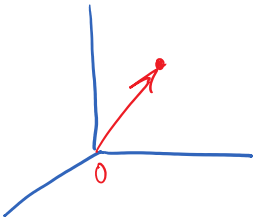
Wednesday, September 3, 2025 11:19 AM

# Linear Algebra

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$




R - matrix - rotation matrix

$$\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} \to (r_{ij})$$

$$a'_i = r_{ij} a_j = \sum_{j=1}^{3} r_{ij} a_j$$

"j" - dummy index

Vector - column matrix $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \to a_i$

Matrix $R \Rightarrow r_{ij}$

Rules $R + Q = S$ : $r_{ij} + q_{ij} = s_{ij}$

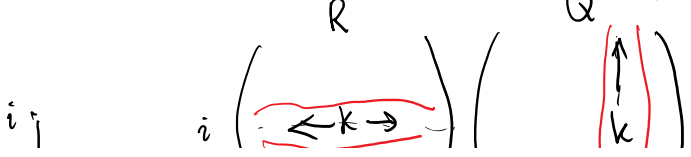
(# of columns and rows are the same for R, Q and S)

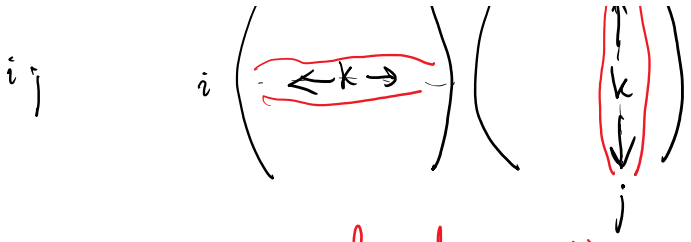
$cR = S$ : $s_{ij} = c r_{ij}$

## Matrix multiplication

$RQ = S$ : $s_{ij} = r_{ik} q_{kj}$

Ex: 3×3 $s_{ij} = r_{i1} q_{1j} + r_{i2} q_{2j} + r_{i3} q_{3j}$






second dimension of R must be the same as the first dimension of Q

$R(1 \times n)$ $Q(n \times 1)$

row

$b_1 \dots b_n$

transposed vector

$\begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} = a_1 b_1 + a_2 b_2 + \dots + a_n b_n$

vector

↑ dot product

$(1 \times n) \cdot (n \times 1)$

dummy

$r_{1k} q_{k1} = s_{11}$ → scalar product

$\vec{a} \cdot \vec{b} = A^T B$

$\Rightarrow (1)$

Transposition $A^T(A)$ : $A = a_{ij}$

$A^T = a_{ji}$