Dynamic Programming Problem Set

CIS 320

1 Problem 1: Longest Common Subsequence

Given two strings, X and Y, of lengths m and n respectively, define a dynamic programming algorithm to find the length of their longest common subsequence (LCS).

1. Write the recursive relation for the LCS problem.
2. Construct the dynamic programming table for the strings $X = AGGTAB$ and $Y = GXTXAYB$.
3. What is the time complexity of your algorithm?

2 Problem 2: Knapsack Problem

You are given a set of n items, each with a weight w_i and a value v_i. You have a knapsack that can carry a maximum weight W.

1. Formulate the 0/1 knapsack problem using dynamic programming.
2. Describe the state transition and the base cases.
3. If $n = 4$, $W = 7$, and the items are $(w_1, v_1) = (1, 1)$, $(w_2, v_2) = (3, 4)$, $(w_3, v_3) = (4, 5)$, $(w_4, v_4) = (5, 7)$, compute the maximum value that can be obtained.

3 Problem 3: Coin Change Problem

You are given an infinite supply of coins of denominations $d_1, d_2, \ldots, d_m$ and a total amount A.

1. Define a dynamic programming approach to find the minimum number of coins needed to make the amount A.
2. Write the recursive formula for the problem.
3. If the denominations are $d = \{1, 3, 4\}$ and $A = 6$, what is the minimum number of coins required?

4 Problem 4: Matrix Chain Multiplication

You are given a sequence of matrices $A_1, A_2, \ldots, A_n$ with dimensions $p_0, p_1, \ldots, p_n$.

1. Describe the dynamic programming approach to find the minimum number of scalar multiplications needed to multiply the chain of matrices.

2. Write down the recursive relation for the cost of multiplying matrices.

3. For matrices with dimensions 10×30, 30×5, and 5×60, compute the minimum multiplication cost.

5 Problem 5: Edit Distance

Given two strings A and B, define the edit distance as the minimum number of operations (insertions, deletions, substitutions) required to transform A into B.

1. Formulate the dynamic programming solution for computing the edit distance.

2. Provide the recursive relation and the base cases.

3. If $A = kitten$ and $B = sitting$, calculate the edit distance.