Prácticas de Matlab

Diagrama de eficiencia con métodos monopaso explícitos

Hoja 4

Consideramos el siguiente problema lineal

$$y'(t) = Ay(t) + B(t) \quad \text{para} \quad 0 \leq t \leq 10, \quad y(0) = (2,3)^T \tag{1}$$

$$A = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} \qquad B(t) = \begin{pmatrix} 2\sin(t) \\ 2(\cos(t) - \sin(t)) \end{pmatrix} \tag{2}$$

La solución exacta es:

$$y = 2e^{-t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} \sin(t) \\ \cos(t) \end{pmatrix} \tag{3}$$

1.1 Solución

1.1.1 El método de Euler

$$\begin{aligned}
N_{vect} &= (200, 400, 800, 1600, 3200, 6400, 12800,) \\
h_{vect} &= (0.05, 0.025, 0.0125, 0.00625, 0.003125, 0.0015625, 0.00078125,) \\
error_{vect} &= (0.0176883757065681, 0.0087989544665164, 0.0043882782244450, 0.0021913796558545, 0.0010949926873401, 0.0005473226335860, 0.0002736178571274,) \\
Ev_{vect} &= (200, 400, 800, 1600, 3200, 6400, 12800,)
\end{aligned}$$

1.1.2 El método de Euler mejorado

$$\begin{aligned}
N_{vect} &= (200, 400, 800, 1600, 3200, 6400, 12800,) \\
h_{vect} &= (0.05, 0.025, 0.0125, 0.00625, 0.003125, 0.0015625, 0.00078125,) \\
error_{vect} &= (0.0006281932787013, 0.0001527684751906, 0.0000376765518920, 0.0000093561302636, 0.0000023312324346, 0.0000005818377535, 0.0000001453386120,) \\
Ev_{vect} &= (400, 800, 1600, 3200, 6400, 12800, 25600,)
\end{aligned}$$

1.1.3 El método de Euler modificado

$$\begin{aligned}
N_{vect} &= (200, 400, 800, 1600, 3200, 6400, 12800,) \\
h_{vect} &= (0.05, 0.025, 0.0125, 0.00625, 0.003125, 0.0015625, 0.00078125,) \\
error_{vect} &= (0.0003581947889174, 0.0000869474740182, 0.0000214239992978, 0.0000053178132343, 0.0000013247238204, 0.0000003305924434, 0.0000000825750520,) \\
Ev_{vect} &= (400, 800, 1600, 3200, 6400, 12800, 25600,)
\end{aligned}$$

1.1.4 El método de Runge-Kutta 4

$$\begin{aligned}
N_{vect} &= (200, 400, 800, 1600, 3200, 6400, 12800,) \\
h_{vect} &= (0.05, 0.025, 0.0125, 0.00625, 0.003125, 0.0015625, 0.00078125,) \\
error_{vect} &= (0.0000004254488192, 0.0000000256448106, 0.0000000015739424, 0.0000000000974948, 0.0000000000059887, 0.0000000000004803, 0.0000000000005748,) \\
Ev_{vect} &= (800, 1600, 3200, 6400, 12800, 25600, 51200,)
\end{aligned}$$