

# Effect of pavement materials' damping properties on tyre/road noise characteristics



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## HIGHLIGHTS

- The effect of vibroacoustical damping properties of asphalt surfaces on tyre/road noise characteristics were quantified.
- A total of 168 asphalt mixtures covering 5040 data points were used in the study.
- The damping capacities of different asphalt mixtures with respect to classic acoustical control regions were established.
- Excellent correlations were found between the various tyre/pavement damping acoustical parameters of the asphalt mixtures.

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## ABSTRACT

The objective of this study was to investigate and quantify the effect of fundamental pavement vibro-acoustical damping properties of two different asphalt surfaces on tyre/road noise characteristics. A total of 159 conventional dense graded asphalt (CONV) and 9 asphalt rubber friction course (ARFC) mixtures, resulting in over 5000 data points were utilized to estimate the various physical damping parameters. Previously, viscoelastic phase angle properties were obtained for these complex modulus materials and **relationship between phase angle and highway noise** was established. Damping ratios, dynamic magnification factors, and transmissibility at resonance were estimated for these mixes. The calculations indicated that ARFC mixes provided higher noise-damping response than the CONV mixes due to the extra binder, higher porosity, rubber inclusions, which respectively rendered extra viscodamping effect, higher noise-absorption potential, and higher vibroacoustical damping capacity. Overall, this study is envisioned to aid in the discernment of the variants of asphalt pavement materials' noise damping capabilities in a fundamental form based on the relationships between vibroacoustical and mixture materials' properties.

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## 1. Introduction

Rapid urbanization has myriad outcomes such as societal development, building of new and heavy structures such as highways, generation of new jobs, improvement in quality of life standards, and so forth; but, at the same time, it creates a host of effects on the environment such as energy and water demands, need for sustainable materials, and generation of noise. One of such a sustainable aspect: highway noise and in particular, tyre/pavement noise has become a growing problem in urban areas, especially, in the neighborhoods of highways and arterial roads. More so, a significant increase in traffic volume has continually been adding more noise pollution. As per the World Bank, even when it is not perceived consciously, chronic exposure to road noise can affect human welfare in varying degrees, both physiologically and psychologically [1].

The wider use of “quieter” pavements across the globe has been an important quality of life interest in reducing the overall noise exposure due to high cost of other mitigation strategies such as noise barriers [2]. Several studies have indicated that tyre/pavement interaction noise contributes significantly to the overall roadway noise, i.e., at vehicle speeds greater than 40 km/h (25 mph) as indicated by [3,4]. Researchers across the world have listed a multitude of factors influencing tyre/road noise such as pavement material type and properties, porosity, texture, pavement age, thickness, tyre rolling resistance and vibration, and pavement friction [5–8].

In case of an asphalt concrete mixture, the viscoelastic property has been found to be one of the main factors influencing tyre/pavement noise [9–11]. The study also concluded that Asphalt Rubber Friction Course (ARFC) mixes reduce tyre/pavement noise (comparing to the conventional dense graded asphalt mixtures) because they act as an acoustic absorber due to the increased viscoelastic nature of the asphalt mix. The increased viscoelastic characteristics of an ARFC come from much higher asphalt binder content (9–10%

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by weight of the total mix) and, possibly, the inclusions of crumb rubber (20% by weight of the asphalt binder).

Field evaluation of the effects of material properties such as texture, porosity and durability on the measured noise levels in California, USA indicated a relationship of pavement surface texture with frequency related acoustic impact and shock mechanisms; the same studies also indicated that the rubberized asphalt concrete surfaces had lower tyre/pavement noise levels than conventional asphalt mixes by about 2–3 dB [12,13].

The aforementioned studies have been directed towards characterizing road noise by means of evaluating and establishing the relationships of the influence of pavement materials' properties on noise levels either by performing laboratory experiments or field measurements. However, understanding the fundamental vibroacoustical elements of pavement materials' noise-damping parameter(s) that attenuate/generate noise from a purely science perspective needs quite an amount of research and development.

One parameter that is extensively used in the design process of heavy machines and large structural members to study the effect of damping is the "damping ratio". Research studies have made use of the damping ratio parameter in embankments and bridges to distinguish between two different materials' vibration attenuation capacity [14,15]. Furthermore, damping ratios of the asphalt mixtures (including asphalt overlays for rehabilitation) were estimated in various other studies to understand damping and vibratory mechanisms along with the estimation of moduli (or stiffness) of those mixtures [16–18]. Damping ratio has also been well-utilized in railway track beds and foundation designs. Zeng et al. and Zeng conducted resonant column tests to measure stiffness and damping ratio of rubber-modified asphalt with different rubber contents, and concluded that rubber-modified mixtures can potentially be used as a foundation material for high-speed railway track-beds owing to their higher damping capacities than a conventional mix and soil subgrade [19–21]. Zhaoyu et al. also reported that an increase in rubber content from 30% to 80% increased damping by about 20% in waste granular rubber and cement soil mixtures [22]. Researchers also have utilized the concept of damping ratios as a means to calculate asphalt mixtures' temperature and frequency related material properties [23,24].

It is noteworthy that although many studies have made use of "damping ratio", a fundamental vibroacoustical parameter to determine damping capacities of different structural members, no similar work has been undertaken in tyre/road noise areas. A careful understanding and determination of the basic fundamental principles of physical acoustics such as damping ratio and its associated parameters would lead to an analogy where the acoustic wave traversing through the roadway material and hence generating noise in a pavement system is akin to a vibratory system such as heavy structures, bridges, dams, railway track beds, and foundations. This noise generating pavement system is potentially capable of storing (mass-storage capacity) and dissipating (damping or magnifying capacity) acoustical energy. Both physical and mechanical science aspects behind noise generation and attenuation in the different pavement systems are not fully understood. Therefore, this paper provides thrust to understanding the science of noise generation at source that is aimed at investigating the effect of pavement system's damping capacity on tyre/road noise characteristics. The approach taken in this study is first of its kind within the framework of tyre/road noise research and development.

The major objective of this study was to investigate and quantify the effect of fundamental pavement vibroacoustical damping properties of two different asphalt surfaces on tyre/road noise characteristics. This study is envisioned to aid in the discernment of the variants of asphalt pavement materials' noise damping

(reduction) capabilities in a fundamental form based on the relationships between vibroacoustical and mixture materials' properties. The research scope of work included four distinct parts:

- Conduct literature search regarding the various basic theories about acoustics, damping, and vibrations, and develop theoretical understanding of the salient parameters involved in estimating or quantifying the damping capacity of the different materials (Section 2).
- Collect the historical asphalt pavement material characterization database (commonly known as Arizona State University (ASU)  $\phi$ - $E^*$  database), which includes a wide range of asphalt mixtures representing conventional and asphalt-rubber (Section 3).
- Calculate and quantify the fundamental acoustical damping parameters of all the mixtures in the  $\phi$ - $E^*$  database, and develop relationships for the different acoustical damping parameters estimated for various temperatures and frequencies (Section 4.1).
- Develop a final relationship of the damping capacity parameters of the asphalt mixtures with already established field highway noise measurement (Section 4.2).

## 2. Theoretical background

This section documents the general basic vibroacoustical theories and analogy, and the test methodology used in this study to characterize asphalt mixtures' fundamental damping properties and their influence on road noise characteristics.

### 2.1. General basic acoustics for engineers

In a single degree of freedom vibration system, three different quantities, namely, mass ( $m$ ), viscous damping coefficient ( $c$ ) and stiffness ( $k$ ) characterize vibration (displacement or transmission,  $x$ ) of an acoustic component [25,26]. This analogy can be as well applied to the transmission of an acoustic wave through the pavement material and the noise damping response can be characterized by mass and stiffness of the material along with its inherent viscous damping characteristics. Fig. 1 presents the schematic of a fundamental theoretical vibration model which may look different in practice. The quantities can be modelled in terms of a homogeneous differential equation whose solution is given by:

$$x(t) = e^{-\zeta\omega_d t} X \sin(\omega_d t + \phi) \quad (1)$$

where  $\zeta$  is the damping ratio,  $X$  is the amplitude constant for motion,  $\omega_d$  the damped natural frequency and  $\phi$  is the phase angle or lag.

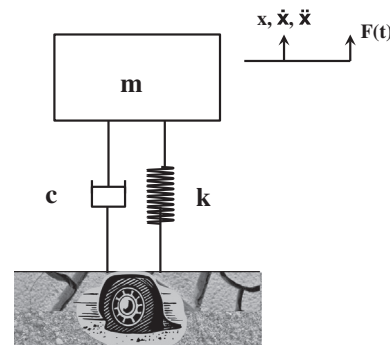


Fig. 1. Schematic of the fundamental theoretical vibration model.

$$\text{and } \omega_1 = \sqrt{\frac{k}{m}} \quad (2)$$

where  $\omega_1$  is the circular natural frequency = undamped natural frequency.

If the transmission of the acoustic wave can be monitored for its diminishing properties (decay of the wave), then  $\zeta$  can be easily calculated. Fig. 2 provides an example of such a plot of exponential decay of vibration following an acoustic impulse. In a system of forced vibration (harmonic loading condition), for instance, when the tyre rolls over a pavement material, an acoustic wave transmits along a certain continuous steady-state path, which is given by:

$$x(t) = \frac{F_0}{k} \text{DMF} \cos(\omega_0 t + \phi) \quad (3)$$

where  $F_0$  is the loading or force amplitude, DMF is the dynamic magnification factor and  $\omega_0$  is the input frequency of loading.

The phase angle,  $\phi$  by which the sinusoidal acoustic motion lags behind the sinusoidal acoustic force is also termed the phase lag. The non-dimensional constant, DMF describes how much greater the transmission is, dynamically, than it would be under a static load of  $F_0$ . DMF basically defines how much acoustic energy is magnified/attenuated and is given by:

$$\text{DMF} = 1 / \sqrt{\left\{ 1 - \left( \frac{\omega_0}{\omega_1} \right)^2 \right\}^2 + 4\zeta^2 \left( \frac{\omega_0}{\omega_1} \right)^2} \quad (4)$$

and

$$\tan \phi = \frac{2\zeta \frac{\omega_0}{\omega_1}}{\left[ 1 - \left( \frac{\omega_0}{\omega_1} \right)^2 \right]} \quad (5)$$

The maximum value of DMF is proportional to the inverse of  $\zeta$  and under these conditions,  $\phi = 90^\circ$ , that is, at resonance,

$$\text{DMF}_{\max} \cong 1/2\zeta \quad (6)$$

The fraction of the applied force amplitude that is transmitted through the system to the support is termed Transmissibility,  $T$ . In other words,  $T$  represents the ratio of the amplitude of the force transmitted to the supporting structure to that of the exciting force and is defined by:

$$T = \sqrt{\frac{1 + (2\zeta(\omega_0/\omega_1))^2}{[1 - (\omega_0/\omega_1)^2]^2 + (2\zeta(\omega_0/\omega_1))^2}} \quad (7)$$

At resonance, when  $\omega_0 = \omega_1$ , Eq. (7) can be represented as:

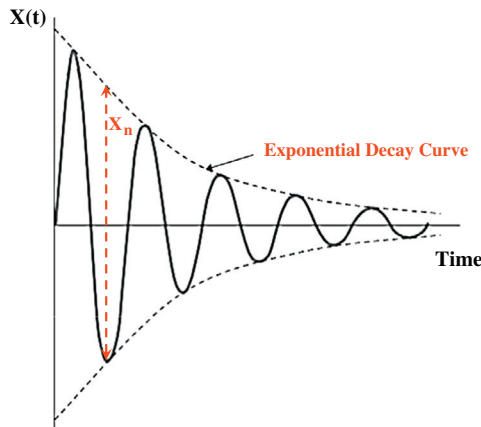


Fig. 2. Exponential decay of vibration for an acoustic impulse (Damping).

$$T_{\text{crit}} = \sqrt{\frac{1 + 4\zeta^2}{4\zeta^2}} \quad (8)$$

## 2.2. Dynamic modulus $E^*$ test

In recent development, the Dynamic Modulus ( $E^*$ ) laboratory test has become one of the most important material properties for asphalt mixtures. It was recommended as a Simple Performance Test to complement the mixture design process under the National Cooperative Highway Research Program (NCHRP) Project 9-19 of the United States [27]. The  $E^*$  is also an important input property in the Mechanistic Empirical Pavement Design Guide (MEDPG) for the Design of New and Rehabilitated Pavement Structures in the United States [28]. Because of its importance, a considerable amount of  $E^*$  tests have been performed by many researchers on many conventional and non-conventional asphalt mixtures.

For linear viscoelastic materials such as asphalt mixes, the stress-strain relationship under a continuous sinusoidal loading is defined by a complex number called the complex modulus  $E^*$  [27]. By current practice [29], dynamic modulus  $E^*$  testing of asphalt materials is conducted on unconfined cylindrical specimens having a height to diameter ratio greater than or equal to 1.5 and uses a uniaxially applied sinusoidal load.  $E^*$  test is performed in the laboratory at  $-10, 4, 21, 38$  and  $54^\circ\text{C}$  ( $14, 40, 70, 100$  and  $130^\circ\text{F}$ ), and at  $25, 10, 5, 1, 0.5$  and  $0.1$  Hz.

$E^*$  has a real ( $E'$ ) and an imaginary ( $E''$ ) part that define the elastic and viscous behavior of the linear viscoelastic material, respectively. The absolute value of the complex modulus  $|E^*|$  is defined as the dynamic modulus. The loss tangent ( $\tan \phi$ ) is the ratio of the energy lost to the energy stored in a cyclic deformation and is equal to:

$$\tan \phi = E''/E' \quad (9)$$

where  $\phi$  is the phase angle or lag (degrees);  $0^\circ$  (purely elastic)  $\leq \phi \leq 90^\circ$  (purely viscous).

$\phi$  is obtained (calculated) simultaneously with the  $E^*$  test. This parameter determines the elastic or viscous properties of an asphalt mix.  $\phi$  was determined to be a parameter of interest to evaluate the noise characteristics in the field [9]. Furthermore, the study effort focused on the utilization of the  $\phi$  database in the development of a relationship between  $\phi$  parameter and pavement materials' properties by means of master curves and predictive equations.

For simple harmonic excitation (e.g., tyre rolling on a pavement surface) in a complex modulus material (such as asphalt mixtures), studies have shown that  $\phi$  and  $\zeta$  can be developed and related [30,31] based on the following inter-relationships.

Loss tangent or  $\tan \phi$  is related to both damping coefficient and stiffness of the material based on the following relation:

$$\tan \phi = \frac{c}{k} \quad (10)$$

Also, Bert [30] defines a relationship for the critical material damping coefficient ( $c_c$ ) to be:

$$\left( \frac{c_c}{2m\omega_1} \right)^2 = \frac{k}{m} = \omega_1^2 \quad (11)$$

$$\text{and } c_c = 2m\omega_1^2 \quad (12)$$

Also, one can obtain a relationship between  $c$  and  $\zeta$  as follows:

$$c = 2\zeta m\omega_1^2 \quad (13)$$

Inserting Eqs. (2) and (12) into Eq. (10), the following important relationship can be derived:

$$\tan \phi = 2\zeta \quad (14)$$

$$\text{or } \zeta = \frac{\tan \phi}{2} \quad (15)$$

$$\text{At resonance, when } \omega_o = \omega_1, \quad \zeta \equiv \zeta_{crit} = \frac{\tan \phi}{2} \quad (16)$$

Furthermore, % critical damping = 100 times the value obtained from Eq. (16).

It is very important to note that an explanation of the fact that loss tangent shown in Eq. (5) is the same as that of Eqs. (10) and (16). Also, one can see that if the numerator term in Eq. (5) is made equal to the loss tangent or  $\tan \phi$ , Eq. (16) can still be obtained as shown before (i.e., damping ratio is equal to half of the loss tangent value).

### 3. Phase angle database – materials and experimental program

The  $\phi$  database [9] used in this study represents a comprehensive set of test data that encompassed dynamic modulus  $E^*$  testing performed under several NCHRP projects for conventional and modified asphalt mixtures; in addition, it included test data for asphalt rubber mixtures from studies conducted for the Arizona Department of Transportation (ADOT), United States of America. The  $\phi$  database used in this study included  $E^*$  test data for 159 dense-graded conventional mixtures (hereafter, referred to as CONV) whose asphalt content ranged from 3.9% to 8.9% (~5% traditionally used). Three samples were used as replicates per mix type, resulting in a total of 4770 data points (i.e., equal to 159 CONV mixtures  $\times$  5 temperatures  $\times$  6 frequencies). The air voids of these mixtures ranged between 4% and 8%. In addition, the database included  $E^*$  test data for 9 Asphalt Rubber Friction Course (ARFC) open graded mixes (asphalt content: 8.3–9.7%). The air voids were in the range of about 18% for the ARFC mixes. Three samples per ARFC mix were used, resulting in a total of 270 data points (i.e., 9 ARFC mixtures  $\times$  5 temperatures  $\times$  6 frequencies).

### 4. Data analyses

The historical  $\phi$  master database was utilized to calculate and quantify the fundamental vibroacoustical damping parameters of both the CONV and ARFC mixtures. Relationships between the various damping parameters were established which entailed quantification of the damping capacities of the different asphalt mixtures. A final relationship between the major damping parameters for the two different asphalt mixtures was established which provided an understanding on how one mix type was more noise-damping than the other. This section provides the analytical procedure adopted to accomplish all of these tasks.

#### 4.1. Estimation of damping parameters

Based on the measured  $\phi$  from the  $E^*$  test, the various fundamental physical vibroacoustical damping parameters were estimated at resonance ( $\omega_o = \omega_1$ ) using the various derived Equations presented in Section 3. The resonance indicator was so chosen because at that frequency, even small periodic driving acoustic forces (here, the tyre) can produce large amplitude oscillations, because the system (here, the pavement material) stores vibrational energy. Also, it is a well-known fact that resonance occurs when a system is able to store and also transfer energy between any two storage modes. This is similar to what happens when tyre rolls over the pavement surface wherein the energy is transferred during the interaction between the tyre and the road, causing damping and undergoing losses at resonance.

$\zeta_{crit}$  was calculated using Eq. (16), and the corresponding % critical damping for each mixture separately for five temperatures and six frequencies. The value of  $DMF_{max}$  at resonance was calculated using Eq. (6), and  $T_{crit}$  was estimated using Eq. (8). Note that for this study, average  $\phi$  (hereafter called  $\phi_{avg}$ ) was estimated for the two

mixes (CONV and ARFC) at different test temperatures and frequencies.

The average values of  $\phi_{avg}$ ,  $\zeta_{crit}$ ,  $DMF_{max}$ , and  $T_{crit}$  for both CONV mixtures ( $n = 4770$ ), and ARFC mixtures ( $n = 270$ ) are shown in Table 1. It must be understood that the estimations of  $\zeta_{crit}$ ,  $DMF_{max}$ , and  $T_{crit}$  are based on the  $\phi_{avg}$  of the two different mixtures as noted previously. Based on the  $\phi$  database, the standard deviations of the  $\phi$  for CONV (159 mixes) are between 2° and 5°, and for the ARFC (9 mixes), the standard deviations are between 5° and 15° depending on temperature and frequency combinations. Also, it was verified in previous studies that the average  $\phi$  between CONV and ARFC are significantly different [9,10].

From Table 1, it was observed that the change in the values of  $\zeta_{crit}$  between CONV and ARFC is anywhere between 1% and 63%, prominent especially at low temperatures (−10 to 4 °C) and very high temperatures (54 °C). For almost all of the temperature–frequency combinations, the average ( $\zeta_{crit}$ )–ARFC was greater than ( $\zeta_{crit}$ )–CONV. The change is in line with the change in  $\phi_{avg}$  values for both the mix types. At the same time, the average  $DMF_{max}$  and  $T_{crit}$  of ARFC were lower than the CONV mixes. These preliminary findings were indicative of the fact that ARFC mixes that have at least on average 4% higher asphalt binder content (and higher porosity), along with crumb rubber inclusions provided higher noise-damping response than the CONV mixes (with no rubber modification). The softness of the ARFC material comes from a higher binder content that provides higher viscous response in the material, which is directly influencing the damping properties as well. It is also noteworthy that crumb rubber particles which are basically elastic in nature provide a cushioning effect in the ARFC mixes only to support the binder to absorb acoustical energy.

Based on the principles illustrated previously, the following theoretical relationships for the different damping parameters can be obtained:

Eq. (16) provides a theoretical relation for  $\zeta_{crit}$  and  $\phi$  in the form:

$$\zeta_{crit} = 0.5 \tan \phi_{avg} \quad (17)$$

Eq. (6) shows a theoretical relation for  $DMF_{max}$  and  $\zeta_{crit}$  as follows:

$$DMF_{max} = 1/(2\zeta_{crit}) \quad (18)$$

Based on Eqs. (5), (7), and (8), one can obtain a theoretical relation between  $T_{crit}$  and  $\phi_{avg}$  as follows:

$$T_{crit} = 1/\sin \phi_{avg} \quad (19)$$

Another important relationship between  $T_{crit}$  and  $DMF_{max}$  can be obtained by manipulating the Eqs. (4) and (7), which is given by:

$$T_{crit} = \sqrt{1 + DMF_{max}^2} \quad (20)$$

Using the Eqs. (17)–(20), both theoretical curves and the actual estimated damping parameters (obtained from Table 1) for both CONV and ARFC mixes were plotted for the following relationships:

- $\zeta_{crit}$  versus  $\phi_{avg}$  (Fig. 3a).
- $DMF_{max}$  versus  $\zeta_{crit}$  (Fig. 3b).
- $T_{crit}$  versus  $\phi_{avg}$  (Fig. 4a).
- $T_{crit}$  versus  $DMF_{max}$  (Fig. 4b).

As shown in Fig. 3a, for both the mixes, with an increase in  $\phi_{avg}$ , there was an increase in  $\zeta_{crit}$ . The theoretical curve plotted using Eq. (17) is representative of the fact that damping ratio is half of the loss tangent. One can observe that the CONV mix's relationship is completely in line with the theoretical curve, which has a range of  $\phi_{avg}$  from 6° to 36°, with corresponding  $\zeta_{crit}$  between 0.06 and



**Table 1**

Fundamental damping properties for conventional dense graded asphalt (CONV) and asphalt rubber friction course (ARFC) mix types.

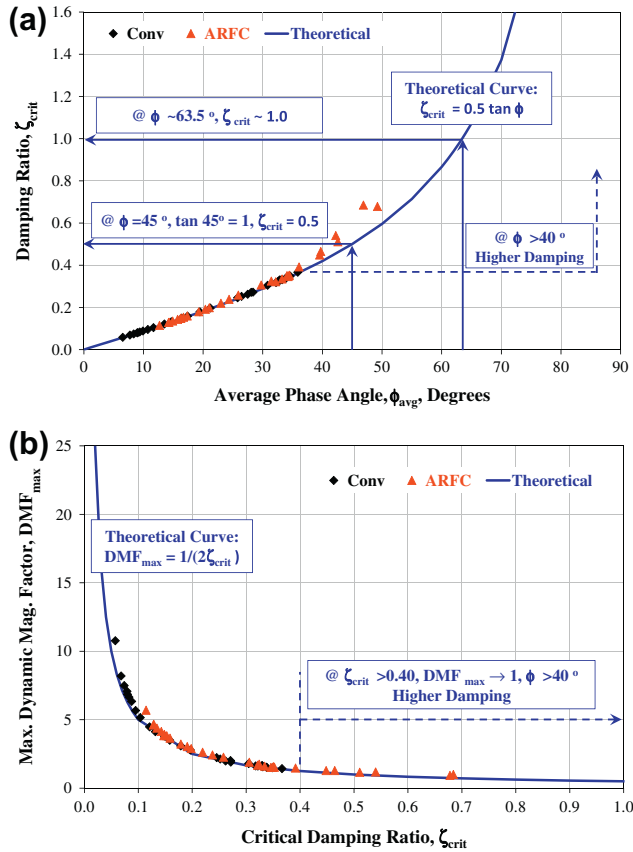
Temp (°C)	$\omega_1$ (Hz)	$\omega_o$ (Hz)	$\phi_{avg}$ (°)		$\zeta_{crit} = \tan \phi/2$ at $\omega_o = \omega_1$		DMF <sub>max</sub>		$T_{crit}$	
			CONV	ARFC	CONV	ARFC	CONV	ARFC	CONV	ARFC
–10	25	25	6.53	12.69	0.0576	0.1144	10.77	5.69	10.82	5.80
	10	10	7.74	14.22	0.0682	0.1281	8.19	4.61	8.25	4.73
	5	5	8.38	14.90	0.0739	0.1347	7.49	4.38	7.56	4.51
	1	1	9.05	15.72	0.0799	0.1428	6.85	4.13	6.93	4.27
	0.5	0.5	9.38	16.30	0.0829	0.1484	6.64	3.92	6.72	4.06
	0.1	0.1	9.91	16.72	0.0877	0.1529	6.34	3.82	6.43	3.97
4	25	25	8.96	14.30	0.0791	0.1290	7.06	4.66	7.14	4.78
	10	10	10.75	16.23	0.0952	0.1475	5.66	3.83	5.76	3.97
	5	5	11.69	17.34	0.1038	0.1590	5.15	3.65	5.25	3.80
	1	1	13.53	19.29	0.1210	0.1791	4.48	3.22	4.59	3.39
	0.5	0.5	14.68	20.38	0.1319	0.1908	4.12	3.03	4.25	3.21
	0.1	0.1	17.40	22.99	0.1584	0.2193	3.49	2.62	3.64	2.82
21	25	25	16.40	21.04	0.1487	0.1982	3.80	2.88	3.94	3.07
	10	10	19.48	24.37	0.1789	0.2378	3.08	2.42	3.25	2.64
	5	5	21.21	25.90	0.1966	0.2582	2.79	2.26	2.97	2.49
	1	1	25.79	29.75	0.2454	0.3060	2.23	1.89	2.46	2.15
	0.5	0.5	28.06	31.45	0.2712	0.3241	2.01	1.74	2.26	2.03
	0.1	0.1	32.83	36.09	0.3297	0.3917	1.64	1.46	1.94	1.79
38	25	25	30.82	32.19	0.3048	0.3212	1.79	1.67	2.07	1.96
	10	10	32.96	33.24	0.3288	0.3344	1.60	1.60	1.89	1.89
	5	5	32.54	34.08	0.3219	0.3506	1.60	1.56	1.90	1.86
	1	1	33.88	39.73	0.3381	0.4644	1.51	1.28	1.82	1.65
	0.5	0.5	33.86	42.24	0.3379	0.5401	1.52	1.18	1.82	1.57
	0.1	0.1	33.26	46.88	0.3309	0.6844	1.56	0.99	1.86	1.43
54	25	25	35.87	34.26	0.3665	0.3459	1.42	1.51	1.74	1.82
	10	10	34.61	34.39	0.3485	0.3496	1.48	1.52	1.79	1.83
	5	5	32.24	34.47	0.3176	0.3524	1.62	1.52	1.90	1.83
	1	1	28.37	39.54	0.2715	0.4483	1.89	1.28	2.15	1.64
	0.5	0.5	27.48	42.59	0.2619	0.5107	1.97	1.15	2.22	1.55
	0.1	0.1	26.45	49.24	0.2519	0.6781	2.12	0.92	2.36	1.39

0.36. The ARFC mixes'  $\zeta_{crit}$  versus  $\phi_{avg}$  relationship also fell on the theoretical curve up until a  $\phi_{avg}$  of 36°, but deviated from the line beyond that threshold value. The highest  $\phi_{avg}$  of the ARFC mixes was about 50° with a corresponding  $\zeta_{crit}$  of around 0.68. Referring to the Figure again, one can clearly see that there is a deviation of the ARFC mix's data between 40° and 50° with respect to  $\phi_{avg}$ , along with a corresponding  $\zeta_{crit}$  between 0.43 and 0.68. One may recall that  $\tan 45^\circ$  is equal to 1, which also happens to be a threshold point when the storage ( $E'$ ) and loss ( $E''$ ) moduli are equal to each other ( $E' \cos 45^\circ = E'' \sin 45^\circ$ ). At the same threshold, the estimated  $\zeta_{crit}$  is equal to 0.5. The reason for this kind of non-theoretical behavior (deviation from the theoretical curve) of the ARFC mixes comes from the fact that the extra binder (additional 4% higher than CONV mix) in these mixes, higher porosity (18% when compared to 7% for CONV), rubber inclusions, respectively, provide sufficient extra visco-damping effect, higher noise-absorption potential, and higher vibroacoustical damping capacity. This extra damping range only means that ARFC mixes would attenuate higher amount of noise or provide higher noise-damping responses than the CONV mixes. Although both CONV and ARFC mixes are categorized as viscoelastic materials, that mix which can provide higher damping in the range of  $\phi_{avg}$  from 40° to 60°, with corresponding  $\zeta_{crit}$  equal to a range of 0.4–1 would appropriately be a visco-damping material.

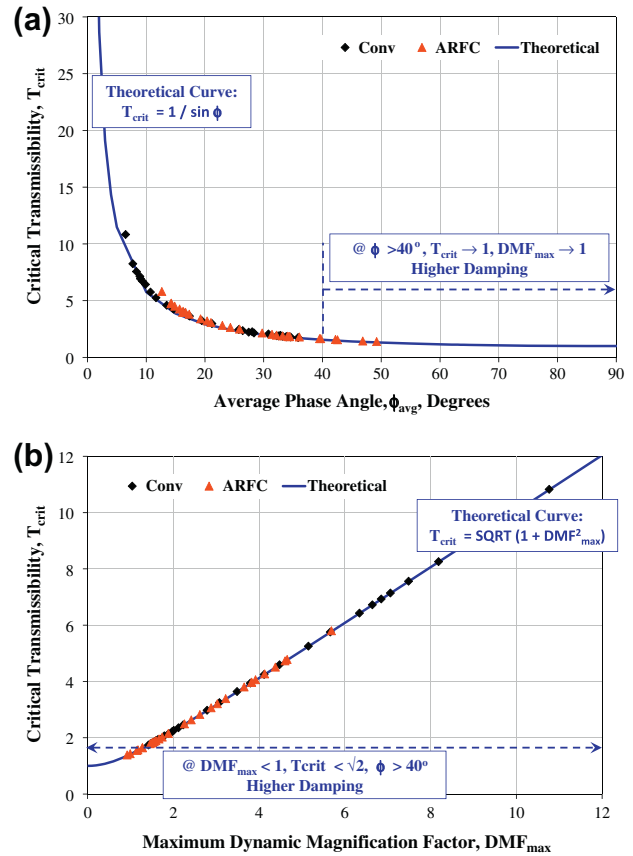
A correlation between  $\zeta_{crit}$  and DMF<sub>max</sub> corroborated the findings of the previous relationship in that DMF<sub>max</sub> which is inversely related to  $\zeta_{crit}$  (Eq. (18)) decreased with increasing  $\zeta_{crit}$ , and further with increasing  $\phi_{avg}$  as shown in Fig. 3b. As mentioned previously (Fig. 3a), for  $\phi_{avg}$  of ARFC mix beyond 40°, the corresponding  $\zeta_{crit}$  was higher than 0.4 and subsequently the mix possesses a higher damping capacity. Inversely, DMF<sub>max</sub> values decreased from 1.28

to 0.92 for the corresponding  $\zeta_{crit}$  range of 0.4 and 0.68, which also is the range of the  $\phi_{avg}$  to be between 40° and 50°. Also, by definition, DMF<sub>max</sub> describes how much greater the transmission is dynamically when loaded (impinged with acoustic wave motion), and defines how much acoustic energy is magnified/attenuated. The results showed that ARFC had much lower DMF<sub>max</sub> values than CONV mix indicative of a greater noise damping effect. A careful observation of the theoretical curve reveals that the value of DMF<sub>max</sub> tends to get smaller and smaller with increase in both  $\zeta_{crit}$  and  $\phi_{avg}$  indicating that higher noise damping is achieved with higher viscous response (higher phase angle due to additional asphalt binder in ARFC), and cushioning effect due to rubber inclusions. The actual available data for  $\phi_{avg}$  of the ARFC and CONV mixes also show the same correlation trend for the two damping parameters. However, one can observe from the Figure that almost all of the CONV data points fall in line with the theoretical curve, but there is a deviation of the ARFC data points from the curve at some coordinates.

Furthermore, theoretical curves of Transmissibility versus phase angle and dynamic magnification factor were also drawn to understand the transmission effect of the acoustic wave through the pavement system. Fig. 4a and b shows the theoretical curves for those correlations calculated using Eqs. (19) and (20), respectively. Similar to the explanatory notes provided previously for Fig. 3, it is observed in Fig. 4a that  $T_{crit}$  decreases with an increase in  $\phi_{avg}$ , especially showing a pronounced effect between 40° and 50°, which happens to be the case of the ARFC mix. The theoretical curve is basically an inverse of the sine of phase angle, which means that  $T_{crit}$  can simply be asymptotic at very low and high  $\phi_{avg}$ . Thus, higher the  $\phi_{avg}$ , higher the visco-damping effect, higher the noise attenuation, and lower the



**Fig. 3.** (a) Critical damping ratio versus average phase angle (Degrees); and (b) maximum dynamic magnification factor versus critical damping ratio for conventional dense graded asphalt (CONV) and asphalt rubber friction course (ARFC) mixtures.



**Fig. 4.** Critical transmissibility versus (a) average phase angle (Degrees); and (b) maximum dynamic magnification factor for conventional dense graded asphalt (CONV) and asphalt rubber friction course (ARFC) mixes.

value of  $T_{crit}$  or transmission of the acoustic wave through the pavement system.

A theoretical relationship between  $T_{crit}$  and  $DMF_{max}$  indicates that for higher damping in the pavement system to take place, both the parameters should be as low as possible, albeit both are not linearly related to each other. It is very interesting to note that theoretically, for a value of  $DMF_{max}$  equal to zero,  $T_{crit}$  still is equal to 1, and will remain positive even if  $DMF_{max}$  is negative (Eq. (20)). In practical scenarios, a negative  $DMF_{max}$  is not possible for pavement systems since it is very difficult to deviate from stiffness and damping control regions of vibroacoustical damping and move into a mass control region where input frequency is greater than natural frequency. From the actual data of both the mixes under consideration, lower  $T_{crit}$  were shown by ARFC mixes than the CONV ones.

It must be noted that all calculations and explanations provided so far represented resonance, meaning, when input ( $\omega_0$ ) and natural ( $\omega_1$ ) frequencies are equal, and when high damping and losses can be expected at that condition.

Another way of understanding the effect of damping in the two different asphalt concrete mixes would be to establish the relationships between DMF and frequency ratios ( $\omega_0/\omega_1$ ) at different  $\zeta$ . Using the same six test frequencies (25, 10, 5, 1, 0.5, and 0.1 Hz) applied in the  $E^*$  test, and utilizing them to be input and natural frequencies, and for the already estimated  $\zeta$ , DMF for various ( $\omega_0/\omega_1$ ) was calculated using Eq. (4). In other words, for  $\omega_1 = 25$  Hz, and  $\omega_0 = 25, 10, 5, 1, 0.5$  and 0.1 Hz, DMF was estimated. Similarly, a matrix was prepared to calculate DMF for the other natural frequencies alike. Based on the calculations, Fig. 5a and b present

the relationship of DMF versus ( $\omega_0/\omega_1$ ) for CONV and ARFC mixes, respectively, for selected  $\zeta$  and five test temperatures.

As observed, for both the mix types, an increase in the temperature yielded lower DMF for the different  $\zeta$  values. Same temperatures were chosen for both the mixes for comparative purposes on similar scales. This is clear when one observes the change in DMF values for  $0.1 < (\omega_0/\omega_1) < 10$  as it is very difficult to read the other data points beyond this range. However, the other data points are marked on the plots for completeness purposes only. Additionally, DMF for ARFC mix was lower than for the CONV mix at any considered temperature and for closely resembling  $\zeta$  values for comparison. This supports the earlier relationships in that higher the value of  $\zeta$ , lower is the DMF (also, theoretically proven by Eq. (18)). This is indicative of the fact that there would be higher noise damping when there is lower dynamic magnification of the acoustical energy through the pavement system. Similar relationships may be prepared between  $T$  and ( $\omega_0/\omega_1$ ), which will yield similar relationships and trends.

An additional feature of these classic relationships is that DMF is lower when input frequency  $\omega_0$  or simply the traffic loading speed is high (a case of arterial roads). However, the natural frequency  $\omega_1$ , or the amplitude of the load transmitted to the supporting structure at which frequency one desires to attenuate the acoustic wave propagation at source, must be lower to achieve lower transmission or equivalently DMF. This is clearly accomplished by ARFC mixes that have lower DMF throughout, even so at lower  $\omega_1$ . Note that when  $\omega_0$  is equal to  $\omega_1$ , resonance occurs and the material's critical damping parameters can be estimated.

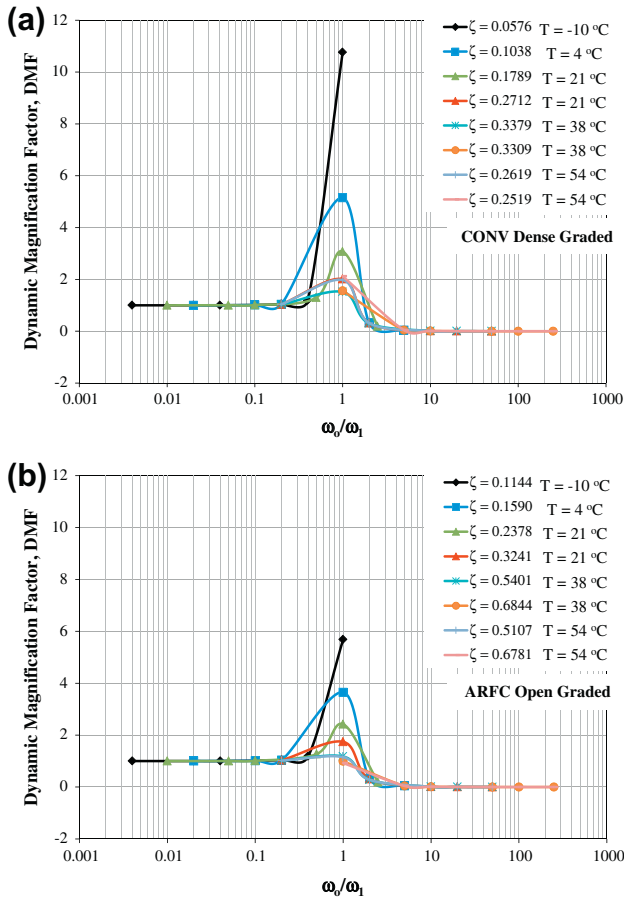


Fig. 5. Dynamic magnification factor versus frequency ratio for (a) conventional dense graded asphalt (CONV); (b) asphalt rubber friction course (ARFC) mixtures.

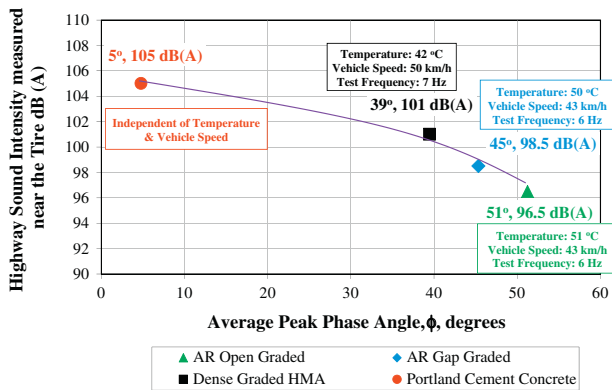


Fig. 6. Relationship between measured highway sound intensity [32] and average peak phase angle  $\phi$  of the pavement materials measured in the dynamic modulus  $E^*$  test [9,10].

#### 4.2. Relationship between damping parameters and control regions

Fig. 6 presents relationship between the average peak  $\phi$  for the various mixture types plotted against average highway noise measured by the sound intensity measurement technique [10]. The plot shows a rational non-linear trend: the higher the  $\phi$ , the lower the sound intensity measured using a close-proximity (CPX) noise trailer [32]. The plot also includes typical test temperatures and

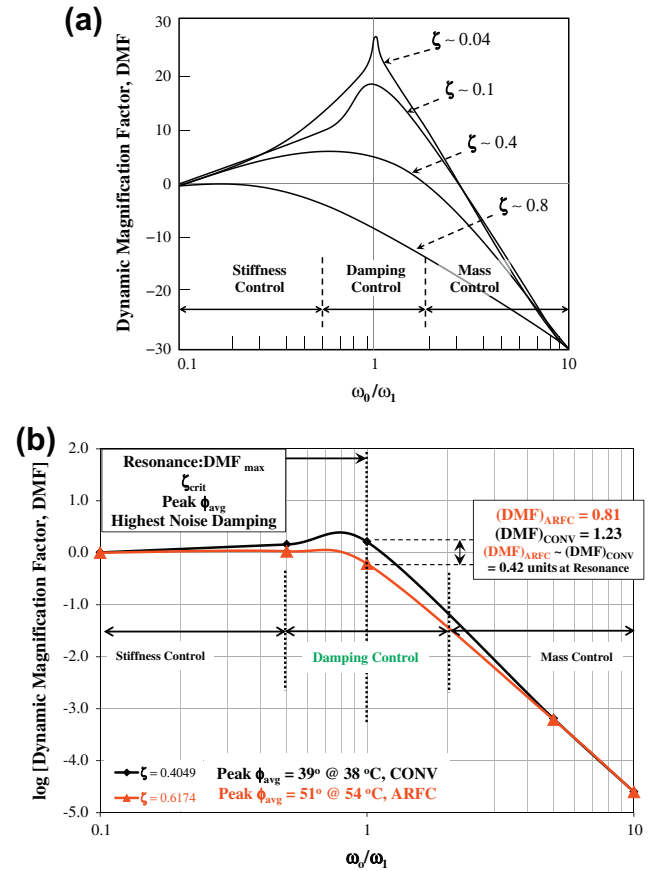


Fig. 7. (a) Classic relationship between dynamic magnification factor and frequency ratio (b) Actual dynamic magnification factor versus frequency ratio for the conventional dense graded asphalt (CONV) and asphalt rubber friction course (ARFC) mixtures.

frequencies at which these peak  $\phi$  occur as measured during an  $E^*$  test. These peak  $\phi$  occurring at the corresponding laboratory test frequencies hold a significant relation with vehicle speed, pavement temperature, and the measured tyre/pavement noise intensity as shown in the Figure. Note that the test frequencies and vehicle speeds have been previously correlated in [27], which was used to get assistance in arriving at the traffic speeds for the corresponding peak  $\phi$  frequencies.

Having the various relationships obtained based on theoretical fundamental physical vibroacoustics, investigating the damping capacities of the two different asphalt mixtures with respect to the three classic acoustical control regions was deemed essential. This exercise was accomplished for a complete understanding of the subject, which has not been previously studied by road engineers who intend to design low-noise or noise-reducing pavements.

Fig. 7a presents a typical classic relationship between DMF and  $(\omega_0/\omega_1)$  for different  $\zeta$  for three noise control regions. Since the vibration is dependent on the physical characteristics of the system, namely, mass, stiffness and damping, the system vibration can be controlled by altering any of these characteristics or by using a damping material. Examples of damping materials introduced in an asphalt mix are additional asphalt binder, increasing air voids (also called porosity), and/or modifying the mix with a noise damper such as crumb rubber particles. Regardless of an addition of a damper in a complex modulus viscoelastic material such as asphalt concrete, three controlled conditions are possible which are described as follows.

- **Stiffness control region:** For  $(\omega_0/\omega_1) < 1$ , for instance, at lower temperatures, when the asphalt material is stiff, higher force is required to reduce the impulsive motion, and hence, one can expect a lower or almost negligible attenuation of acoustic wave propagation. This would mean a no-change in the DMF value. Hence, at lower temperatures, pavement material would produce higher noise or the mix by itself behaves similar to a rock-like stiff material.
- **Mass control region:** When  $(\omega_0/\omega_1) > 1$ , mass needs to be controlled. This is often not possible in real field scenario and will not be discussed further.
- **Damping control region:** When  $(\omega_0/\omega_1)$  gets closer to 1, for example  $\omega_0 \approx \omega_1$ , one can definitely expect a higher attenuation of acoustic energy. As regards to pavement materials' noise damping response, the conditions such as higher temperature aided with softer or viscous (introducing additional binder that increases asphalt  $\phi$ ) behavioral response would produce lower noise transmission through the material. It is noteworthy that the region where  $(\omega_0/\omega_1)$  is between 0.6 and 1, highest noise damping response through the pavement material can be expected.

Fig. 7b presents the actual (calculated) relationship between  $(\omega_0/\omega_1)$  and DMF for the two asphalt mixtures. The Figure also shows the three noise control regions. The curves of DMF versus  $(\omega_0/\omega_1)$  for CONV and ARFC mixes were developed for peak  $\phi_{avg}$  (refer Fig. 6 and the corresponding documentation illustrated previously) and the corresponding natural frequencies marked on the Figure, i.e.,  $\omega_1 = 7$  Hz for the CONV and  $\omega_1 = 6$  Hz for the ARFC mix. Firstly, the DMF for the ARFC is lower than the CONV mix in all the three control regions at all frequency ratios. Secondly, one can observe that at resonance, for the peak  $\phi_{avg}$  values of the two mixes, there is a difference of DMF<sub>max</sub> equal to 0.42 units between ARFC and CONV mixes. Notice that the ordinate in the figure is in logarithmic scale so a conversion of the values to linear scale is required before noting down the values. The value of  $\zeta_{crit}$  for ARFC was 0.6174 at a peak  $\phi_{avg}$  of 51°, a value that is approximately 66% higher than CONV mix's  $\zeta_{crit}$  equal to 0.4049 at a peak  $\phi_{avg}$  of 39°. For a difference of about 10° of the peak  $\phi_{avg}$  values between the CONV and ARFC mixes, there was a difference of about 0.21 units of the  $\zeta_{crit}$  between the two mixes, translating into a difference of 0.42 units of DMF<sub>max</sub> between a “quiet” ARFC (noise level = 96 dB) and a CONV (noise level = 101 dB) mixture. As previously indicated, in the damping control region, higher amount of noise damping can be expected, which is clearly illustrated by the estimations made in this study.

## 5. Discussion

A significant and exclusive contribution of this study is that the final outcome could be well-utilized as a scientific tool measure towards a comprehensive understanding of the fundamental physical damping properties of roadway construction materials. This tool(s) could be useful under the following major heads in future works in the tyre/road noise areas:

- **Concept:** The framework of this study by itself is fundamental in nature. This supports the basic theories of damping and vibroacoustics pertinent to roadway materials' noise characteristics. The development of this concept is truly based on science and its advanced application(s).  $\phi$ , which is one of the most important viscoelastic parameters renders tremendous amount of information as regards pavements-related structural and functional performance characteristics. However, this parameter

has not been harnessed for its potentiality in understanding many problems. In this study,  $\phi$  was well-utilized to investigate the engineering applications of the built roadway surfaces for their damping capacities and a concept was developed to evaluate its power and applicability to formulate tyre/road noise research framework of the two different pavement types. It is noteworthy that although the study made use of only two pavement types, there is no reason why this study could not be extended to conceptualize other surface types in a similar manner.

- **Formulation:** Based on the fundamental vibroacoustics principles, the salient damping parameters such as damping ratio, critical damping, dynamic magnification factor, and transmissibility were employed to formulate the relevant vibroacoustical relationships, both at individual and interactive levels. The formulation of the noise damping problem within the highway materials' types in this study is an important step to creating a venue for scientific understanding of the actual underneath noise-reduction effects of some surfaces versus the others. This study provides such a flexible platform that the formulation of one noise damping parametric index is interconnected to another, thus making it a user-friendly algorithm.
- **Relationships:** A host of relationships between the different damping parameters were developed in this study with the basic understanding that one parameter interacts with the other. The relations and plots produced helped assist the estimation of the magnitude of damping of one pavement material over the other at theoretical (mathematical) level. It must be clearly understood that any problem is easily comprehended using scientific basis. Thus, the significance of numerical outcomes clarified many inherent doubts as to how one material was quieter than the other. The correlations exemplified the theoretical behavior of the pavement materials corresponding to their damping parameters deduced from fundamental principles. There is no doubt that extensive work could be carried out in future to derive many more similar damping relationships, but this study illustrated that it is certainly possible to advancing the state-of-the-art and knowledge appropriate to tyre/road noise research.
- **Findings:** Apart from the relationships developed between the different damping parameters, a major contribution of this study was to investigate the mutual connection between the damping indices and the classic acoustical control regions. It is very important to note that the control regions genuinely determine the position of the physical noise properties of any material. The findings of the study revealed interesting results in that the real noise control engineering problems could be solved once a thorough understanding of the behavioral noise transmission response in a material is investigated. The study clearly depicted that damping control region is extremely consequential and of utmost significance in tyre/road noise areas. However, efforts must be made to disseminate the findings of the study through appropriate forums by providing hands-on-experience to acoustics and engineers to devise noise-reducing road surfaces in the form of software tools embedded with the framework developed in this research task.

## 6. Conclusions and recommendations

The main objective of this study was to investigate and quantify the effect of fundamental pavement vibroacoustical damping properties of two different asphalt surfaces on tyre/road noise characteristics. The approach taken in this study is first of its kind within the framework of tyre/road noise research and development. A total of 159 CONV and 9 ARFC mixes resulting in a total of 5040 data points were utilized to calculate the fundamental



vibroacoustical damping parameters at resonance:  $\zeta_{crit}$ ,  $DMF_{max}$ , and  $T_{crit}$ . In a previous investigation, viscoelastic phase angle ( $\phi_{avg}$ ) properties were obtained for these complex modulus materials, and relationship between  $\phi_{avg}$  and highway noise was established. Based on the estimations of damping parameters made for the 168 asphalt mixtures in this study, the major conclusions and recommendations for future research are as follows:

- **Damping parameter estimates:** For almost all of the temperature–frequency combinations, the average ( $\zeta_{crit}$ )–ARFC was greater than ( $\zeta_{crit}$ )–CONV. Furthermore, relationships between  $\phi_{avg}$  and the estimated parameters were also established. The change of  $\zeta_{crit}$  was in line with the change in  $\phi_{avg}$  values for both the mix types. At the same time, the average  $DMF_{max}$  and  $T_{crit}$  values of ARFC were lower than the CONV mixes. These preliminary findings were indicative of the fact that ARFC mixes that have at least on average 4% higher asphalt binder content (and higher porosity), along with crumb rubber inclusions provided higher noise-damping response than the non-modified CONV mixes.
- **Theoretical curves of damping parameters:** The actual estimates for all the damping relationships of the CONV mixes fell in line with the theoretical curves unlike the ARFC mixes that had deviations from those theoretical relationships, specifically, at higher  $\phi_{avg}$  (40°–50° range). The reason for this kind of non-theoretical behavior of the ARFC mixes comes from the fact that the extra binder (additional 4% higher than CONV mix), higher porosity (18% when compared to 7% for CONV), rubber inclusions, respectively, provide sufficient extra visco-damping effect, higher noise-absorption potential, and higher vibroacoustical damping capacity. Essentially, the mix which can provide higher damping in the  $\phi_{avg}$  range of 40°–60°, with corresponding  $\zeta_{crit}$  equal to a range of 0.4–1 would appropriately be a viscodamping material. Relationships between DMF and frequency ratios ( $\omega_0/\omega_1$ ) at different  $\zeta$  and temperatures for CONV and ARFC mixes revealed that an increase in the temperature yielded lower DMF for the different  $\zeta$  values. Additionally, DMF for ARFC mix was lower than for the CONV mix at any considered temperature and for closely resembling  $\zeta$  values for comparison.
- **Damping parameters at control regions:** The actual (calculated) relationship between ( $\omega_0/\omega_1$ ) and DMF within the three noise control regions were established for peak  $\phi_{avg}$  for each mix type. The DMF for the ARFC was lower than the CONV mix in all the three control regions at all frequency ratios. At resonance, for a difference of about 10° of the peak  $\phi_{avg}$  between the CONV and ARFC mixes, there was a difference of about 0.21 units of the  $\zeta_{crit}$  between the two mixes, translating into a difference of 0.42 units of  $DMF_{max}$  between a “quiet” ARFC (noise level = 96 dB) and a CONV (noise level = 101 dB) mixture. Classically, in the damping control region, a higher amount of noise damping is expected, which was clearly indicated by the estimations made in this study.
- **Future scope of research:** Overall, this study is envisioned to aid in the discernment of the variants of asphalt pavement materials’ noise damping capabilities in a fundamental form based on the relationships between vibroacoustical and mixture materials’ properties. With the basic understanding of the various fundamental vibroacoustical damping parameters obtained in this study, nomographs or charts could be prepared to further the state-of-the-knowledge of the different pavement materials at various actual vehicle speeds (frequencies) so that a road construction designer would be able to account for acoustical design as well while producing a “quiet” pavement materials system.

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