

# Adaptive Backstepping Active Fault-Tolerant Control with Nonlinear Adaptive Observer-Based for Quadrotor UAV under Actuator Faults and Disturbances

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**Abstract:** This paper presents a robust active fault-tolerant control (AFTC) strategy for quadrotor unmanned aerial vehicles (UAVs) that addresses actuator faults and external disturbances within a nonlinear system. The proposed approach involves using a comprehensive nonlinear model of the quadrotor. To enable simultaneous estimation of system states and actuator faults, a nonlinear adaptive observer (AO) is designed. This observer does not require the conventional observer matching condition and leverages an LMI-based optimization approach to simplify the design process. Building on this, an adaptive backstepping FTC (ABFTC) controller is proposed, utilizing AO-based fault estimation (FE) to compensate for actuator faults and adaptive control for external disturbances. Furthermore, an adaptive algorithm is integrated into the FE unit to decouple disturbances from fault estimates. The stability of the closed-loop system is validated through the Lyapunov theory. The effectiveness of the proposed FTC strategy is validated through MATLAB simulations.

**Keywords:** Active fault-tolerant control, quadrotor unmanned aerial vehicles, nonlinear adaptive observer, actuator faults, external disturbances, adaptive backstepping.

## 1. Introduction

In the last few years, quadrotors, as unmanned aerial vehicles (UAVs), have garnered significant attention and widespread adoption across various sectors, including commercial, industrial, and military applications. However, despite their many advantages, quadrotors are vulnerable to actuator faults and external disturbances, which can critically affect their stability. To address these challenges, actuator fault-tolerant control (FTC) systems have become a focal point of research.

FTC systems are categorized into passive designs, which treat faults as perturbations and use general optimization methods, which can be limiting. Active designs, however, utilize detailed fault estimation (FE) using observer-based methods (Lan & Patton, 2016) and a recovery module that takes the necessary actions to correct the faulty system for precise control adjustments and adaptability (Jain, J. Yamé, & Sauter, 2018). Among the most commonly used techniques in the recovery module of FTC systems are sliding mode control (SMC) and backstepping.

Backstepping has become an attractive control technique to deal with issues associated with underactuated systems, such as quadrotors.

Many control techniques based on backstepping have been proposed without considering the impacts of faults or disturbances, for example (Bouadi, Tadjine, & Bouchoucha, 2007) and (Saibi, Boushaki, & Belaidi, 2022).

There have been several ideas for backstepping control systems to address the impacts of disturbances such as in (Huo, Huo, & Karimi, 2014), (Xuan-Mung & Hong, 2019), (Maaruf, Hamanah, & Abido, 2023), and (Karahana, Kasnakoglu, & Nuri Akay, 2023). However, these papers do not address faults within the system.

In (Zhang et al., 2010) and (Khebbache et al., 2012), (Mlayeh & Ben Othman, 2022), (Mlayeh & Khedher, 2024), the authors propose a passive FTC technique based on a backstepping approach to overcome the effect of actuator faults without considering the effect of disturbances.

Existing control techniques based on backstepping have made significant strides, but many either do not account for both disturbances and faults or focus on one aspect while neglecting the other. And the proposed backstepping-based FTC are mostly passive strategies.

Despite its advantages, traditional backstepping, unlike conventional SMC, is sensitive to disturbances. However, practically, the upper bound of external disturbances, which is necessary for the traditional SMC, is challenging to determine perfectly a priori. These limitations highlight the necessity of improving backstepping by integrating it with other techniques, such as adaptive control, fuzzy logic, or neural networks. Such enhancements can help mitigate the impact of disturbances and improve the controller's robustness.

In this study, these gaps are addressed by proposing a novel backstepping-based active FTC strategy that simultaneously tackles both external disturbances and actuator faults.

In the field of active FTC, there is now an extensive amount of research on FE techniques, principally based on adaptive observers (AO) and sliding-mode observers (SMO) (Lan & Patton, 2016). AO enables us to estimate faults when they are modeled as changes in parameters.

For FE, several authors have used adaptive observers (Wang & Daley, 1996), (Besançon, 2007), (Zhang, Jiang, & Cocquempot, 2008), and (Gao & Duan, 2012). However, they make the assumption that the number of measured outputs and the transfer functions between faults are strictly positive real (SPR), however, for many real-world systems, including quadrotors, this is not the case. New adaptation law with relaxed SPR is proposed by (Oucief, Tadjine, & Labiod, 2016a).

This paper is based on a joint use of an AO for FE, and an **adaptive backstepping FTC (ABFTC) technique** for stabilizing the faulty system and disturbance estimation and compensation.

The primary contributions of this study are: (1) using a comprehensive nonlinear model of the quadrotor UAV that accounts for the system's nonlinearities and high-order nonholonomic constraints; (2) a nonlinear AO is used to estimate both system states and actuator faults simultaneously, without requiring the traditional observer matching condition. This approach effectively handles faults that affect unmeasured state dynamics. The observer design is simplified through a linear matrix inequality (LMI) optimization approach; (3) using the AO-based FE, an **ABFTC** controller is presented for

compensating for external disturbances and actuator faults; (4) the ABFTC is intended to estimate the unknown external disturbance and compensate for their effect without the need for a precise upper bound for external disturbances; (5) an adaptive law is designed to decouple disturbances from actuator fault estimates to obtain a more accurate FE.

**The remainder of this study is organized as follows:** Section II provides a short explanation of the system's nonlinear dynamic model. In Section III, a nonlinear AO is introduced to estimate the faults. Next, a novel law is designed to decouple disturbances from actuator fault estimates. Section IV provides a robust adaptive backstepping control technique that is used to handle external disturbances and failure effects. Finally, Section V shows the validation of the FTC strategy using MATLAB simulations.

## 2. Quadrotor Modeling

Let's consider an inertial frame E (O, X, Y, Z), and let B (o, x, y, z) designate a frame that is permanently coupled to the quadrotor, as illustrated in Figure 1. The quadrotor's absolute position is defined by the three coordinates  $(x, y, z)$  and its attitude by the three Euler's angles  $(\varphi, \theta, \psi)$  named roll, pitch, and yaw.

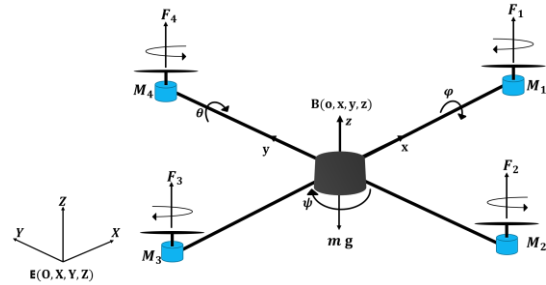


Figure 1. Quadrotor configuration

The quadrotor model is provided as in (Bouadi, Tadjine, & Bouchoucha, 2007) by:

$$\ddot{\varphi} = \frac{1}{I_x} \left( \dot{\theta} \dot{\psi} (I_y - I_z) - K_{fax} \dot{\varphi}^2 - J_r \bar{\Omega} \dot{\theta} + dU_2 \right) \quad (1a)$$

$$\ddot{\theta} = \frac{1}{I_y} \left( \dot{\varphi} \dot{\psi} (I_z - I_x) - K_{fay} \dot{\theta}^2 + J_r \bar{\Omega} \dot{\varphi} + dU_3 \right) \quad (1b)$$

$$\ddot{\psi} = \frac{1}{I_z} \left( \dot{\varphi} \dot{\theta} (I_x - I_y) - K_{faz} \dot{\psi}^2 + U_4 \right) \quad (1c)$$

$$\ddot{x} = \frac{1}{m} \left( (C\varphi S\theta C\psi + S\varphi S\psi)U_1 - K_{ftx} \dot{x} \right) \quad (1d)$$

$$\ddot{y} = \frac{1}{m} \left( (C\varphi S\theta S\psi - S\varphi C\psi)U_1 - K_{fyy} \dot{y} \right) \quad (1e)$$

$$\ddot{z} = \frac{1}{m} \left( C\varphi C\theta U_1 - K_{fz} \dot{z} \right) - g \quad (1f)$$

where:

- $C$  is the cosine function, and  $S$  is the sine function.
- $I_x$ ,  $I_y$  and  $I_z$  are the constant's inertia.
- $K_{fax}$ ,  $K_{fay}$  and  $K_{faz}$  are the aerodynamic friction coefficients around X, Y, and Z.
- $J_r$  is the rotor inertia.
- $\bar{\Omega}$  is the disturbance due to the rotor imbalance.
- $d$  is the distance between the quadrotor centre of gravity and the rotation axis of propellers.
- $m$  is the quadrotor's mass.
- $g$  is the gravity acceleration constant.
- $K_{fix}$ ,  $K_{fyz}$  and  $K_{fyz}$  represent the translation drag coefficients.
- $U_1$ ,  $U_2$ ,  $U_3$ , and  $U_4$  represent the control inputs of the system.

$U_1$ ,  $U_2$ ,  $U_3$ , and  $U_4$  are expressed based on the angular speeds  $\omega_i$  (for  $i \in \{1, 2, 3, 4\}$ ) of the four propellers as follows:

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} K_p & K_p & K_p & K_p \\ -K_p & 0 & K_p & 0 \\ 0 & -K_p & 0 & K_p \\ K_d & -K_d & K_d & -K_d \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} \quad (2)$$

$$\bar{\Omega} = \omega_1 - \omega_2 + \omega_3 - \omega_4 \quad (3)$$

The control inputs remain restricted by the motors' maximum rotational speeds  $\omega_{\max}$ , which are illustrative of their physical constraints:

$$\begin{aligned} 0 &\leq U_1 \leq 4K_p \omega_{\max}^2 \\ -K_p \omega_{\max}^2 &\leq U_2 \leq K_p \omega_{\max}^2 \\ -K_p \omega_{\max}^2 &\leq U_3 \leq K_p \omega_{\max}^2 \\ -2K_d \omega_{\max}^2 &\leq U_4 \leq 2K_d \omega_{\max}^2 \end{aligned} \quad (4)$$

The high-order nonholonomic constraints may be obtained from the translation dynamics equations in (1):

$$\sin \varphi = \frac{\left( \ddot{x} + \frac{K_{fix}}{m} \dot{x} \right) S\psi - \left( \ddot{y} + \frac{K_{fyz}}{m} \dot{y} \right) C\psi}{\sqrt{\left( \ddot{x} + \frac{K_{fix}}{m} \dot{x} \right)^2 + \left( \ddot{y} + \frac{K_{fyz}}{m} \dot{y} \right)^2 + \left( \ddot{z} + g + \frac{K_{fz}}{m} \dot{z} \right)^2}} \quad (5a)$$

$$\tan \theta = \frac{\left( \ddot{x} + \frac{K_{fix}}{m} \dot{x} \right) C\psi + \left( \ddot{y} + \frac{K_{fyz}}{m} \dot{y} \right) S\psi}{\ddot{z} + g + \frac{K_{fz}}{m} \dot{z}} \quad (5b)$$

This corrective block will be used to generate the intended roll ( $\varphi_d$ ) and pitch ( $\theta_d$ ).

### 3. Adaptive observer design

#### 3.1. Adaptive observer design

The system (1) state space, including actuator faults, is given by:

$$\begin{cases} \dot{x}(t) = Ax(t) + B\Phi(x, u) + \eta(y, u) + Ef(x) \\ y(t) = Cx(t) \end{cases} \quad (6)$$

where  $x \in \mathbb{R}^{12}$  is the state vector of the system, such as:

$$x = [x_1, \dots, x_{12}]^T = [\varphi, \theta, \psi, x, y, z, \dot{\varphi}, \dot{\theta}, \dot{\psi}, \dot{x}, \dot{y}, \dot{z}]^T \quad (7)$$

where  $x_i$  for  $i \in \{1, 12\}$  are the system states.

$A \in \mathbb{R}^{12 \times 12}$ ,  $B \in \mathbb{R}^{12 \times 14}$ ,  $E \in \mathbb{R}^{12 \times 4}$ , and  $C \in \mathbb{R}^{6 \times 12}$  are known constant matrices.  $f(x) = \sigma(x)f_a(t)$ , is

the resultant of the actuator faults, where  $f_a \in \mathbb{R}^4$  represent the actuator faults vector, with

$f_a = [f_{a1}, f_{a2}, f_{a3}, f_{a4}]^T$ .  $\sigma(x): \mathbb{R}^{12} \rightarrow \mathbb{R}^{4 \times 4}$  is a

known function matrix that might have nonlinear dependencies on  $x$ . And  $\eta(y, u)$  and  $\Phi(x, u)$  are known nonlinear functions vectors.

$u = [U_1, U_2, U_3, U_4]^T$  is the input control vector,

$y \in \mathbb{R}^6$  is the system output giving by

$$y = [\varphi, \theta, \psi, x, y, z]^T.$$

Throughout this article, system model (6) satisfies the following conditions:

**C0:** The pair  $(C, A)$  is observable;

**C1:**  $\eta(y, u)$  is continuous in  $y$  and  $u$ ;

**C2:**  $\sigma(x)$  and  $\Phi(x, u)$  fulfill the Lipschitz property, i.e. there exist positive constants  $\gamma_1$  and  $\gamma_2$  such that for all  $x, \hat{x} \in \mathbb{R}^{12}$ :

$$\|\Phi(x, u) - \Phi(\hat{x}, u)\| \leq \gamma_1 \|x - \hat{x}\| \quad (8a)$$

$$\|\sigma(x) - \sigma(\hat{x})\| \leq \gamma_2 \|x - \hat{x}\| \quad (8b)$$

**C3:** The actuator fault vector  $f_a$  is bounded and piecewise constant:

$$\|f_a(t)\| \leq \gamma_3 \quad (9)$$

where  $\gamma_3$  is a known positive constant.

**C4:** The matrix  $E \sigma(x)$  is persistently exciting, such that for all  $t \geq 0$ :

$$n_1 I_{12} \geq \int_t^{t+\tau} E \sigma(x) \sigma(x)^T E^T dt \geq n_2 I_{12} \quad (10)$$

where  $\tau$ ,  $n_1$  and  $n_2$  are positive constants.

$I_{12} \in \mathbb{R}^{12 \times 12}$  represent the identity matrix.

The standard form of the AO for the system (6) is given in (Cho & Rajamani, 1997). To be used in FE, the system must be satisfying the famous matching condition given by:

$$E^T P = FC \quad (11)$$

The equality  $E^T P = FC$  hold if and only if (Corless & Tu, 1998):

$$\text{rank}(CE) = \text{rank}(E) \quad (12)$$

In the case of our model given by (1) and (6),  $\text{rank}(CE) = 0$  and  $\text{rank}(E) = 4$ . Unfortunately,

the observer matching requirement (12) is not fulfilled for our system, and therefore the standard form of the AO cannot be used.

In (Oucief, Tadjine, & Labiod, 2016a), authors have proposed a novel approach for developing an AO for a particular class of nonlinear systems.

For developing the considered adaptive observer, in addition to conditions C0, C1, C2 and C3 the system model (6) has to fulfill the following conditions.

**C5:** The matrices  $A$ ,  $B$ ,  $C$  and  $E$  must satisfy:

$$CB = 0_{6 \times 14} \quad (13a)$$

$$CE = 0_{6 \times 4} \quad (13b)$$

$$\text{rank}(CAE) = \text{rank}(E) \quad (13c)$$

**C6:** Given bounded  $x$ , the first derivative in time of  $\sigma(x)$  is continuous and bounded.

The system state space  $x = [x_1, \dots, x_{12}]^T$ , as given by (7), is rearranged to satisfy condition C5 as follows:

$$\begin{aligned} \dot{x}_1 &= x_7 \\ \dot{x}_2 &= x_8 \\ \dot{x}_3 &= x_9 \\ \dot{x}_4 &= x_{10} \\ \dot{x}_5 &= x_{11} \\ \dot{x}_6 &= x_{12} \\ \dot{x}_7 &= a_1 x_8 x_9 + a_2 x_7^2 + a_3 \bar{\Omega} x_8 + b_1 (U_2 + f_{a1}) \end{aligned} \quad (14)$$

$$\dot{x}_8 = a_4 x_7 x_9 + a_5 x_8^2 + a_6 \bar{\Omega} x_7 + b_2 (U_3 + f_{a2})$$

$$\dot{x}_9 = a_7 x_7 x_8 + a_8 x_9^2 + b_3 (U_4 + f_{a3})$$

$$\dot{x}_{10} = a_9 x_{10} + U_x \frac{U_1}{m}$$

$$\dot{x}_{11} = a_{10} x_{11} + U_y \frac{U_1}{m}$$

$$\dot{x}_{12} = a_{11} x_{12} - g + \frac{\cos(x_1) \cos(x_2)}{m} (U_1 + f_{a4})$$

where

$$\begin{aligned} a_1 &= \frac{I_y - I_z}{I_x} & a_2 &= \frac{-K_{fax}}{I_x} & a_3 &= \frac{-J_r}{I_x} \\ a_4 &= \frac{I_z - I_x}{I_y} & a_5 &= \frac{-K_{fay}}{I_y} & a_6 &= \frac{J_r}{I_y} \\ a_7 &= \frac{I_x - I_y}{I_z} & a_8 &= \frac{-K_{faz}}{I_z} & a_9 &= \frac{-K_{fzx}}{m} \\ a_{10} &= \frac{-K_{fzy}}{m} & a_{11} &= \frac{-K_{fyz}}{m} & b_1 &= \frac{d}{I_x} \\ b_2 &= \frac{d}{I_y} & b_3 &= \frac{1}{I_z} \end{aligned} \quad (15)$$

When these conditions are satisfied, a stable observer for the system (6) has the form (Oucief, Tadjine, & Labiod, 2016a):

$$\dot{\hat{x}} = A\hat{x} + B\Phi(\hat{x}, u) + \eta(y, u) + E\hat{f}(\hat{x}) + L(y - C\hat{x}) \quad (16a)$$

$$\hat{f}_a = W + \Gamma \sigma^T(\hat{x}) Hy \quad (16b)$$

$$\dot{W} = -\Gamma \frac{d\sigma^T(\hat{x})}{dt} Hy - \Gamma \sigma^T(\hat{x}) \begin{bmatrix} HC(A\hat{x} + \eta(y, u)) + \\ G(y - C\hat{x}) \end{bmatrix} \quad (16c)$$

where  $\hat{f}(\hat{x}) = \sigma(\hat{x}) \hat{f}_a(t)$ , and  $\hat{f}_a$  is the actuator FE vector.  $\Gamma = \Gamma^T > 0$  is the learning rate matrix, while  $H$  and  $G$  are constant matrices that need to be found.

**Theorem 1.** Under conditions C1, C2, C3, C5, and C6 the state  $\hat{x}$  estimate converges to the real one  $x$  asymptotically, and  $E\hat{f}(\hat{x})$  converges to  $Ef(x)$  if there are positive real constants  $\varepsilon_1$  and  $\varepsilon_2$ , and matrices  $P = P^T > 0 \in \mathbb{R}^{12 \times 12}$ ,  $G \in \mathbb{R}^{4 \times 6}$ , and  $H \in \mathbb{R}^{4 \times 6}$  such that:

$$\begin{aligned} (A - LC)^T P + P(A - LC) + \varepsilon_1 PBB^T P + \varepsilon_2 PEE^T P \\ + \varepsilon_1^{-1} \gamma_1^2 I_{12} + \varepsilon_2^{-1} \gamma_2^2 \gamma_3^2 I_{12} < 0 \end{aligned} \quad (17a)$$

$$HCA - GC = E^T P \quad (17b)$$

Furthermore, if  $E \sigma(x)$  fulfills the PE condition given by C4, then  $\hat{f}_a$  converges to  $f_a$ .

Conditions C0 to C6 are met for our system, hence the adaption law (15) is possible. To find the observer gains, (17a) and (17b) can be turned into an LMI optimization problem. Applying the Schur complement (Boyd et al., 1994) and by considering  $L = P^{-1}M$  for inequality (17a), the following LMI is obtained:

$$\begin{bmatrix} \Lambda & PB & PE \\ B^T P & -\varepsilon_1^{-1} I_{14} & 0_{14 \times 4} \\ E^T P & 0_{4 \times 14} & -\varepsilon_2^{-1} I_4 \end{bmatrix} < 0 \quad (18)$$

with  $\Lambda = A^T P + PA - C^T M^T - MC + \varepsilon_1^{-1} \gamma_1^2 I_{12} + \varepsilon_2^{-1} \gamma_2^2 \gamma_3^2 I_{12}$ .

Also, by applying the same technique utilized in (Corless & Tu, 1998), (17b) may be solved using:

Minimize  $\delta$  subject to:

$$\begin{bmatrix} \delta I_4 & \Pi \\ \Pi^T & \delta I_{12} \end{bmatrix} \geq 0 \quad (19)$$

where  $\Pi = HCA - GC - E^T P$ , and  $\delta$  is a positive scalar.

### 3.2. Disturbances decoupling

Suppose the attitude system of the quadrotor is subject to external disturbances  $d = [d_\varphi, d_\theta, d_\psi]^T$ . The states  $x_7$ ,  $x_8$ , and  $x_9$  in (14) becomes.

$$\dot{x}_7 = a_1 x_8 x_9 + a_2 x_7^2 + a_3 \bar{\Omega} x_8 + b_1 (U_2 + f_{a1}) + \frac{1}{I_x} d_\varphi \quad (20a)$$

$$\dot{x}_8 = a_4 x_7 x_9 + a_5 x_8^2 + a_6 \bar{\Omega} x_7 + b_2 (U_3 + f_{a2}) + \frac{1}{I_y} d_\theta \quad (20b)$$

$$\dot{x}_9 = a_7 x_7 x_8 + a_8 x_9^2 + b_3 (U_4 + f_{a3}) + \frac{1}{I_z} d_\psi \quad (20c)$$

As a result, disturbances will significantly impair the accuracy of FE, resulting in inaccurate FE and potential false alarms. To tackle this issue, an adaptive law is created to separate disturbances from actuator fault estimates.

$$\hat{f}_a^\dagger(t) = \hat{f}_a(t) - D\hat{d}(t) \quad (21)$$

where  $\hat{f}_a^\dagger$  represents the improved fault estimate,  $\hat{f}_a$  is the original fault estimate given by (16),  $\hat{d} = [\hat{d}_\varphi, \hat{d}_\theta, \hat{d}_\psi]^T$  is the disturbance estimation vector, and  $D = \text{diag}(\lambda_1, \lambda_2, \lambda_3)$  is a diagonal matrix with  $\lambda_i \geq 0$  for  $i \in \{1, 2, 3\}$ . The parameters  $\lambda_i$  are determined through a tuning

process based on simulation to optimize the disturbance decoupling and enhance the accuracy of FE.

Based on (21), the improved attitude actuator faults estimate becomes:

$$\begin{aligned} \hat{f}_{a1}^\dagger &= \hat{f}_{a1} - \lambda_1 \hat{d}_\varphi \Theta_1 \\ \hat{f}_{a2}^\dagger &= \hat{f}_{a2} - \lambda_2 \hat{d}_\theta \Theta_2 \\ \hat{f}_{a3}^\dagger &= \hat{f}_{a3} - \lambda_3 \hat{d}_\psi \Theta_3 \end{aligned} \quad (22)$$

where  $\Theta_i$  for  $i \in \{1, 2, 3\}$  are activation parameters given by:

$$\Theta_i = \begin{cases} 0 & \text{if } |\hat{f}_{ai}| \leq \Delta_i \\ 1 & \text{if } |\hat{f}_{ai}| > \Delta_i \end{cases} \quad (23)$$

where  $\Delta_i$  for  $i \in \{1, 2, 3\}$  is the threshold parameter.

## 4. FTC strategy of the quadrotor

The proposed control approach is based on two loops, the internal loop has four control laws ( $U_1$ ,  $U_2$ ,  $U_3$ , and  $U_4$ ), and the external loop has two virtual control laws ( $U_x$  and  $U_y$ ). The synoptic scheme (Figure 2) below illustrates this control strategy.

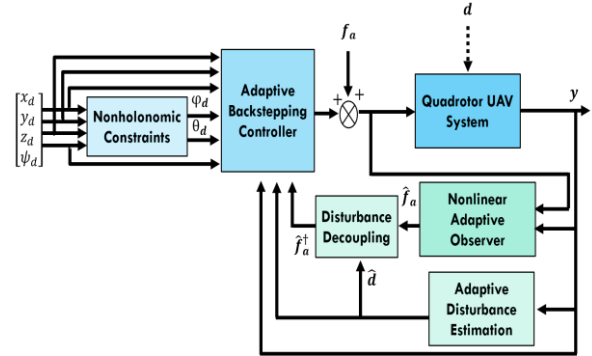


Figure 2. Proposed FTC structure

All phases of computation concerning Lyapunov functions and tracking errors are summarized as follows:

$$e_i = \begin{cases} x_i - x_{id} & i \in \{1, 2, 3, 4, 5, 6\} \\ x_i - \dot{x}_{(i-6)d} + k_{(i-6)} e_{(i-6)} & i \in \{7, 8, 9, 10, 11, 12\} \end{cases} \quad (24)$$

The related Lyapunov functions are provided by

$$V_i = \begin{cases} \frac{1}{2} e_i^2 & i \in \{1, 2, 3, 4, 5, 6\} \\ V_{i-6} + \frac{1}{2} e_i^2 & i \in \{7, 8, 9, 10, 11, 12\} \end{cases} \quad (25)$$

The synthesized stabilizing control laws are described in the following:

$$U_2 = \frac{1}{b_1} \left[ \ddot{\varphi}_d - a_1 x_8 x_9 - a_2 x_7^2 - a_3 \bar{\Omega} x_8 - k_1 (-k_1 e_1 + e_7) - e_1 - k_7 e_7 - \frac{1}{I_x} \hat{d}_\varphi \right] - \hat{f}_{a1}^\dagger - k_{a1} \text{sign}(e_7) \quad (26a)$$

$$U_3 = \frac{1}{b_2} \left[ \ddot{\theta}_d - a_4 x_7 x_9 - a_5 x_8^2 - a_6 \bar{\Omega} x_7 - k_2 (-k_2 e_2 + e_8) - e_2 - k_8 e_8 - \frac{1}{I_y} \hat{d}_\theta \right] - \hat{f}_{a2}^\dagger - k_{a2} \text{sign}(e_8) \quad (26b)$$

$$U_4 = \frac{1}{b_3} \left[ \ddot{\psi}_d - a_7 x_7 x_8 - a_8 x_9^2 - k_3 (-k_3 e_3 + e_9) - e_3 - k_9 e_9 - \frac{1}{I_z} \hat{d}_\psi \right] - \hat{f}_{a3}^\dagger - k_{a3} \text{sign}(e_9) \quad (26c)$$

$$U_x = \frac{m}{U_1} \left[ \ddot{x}_d - a_9 x_{10} - k_4 (-k_4 e_4 + e_{10}) - e_4 - k_{10} e_{10} \right] \quad (26d)$$

$$U_y = \frac{m}{U_1} \left[ \ddot{y}_d - a_{10} x_{11} - k_5 (-k_5 e_5 + e_{11}) - e_5 - k_{11} e_{11} \right] \quad (26e)$$

$$U_1 = \frac{m}{c x_1 c x_2} \left[ \ddot{z}_d - a_{11} x_{12} + g - k_6 (-k_6 e_6 + e_{12}) - e_6 - k_{12} e_{12} \right] - \hat{f}_{a4}^\dagger - k_{a4} \text{sign}(e_{12}) \quad (26f)$$

where  $k_i > 0$  for  $i \in \{1, 12\}$  and  $k_{ai} > 0$  for  $i \in \{1, 2, 3, 4\}$ .

Disturbances estimates are given by:

$$\dot{\hat{d}}_\varphi = \beta_1 e_7 \quad \dot{\hat{d}}_\theta = \beta_2 e_8 \quad \dot{\hat{d}}_\psi = \beta_3 e_9 \quad (27)$$

where  $\beta_i > 0$  for  $i \in \{1, 2, 3\}$ .

### Proof.

Let's demonstrate the expression of  $U_2$ , considering the roll ( $\varphi$ ) subsystem:

$$\begin{cases} \dot{x}_1 = x_7 \\ \dot{x}_7 = \Phi_1(x) + b_1 U_2(t) + b_1 f_{a1}(t) + \frac{1}{I_x} d_\varphi(t) \end{cases} \quad (28)$$

where  $\Phi_1(x) = a_1 x_4 x_6 + a_2 x_2^2 + a_3 \bar{\Omega} x_4 \cdot f_{a1}$  and  $d_\varphi$  are the actuator fault and disturbances cannot be measured respectively. The calculation of the law control  $U_2$  is done in two steps.

**Step 1:** The first tracking error  $e_1$  is examined, and provided by

$$e_1 = x_1 - x_{1d} \quad (29)$$

Let the first Lyapunov function candidate:

$$V_1(e_1) = \frac{1}{2} e_1^2 \quad (30)$$

The time derivative of (30) is given by:

$$\dot{V}_1(e_1) = e_1 \dot{e}_1 = e_1 (x_7 - \dot{x}_{1d}) \quad (31)$$

According to Lyapunov's theory, the stability of  $e_1$  can be achieved by incorporating a new virtual control  $(x_7)_d$  which represents the desired value of  $x_7$ :

$$(x_7)_d = \alpha_1 = \dot{x}_{1d} - k_1 e_1 \quad (k_1 > 0) \quad (32)$$

The equation (31) is then  $\dot{V}_1(e_1) = -k_1 e_1^2 \leq 0$

**Step 2:** As  $x_7$  is not a real command, the following tracking-error variable  $e_7$  is defined between  $x_7$  and  $\alpha_1$ .

$$e_7 = x_7 - \alpha_1 = x_7 - \dot{x}_{1d} + k_1 e_1 \quad (33)$$

The augmented Lyapunov function is given by:

$$V_7(e_1, e_7) = V_1 + \frac{1}{2} e_7^2 \quad (34)$$

The time derivative of  $V_7$  is given by:

$$\dot{V}_7(e_1, e_7) = e_1 \dot{e}_1 + e_7 \dot{e}_7 \quad (35)$$

Using  $\dot{e}_1 = -k_1 e_1 + e_7$ , (35) becomes:

$$\dot{V}_7(e_1, e_7) = e_1 (-k_1 e_1 + e_7) + e_7 [\dot{x}_7 - \ddot{x}_{1d} + k_1 (-k_1 e_1 + e_7)] \quad (36)$$

Substituting  $\dot{x}_7$  by its expression, (36) yields

$$\dot{V}_7(e_1, e_7) = e_1 (-k_1 e_1 + e_7) + e_7 [(\Phi_1 + b_1 U_2 + b_1 f_{a1} + \frac{1}{I_x} d_\varphi - \ddot{x}_{1d} + k_1 (-k_1 e_1 + e_7))] \quad (37)$$

The stability of  $(e_1, e_7)$  may be achieved by adding the real input control  $U_2$ . Based on the principle of certain equivalence,  $f_{a1}$  and  $d_\varphi$  are replaced by their estimates:

$$U_2 = \frac{1}{b_1} \left( \ddot{\varphi}_d - \Phi_1 - k_1 (-k_1 e_1 + e_7) - e_1 - k_7 e_7 - b_1 \hat{f}_{a1}^\dagger - b_1 \Gamma_1 - \frac{1}{I_x} \hat{d}_\varphi \right) \quad (38)$$

The equation (37) becomes

$$\dot{V}_7(e_1, e_7) = -k_1 e_1^2 - k_7 e_7^2 + e_7 (b_1 \tilde{f}_{a1} - b_1 \Gamma_1 + \frac{1}{I_x} \tilde{d}_\varphi) \quad (39)$$

where  $\tilde{f}_{a1} = f_{a1} - \hat{f}_{a1}^\dagger$  and  $\tilde{d}_\varphi = d_\varphi - \hat{d}_\varphi$  (assuming  $\dot{d}_\varphi \approx 0$ ). The presence of term error  $\tilde{d}_\varphi$  in the expression of  $\dot{V}_7$  does not allow the determination of its sign. The Lyapunov function (34) is increased by a square term to  $\tilde{d}_\varphi$ .

$$V_7^\dagger(e_1, e_7) = V_7(e_1, e_7) + \frac{1}{2\beta_1 I_x} \tilde{d}_\varphi^2 \quad (\beta_1 > 0) \quad (40)$$

The time derivative of  $V_7^\dagger$ :

$$\dot{V}_7^\dagger(e_1, e_7) = \dot{V}_7(e_1, e_7) - \frac{1}{\beta_1 I_x} \tilde{d}_\varphi \dot{\tilde{d}}_\varphi \quad (41)$$

$$= -k_1 e_1^2 - k_7 e_7^2 + b_1 e_7 (\tilde{f}_{a1} - \Gamma_1) + \frac{1}{I_x} \tilde{d}_\varphi \left( e_7 - \frac{1}{\beta_1} \dot{\tilde{d}}_\varphi \right)$$

Selecting a proper law of adaptation for the estimate  $\hat{d}_\varphi$  will eliminate the term of uncertainty. Choosing

$$\dot{\tilde{d}}_\varphi = \beta_1 e_7 \quad (42)$$

The expression (41) becomes

$$\dot{V}_7^\dagger(e_1, e_7) = -k_1 e_1^2 - k_7 e_7^2 - b_1 e_7 (\Gamma_1 - \tilde{f}_{a1}) \quad (43)$$

The presence of term errors  $\tilde{f}_{a1}$  in the expression of  $\dot{V}_7^\dagger$  does not allow the determination of its sign. **By choosing**

$$\Gamma_1 = k_{a1} \text{sign}(e_7) \quad (k_{a1} > 0) \quad (44)$$

The equation (43) is then

$$\dot{V}_7^\dagger(e_1, e_7) \leq -k_1 e_1^2 - k_7 e_7^2 - b_1 |e_7| (k_{a1} - |\tilde{f}_{a1}|) \quad (45)$$

Suppose there exists an unknown parameter  $k_{a1} > 0$ , such that:

$$|\tilde{f}_{a1}| \leq k_{a1} \quad (46)$$

Finally, the equation (45) becomes

$$\dot{V}_7^\dagger(e_1, e_7) \leq -k_1 e_1^2 - k_7 e_7^2 \leq 0 \quad (47)$$

Finally

$$U_2 = \frac{1}{b_1} \begin{bmatrix} \ddot{\varphi}_d - a_1 x_8 x_9 - a_2 x_7^2 - a_3 \ddot{\Omega} x_8 - k_1 (-k_1 e_1 + e_7) - \\ e_1 - k_7 e_7 - b_1 \hat{f}_{a1} - b_1 k_{a1} \text{sign}(e_7) - \frac{1}{I_x} \hat{d}_\varphi \end{bmatrix} \quad (48)$$

Following the same steps,  $U_3$ ,  $U_4$ ,  $U_x$ ,  $U_y$  and  $U_1$  can be extracted.

High-frequency switching of the control signal can cause chattering, which wears down actuators and degrades system performance, for this reason, the sign function may be substituted with another smooth function. **In this paper, the following function is chosen:**

$$\text{sign}(e_i) = \frac{e_i}{e_i + \varepsilon} \quad i \in \{7, 8, 9, 12\} \quad (49)$$

where  $\varepsilon$  is a given small positive constant.

## 5. Simulation results and analysis

To evaluate the performance of the proposed ABFTC, simulations were executed in MATLAB/Simulink with a fundamental sample time of 0.001s. The quadrotor subject of our study is the Draganfly IV, manufactured by "Draganfly Innovations". Parameter identification is studied in (Derafa, Madani, & Benallegue, 2006) and summarized below:

**Table 1.** Quadrotor model parameters

Parameter	Value
$m$	400 g
$g$	9.81 m.s <sup>-2</sup>
$d$	25.5 cm
$J_r$	$2.8385 \times 10^{-5}$ N.m / rad / s <sup>2</sup>
$K_p$	$2.9842 \times 10^{-5}$ N / rad / s
$K_d$	$3.232 \times 10^{-7}$ N.m / rad / s
$I_x, I_y, I_z$	$(3.828, 3.828, 7.135) \times 10^{-3}$ N.m / rad / s <sup>2</sup>
$K_{fTx}, K_{fTy}, K_{fTz}$	$(3.2, 3.2, 4.8) \times 10^{-2}$ N / m / s
$K_{fax}, K_{fay}, K_{faz}$	$(5.567, 5.567, 6.354) \times 10^{-4}$ N / rad / s

### 5.1. Observer design

In this subsection, the parameters and the matrices of the observer will be determined.

**The values  $\gamma_1 = 35$ ,  $\gamma_2 = 2.06$ , and  $\gamma_3 = 10.2$  were chosen.** Let  $\varepsilon_1 = 76$  and  $\varepsilon_2 = 80$ .

The observer matrices are given by

$$\begin{aligned} P &= 10^3 \times \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & 0_{6 \times 6} \end{bmatrix} \\ P_{11} &= \text{diag}(8.598, 8.598, 7.683, 5.823, 5.823, 6.715) \\ P_{12} &= -10^{-2} \times \text{diag}(1.11, 1.11, 1.11, 1.19, 1.19, 1.12) \\ M &= 10^4 \times [M_1 \quad M_2]^T \\ M_1 &= \text{diag}(2.881, 2.881, 2.336, 0.709, 0.709, 1.240) \\ M_2 &= \text{diag}(0.846, 0.846, 0.755, 0.572, 0.572, 0.661) \\ H &= [H_1 \quad 0_{4 \times 1} \quad H_2] \\ H_1 &= \text{diag}(0.0286, 0.0286, 0.0342, 0) \\ H_2 &= [0 \quad 0 \quad 0 \quad 0.0475]^T \\ L &= 10^5 \times [L_1 \quad L_2]^T \\ L_1 &= 10^{-3} \times \text{diag}(7.6, 7.6, 6.1, 1.6, 1.6, 3.9) \\ L_2 &= \text{diag}(5.916, 5.916, 4.181, 0.795, 0.795, 2.307) \\ \Gamma &= 9.10^3 \times I_4 \quad \delta = 1.0001 \times 10^{-5} \end{aligned} \quad (50)$$

### 5.2. Simulation parameters

In addition, to obtain realistic simulations, additive Gaussian noise is introduced during all



simulations in both disturbances and injected faults.

To simulate the nonlinear dynamics of the used quadrotor in a real-world environment, control inputs are constrained according to equation (4), based on the assumption that the maximum rotor speed is 8000 rpm. The linear velocities ( $\dot{x}$ ,  $\dot{y}$ , and  $\dot{z}$ ) and angular velocities ( $\dot{\phi}$ ,  $\dot{\theta}$ , and  $\dot{\psi}$ ) are bounded in practical values.

The proposed paths in the following simulations are selected to effectuate a helicoidal trajectory:

$$\psi_d(t) = \begin{cases} \frac{\pi}{6} & 15 \leq t < 55 \\ -\frac{\pi}{6} & 70 \leq t < 110 \end{cases} \quad (51)$$

$$x_d(t) = 22\sin(0.6t), \quad y_d(t) = 22\cos(0.6t)$$

and  $z_d(t) = 0.5t$ .

The simulations were carried out in two scenarios, a fault-free case and a faulty case. Two time-varying actuator faults  $f_{a1}$  and  $f_{a4}$  associated with roll  $\phi$  and altitude  $z$  commands are introduced. In addition, additive gaussian

noise  $\mathcal{N}_{a1}(0.05, 0.01^2)$  is added to  $f_{a1}$  and  $\mathcal{N}_{a4}(0.01, 0.02^2)$  is added to  $f_{a4}$  with sample time of  $0.02s$  in both cases.

In both scenarios, the system is assumed to be suffering from attitude disturbances, given as:

$$d_\phi = d_\theta = 1\sin(0.2t) + \mathcal{N}_{\phi,\theta}(0.0005, 0.0005^2)$$

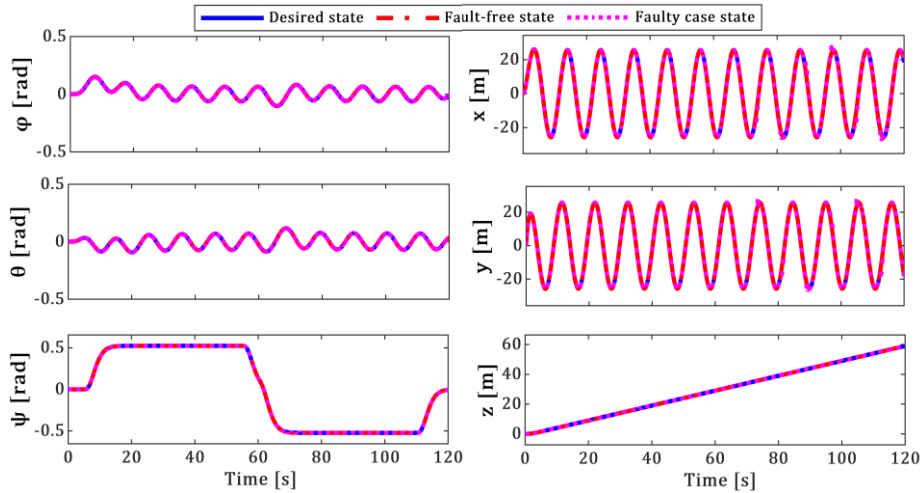
$$d_\psi = 0.2\sin(0.2t) + \mathcal{N}_\psi(0.0001, 0.00005^2)$$

Disturbance decoupling parameters are sited as:  $\lambda_1 = 3.95$ ,  $\lambda_2 = 3.95$ , and  $\lambda_3 = 1.04$ .

### 5.3. Simulation results

Using the proposed ABFTC, Figure 3 provides a comparative analysis of the system's attitude and position states under fault-free and actuator fault conditions. It reveals that, despite the introduction of faults, the states consistently converge to the desired values, indicating the ABFTC's efficacy in fault handling.

As observed, the roll and pitch angles are maintained within a moderate range of  $\pm 5^\circ$ , these values effectively facilitate the quadrotor's smooth movement.



**Figure 3.** Attitude and position trajectories in fault-free and faulty case

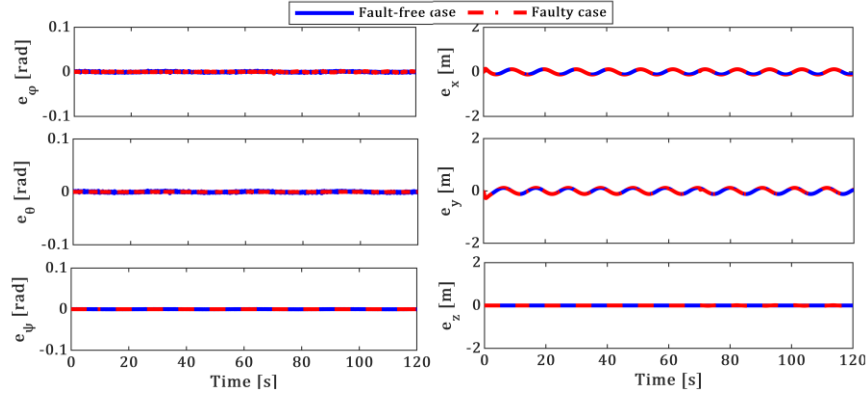
Figure 4 depicting tracking errors in attitude and position demonstrates the system's performance in both fault-free and faulty scenarios.

Figure 5 illustrates the disturbances estimation performance given by the adaptive law control, the estimated disturbances align closely with the real ones, facilitating effective decoupling from the system dynamics.

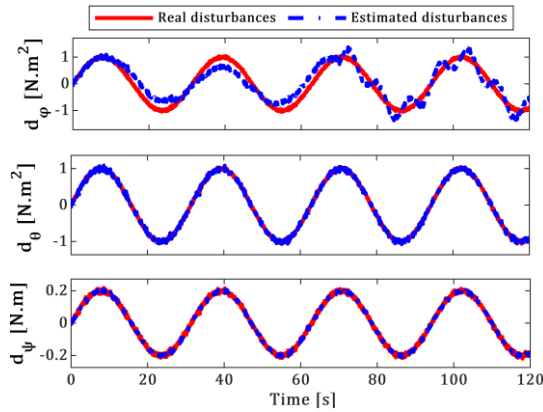
In Figure 6, the first figure presents the real actuator fault  $f_{a1}$  alongside its estimates with and without disturbance decoupling. It is evident that disturbance decoupling significantly enhances the accuracy of FE. The second figure corroborates this by showing the estimation of  $f_{a4}$ , which closely matches the real fault.

Figure 7 displays control inputs in the faulty case.

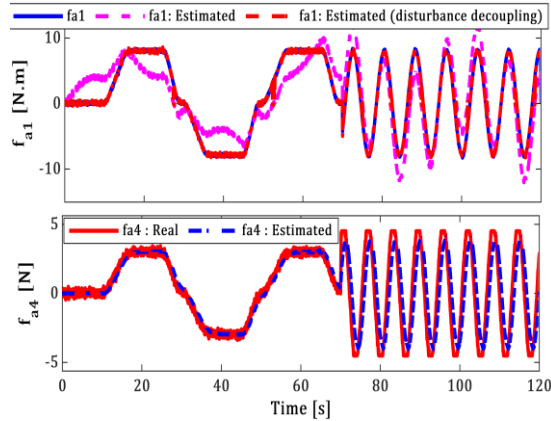




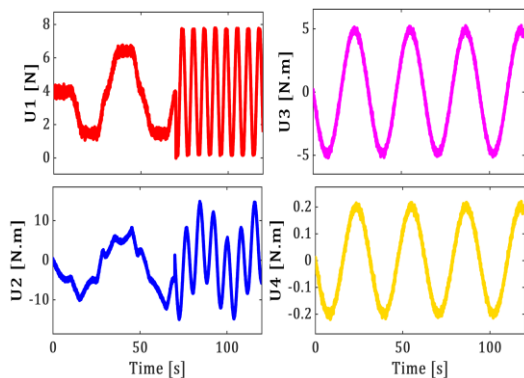
**Figure 4.** Tracking errors in fault-free and faulty case



**Figure 5.** Attitude disturbance estimation



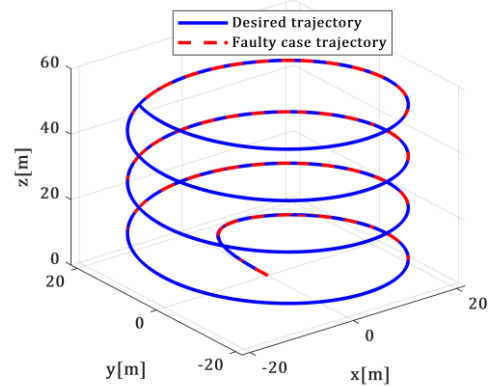
**Figure 6.** Actuator fault estimation performance



**Figure 7.** Control inputs of actuators in faulty case  
<http://www.sic.ici.ro>

Figure 7 reveals how the system adapts its inputs to manage faults effectively. Despite this, the quadrotor's closed-loop dynamics remain stable. Furthermore, this control approach provides input control signals that are both acceptable and physically achievable, reflecting the robustness and practicality of the proposed FTC approach. And also preserves a low energy consumption with small control inputs.

Figure 8 illustrates the quadrotor aircraft's 3D trajectory during its flight.



**Figure 8.** The quadrotor global trajectory in 3D in free and faulty case

The simulation results show good performance and resilience along trajectory tracking even after actuator faults occur.

To numerically evaluate the results obtained from the simulations, the RMSE (Root Mean Square Error) will be calculated using established numerical criteria. (Table 2).

While faults introduced in the roll ( $\varphi$ ) and vertical ( $z$ ) axes led to moderate increases in RMSE, pitch ( $\theta$ ), yaw ( $\psi$ ), and horizontal ICI Bucharest © Copyright. All rights reserved

positions  $(x, y)$  remained unaffected, highlighting the controller's robustness. The relatively low increase in RMSE, even in the faulty case, showcases the method's ability to isolate faults and maintain stable performance.

**Table 2.** RMSE values of attitude (on rad) and position (on m) trajectories

	Fault-free case	Faulty case
$RMSE_{\varphi}$	$9,4 \cdot 10^{-5}$	$3,4 \cdot 10^{-4}$
$RMSE_{\theta}$	$9,4 \cdot 10^{-5}$	$9,4 \cdot 10^{-5}$
$RMSE_{\psi}$	$5,4 \cdot 10^{-6}$	$5,4 \cdot 10^{-6}$
$RMSE_x$	$2,2 \cdot 10^{-2}$	$2,2 \cdot 10^{-2}$
$RMSE_y$	$2,3 \cdot 10^{-2}$	$2,3 \cdot 10^{-2}$
$RMSE_z$	$7,7 \cdot 10^{-6}$	$2,4 \cdot 10^{-3}$

**Table 3.** RMSE values of actuator FE under different conditions

	$\hat{f}_{a1}$	$\hat{f}_{a1}^{\dagger}$	$\hat{f}_{a4}$
Absence of $f_a$	2.85	0.07	0.04
Absence of $d_i$	0.13	0.13	0.93
Presence of $f_a$ and $d_i$	2.85	0.36	0.93

In Table 3, in the fault-free case, where the system operates without faults but with disturbances, disturbance decoupling significantly enhances FE accuracy, with an RMSE of  $0.07 \text{ rad}$  for  $\hat{f}_{a1}^{\dagger}$ , compared to  $2.85 \text{ rad}$  for  $\hat{f}_{a1}$  without decoupling. This improvement helps in accurately distinguishing between faults and disturbances, thus preventing false alarms and avoiding incorrect fault declarations.

In the absence of disturbances, where faults are present but disturbances are not, the RMSE values for improved and normal FE ( $\hat{f}_{a1}^{\dagger}$  and  $\hat{f}_{a1}$ ) are equal, indicating that disturbance decoupling does not affect the FE accuracy when there are no disturbances present. In the presence of faults and disturbances, the RMSE for  $\hat{f}_{a1}^{\dagger}$  increases to  $0.36 \text{ rad}$  and RMSE for  $\hat{f}_{a1}$  is significantly higher at  $2.85 \text{ rad}$ , underscoring the critical importance of disturbance decoupling for maintaining accurate FE.

For  $\hat{f}_{a4}$ , the RMSE is very low in the fault-free case ( $0.04 \text{ m}$ ). However, in the faulty case scenario, the RMSE for  $\hat{f}_{a4}$  increases to  $0.93 \text{ m}$ .

## 6. Conclusion

This paper introduces a new active FTC strategy for diagnosing actuator faults in the quadcopter in the presence of external disturbances. Firstly, **A complete nonlinear quadrotor's dynamical model was introduced**, taking into account several physics phenomena that might impact our system's navigation in space. Secondly, to estimate the actuator faults, an AO has been developed, which does not need the system to meet the traditional observer matching requirement. This approach effectively handles faults that affect unmeasured state dynamics. **Thirdly, a new ABFTC controller was presented**, in the presence of actuator faults and external disturbances, based on the adaptive backstepping technique. This controller utilizes the AO-based FE to compensate for actuator faults, and external disturbances were estimated using an adaptive law. To decouple disturbances from actuator fault estimates a novel adaptive FE law was proposed. Finally, several simulations in MATLAB were run to verify the effectiveness of the suggested strategy with a defective system. Two time-varying actuator faults related to roll  $\varphi$ , and altitude  $z$  commands are introduced. In addition, disturbances and faults were coupled with additive Gaussian noise to simulate a realistic flight environment. The Root Mean Square Error (RMSE) was used to numerically assess the accuracy of the simulation results, providing a measure of the differences between estimated and actual values.

The simulation results demonstrate the success of the proposed strategy. It allowed for precise FE even in the presence of external disturbances and noise, as well as stability and trajectory tracking. Moreover, this control method offers physically realizable input control signals. The fact that the highest RMSE for attitude in the faulty case is lower than  $10^{-4} \text{ rad}$ , and the highest RMSE for the position is lower than  $10^{-2} \text{ m}$  further highlights the method's effectiveness in maintaining accurate control performance.

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