Firm Dynamics and Innovation: Evidence from Decomposing Top Sales Shares

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What do changes in top sales shares signal about changes in firm dynamics? I use an accounting decomposition to identify two sources of top sales shares growth: (i) incumbent top firms grow bigger; (ii) new top firms replace old top firms. Over the 1950-2019 period, incumbent top firms contribute about 3.5 times as much as new top firms to the growth of sales share accrued to the top 0.01% firms in the US economy. I then build an analytical framework to estimate a firm dynamics process in which firms grow in response to an own innovation shock and shrink at the impact of a creative destruction shock using the empirical decomposition terms of top sales shares growth. I find that own innovation is the major force that drives top sales shares growth. The implied aggregate productivity growth is highest over the 1980-1995 period. The 1995-2010 period and the 2010-2019 period feature a lower productivity growth due to the decline in both own innovation and creative destruction rates over time.

1 Introduction

Recently there has been a revival of interest in the rise of large firms in the economy. Autor et al. (2020) documents that leading firms have captured an increasing share of industry sales during the 1990-2010 period and this leads to the rise of "superstar firms" which can explain several important recent macroeconomic trends. As top firms amass a larger share of total sales, what does it signal about the underlying firm dynamics process that drives the rise and fall of large firms? What do changes in the top sales shares imply about aggregate productivity growth?

In this paper, I answer these questions by separately accounting for two potential sources of the growth of top sales shares: (i) incumbent top firms grow bigger; (ii) new top firms replace old top firms. These two sources of top sales share growth connect naturally to the underlying forces that make firms grow and shrink. Intuitively, forces that drive the growth of incumbent top firms increase the first component of top sales share growth, whereas forces that make incumbent top firms shrink will dampen the growth effect by existing top firms. By contrast, as destructive forces make incumbent top firms shrink, they create opportunities for lower-ranked firms to surpass existing top firms, thereby promoting top sales shares growth via its second component.

Taking advantage of the mechanism that forces that make firms grow or shrink have differential impact on the two components of top share growth, I infer the underlying process that drives firm dynamics based on the decomposition of top sales share growth. Depending on whether the rise in top sales share comes from growth of incumbent top firms or new top firms displacing old top firms, different underlying processes that drive the rise and fall of large firms may be at work.

I start by applying the accounting framework developed in Gomez (2020) to decompose the growth of sales shares accrued to the top 0.01% firms (henceforth top sales share) of the US economy over the 1950-2019 period. I focus on two key components of the growth of top sales shares from this decomposition. The first term, the *within* term, is the contribution to top sales shares growth by firms that belong to the top percentile at the beginning of period, whether or not they remain in the top by the end of period (i.e., holding constant the composition of top firms). The second term, the *displacement* term, measures the effect of compositional change of top firms on top sales shares growth.

During the whole sample period (1950-2019), the within term is about 3.5 fold the magnitude of the displacement term. That is, the growth of top sales shares is to a large extent due to incumbent top firms growing bigger. However, the displacement of old top firms by new top firms also makes important positive contributions to the growth of top sales shares.

Equipped with the empirical decomposition results, I estimate the underlying firm dynamics process which features a positive shock that makes a firm grow and a negative shock that makes a firm shrink. For instance, these shocks can be interpreted as innovation shocks that take place in different forms. The positive shock can be viewed as an "own innovation" shock in response to which firms improve on their own products and grow larger. In contrast, the negative shock can be viewed as a "creative destruction" shock which occurs to a firm when its product is improved upon by its competitor.

Through the lens of a firm random growth model in which the dynamics of individual firms are subject to these two shocks, I obtain closed-form formulas for the components of top sales shares growth that can be mapped to the within term and the displacement term obtained from the empirical decomposition. This mapping between the theoretical and empirical components of the top sales share growth allows me to estimate the underlying process that drives the dynamics of large firms.

With regard to the dynamic process that drives the rise and fall of large firms, I find that prior to 1980, the rate at which large firms are hit by own innovation shocks is low. However, when the shocks occur, they make large firms grow at a large step size. In contrast, in the post 1980s, the rate of own innovation is much higher with a smaller step size. The rate of creative destruction has been declining from 16% prior to the 1980s to 10% in more recent years. However, the step size at which firms shrink at the impact of creative destruction shocks has become larger over the years.

Across periods, increases in own innovation plays a prominent role in the rise of top sales share growth, while changes in creative destruction can offset a small amount of top sales share growth. In addition, I find that the implied aggregate productivity growth is highest over the 1980-1995 period when both own innovation and creative destruction rates are fairly high.

Related literature. Existing studies have documented that from the late 1990s to early 2010s, top firms have captured an increasing proportion of industry sales. For instance, Dorn et al. (2017) and Autor et al. (2020) report an upward trend in the sales share accrued to top 4 and top 20 firms across four-digit industries by major sector.

I extend the analysis to a longer time horizon covering the 1950-2019 period and document the evolution of top sales shares at the aggregate level by measuring the sales share accrued to the top 0.01% firms in the entire economy. Beyond the year-by-year snapshots depicted by the overall time series, I add a dynamic perspective to the top sales share by taking into account the entry/exit of firms in and out of the top percentile via tracking a panel of top firms.

My study connects to a growing literature that seeks to understand the forces behind the rise of large firms. One prevalent view in this literature is that star firms result from the reallocation of market share to more productive firms. This reallocation can operate via several channels: (i) network effect or platform competition foster a "winner-takes-most" market structure (Van Reenen (2018); Autor et al. (2020)); (ii) rising importance of intangible capital that benefits large firms disproportionately (Crouzet and Eberly (2018); Haskel and Westlake (2017)); (iii) technology change that favors large firms more either via a large fixed cost that deters small competitors or a marginal benefit that rise with scale (e.g.: Information and Communication Technology (ICT) (Bessen (2017); Aghion et al. (2019)), digital capital(Tambe et al. (2020)) and Artificial Intelligence (AI) (Babina et al. (2020))); (iv) increasing levels of product differentiation that can result from better search technology and the pervasive power of data analytics (Calligaris et al. (2018)).

The literature remains unclear on whether the above-mentioned economic forces that favors large firms disproportionately will make existing large firms more entrenched or launch new superstar firms. I contribute to these work by examining the forces that drive the rise and fall of large firms in different periods when different technological environments are in place.

My paper also connects to the growth literature that emphasizes different forms of innovation. Own innovation and creative destruction are two major innovation forms that appear in growth models either separately or jointly. [1]

To determine the extent to which growth comes from own innovation as opposed to creative destruction, previous studies have directly sought to observe the substitution of existing products by new products (Christensen (2013), Broda and Weinstein (2010) and Hottman et al. (2016)) or looked at patent citations by same firm or other firm and breakthrough patents (Akcigit and Kerr (2018) and Kelly et al. (2021)). From an indirect perspective, through the lens of an exogenous growth model in

[1]Classic examples of creative destruction include Stokey (1988); Grossman and Helpman (1991); Aghion and Howitt (1990); Klette and Kortum (2004). Theories focusing on own innovation include Krusell (1998); Lucas Jr and Moll (2014). Some models combine creative destruction and own innovation (e.g., Akcigit and Kerr (2018)).

which compared to own innovation, creative destruction leaves a more polarized job creation/destruction rate in the data, Garcia-Macia et al. (2019) and Klenow and Li (2020) infer which forms of innovation play a dominant role on aggregate growth.

I also take the indirect approach to infer on the innovation process that drives firm dynamics. Relative to the existing studies, rather than using employment moments which focus more on smaller firms and new firms into the economy, my inference differs by focusing on moments of the right tail of firm size distribution such as components of top share change resulting from compositional changes of top firms and turnover rate due to entry and exit of firms into the top percentile. A significant amount of innovation are conducted by large firms and their products are also subject to challenges from innovative competitors. As a result, disciplining the firm innovation process with moments of top sales share can provide a new angle in comparison to focusing on moments generated by firms at the lower end of firm size distribution.

By bridging changes in concentration to changes in aggregate productivity via the underlying innovation processes, my paper is related to previous work on innovation and productivity growth. The literature on the dynamics of U.S. productivity seeks to explain its surge in the 1990s to early 2000s and the subsequent slowdown (Sparque (2021)). Potential factors that has been a drag on aggregate productivity include: (1) decline in job reallocation due to weaker responsiveness of businesses to productivity shocks (Decker et al. (2020)); (2) decline in competition (Grullon et al. (2019)); (3) decline in technological diffusion between frontier firms and laggard firms (Andrews et al. (2015)); (4) diminishing returns to innovation (Bloom et al. (2020)). Innovation can prompt a surge in entry of new firms which first increases productivity dispersion and then leads to productivity growth (Foster et al. (2018)). However, the connection between innovation and productivity growth maybe more subtle since it may lead to a burst in productivity growth at first but with a subsequent productivity slowdown (Klenow et al. (2019)). Relative to this literature, I explore on whether changes in different forms of innovation account for changes in aggregate productivity. Existing studies have shown that, aside from own innovation forces which make existing firms grow, creative destruction also contribute to aggregate productivity growth via resource reallocation (Fujita et al. (2008); Foster et al. (2006) and Petrin et al. (2011)). Equipped with rates of own innovation and creative destruction calibrated from components of top share change and viewing through a firm random growth model, I infer on the relative contribution to productivity growth by own innovation as opposed to creative destruction.

I study concentration from the evolution and compositional changes of firms that belong to the top percentile of the whole economy. This connects to studies on the characteristics of star firms and how long they stay as top firms. Large firms may escape the Schumpeterian process of destruction and remains large via exploiting their political connections to stifle entry, an idea tracing back to earlier works by Steffens (1906), carried through by Stigler (1971), revived recently in empirical work by Faccio and McConnell (2020). There is also recent evidence that large incumbents impede innovation of their rival startups through acquisition (Kamepalli et al. (2020); Cunningham et al. (2021)). A separate strand of literature approach the question of leadership turnover from an industrial organization (IO) perspective. Papers in this category feature game theoretical elements on the strategic behavior of one or two market leaders and examine how their gap in market share evolves (e.g., Sutton (2007); Dou et al. (2020); Liu et al. (2019)). My work complements these two strands of literature by examining entry/exit rate into a group of top firms of the whole economy (rather than change of leadership for a few firms at the industry level) and comparing among firms that enter, exit and stay at the top.

2 Data

Top sales shares. Sales data of firms with headquarter in the US are from *Compustat*. Total sales are proxied by the gross output of all industries from Bureau of Economic Analysis (BEA). [2] Gross output has not been released by the BEA yet, therefore I impute the gross output in 2020 from the gross output in 2019 and the net GDP growth rate from 2019 to 2020 [3]. The total number of firms in 1978-2018 are from Census Business Dynamics Statistics (BDS) (TableID: BDSNAICS). Number of firms in 1950-1977 are imputed by assuming that the number of firms is a constant fraction of population size [4], where the firm number-to-population ratio is calculated as the average firm number-to-population ratio between 1978 to 1982. Number of firms in 2019 is imputed from the number of firms in 2018 and the growth rate of firm number from 2017 to 2018. With the pandemic outbreak in 2020, I assume the firm number does not grow and remains the same as 2019. Sales and gross output are deflated using US GDP deflator line 1 from NIPA Table 1.1.9.

Merger and acquisition. The merger and acquisition (M&A) data comes from the SDC platinum database. I keep the M&A deals whose status have been completed, remove the deals in which the target and the acquirer have the same name or CUSIP to avoid counting in the share buybacks by the same firm, and keep the deals after which the acquirer owns 100% share of the target. I merge this M&A dataset with the Compustat firm sales data to label firms in the top 0.01% which have been involved in M&A transactions each year, that is, a top firm that is either an acquirer or a target in an M&A deal effective in that year. Since there is no common firm identifier

[2] Source: https://www.bea.gov/industry/io-histannual and https://apps.bea.gov/iTable/iTable.cfm?reqid=150&step=2&isuri=1&categories=ugdpxind

[3] source: https://fred.stlouisfed.org/series/GDPA

[4] Source: http://www.demographia.com/db-uspop1900.htm

to directly merge between the SDC platinum database and the Compustat firms, I generate the matching in a number of ways: (1) use deal number (in SDC platinum) to acquirer and target GVKEY (in Compustat) crosswalk constructed from Phillips and Zhdanov (2013) and Ewens et al. (2019) [5]; (2) Combine the match with CUSIP (6-digit in SDC platinum and 9-digit in Compustat) and the fuzzy name match to crosswalk the deals without deal number in SDC platinum into Compustat.

Firm-level data. I use Fama-French 48 industry definition to classify industries [6]. It can be matched to Compustat database via crosswalk between the 4-digit SIC code in Compustat and Fama French 48 industry code [7]. Firm-level patent value is from Kogan et al. (2017) [8]. Markup of firms is measured following De Loecker et al. (2020). Variables such as sales, assets, costs of goods sold come from accounting data from Compustat database. The firm patent value data from Kogan et al. (2017) is matched with firm accounting data using the "PERMNO" firm identifier in CRSP-Compustat Merged (CCM) database.

3 Three facts about top sales shares

In this section, I document three empirical facts on concentration in the upper tail of firm size distribution measured by the share of sales accrued to the top 0.01% firms (henceforth "top sales share") in the US economy over the 1950-2020 period. Starting from the year-by-year static snapshots of top sales shares, these facts further examine the evolution of top sales shares from the perspective of large firm dynamics by taking into account the entry and exit of firms into the top 0.01% percentile.

[5] Source: https://github.com/michaelewens/SDC-to-Compustat-Mapping

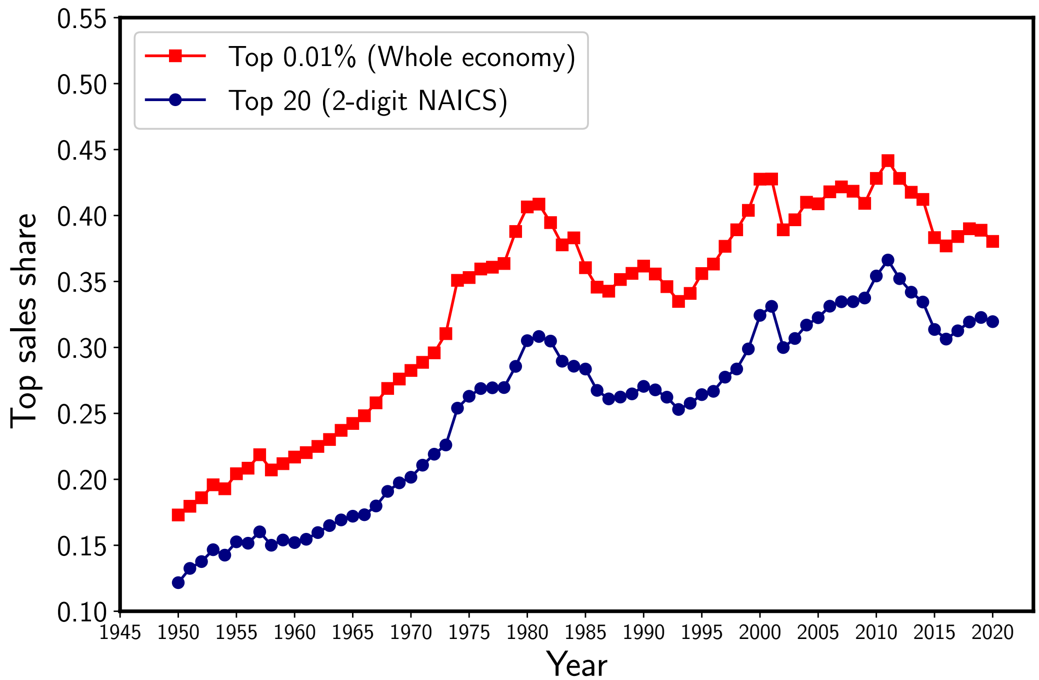
[6] Source: https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/Data_Library/det_48_ind_port.html

[7] Source: the ffind.ado file from https://sites.google.com/site/judsoncaskey/data

[8] Source: https://github.com/KPSS2017/Technological-Innovation-Resource-Allocation-and-Growth-Extended-

Fact 1: The sales share accrued to the top 0.01% firms of the US economy has doubled from 17% in 1950 to around 35% in 2020. Over the 1950-1980 period, the top sales share has been on a steady upward trend. In the post-1980s, the trend has become flatter and exhibits more fluctuations, with two declining phases (1980-1995 and 2010-2020) and an increasing phase (1995-2010).

Figure 1: Aggregate and within-industry top sales share



Note: "Top 0.01% (Whole economy)" plots the sales share accrued to the top 0.01% firms of the US economy. "Top 20 (2-digit NAICS)" plots the average sales share of the top 20 firms of 2-digit NAICS industries (weighted by industry sales). Data are from *Compustat* (sales of top firms), BEA (gross output of all industries) and Census BDS (TableID: BDSNAICS).

To construct the top sales shares [9], I first obtain the total number of firms in the U.S. from the Census Business Dynamics Statistics (BDS). Depending on the total number

[9] Section 2 provides more details on the construction of top sales shares.

of firms in the US economy, the top 0.01% group include from a range of 246 firms in 1950 to 532 firms in 2020. I obtain the sales of firms in this top 0.01% group from *Compustat*. Since *Compustat* only contains sales data of firms that are publicly traded, in doing so, I make the assumption that all firms that are ranked in the top 0.01% by sales are public firms. The total sales of all firms in the US economy are proxied by the gross output of all industries from the Bureau of Economic Analysis (BEA). By assembling the above pieces of data, I can calculate the top sales share as the ratio of the sum of sales of the top 0.01% firms to the total sales of all firms in the US.

During the entire sample period (1950-2020), the top sales share has doubled from 17% in 1970 to 37% in 2020. A closer examination of Figure 1 shows that the top sales share starts low at 17% in 1950, has risen steadily to 40% in 1980, then declines and remains fairly low around 35% over the 1985-1995 period. Starting from the late 1990s, the top sales share rises from 35% to a historical peak at 44% in 2011, then drops to 37% in 2020.

Fact 2: The aggregate trend of top sales share reflects within industry changes.

One question that arises from Fact 1 is whether the aggregate trend reflects changes in the top sales share that have happened within industries or it stems from across-industry effects where top firms in some industries are capturing more sales relative to those in other industries. Figure 1 shows that the sales-weighted average of the within-industry top 20 firms sales share tracks well the aggregate trend of the sales share accrued to the top 0.01% firms of the US economy. This indicates that the aggregate trend in the top sales shares mainly reflects within-industry top sales share changes rather than across-industry effects.

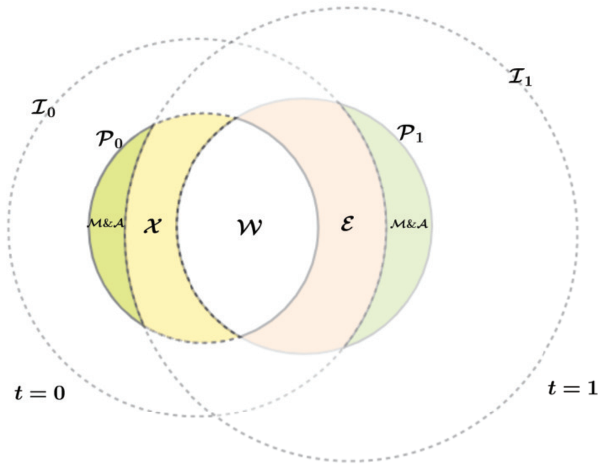
The broad-based rise of top firm sales share at the industry level from the late 1990s to the early 2010s echos the findings from the existing studies that the average sales share accrued to top 4 and top 20 firms across four-digit industries by major sector has risen during the same period (Dorn et al. (2017); Autor et al. (2020)).

Fact 3: The growth of the top 0.01% firms sales shares can come from: (i) incumbent top firms grow bigger; (ii) new top firms replace old top firms.

The top 0.01% percentile does not comprise the same firms over time. The rise and fall of large firms can lead to compositional changes of firms that belong to the top 0.01% group. As a result, both changes in the share of firms that remain inside the top and changes due to the entry/exit of firms into the top can contribute to changes in the top sales share. To understand what is the major driving force behind the aggregate trend of top sales shares and how that evolves over time, I apply an accounting decomposition framework which is developed in Gomez (2020) to decompose the changes in sales share of top firms.

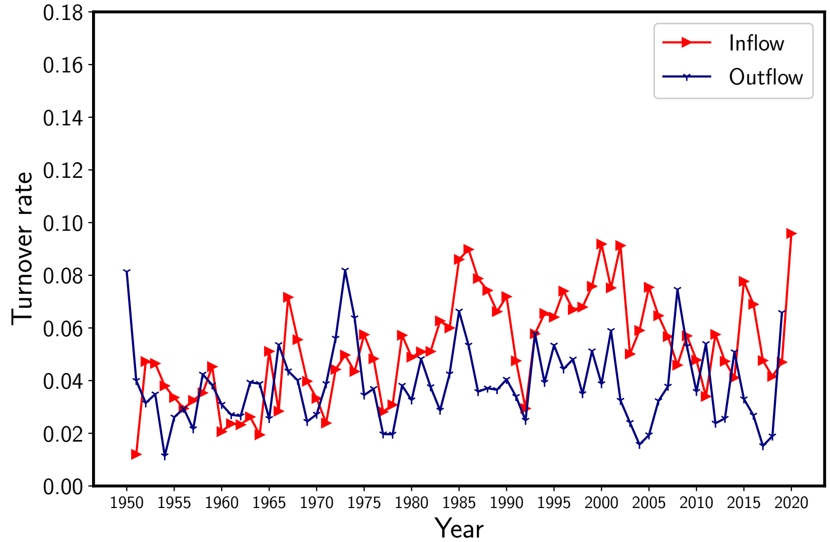
Accounting Framework. To illustrate the accounting framework, one can start by considering the different sets of firms that comprise the top percentile firms in two periods. As demonstrated in the Venn diagram (Figure 2), consider a given top sales percentile $p \in (0, 1]$ and denote $\mathcal{P}_t \subset \mathcal{I}_t$ the subset of firms who are in the top percentile at time t ($t \in \{0, 1\}$). Denote $\mathcal{W}$ the set of firms who remain in the top percentile in both periods. Let $\mathcal{X}$ be the set of firms who exit the top percentile but remains in the economy at $t = 1$ and $\mathcal{E}$ the set of firms who are in the economy at $t = 0$ and enter the top percentile at $t = 1$. Top firms who are targets or new firms that emerge from merger and acquisition (M&A) transactions also lead to compositional changes in the top firms and I group them as the $\mathcal{M}\&\mathcal{A}$ sets. Let q_t be the sales of the firm at the percentile threshold (i.e., the top p quantile).

Figure 2: Venn Diagram Representing Composition Changes in the Top Percentile



Firms in the sets $\mathcal{X}$ and $\mathcal{E}$ contribute to the compositional change effect that new top firms displace old top firms. In particular, this effect represents that lower-ranked firms surpass old top firms via organic growth rather than mechanical forces such as M&A. Figure 3 plots the fraction of top firms that belongs to the set $\mathcal{E}$ (Inflow) and the set $\mathcal{X}$ (Outflow) and show that around 2-8% of top firms each year enter and exit the top and contribute to this displacement effect.

Figure 3: Turnover rate (Inflow and Outflow)



For a set of firms $\mathcal{G}$, let $|\mathcal{G}|$ be the number of firms in $\mathcal{G}$ and $\overline{y}_{\mathcal{G},t}$ be the average sales of firms in $\mathcal{G}$ at time t. Then the top percentile's sales share $\mathcal{S}_t$ can be expressed as:

$$\mathcal{S}_t = \frac{\sum\limits_{i \in \mathcal{P}_t} y_{it}}{\sum\limits_{i \in \mathcal{I}_t} y_{it}} = \frac{\sum\limits_{i \in \mathcal{P}_t} \frac{y_{it}}{\overline{y}_{\mathcal{I}_t}}}{\sum\limits_{i \in \mathcal{I}_t} \frac{y_{it}}{\overline{y}_{\mathcal{I}_t}}} = \frac{|\mathcal{P}_t|\overline{u}_{\mathcal{P}_t}}{|\mathcal{I}_t|} = p\overline{u}_{\mathcal{P}_t} \tag{1}$$

where $\overline{u}_{\mathcal{P}_t} = \frac{\sum_{i \in \mathcal{P}_t} \frac{y_{it}}{\overline{y}_{\mathcal{I}_t}}}{|\mathcal{P}_t|}$. Therefore, $\frac{\mathcal{S}_t}{\mathcal{S}_{t-1}} = \frac{\overline{u}_{\mathcal{P}_t}}{\overline{u}_{\mathcal{P}_{t-1}}}$, that is, when firm sales is normalized by the average sales of all firms, then the growth of top sales share is equal to the growth of the average of *normalized* sales of the top percentile p.

Proposition 1. *The growth of the average sales in a top percentile between time t = 0 and t = 1 can be decomposed as follows:*

$$
\underbrace{\frac{\overline{y}_{\mathcal{P}_{1,1}} - \overline{y}_{\mathcal{P}_{0,0}}}{\overline{y}_{\mathcal{P}_{0,0}}}}_{\text{Change in top share}} = \underbrace{\left(\frac{\overline{y}_{\mathcal{P}_0 \setminus \mathcal{D},1} - \overline{y}_{\mathcal{P}_0 \setminus \mathcal{D},0}}{\overline{y}_{\mathcal{P}_0 \setminus \mathcal{D},0}}\right)}_{\text{Within}}
$$

$$
+ \underbrace{\underbrace{\frac{|\mathcal{E}|}{|\mathcal{P}_1|}\left(\frac{\overline{y}_{\mathcal{E},1} - q_1}{\overline{y}_{\mathcal{P}_0,0}}\right)}_{\text{Inflow}} + \underbrace{\frac{|\mathcal{X}|}{|\mathcal{P}_1|}\left(\frac{q_1 - \overline{y}_{\mathcal{X},1}}{\overline{y}_{\mathcal{P}_0,0}}\right)}_{\text{Outflow}}}_{\text{Displacement}} \tag{2}
$$

$$
+ \text{M\&A term} + \Delta \text{ Firm No. term.}
$$

The M&A term and the Firm number change term can be further expanded into:

$$
\text{M\&A term} + \Delta \text{ Firm No. term} = \underbrace{\underbrace{\left(\frac{\overline{y}^{MA}_{\mathcal{P}_0 \setminus \mathcal{D},1}}{\overline{y}_{\mathcal{P}_0 \setminus \mathcal{D},0}}\right)}_{\text{Within M\&A}} + \underbrace{\frac{|\mathcal{E}|}{|\mathcal{P}_1|}\left(\frac{\overline{y}^{MA}_{\mathcal{E},1}}{\overline{y}_{\mathcal{P}_0,0}}\right)}_{\text{Inflow M\&A}} + \underbrace{\frac{|\mathcal{B}|}{|\mathcal{P}_1|}\left(\frac{\overline{y}_{\mathcal{B},1} - q_1}{\overline{y}_{\mathcal{P}_0,0}}\right)}_{\text{Birth M\&A}}}_{\text{M\&A growth}}
$$

$$
- \underbrace{\frac{|\mathcal{D}|}{|\mathcal{P}_1|}\left(\frac{\frac{\overline{y}_{\mathcal{P}_0 \setminus \mathcal{D},1}}{\overline{y}_{\mathcal{P}_0 \setminus \mathcal{D},0}}\overline{y}_{\mathcal{D},0} - q_1}{\overline{y}_{\mathcal{P}_0,0}}\right)}_{\text{M\&A shrink}}
$$

$$
- \underbrace{\frac{|\mathcal{P}_1| - |\mathcal{P}_0|}{|\mathcal{P}_1|}\left(\frac{\frac{\overline{y}_{\mathcal{P}_0 \setminus \mathcal{D},1}}{\overline{y}_{\mathcal{P}_0 \setminus \mathcal{D},0}}\overline{y}_{\mathcal{P}_0,0} - q_1}{\overline{y}_{\mathcal{P}_0,0}}\right)}_{\text{Firm number change}} \tag{3}
$$

Discuss each term. The within term measures the average growth of firms from $t = 0$ to $t = 1$ in the set $\mathcal{P}_0 \setminus \mathcal{D}$ which are the firms that are in the top percentile at $t = 0$ and do not exit the economy at $t = 1$. This term can be either positive or negative. The displacement term accounts for the change in total top share growth induced by the compositional change of the top percentile between $t = 0$ and $t = 1$. It can be further separated into an inflow term and an outflow term: the inflow term captures the effect of new entrants into the top percentile displacing firms that are close to the percentile threshold and the outflow term reflects that firms exiting the top percentile are replaced by firms just below the percentile threshold. The displacement term is always positive.

Firms who are not in the economy at $t = 0$ may emerge at $t = 1$ as top firms as a consequence of breakups, spinoffs or M&As of existing top firms. This positive effect is accounted for in the M&A growth term. Firms that are at the top percentile at $t = 0$ may exit the economy at $t = 1$, potentially due to merger and acquisition, public firms going private, etc. This negative effect is summarized in the M&A shrink term. Lastly, the total number of firms in the economy changes over time. An increase in the number of firms will add more firms to the top percentile. Since the firms that enter the top due to an expansion in the number of firms have less sales than the existing firms at the top, the firm number change term is negative when there is an increase in the total firm number. Except for the 2008 financial crisis period, firm number has always been on the rise. Therefore, the firm number change term is negative most of the time.

Decompose top shares in data. I apply the accounting framework (Equation (2)) to decompose the annual growth of the average normalized sales of top 0.01% firms in the US economy over the 1950-2019 period. By Equation (1), this is equivalent to decomposing the growth of top sales share.

Table 1 presents each term geometrically averaged over each period. During the whole sample period, the total 1.13% annual growth is the sum of a within term equal to 1.22%, a displacement term equal to 0.35%, a M&A term equal to 0.36% and a firm number change term equal to -0.81%. During the whole sample period (1950-2019), the within term is about 3.5 fold the magnitude of the displacement term. That is, the growth of top sales shares is to a large extent due to incumbent top firms growing bigger. However, the displacement of old top firms by new top firms also makes important positive contribution to the growth of top sales shares.

The contribution to the total top share growth by the within term and the displacement term vary across periods. The within term is high at 2.95% over the 1950-1980 period. Turning into the 1980-1995 period, the within term is negative (-0.37%) and the positive change in top sales shares is mainly driven by the displacement term. Since this positive contribution (0.37%) is less than the negative contributions from the residual term (-0.71%), the total top sales shares are declining during this period. Over the 1995-2010 period, the displacement term peaks at 0.43% and the within term has climbed to 1.05%. The rise in the top sales share during this period comes from increases in both the within term and the displacement term.

Table 1: Decompose annual growth of top 0.01% firms sales share

Period	Total(%)	Within(%)	Displacement(%)			Residual (%)			
			Total	Inflow	Outflow	Total	M&A Growth	M&A Shrink	Δ Firm No. (%)
1950-2019	1.13	1.22	0.35	0.22	0.13	-0.44	1.32	-0.96	-0.81
1950-1980	2.82	2.95	0.33	0.23	0.1	-0.46	0.67	-0.12	-1
1980-1995	-0.71	-0.37	0.37	0.21	0.17	-0.71	1.56	-1.21	-1.07
1995-2010	1.35	1.05	0.43	0.25	0.19	-0.13	2.34	-2.17	-0.33

Note: Table shows the geometric average of the growth rate of top 0.01% firms sales share, the within, displacement, M&A and firm number change terms. Data are from Census BDS, BEA, *Compustat* and *SDC Platinum* databases.

Figure 4: Decomposing Annual Growth of top sales shares

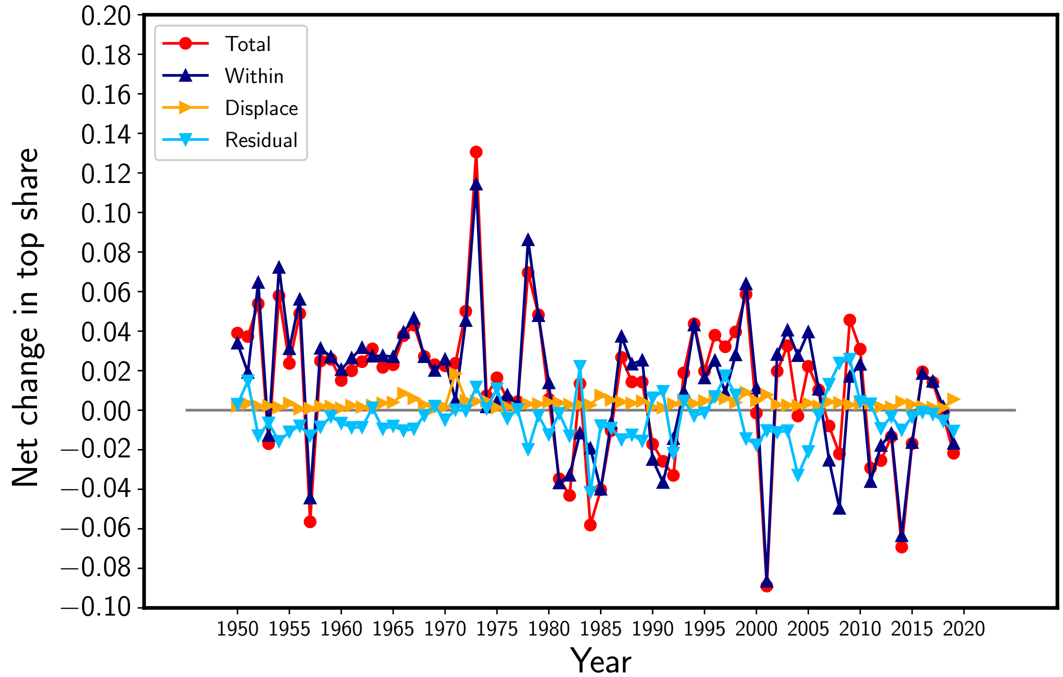
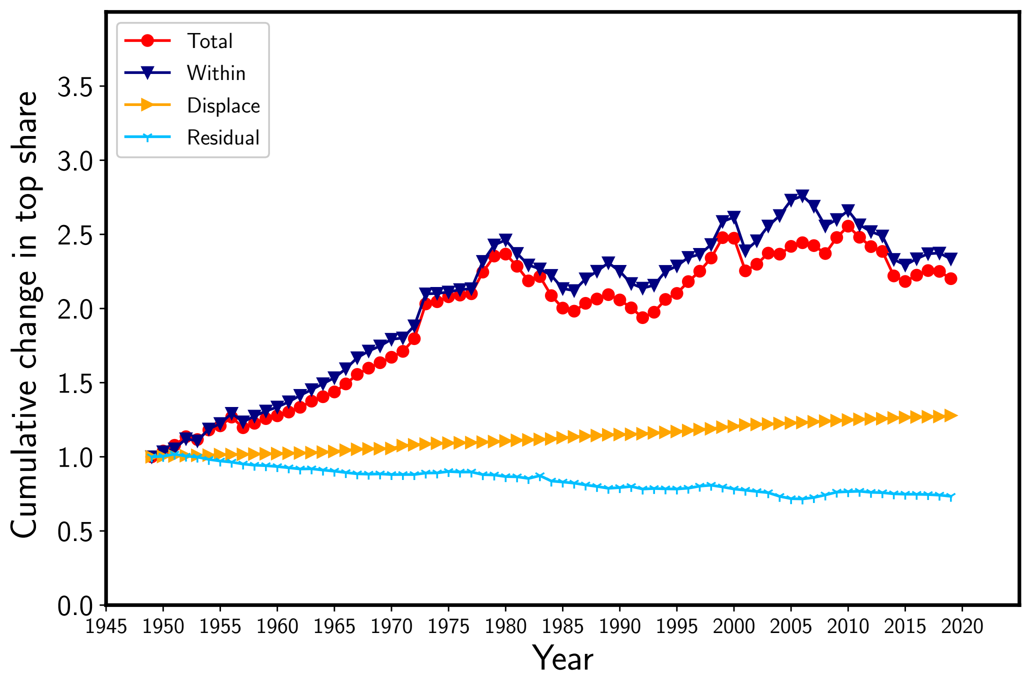


Figure 4 plots the decomposition term each year over the 1950-2019 period. In a broad sense, the decompositions terms exhibit different features prior to and after 1980. Prior to 1980, the within term is consistently positive and dominates the displacement term which is much smaller in magnitude. Hence, the growth of top share is mainly driven by the within term and this corresponds to the steady rise in the top sales shares during the period. After 1980, the decomposition time series become more volatile. The within term is mostly negative during 1980-1985, 1990-1993, 2010-2020 and around the 2000 and 2008 crisis periods. The displacement term is small in magnitude but remains fairly stable over time and it becomes the dominant force driving up top sales share growth when the within term is negative. Figure 5 plots the cumulative sum of each term over time and shows that the cumulative contribution by the within term first rises steadily till 1980, remains roughly stable during 1980-1995, then rise steeply from 1995 to 2005 and declines afterwards. Although smaller in magnitude, the displacement term has consistently made positive contributions to the cumulative growth of top sales shares.

Figure 5: Decomposing Cumulative Growth



To further understand when each decomposition terms exhibit significant changes, I test for structural breaks in each yearly time series of decomposition terms. Table 2 shows that the within term mirrors the total change in top sales share and has a break point at 1975. The displacement term undergoes structural break around 1963 and this is reflected in the time series of the displacement term plotted in Figure 6 where it remains low prior to 1963 and becomes high and more volatile after 1963.

Table 2: Test for structural break of decomposition terms

Decomposition term	χ^2	Break point
Total	24.950***	1975
Within	19.325***	1975
Displacement	16.969***	1963

Note: The table reports the Wald statistic from structural break test. Significance levels: * (p<0.1), ** (p<0.05), *** (p<0.01).

Figure 6: Decomposing Displacement Term into Inflow v.s. Outflow

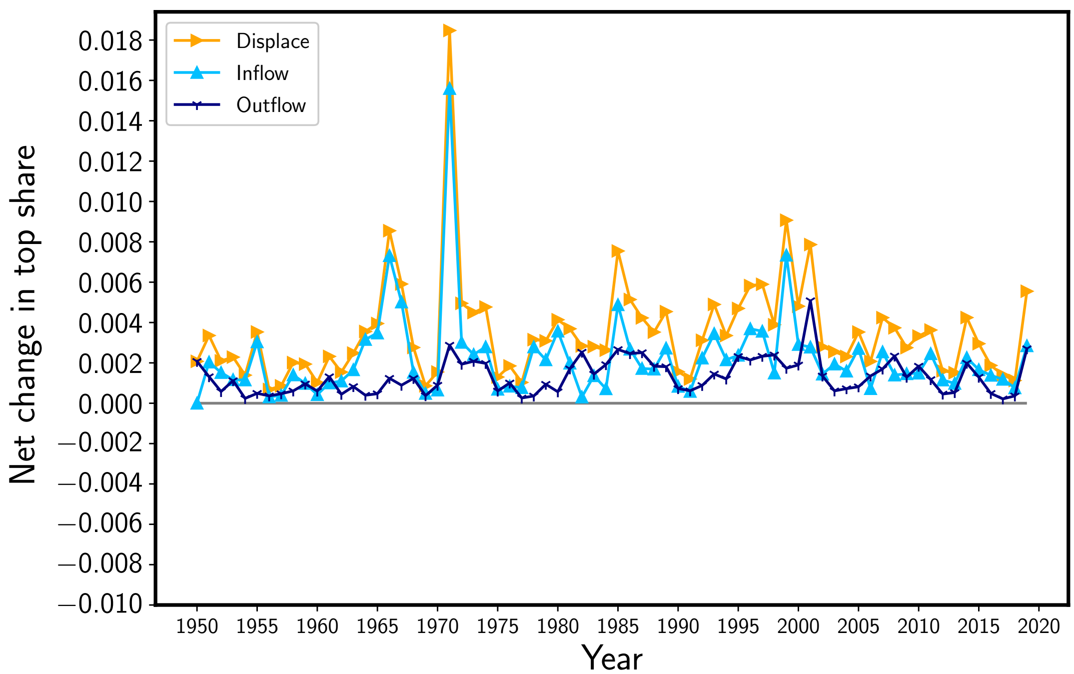
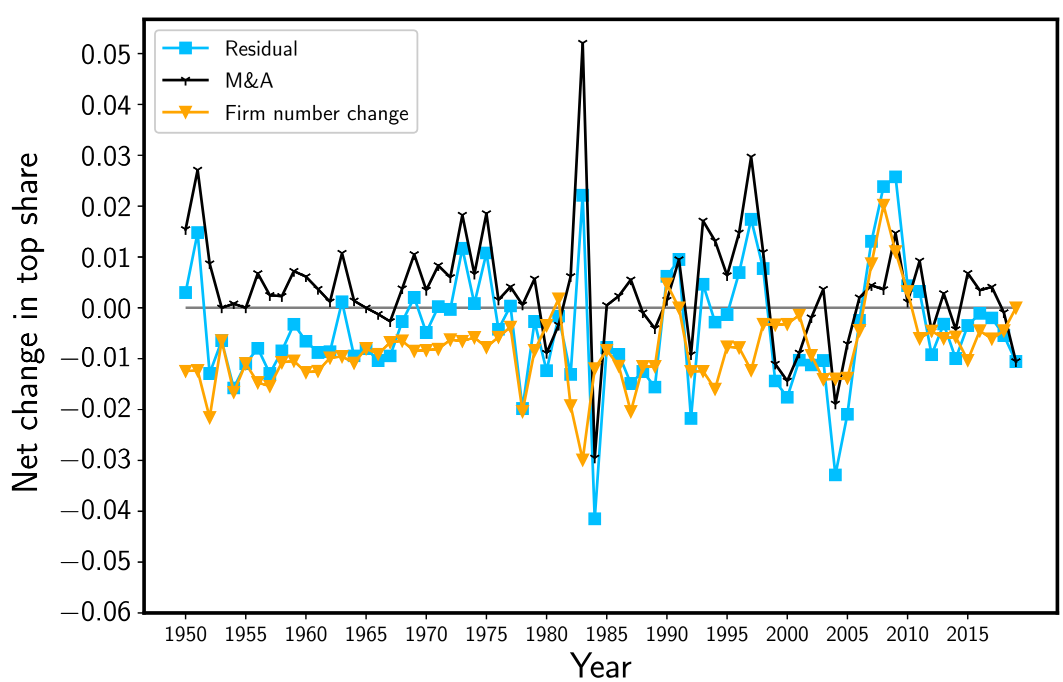


Figure 6 shows that aside from two spikes around 1970, displacement terms are high during the mid-1980s and the mid-1990s and stay at a lower level after 2000 for the recent two decades. The accounting framework breaks down the displacement term further into an inflow term and an outflow term. The inflow term reflects the positive contribution from new entrants displacing firms that are close to the percentile threshold. The outflow term reflects the positive effect to top share from the replacement of exiting firms by firms just below the percentile threshold. The inflow term is larger than the outflow term. The decline in the displacement term since 2000 reflects declines in both the inflow term and the outflow term. Figure 7 separates the residual term into the M&A term and the firm number change term. The M&A term is high during the early-1980s and the mid-1990s. The firm number change term has been negative for most of the time except during the years around the 2008 Financial Crisis period when the total number of firms in the economy has reduced.

Figure 7: Decomposing Residual Term into M&A vs Firm Population Growth



4 Implications for Firm Dynamics

4.1 Firm dynamics and top sales shares.

Equipped with the empirical decomposition result which accounts for different sources of top sales share growth, I now discuss its implication for the dynamics of large firms. To do so, I build an analytical framework to map individual firm dynamics into the two components of the dynamics of the changes in top sales shares, namely the growth of the incumbent top firms and the growth from entry/exit of firms into and out of the top percentile. This mapping between the theoretical and empirical components of the top sales share growth allows me to estimate the underlying process that drives the dynamics of large firms.

To be more specific, I model firm dynamics as subject to a positive shock which makes a firm grow and a negative shock which makes a firm shrink. For instance, these shocks can be interpreted as innovation shocks that take place in different forms. The positive shock can be viewed as an "own innovation" shock in response to which firms improve on their own products and grow larger. In contrast, the negative shock can be viewed as a "creative destruction" shock which occurs to a firm when its product is improved upon by its competitor. When the dynamics of individual firms follow a random growth process subject to these two shocks, I obtain closed-form formulas for the components of top sales share growth that can be mapped to the within term and the displacement term obtained from the empirical decomposition of top sales share growth.

A model of firm growth. Consider a firm dynamics process in which a firm i's sales y_{it} follows the random growth process:

$$\frac{dy_{it}}{y_{it}} = \mu dt + G_\lambda dN_{it}^{\lambda} - G_\delta dN_{it}^{\delta}, \tag{4}$$

where μ is the growth rate of firm i relative to the average growth rate of all firms in the economy in absence of any shocks, dN_{it}^{λ} is the poisson process for own innovation shocks which occur at rate λ, G_λ is the net percentage increase in firm sales when own innovation occurs for firm i, dN_{it}^{δ} is the poisson process for creative destruction which occurs at rate δ, G_δ is the (absolute) net percentage decrease in firm sales when creative destruction occurs for firm i.

Proposition 2. (Decompose top sales shares) *When the sales of individual firms follows the jump process in Equation (4) and the distribution of firm sales is Pareto (with Pareto exponent ζ), the average firm size in the top percentile $\overline{y}$ follows the law of motion:*

$$\begin{aligned}\frac{d\overline{y}_t}{\overline{y}_t} = {} & \underbrace{(\mu + \lambda G_\lambda - \delta G_\delta)\, dt}_{\text{within}} \\ & + \underbrace{\underbrace{\lambda \left(\frac{(1+G_\lambda)^\zeta - 1 - \zeta G_\lambda}{\zeta} \right) dt}_{\text{inflow}} + \underbrace{\delta \left(\frac{(1-G_\delta)^\zeta - 1 + \zeta G_\delta}{\zeta} \right) dt}_{\text{outflow}}}_{\text{displacement}}\end{aligned} \tag{5}$$

As mentioned previously in Equation (1), when firm size is normalized by the average firm size, this is equivalent to the law of motion of top share S_t since $\frac{dS_t}{S_t} = \frac{d\overline{y}_t}{\overline{y}_t}$.

Proposition 3. (Decompose turnover) *When individual firm size follows the jump process (4) and the distribution of firm sales is Pareto (with Pareto exponent ζ), the turnover rate in the top percentile is:*

$$\begin{aligned} \text{Turnover Rate (Inflow)} &= \lambda \left((1 + G_\lambda)^\zeta - 1 \right) \\ \text{Turnover Rate (Outflow)} &= \delta \left(1 - (1 - G_\delta)^\zeta \right) \end{aligned} \tag{6}$$

The model predicts that a rise in the top sales share could be driven by a change in any of these two forces: either an increase in own innovation or a decrease in creative destruction. The within term increases with own innovation and decreases with creative destruction. The displacement term can be further separated into an inflow term and an outflow term. When own innovation increases, the inflow term increases. The outflow term increases as creative destruction increases. Therefore, creative destruction has negative effect on the growth contribution by the incumbent firms (the within term) and has positive effect on the growth contribution through exit of firms whose sales go below the percentile threshold (the outflow term). Taking advantage of the fact that creative destruction has opposite effects on the decomposition terms, one can tease out how much a change in top share growth can be attributed to a change in own innovation versus a change in creative destruction.

4.2 Estimating the firm dynamics process

The above model provides closed-form solutions to the key components of changes in the top sales shares including the growth of incumbent firms (the within term), the growth through entry/exit of firms into the top percentile (the displacement term), the turnover rates which measure the entry/exit rate into and out of the top percentile. The displacement term can be separately characterized by an inflow term that results from own innovation shocks and an outflow term that results from creative destruction shocks.

The within term increases with own innovation probability λ and step size G_λ and decreases with rate of creative destruction δ and step size G_δ. This corresponds to the fact that own innovation drives growth of incumbent top firms whereas creative destruction makes top firms shrink and undermine incumbent growth. The inflow component of the displacement term is driven by own innovation parameters and increases with λ and G_λ, reflecting the effect that lower-ranked firms can displace incumbent top firms after experiencing positive own innovation shocks. On the other hand, displacement also increases through the outflow term triggered by creative destruction shocks. The turnover rate due to entry of firms into the top percentile increases with own innovation rate λ and step size G_λ, whereas the turnover rate due to exit of firms out of the top percentile increases with creative destruction rate δ and (absolute) step size G_δ.

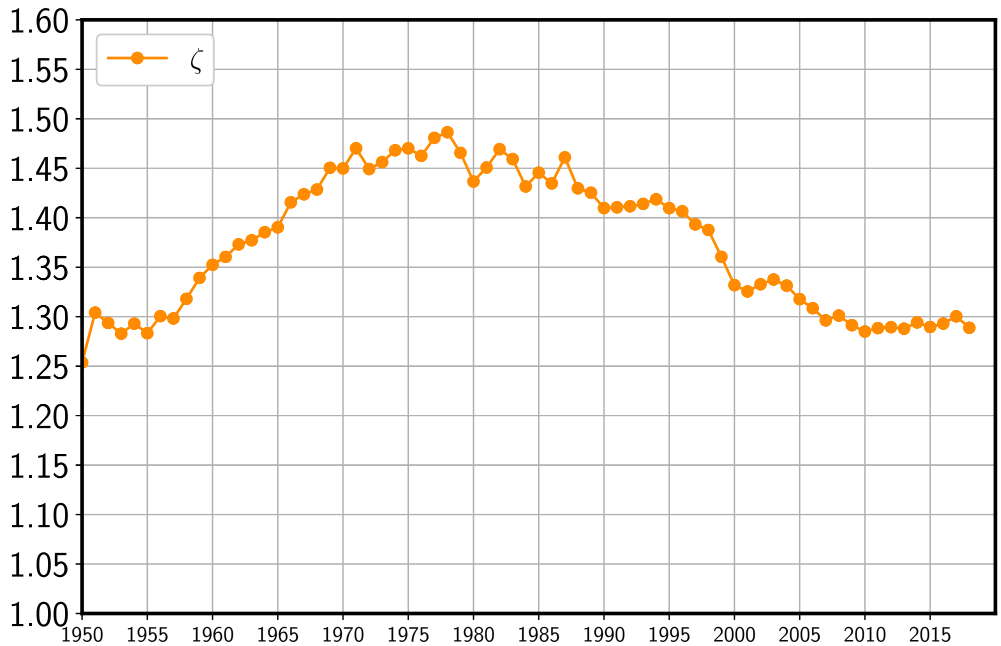
Effect of inequality on displacement. It is also noteworthy that displacement is mediated by the top tail inequality of firm size distribution. When firm size follows Pareto distribution, this inequality is characterized by Pareto exponent ζ, which comes from the density function $g(y) = y^{-\zeta-1}$. From the equations that give top percentile $p = \int_q^\infty y^{-\zeta-1}dy$ and top share $S = \int_q^\infty y \cdot y^{-\zeta-1}dy$. Eliminating q, we get

$$\frac{S/p}{q} = \frac{\zeta}{\zeta - 1}, \tag{7}$$

that is, $\frac{\zeta}{\zeta-1}$ is equal to the ratio of the average sales of top percentile p and the $1 - p$ percentile quantile q. ζ reflects the degree of inequality at the upper tail of top distribution: when ζ is low, $\frac{S/p}{q}$ is high, which implies that the average sales of top percentile is pulling far ahead from the sales of the firm that just reaches the top percentile. When ζ is low, both the inflow and outflow terms are low, which means that the displacement term decreases as inequality at the top tail of the firm size distribution increases. Intuitively, when inequality is high as measured by a more dispersed upper tail distribution, the effect of displacement on top share growth is small and turnover is lower because larger jump size is required to get into the top percentile.

Estimate Pareto exponent ζ. I estimate the ratio of the average sales of top percentile p and the $1-p$ percentile quantile q from the top 0.01% percentile firms and calculate ζ from equation (7). Figure 8 shows that the estimated ζ has declined from its peak close to 1.5 in the pre-1980s to around 1.3 in recent years. This suggests that the average sales of firms above the threshold at the top 0.01% percentile is roughly three times the sales of the firm at the threshold around 1980. This ratio has increased to 4.3 due to the increased inequality among top firms over the past three decades.

Figure 8: Pareto tail index ζ



Estimate innovation parameters. After estimating the Pareto exponent ζ, I use the model-implied formulas for the five moments (Within, Inflow, Outflow, Turnover (Inflow) and Turnover (Outflow)) in equation (8) to calibrate the five parameters $(\mu, \lambda, G_\lambda, \delta, G_\delta)$ underlying the firm dynamics process.

$$\begin{cases} \text{Within} = \mu + \lambda G_\lambda - \delta G_\delta \\ \text{Inflow} = \lambda \left(\frac{(1+G_\lambda)^\zeta - 1 - \zeta G_\lambda}{\zeta} \right) \\ \text{Outflow} = \delta \left(\frac{(1-G_\delta)^\zeta - 1 + \zeta G_\delta}{\zeta} \right) \\ \text{Turnover Rate (Inflow)} = \lambda \left((1+G_\lambda)^\zeta - 1 \right) \\ \text{Turnover Rate (Outflow)} = \delta \left(1 - (1-G_\delta)^\zeta \right) \end{cases} \tag{8}$$

Based on Figure 1, the aggregate trend in the sales share of top 0.01% firms can be roughly delineated into three periods: (1) the 1950-1980 period when top sales share has steadily increased; (2) the 1980-1995 period when it has declined; (3) the 1995-2010 period when the top sales share has risen again. For each period, I estimate the innovation parameters that govern the firm dynamics process. Table 3 reports the moments used in the estimation.

Table 3: Moments used in estimation

Period	Within(%)	Inflow(%)	Outflow(%)	Turnover (Inflow)(%)	Turnover (Outflow)(%)
1950-2019	1.22	0.22	0.13	5.14	3.49
1950-1980	2.95	0.23	0.1	3.78	3.75
1980-1995	-0.37	0.21	0.17	6.27	4.1
1995-2010	1.05	0.25	0.19	6.65	4.09

Note: Table shows the geometric average of the growth rate of top 0.01% firms sales share, the within, inflow and outflow terms within relevant periods. Turnover (Inflow) and Turnover (Outflow) are the average turnover rate due to entry and exit of firms in and out of the top 0.01% percentile within relevant periods. Data are from Census BDS, BEA, *Compustat* and *SDC Platinum* databases.

Discuss estimated innovation process. Table 4 reports the estimated innovation parameters λ, G_λ, δ and G_δ. From the estimation results, we learn that changes in the growth rate of top share across different periods reflect changes in the underlying innovation processes. During the whole sample period, the rate of own innovation is similar to the rate of creative destruction with own innovation occurring at a larger step size than creative destruction. The 1950-1980 period is featured by a low rate of own innovation is around 4%, but when own innovation shock occurs to a firm, it makes the firm grow at a large step size ($\sim$ 50% net percentage change). Turning to the 1980-1995 and the 1995-2010 periods, the rate of own innovation becomes higher ($\sim$ 15%), while the step size has dropped to around $20-30\%$. Across the three periods, the rate of creative destruction has been declining from 16% prior to the 1980s to 10% in more recent years. However, the step size at which firms shrink at the impact of creative destruction shock has become larger over the years.

Although not an innovation parameter, the relative growth rate of top firms to average growth rate of the economy μ also plays an important role in the growth of top sales shares. The period prior to 1980 is a time when large firms is growing at a much higher speed in absence of shocks relative to the rest of the economy. And this trend is reversed after 1980 when μ becomes negative.

Table 4: Estimated innovation parameters

Period	μ(%)	λ(%)	G_λ(%)	δ(%)	G_δ(%)
1950-2019	0.29	8.52	44.21	9.55	29.72
1950-1980	3.25	4.69	53.14	16.49	16.95
1980-1995	-1.51	17.38	23.94	12.56	24.08
1995-2010	-0.42	14.32	32.94	10.31	31.48

Note: μ, λ, δ, G_λ, G_δ are estimated using the five moments: “Within”, “Inflow”, “Outflow”, “Turnover rate (Inflow)”, “Turnover rate (Outflow)”.

Changes in innovation and top sales share. I then use the model-implied innovation parameters to quantify how much changes in the top sales share growth from period to period comes from changes in own innovation as opposed to changes in creative destruction. The following equations quantify how much change in the growth rate of top share can be attributed to changes in own innovation as opposed to changes in creative destruction:

1. The decrease in top share growth rate from 2.82% during the 1950-1980 period to -0.71% during the 1980-1995 period can be decomposed to:

$$\underbrace{-3.53\%}_{\text{Change in top share growth}} \approx \underbrace{-4.77\%}_{\text{drift}} \underbrace{+1.62\%}_{\text{own innovation}} \underbrace{-0.18\%}_{\text{creative destruction}} \tag{9}$$

2. The increase in top share growth rate from -0.71% during the 1980-1995 period to 1.35% during the 1995-2010 period can be decomposed

$$\underbrace{+2.06\%}_{\text{Change in top share growth}} \approx \underbrace{+1.09\%}_{\text{drift}} \underbrace{+0.67\%}_{\text{own innovation}} \underbrace{-0.15\%}_{\text{creative destruction}} \tag{10}$$

The above quantification reveals a prominent role by increases in own innovation in leading to the rise of top sales share growth. Changes in creative destruction can offset a small amount of top sales share growth across periods.

5 Implications for Aggregate Growth

To study the implications of the estimated innovation process that drives firm dynamics on aggregate productivity growth, I now nest this firm dynamics process in a growth model by Jones and Kim (2018). Jones and Kim (2018) uses a growth model with quality ladders in the traditions of Grossman and Helpman (1991) to study how entrepreneurial effort and creative destruction jointly shape income inequality. For my purpose, I abstract from how optimal innovation rate is determined by endogenous entrepreneurial effort and consider the case where own innovation and creative destruction shocks arrive at exogenous poisson rates.

Production. There is a unit measure of varieties in the economy and varieties combine to produce a single final output good:

$$Y = \left(\int_0^1 Y_i^{\theta} di \right)^{1/\theta}, 0 < \theta < 1. \tag{11}$$

Each variety i is produced by a firm i with constant return to scale in labor L_i, and n_t is the step on quality ladder, γ is the step size of quality ladder:

$$Y_{it} = \underbrace{(1+\gamma)^{n_t}}_{\text{aggregate}} \underbrace{x_{it}^{1-\theta}}_{\text{idiosyncratic}} L_{it}, \tag{12}$$

where firm productivity x_{it} follows a jump process:

$$\frac{dx_{it}}{x_{it}} = \mu dt + G_{\lambda} dN_{it}^{\lambda} - G_{\delta} dN_{it}^{\delta}, \tag{13}$$

Assume when innovation occurs for one variety, it generates spillovers that move all other varieties up the quality ladder by one step:

$$\dot{n}_t = \lambda + \delta \tag{14}$$

Proposition 4. Firm sales and total output: *Let Y_{it} denote the amount of sales for firm i.*

Table 5: Contribution to aggregate growth by each form of innovation

Period	Own innovation (%)	Creative Destruction (%)
1950-2019	0.62	0.69
1950-1980	0.34	1.19
1980-1995	1.26	0.91
1995-2010	1.04	0.75

Sales of each firm are given by:

$$Y_{it} = (1+\gamma)^{n_t} X_t^{-\theta} L_t x_{it}, \tag{15}$$

Aggregate output is given by:

$$Y_t = (1+\gamma)^{n_t} X_t^{1-\theta} L_t, \tag{16}$$

where $X_t \equiv (\int_0^1 x_{it}^{\theta} di)^{\frac{1}{\theta}}$ *is the CES aggregate of the productivity distribution across firms and* $L_t \equiv \int_0^1 L_{it} di$ *is total labor supply.*

On balanced growth path, aggregate growth (output per person) is

$$\begin{aligned}\Delta log(\frac{Y}{L}) &= \dot{n}_t log(1+\gamma) \\ &= \underbrace{\lambda log(1+\gamma)}_{\text{own innovation}} + \underbrace{\delta \log(1+\gamma)}_{\text{creative destruction}}\end{aligned}$$

I take $\gamma = 7.5\%$ following Garcia-Macia et al. (2019) and report the estimated contribution to aggregate productivity growth by own innovation and creative destruction in Table 5.

I find that the implied aggregate productivity growth is highest over the 1980-1995 period. Prior to 1980, the contribution to aggregate productivity growth from own innovation is much smaller than that by creative destruction due to a low own innovation rate. Over the 1995-2010 period, the aggregate productivity growth is lower than that during the 1980-1995 period because both own innovation and creative destructions rates have declined.

6 Robustness

I now examine the robustness of my decomposition exercise. First, I explore how the decomposition terms would change if I decompose over longer windows: a 5-year and a 10-year horizon. Second, I check whether the decomposition terms are robust to removing financial firms from the sample.

Decompose over 5-year horizon. In the baseline decomposition exercise, I decompose the one-year growth of the top 0.01% firms top sales into the within term and the displacement term. In doing so, I count firms that are new entrants into the top percentile over the one-year window into the group of firms that generate the displacement effect. However, it may take a longer time for new top firms to grow bigger and contribute to the displacement effect. Therefore, I explore a scenario in which I decompose the growth of the top sales shares over a 5-year window. This allows me to classify all new entries and exits over the 5-year horizon as contributions to the displacement term.

Table 6 reports the decomposition terms of the growth of the top 0.01% firms sales shares over 5-year horizons. To facilitate comparison with the yearly decomposition in the baseline scenario, I report in this table the annualized terms that result from the decomposition over 5-year windows.

Table 6: Decompose the growth of the top 0.01% firms sales shares over 5-year horizon

Period	Total(%)	Within(%)	Displacement(%)			Residual (%)			
			Total	Inflow	Outflow	Total	M&A Growth	M&A Shrink	Δ Firm No. (%)
1950-2019	1.15	1.63	0.44	0.33	0.11	-0.93	0.83	-0.94	-0.88
1950-1980	2.89	2.95	0.32	0.22	0.1	-0.37	0.74	-0.07	-1.08
1980-1995	-0.88	-0.07	0.53	0.41	0.12	-1.35	0.74	-1	-1.13
1995-2010	1.24	2.03	0.62	0.5	0.12	-1.42	1.25	-2.42	-0.44

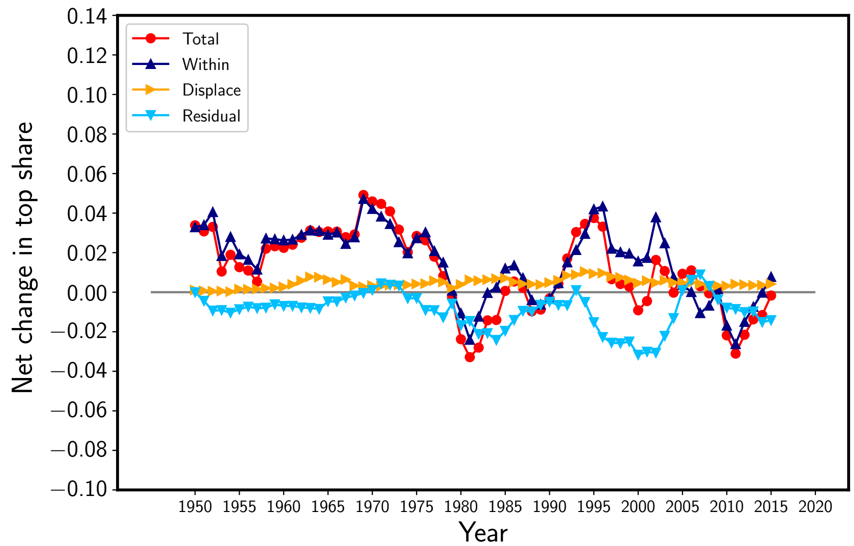
Note: Table shows the annualized 5-year growth rate of the top 0.01% firms sales shares, the within, displacement, M&A and firm number change terms. Data are from Census BDS, BEA, *Compustat* and *SDC Platinum* databases.

Over the 1950-2019 period, decomposition over the 5-year window yields a similar set of the within term and the displacement term as in the yearly decomposition. Compared with the decomposition over a one-year window, both the within term and the displacement term become larger during the two periods: 1980-1995 and 1995-2010. The increase in the displacement terms is attributable to a larger inflow term, which results from the fact that new entrants make larger contribution to top sales shares growth given the longer 5-year horizon.

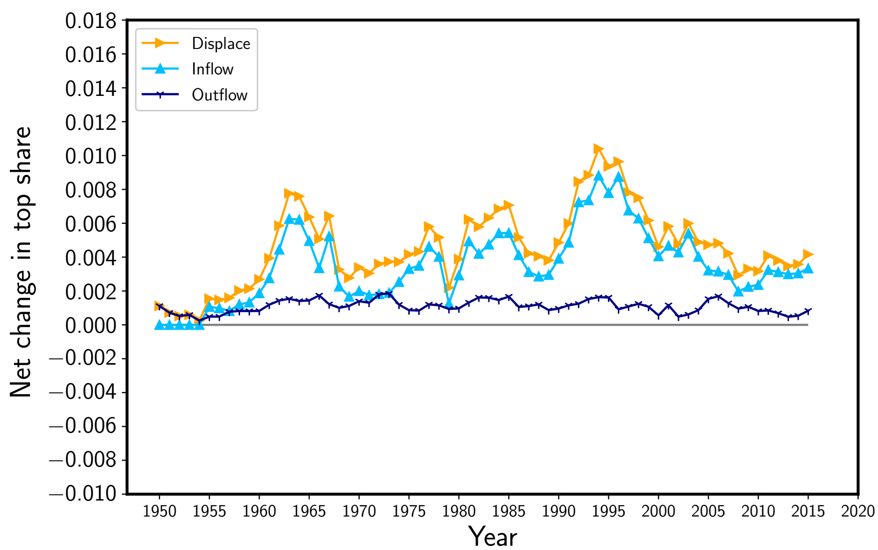
Figure 9a plots the annualized decomposition terms that are components of the top sales shares growth over 5-year horizons for each starting year. When converted to annual frequency, the terms that result from decomposition over 5-year horizons exhibit similar patterns (in terms of sign and magnitude) as the terms obtained from the decomposition exercise at the one-year frequency (Figures 4 and 9a). The time series for the annualized 5-year decomposition terms are smoother than the 1-year decomposition series which contain more year-by-year fluctuations. It shows that firms that are in the top 0.01% percentile at starting years in the intervals [1950,1975] and [1990, 2005] made positive contributions to the growth of top sales shares over 5-year horizons. The cohorts of firms that are in the top at the beginning years during the 1980s and the post-2005 period have shrunk and thus have contributed negatively to top sales shares growth.

In terms of the displacement effect, new top firms that have entered the top percentile during the mid-1960s, the mid-1980s and the mid-1990s made large contributions to the top sales shares growth over the next 5 years after entering the top. Firms that have entered the top during the 2000-2015 period have had a lower displacement effect compared with the cohorts of firms that have entered the top during the two decades before 2000.

Figure 9: Annualized decomposition terms of top sales shares growth over 5-year horizon



(a) All terms



(b) Displacement term

Decompose over 10-year horizon. Appendix Figure A1a shows that when the decomposition is done over 10-year horizons, firms that are already within the top prior to 1970 have large contributions to the top sales share growth over the subsequent 10-year periods. This effect from the within group remains close to zero for firms that are within the top at starting years in the interval [1975, 1990]. The within term then peaks around the late 1990s. This suggests that the incumbent top firms during the late 1990s has the largest contribution to top sales shares growth over 10-year horizons. Appendix Figure A1b shows that over 10-year horizons, the cohorts of firms that enter the top in the early 1990s have the largest contribution to top sales shares growth.

Remove financial firms. The next set of robustness checks involves the removal of financial firms. Since the sales of financial firms are measured differently from that of non-financial firms, I check how the decomposition results would change if I remove all financial firms from my sample. Table 7 shows that the decomposition terms obtained from non-financial firms are similar in magnitude as the ones obtained from the baseline decomposition using the whole sample of firms. Appendix figures A2a and A2b show that the yearly decomposition terms also resemble those obtained from the baseline decomposition.

Table 7: Decomposition terms (without financial firms)

Period	Total(%)	Within(%)	Inflow(%)	Outflow(%)	Residual(%)
1950-1980	2.58	2.9	0.16	0.08	-0.55
1980-1995	-1.07	-0.42	0.19	0.12	-0.96
1995-2010	1.23	0.93	0.25	0.16	-0.12

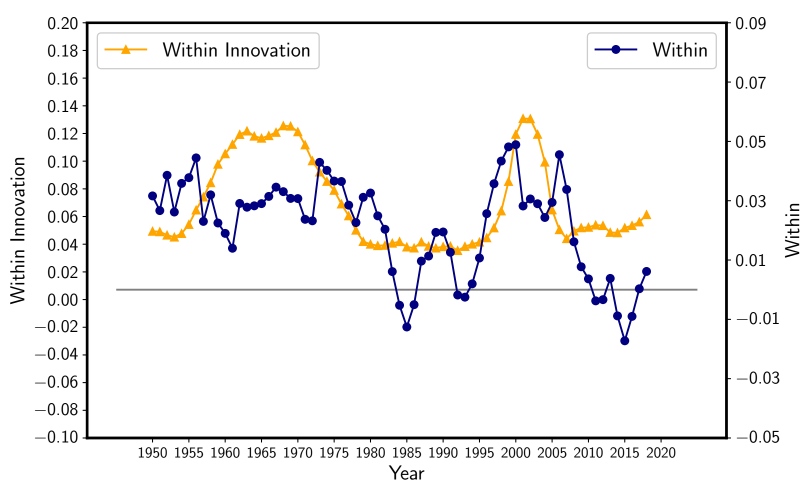
Note: Decomposition terms are obtained after removing financial firms. Residual is the sum of the rest of decomposition terms, including the M&A growth, M&A shrink and firm number change terms.

7 Characteristics of firms that enter, exit and stay in the top

I have estimated a firm dynamics process in which the growth of firms are driven by innovation shocks in section 4. In reality, the growth of firms could be driven by many factors, and innovation is one of them. Now I provide some empirical evidence showing that the growth of incumbent top firms are correlated with their innovating activities.

Within term and innovation. I show that at low-frequency, the growth from incumbent top firms is positively correlation with the total innovation by the group of incumbent top firms. I measure the total innovation by the group of incumbent top firms as the sum of the patent values of all firms in this group divided by their total assets. And I obtain patent values at the firm level from Kogan et al. (2017). Figure 10 shows that the 5-year rolling average of the within term and the total innovation by incumbent top firms are positively correlated (with correlation coefficient +0.436). This suggests that at low-frequency, the fluctuations of the within term is correlated with the innovation activities by the incumbent top firms.

Figure 10: 5-year Rolling Average of Innovation and Within term



Entry, exit vs stayer: innovation. By comparing the innovation of firms that enter, exit and stay in the top, I find that firms that stay in the top have higher innovation than firms that exit the top, suggesting that innovation is an important factor for firms to stay in the top percentile.

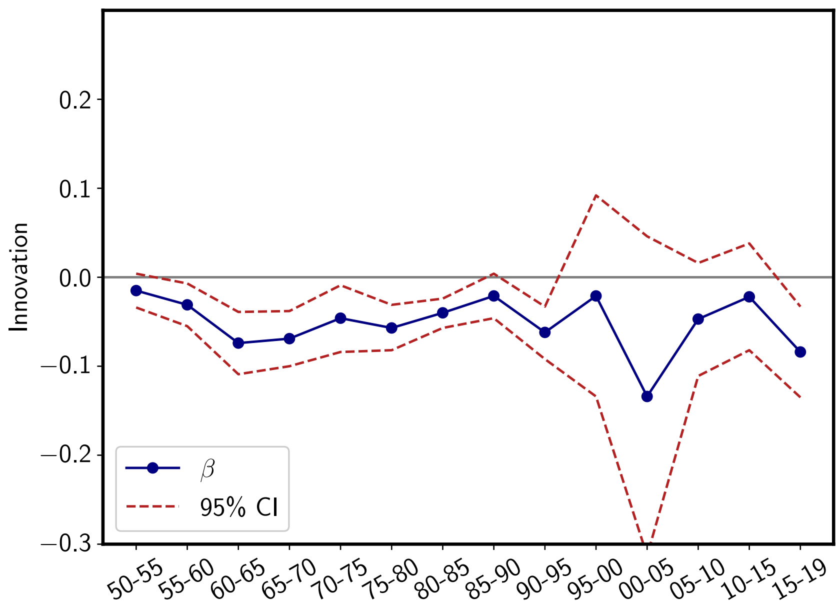
I compare the characteristics of firms that enter, exit and stay in the top 0.01% percentile by running Regression 17. The innovation of a firm i in year t, Innovation_{it}, is measured as the ratio of a firm's patent value to its total asset. Since a firm may get approval on several patents in a certain year while in other years there are no patents accredited to the firm, I smooth out firm-level innovation by using the 5-year rolling average of the patent value-to-asset ratio. λ_t is year fixed effect to absorb the effect of time-varying factors. T is the relevant 5-year window within the whole 1950-2019 sample period.

$$\text{Innovation}_{it}^{T} = \alpha + \beta \text{Entry/Exit vs Stayer}_{it} + \lambda_t + \epsilon_{it} \tag{17}$$

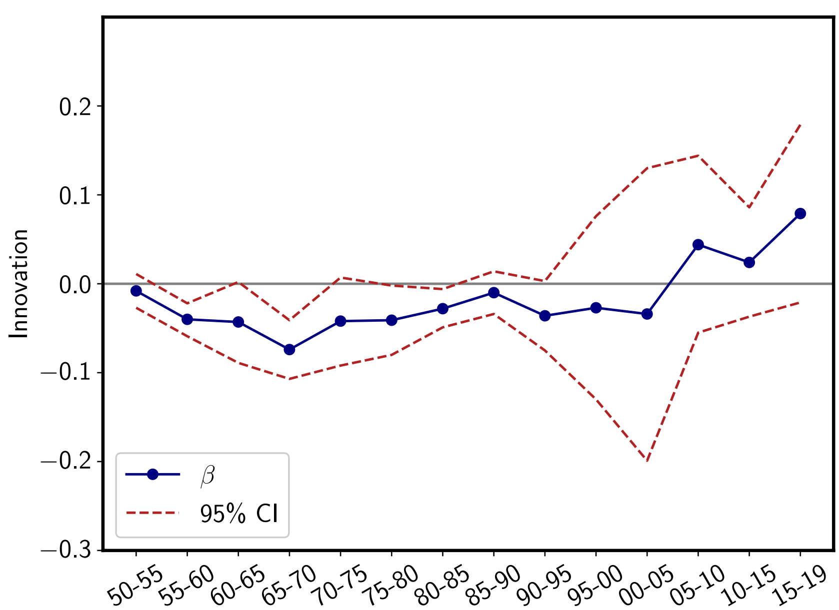
Figure 11a shows that over each 5-year window throughout the whole 1950-2019 period, firms that exit from the top have lower innovation than firms that remain in the top. This suggests that firms that remain in the top are more innovative than the old top firms that are being displaced from the top by new firms. Figure 11b shows that firms that remain in the top have innovated more than the firms that are new entrants to the top in the early years prior to 1980. However, this innovation gap has been closing over time and during recent years (2005-2019), new entrants have higher innovation than incumbent top firms.

Taken together, innovation is an important driving force for firms to stay in the top. Firms that do not innovate enough may get displaced out of the top percentile. Compared with old top firms, new top firms have become more innovative over time.

Figure 11: Compare innovation of firms that enter, exit and stay in the top 0.01% percentile



(a) Innovation: Exit vs Stayer



(b) Innovation: Entry vs Stayer

8 Conclusion

While the broad-based rise in top sales share since the late-1990s has been documented in the literature. What it implies about the underlying firm dynamics process is still unclear. In this paper, I apply the accounting framework developed in Gomez (2020) to decompose the growth of sales shares accrued to top 0.01% firms in a longer time horizon over the 1950-2019 period.

I obtain on two key components of the growth of top sales shares from this decomposition. The first term, the *within* term, is the contribution to top sales shares growth existing top firms. The second term, the *displacement* term, measures the effect of compositional change of top firms on top sales shares growth. I find that during the whole sample period (1950-2019), the within term is larger in magnitude than the displacement term. That is, the growth of top sales shares is to a large extent due to incumbent top firms growing bigger. However, the displacement of old top firms by new top firms also makes important positive contributions to the growth of top sales shares.

Through the lens of a firm random growth model in which the dynamics of individual firms are subject to own innovation and creative destruction shocks, I obtain a mapping between the theoretical and empirical components of the top sales share growth. This allows me to estimate the underlying innovation process that drives the dynamics of large firms. I find that the rate of own innovation is highest over the 1980-1995 period, whereas the rate of creative destruction has been declining over time. I also find that the implied aggregate productivity growth is highest over the 1980-1995 period.

References

Aghion, Philippe and Peter Howitt, "A model of growth through creative destruction," 1990.

—, Antonin Bergeaud, Timo Boppart, Peter J Klenow, and Huiyu Li, "A theory of falling growth and rising rents," Technical Report, National Bureau of Economic Research 2019.

Akcigit, Ufuk and William R Kerr, "Growth through heterogeneous innovations," *Journal of Political Economy*, 2018, *126* (4), 1374–1443.

Andrews, Dan, Chiara Criscuolo, and Peter N Gal, "Frontier firms, technology diffusion and public policy: Micro evidence from OECD countries," 2015.

Autor, David, David Dorn, Lawrence F Katz, Christina Patterson, and John Van Reenen, "The fall of the labor share and the rise of superstar firms," *The Quarterly Journal of Economics*, 2020, *135* (2), 645–709.

Babina, Tania, Anastassia Fedyk, Alex Xi He, and James Hodson, "Artificial intelligence, firm growth, and industry concentration," *Firm Growth, and Industry Concentration (July 14, 2020)*, 2020.

Bessen, James, "Information technology and industry concentration," 2017.

Bloom, Nicholas, Charles I Jones, John Van Reenen, and Michael Webb, "Are ideas getting harder to find?," *American Economic Review*, 2020, *110* (4), 1104–44.

Broda, Christian and David E Weinstein, "Product creation and destruction: Evidence and price implications," *American Economic Review*, 2010, *100* (3), 691–723.

Calligaris, Sara, Chiara Criscuolo, and Luca Marcolin, "Mark-ups in the digital era," 2018.

Christensen, Clayton M, *The innovator's dilemma: when new technologies cause great firms to fail*, Harvard Business Review Press, 2013.

Crouzet, Nicolas and Janice Eberly, "Intangibles, investment, and efficiency," in "AEA Papers and Proceedings," Vol. 108 2018, pp. 426–31.

Cunningham, Colleen, Florian Ederer, and Song Ma, "Killer acquisitions," *Journal of Political Economy*, 2021, *129* (3), 649–702.

Decker, Ryan A, John Haltiwanger, Ron S Jarmin, and Javier Miranda, "Changing business dynamism and productivity: Shocks versus responsiveness," *American Economic Review*, 2020, *110* (12), 3952–90.

Dorn, David, Lawrence F Katz, Christina Patterson, and John Van Reenen, "Concentrating on the Fall of the Labor Share," *American Economic Review*, 2017, *107* (5), 180–85.

Dou, Winston Wei, Yan Ji, and Wei Wu, "Competition, profitability, and discount rates," *Journal of Financial Economics*, 2020.

Ewens, Michael, Ryan H Peters, and Sean Wang, "Acquisition prices and the measurement of intangible capital," Technical Report 2019.

Faccio, Mara and John J McConnell, "Impediments to the Schumpeterian Process in the Replacement of Large Firms," Technical Report, National Bureau of Economic Research 2020.

Foster, Lucia, Cheryl Grim, John C Haltiwanger, and Zoltan Wolf, "Innovation, productivity dispersion, and productivity growth," Technical Report, National Bureau of Economic Research 2018.

_ , John Haltiwanger, and Cornell J Krizan, "Market selection, reallocation, and restructuring in the US retail trade sector in the 1990s," *The Review of Economics and Statistics*, 2006, *88* (4), 748–758.

Fujita, Shigeru et al., "Creative destruction and aggregate productivity growth," *Business Review (Federal Reserve Bank of Philadelphia)*, 2008.

Garcia-Macia, Daniel, Chang-Tai Hsieh, and Peter J Klenow, "How destructive is innovation?," *Econometrica*, 2019, *87* (5), 1507–1541.

Gomez, Matthieu, "Decomposing the Rise in Top Wealth Shares," 2020.

Grossman, Gene M and Elhanan Helpman, "Trade, knowledge spillovers, and growth," *European economic review*, 1991, *35* (2-3), 517–526.

Grullon, Gustavo, Yelena Larkin, and Roni Michaely, "Are US industries becoming more concentrated?," *Review of Finance*, 2019, *23* (4), 697–743.

Haskel, Jonathan and Stian Westlake, *Capitalism without capital*, Princeton University Press, 2017.

Hottman, Colin J, Stephen J Redding, and David E Weinstein, "Quantifying the sources of firm heterogeneity," *The Quarterly Journal of Economics*, 2016, *131* (3), 1291–1364.

Jones, Charles I and Jihee Kim, "A Schumpeterian model of top income inequality," *Journal of Political Economy*, 2018, *126* (5), 1785–1826.

Jr, Robert E Lucas and Benjamin Moll, "Knowledge growth and the allocation of time," *Journal of Political Economy*, 2014, *122* (1), 1–51.

Kamepalli, Sai Krishna, Raghuram Rajan, and Luigi Zingales, "Kill zone," Technical Report, National Bureau of Economic Research 2020.

Kelly, Bryan, Dimitris Papanikolaou, Amit Seru, and Matt Taddy, "Measuring technological innovation over the long run," *American Economic Review: Insights*, 2021, *3* (3), 303–20.

Klenow, Peter J and Huiyu Li, "Innovative Growth Accounting," in "NBER Macroeconomics Annual 2020, volume 35," University of Chicago Press, 2020.

—, —, Theodore Naff et al., "Is Rising Concentration Hampering Productivity Growth?," *FRBSF Economic Letter*, 2019, *2019*, 28.

Klette, Tor Jakob and Samuel Kortum, "Innovating firms and aggregate innovation," *Journal of political economy*, 2004, *112* (5), 986–1018.

Kogan, Leonid, Dimitris Papanikolaou, Amit Seru, and Noah Stoffman, "Technological innovation, resource allocation, and growth," *The Quarterly Journal of Economics*, 2017, *132* (2), 665–712.

Krusell, Per, "How is R&D Distributed Across Firms? A Theoretical Analysis," *unpublished paper*, 1998.

Liu, Ernest, Atif Mian, and Amir Sufi, "Low interest rates, market power, and productivity growth," Technical Report, National Bureau of Economic Research 2019.

Loecker, Jan De, Jan Eeckhout, and Gabriel Unger, "The rise of market power and the macroeconomic implications," *The Quarterly Journal of Economics*, 2020, *135* (2), 561–644.

Petrin, Amil, T Kirk White, and Jerome P Reiter, "The impact of plant-level resource reallocations and technical progress on US macroeconomic growth," *Review of Economic Dynamics*, 2011, *14* (1), 3–26.

Phillips, Gordon M and Alexei Zhdanov, "R&D and the Incentives from Merger and Acquisition Activity," *The Review of Financial Studies*, 2013, *26* (1), 34–78.

Reenen, John Van, "Increasing differences between firms: Market power and the macro-economy," 2018.

Sparque, Shawn, "The US productivity slowdown: an economy-wide and industry-level analysis," *Monthly Lab. Rev.*, 2021, *144*, 1.

Steffens, Lincoln, *The Struggle for Self-government: Being an Attempt to Trace American Political Corruption to Its Sources in Six States of the United States*, McClure, Phillips & Company, 1906.

Stigler, George J, "The theory of economic regulation," *The Bell journal of economics and management science*, 1971, pp. 3–21.

Stokey, Nancy L, "Learning by doing and the introduction of new goods," *journal of Political Economy*, 1988, *96* (4), 701–717.

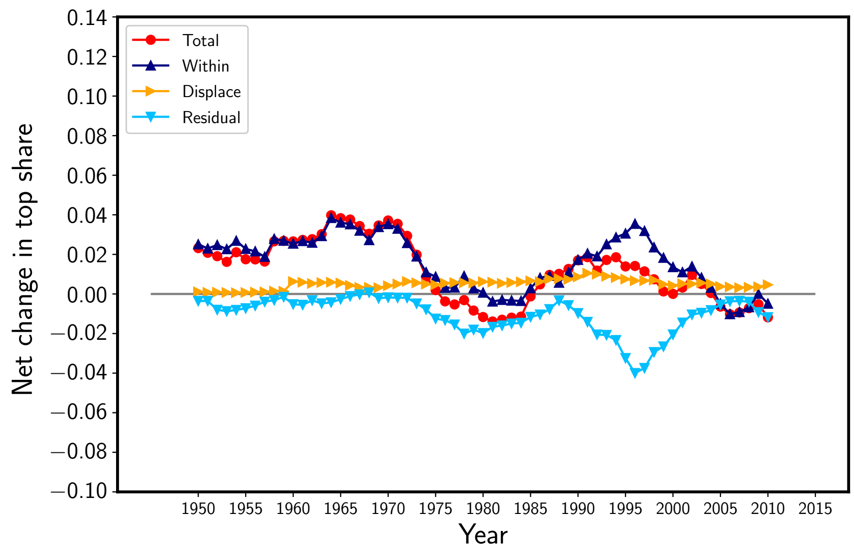
Sutton, John, "Market share dynamics and the" persistence of leadership" debate," *American Economic Review*, 2007, *97* (1), 222–241.

Tambe, Prasanna, Lorin Hitt, Daniel Rock, and Erik Brynjolfsson, "Digital Capital and Superstar Firms," Technical Report, National Bureau of Economic Research 2020.

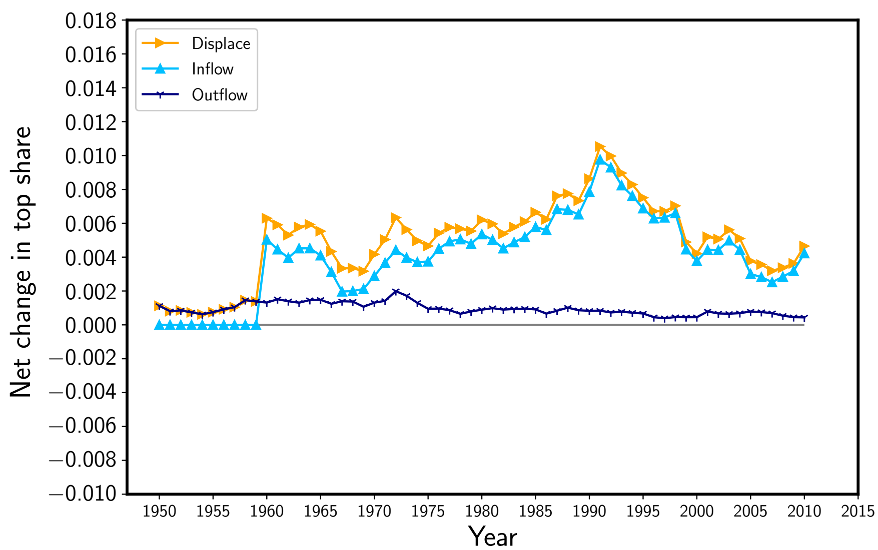
Appendices

A Figures

Figure A1: Annualized decomposition terms of top sales shares growth over 10-year horizon

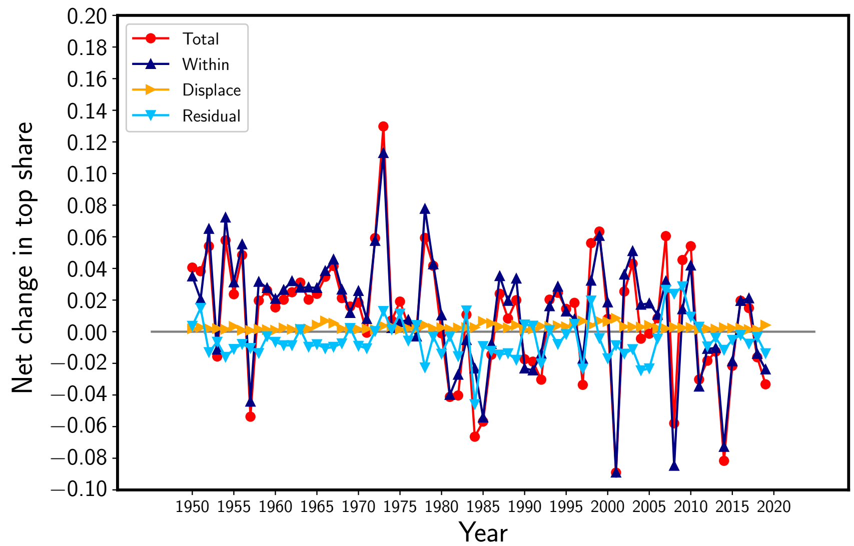


(a) All terms

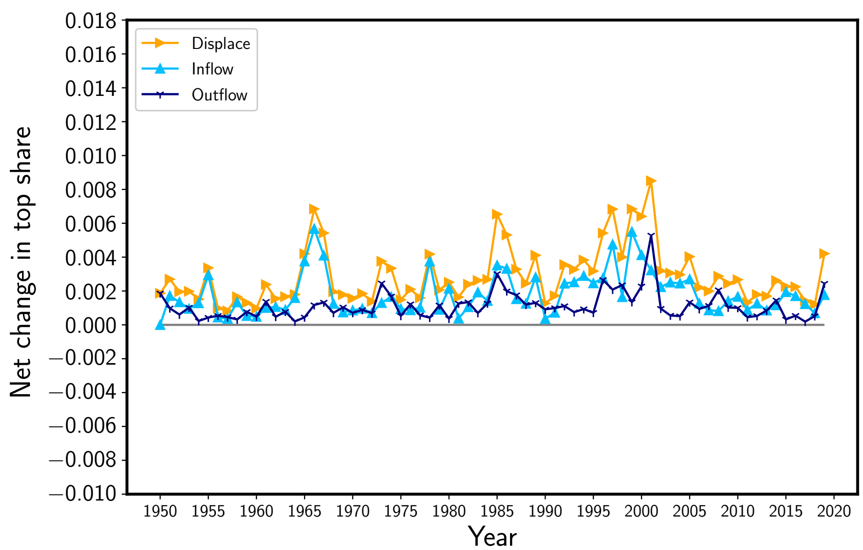


(b) Displacement term

Figure A2: Decomposition terms of top sales shares growth (without financial firms)



(a) All terms



(b) Displacement term

B Proofs

B.1 Proof of Proposition 4

The optimization problem for the final goods sector and the firm's monopoly decisions are static. A perfectly competitive final goods sector combines varieties $i \in [0,1]$ with price p_i to produce the final good Y. A representative firm in the final goods sector solves:

$$\min_{\{Y_i\}} \int_0^1 p_i Y_i d_i$$

s.t.

$$Y = \left(\int_0^1 Y_i^{\theta} di \right)^{\frac{1}{\theta}}$$

The demand for each variety i is

$$\left(\frac{Y}{Y_i} \right)^{1-\theta} = p_i. \tag{18}$$

Each firm i chooses Y_i:

$$\max_{\{Y_i\}} p_i(Y_i)Y_i - wL_i = Y^{1-\theta} Y_i^{\theta} - \frac{w}{(1+G)^n x_i^(1-\theta)} Y_i$$

The solution to Y_i is:

$$Y_i = \left(\frac{1}{\theta} \cdot \frac{w}{(1+G)^n} \right)^{\frac{1}{\theta-1}} Y x_i \tag{19}$$

Plugging equation (19) into the final goods production function $Y = \left(\int_0^1 Y_i^{\theta} di \right)^{\frac{1}{\theta}}$, we obtain the equilibrium real wage:

$$w = \theta(1+G)^n \left(\int_0^1 x_i^{\theta} di \right)^{\frac{1-\theta}{\theta}} \tag{20}$$

Define $X \equiv \left(\int_0^1 x_i^{\theta} d_i \right)^{\frac{1}{\theta}}$, the equilibrium wage equation is:

$$w = \theta(1+G)^n X^{1-\theta}. \tag{21}$$

Plug this into equation (19), we get

$$Y_i = \left(\frac{x_i}{X} \right) Y \tag{22}$$

Joining equation (22) with firm i's production function $Y_i = (1+G)^{n_t} x_i^{1-\theta} L_i$, we get employment by firm i:

$$L_i = \frac{Y_i}{(1+G)^{n_t} x_i^{\alpha}} = (1+G)^{-n_t} \frac{x_i^{\theta}}{X} Y \tag{23}$$

From the labor market clearing condition $L = \int_0^1 L_i di$ and equation (23), we get aggregate output:

$$Y = (1+G)^{n_t} X^{1-\theta} L \tag{24}$$

Firm sales, aggregate output and wage in proposition 4 are given by equations (22, 24), and (21), respectively. Q.E.D.

B.2 Proof of Proposition 2

Assume that firm size follows the law of motion:

$$\frac{dx_t}{x_{t-}} = \mu dt + \sum_{i=1}^{2} (e^{U_i} - 1) dN_{it}, \tag{25}$$

where N_{1t} and N_{2t} are Poisson processes with intensity λ and δ, respectively. Under this set up, firm size can jump with either a positive log size U_1, or a negative log size U_2.

Step 1. Suppose h($\cdot$) is a twice-differentiable scalar function, by Ito's formula for jump process:

$$\begin{aligned} dh(x_t) &= h'(x_t)\mu x_t dt + \frac{d}{d_t} \sum_{i=1}^{2} \int_0^t [h(x_s) - h(x_{s-})] dN_{is} \\ &= h'(x_t)\mu x_t dt + \sum_{i=1}^{2} [h(x_t) - h(x_{t-})] dN_{it} \end{aligned} \tag{26}$$

Let $g_t(x) = p(x_0, 0; x, t)$ be the density function of x_t: integrate from 0 to T,

$$\begin{aligned} h(x_T) - h(x_0) &= \int_0^T h'(x_t)\mu x_{t-} dt + \sum_{i=1}^{2} \int_0^T [h(x_t) - h(x_{t-})] dN_{it} \\ &= \int_0^T h'(x_t)\mu x_t dt + \sum_{i=1}^{2} \int_0^T [h(x_t) - h(x_{t-})] dN_{it}, \end{aligned} \tag{27}$$

and this step uses the fact that $\int_0^T h'(x_t)\mu x_{t-} dt = \int_0^T h'(x_t)\mu x_t dt$ since jumps at finite number of points does not affect the value of integral.

Let $d\tilde{N}_{it}$ is compensated poisson process for dN_{it}, that is, $d\tilde{N}_{1t} = dN_{1t} - \lambda_t$ and $d\tilde{N}_{2t} = dN_{2t} - \delta_t$. Take expectation on both side of equation (27) and plug in $x_t = e^U x_{t-}$, we get

$$\begin{aligned} E[h(x_T) - h(x_0)] &= \int_0^T \int_0^\infty h'(y)\mu y g_t(y) dy dt + E[\sum_{i=1}^{2} \int_0^T [h(e^{U_i} x_{t-}) - h(x_{t-})] dN_{it}] \\ &= \int_0^T \int_0^\infty h'(y)\mu y g_t(y) dy dt + \sum_{i=1}^{2} E[\int_0^T [h(e^{U_i} x_{t-}) - h(x_{t-})] d\tilde{N}_{it}] \\ &\quad + E[\int_0^T [h(e^{U_1} x_{t-}) - h(x_{t-})] \lambda d_t] + E[\int_0^T [h(e^{U_2} x_{t-}) - h(x_{t-})] \delta d_t] \\ &= \int_0^T \int_0^\infty h'(y)\mu y g_t(y) dy dt \\ &\quad + E[\int_0^T [h(e^{U_1} x_t) - h(x_t)] \lambda d_t] + E[\int_0^T [h(e^{U_2} x_t) - h(x_t)] \delta d_t], \end{aligned} \tag{28}$$

and this step uses $E[\int_0^T [h(e^{U_i} x_{t-}) - h(x_{t-})] d\tilde{N}_{it}] = 0$ since the compensated poisson process $d\tilde{N}_{it}$ is a martingale.

Expand on the LHS and RHS of equation (28), we get

$$\begin{aligned} \int_0^{+\infty} h(y) g_T(y) dy - h(x_0) = \int_0^T \int_0^\infty h'(y)\mu y g_t(y) dy dt + \lambda \int_0^T \int_0^\infty [h(e^{U_1} y) - h(y)] g_T(y) dy dt \\ + \delta \int_0^T \int_0^\infty [h(e^{U_2} y) - h(y)] g_T(y) dy dt \end{aligned} \tag{29}$$

Take derivative w.r.t. to T on both sides:

$$\begin{aligned} \int_0^\infty h(y) d g_T(y) dy = \int_0^T [h'(y)\mu y g_T(y)] dy \\ + \lambda \int_0^\infty [h(e^{U_1} y) - h(y)] g_T(y) dy + \delta \int_0^{+\infty} [h(e^{U_2} y) - h(y)] g_T(y) dy \end{aligned} \tag{30}$$

Step 2. – Law of motion of average firm size in the top percentile The average firm size in the top percentile p is

$$\bar{x}_t = \frac{1}{p} \int_{q_t}^\infty x g_t(x) dx \tag{31}$$

Denote $\gamma_t(\cdot)$ as density of log firm size, then $\gamma_t(z) = e^z g_t(e^z)$. With a change of variable $x = e^z$, we can rewrite $\overline{x}_t$ as:

$$\begin{aligned}\overline{x}_t &= \frac{1}{p}\int_{q_t}^{\infty} x g_t(x)dx \\ &= \frac{1}{p}\int_{log q_t}^{\infty} e^z g_t(e^z)e^z dz \\ &= \frac{1}{p}\int_{\chi_t}^{\infty} \gamma_t(z)e^z dz\end{aligned} \tag{32}$$

Since γ_t is positive everywhere, the function $\chi \to \int_{\chi}^{\infty}\gamma_t(z)dz$ is strictly decreasing, therefore, $\forall t \geq 0$, there exists a unique (log) quantile χ_t s.t. $p = \int_{\chi_t}^{\infty}\gamma_t(z)dz$.

From the set of equations:

Step 3. In this step, I apply the general formula derived in equation (30) to solve for the dynamics of average size of top percentile p firms, which is

$$d\overline{x}_t = \frac{1}{p}\int_{q_t}^{\infty}(x - q_t)dg_t(x)dx \tag{33}$$

Let $h(x) = (x - q_t)^+$, then $h'(x) = 1$ if $x \geq q_t$. Rewrite $d\overline{x}_t$ in terms of $h(x)$ and plug in the equation (30), we get

$$\begin{aligned}d\overline{x}_t =& \frac{1}{p}\int_{q_t}^{\infty}(x - q_t)dg_t(x)dx \\ =& \frac{1}{p}\int_{q_t}^{\infty} h(x)dg_t(x)dx \\ =& \frac{1}{p}\int_{q_t}^{\infty}\mu x g_t(x)dx + \frac{\lambda}{p}\int_0^{\infty}[(e^{U_1}x - q_t)^+ - (x - q_t)^+]g_t(x)dx \\ &+ \frac{\delta}{p}\int_0^{\infty}[(e^{U_2}x - q_t)^+ - (x - q_t)^+]g_t(x)dx \\ =& \frac{1}{p}\int_{q_t}^{\infty}\mu x g_t(x)dx \\ &+ \frac{\lambda}{p}\int_{e^{-U_1}q_t}^{\infty}(e^{U_1}y - q_t)g_t(x)dx - \frac{\lambda}{p}\int_{q_t}^{\infty}(x - q_t)g_t(x)dx \\ &+ \frac{\delta}{p}\int_{e^{-U_2}q_t}^{\infty}(e^{U_2}y - q_t)g_t(x)dx - \frac{\delta}{p}\int_{q_t}^{\infty}(x - q_t)g_t(x)dx\end{aligned} \tag{34}$$

Joining equations (31) and (34), together with $p = \int_{q_t}^{\infty} g_t(x)dx$, we get the law of motion of the average firm size in the top percentile $\overline{x}_t$:

$$
\begin{aligned}
\frac{d\overline{x}_t}{\overline{x}_t} =& \frac{1}{p\overline{x}_t}\int_{q_t}^{\infty} \mu x g_t(x)dx \\
&+ \frac{\lambda}{p\overline{x}_t}\int_{e^{-U_1}q_t}^{\infty}(e^{U_1}x - q_t)g_t(x)dx - \frac{\lambda}{p\overline{x}_t}(\overline{x}_t p - q_t p) \\
&+ \frac{\delta}{p\overline{x}_t}\int_{e^{-U_2}q_t}^{\infty}(e^{U_2}x - q_t)g_t(x)dx - \frac{\delta}{p\overline{x}_t}(\overline{x}_t p - q_t p)
\end{aligned} \tag{35}
$$

When the stationary distribution of the density $g_t(x)$ follows Pareto distribution with exponent ζ, i.e., $g_t(x) = x^{-\zeta-1}$, then $\frac{d\overline{x}_t}{\overline{x}_t}$ becomes:

$$
\begin{aligned}
\frac{d\overline{x}_t}{\overline{x}_t} =& \mu + \frac{\lambda}{p\overline{x}_t}\int_{e^{-U_1}q_t}^{\infty}(e^{U_1}x - q_t)x^{-\zeta-1}dx - \frac{\lambda}{p\overline{x}_t}(\overline{x}_t p - q_t p) \\
&+ \frac{\delta}{p\overline{x}_t}\int_{e^{-U_2}q_t}^{\infty}(e^{U_2}x - q_t)x^{-\zeta-1}dx - \frac{\delta}{p\overline{x}_t}(\overline{x}_t p - q_t p) \\
=& \mu + \frac{\lambda e^{U_1}}{p\overline{x}_t}\frac{e^{U_1(\zeta-1)}}{\zeta-1}q_t^{-\zeta+1} - \frac{\lambda}{p\overline{x}_t}\frac{e^{U_1\zeta}}{\zeta}q_t^{-\zeta+1} - \lambda + \frac{\lambda q_t}{\overline{x}_t} \\
&+ \frac{\delta e^{U_2}}{p\overline{x}_t}\frac{e^{U_2(\zeta-1)}}{\zeta-1}q_t^{-\zeta+1} - \frac{\delta}{p\overline{x}_t}\frac{e^{U_2\zeta}}{\zeta}q_t^{-\zeta+1} - \delta + \frac{\delta q_t}{\overline{x}_t}
\end{aligned} \tag{36}
$$

From the set of equations:

$$
\begin{cases}
p = \int_{q_t}^{\infty} g_t(x)dx = \int_{q_t}^{\infty} x^{-\zeta-1}dx = \frac{q_t^{-\zeta}}{\zeta} \\
\overline{x}_t = \frac{1}{p}\int_{q_t}^{\infty} x g_t(x)dx = \frac{1}{p}\int_{q_t}^{\infty} x^{-\zeta}dx = \frac{q_t^{-\zeta+1}}{p(\zeta-1)},
\end{cases} \tag{37}
$$

we can get

$$
\frac{q_t}{\overline{x}_t} = \frac{\zeta-1}{\zeta} \tag{38}
$$

Plug equation (38) into equation (36), we can simplify $\frac{d\overline{x}_t}{\overline{x}_t}$ into:

$$
\frac{d\overline{x}_t}{\overline{x}_t} = \mu + \lambda\frac{e^{U_1\zeta}-1}{\zeta} + \delta\frac{e^{U_2\zeta}-1}{\zeta} \tag{39}
$$

The within term is the average growth of firms in the top percentile holding constant the composition of top firms, which is equal to $\mu + \lambda(e^{U_1} - 1) + \delta(e^{U_2} - 1)$. Therefore, we can separate the growth of the average firm size into a within term and a displacement term which comes as the residual from the total term minus the within term:

$$
\begin{aligned}
\frac{d\overline{x}_t}{\overline{x}_t} =& \mu + \lambda\frac{e^{U_1\zeta}-1}{\zeta} + \delta\frac{e^{U_2\zeta}-1}{\zeta} \\
=& \underbrace{\mu + \lambda(e^{U_1}-1) + \delta(e^{U_2}-1)}_{\text{Within}} \\
& \underbrace{+\lambda\frac{e^{U_1\zeta}-1}{\zeta} + \delta\frac{e^{U_2\zeta}-1}{\zeta} - \lambda(e^{U_1}-1) - \delta(e^{U_2}-1)}_{\text{Displacement}}
\end{aligned} \tag{40}
$$

Let $G_\lambda = e^{U_1} - 1$ and $G_\delta = e^{U_2} - 1$, following the law of motion in Proposition 2, $G_1 = G$ and $G_2 = -1$, we can further simplify equation (40) into:

$$
\begin{aligned}
\frac{d\overline{x}_t}{\overline{x}_t} =& \underbrace{(\mu + \lambda G_\lambda - \delta G_\delta)\, dt}_{\text{within}} \\
& \underbrace{\underbrace{+\lambda\left(\frac{(1+G_\lambda)^\zeta - 1 - \zeta G_\lambda}{\zeta}\right) dt}_{\text{inflow}} + \underbrace{\delta\left(\frac{(1-G_\delta)^\zeta - 1 + \zeta G_\delta}{\zeta}\right) dt}_{\text{outflow}}}_{\text{displacement}}
\end{aligned} \tag{41}
$$

The dN_{1t} is the poisson process for the positive shock to firm size, it corresponds to the inflow term which is the contribution by firms entering the top after experiencing positive shocks. And dN_{2t} is the poisson process for the negative shock to firm size, it corresponds to the outflow term which is the contribution by firms exiting the top after experiencing negative shocks. Therefore, equation (41) gives Proposition 2. Q.E.D.

B.3 Proof of Proposition 3

Turnover rate (Inflow) is calculated as the ratio of the number of firms that enters the top percentile p from $t-1$ to t to the number of firms that are inside the top percentile p. Since per unit of time, the quantile q increases by a net step size G_λ with probability λ, we can write turnover rate from inflow as:

$$
\begin{aligned}
&\lambda \frac{\int_{q/(1+G_\lambda)}^{\infty} x^{-\zeta-1}dx - \int_{q}^{\infty} x^{-\zeta-1}dx}{\int_{q}^{\infty} x^{-\zeta-1}dx} \\
=&\lambda \frac{\int_{q/(1+G_\lambda)}^{q} x^{-\zeta-1}dx}{\int_{q}^{\infty} x^{-\zeta-1}dx} \\
=&\lambda \left((1+G_\lambda)^{\zeta} - 1 \right)
\end{aligned}
\tag{42}
$$

Turnover rate (Outflow) is calculated as the ratio of the number of firms that exits the top percentile p from $t-1$ to t to the number of firms that are inside the top percentile p. Since per unit of time, the quantile q increases by a net step size G_δ with probability δ, we can write turnover rate from inflow as:

$$
\begin{aligned}
&\lambda \frac{\int_{q}^{\infty} x^{-\zeta-1}dx - \int_{q/(1-G_\delta)}^{\infty} x^{-\zeta-1}dx}{\int_{q}^{\infty} x^{-\zeta-1}dx} \\
=&\lambda \frac{\int_{q}^{q/(1-G_\delta)} x^{-\zeta-1}dx}{\int_{q}^{\infty} x^{-\zeta-1}dx} \\
=&\delta \left(1 - (1-G_\delta)^{\zeta} \right)
\end{aligned}
\tag{43}
$$

This gives Proposition 3. Q.E.D.