995 and 1995-2010. The increase in the displacement terms is attributable to a larger inflow term, which results from the fact that new entrants make larger contribution to top sales shares growth given the longer 5-year horizon.

Figure 2a plots the annualized decomposition terms that are components of the top sales shares growth over 5-year horizons for each starting year. When converted to annual frequency, the terms that result from decomposition over 5-year horizons exhibit similar patterns (in terms of sign and magnitude) as the terms obtained from the decomposition exercise at the one-year frequency (Figures 5 and 2a). The time series for the annualized 5-year decomposition terms are smoother than the 1-year decomposition series which contain more year-by-year fluctuations. It shows that firms that are in the top 0.01% percentile at starting years in the intervals [1950,1975] and [1990, 2005] made positive contributions to the growth of top sales shares over 5-year horizons. The cohorts of firms that are in the top at the beginning years during the 1980s and the post-2005 period have shrunk and thus have contributed negatively to top sales shares growth.

In terms of the displacement effect, new top firms that have entered the top percentile during the mid-1960s, the mid-1980s and the mid-1990s made large contributions to the top sales shares growth over the next 5 years after entering the top. Firms that have entered the top during the 2000-2015 period have had a lower displacement effect compared with the cohorts of firms that have entered the top during the two decades before 2000.

Decompose over 10-year horizon. Appendix Figure 3a shows that when the decomposition is done over 10-year horizons, firms that are already within the top prior to 1970 have large contributions to the top sales share growth over the subsequent 10-year periods. This effect from the within group remains close to zero for firms that are within the top at starting years in the interval [1975, 1990]. The within term then peaks around the late 1990s. This suggests that the incumbent top firms during the late 1990s has the largest contribution to top sales shares growth over 10-year horizons. Appendix Figure 3b shows that over 10-year horizons, the cohorts of firms that enter the top in the early 1990s have the largest contribution to top sales shares growth.

To get a sense on which firms make the largest contribution to top sales shares growth, I report in Appendix Table 1 the ten firms with largest increase in sales during each decade from 1950 to 2020. The sales changes are deflated to 2012 dollars so that they are comparable across periods. The firms that are among top 10 in terms contribution to top sales shares are mostly already incumbent top 0.01% firms at the beginning of the decade. Prior to 1990, firms that have expanded the most in terms of sales mainly come from the manufacture sector. Starting from 1990s, the top 10 firms that have the largest expansion in sales come from the retail, wholesale and finance sectors. The post-1990s period also see fast expansion of several big firms into gigantic firms (e.g.: Walmart, Amazon, Apple and Google etc.) The magnitude of the expansions of these star firms over a 10-year horizon is very rare prior to 1990.

With regards to the shrinkage of top firms, Appendix Table 2 shows the ten firms with largest decrease in sales during each decade from 1950 to 2020. During the pre-1980s period, top firms only exhibited mild shrinkage in sales. After 1980, there have been large decline in top firms marked by the fall of oil and automobile giants such as Exon Mobil and General Motors.

Remove financial firms. The next set of robustness checks involves the removal of financial firms. Since the sales of financial firms are measured differently from that of non-financial firms, I check how the decomposition results would change if I remove all financial firms from my sample. Table 2 shows that the decomposition terms obtained from non-financial firms are similar in magnitude as the ones obtained from the baseline decomposition using the whole sample of firms. Appendix figures 4a and 4b show that the yearly decomposition terms also resemble those obtained from the baseline decomposition.

Table 2: Decomposition terms (without financial firms)

Period	Total(%)	Within(%)	Inflow(%)	Outflow(%)	Residual(%)
1950-1980	2.58	2.9	0.16	0.08	-0.55
1980-1995	-1.07	-0.42	0.19	0.12	-0.96
1995-2010	1.23	0.93	0.25	0.16	-0.12

Note: Decomposition terms are obtained after removing financial firms. Residual is the sum of the rest of decomposition terms, including the M&A growth, M&A shrink and firm number change terms.

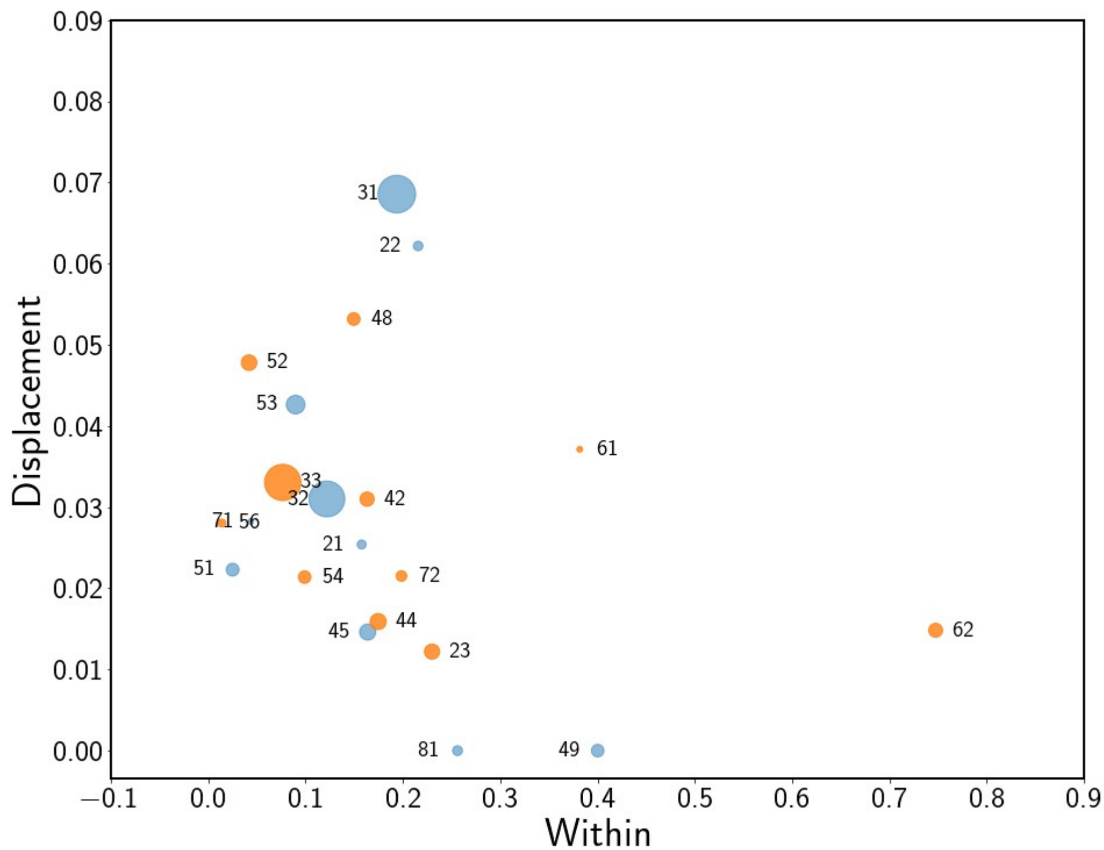
B Decomposition at the industry level

The analysis at the aggregate level in the main text reveals that a higher level of innovation may or may not result in larger top sales shares growth, depending on the whether own innovation or creative destruction is the dominant innovation force at play. I now bring this insight into the analysis at the industry level where I can generate a panel data for the 2-digit NAICS industries with components of top sales shares growth and innovation. I use this panel data to examine whether industries with higher innovation have larger top sales shares growth and see how they vary across periods. This sheds light on to what extent innovation works through own innovation as opposed to creative destruction in different industries and during different time periods.

I decompose the 5-year growth of top sales shares accrued to the top 0.01% firms of the 2-digit NAICS industries by applying the same accounting framework in Equation (2). This decomposition yields the within term and the displacement term for each (industry, 5-year period) pair. Figure 12 plots the average within and displacement terms obtained from the accounting decomposition (Equation 2) of the 5-year top sales shares growth of the 2-digit NAICS industries during the 1950-2018 period.

At the industry level, the within term is larger than the displacement term. However, the relative magnitude of the two terms varies across industries. For manufacturing industries, the within term is roughly three times the magnitude of the displacement term. For retail and wholesale industries, the within term is much larger than the displacement term, suggesting that in these industries the growth of top sales shares is largely driven by the growth incumbent top firms. In contrast, in the information industry (e.g.: Publishing, Motion Picture, Telecommunication, Software and Data processing) and the finance and insurance industry, the displacement term contributes equally to the growth of top sales shares as the within term.

Figure 12: Industry-level 5-year top sales share growth: Within vs Displacement (average over the 1950-2018 period)



Note: Figure plots the average within and displacement terms obtained from the accounting decomposition (Equation 2) of the 5-year top sales shares growth of the 2-digit NAICS industries during the 1950-2018 period. The size of scatter points represents the average share of sales of the 2-digit NAICS industries over the 1950-2018 period.The 2-digit NAICS industry codes correspond to the following industries: 21 - Mining; 22 - Utilities; 23 - Construction; 31, 32, 33 - Manufacturing; 42 - Wholesale trade; 44, 45 - Retail trade; 48, 49 - Transportation and Warehousing; 51 - Information; 52 - Finance and Insurance; 53 - Real Estate; 54 - Professional Scientific and Technical Services; 56 - Administrative and Waste Management Services; 61 - Educational Services; 62 - Health care and Social assistance; 71 - Arts, Entertainment, and Recreation; 72 - Accommodation and Food Services; 81 - Other Service (except government).

Applying the methods described in Section 4, I estimate the innovation parameters for each (industry, 5-year period) pair over the past seventy years. Equipped with the estimated innovation parameters, I obtain the component of top sales shares growth that is due to innovation: within term $-$ drift (μ) $+$ displacement term.

To measure innovation at the industry level, I first obtain the patent value for each public firm in the industry from Kogan et al. (2017). I calculate the innovation of an industry as the ratio of the 5-year rolling average of an industry's total patent value (i.e., the sum of the patent value of all public firms in the industry) and the 5-year rolling average of its total asset value.

Innovation and top sales shares growth. To test whether industries with higher levels of innovation have larger components of the top sales share growth from innovation, I run the following regression:

$$Y_{jt}^{T} = \alpha + \beta \log(\text{Innovation})_{jt} + \lambda_t + \epsilon_{jt}, \tag{19}$$

where Y_{jt}^{T} can be the components of top sales shares growth due to innovation (i.e., within term $-$ drift (μ) $+$ displacement term) of industry j over the time window [t, t+5], or a further breakdown of these components into the within component due to innovation (i.e., within term - drift (μ)), and the displacement component. T can be each sub-periods: 1950-1980, 1980-1995 and 1995-2010. Innovation$_{jt}$ is innovation (patent value-to-asset) of industry j over the time window [t, t+5]. λ_t is year fixed effect to absorb the effect of time-varying factors.

I find that industries with higher innovation have larger growth in the top sales shares in terms of the model-implied component due to innovation during the 1950-1980 and the 1980-1995 periods, but not the 1995-2010 period (Appendix Table 1). A further breakdown in Appendix Table 2 shows that during the 1950-1980 and the 1980-1995 periods, industries with higher innovation have larger within component of the top sales shares growth (i.e., the within term minus drift (μ) to remove the effect not related to innovation). However, the regression coefficient becomes negative during the 1995-2010 period.

Table 1: Δ Top Share (Within + Displacement) vs Innovation of 2-digit NAICS industries

Within - μ + Displacement	1950 to 1980 (1)	1980 to 1995 (2)	1995 to 2010 (3)
log(Innovation)	0.027***	0.007	-0.007
	(0.005)	(0.005)	(0.007)
Constant	0.114***	0.070***	0.076*
	(0.030)	(0.021)	(0.044)
Number of observations	175	125	142
R^2	0.389	0.202	0.200
Period FE	Yes	Yes	Yes

Note: Within $-\ \mu\ +$ Displacement is the model-implied component of top sales shares growth due to innovation. log(Innovation) is the log of the 5-year rolling average of the patent value-to-total asset ratio of each industry. Industries are weighted by their sales in the initial year of the period. Robust standard errors clustered at the industry level are in parentheses. Significance levels: * (p<0.1), ** (p<0.05), *** (p<0.01).

Recall from previous analyses that the model-implied formula for the within term minus drift (μ) is $\lambda G_\lambda - \delta G_\delta$, where λ and G_λ are the rate and step size for own innovation and δ and G_δ are the rate and step size for creative destruction. A positive within minus drift (μ) term implies that the effect of own innovation on the within component of top sales shares growth outweighs that of creative destruction and vice versa. Industries with different levels of innovation do not have significant differences in the displacement term (Appendix Table 3).

Table 2: Δ Top Share (Within) vs Innovation of 2-digit NAICS industries

Within - μ	1950 to 1980 (1)	1980 to 1995 (2)	1995 to 2010 (3)
log(Innovation)	0.025***	0.009*	-0.008
	(0.006)	(0.005)	(0.006)
Constant	0.058**	0.039**	0.022
	(0.022)	(0.016)	(0.030)
Number of observations	175	125	142
R^2	0.402	0.154	0.212
Period FE	Yes	Yes	Yes

Note: Within $-\mu$ is the model-implied within component of top sales shares growth due to innovation. log(Innovation) is the log of the 5-year rolling average of the patent value-to-total asset ratio of each industry. Industries are weighted by their sales in the initial year of the period. Robust standard errors clustered at the industry level are in parentheses. Significance levels: * (p<0.1), ** (p<0.05), *** (p<0.01).

Table 3: Δ Top Share (Displacement) vs Innovation of 2-digit NAICS industries

Displacement	1950 to 1980 (1)	1980 to 1995 (2)	1995 to 2010 (3)
log(Innovation)	0.002	-0.002	0.002
	(0.005)	(0.003)	(0.003)
Constant	0.056***	0.030**	0.054***
	(0.017)	(0.013)	(0.017)
Number of observations	175	125	142
R^2	0.263	0.105	0.090
Period FE	Yes	Yes	Yes

Note: Displacement is the component of top sales shares growth due to compositional changes of top firms by performing the accounting decomposition exercise at the industry level. log(Innovation) is the log of the 5-year rolling average of the patent value-to-total asset ratio of each industry. Industries are weighted by their sales in the initial year of the period. Robust standard errors clustered at the industry level are in parentheses. Significance levels: * (p<0.1), ** (p<0.05), *** (p<0.01).

The above results show that industries with higher innovation have larger top sales shares growth due to innovation during the 1950-1980 and the 1980-1995 periods, but not the 1995-2010 period. This suggests that innovative industries feature different combinations of own innovation and creative destruction across different periods. I further test this mechanism by running the regression that takes the same form as in Equation (19) and replace Y_{jt}^{T} with the estimated rates of own innovation, creative destruction and their sum.

Appendix Table 4 shows that, in all periods, more innovative industries have higher total innovation rates (i.e., the sum of own innovation and creative destruction). Appendix tables 5 and 6 further regresse the estimated rates of own innovation and creative destruction on innovation of industries. During the 1950-1980 period, industries with higher innovation have higher rates of own innovation, but are not significantly different in the rates of creative destruction. During the 1980-1995 period, more innovative industries have both higher rates of own innovation and creative destruction. During the 1995-2010 period, industries with more innovation have higher rates of creative destruction, but not higher rates of own innovation.

Table 4: Innovation rate (OI plus CD) vs Innovation of 2-digit NAICS industries

Innovation rate (OI plus CD)	1950 to 1980 (1)	1980 to 1995 (2)	1995 to 2010 (3)
log(Innovation)	0.071**	0.054**	0.021***
	(0.027)	(0.021)	(0.005)
Constant	0.714***	0.765***	0.428***
	(0.129)	(0.135)	(0.019)
Number of observations	175	125	142
R^2	0.329	0.268	0.242
Period FE	Yes	Yes	Yes

Note: Δ is change of variable over a 5-year period. Industries are weighted by their sales in the initial year of the period. Robust standard errors clustered at the industry level are in parentheses. Significance levels: * (p<0.1), ** (p<0.05), *** (p<0.01).

Table 5: Innovation rate (OI) vs Innovation of 2-digit NAICS industries

Innovation rate (OI)	1950 to 1980 (1)	1980 to 1995 (2)	1995 to 2010 (3)
log(Innovation)	0.059***	0.038**	-0.004
	(0.018)	(0.017)	(0.007)
Constant	0.359***	0.405***	0.136***
	(0.079)	(0.104)	(0.019)
Number of observations	175	125	142
R^2	0.307	0.282	0.056
Period FE	Yes	Yes	Yes

Note: Innovation rate (OI) is the estimated own innovation rate at the industry level. log(Innovation) is the log of the 5-year rolling average of the patent value-to-total asset ratio of each industry. Industries are weighted by their sales in the initial year of the period. Robust standard errors clustered at the industry level are in parentheses. Significance levels: * ($p<0.1$), ** ($p<0.05$), *** ($p<0.01$).

Table 6: Innovation rate (CD) vs Innovation of 2-digit NAICS industries

Innovation rate (CD)	1950 to 1980 (1)	1980 to 1995 (2)	1995 to 2010 (3)
log(Innovation)	0.012	0.016**	0.025***
	(0.011)	(0.007)	(0.005)
Constant	0.355***	0.360***	0.292***
	(0.054)	(0.038)	(0.020)
Number of observations	175	125	142
R^2	0.341	0.186	0.345
Period FE	Yes	Yes	Yes

Note: Innovation rate (CD) is the estimated creative destruction rate at the industry level. log(Innovation) is the log of the 5-year rolling average of the patent value-to-total asset ratio of each industry. Industries are weighted by their sales in the initial year of the period. Robust standard errors clustered at the industry level are in parentheses. Significance levels: * ($p<0.1$), ** ($p<0.05$), *** ($p<0.01$).

From the above analysis, we learn that innovation at the industry level implies a stronger force from own innovation prior to 1980, and works through a dominant creative destruction force over the 1995-2010 period. Which form of innovation is the dominant force at play also has implications on the turnover rate of the industry's top firms. Appendix tables 7 and 8 regresse the turnover rates due to inflow and outflow on the innovation of industries in the form of Equation (19). During the 1950 -1980 period, more innovative industries have higher rates of turnover by promoting more previously lower-ranked firms into the top. During the 1995-2010 period, more innovative industries have larger turnover rates by making incumbent top firms shrink so that they no longer qualify as top firms.

Table 7: Turnover (Inflow) vs Innovation of 2-digit NAICS industries

Turnover (Inflow)	1950 to 1980 (1)	1980 to 1995 (2)	1995 to 2010 (3)
log(Innovation)	0.033***	0.006	0.004
	(0.009)	(0.006)	(0.010)
Constant	0.293***	0.217***	0.264***
	(0.043)	(0.034)	(0.060)
Number of observations	175	125	142
R^2	0.370	0.197	0.221
Period FE	Yes	Yes	Yes

Note: Turnover (Inflow) is the fraction of new entrants that enters the top 0.01% percentile of an industry due to inflow. log(Innovation) is the log of the 5-year rolling average of the patent value-to-total asset of each industry. Industries are weighted by their sales in the initial year of the period. Robust standard errors clustered at the industry level are in parentheses. Significance levels: * (p<0.1), ** (p<0.05), *** (p<0.01).

Table 8: Turnover (Outflow) vs Innovation of 2-digit NAICS industries

Turnover (Outflow)	1950 to 1980 (1)	1980 to 1995 (2)	1995 to 2010 (3)
log(Innovation)	-0.004	-0.004	0.013***
	(0.007)	(0.003)	(0.004)
Constant	0.132***	0.117***	0.160***
	(0.026)	(0.013)	(0.017)
Number of observations	175	125	142
R^2	0.250	0.129	0.290
Period FE	Yes	Yes	Yes

Note: Turnover (Outflow) is the fraction of top firms that exits the top 0.01% percentile of an industry due to outflow. log(Innovation) is the log of the 5-year rolling average of the patent value-to-total asset ratio of each industry. Industries are weighted by their sales in the initial year of the period. Robust standard errors clustered at the industry level are in parentheses. Significance levels: * ($p<0.1$), ** ($p<0.05$), *** ($p<0.01$).

To summarize, at the industry level, higher innovation measured as the patent value-to-asset ratio is associated with different combinations of own innovation and creative destruction across different periods. Prior to 1980, industries with higher innovation demonstrate a stronger own innovation force relative to the creative destruction force. Over the 1980-1995 period, industries that have higher innovation feature both higher levels of own innovation and creative destruction. During the 1995-2010 period, more innovative industries have more creative destruction at play than own innovation.

C Tables

Table 1: Firms with largest sales increase for each decade over the 1950-2020 period

Period	Firm	Δ sales	Sector	Is incumbent
1950-1960	EXXON MOBIL CORP	24	Manufacture	✓
	AT&T CORP	23	Communication	✓
	GENERAL MOTORS CO	19	Manufacture	✓
	MOBIL CORP	9	Manufacture	✓
	TEXACO INC	8	Manufacture	✓
	GENERAL ELECTRIC CO	8	Conglomerate	✓
	GULF CORP	8	Manufacture	✓
	GREAT ATLANTIC & PAC TEA CO	7	Retail	✓
	BOEING CO	7	Manufacture	✓
	INTL BUSINESS MACHINES CORP	7	Service	✓
1960-1970	FORD MOTOR CO	38	Manufacture	✓
	AT&T CORP	31	Communication	✓
	EXXON MOBIL CORP	28	Manufacture	✓
	INTL BUSINESS MACHINES CORP	26	Service	✓
	ITT INC	24	Manufacture	✓
	SEARS ROEBUCK & CO	18	Retail	✓
	LTV CORP	16	Manufacture	
	GENERAL ELECTRIC CO	15	Conglomerate	✓
	MOBIL CORP	14	Manufacture	✓
	CHRYSLER CORP	14	Manufacture	✓
1970-1980	EXXON MOBIL CORP	168	Manufacture	✓
	MOBIL CORP	107	Manufacture	✓
	TEXACO INC	92	Manufacture	✓
	CHEVRON CORP	76	Manufacture	✓
	CITIGROUP GLOBAL MKTS HLDGS	56	Finance	✓
	GENERAL MOTORS CO	50	Manufacture	✓
	AMOCO CORP	45	Manufacture	✓
	ATLANTIC RICHFIELD CO	44	Manufacture	✓
	AT&T CORP	42	Communication	✓
	GULF CORP	38	Manufacture	✓
1980-1990	FORD MOTOR CO	66	Manufacture	✓
	GENERAL MOTORS CO	57	Manufacture	✓
	ALTRIA GROUP INC	52	Manufacture	✓
	WALMART INC	47	Retail	
	INTL BUSINESS MACHINES CORP	46	Service	✓
	GENERAL ELECTRIC CO	32	Conglomerate	✓
	DU PONT (E I) DE NEMOURS	30	Manufacture	✓
	SEARS ROEBUCK & CO	28	Retail	✓

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Table 1 – continued from previous page

Period	Firm	Δ sales	Sector	Is incumbent
	CONAGRA BRANDS INC	27	Manufacture	
	CITICORP	27	Finance	✓
1990-2000	WALMART INC	195	Retail	✓
	CITIGROUP INC	134	Finance	✓
	ENRON CORP	108	Wholesale	✓
	EXXON MOBIL CORP	98	Manufacture	✓
	GENERAL ELECTRIC CO	73	Conglomerate	✓
	FORD MOTOR CO	64	Manufacture	✓
	VERIZON COMMUNICATIONS INC	64	Communication	✓
	BANK OF AMERICA CORP	64	Finance	✓
	JPMORGAN CHASE & CO	63	Finance	✓
	GENERAL ELECTRIC CAPITAL SVC	61	Finance	✓
2000-2010	WALMART INC	191	Retail	✓
	CONOCOPHILLIPS	156	Mining	✓
	CHEVRON CORP	138	Manufacture	✓
	FEDERAL NATIONAL MORTGA ASSN	105	Finance	✓
	BERKSHIRE HATHAWAY	97	Conglomerate	✓
	EXXON MOBIL CORP	91	Manufacture	✓
	CVS HEALTH CORP	75	Retail	✓
	UNITEDHEALTH GROUP INC	71	Finance	✓
	HP INC	69	Manufacture	✓
	AMERISOURCEBERGEN CORP	66	Wholesale	✓
2010-2020	AMAZON.COM INC	304	Retail	✓
	APPLE INC	174	Manufacture	✓
	CVS HEALTH CORP	136	Retail	✓
	ALPHABET INC	130	Service	✓
	UNITEDHEALTH GROUP INC	128	Finance	✓
	CIGNA CORP	119	Finance	✓
	BERKSHIRE HATHAWAY	111	Conglomerate	✓
	CENTENE CORP	93	Finance	
	MCKESSON CORP	93	Wholesale	✓
	AMERISOURCEBERGEN CORP	86	Wholesale	✓

Note: Table shows firms which are in the top 0.01% percentile at the end of the corresponding period that have experienced the largest (top 10) growth in sales. Δ sales is the change in sales (deflated to 2012 dollars, unit: billion dollars) from the beginning to the end of the corresponding period. "Is incumbent" indicates whether the firm remains in the top 0.01% percentile during the relevant period. Data are from Compustat database.

Table 2: Firms with largest sales decrease for each decade over the 1950-2020 period

Period	Firm	Δ sales	Sector	Remains in top
1950-1960	CUDAHY CO	-2.4	Manufacture	✓
	ESMARK INC-OLD	-2.2	Manufacture	✓
	ASARCO INC	-0.9	Manufacture	✓
	ADMIRAL CORP	-0.6	Manufacture	
	SCHENLEY INDUSTRIES INC	-0.6	Manufacture	
	WALLACE-MURRAY CORP	-0.6	Manufacture	
	USX CORP-CONSOLIDATED	-0.6	Manufacture	✓
	UNITED STATES STEEL CORP	-0.6	Manufacture	✓
	REPUBLIC STEEL CORP	-0.4	Manufacture	✓
	EASTERN ENTERPRISES	-0.4	Communication	
1960-1970	GREAT ATLANTIC & PAC TEA CO	-5.4	Retail	✓
	MARTIN MARIETTA CORP	-1.8	Manufacture	✓
	GENERAL DYNAMICS CORP	-1.7	Manufacture	✓
	ANDERSON CLAYTON & CO	-1.5	Manufacture	✓
	AMERICAN MOTORS CORP	-1.3	Manufacture	✓
	AEROJET GENERAL CORP	-0.9	Manufacture	
	HYGRADE FOOD PRODUCTS CORP	-0.9	Manufacture	
	R S N PROJECTS INC	-0.7	Manufacture	
	RATH PACKING CO	-0.5	Manufacture	
	ESMARK INC-OLD	-0.5	Manufacture	✓
1970-1980	CHRYSLER CORP	-10.5	Manufacture	✓
	GREAT ATLANTIC & PAC TEA CO	-9.6	Retail	✓
	ESMARK INC-OLD	-7.3	Manufacture	✓
	REVLON GROUP INC	-5.5	Manufacture	
	MCCRORY PARENT CORP	-4.4	Retail	✓
	GENESCO INC	-3.8	Retail	
	BICOASTAL CORP	-3.2	Manufacture	✓
	ALLIED SUPERMARKETS	-3.1	Retail	
	U S INDUSTRIES	-3	Manufacture	
	VORNADO REALTY TRUST	-2.3	Finance	
1980-1990	EXXON MOBIL CORP	-78.3	Manufacture	✓
	TEXACO INC	-56.9	Manufacture	✓
	MOBIL CORP	-50	Manufacture	✓
	CHEVRON CORP	-35.1	Manufacture	✓
	AT&T CORP	-32.2	Communication	✓
	ATLANTIC RICHFIELD CO	-27.9	Manufacture	✓
	UNITED STATES STEEL CORP	-20	Manufacture	✓
	AMOCO CORP	-17.8	Manufacture	✓
	SAFEWAY INC	-12.4	Retail	✓
	SUNOCO INC	-12.1	Manufacture	✓

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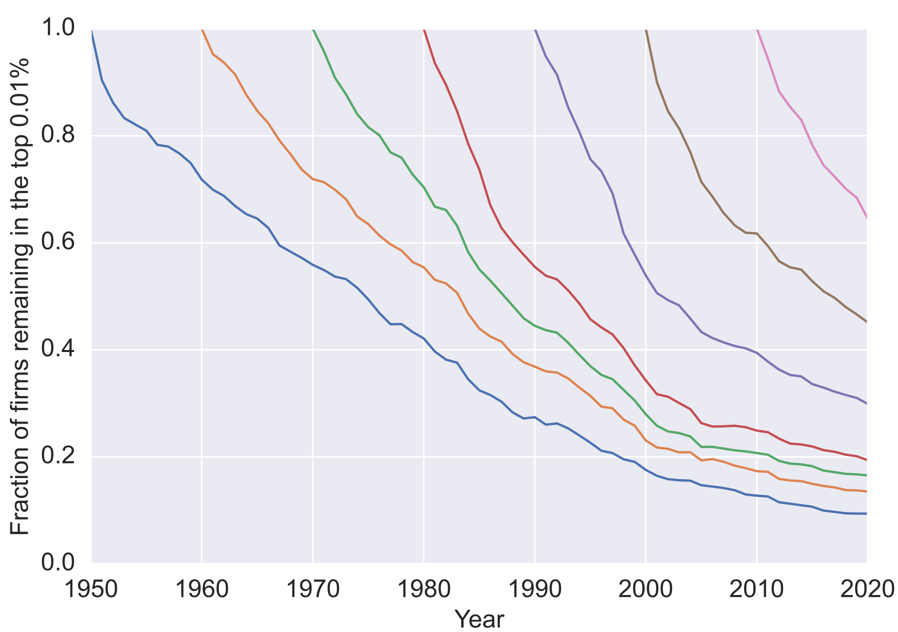
Table 2 – continued from previous page

Period	Firm	Δ sales	Sector	Remains in top
1990-2000	SEARS ROEBUCK & CO	-35.5	Retail	✓
	ITT INC	-26.2	Manufacture	✓
	DU PONT (E I) DE NEMOURS	-25.7	Manufacture	✓
	TENNESSEE GAS PIPELINE CO	-20.7	Wholesale	
	TENNECO INC	-18.2	Manufacture	
	CITIGROUP GLOBAL MKTS HLDGS	-17.1	Finance	✓
	OCCIDENTAL PETROLEUM CORP	-16.7	Mining	✓
	EASTMAN KODAK CO	-11.8	Manufacture	✓
	ROCKWELL AUTOMATION	-10.3	Manufacture	✓
	HOST HOTELS & RESORTS INC	-10.1	Finance	
2000-2010	GENERAL MOTORS CO	-90.2	Manufacture	✓
	FORD MOTOR CO	-83.6	Manufacture	✓
	ALTRIA GROUP INC	-63.5	Manufacture	✓
	DUKE ENERGY CORP	-47.8	Communication	✓
	DYNEGY INC	-35.3	Mining	
	GENERAL ELECTRIC CAPITAL SVC	-32.2	Finance	✓
	CENTERPOINT ENERGY INC	-28.4	Communication	✓
	MOTOROLA SOLUTIONS INC	-28.1	Manufacture	✓
	CITIGROUP INC	-27.2	Finance	✓
	EL PASO CORP	-23.3	Communication	
2010-2020	EXXON MOBIL CORP	-198.2	Manufacture	✓
	CONOCOPHILLIPS	-166.3	Mining	✓
	CHEVRON CORP	-114.1	Manufacture	✓
	GENERAL ELECTRIC CO	-85	Conglomerate	✓
	PHILLIPS 66	-81.8	Manufacture	✓
	HP INC	-81.3	Manufacture	✓
	MARATHON OIL CORP	-67	Mining	
	FEDERAL NATIONAL MORTGA ASSN	-65.3	Finance	✓
	BANK OF AMERICA CORP	-57.1	Finance	✓
	FEDERAL HOME LOAN MORTG CORP	-44.5	Finance	✓

Note: Table shows firms which are in the top 0.01% percentile at the beginning of the corresponding period that have experienced the largest (top 10) decrease in sales. Δ sales is the change in sales (deflated to 2012 dollars, unit: billion dollars) from the beginning to the end of the corresponding period. "Remains in top" indicates whether the firm remains in the top 0.01% percentile after experiencing shrinkage of sales during the relevant period. Data are from Compustat database.

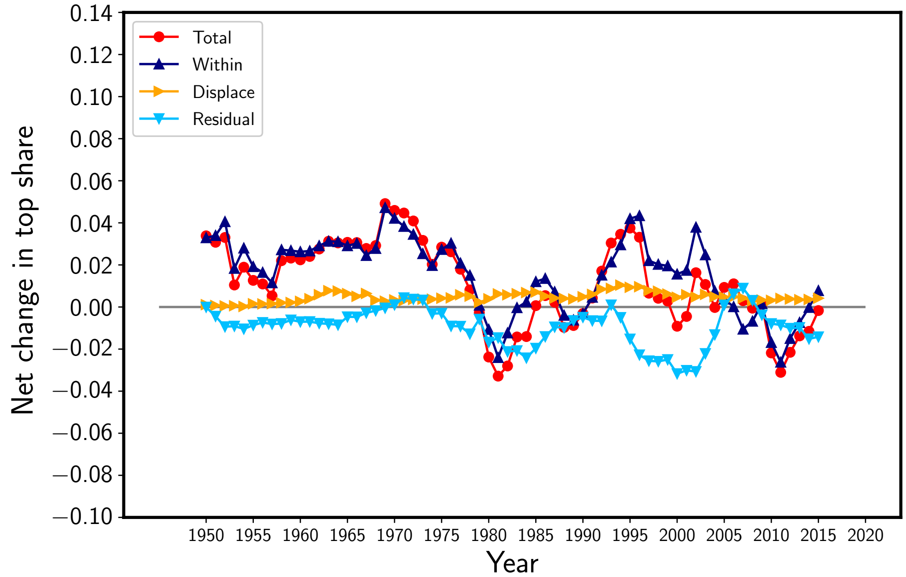
D Figures

Figure 1: Fraction of firms remaining in the top 0.01% percentile (ranked by sales)

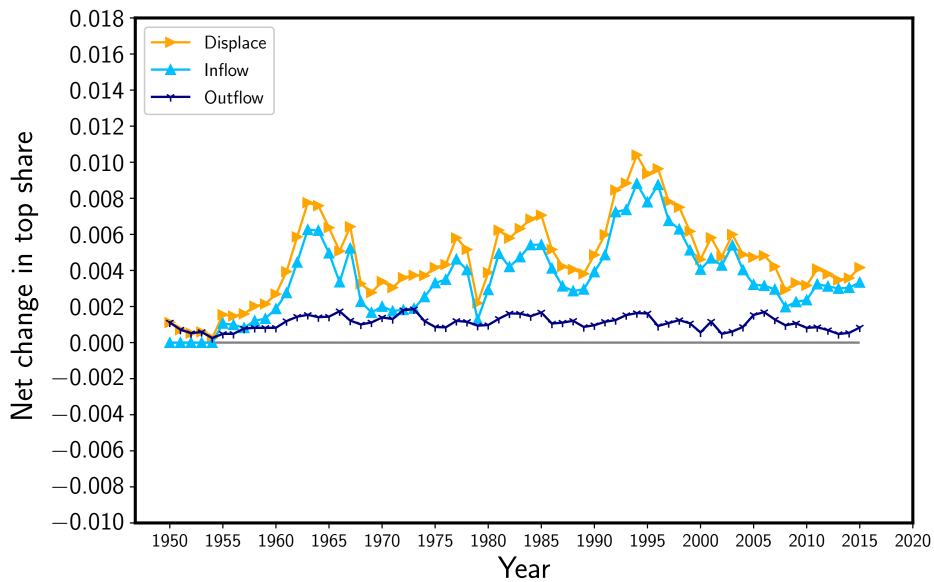


Note: For each cohort of firms that are in the top 0.01% percentile (ranked by sales) at the starting years 1950, 1960, ..., 2010, Figure 1 plots the fraction of these cohorts of top firms remaining in the top 0.01% percentile in subsequent years during the 1950-2020 period.

Figure 2: Annualized decomposition terms of top sales shares growth over 5-year horizon



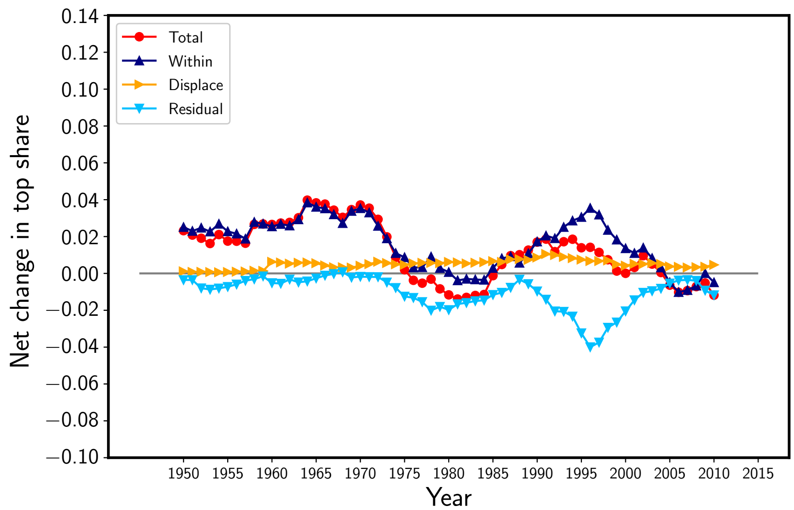
(a) All terms



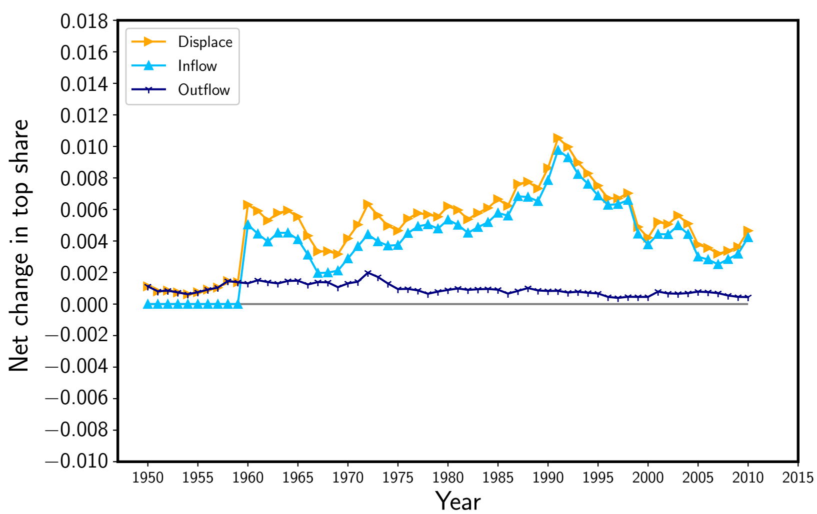
(b) Displacement term

Note: Figure 2a plots the annualized 5-year growth of the sales shares accrued to the top 0.01% firms over the 1950-2019 period, along with the within, displacement and residual (combining the M&A term and the firm number change term) terms defined in accounting framework (2). Figure 2b plots the inflow and the outflow terms which comprise the displacement term from the same decomposition. Data are from Compustat (sales of top firms), BEA (gross output of all industries) and Census BDS (total firm number), and SDC Platinum (mergers and acquisitions).

Figure 3: Annualized decomposition terms of top sales shares growth over 10-year horizon



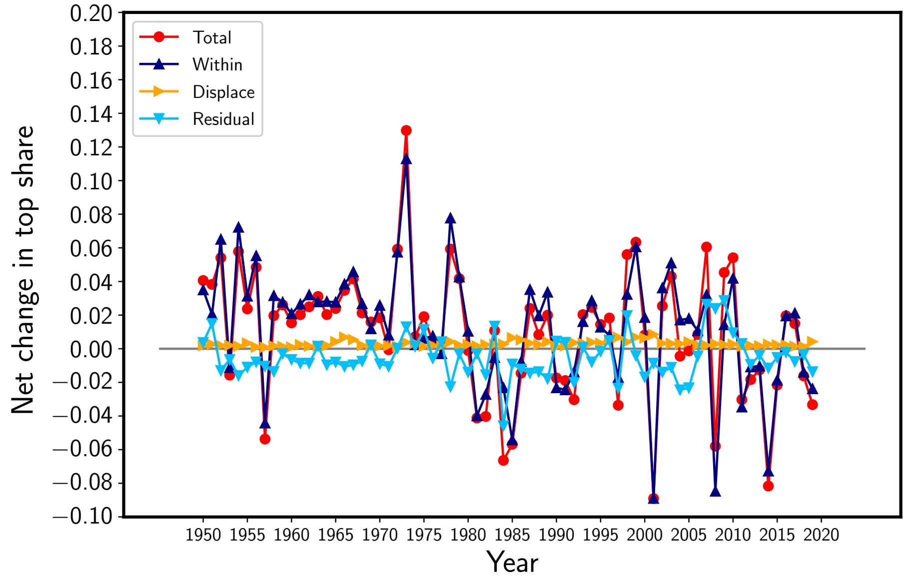
(a) All terms



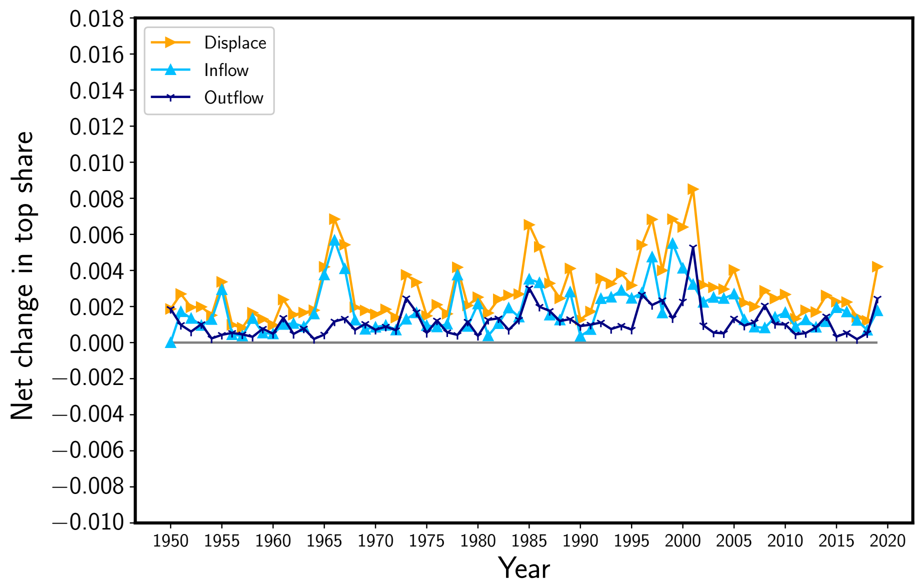
(b) Displacement term

Note: Figure 3a plots the annualized 10-year growth of the sales shares accrued to the top 0.01% firms over the 1950-2019 period, along with the within, displacement and residual (combining the M&A term and the firm number change term) terms defined in accounting framework (2). Figure 3b plots the inflow and the outflow terms which comprise the displacement term from the same decomposition. Data are from Compustat (sales of top firms), BEA (gross output of all industries) and Census BDS (total firm number), and SDC Platinum (mergers and acquisitions).

Figure 4: Decomposition terms of top sales shares growth (without financial firms)



(a) All terms



(b) Displacement term

Note: Figure 4a plots the annual growth of the sales shares accrued to the top 0.01% firms (excluding financial firms) over the 1950-2019 period, along with the within, displacement and residual (combining the M&A term and the firm number change term) terms defined in accounting framework (2). Figure 4b plots the inflow and the outflow terms which comprise the displacement term from the same decomposition. Data are from Compustat (sales of top firms), BEA (gross output of all industries) and Census BDS (total firm number), and SDC Platinum (mergers and acquisitions).

E Proofs

A Proof of Proposition 5

The optimization problem for the final goods sector and the firm's monopoly decisions are static. A perfectly competitive final goods sector combines varieties $i \in [0,1]$ with price p_i to produce the final good Y. A representative firm in the final goods sector solves:

$$\min_{\{Y_i\}} \int_0^1 p_i Y_i d_i$$

s.t.

$$Y = \left(\int_0^1 Y_i^\theta di \right)^{\frac{1}{\theta}}$$

The demand for each variety i is

$$\left(\frac{Y}{Y_i} \right)^{1-\theta} = p_i. \tag{20}$$

Each firm i chooses Y_i:

$$\max_{\{Y_i\}} p_i(Y_i)Y_i - wL_i = Y^{1-\theta} Y_i^\theta - \frac{w}{(1+G)^n x_i^(1-\theta)} Y_i$$

The solution to Y_i is:

$$Y_i = \left(\frac{1}{\theta} \cdot \frac{w}{(1+G)^n} \right)^{\frac{1}{\theta-1}} Y x_i \tag{21}$$

Plugging equation (21) into the final goods production function $Y = \left(\int_0^1 Y_i^\theta di \right)^{\frac{1}{\theta}}$, we obtain the equilibrium real wage:

$$w = \theta (1+G)^n \left(\int_0^1 x_i^\theta di \right)^{\frac{1-\theta}{\theta}} \tag{22}$$

Define $X \equiv \left(\int_0^1 x_i^\theta d_i\right)^{\frac{1}{\theta}}$, the equilibrium wage equation is:

$$w = \theta(1+G)^n X^{1-\theta}. \tag{23}$$

Plug this into equation (21), we get

$$Y_i = \left(\frac{x_i}{X}\right) Y \tag{24}$$

Joining equation (24) with firm i's production function $Y_i = (1+G)^{n_t} x_i^{1-\theta} L_i$, we get employment by firm i:

$$L_i = \frac{Y_i}{(1+G)^{n_t} x_i^\alpha} = (1+G)^{-n_t} \frac{x_i^\theta}{X} Y \tag{25}$$

From the labor market clearing condition $L = \int_0^1 L_i di$ and equation (25), we get aggregate output:

$$Y = (1+G)^{n_t} X^{1-\theta} L \tag{26}$$

Firm sales, aggregate output and wage in proposition 5 are given by equations (24, 26), and (23), respectively. Q.E.D.

B Proof of Proposition 1

Since the set of top percentile firms at $t = 0$ is $\mathcal{P}_0 = (\mathcal{P}_0 \backslash \mathcal{D}) \cup \mathcal{D}$, and the set of top percentile firms at $t = 1$ is $\mathcal{P}_1 = (\mathcal{P}_0 \backslash \mathcal{D}) \cup \mathcal{E} \cup \mathcal{B} \backslash \mathcal{X}$, we can write

$$\sum_{i \in \mathcal{P}_0} y_{i0} = \sum_{i \in \mathcal{P}_0 \backslash \mathcal{D}} y_{i0} + \sum_{i \in \mathcal{D}} y_{i0}, \tag{27}$$

and

$$\sum_{i\in\mathcal{P}_1} y_{i1} = \sum_{i\in\mathcal{P}_0\setminus\mathcal{D}} y_{i1} - \sum_{i\in\mathcal{X}} y_{i1} + \sum_{i\in\mathcal{E}} y_{i1} + \sum_{i\in\mathcal{B}} y_{i1}. \tag{28}$$

In the following parts, we are interested in separating out the growth of firms that are due to its own organic growth from the growth effect due to mergers and acquisitions. For each firm j, we denote its total sales by y_j^{ALL}. We account for its sales increase from acquiring other firms, (i.e., the sales growth due to M&A), as y_j^{MA}. Then the sales increase of firm j due to organic growth $y_j^{OG} = y_j^{ALL} - y_j^{MA}$.

Dividing equations (27) and (28) by $|\mathcal{P}_1|$:

$$\overline{y}_{\mathcal{P}_1,1} = \frac{\sum_{i\in\mathcal{P}_1} y_{i1}}{|\mathcal{P}_1|} = \frac{|\mathcal{P}_0\setminus\mathcal{D}|\ \overline{y}^{ALL}_{\mathcal{P}_0\setminus\mathcal{D},1}}{|\mathcal{P}_1|} - \frac{|\mathcal{X}|\overline{y}_{\mathcal{X},1}}{|\mathcal{P}_1|} + \frac{|\mathcal{E}|\ \overline{y}^{ALL}_{\mathcal{E},1}}{|\mathcal{P}_1|} + \frac{|\mathcal{B}|\ \overline{y}^{ALL}_{\mathcal{B},1}}{|\mathcal{P}_1|} \tag{29}$$

$$\frac{|\mathcal{P}_0|\overline{y}_{\mathcal{P}_0,0}}{|\mathcal{P}_1|} = \underbrace{\frac{|\mathcal{P}_0\setminus\mathcal{D}|\overline{y}_{\mathcal{P}_0\setminus\mathcal{D},0}}{|\mathcal{P}_1|} + \frac{|\mathcal{D}|\overline{y}_{\mathcal{D},0}}{|\mathcal{P}_1|}}_{(2)} \tag{30}$$

Plugging (30) into (29), we get

$$\begin{aligned}
\overline{y}_{\mathcal{P}_1,1} =& \frac{|\mathcal{P}_0\setminus\mathcal{D}|}{|\mathcal{P}_1|}\frac{\overline{y}^{ALL}_{\mathcal{P}_0\setminus\mathcal{D},1}}{\overline{y}_{\mathcal{P}_0\setminus\mathcal{D},0}}\overline{y}_{\mathcal{P}_0\setminus\mathcal{D},0} - \frac{|\mathcal{X}|\overline{y}_{\mathcal{X},1}}{|\mathcal{P}_1|} + \frac{|\mathcal{E}|\ \overline{y}^{ALL}_{\mathcal{E},1}}{|\mathcal{P}_1|} + \frac{|\mathcal{B}|\ \overline{y}^{ALL}_{\mathcal{B},1}}{|\mathcal{P}_1|} \\
=& \frac{\overline{y}^{ALL}_{\mathcal{P}_0\setminus\mathcal{D},1}}{\overline{y}_{\mathcal{P}_0\setminus\mathcal{D},0}}\Big(\frac{|\mathcal{P}_0|}{|\mathcal{P}_1|}\overline{y}_{\mathcal{P}_0,0} - \frac{|\mathcal{D}|}{|\mathcal{P}_1|}\overline{y}_{D,0}\Big) - \frac{|\mathcal{X}|\overline{y}_{\mathcal{X},1}}{|\mathcal{P}_1|} + \frac{|\mathcal{E}|\ \overline{y}^{ALL}_{\mathcal{E},1}}{|\mathcal{P}_1|} + \frac{|\mathcal{B}|\ \overline{y}^{ALL}_{\mathcal{B},1}}{|\mathcal{P}_1|}
\end{aligned}$$

Adding and subtracting the term $\frac{\overline{y}^{ALL}_{\mathcal{P}_0\backslash\mathcal{D},1}}{\overline{y}_{\mathcal{P}_0\backslash\mathcal{D},0}}\overline{y}_{\mathcal{P}_0,0}$, it follows then

$$\begin{aligned}
\overline{y}_{\mathcal{P}_1,1}-\overline{y}_{\mathcal{P}_0,0}=&\frac{\overline{y}^{ALL}_{\mathcal{P}_0\backslash\mathcal{D},1}}{\overline{y}_{\mathcal{P}_0\backslash\mathcal{D},0}}\frac{|\mathcal{P}_0|}{|\mathcal{P}_1|}\overline{y}_{\mathcal{P}_0,0}+\frac{\overline{y}^{ALL}_{\mathcal{P}_0\backslash\mathcal{D},1}}{\overline{y}_{\mathcal{P}_0\backslash\mathcal{D},0}}\overline{y}_{\mathcal{P}_0,0}-\frac{\overline{y}^{ALL}_{\mathcal{P}_0\backslash\mathcal{D},1}}{\overline{y}_{\mathcal{P}_0\backslash\mathcal{D},0}}\overline{y}_{\mathcal{P}_0,0}-\overline{y}_{\mathcal{P}_0,0}\\
&-\frac{\overline{y}_{\mathcal{P}_0\backslash\mathcal{D},1}}{\overline{y}_{\mathcal{P}_0\backslash\mathcal{D},0}}\frac{|\mathcal{D}|}{|\mathcal{P}_1|}\overline{y}_{D,0}-\frac{|\mathcal{X}|\overline{y}_{\mathcal{X},1}}{|\mathcal{P}_1|}+\frac{|\mathcal{E}|\ \overline{y}^{ALL}_{\mathcal{E},1}}{|\mathcal{P}_1|}+\frac{|\mathcal{B}|\ \overline{y}^{ALL}_{\mathcal{B},1}}{|\mathcal{P}_1|}\\
=&\Big(\frac{\overline{y}^{ALL}_{\mathcal{P}_0\backslash\mathcal{D},1}}{\overline{y}_{\mathcal{P}_0\backslash\mathcal{D},0}}-1\Big)\overline{y}_{\mathcal{P}_0,0}-\Big(\frac{|\mathcal{P}_1|-|\mathcal{P}_0|}{|\mathcal{P}_1|}\Big)\frac{\overline{y}^{ALL}_{\mathcal{P}_0\backslash\mathcal{D},1}}{\overline{y}_{\mathcal{P}_0\backslash\mathcal{D},0}}\overline{y}_{\mathcal{P}_0,0}\\
&-\frac{\overline{y}_{\mathcal{P}_0\backslash\mathcal{D},1}}{\overline{y}_{\mathcal{P}_0\backslash\mathcal{D},0}}\frac{|\mathcal{D}|}{|\mathcal{P}_1|}\overline{y}_{D,0}-\frac{|\mathcal{X}|\overline{y}_{\mathcal{X},1}}{|\mathcal{P}_1|}+\frac{|\mathcal{E}|\ \overline{y}^{ALL}_{\mathcal{E},1}}{|\mathcal{P}_1|}+\frac{|\mathcal{B}|\ \overline{y}^{ALL}_{\mathcal{B},1}}{|\mathcal{P}_1|}
\end{aligned}$$

Observe that $|\mathcal{P}_1|=|\mathcal{P}_0|-|\mathcal{D}|-|\mathcal{X}|+|\mathcal{E}|+|\mathcal{B}|$. It follows that

$$\begin{aligned}
\overline{y}_{\mathcal{P}_1,1}-\overline{y}_{\mathcal{P}_0,0}=&\Big(\frac{\overline{y}^{ALL}_{\mathcal{P}_0\backslash\mathcal{D},1}}{\overline{y}_{\mathcal{P}_0\backslash\mathcal{D},0}}-1\Big)\overline{y}_{\mathcal{P}_0,0}-\frac{|\mathcal{P}_1|-|\mathcal{P}_0|}{|\mathcal{P}_1|}\Big(\frac{\overline{y}_{\mathcal{P}_0\backslash\mathcal{D},1}}{\overline{y}_{\mathcal{P}_0\backslash\mathcal{D},0}}\overline{y}_{\mathcal{P}_0,0}-q_1\Big)\\
&+\frac{|\mathcal{E}|}{|\mathcal{P}_1|}(\overline{y}^{ALL}_{\mathcal{E},1}-q_1)+\frac{|\mathcal{B}|}{|\mathcal{P}_1|}(\overline{y}^{ALL}_{\mathcal{B},1}-q_1)+\frac{|\mathcal{X}|}{|\mathcal{P}_1|}(q_1-\overline{y}_{\mathcal{X},1})-\frac{|\mathcal{D}|}{|\mathcal{P}_1|}\Big(\frac{\overline{y}_{\mathcal{P}_0\backslash\mathcal{D},1}}{\overline{y}_{\mathcal{P}_0\backslash\mathcal{D},0}}\overline{y}_{D,0}-q_1\Big)
\end{aligned}$$

where

$$\overline{y}^{ALL}_{\mathcal{P}_0\backslash\mathcal{D},1}=\frac{\sum_{i\in\mathcal{P}_0\backslash\mathcal{D}}y_{i1}}{|\mathcal{P}_0\backslash\mathcal{D}|}=\frac{\sum_{i\in\mathcal{P}_0\backslash\mathcal{D}}(y^{OG}_{i1}+y^{MA}_{i1})}{|\mathcal{P}_0\backslash\mathcal{D}|}=\overline{y}_{\mathcal{P}_0\backslash\mathcal{D},1}+\overline{y}^{MA}_{\mathcal{P}_0\backslash\mathcal{D},1},$$

we here denote $\overline{y}_{\mathcal{P}_0\backslash\mathcal{D},1}=\frac{\sum_{i\in\mathcal{P}_0\backslash\mathcal{D}}y^{OG}_{i1}}{|\mathcal{P}_0\backslash\mathcal{D}|}$.

And

$$\overline{y}^{ALL}_{\mathcal{E},1}=\frac{\sum_{i\in\mathcal{E}}y_{i1}}{|\mathcal{E}|}=\frac{\sum_{i\in\mathcal{E}}(y^{OG}_{i1}+y^{MA}_{i1})}{|\mathcal{E}|}=\overline{y}_{\mathcal{E},1}+\overline{y}^{MA}_{\mathcal{E},1},$$

together with

$$\overline{y}^{ALL}_{\mathcal{X},1}=\frac{\sum_{i\in\mathcal{X}}y_{i1}}{|\mathcal{X}|}=\frac{\sum_{i\in\mathcal{X}}(y^{OG}_{i1}+y^{MA}_{i1})}{|\mathcal{X}|}=\overline{y}_{\mathcal{X},1}+\overline{y}^{MA}_{\mathcal{X},1}.$$

Therefore

$$
\begin{aligned}
\frac{\overline{y}_{\mathcal{P}_1,1}-\overline{y}_{\mathcal{P}_0,0}}{\overline{y}_{\mathcal{P}_0,0}} = & \underbrace{\Big(\frac{\overline{y}_{\mathcal{P}_0\setminus\mathcal{D},1}}{\overline{y}_{\mathcal{P}_0\setminus\mathcal{D},0}}-1\Big)}_{\text{Within}} + \underbrace{\frac{|\mathcal{E}|}{|\mathcal{P}_1|}\Big(\frac{\overline{y}_{\mathcal{E},1}-q_1}{\overline{y}_{\mathcal{P}_0,0}}\Big)}_{\text{Inflow}} + \underbrace{\frac{|\mathcal{X}|}{|\mathcal{P}_1|}\Big(\frac{q_1-\overline{y}_{\mathcal{X},1}}{\overline{y}_{\mathcal{P}_0,0}}\Big)}_{\text{Outflow}} \\
& + \underbrace{\frac{\overline{y}^{MA}_{\mathcal{P}_0\setminus\mathcal{D},1}}{\overline{y}_{\mathcal{P}_0\setminus\mathcal{D},0}} + \frac{|\mathcal{E}|}{|\mathcal{P}_1|}\frac{\overline{y}^{MA}_{\mathcal{E},1}}{\overline{y}_{\mathcal{P}_0,0}} + \frac{|\mathcal{B}|}{|\mathcal{P}_1|}\Big(\frac{\overline{y}_{\mathcal{B},1}-q_1}{\overline{y}_{\mathcal{P}_0,0}}\Big)}_{\text{M\&A growth}} \\
& \underbrace{-\frac{|\mathcal{D}|}{|\mathcal{P}_1|}\Big(\frac{\overline{y}_{\mathcal{P}_0\setminus\mathcal{D},1}}{\overline{y}_{\mathcal{P}_0\setminus\mathcal{D},0}}\overline{y}_{D,0}-q_1\Big)}_{\text{M\&A shrink}} - \underbrace{\frac{|\mathcal{P}_1|-|\mathcal{P}_0|}{|\mathcal{P}_1|}\Big(\frac{\frac{\overline{y}_{\mathcal{P}_0\setminus\mathcal{D},1}}{\overline{y}_{\mathcal{P}_0\setminus\mathcal{D},0}}\overline{y}_{\mathcal{P}_0,0}-q_1}{\overline{y}_{P_0,0}}\Big)}_{\text{Firm number change}}
\end{aligned}
$$

This gives Proposition 1. Q.E.D.

C Proof of Propositions 2 and 3

Assume that firm size follows the law of motion:

$$
\frac{dy_{it}}{y_{it-}} = \mu dt + (e^{U_1}-1)dN^{\lambda}_{it} + (e^{U_2}-1)dN^{\delta}_{it}, \tag{31}
$$

where N^{λ}_{it} and N^{δ}_{it} are Poisson processes with intensity λ and δ, respectively. Under this set up, firm size can jump with either a positive log size U_1, or a negative log size U_2.

C.1 Step 1: Characterize stationary firm size distribution.

Suppose h(·) is a twice-differentiable scalar function, by Ito's formula for jump process:

$$
\begin{aligned}
dh(y_t) &= h'(y_t)\mu y_t dt + \frac{d}{d_t}\sum_{i=1}^{2}\int_0^t [h(y_s)-h(y_{s-})]dN_{is} \\
&= h'(y_t)\mu y_t dt + \sum_{i=1}^{2}[h(y_t)-h(y_{t-})]dN_{it}
\end{aligned}
\tag{32}
$$

Let $g_t(y) = p(y_0, 0; y, t)$ be the density function of y_t: integrate from 0 to T,

$$\begin{aligned} h(y_T) - h(y_0) &= \int_0^T h'(y_t)\mu y_{t-} dt + \sum_{i=1}^{2} \int_0^T [h(y_t) - h(y_{t-})] dN_{it} \\ &= \int_0^T h'(y_t)\mu y_t dt + \sum_{i=1}^{2} \int_0^T [h(y_t) - h(y_{t-})] dN_{it}, \end{aligned} \tag{33}$$

and this step uses the fact that $\int_0^T h'(y_t)\mu y_{t-} dt = \int_0^T h'(y_t)\mu y_t dt$ since jumps at finite number of points does not affect the value of integral.

Let $d\tilde{N}_{it}$ is compensated poisson process for dN_{it}, that is, $d\tilde{N}_{it}^{\lambda} = dN_{it}^{\lambda} - \lambda dt$ and $d\tilde{N}_{it}^{\delta} = dN_{it}^{\delta} - \delta dt$. Take expectation on both side of equation (33) and plug in $y_t = e^U y_{t-}$, we get

$$\begin{aligned} E[h(y_T) - h(y_0)] &= \int_0^T \int_0^{\infty} h'(y)\mu y g_t(y) dy dt + E[\sum_{i=1}^{2} \int_0^T [h(e^{U_i} y_{t-}) - h(y_{t-})] dN_{it}] \\ &= \int_0^T \int_0^{\infty} h'(y)\mu y g_t(y) dy dt + \sum_{i=1}^{2} E[\int_0^T [h(e^{U_i} y_{t-}) - h(y_{t-})] d\tilde{N}_{it}] \\ &\quad + E[\int_0^T [h(e^{U_1} y_{t-}) - h(y_{t-})] \lambda d_t] + E[\int_0^T [h(e^{U_2} y_{t-}) - h(y_{t-})] \delta d_t] \\ &= \int_0^T \int_0^{\infty} h'(y)\mu y g_t(y) dy dt \\ &\quad + E[\int_0^T [h(e^{U_1} y_t) - h(y_t)] \lambda d_t] + E[\int_0^T [h(e^{U_2} y_t) - h(y_t)] \delta d_t], \end{aligned} \tag{34}$$

and this step uses $E[\int_0^T [h(e^{U_i} y_{t-}) - h(y_{t-})] d\tilde{N}_{it}] = 0$ since the compensated poisson process $d\tilde{N}_{it}$ is a martingale.

Expand on the LHS and RHS of equation (34), we get

$$\begin{aligned} \int_0^{+\infty} h(y) g_T(y) dy - h(y_0) &= \int_0^T \int_0^{\infty} h'(y)\mu y g_t(y) dy dt + \lambda \int_0^T \int_0^{\infty} [h(e^{U_1} y) - h(y)] g_T(y) dy dt \\ &\quad + \delta \int_0^T \int_0^{\infty} [h(e^{U_2} y) - h(y)] g_T(y) dy dt \end{aligned} \tag{35}$$

Take derivative w.r.t. T on both sides:

$$\int_0^\infty h(y)dg_T(y)dy = \int_0^T [h'(y)\mu y g_T(y)]dy \\ + \lambda \int_0^\infty [h(e^{U_1}y) - h(y)]g_T(y)dy + \delta \int_0^\infty [h(e^{U_2}y) - h(y)]g_T(y)dy \tag{36}$$

Performing integration by parts on Equation (36), it follows that

$$\int_0^\infty h(y)dg_T(y)dy = \int_0^\infty h(y)[-\mu y g_T(y)]'dy + \lambda \int_0^\infty h(y)[e^{-U_1}g_T(e^{-U_1}y) - g_T(y)]dy \\ + \delta \int_0^{+\infty} h(y)[e^{-U_2}g_T(e^{-U_2}y) - g_T(y)]dy \tag{37}$$

for any $h(y)$ satisfying $h(0) = h(\infty) = 0$. Since (37) holds for any such $h(y)$, we get the following partial differential equation for $g(y,t) = g_t(y)$

$$\frac{\partial g_t}{\partial t}(y) = \frac{\partial}{\partial y}[-\mu y g_t(y)] + \lambda[e^{-U_1}g_t(e^{-U_1}y) - g_t(y)] + \delta[e^{-U_2}g_t(e^{-U_2}y) - g_t(y)]. \tag{38}$$

Multiplying both sides of (38) by y and setting $\tilde{g}_t(y) = y g_t(y)$, we get the equation

$$\frac{\partial \tilde{g}_t}{\partial t}(y) = y\frac{\partial}{\partial y}[-\mu \tilde{g}_t(y)] + \lambda[\tilde{g}_t(e^{-U_1}y) - \tilde{g}_t(y)] + \delta[\tilde{g}_t(e^{-U_2}y) - \tilde{g}_t(y)]. \tag{39}$$

Since $\int_0^\infty g_t(y)dy$ is finite, it must hold that $\lim_{y\to\infty} y g_t(y) = 0$ for any t. We first look for the stationary solution to (39), which is a function $\tilde{g}(y)$ satisfying (39) with the left-hand side being 0, that is

$$-\mu y \frac{\partial \tilde{g}(y)}{\partial y} + \lambda\big(\tilde{g}(e^{-U_1}y) - \tilde{g}(y)\big) + \delta\big(\tilde{g}(e^{-U_2}y) - \tilde{g}(y)\big) = 0. \tag{40}$$

We are interested in the behavior of $\tilde{g}(y)$ for y large enough. Considering that $\lim_{y\to\infty} \tilde{g}(y) =$

0, we take the expansion of $\tilde{g}(y)$ around $y = \infty$, i.e. $y^{-1} = 0$, so for some $s \geq 0$

$$\tilde{g}(y) = y^{-s} \sum_{n=0}^{\infty} a_n y^{-n}, \tag{41}$$

with the coefficients a_n to be determined and $a_0 \neq 0$.

Substituting (41) into Equation (40) we obtain

$$\sum_{n=0}^{\infty} \Big(a_n \mu(n+s) + a_n \lambda [e^{U_1(n+s)} - 1] + a_n \delta [e^{U_2(n+s)} - 1] \Big) y^{-n-s} = 0, \tag{42}$$

which implies that

$$\Big(\mu(n+s) + \lambda [e^{U_1(n+s)} - 1] + \delta [e^{U_2(n+s)} - 1] \Big) a_n = 0, \quad \forall n = 0, 1, 2, \cdots \tag{43}$$

Since $a_0 \neq 0$ we have the indicial equation for s

$$\Psi(s) := \mu s + \lambda [e^{U_1 s} - 1] + \delta [e^{U_2 s} - 1] = 0, \tag{44}$$

which determines the number $\zeta \geq 0$ by $\Psi(\zeta) = 0$. To guarantee $\lim_{y \to \infty} \tilde{g}(y) = 0$, we need $\zeta > 0$.

Claim: Under the assumption that

$$U_1 > 0,\, U_2 < 0, \quad \text{and} \quad \Psi(1) = \mu + \lambda [e^{U_1} - 1] + \delta [e^{U_2} - 1] < 0,$$

together with

$$\Psi'(0) = \mu + \lambda U_1 + \delta U_2 < 0,$$

$\Psi(s) = 0$ has a unique positive root $\zeta > 1$.

We now verify that this is indeed the case. Observe that $\Psi(0) = 0$, $U_1 > 0$, we have $\lim_{s \to \infty} \Psi(s) = \infty$, and $\Psi''(s) = \lambda U_1^2 e^{sU_1} + \delta U_2^2 e^{sU_2} > 0$. Therefore, $\Psi(s)$ is strictly convex when $s > 0$. By intermediate value theorem, $\Psi(1) < 0$ implies that $\Psi(s) = 0$ has a unique positive root $\zeta > 1$, which is the one we are seeking for. We observe that

$$\Psi(s) < 0 \text{ for } s \in (0, \zeta), \quad \text{and} \quad \Psi(s) > 0 \text{ for } s > \zeta. \tag{45}$$

With this choice of ζ, Equation (42) reads $\Psi(n+\zeta)a_n = 0$, hence $a_n = 0$ for any $n \geq 1$. We thus obtain the solution $\tilde{g}(y) = a_0 y^{-\zeta}$ of (40). The positive constant a_0 can be determined by $\int g(y)dy = \int y^{-1}\tilde{g}(y)dy = 1$.

That is, when individual firm dynamics follows the jump (i.e., compound poisson) process specified in Equation (31), the stationary distribution of the firm size distribution is a Pareto distribution with exponent ζ, and its density function is $g(y) = a_0 y^{-\zeta-1}, a_0 > 0$.

This gives Proposition 2. Q.E.D.

C.2 Step 2: solving for the transition dynamics

We now go back to the evolving equation (39) and we wish to find a solution $\tilde{g}_t(y)$ that converges to the stationary one $\tilde{g}(y) = a_0 y^{-\zeta}$. Note that such solutions may not be unique, and it suffices to choose one. We take a similar approach as above by setting $\tilde{g}_t(y) = y^{-s}\sum_{n=0}^{\infty} a_n(t) y^{-n}$. Then Equation (39) is reduced to

$$\frac{\partial a_n(t)}{\partial t} = a_n(t)\Psi(n+s), \quad \forall n \geq 0.$$

One candidate is to choose $a_n(t) = 0$ for any $n \geq 1$ and $a_0(t) = e^{\Psi(s)t}$. To make sure $a_0(t)$ converges as $t \to \infty$, we need $\Psi(s) < 0$. By Equation (45) this is possible if we choose $s = \hat{\zeta} \in (0, \zeta)$. Furthermore, we also fiy a $\check{\zeta} \in (0, \zeta)$ which satisfies $\Psi(\hat{\zeta}) = \Psi(\check{\zeta}) < 0$ and $\hat{\zeta} < \check{\zeta}$.

Thus we consider one solution to Equation (39)

$$\tilde{g}_t(y) = a_0 y^{-\zeta} + \varepsilon e^{\Psi(\hat{\zeta})t}\hat{\zeta} y^{-\hat{\zeta}} - \varepsilon e^{\Psi(\check{\zeta})t}\check{\zeta} y^{-\check{\zeta}}, \tag{46}$$

for some suitably chosen $\varepsilon > 0$ to guarantee $\tilde{g}(y) > 0$. We now argue that this is possible.

Let $\bar{y} > 1$ be the unique root of $\hat{\zeta} y^{-\hat{\zeta}} - \check{\zeta} y^{-\check{\zeta}} = 0$. It is clear that $\varepsilon e^{\Psi(\hat{\zeta})t} \hat{\zeta} y^{-\hat{\zeta}} - \varepsilon e^{\Psi(\check{\zeta})t} \check{\zeta} y^{-\check{\zeta}} > 0$, for $y > \bar{y}$. On the other hand, for $y \in [1, \bar{y})$

$$a_0 y^{-\zeta} - \varepsilon e^{\Psi(\check{\zeta})t} \check{\zeta} y^{-\check{\zeta}} \geq a_0 - \varepsilon \check{\zeta} \bar{y}^{-\check{\zeta}} > 0,$$

if we take $\varepsilon = a_0 \bar{y}^{\check{\zeta}} \check{\zeta}^{-1}/2$.

The reason we choose $\tilde{g}_t(y)$ in the form of (46) is that for $g_t(y) = y^{-1} \tilde{g}_t(y)$,

$$\int_1^\infty g_t(y) dy = a_0 \zeta^{-1} + \varepsilon e^{\Psi(\hat{\zeta})t} \int_1^\infty (\hat{\zeta} y^{-\hat{\zeta}-1} - \check{\zeta} y^{-\check{\zeta}-1}) dy = a_0 \zeta^{-1}$$

is independent of t. We may fiy $a_0 = \zeta$ to make this integral be 1.

It is clear that $\tilde{g}_t \to \tilde{g}$ as $t \to \infty$. Equivalently, $g_t = y^{-1} \tilde{g}_t(y)$ satisfies Equation (38) and $g_t \to g(y) = a_0 y^{-1-\zeta}$. Therefore, $g_t = y^{-1} \tilde{g}_t(y)$ is one solution to the density function in transition dynamics which converges to the stationary Pareto distribution with exponent ζ. We will use these solutions in the following sections to characterize the dynamics of top sales shares..

C.3 Step 3: Law of motion of average firm size in the top percentile

The average firm size in the top percentile p is

$$\overline{y}_t = \frac{1}{p} \int_{q_t}^\infty y g_t(y) dy \tag{47}$$

Denote $\gamma_t(\cdot)$ as density of log firm size, then $\gamma_t(z) = e^z g_t(e^z)$. With a change of variable $y = e^z$, we can rewrite $\overline{y}_t$ as:

$$
\begin{aligned}
\bar{y}_t &= \frac{1}{p}\int_{q_t}^{\infty} y g_t(y) dy \\
&= \frac{1}{p}\int_{log q_t}^{\infty} e^z g_t(e^z) e^z dz \\
&= \frac{1}{p}\int_{\chi_t}^{\infty} \gamma_t(z) e^z dz
\end{aligned} \tag{48}
$$

Since γ_t is positive everywhere, the function $\chi \to \int_{\chi}^{\infty} \gamma_t(z)dz$ is strictly decreasing, therefore, $\forall t \geq 0$, there eyists a unique (log) quantile χ_t s.t. $p = \int_{\chi_t}^{\infty} \gamma_t(z)dz$.

In this step, I apply the general formula derived in equation (36) to solve for the dynamics of average size of top percentile p firms, which is

$$
d\bar{y}_t = \frac{1}{p}\int_{q_t}^{\infty} (y - q_t) dg_t(y) dy \tag{49}
$$

Let $h(y) = (y - q_t)^+$, then $h'(y) = 1$ if $y \geq q_t$. Rewrite $d\bar{y}_t$ in terms of $h(y)$ and plug it into Equation (36), we get

$$
\begin{aligned}
d\bar{y}_t =& \frac{1}{p}\int_{q_t}^{\infty} (y - q_t) dg_t(y) dy \\
=& \frac{1}{p}\int_{q_t}^{\infty} h(y) dg_t(y) dy \\
=& \frac{1}{p}\int_{q_t}^{\infty} \mu y g_t(y) dy + \frac{\lambda}{p}\int_{0}^{\infty} [(e^{U_1} y - q_t)^+ - (y - q_t)^+] g_t(y) dy \\
& + \frac{\delta}{p}\int_{0}^{\infty} [(e^{U_2} y - q_t)^+ - (y - q_t)^+] g_t(y) dy \\
=& \frac{1}{p}\int_{q_t}^{\infty} \mu y g_t(y) dy \\
& + \frac{\lambda}{p}\int_{e^{-U_1} q_t}^{\infty} (e^{U_1} y - q_t) g_t(y) dy - \frac{\lambda}{p}\int_{q_t}^{\infty} (y - q_t) g_t(y) dy \\
& + \frac{\delta}{p}\int_{e^{-U_2} q_t}^{\infty} (e^{U_2} y - q_t) g_t(y) dy - \frac{\delta}{p}\int_{q_t}^{\infty} (y - q_t) g_t(y) dy
\end{aligned} \tag{50}
$$

Joining equations (47) and (50), together with $p = \int_{q_t}^{\infty} g_t(y)dy$, we get the law of motion of the average firm size in the top percentile $\overline{y}_t$:

$$\begin{aligned}\frac{d\overline{y}_t}{\overline{y}_t} =&\frac{1}{p\overline{y}_t}\int_{q_t}^{\infty}\mu y g_t(y)dy\\&+\frac{\lambda}{p\overline{y}_t}\int_{e^{-U_1}q_t}^{\infty}(e^{U_1}y-q_t)g_t(y)dy-\frac{\lambda}{p\overline{y}_t}(\overline{y}_t p-q_t p)\\&+\frac{\delta}{p\overline{y}_t}\int_{e^{-U_2}q_t}^{\infty}(e^{U_2}y-q_t)g_t(y)dy-\frac{\delta}{p\overline{y}_t}(\overline{y}_t p-q_t p)\end{aligned} \tag{51}$$

By the choice of the evolving density function $g_t(y)$ that converges to the stationary distribution which follows Pareto distribution with exponent ζ, i.e. Equation (46) reads $g_t(y)=a_0 y^{-\zeta-1}+\varepsilon e^{\Psi(\hat{\zeta})t}y^{-1-\hat{\zeta}}-\varepsilon e^{\Psi(\check{\zeta})t}y^{-1-\check{\zeta}}$, then $\frac{d\overline{y}_t}{\overline{y}_t}$ becomes:

$$\begin{aligned}\frac{d\overline{y}_t}{\overline{y}_t} =&\mu+\frac{a_0\lambda}{p\overline{y}_t}\int_{e^{-U_1}q_t}^{\infty}(e^{U_1}y-q_t)y^{-\zeta-1}dy-\frac{\lambda}{p\overline{y}_t}(\overline{y}_t p-q_t p)\\&+\frac{a_0\delta}{p\overline{y}_t}\int_{e^{-U_2}q_t}^{\infty}(e^{U_2}y-q_t)y^{-\zeta-1}dy-\frac{\delta}{p\overline{y}_t}(\overline{y}_t p-q_t p)\\&+\varepsilon\frac{e^{\Psi(\hat{\zeta})t}\lambda}{p\overline{y}_t}\int_{e^{-U_1}q_t}^{\infty}(e^{U_1}y-q_t)y^{-\hat{\zeta}-1}dy+\varepsilon\frac{e^{\Psi(\hat{\zeta})t}\delta}{p\overline{y}_t}\int_{e^{-U_2}q_t}^{\infty}(e^{U_2}y-q_t)y^{-\hat{\zeta}-1}dy\\&+\varepsilon\frac{e^{\Psi(\check{\zeta})t}\lambda}{p\overline{y}_t}\int_{e^{-U_1}q_t}^{\infty}(e^{U_1}y-q_t)y^{-\check{\zeta}-1}dy+\varepsilon\frac{e^{\Psi(\check{\zeta})t}\delta}{p\overline{y}_t}\int_{e^{-U_2}q_t}^{\infty}(e^{U_2}y-q_t)y^{-\check{\zeta}-1}dy\\=&\mu+\frac{a_0\lambda}{p\overline{y}_t}\frac{e^{U_1\zeta}q_t^{-\zeta+1}}{\zeta(\zeta-1)}-\lambda+\frac{\lambda q_t}{\overline{y}_t}+\frac{a_0\delta}{p\overline{y}_t}\frac{e^{U_2\zeta}q_t^{-\zeta+1}}{\zeta(\zeta-1)}-\delta+\frac{\delta q_t}{\overline{y}_t}\\&+\varepsilon\frac{e^{\Psi(\hat{\zeta})t}\lambda}{p\overline{y}_t}\frac{e^{U_1\hat{\zeta}}q_t^{-\hat{\zeta}+1}}{\hat{\zeta}(\hat{\zeta}-1)}+\varepsilon\frac{e^{\Psi(\hat{\zeta})t}\delta}{p\overline{y}_t}\frac{e^{U_2\hat{\zeta}}q_t^{-\hat{\zeta}+1}}{\hat{\zeta}(\hat{\zeta}-1)}\\&-\varepsilon\frac{e^{\Psi(\check{\zeta})t}\lambda}{p\overline{y}_t}\frac{e^{U_1\check{\zeta}}q_t^{-\check{\zeta}+1}}{\check{\zeta}(\check{\zeta}-1)}-\varepsilon\frac{e^{\Psi(\check{\zeta})t}\delta}{p\overline{y}_t}\frac{e^{U_2\check{\zeta}}q_t^{-\check{\zeta}+1}}{\check{\zeta}(\check{\zeta}-1)}\end{aligned} \tag{52}$$

From the set of equations:

$$\begin{cases}p=\int_{q_t}^{\infty}g_t(y)dy=\frac{a_0}{\zeta}q_t^{-\zeta}+\varepsilon\frac{e^{\Psi(\hat{\zeta})t}}{\hat{\zeta}}q_t^{-\hat{\zeta}}-\varepsilon\frac{e^{\Psi(\check{\zeta})t}}{\check{\zeta}}q_t^{-\check{\zeta}}\\ \overline{y}_t=\frac{1}{p}\int_{q_t}^{\infty}yg_t(y)dy=\frac{1}{p}\left(a_0\frac{q_t^{-\zeta+1}}{\zeta-1}+\varepsilon e^{\Psi(\hat{\zeta})t}\frac{q_t^{-\hat{\zeta}+1}}{\hat{\zeta}-1}-\varepsilon e^{\Psi(\check{\zeta})t}\frac{q_t^{-\check{\zeta}+1}}{\check{\zeta}-1}\right),\end{cases} \tag{53}$$

Plugging (53) into (52) we obtain

$$
\begin{aligned}
\frac{d\overline{y}_t}{\overline{y}_t} =&\mu + \Big(\frac{a_0\lambda e^{U_1\zeta}}{\zeta(\zeta-1)} + \frac{a_0\delta e^{U_2\zeta}}{\zeta(\zeta-1)}\Big)\frac{q_t^{-\zeta+1}}{\frac{a_0 q_t^{-\zeta+1}}{\zeta-1} + \varepsilon\frac{e^{\Psi(\hat{\zeta})t}q_t^{-\hat{\zeta}+1}}{\hat{\zeta}-1} - \varepsilon\frac{e^{\Psi(\check{\zeta})t}q_t^{-\check{\zeta}+1}}{\check{\zeta}-1}} \\
&-(\lambda+\delta)\frac{\frac{a_0 q_t^{-\zeta+1}}{\zeta(\zeta-1)} + \varepsilon\frac{e^{\Psi(\hat{\zeta})t}q_t^{-\hat{\zeta}+1}}{\hat{\zeta}(\hat{\zeta}-1)} - \varepsilon\frac{e^{\Psi(\check{\zeta})t}q_t^{-\check{\zeta}+1}}{\check{\zeta}(\check{\zeta}-1)}}{\frac{a_0 q_t^{-\zeta+1}}{\zeta-1} + \varepsilon\frac{e^{\Psi(\hat{\zeta})t}q_t^{-\hat{\zeta}+1}}{\hat{\zeta}-1} - \varepsilon\frac{e^{\Psi(\check{\zeta})t}q_t^{-\check{\zeta}+1}}{\check{\zeta}-1}} \\
&+\varepsilon\Big(\frac{e^{\Psi(\hat{\zeta})t}\lambda e^{U_1\hat{\zeta}}}{\hat{\zeta}(\hat{\zeta}-1)} + \frac{e^{\Psi(\hat{\zeta})t}\delta e^{U_2\hat{\zeta}}}{\hat{\zeta}(\hat{\zeta}-1)}\Big)\frac{q_t^{-\hat{\zeta}+1}}{\frac{a_0 q_t^{-\zeta+1}}{\zeta-1} + \varepsilon\frac{e^{\Psi(\hat{\zeta})t}q_t^{-\hat{\zeta}+1}}{\hat{\zeta}-1} - \varepsilon\frac{e^{\Psi(\check{\zeta})t}q_t^{-\check{\zeta}+1}}{\check{\zeta}-1}} \\
&+\varepsilon\Big(\frac{e^{\Psi(\check{\zeta})t}\lambda e^{U_1\check{\zeta}}}{\check{\zeta}(\check{\zeta}-1)} + \frac{e^{\Psi(\check{\zeta})t}\delta e^{U_2\check{\zeta}}}{\check{\zeta}(\check{\zeta}-1)}\Big)\frac{q_t^{-\check{\zeta}+1}}{\frac{a_0 q_t^{-\zeta+1}}{\zeta-1} + \varepsilon\frac{e^{\Psi(\hat{\zeta})t}q_t^{-\hat{\zeta}+1}}{\hat{\zeta}-1} - \varepsilon\frac{e^{\Psi(\check{\zeta})t}q_t^{-\check{\zeta}+1}}{\check{\zeta}-1}} \\
=&\mu + \frac{\lambda e^{U_1\zeta}}{\zeta} - \frac{\lambda}{\zeta} + \frac{\delta e^{U_2\zeta}}{\zeta} - \frac{\delta}{\zeta} + O(e^{\Psi(\hat{\zeta})t})
\end{aligned}
\tag{54}
$$

where the last term $O(e^{\Psi(\hat{\zeta})t})$ denotes a function that is bounded by $Ce^{\Psi(\hat{\zeta})t}$ for some $C > 0$. By the choice $\Psi(\hat{\zeta}) < 0$, this term decays to 0 exponentially fast as $t \to \infty$. Therefore, it becomes as small as possible when t is large enough.

As $t \to \infty$,

$$\lim_{t\to\infty}\frac{d\overline{y}_t}{\overline{y}_t} = \lim_{t\to\infty}\zeta^{-1}\Psi(\zeta) + O(e^{\Psi(\hat{\zeta})t}) = 0,$$

since ζ is the root to $\Psi(\zeta) = 0$ and $\lim_{t\to\infty} O(e^{\Psi(\hat{\zeta})t}) = 0$, which characterizes the stationary case.

With a little abuse of notation, I now expand $d\overline{y}_t$ as $\frac{d\overline{y}_t}{dt}$ and further elaborate on the transition dynamics depicted by Equation (54). The within term is the average growth of firms in the top percentile holding constant the composition of top firms, which is equal to $[\mu + \lambda(e^{U_1} - 1) + \delta(e^{U_2} - 1)]dt$. Therefore, we can separate the growth of the average firm size into a within term and a displacement term which comes as the residual from the total term minus the within term:

$$
\begin{aligned}
\frac{d\overline{y}_t}{\overline{y}_t} =& \mu dt + \lambda \frac{e^{U_1\zeta}-1}{\zeta} dt + \delta \frac{e^{U_2\zeta}-1}{\zeta} dt + O(e^{\Psi(\hat{\zeta})t}) dt \\
=& \underbrace{\mu dt + \Big[\lambda(e^{U_1}-1) + \delta(e^{U_2}-1)\Big] dt}_{\text{Within}} \\
& + \underbrace{\left[\lambda \frac{e^{U_1\zeta}-1}{\zeta} + \delta \frac{e^{U_2\zeta}-1}{\zeta} - \lambda(e^{U_1}-1) - \delta(e^{U_2}-1)\right]}_{\text{Displacement}} dt + \underbrace{O(e^{\Psi(\hat{\zeta})t})}_{\text{error term}} dt
\end{aligned}
\tag{55}
$$

Let $G_\lambda = e^{U_1} - 1$ and $G_\delta = -(e^{U_2} - 1)$, (i.e., the absolute value of the negative jump size), following the law of motion in Proposition 3, we can further simplify Equation (55) into:

$$
\begin{aligned}
\frac{d\overline{y}_t}{\overline{y}_t} =& \underbrace{(\mu + \lambda G_\lambda - \delta G_\delta)\, dt}_{\text{within}} \\
& + \underbrace{\underbrace{\lambda \left(\frac{(1+G_\lambda)^\zeta - 1 - \zeta G_\lambda}{\zeta} \right) dt}_{\text{inflow}} + \underbrace{\delta \left(\frac{(1-G_\delta)^\zeta - 1 + \zeta G_\delta}{\zeta} \right) dt}_{\text{outflow}}}_{\text{displacement}} \\
& + \underbrace{O(e^{\Psi(\hat{\zeta})t})}_{\text{error term}} dt
\end{aligned}
\tag{56}
$$

dN_{1t} is the poisson process for the positive shock to firm size, it corresponds to the inflow term which is the contribution by firms entering the top after experiencing positive shocks. And dN_{2t} is the poisson process for the negative shock to firm size, it corresponds to the outflow term which is the contribution by firms exiting the top after experiencing negative shocks.

Equation (56) approximates the asymptotic behavior of the transition dynamics of the average firm size in the top percentile, up to an error term that is arbitrarily small as $t \to \infty$.

Therefore, equation (56) gives Proposition 3. Q.E.D.

D Proof of Proposition 4

Turnover rate (Inflow) is calculated as the ratio of the number of firms that enter the top percentile p from $t-dt$ to t to the number of firms that are inside the top p percentile at time t. Under the impact of own innovation shock, i.e., positive jump process, when time goes from $t-dt$ to t, the threshold for the top p percentile increases from $q_t/(1+G_\lambda)$ to q_t with probability λ, we can write turnover rate due to inflow as:

$$
\begin{aligned}
&\lambda\frac{\int_{q_t/(1+G_\lambda)}^{\infty} g_t(y)dy - \int_{q_t}^{\infty} g_t(y)dy}{\int_{q_t}^{\infty} g_t(y)dy} \\
=&\lambda\frac{\int_{q_t/(1+G_\lambda)}^{q_t}[a_0 y^{-\zeta-1} + \varepsilon e^{\Psi(\hat{\zeta})t} y^{-1-\hat{\zeta}} - \varepsilon e^{\Psi(\check{\zeta})t} y^{-1-\check{\zeta}}]dy}{\int_{q_t}^{\infty}[a_0 y^{-\zeta-1} + \varepsilon e^{\Psi(\hat{\zeta})t} y^{-1-\hat{\zeta}} - \varepsilon e^{\Psi(\check{\zeta})t} y^{-1-\check{\zeta}}]dy} \\
=&\lambda\left((1+G_\lambda)^{\zeta}-1\right) + O(e^{\Psi(\hat{\zeta})t})
\end{aligned}
\tag{57}
$$

where the last term $O(e^{\Psi(\hat{\zeta})t})$ denotes a function that is bounded by $C_1 e^{\Psi(\hat{\zeta})t}$ for some $C_1 > 0$. By the choice $\Psi(\hat{\zeta}) < 0$, this term decays to 0 exponentially fast as $t \to \infty$. Therefore, it becomes as small as possible when t is large enough.

Turnover rate (Outflow) is calculated as the ratio of the number of firms that exit the top percentile p from $t-dt$ to t to the number of firms that are inside the top p percentile. The creative destruction shock, i.e., negative jump process, maintains a higher threshold of the top percentile by removing firms with sales no longer above the threshold from the top. When time goes from $t-dt$ to t, quantile q_{t-dt} increases to $q_{t-dt}/(1-G_\delta)$ with probability δ. Therefore, we can write turnover rate due to outflow as:

$$
\begin{aligned}
&\delta\frac{\int_{q_{t-dt}}^{\infty} g_t(y)dy - \int_{q_{t-dt}/(1-G_\delta)}^{\infty} g_t(y)dy}{\int_{q_{t-dt}}^{\infty} g_t(y)dy} \\
=&\delta\frac{\int_{q_{t-dt}}^{q_{t-dt}/(1-G_\delta)}[a_0 y^{-\zeta-1} + \varepsilon e^{\Psi(\hat{\zeta})t} y^{-1-\hat{\zeta}} - \varepsilon e^{\Psi(\check{\zeta})t} y^{-1-\check{\zeta}}]dy}{\int_{q_{t-dt}}^{\infty}[a_0 y^{-\zeta-1} + \varepsilon e^{\Psi(\hat{\zeta})t} y^{-1-\hat{\zeta}} - \varepsilon e^{\Psi(\check{\zeta})t} y^{-1-\check{\zeta}}]dy} \\
=&\delta\left(1-(1-G_\delta)^{\zeta}\right) + O(e^{\Psi(\hat{\zeta})t})
\end{aligned}
\tag{58}
$$

where the last term $O(e^{\Psi(\hat{\zeta})t})$ denotes a function that is bounded by $C_2 e^{\Psi(\hat{\zeta})t}$ for some $C_2 > 0$. By the choice $\Psi(\hat{\zeta}) < 0$, this term decays to 0 exponentially fast as $t \to \infty$. Therefore, it becomes as small as possible when t is large enough.

This gives Proposition 4. Q.E.D.