

Prácticas de Matlab
Resolución de EDO con métodos implícitos
Hoja 5 A

1.1 Práctica 3 (Ecuación no rígida con Euler implícito)

1.1.1 Objetivo

Vamos a comparar, mediante diagramas de eficiencia, diferentes implementaciones del método del Euler implícito

$$y_{k+1} = y_k + h(f(t_{k+1}, y_{k+1})) \quad (1)$$

mediante:

- Punto fijo
- Punto fijo+Euler (PC)
- Newton
- Newton + Euler (PC)

Lo vamos a hacer para una ecuación no rígida y una rígida.

1.1.2 Caso no rígido

Consideramos el siguiente sistema

$$y'(t) = Ay(t) + B(t) \quad t \in [0, 10] \quad (2)$$

$$A = \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} \quad B(t) = \begin{pmatrix} 2 \sin(t) \\ 2(\cos(t) - \sin(t)) \end{pmatrix} \quad (3)$$

$$y(0) = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad (4)$$

La solución exacta es:

$$y = 2e^{-t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} \sin(t) \\ \cos(t) \end{pmatrix} \quad (5)$$

1.1.3 Resultados de los errores

| h_{vect} | 0.05 | 0.025 | 0.0125 | 0.00625 | 0.003125 | 0.0015625 | 0.00078125 | 0.000390625 |
|----------------------------------|-----------|------------|------------|------------|-------------|-------------|-------------|-------------|
| $\text{err}_{\text{elimfpx}}$ | 0.0152631 | 0.00784367 | 0.00510129 | 0.00360051 | 0.00311607 | 0.00180795 | 0.000964515 | 0.000482232 |
| $\text{err}_{\text{elimfpxpc}}$ | 0.0175537 | 0.00840571 | 0.00406073 | 0.00199587 | 0.000989447 | 0.000492617 | 0.000245784 | 0.000122761 |
| $\text{err}_{\text{elimpnwt}}$ | 0.0154744 | 0.00779209 | 0.00391016 | 0.00195858 | 0.000980165 | 0.000490302 | 0.000245206 | 0.000122617 |
| $\text{err}_{\text{elimpnwtpc}}$ | 0.0154744 | 0.00779209 | 0.00391016 | 0.00195858 | 0.000980165 | 0.000490302 | 0.000245206 | 0.000122617 |

1.1.4 Gráficas

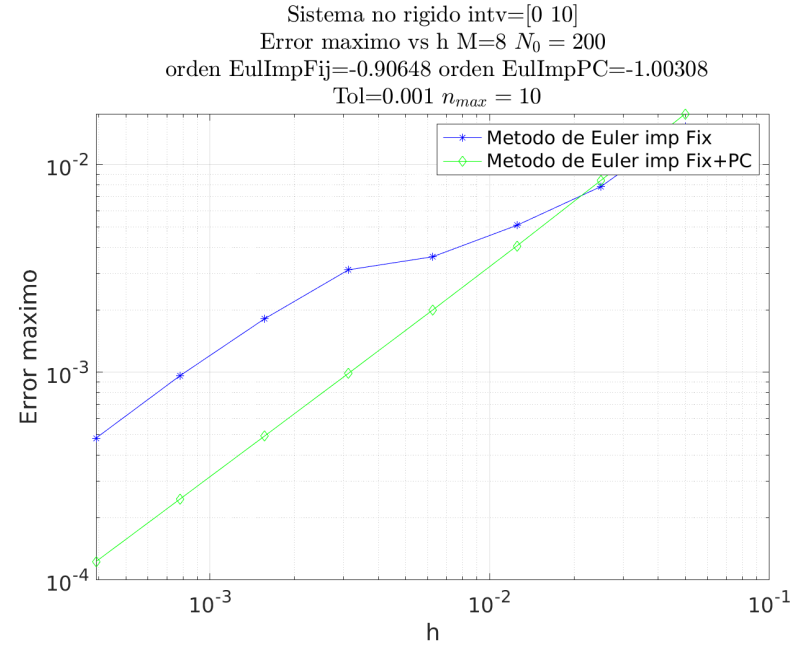


Figure 1: El método del Euler implícito punto fijo vs PC

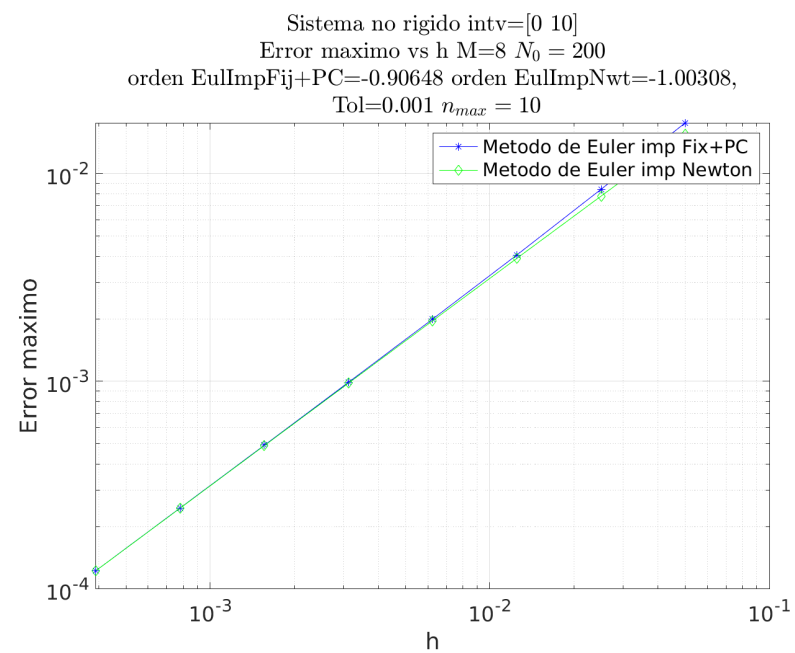


Figure 2: El método del Euler implícito-punto-fijo-PC vs Euler-Imp-Nwt

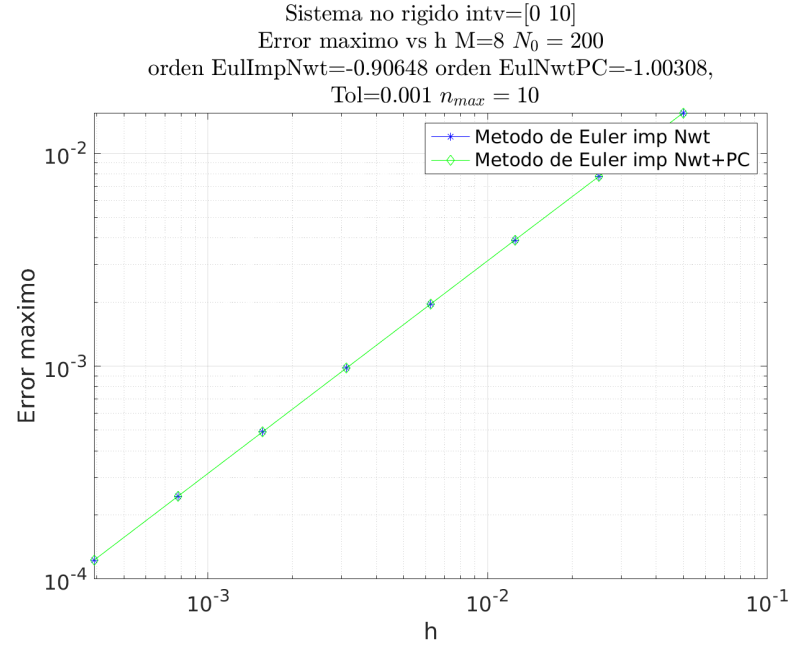


Figure 3: El método del Euler implícito Nwt vs Euler-imp Nwt-PC

1.2 Práctica 5 (Ecuación rígida con Euler implícito)

$$y'(t) = Ay(t) + B(t) \quad t \in [0, 10] \quad (6)$$

$$\left(A = \begin{pmatrix} -2 & 1 \\ 998 & -999 \end{pmatrix} \right) \quad B(t) = \begin{pmatrix} 2 \sin(t) \\ 999(\cos(t) - \sin(t)) \end{pmatrix} \quad B(t) = \begin{pmatrix} 2 \sin(t) \\ 2(\cos(t) - \sin(t)) \end{pmatrix} \quad (7)$$

$$y(0) = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \quad (8)$$

La solución exacta es:

$$y = 2e^{-t} \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} \sin(t) \\ \cos(t) \end{pmatrix} \quad (9)$$

1.2.1 Resultados de los errores

| | | | | | | | | |
|-----------------------------------|-----------|-----------|------------|------------|------------|------------|-------------|-------------|
| h_{vect} | 0.1 | 0.05 | 0.025 | 0.0125 | 0.00625 | 0.003125 | 0.0015625 | 0.00078125 |
| $\text{err}_{\text{relimpfxpc}}$ | Inf | Inf | Inf | Inf | Inf | Inf | Inf | 0.000275157 |
| $\text{err}_{\text{relimpnwt}}$ | 0.0342405 | 0.0173753 | 0.00883207 | 0.00487825 | 0.00303301 | 0.0040913 | 0.00221767 | 0.00110785 |
| $\text{err}_{\text{relimpnwtpc}}$ | 0.0337565 | 0.0172859 | 0.00830651 | 0.00427411 | 0.00216756 | 0.00109144 | 0.000547636 | 0.000274298 |

1.2.2 Gráficas

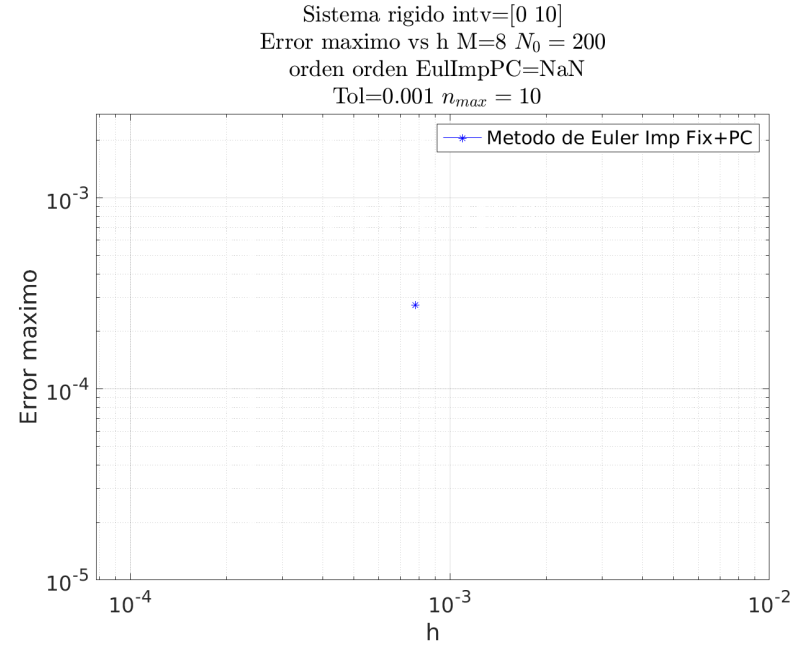


Figure 4: Euler Implicit Punto fijo+ PC

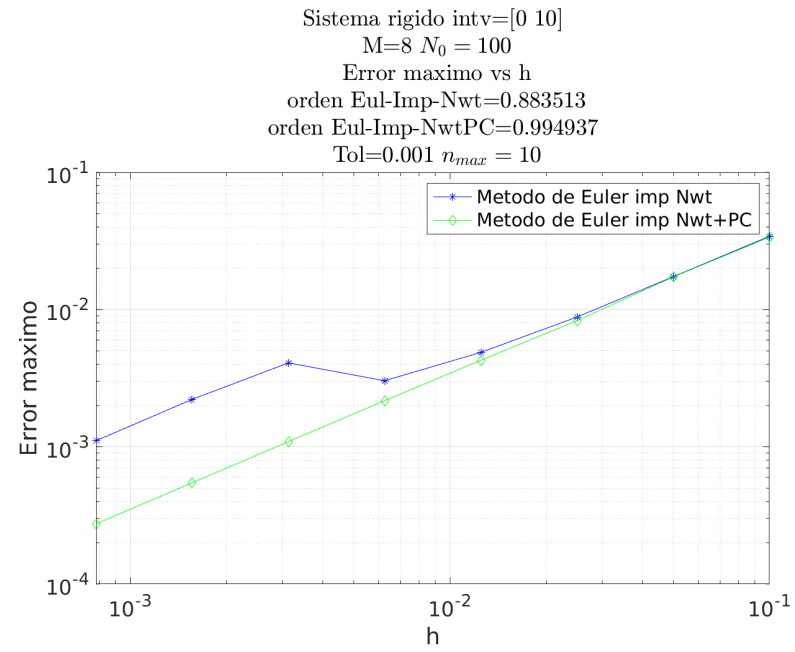


Figure 5: Euler Implicito Newton+ Newton-PC