19th INTERNATIONAL CONGRESS ON ACOUSTICS
MADRID, 2-7 SEPTEMBER 2007

ON THE DYNAMIC STIFFNESS OF MATERIALS USED UNDER FLOATING FLOORS: ANALYSIS OF THE RESONANT FREQUENCY DEPENDENCE BY EXCITATION FORCE AMPLITUDE USING DIFFERENT MEASUREMENT TECHNIQUES

PACS: 43.58.-e

F. Bettarello[1], P. Fausti[2], A. Schiavi[3],
[1] ENDIF, University of Ferrara, Via Saragat 1, 44100 Ferrara, Italy; federica.bettarello@unife.it
[2] ENDIF, University of Ferrara, Via Saragat 1, 44100 Ferrara, Italy; patrizio.fausti@unife.it
[3] INRIM, Strada. delle Cacce 91, 10135 Torino, Italy; schiavi@inrim.it

ABSTRACT

ISO 9052-1 provides for the determination of the dynamic stiffness of materials used under floating floors in dwellings. The measurement technique is based on the resonant frequency of the fundamental vertical vibration of a spring-mass system achieved in the test. Either sinusoidal, white noise or pulse signals are used. When using a sinusoidal excitation signal, the tested material shows a dependence between the resonant frequency and the amplitude of the excitation force. A specific procedure for this calculation is given in the standard. There is no similar procedure for the pulse signal method, even though the amplitude of the excitation pulse may affect the resonant frequency. In this paper, a measurement procedure using both sinusoidal and pulse signals techniques is described and analyzed in order to determine the correct value of the dynamic stiffness of a material in relation to the real dependence of the resonant frequency by excitation force amplitude.
The tests using sinusoidal signals show that the relation between the resonant frequency and the force amplitude is inverse with a logarithmic trend. The results are in agreement with those of the pulse signals, which underwent a similar analysis.

DETERMINATION OF RESONANT FREQUENCY TO DETERMINE THE APPARENT DYNAMIC STIFFNESS VALUE OF RESILIENT MATERIALS

In order to determine the resonant frequency of resilient materials, ISO 9052-1 [1] recommends the use of the following excitation techniques: sinusoidal, white noise and pulse signals.
In the case of sinusoidal signals, if the resonant frequency depends on the amplitude of the excitation force, this dependence shall be set to as low a value as possible, and the resonant frequency shall be extrapolated to zero force amplitude.
Depending on the expected stiffness value *s'*, the measurement interval used as the basis for extrapolation is as follows:

$$200 \text{ mN} = F = 800 \text{ mN where } s' > 50 \text{ MN/m}^3$$
$$100 \text{ mN} = F = 400 \text{ mN where } s' = 50 \text{ MN/m}^3$$

Within these intervals, measurements shall be taken at least at three points. There are no specific procedures given for using white noise and pulse signals. Previous results [2] have demonstrated how this investigation could be undertaken when using pulse signal techniques.
The sinusoidal and white noise signals can be applied with a specific vibration exciter attached to the structure, the correct measurement procedure is described in ISO 7626-2 [3]. An impactor with a built-in force transducer, as described in ISO 7626-5 [4], can be used for the pulse signals.
The spectrum of a sinusoidal signal is composed of a single frequency and measuring the amplitude of the wave form is simple. In the case of white noise and pulse signals, the energy is spread to all frequencies. Thus determining the amplitude of the total excitation force value requires an elaborate system of instrumentation in order to process and analyze the signals.

The extrapolation of the resonant frequency value at zero force amplitude for a spring-mass system is derived from Newton's second law of motion for a spring-mass damping system under a force, as described in [5]. For damping values of less than the critical damping value, the elastic constant *k* of a spring mass system with motion only in the vertical axis *z* could be calculated as follows:

$$k = (2pf)^2 \cdot m \qquad \text{(Eq.1)}$$

For a real spring-mass system, the elastic constant value is obtained from the resonant frequency measurement using an amplitude force value of near zero but equal to $P_0 \sin wt$, where P_0 is the amplitude value and ω is the pulse of the system. Furthermore, the system is not linear but extends to a surface.
The motion equation of the real system is more complex, and the parallel components *x* and *y* of the system could be affected by the shear stiffness phenomenon, which could create significant torsions.
For a system with *n*-degrees of freedom, subjected to a harmonic force, the matrix form of the motion equation becomes:

$$[m]\{\ddot{x}(t)\} + [c]\{\dot{x}(t)\} + [k]\{x(t)\} = \{F(t)\} \qquad \text{(Eq.2)}$$

Considering:

$$\{x(t)\} = \{X(w)\}e^{iwt} \qquad \text{(Eq.3)}$$

The solution of equation 2 becomes:

$$\{X(w)\} = \left[-w^2[m] + iw[c] + [k]\right]^{-1}\{F_0\} = \left[H^*(w)\right]\{F_0\} \qquad \text{(Eq.4)}$$

The matrix $[H^*(w)]$ is the characteristic harmonic function response of the system and the constant *k*, expressed by equation 1, is the general function of equation 4. In extrapolating the real pulse ω of the system with a decreasing amplitude of excitation force, the stiffness and damping on the *x* and *y* axes are not significant, and the resonant frequency value of the real system increases, but not linearly, as it would if the motion was on a single axis.

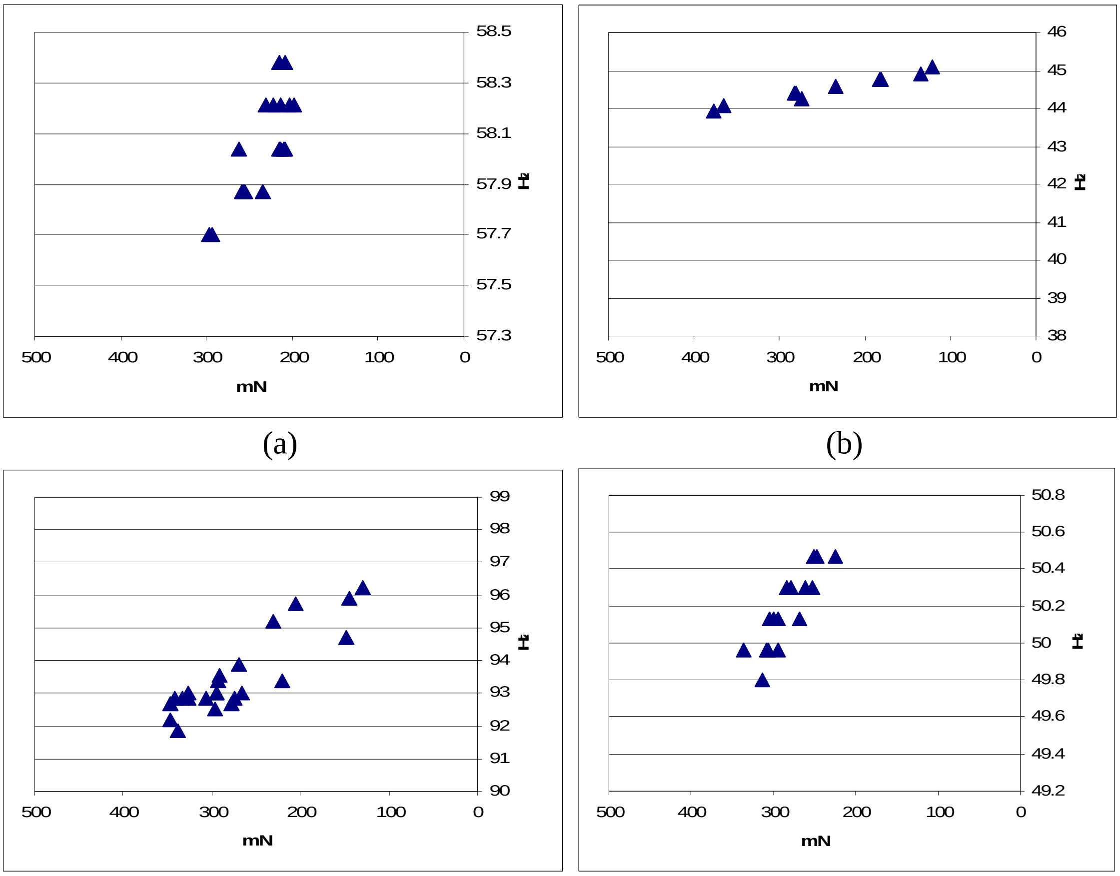
PULSE EXCITATION SIGNAL

The following analysis using the pulse signal technique was performed by the ENDIF acoustic laboratory at the University of Ferrara (Italy). The use of pulse signals as an impactor with a built-in force transducer requires an elaborate system of instrumentation provides a simple way in which to bring vibration solicitations to a structure. Impact excitation offers the following intrinsic advantages compared to the use of an attached exciter: measurement speed, ease of installation, ease of relocating the excitation point and minimal structural loading by the exciter. But the following limitations must also be taken into account: nonlinearity restrictions, signal-to-noise problems and limited frequency resolution, damping restrictions and dependence on operator skill. ISO 7626-5 suggests that averaging three to five impacts is usually sufficient to verify data quality.
The measurement set-up is composed of:

- impactor with built-in force transducer;
- charge conditioner;
- accelerometer;
- power amplifier
- signal elaboration system
- software for FFT analysis implemented by Matlab® code.

Force and acceleration signals are simultaneously acquired after the measurement system calibration. The signals are processed with FFT analysis software in order to obtain for each pulse the force and acceleration spectrum and to determine the peak and rms values. The rms value is obtained as the square root of the sum of all the spectrum values. The resonant frequency value of the spring-mass system is obtained by the acceleration signal spectrum as a maximum value in the frequency range of 0-2 kHz, at 44100 Hz sampling frequency, with frequency resolution of 0,673 Hz. The analysis is conducted by imparting many pulses to the load mass and then choosing only those pulses with amplitude force values within the range described in ISO 9052-1 for sinusoidal signals.

Figure 1 shows the results obtained by the analysis of the resonant frequency dependence by the amplitude of the excitation force for four different material types using the pulse excitation technique.Only pulses imparted with amplitude values of a pulse force within the range proposed by ISO 9052-1 for sinusoidal signals were chosen for the analysis.



(a) (b) (c) (d)

Figure 1.- investigation with pulse signals for four different material types.

For the four tested material types of figure 1, the resonant frequency value increases when the amplitude of excitation force decreases, and the link between the two quantities seems to be linear. This means that the averaging technique, as proposed by ISO 7626-5, may not be sufficient to determine the frequency-.force dependence, and also that the average of the resonant frequency values obtained by different impacts can lead to the determination of a not unambiguous apparent dynamic stiffness value.

Table 1 shows the equations obtained by a linear regression for each material type, the respective linear regression coefficients, the resonant frequency values derived from extrapolation to zero force amplitude and the values obtained by averaging the imparted impacts.

Table I.- linear regression equations, linear regression coefficients, resonant frequency at zero force amplitude extrapolation and by averaged imparted impacts

Material type	linear regression eq.	r^2	f_r (*F=0*) [Hz]	f_r (*average*) [Hz]
(a)	y = -0.0055x + 59.322	0.65	59.3	58.0
(b)	y = -0.0041x + 45.525	0.96	45.5	44.5
(c)	y = -0.0163x + 97.97	0.78	98.0	93.5
(d)	y = -0.0062x + 51.915	0.76	51.9	50.2

SINUSOIDAL EXCITATION SIGNAL

The following analysis with sinusoidal excitation signal techniques was performed by the INRIM acoustic laboratory of Torino (Italy). The measurement set-up is composed of:

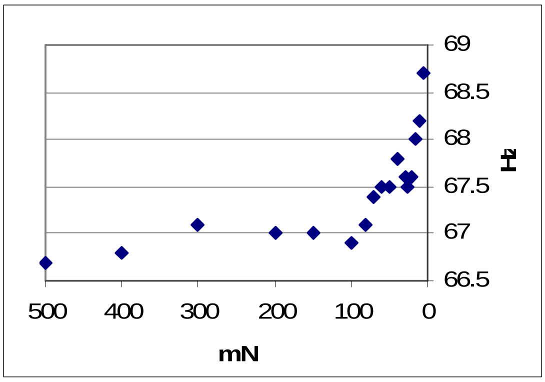
- power amplifier
- electrodynamics shaker
- functions generator
- accelerometer
- signal elaboration system

The resonant frequency is determined by varying the frequency excitation at constant force value and by measuring the maximum acceleration level outbound from the system on the load plate or by calculating the transfer function module between the excitation and vibration signals, as described in ISO 7626-2.

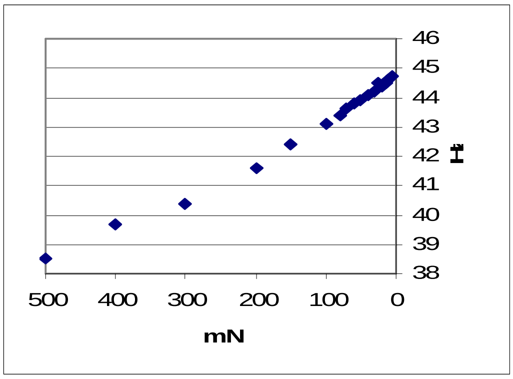
The force transducer is screwed on the load plate and is rigidly connected to the shaker by a drive rod. This configuration guarantees the best connection modality of the excitation system to the load mass. The generated force has an exclusive vertical motion, perpendicular to the load plate, without spurious momentum in the orthogonal axes. The force level was conditioned with a “fine attenuator “ positioned between the functions generator and the power amplifier. The accelerometer is also screwed on the load plate.

The INRIM laboratory has the use of apparatus for measuring the apparent dynamic stiffness, which includes an inertial base made of concrete and connected to the embankment of the laboratory. The mass and the rigidity of the inertial base guarantees a reduced level of background vibrations.

Figure 2 shows the results obtained by the investigation of the resonant frequency dependence by the amplitude of the excitation force, using sinusoidal signal techniques, for the same four material types as seen in Figure 1. The measurement interval used for the analysis was from 1 N to approximately near zero amplitude for each specimen. The resonant frequency values were measured till the acceleration level outbound from the system was greater than that of the background acceleration.



(a)



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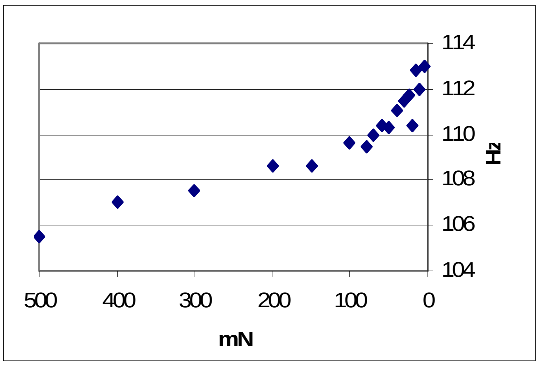

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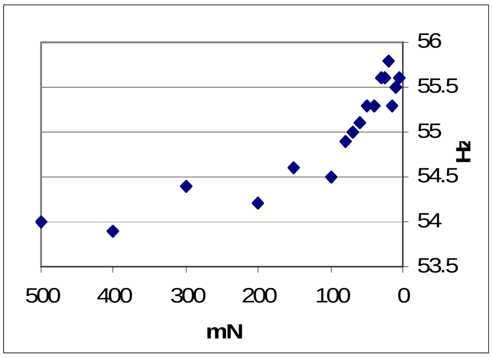

(d)

Figure 2.- investigation with sinusoidal signals for four different material types.

Table II.- logarithmic regression equations and regression coefficients, resonant frequency corresponding to 1 mN amplitude and those obtained by linear regression.

Material type	logarithmic regression eq.	r^2	f_r (*log reg.*) [Hz]	f_r (*lin. reg.*) [Hz]
(a)	y = -0.3354Ln(x) + 68.693	0.62	68.7	67
(b)	y = -2.9533Ln(x) + 57.066	0.99	57.1	44.1
(c)	y = -2.9412Ln(x) + 123.72	0.92	123.7	110
(d)	y = -0.5488Ln(x) + 57.222	0.83	57.2	54.5

COMPARISON OF EXPERIMENTAL RESULTS

The analysis of the sinusoidal excitation signal at near zero amplitude permits an in-depth investigation of the dependence from the resonant frequency value to the amplitude of force excitation.

With the pulse technique, the operator isn't able to control the amplitude force until lower values are reached. With the methodology described here, the results can be carried into the range prescribed by ISO 9052-1 for sinusoidal signals. The dependence of the resonant value from the amplitude of force excitation appeared to be linear, which was verified by good linear regression coefficients.

Using the results obtained with the sinusoidal signal techniques, further analysis on the results obtained with the pulse signal techniques was carried out. In particular, a logarithmic regression on the same values was calculated and the results obtained, with corresponding regression coefficients, are reported in table III.

Table III.- logarithmic regression equations and regression coefficients for pulse signals

Material type	logarithmic regression eq.	r^2
(a)	y = -1.3232Ln(x) + 65.254	0.64
(b)	y = -0.9321Ln(x) + 49.582	0.94
(c)	y = -3.7252Ln(x) + 114.27	0.78
(d)	y = -1.7024Ln(x) + 59.775	0.76

The logarithmic regression coefficients are very similar to those obtained with linear regression, because, as was shown with the sinusoidal measurement technique, the logarithmic trend is clearly visible at force amplitude values of less than 0.1 N. This means that in the force range used with the pulse signals, the logarithmic trend follows its own asymptote and the trend is similar to the linear one.

The resonant frequency values obtained with logarithmic regression with pulse signals were also compared to those obtained with sinusoidal signals at approximately near zero amplitude for each sample.

In order to show how the resonant frequency value f_r obtained with logarithmic regression for both excitation signals could be significant in terms of apparent dynamic stiffness values s'_t and for the reduction of impact sound pressure level *DL* of floating floors, a comparison of the results obtained for the four tested material types is shown in Table IV. The apparent dynamic stiffness value derives from the equation:

$$s'_t = 4p^2 m'_t (f_r)^2 \qquad \text{(Eq. 5)}$$

as described in ISO 9052-1, where m'_t , the total mass per unit area used during the test, is equal to 200 kg/m^2. The reduction of impact sound pressure level *DL* values derives from the equation:

$$\Delta L = 30 \log \left(\frac{f}{f_0} \right) \qquad \text{(Eq. 6)}$$

as described in EN 12354-2 [6], where *f*, the octave or third octave band centre frequency, is equal to 500 Hz and f_0 is the resonant frequency value of the system.

Table IV.- Comparison of the apparent dynamic stiffness values and reduction of impact sound pressure level obtainable with pulse and sinusoidal signals techniques

Material type	s'_t [MN/m^3]		*DL* [dB]	
	Pulse signals	Sinusoidal signals	Pulse signals	Sinusoidal signals
(a)	34	37	26.5	25.9
(b)	19	26	30.1	28.3
(c)	103	121	19.2	18.2
(d)	28	26	27.7	28.2

CONCLUSION

An investigation of the dependence of the resonant frequency value to the amplitude of excitation force for resilient materials is described. The analysis was conducted with both pulse and sinusoidal signals techniques.

First results obtained with pulse signals techniques provided evidence of a unique dependence from resonant frequency values to amplitude excitation forces for each material type.

The results obtained with sinusoidal techniques at approximately near zero amplitude demonstrate that the resonant frequency value logarithmically increases with amplitude of excitation force, and is clearly pronounced for force amplitude values of less than 0.1 N.

It could be difficult to carry out this control in so low a range using the pulse signals technique. The analysis could be easily included into the force amplitude range described in ISO 9052-1. In that range, the relation between resonant frequency and force amplitude values could be either linear or logarithmic

The differences calculated for apparent dynamic stiffness using a logarithmic regression analysis for each excitation signals are variable from 2 to 18 MN/m^3 and are related to the material type. In terms of reduction of impact sound pressure level values, these differences are less than 2 dB for the four tested material types.

References: [1] ISO 9052-1: Acoustics - Determination of dynamic stiffness - Part 1: Materials used under floating floors in dwellings [2]. F. Bettarello, et All. “Sulla determinazione della rigidità dinamica di materiali resilienti: confronto tra diverse tecniche di misura”, Proceedings of the 33° AIA Congress, Ischia 10-12 Maggio 2006. [3] ISO 7626-2: Vibration and shock - Experimental determination of mechanical mobility. Part 2: Measurements using single-point translation excitation with an attached vibration exciter. [4] ISO 7626-5: Vibration and shock - Experimental determination of mechanical mobility. Part.5: measurement using impact excitation with an exciter which is not attached to the structure. [5] Schiavi et All. “ Rigidità dinamica: indagine sperimentale sulla determinazione della frequenza di risonanza di un sistema massa molla a forzante quasi nulla” Proceedings of the 34° AIA Congress, Firenze 13-15 Giugno 2007. [6] EN 12354-2: Building acoustics – Estimation of acoustic performance of buildings from the performance of elements.-Part 2: Impact Sound insulation between rooms.