Sensor Fault Detection of Quadrotor using Nonlinear Parity Space Relations

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***Abstract*—In this study a fault detection and isolation scheme (FDI) is proposed, to detect sensor faults in the dynamic behaviors of the model of the quadrotor vehicle, using nonlinear analytical redundancy (NLAR) relations also called nonlinear parity space relations. This technique is an particularly useful and interesting because it gives us to explicitly derive the maximum possible number of independent tests from the nonlinear state space representation of the system. Finally, the results prove the effectiveness of the proposed scheme.**

***Index Terms*—Fault detection, quadrotor, nonlinear analytical redundancy, residuals, parity space, nonlinear systems, sensor, IMU, UAV, Backstepping.**

I. Introduction

Recently, autonomous Unmanned Aerial Vehicles (UAVs) have caught considerable attention owing to their strong autonomy and capability to achieve complex tasks without human intervention in various civilian and military applications, such as monitoring, science and research, search and rescue in hazardous environment etc.

Quadrotors are a special class of UAVs which have received observable attentions, vertical take-off and hover capabilities along with reduced mechanical complexity make quadrotors potentially adapted for many practical applications.

They are often equipped with low cost and light weight micro-electro-mechanical systems (MEMS) inertial measurement unit (IMU). These sensors represent an important role in most quadrotor navigation and control applications, and provide information such as relative position, which are susceptible to bias faults as a result of temperature variation, component damage/degradation, excessive vibration, etc.

To elevate the safety and reliability, and assure the normal operation of this vehicles, the detection of these sensor faults plays an essential role in the safe operations of quadrotors. The problem of quadrotor IMU sensor fault detection and isolation have investigated by several researchers based on divers observer or estimator techniques, [7], [5], [8], and using NLAR relations [13]. Conceptually, the implementation based on parity relations is more straightforward than the observer based approach.

Various approachs have been proposed to control this vehicle [2], [1], [6], the challenge in controlling a such vehicule is that the quadrotor has six degrees of freedom but are only four control inputs, in this work the backstepping has been chosen because it is a modern controller, powerful and well suited to answer the problems inherent to the dynamics under-actuated of the quadrotor.

This paper addresses the nonlinear parity space relations. This method is frequency used in fault detection. To derive the nonlinear analytical redundancy (NLAR) tests this approach exploits the structure of nonlinear geometric control theory [10] [3], extending the analytical redundancy (AR) principle into the nonlinear realm. This technique of NLAR preserves the same desirable formal guarantees that are generated by AR: the resulting tests represent a minimal span of that space, and arise from spaning an observation subspace. This residual generation approach is important due to it's capability to derive the maximum possible number of independent tests of the consistency of sensor data and past control inputs with the system model. All observable deviation from the dynamic model of the system will be detected, and each test contain some information not observed by other tests. this method will be applied to the quadrotor in objective of fault detection and isolation.

The paper is organized as follows. The next section introduces the general description and flight dynamics of quadrotor. Section III, presents the quadrotor control. Section IV proposes the procedure to generate NLAR (residuals). Section V provides simulation results. Conclusions are presented in section VI.

II. Quadrotor Modelling

The quadrotor nominal system dynamics are derived from the Newton-Euler equations of motion and are given by:

$$\dot{\zeta} = v \tag{1}$$

$$m\ddot{\zeta} = R(\eta) \times \begin{bmatrix} 0 \\ 0 \\ \sum_{i=1}^{4} F_i \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} \tag{2}$$

$$\dot{\eta} = \begin{bmatrix} 1 & S_\phi T_\theta & C_\phi T_\theta \\ 0 & C_\phi & -S_\phi \\ 0 & S_\phi S_\theta & C_\phi S_\theta \end{bmatrix} \Omega \tag{3}$$

$$I\dot{w} = \begin{bmatrix} (I_y - I_z)qr \\ (I_z - I_x)pr \\ (I_x - I_y)pq \end{bmatrix} + \begin{bmatrix} l(F_4 - F_2) \\ l(F_3 - F_1) \\ d(w_1^2 - w_2^2 + w_3^2 - w_4^2) \end{bmatrix} \tag{4}$$

where $\zeta \in \Re^3$ is the inertial position $\zeta = \left[x, y, z\right]^T$, $v \in \Re^3$ is the velocity expressed in the Earth frame, $\eta = \left[\phi, \theta, \psi\right]^T$ are the roll, pitch and yaw Euler angles, respectively, and $\Omega = \left[p, q, r\right]^T$ represents the angular rates, m is the mass of the quadrotor, l is the length of the four arms of the quadrotor, and g is the gravitational acceleration. The terms I_x, I_y and I_z represent the quadrotor inertias about the body x-, y- and z-axis, respectively. Note that the quadrotor is assumed to be symmetric about the xz and yz planes (i.e.the product of inertias is zero).

The rotor forces and moment equations can be written as:

$$\begin{array}{ll} F_i = f\omega_i^2 & \\ M_i = kF_i = kf\omega_i^2 & i \in (1,2,3,4) \end{array} \tag{5}$$

where f and k are positive constants and ω_i is the rotational speed of rotor i. The transformation from the body frame to the Earth fixed frame is given by the rotation matrix R which is defined based on a $3-2-1$ rotation sequence as follows:

$$R(\eta) = \begin{bmatrix} C_\theta C_\psi & S_\phi S_\theta C_\psi - C_\phi S_\psi & C_\phi S_\theta C_\psi + S_\phi S_\psi \\ C_\theta S_\psi & S_\phi S_\theta S_\psi + C_\phi C_\psi & C_\phi S_\theta S_\psi - S_\phi C_\psi \\ S_\theta & S_\phi C_\theta & C_\phi C_\theta \end{bmatrix} \tag{6}$$

In which $T_x = tan(x)$, $C_x = cos(x)$, $S_x = sin(x)$,
Since the inertia matrix of the quadrotor can be considered diagonal with $I_x = I_y$. The roll, pitch and yaw moment equations can be written as:

$$\begin{cases} \dot{p} = \left(\frac{l}{I_x}\right)(F_4 - F_2) + b_2 qr \\ \dot{q} = \left(\frac{l}{I_y}\right)(F_1 - F_3) + b_4 pr \\ \dot{r} = \left(\frac{k}{I_z}\right)(F_2 - F_1 + F_4 - F_3) \end{cases} \tag{7}$$

where $b_2 = (I_z - I_y)/(I_x)$ and $b_4 = (I_x - I_z)/(I_y)$.
The Euler equations are given by:

$$\begin{cases} \dot{\phi} = p + T_\theta S_\phi q + T_\theta C_\phi r \\ \dot{\theta} = C_\phi q - S_\phi r \\ \dot{\psi} = \left(\frac{S_\phi}{C_\theta}\right) q - \left(\frac{C_\phi}{C_\theta}\right) r \end{cases} \tag{8}$$

The acceleration equations written directly in the Earth frame are given by:

$$\begin{cases} \ddot{x} = \left(\frac{1}{m}\right)(C_\psi S_\theta C_\phi + S_\psi S_\phi)F \\ \ddot{y} = \left(\frac{1}{m}\right)(S_\psi S_\theta C_\phi - C_\psi S_\phi)F \\ \ddot{z} = -g + \left(\frac{1}{m}\right) C_\theta C_\phi F \end{cases} \tag{9}$$

where

$$F = F_1 + F_2 + F_3 + F_4 \tag{10}$$

In equations (7), the effects of the rotor forces appear as differences, thus we determine new attitude inputs u_q and u_p as:

$$\begin{array}{l} u_q = F_1 - F_3 \\ u_p = F_4 - F_2 \end{array} \tag{11}$$

In the heading and position dynamics, the effects of rotor forces and moments appear as sums, thus we determine new guidance inputs u_ψ and u_z as:

$$\begin{array}{l} u_\psi = (F_2 + F_4) - (F_1 + F_3) \\ u_z = F = F_1 + F_2 + F_3 + F_4 \end{array} \tag{12}$$

where

$$\underline{F} = T\underline{u} \tag{13}$$

$$T = \frac{1}{4}\begin{bmatrix} 0 & 2 & -1 & 1 \\ -2 & 0 & 1 & 1 \\ 0 & -2 & -1 & 1 \\ 2 & 0 & 1 & 1 \end{bmatrix} \tag{14}$$

$$F = \begin{bmatrix} F_1 & F_2 & F_3 & F_4 \end{bmatrix}^T \tag{15}$$

$$\begin{aligned} u &= \begin{bmatrix} u_p & u_q & u_\psi & u_z \end{bmatrix}^T \\ &= \begin{bmatrix} u_1 & u_2 & u_3 & u_4 \end{bmatrix}^T \end{aligned} \tag{16}$$

III. QUADROTOR CONTROL

In nonlinear control the backstepping is a technique that uses a recursive Lyapunov methodology to ensure the stability of the system.

Because of the quadrotor dynamics, as can be shown in Fig. 1, a cascaded design of two Backstepping controllers is used [9]. An intern control loop can be designed to ensure asymptotic tracking of desired altitude, attitude and heading. although, an external control loop can be designed for quadrotor navigation.

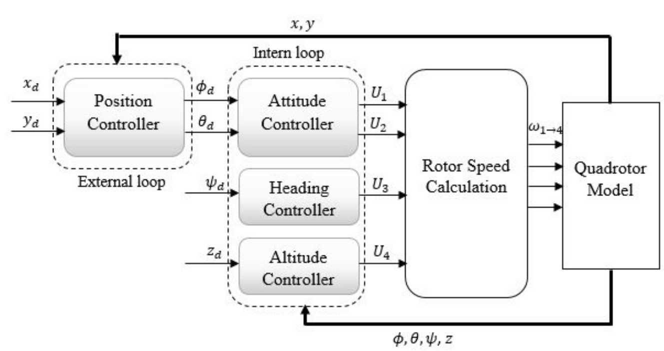

Fig. 1. Control architecture

For position control, a Backstepping controller is implemented to generate the reference roll ϕ_d and pitch θ_d. using the desired waypoint, the position controller calculates the reference roll and pitch angles written as:

$$\begin{cases} \phi_d = arcsin(u_x sin(\psi_d) - u_y cos(\psi_d)) \\ \theta_d = arcsin\left(\frac{(u_x cos(\psi_d) + u_y sin(\psi_d))}{cos(\phi_d)}\right) \end{cases} \tag{17}$$

The reference angles are followed by the attitude, heading and altitude controllers (see Fig. 1) derived using the Back-stepping approach. At the last step, the actual control input is used to stabilize the whole system. The design procedure is systematic and is used in many literature such as [11] and [12]. The control inputs can be written as follows:

$$U_1 = \frac{1}{b_1}(-a_1 x_4 x_6 + \ddot{\phi} + k_1(-k_1 e_1 + e_2) + k_2 e_2 + e_1) \tag{18}$$

$$U_2 = \frac{1}{b_2}(-a_4 x_2 x_6 + \ddot{\theta} + k_3(-k_3 e_3 + e_4) + k_4 e_4 + e_3) \tag{19}$$

$$U_3 = \frac{1}{b_3}(\ddot{\psi} + k_5(-k_5 e_5 + e_6) + k_6 e_6 + e_5) \tag{20}$$

$$U_4 = \frac{m}{cos(x_1)cos(x_3)}(g + \ddot{z}_d + k_{11}(-k_{11} e_{11} + e_{12}) + k_{12} e_{12} + e_{11}) \tag{21}$$

$$U_x = \frac{m}{U_1}(\ddot{x}_d + k_7(-k_7 e_7 + e_8) + k_8 e_8 + e_7) \tag{22}$$

$$U_y = \frac{m}{U_1}(\ddot{y}_d + k_9(-k_9 e_9 + e_{10}) + k_{10} e_{10} + e_9) \tag{23}$$

IV. Nonlinear Analytical Redundancy

Based on the physical model under several assumptions, a nonlinear state space representation mentioned in the physical model in section II can be written as:

$$\begin{cases} \dot{x}(t) = f\left(x(t)\right) + g\left(x(t)\right) u(t) \\ y(t) = h\left(x(t)\right) \end{cases} \tag{24}$$

where $x \in \Re^n$ is the state vector, $u \in \Re^p$ is the vector of control inputs, $y \in \Re^m$ is the output vector, while $f(x)$ is the state function vector, $g(x)$ is the input function matrix and h is the output function vector. The details about this functions may be found in [13].

The NLAR approach can be considered as an extension of the linear AR technique, this method developed by [3]. The core notion of nonlinear analytical redundancy is elegant and intuitive:

– Intuition arises since NLAR exploits the concept of observability, specifically, that the key information which can be obtained about the model based behavior of a system can be inferred from the observation space.

– Elegance follows from the processing of that information in such a way as to generate a formally complete set of residual tests.

These residuals use known control inputs and sensor data histories to detect any deviation from the dynamic or static behaviors of the model in real time fig.2 .

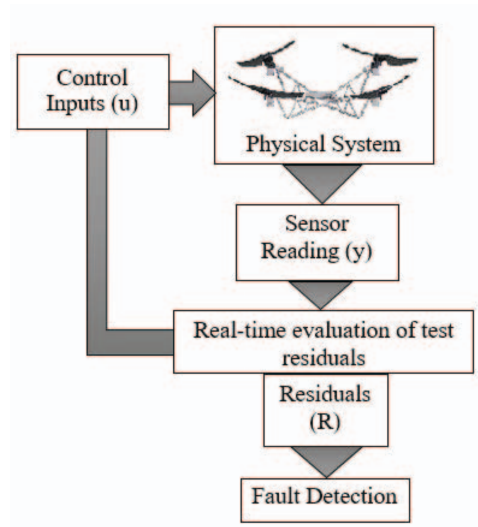

Fig. 2. FDI method

We will concentrate in this section on the details of this approach. Based on the general nonlinear state space representation (24), the triangular nonlinear observability O_Δ is expressed as follows:

$$O_\Delta = \begin{bmatrix} h(x) \\ L_f h + L_g h u \\ L_{ff} h + L_{gf} h u + L_{fg} h u + L_{gg} h u^2 \\ \begin{pmatrix} L_{fff} h + (L_{gff} h + L_{fgf} h + L_{ffg} h) u + \\ (L_{ggf} h + L_{gfg} h + L_{fgg} h + L_{ggg} h) u^2 + \\ (2 L_{gf} h + L_{fg} h) \dot{u} + 3 L_{gg} h u u p + L_g h \ddot{u} \end{pmatrix} \\ \cdots \end{bmatrix} \tag{25}$$

with $L_k h = \sum_{i=1}^{n} \frac{\partial h(x)}{\partial x} k_i(x)$ is the Lie derivative of scalar function h in the direction of vector function k. The recursive Lie derivative is giving such as:

$$L_i\left(L_j(L_k h)\right) = L_{ijk} h \tag{26}$$

where, the parity matrix Ω is deduced via the following expression:

$$\Omega \times O_\Delta = [0] \tag{27}$$

Incorporation of input-output information, the nonlinear dynamically derived observability $O_{\Delta DD}$ reformulated by the observability in terms of multiple control inputs u_i, and sensor readings y_i is needed to complete the analytical redundancy parity equations. In our case, since $h(x) = Cx$, we can write the $O_{\Delta DD}$ as follows:

$$O_{\Delta DD} = \begin{bmatrix} y - 0 \\ \dot{y} - 0 \\ \ddot{y} - \sum \dot{u} L_g \\ \frac{d^3y}{dt^3} - \begin{pmatrix} \sum \ddot{u}_i L_{g_i} + \sum \dot{u}_i L_{\dot{x} g_i} \\ + \sum \dot{u}_i L_{g_i f} + \sum \dot{u}_i L_{f g_i} \\ + \sum u_i \sum \dot{u}_j L_{g_i g_j} \\ + \sum \dot{u}_i \sum u_j L_{g_i g_j} \end{pmatrix} \\ \dots \end{bmatrix} \tag{28}$$

where all of the Lie derivatives used here are with respect to Cx, thus $L_g = L_g(Cx) = gC$. Nonlinear residuals vector $R = [R_i]^T (i = 1, 2, 3...)$ are obtained by multiplying the nonlinear dynamically derived observability vector $O_{\Delta DD}$ with the parity matrix Ω:

$$R = \Omega \times O_{\Delta DD} \tag{29}$$

Note that $O_{\Delta DD}$ is a vector-valued nonlinear function, and cannot be directly checked. However, by considering $\nabla O_{\Delta DD}$, we can infer the information content, It is straightforward to calculate the dimension r_j of the associated observation space for each sensor j (from the associated rank in $\nabla O_{\Delta DD}$), and it is easy to see that these quantities are well-defined and well-behaved.

The number of residuals retained therefore corresponds to the sum of these ranks:

$$N = \sum_{i=1}^{m} r_j + (m - n) \tag{30}$$

We now determines how many rows of $O_{\Delta DD}$ are required and how many of the resulting residuals are independent eq 30. The full NLAR relations generation algorithm is given in [4].

V. Simulation Results

The nonlinear quadrotor system control law and FDI are simulated using Matlab/Simulink.

In this section we tests the performance of the nonlinear parity space relations, when some faults are injected to the IMU sensors. The residuals generated by this technique are supposed to differ from zero in case of faults and to be zero when there are no faults on the sensors

Table II represents the fault signatures matrix for these residuals. Assuming that simultaneous faults cannot occur, we find that the signatures for each of the failures are quite different.

TABLE I
Fault signatures matrix.

	R_1	R_2	R_3	R_4
$nofault$	0	0	0	0
d_1	0	0	1	1
d_2	1	1	0	0

We simulate each system during the time of $T = 25s$ for residuals evolution. It is also noted that all faulty signals are additive and visible.

In order to evaluate the proposed fault detection method, Two simulation scenarios are tested in this section:

The first one in the absence of faults also called fault free system, fig.3 shows that all residuals convergent closely to zero.

In the second one in the presence of sensor faults

– At approximately time $t = 10s$, a constant bias fault d_1 is injected on the pitch angle(ϕ), fig.4 shows that the residuals R_3 and R_4 are different to zero during the time of fault existence.

– a fault d_2 is injected on the roll angle(θ) during the time from $t = 10s$ to $t = 15s$, fig.5 shows that the residuals R_1 and R_2 sensitive to this fault changed from zero during the failure time.

It can be seen that the proposed nonlinear analytical rerundancy procedure succeeds to detect and isolate the faults in the IMU sensors affecting the quadrotor.

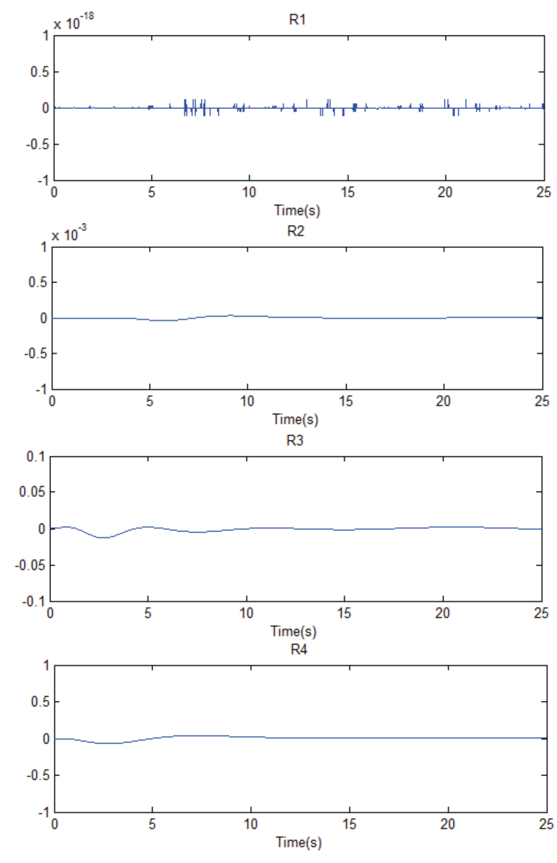

Fig. 3. Residuals evolution of the system without fault

VI. Conclusion

In this paper, an approach for fault detection and isolation based on nonlinear analytical redundancy, for a class of affine nonlinear systems has been proposed, The application is to a quadrotor drone. Through the simulation results, it is shown

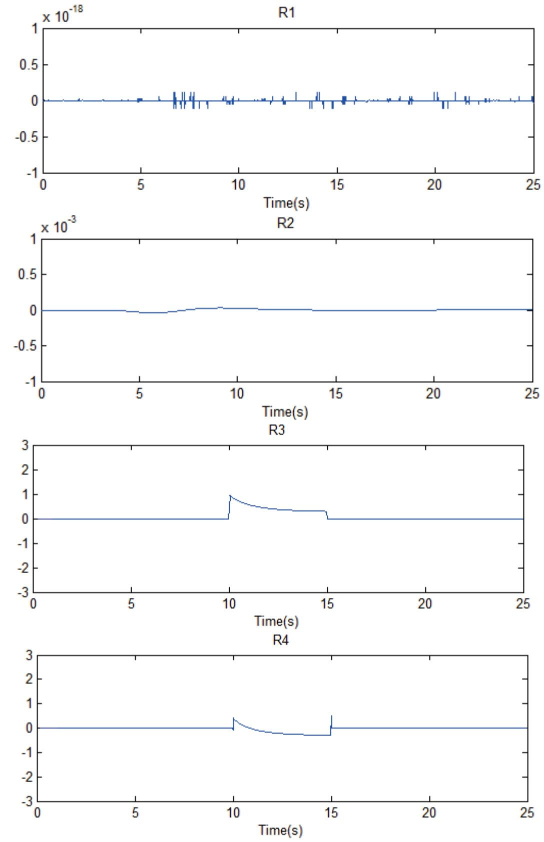

Fig. 4. Residuals evolution of the system with fault d_1

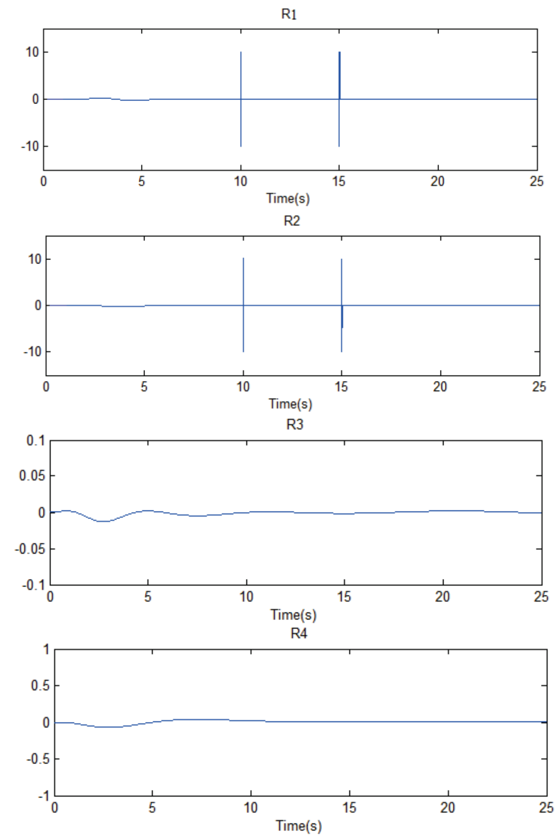

Fig. 5. Residuals evolution of the system with fault d_2

that all faults defined in specifications are detected by the combination of nonlinear independent residuals.

Appendix 1: Rotorcraft Physical Characteristics

A. Physical parameter values

TABLE II
PARAMETER VALUES

Parameter	Description	Value
m	Mass	$500g$
C_d	Body drag coefficient	0.05
$\rho(0)$	Volumetric mass of the air at see level	$1.225kg/m$
S_p	Area of the propellers	$0.005m^2$
r_p	Propeller radius	$0.125m$
C_t	Propeller thrust aerodynamic coefficient	0.297
C_q	Propeller moment aerodynamic coefficient	0.0276
f	Force coefficient	$0.5\rho(0)S_pr_pC_t$
k	Moment coefficient	$0.5\rho(0)S_pr_pC_q$
l	Arms length	$0.25m$
I_x	inertias about body frame's x-axis	$0.007kg.m^2$
I_y	inertias about body frame's y-axis	$0.0137kg.m^2$
I_z	inertias about body frame's z-axis	$0.0073kg.m^2$

B. Rotor engine dynamics

The rotor dynamics is described by the input output relation between the input voltage Va and the angular rate ω. According to S. Waslander, the model of rotor dynamics is such as:

$$\dot{\omega}(t) = -\frac{1}{t}\omega(t) - K_g\omega(t)^2 + \frac{K_{V_a}}{t}V_a(t) \tag{31}$$

With $\omega(0) = \omega_0$ where t, K_g and K_{V_a} are given positive parameters. The voltage input is like as:

$$0 \leq V_a \leq V_{max}$$

C. Numerical values for the rotors parameters

The rotors parameters [14]:
$\tau = 10$, $K_Q = 0.0079$, $K_{V_a} = 1000$, $Vmax = 11V$.
The time response of this generator is negligible. So the rotor dynamics are given by a scalar Riccati equation:

$$\dot{\omega}(t) = -\frac{1}{\tau}\omega(t) - K_Q\omega(t)^2 + \frac{K_{V_a}}{\tau}V_a(t) \tag{32}$$

The solution is given as follows, in the case of a step input Va:

$$\omega(t) = \omega_1 + \frac{1}{\frac{1}{\omega(0)-\omega_1}\exp t/\tau' + K_Q\tau'(\exp t/\tau' - 1)}, t \geq 0 \tag{33}$$

with

$$\sqrt{1+4K_vK_Q\tau}\omega_1 = \frac{1}{2\tau K_Q}(\sqrt{1+4K_vK_Q\tau V_a} - 1) \tag{34}$$

writing $\omega(t)$ as:

$$\omega(t) = \omega_1 + \frac{1}{\frac{1}{\omega(0)-\omega_1} + K_Q\tau'(1-\exp -t/\tau')}, t \geq 0 \tag{35}$$

It appears that the dynamics of the rotor may be close to those of a first order linear system with time constant $\tau^{'}$, but (34) and (35) shows that this value is a function of V_a. If the desired dynamics for the output are like as:

$$\dot{\omega} = -\frac{1}{T}(\omega - \omega_c) \tag{36}$$

where T is a very time constant V_a can be chosen such as:

$$V_a(t) = \frac{1}{K_{V_a}}[(1-\frac{\tau}{T}) + \frac{\tau}{T}\omega_c + \tau K_Q\omega(t)^2] \tag{37}$$

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