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Sliding Mode Active Fault-Tolerant Control for Quadrotor UAV System in Presence of Actuator Faults

Presented by:

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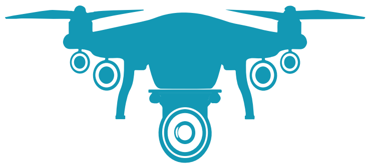

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Problem statement

Faults sources

- Wear and Tear;
- Mechanical Damage;
- Environmental Factors;
- Electrical Issues;

Traditional Methods

- Struggle to handle unexpected issues;
- Typically designed to work under ideal conditions;
- Quadrotor may become unstable, deviate from its intended path, or even crash;

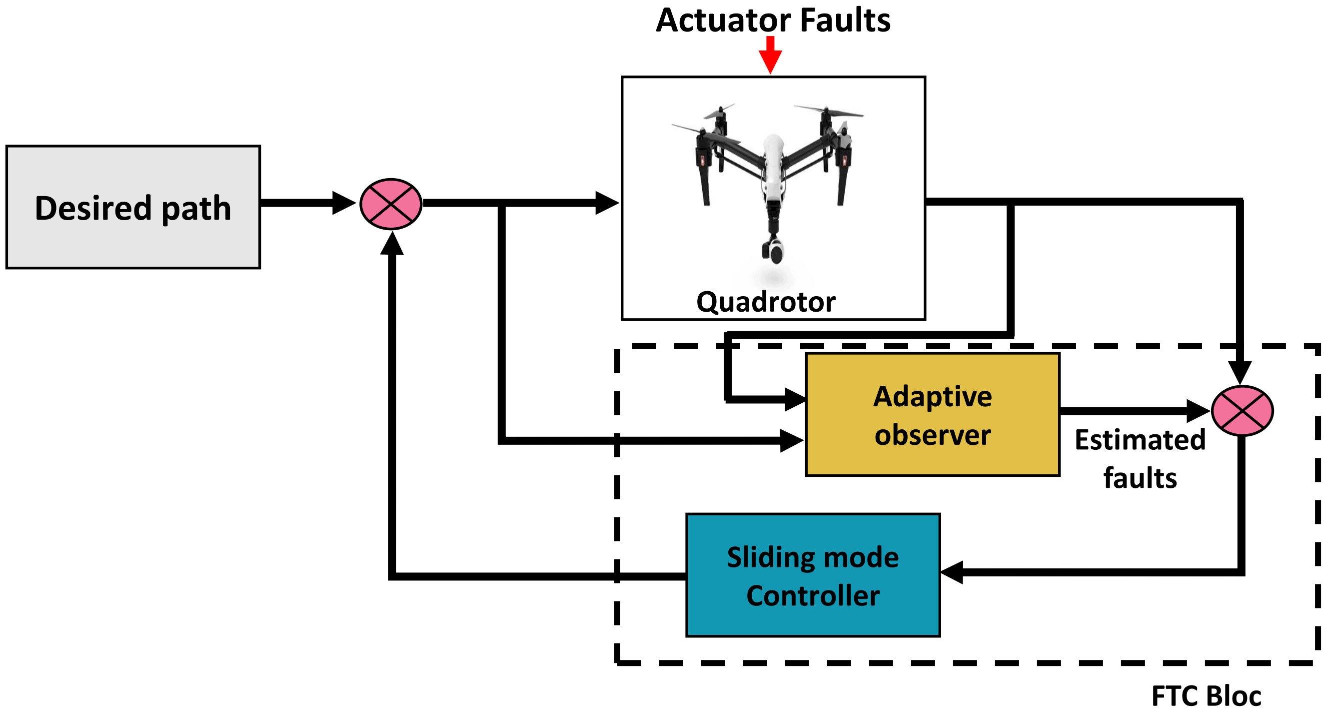
Fault Tolerant Control (FTC)

- FTC is a type of control system to maintain the stability and performance of a system even when faults occur.

Proposed FTC

FTC Structure

- **Adaptive observer:** Estimate the actuator faults
- **Sliding mode controller:** Adjusts the control strategy



Outline

- Quadrotor Nonlinear Dynamical modeling
- Nonlinear adaptive observer design
- Quadrotor FTC in presence of Actuator Faults
- Simulation Results
- Conclusion

NONLINEAR DYNAMICAL MODEL

- E (O, X, Y, Z) designate an inertial frame.
- B (o, x, y, z) a frame permanently coupled to the quadrotor body.
- The absolute position (x, y, z).
- The attitude by the three Euler's angles (φ, θ, ψ).

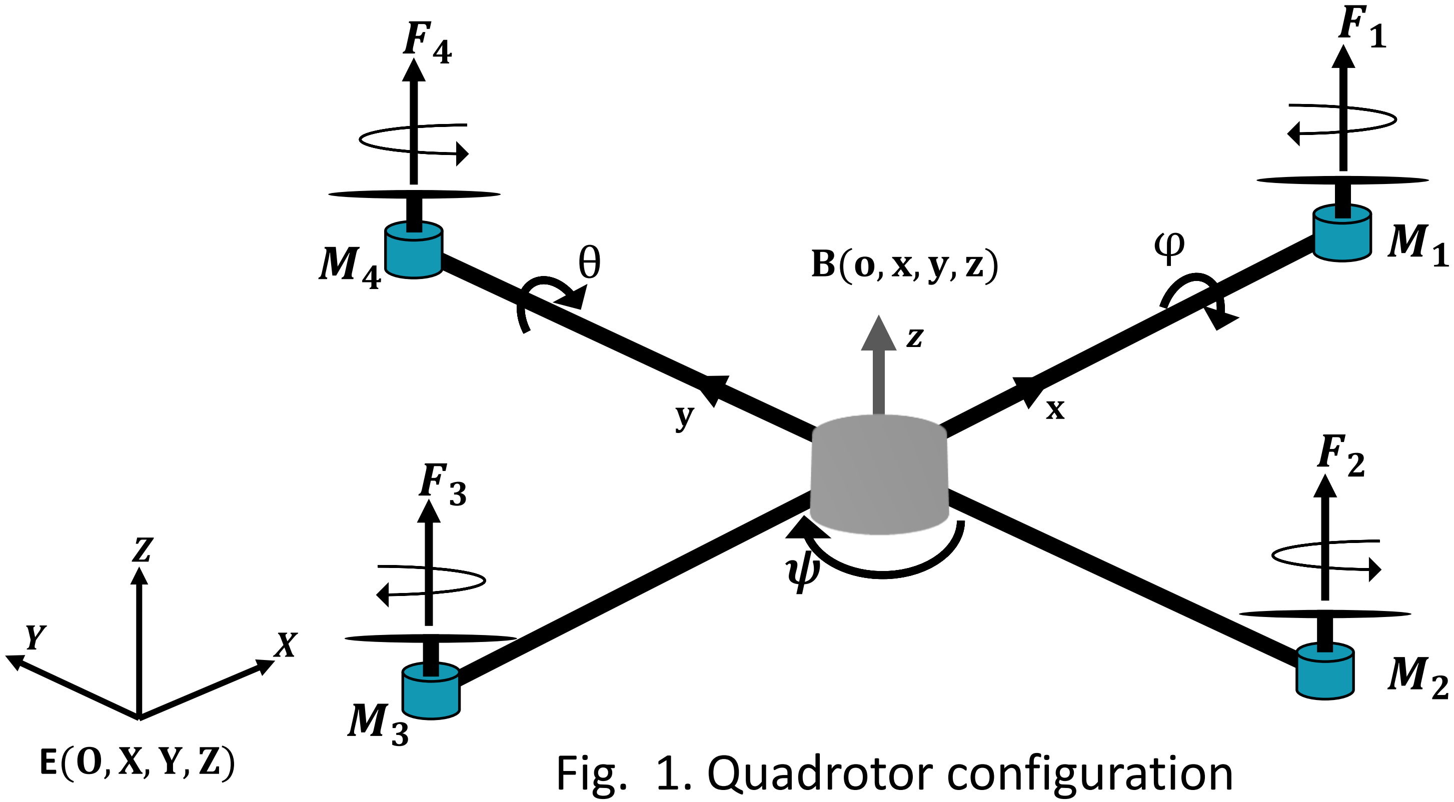


Fig. 1. Quadrotor configuration

The quadrotor dynamical model can be derived using the Newton-Euler formalism.

- $\boldsymbol{F_f}$ is the resultant of the **forces** generated by the four **rotors**
- $\boldsymbol{F_t}$ the resultant of the **drag forces** along (X,Y,Z) axis
- $\boldsymbol{F_g}$ is the **gravity** force

Forces

- $\boldsymbol{\Gamma_f}$ is the moment developed by the quadrotor

Moment

- $\boldsymbol{\Gamma_a}$ is the resultant of **aerodynamics frictions** torques
- $\boldsymbol{\Gamma_g}$ is the resultant of torques due to the **gyroscopic effects**

Torques

Newton-Euler formalism

Attitude coordinates

$(\boldsymbol{\varphi}, \boldsymbol{\theta}, \boldsymbol{\psi})$

Position coordinates

$(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$

The quadrotor's dynamic model:

$$
\begin{cases}
\ddot{\varphi} = \dfrac{1}{I_x}\left(\dot{\theta}\dot{\psi}\left(I_y - I_z\right) - K_{fax}\dot{\varphi}^2 - \mathrm{J}_r\overline{\Omega}\dot{\theta} + d\boldsymbol{U_2}\right) \\
\ddot{\theta} = \dfrac{1}{I_y}\left(\dot{\varphi}\dot{\psi}(I_z - I_x) - K_{fay}\dot{\theta}^2 + \mathrm{J}_r\overline{\Omega}\dot{\varphi} + d\boldsymbol{U_3}\right) \\
\ddot{\psi} = \dfrac{1}{I_z}\left(\dot{\theta}\dot{\varphi}\left(I_x - I_y\right) - K_{faz}\dot{\psi}^2 + \boldsymbol{U_4}\right)
\end{cases}
$$

$$
\begin{cases}
\ddot{x} = \dfrac{1}{m}\Big((C\varphi S\theta C\psi + S\varphi S\psi)U_1 - K_{ftx}\dot{x}\Big) \\
\ddot{y} = \dfrac{1}{m}\Big((C\varphi S\theta S\psi - S\varphi C\psi)U_1 - K_{fty}\dot{y}\Big) \\
\ddot{z} = \dfrac{1}{m}\Big((C\varphi C\theta)\boldsymbol{U_1} - K_{ftz}\dot{z}\Big) - \mathrm{g}
\end{cases}
$$

Outline

- [x] Quadrotor Nonlinear Dynamical modeling
- [ ] Nonlinear adaptive observer design
- [ ] FTC Strategy of Quadrotor in presence of Actuator Faults
- [ ] Simulation Results
- [ ] Conclusion

The complete model may be expressed in state-space form:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\,\boldsymbol{\Phi}(\boldsymbol{x},\boldsymbol{u}) + \boldsymbol{E}\,\boldsymbol{\sigma}(\boldsymbol{x})\,\boldsymbol{f}_{\boldsymbol{a}}(\boldsymbol{t})$$

$$\boldsymbol{y} = \boldsymbol{C}\,\boldsymbol{x}$$

- $\boldsymbol{x}$ is the state vector of the system.
- $\boldsymbol{y}$ is the output vector.
- $\boldsymbol{f}_{\boldsymbol{a}}$ represent the actuator faults vector.
- $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{E}$ and $\boldsymbol{C}$ are known constant matrices.
- $\boldsymbol{\sigma}(\boldsymbol{x})$ and $\Phi(\boldsymbol{x},\boldsymbol{u})$ known nonlinear functions

1. The pair (C, A) is observable.
2. $\Phi(x,u)$ and $\sigma(x)$ satisfy the Lipschitz property with respect to x.
3. $\|f_a(t)\| \leq \gamma_3$
4. $\dot{f}_{ai}(t) \approx 0 \qquad i \in [1,2,3,4]$
5. $CB = 0_{6\times 14}$
$CE = 0_{6\times 4}$
$rank(CAE) = rank(E)$
6. Given that x is bounded, the first derivative of $\sigma(x)$ in time is both continuous and bounded.

Original system

$$\dot{x} = Ax + B\Phi(x,u) + Ef(x)$$
$$y = C\,x$$

$\boldsymbol{x}$

Coordinate change

$$z = T\,x$$

$\boldsymbol{z}$

New system

$$\dot{z} = \bar{A}z + \bar{B}\Phi(T^{-1}z,u) + \bar{E}\sigma(T^{-1}z)f_a$$
$$y = \bar{C}z$$

$\boldsymbol{z}$

Adaptive observer representation

$$\dot{\hat{z}} = \bar{A}\hat{z} + \bar{B}\hat{\Phi} + \bar{E}\hat{\sigma}\hat{\theta} + \bar{L}(y - \hat{z}_1)$$
$$\hat{f}_a = W + \Gamma\hat{\sigma}^T F y$$

$\hat{\boldsymbol{f}}_{\boldsymbol{a}}$

Theorem 1

If the above assumptions are satisfied the observation error is asymptotically stable, and the fault estimation error remains bounded, such that:

1
$$P = \begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix} > 0$$

2
$$Q = \begin{bmatrix} \Lambda & P\bar{B} & P\bar{E} \\ \bar{B}^T P & -\varepsilon_1 I_{14} & 0_{14\times 4} \\ \bar{E}^T P & 0_{4\times 14} & -\varepsilon_2 I_4 \end{bmatrix} < 0$$

3
$$E_2^T P_3 = F A_{12}$$

$P = P^T > 0$, F and $M = M^T$.
And $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$

1 $P > 0$ | **2** $Q < 0$ | **3** $E_2^T P_3 = F A_{12}$

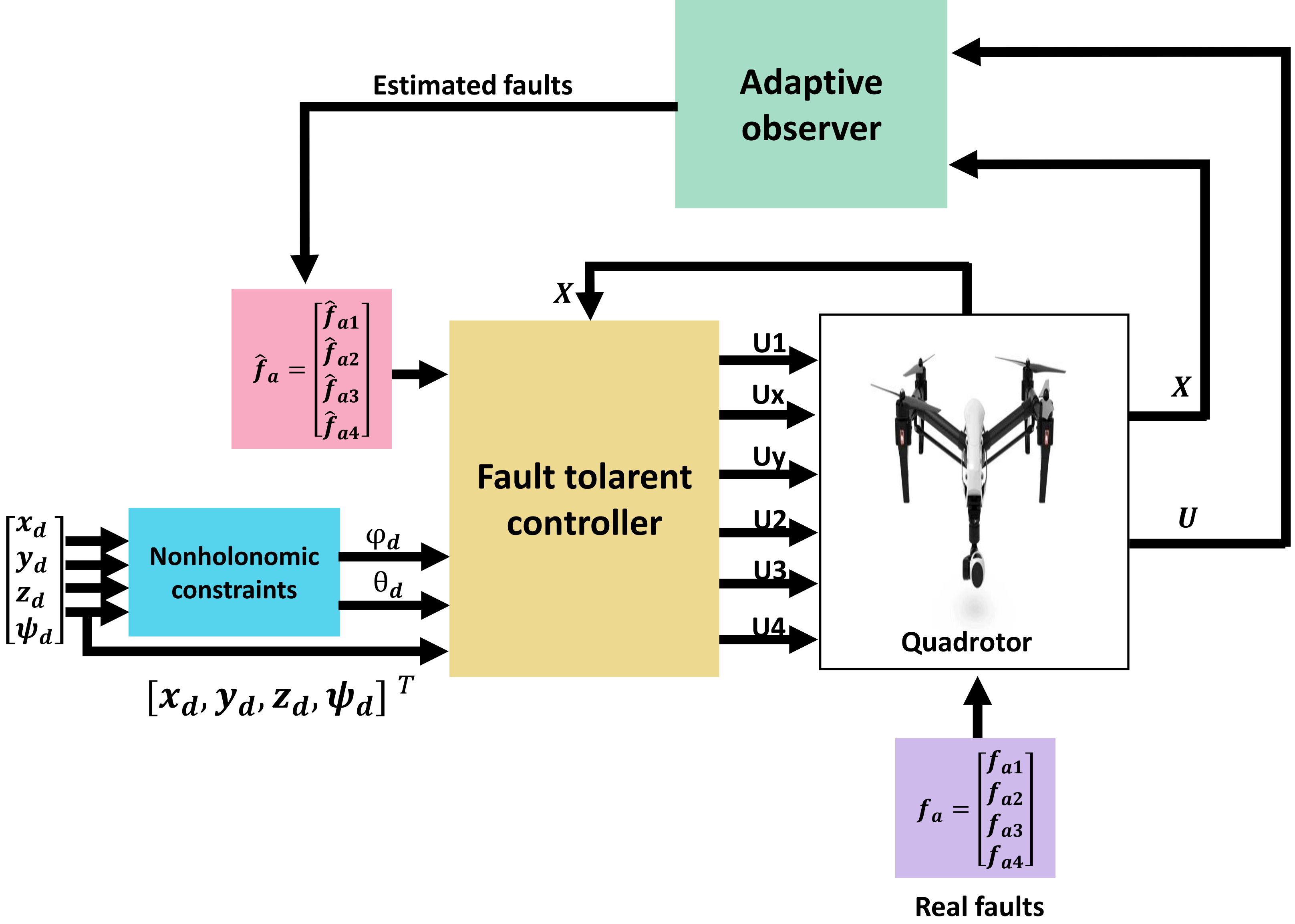
LMI optimization
(Simultaneous solve of 1, 2 and 3)

P, M and F

F | $\bar{L} = P^{-1} M \bar{C}^T$

$$\dot{\hat{z}} = \bar{A}\hat{z} + \bar{B}\hat{\Phi} + \bar{E}\hat{\sigma}\hat{\theta} + \bar{L}(y - \hat{z}_1)$$
$$\hat{f}_a = W + \Gamma \hat{\sigma}^T F y$$

An active fault-tolerant tracking control system approach for actuator faults.



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- An iterative algorithm is employed to synthesize the control laws.

1 $e_i = x_i - x_{id}$

2 $s_i = \dot{e}_i + k_i e_i$

3 $V_i = \frac{1}{2} s_i^2 + \frac{1}{2\alpha_i} \tilde{\Gamma}_i^2$

where $\dot{\hat{\Gamma}}_i = \alpha_i |s_i|$

$e_i = x_i - x_{id}$

↓

$s_i = \dot{e}_i + k_i e_i$

↓

$V_i = \frac{1}{2} s_i^2 + \frac{1}{2\alpha_i} \tilde{\Gamma}_i^2$

↓

SMC Approach

(Find U_i such that $\dot{V}_i \leq 0$)

↓

Extract control laws

U_1, U_2, U_3, U_4

The synthesized stabilizing control laws are as described in the following:

$$U_1 = \frac{m}{cx_1 cx_2}\left[\ddot{z}_d - a_{11}x_{12} + \mathrm{g} - k_6\dot{e}_6 - A_6 s_6 - \hat{\boldsymbol{f}}_{\boldsymbol{a4}} - \hat{\Gamma}_6\, sign(s_6)\right]$$

$$U_2 = \frac{1}{b_1}\left[\ddot{\varphi}_d - a_1x_8x_9 - a_2x_7^2 - a_3\overline{\Omega}x_8 - k_1\,\dot{e}_1 - A_1s_1 - \hat{\boldsymbol{f}}_{\boldsymbol{a1}} - \hat{\Gamma}_1\, sign(s_1)\right]$$

$$U_3 = \frac{1}{b_2}\left[\ddot{\theta}_d - a_4x_7x_9 - a_5x_8^2 - a_6\overline{\Omega}x_7 - k_2\dot{e}_2 - A_2s_2 - \hat{\boldsymbol{f}}_{\boldsymbol{a2}} - \hat{\Gamma}_2\, sign(s_2)\right]$$

$$U_4 = \frac{1}{b_3}\left[\ddot{\psi}_d - a_7x_7x_8 - a_8x_9^2 - k_3\dot{e}_3 - A_3s_3 - \hat{\boldsymbol{f}}_{\boldsymbol{a3}} - \hat{\Gamma}_3\, sign(s_3)\right]$$

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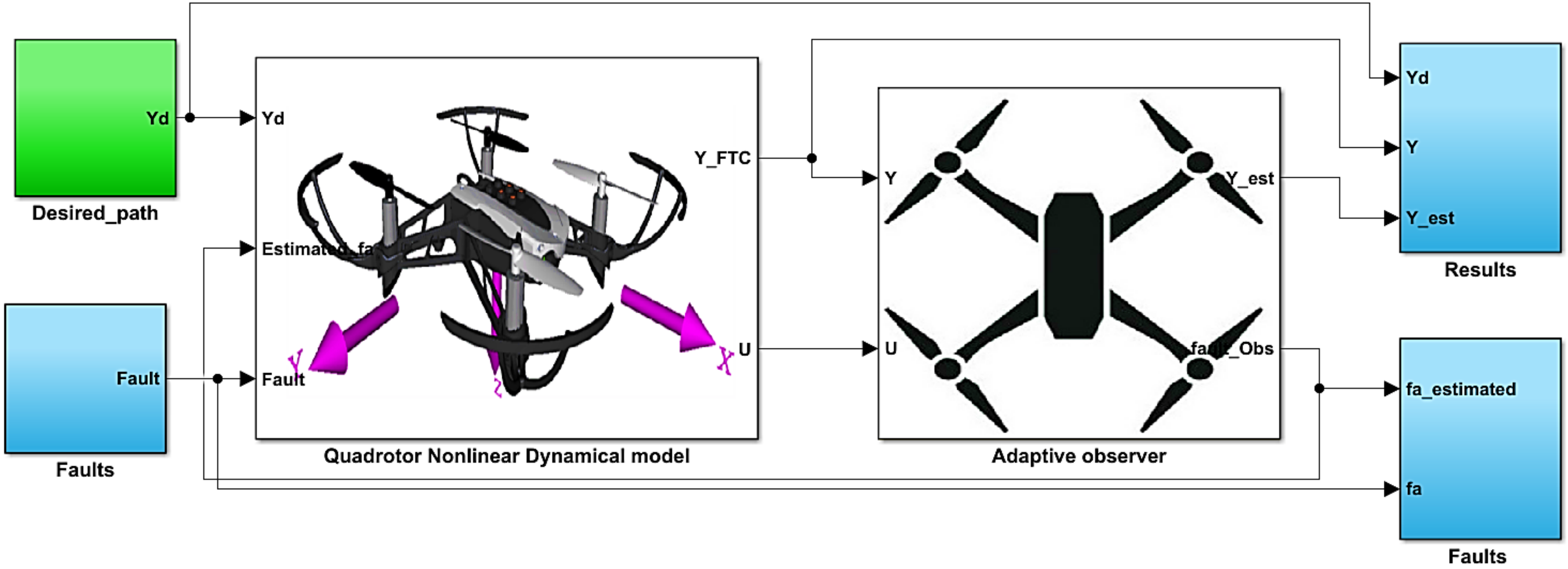
- We executed simulations in MATLAB/SIMULINK® environment.
- The quadrotor object of our study is the Draganfly IV, manufactured by Draganfly Innovations.
- Two actuator faults related to φ (f_{a1}) and z (f_{a4}) are introduced.



A. States estimation

- The estimated states converge to the real ones accurately before and after the fault occurrence.

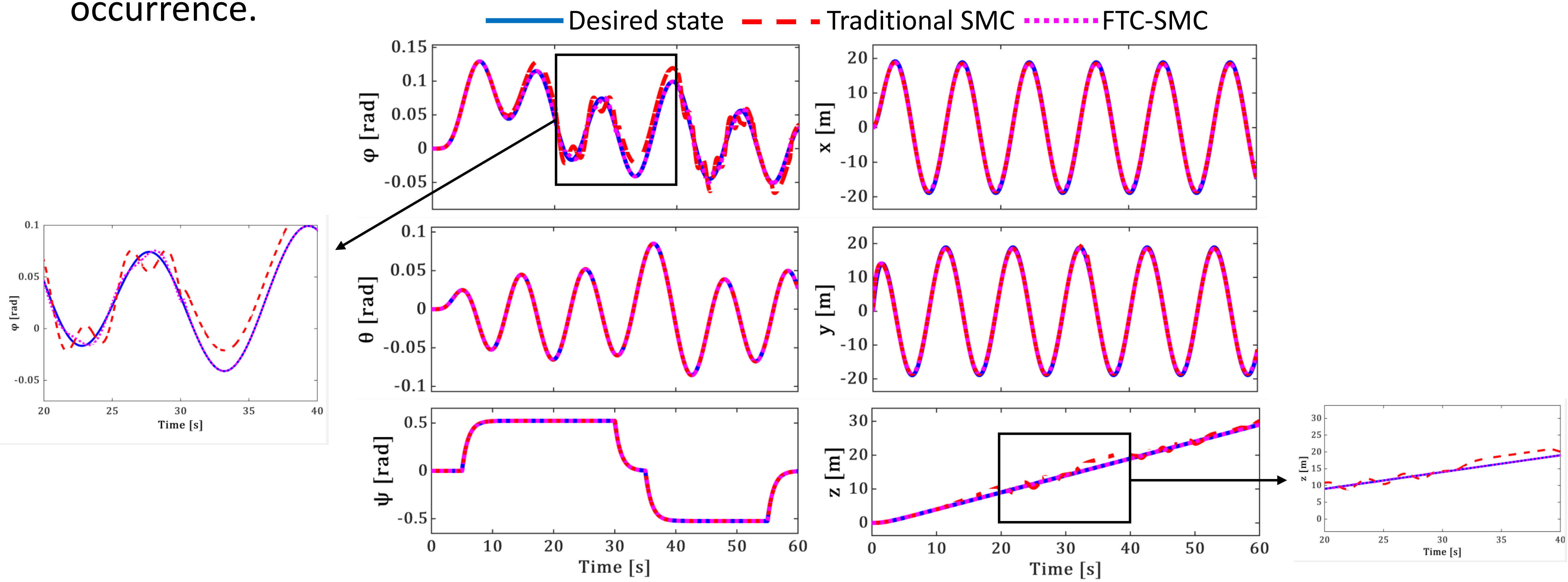


Fig. 2. Trajectories over roll (φ), pitch (θ), yaw angle (ψ), and altitude (z)

A. States estimation

- The estimated states converge to the real ones accurately before and after the fault occurrence.

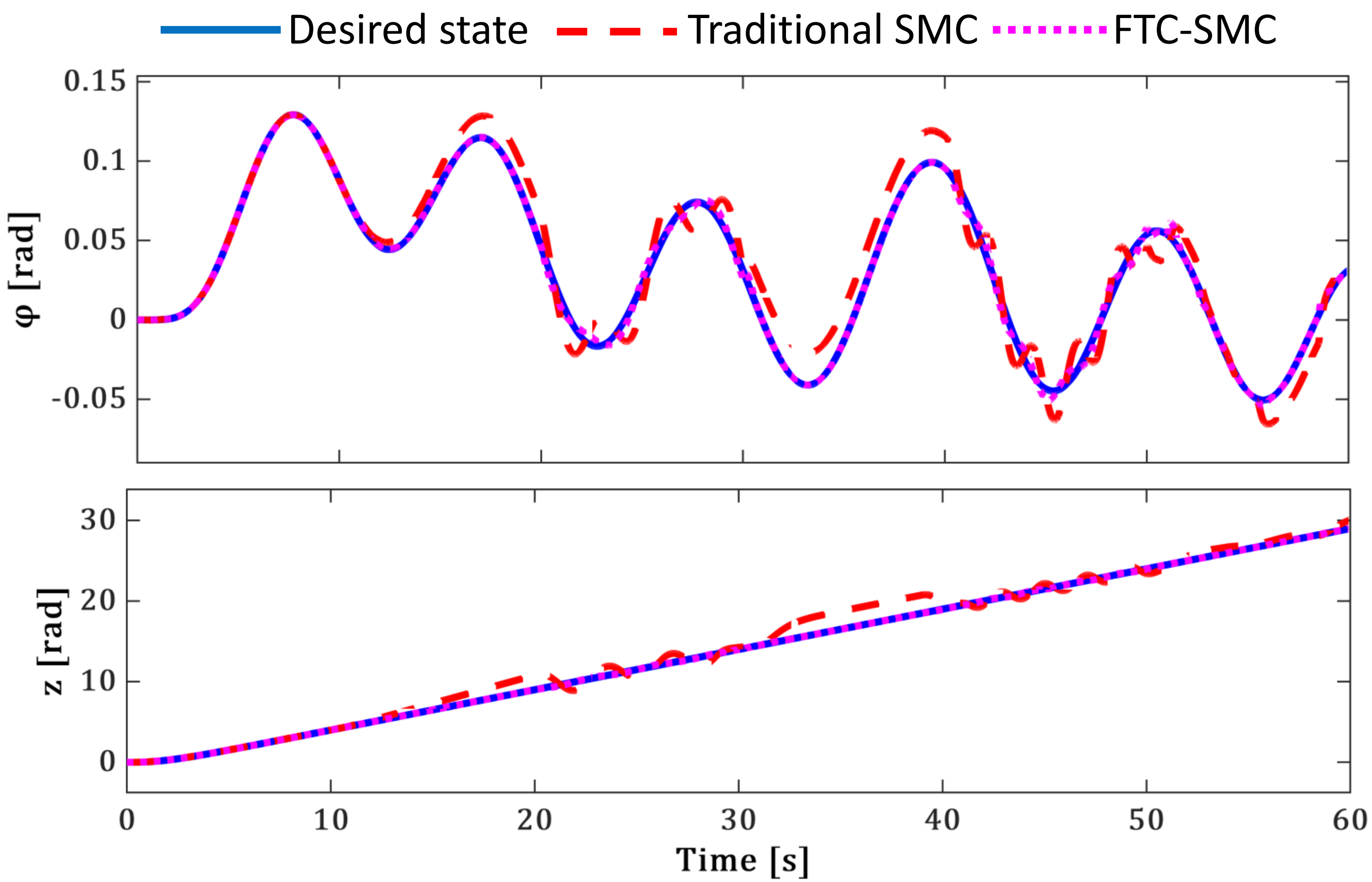


Fig. 3. Trajectories over roll (φ), and altitude (z)

B. Fault estimation

- Convergence time of less than 1 second, ensuring fast fault detection.
- $RMSE\ (f_{a1}) = 0.02\, rad$
- $RMSE\ (f_{a4}) = 0.30\, m$

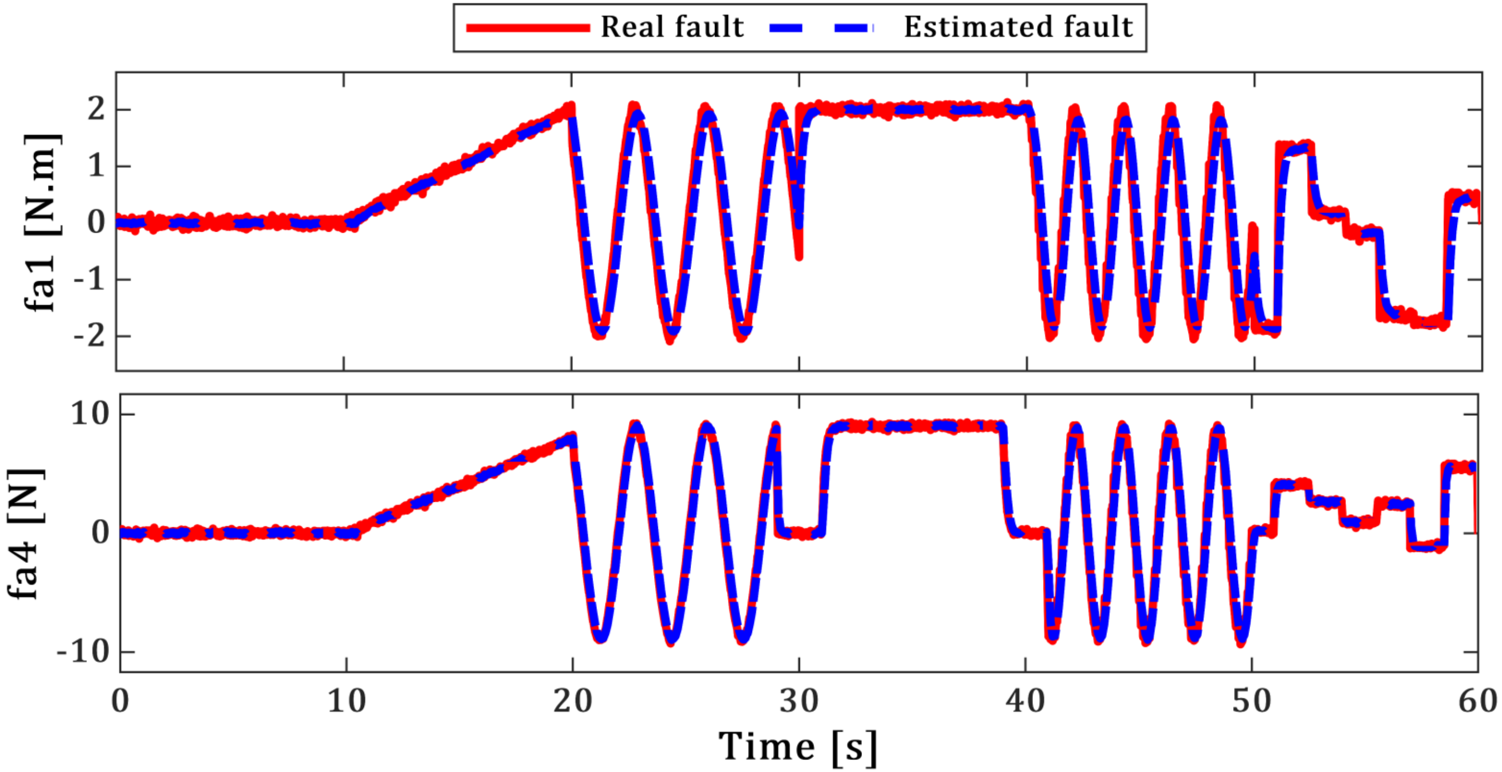


Fig. 4. Fault estimation

C. The quadrotor's global trajectory in 3D

- Excellent performance and resilience towards stability and tracking despite the occurrence of actuator failures.

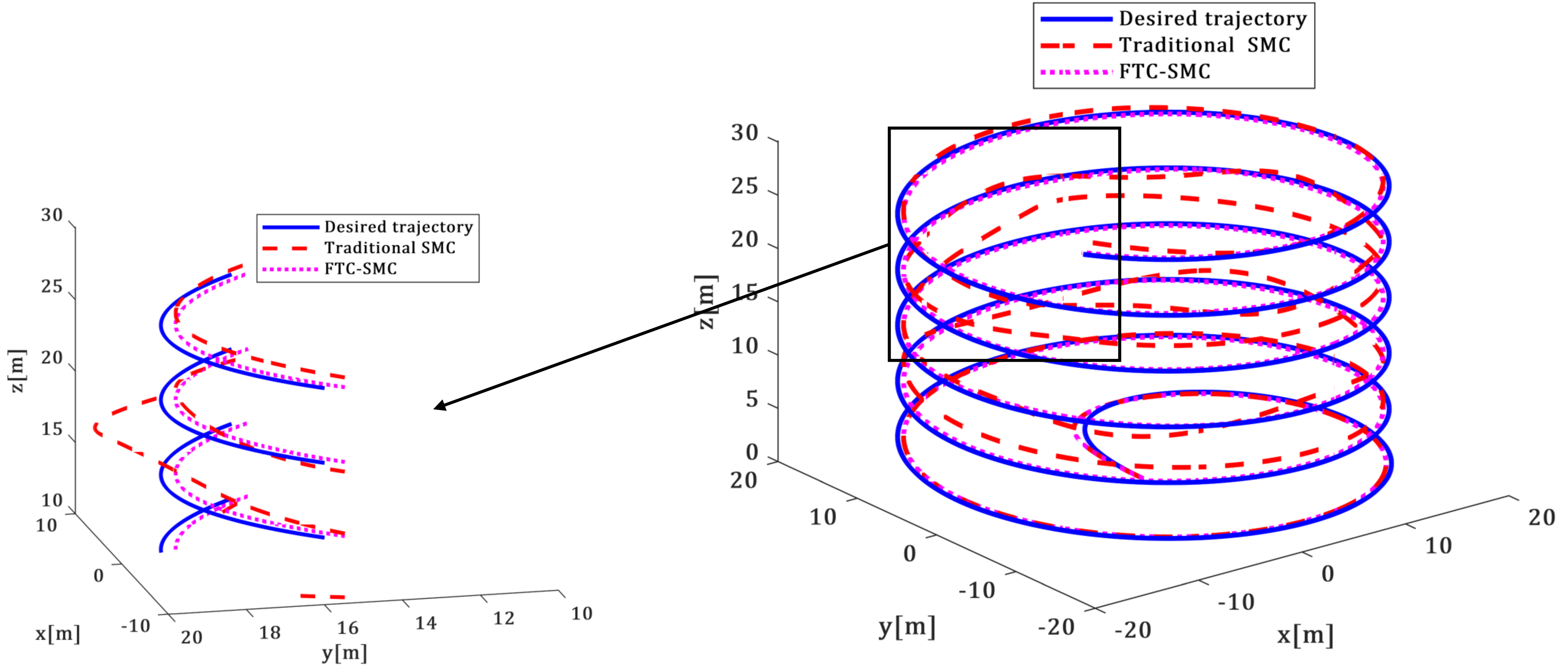


Fig.5. The quadrotor's global trajectory in 3D

D. Estimation errors

	$RMSE(\varphi)$ [rad]	$RMSE(z)$ [m]
Traditional SMC	10^{-3}	0.5
FTC-SMC	10^{-5}	10^{-4}

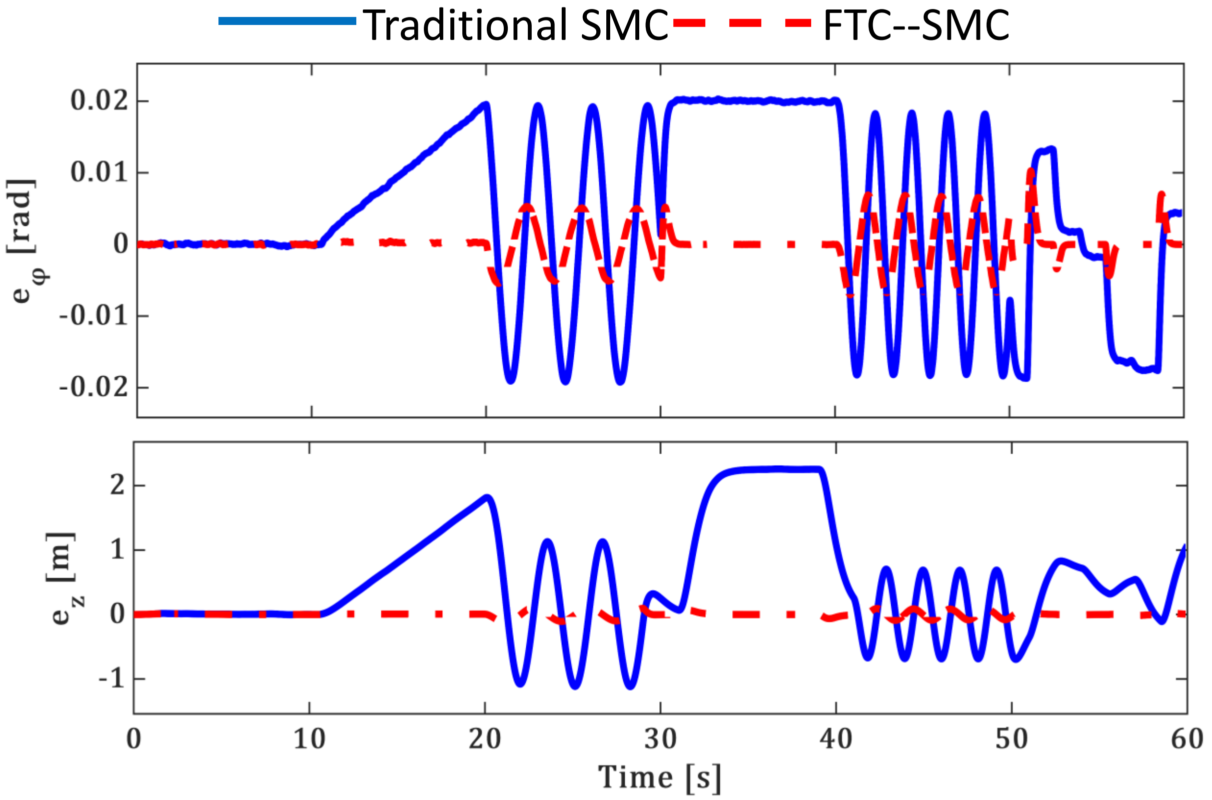


Fig.6. Tracking errors

E. Control inputs

- The input control signals are acceptable and physically realizable.

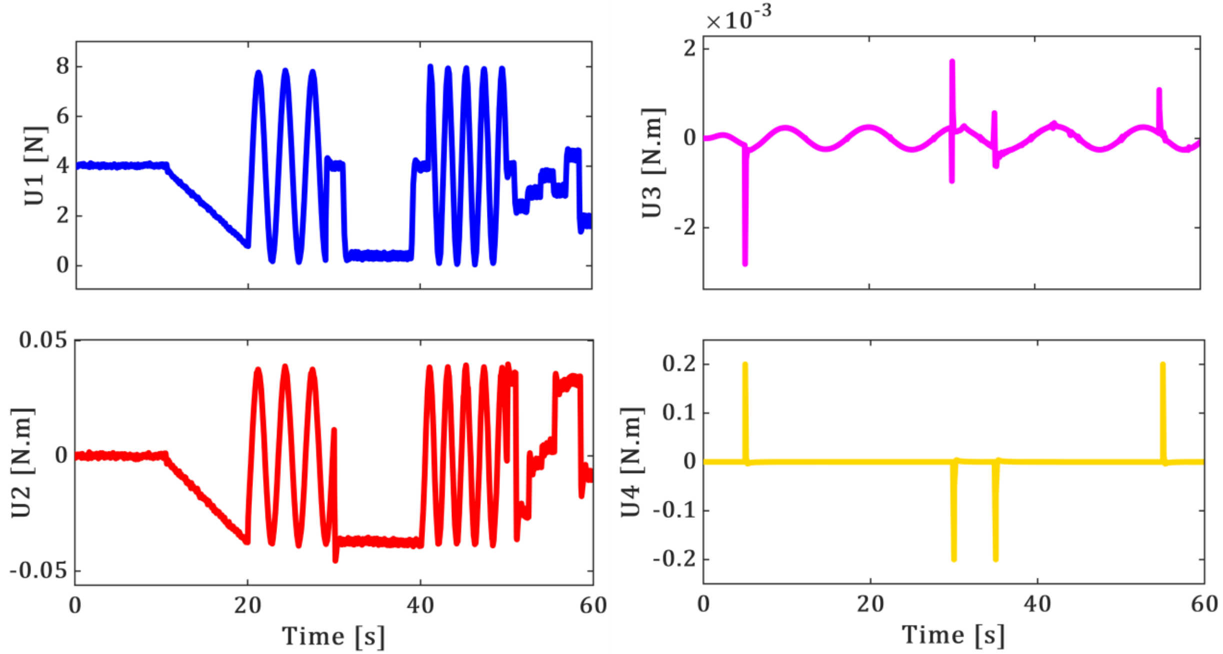


Fig.7. Control inputs of actuators in faulty case

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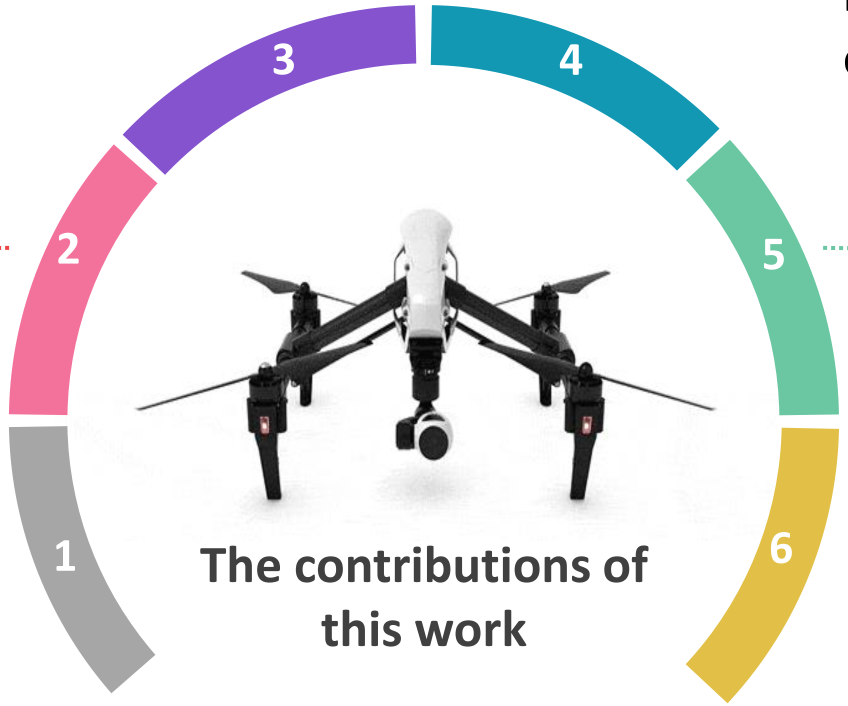
The used observer does not require that the system structure meets the required standard observer matching requirement for the traditional ASO.

Both system state and the actuator faults can be simultaneously estimated by the used AO.

The non-linearity of the quadrotor and the high-order non-holonomic constraints are not neglected.



Regardless of the number of tracked outputs, both additive and multiplicative faults can be estimated.

RMSE values in the faulty case less than $10^{-4}\ rad$.

It made it possible to precisely estimate the faults and ensure stability and trajectory tracking using an LMI optimization problem.

Thank you for your attention

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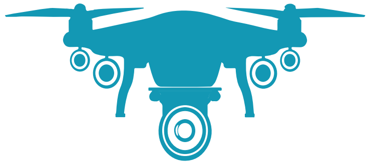

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Annex

Parameters

- C and S represent the trigonometrical functions cosines and sines respectively.
- m is the total mass of the quadrotor.
- g is the gravity acceleration constant.
- K_d is the drag coefficient.
- I_x, I_y and I_z are the constants inertia.
- K_{ftx}, $K_{fty}$and K_{ftz} are the translation drag coefficients.
- K_{fax}, $K_{fay}$and K_{faz} are the aerodynamic friction coefficients around x, y, and z.
- d is the distance between the quadrotor center of mass and the rotation axis of propellers.
- J_r is the rotor inertia.
- $\overline{\Omega}$ is the disturbance due to the rotor imbalance.
- U_1, U_2, U_3, and U_4 represent the control inputs of the system.

Based on the angular speeds of the four rotors, the control inputs and the disturbance $\overline{\Omega}$ are expressed as follows:

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} = \begin{bmatrix} K_p & K_p & K_p & K_p \\ -K_p & 0 & K_p & 0 \\ 0 & -K_p & 0 & K_p \\ K_d & -K_d & K_d & -K_d \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} \tag{2}$$

$$\overline{\Omega} = \omega_1 - \omega_2 + \omega_3 - \omega_4 \tag{3}$$

The state space correspondent to the model (1) is rearranged as follow:

$$
\begin{aligned}
\dot{x}_1 &= x_7 \\
\dot{x}_2 &= x_8 \\
\dot{x}_3 &= x_9 \\
\dot{x}_4 &= x_{10} \\
\dot{x}_5 &= x_{11} \\
\dot{x}_6 &= x_{12} \\
\dot{x}_7 &= a_1 x_8 x_9 + a_2 x_7^2 + a_3 \overline{\Omega} x_8 + b_1 U_2 + f_{a1} \\
\dot{x}_8 &= a_4 x_7 x_9 + a_5 x_8^2 + a_6 \overline{\Omega} x_7 + b_2 U_3 + f_{a2} \\
\dot{x}_9 &= a_7 x_7 x_8 + a_8 x_9^2 + b_3 U_4 + f_{a3} \\
\dot{x}_{10} &= a_9 x_{10} + U_x \frac{U_1}{m} \\
\dot{x}_{11} &= a_{10} x_{11} + U_y \frac{U_1}{m} \\
\dot{x}_{12} &= a_{11} x_{12} - \mathrm{g} + \frac{cos(x_1)\, cos(x_2)}{m} U_1 + f_{a4}
\end{aligned}
\qquad (16)
$$

$$
\begin{aligned}
a_1 &= \frac{I_y - I_z}{I_x} & a_2 &= \frac{-K_{fax}}{I_x} & a_3 &= \frac{-\mathrm{J}_r}{I_x} \\
a_4 &= \frac{I_z - I_x}{I_y} & a_5 &= \frac{-K_{fay}}{I_y} & a_6 &= \frac{\mathrm{J}_r}{I_y} \\
a_7 &= \frac{I_x - I_y}{I_z} & a_8 &= \frac{-K_{faz}}{I_z} & a_9 &= \frac{-K_{fax}}{m} \\
a_{10} &= \frac{-K_{fay}}{m} & a_{11} &= \frac{-K_{faz}}{m} & & \\
b_1 &= \frac{d}{I_x} & b_2 &= \frac{d}{I_y} & b_3 &= \frac{1}{I_z}
\end{aligned}
$$

And

$$
\begin{aligned}
U_x &= Cx_1 Sx_2 Cx_3 + Sx_1 Sx_3 \\
U_y &= Cx_1 Sx_2 Sx_3 - Sx_1 Cx_3
\end{aligned}
$$