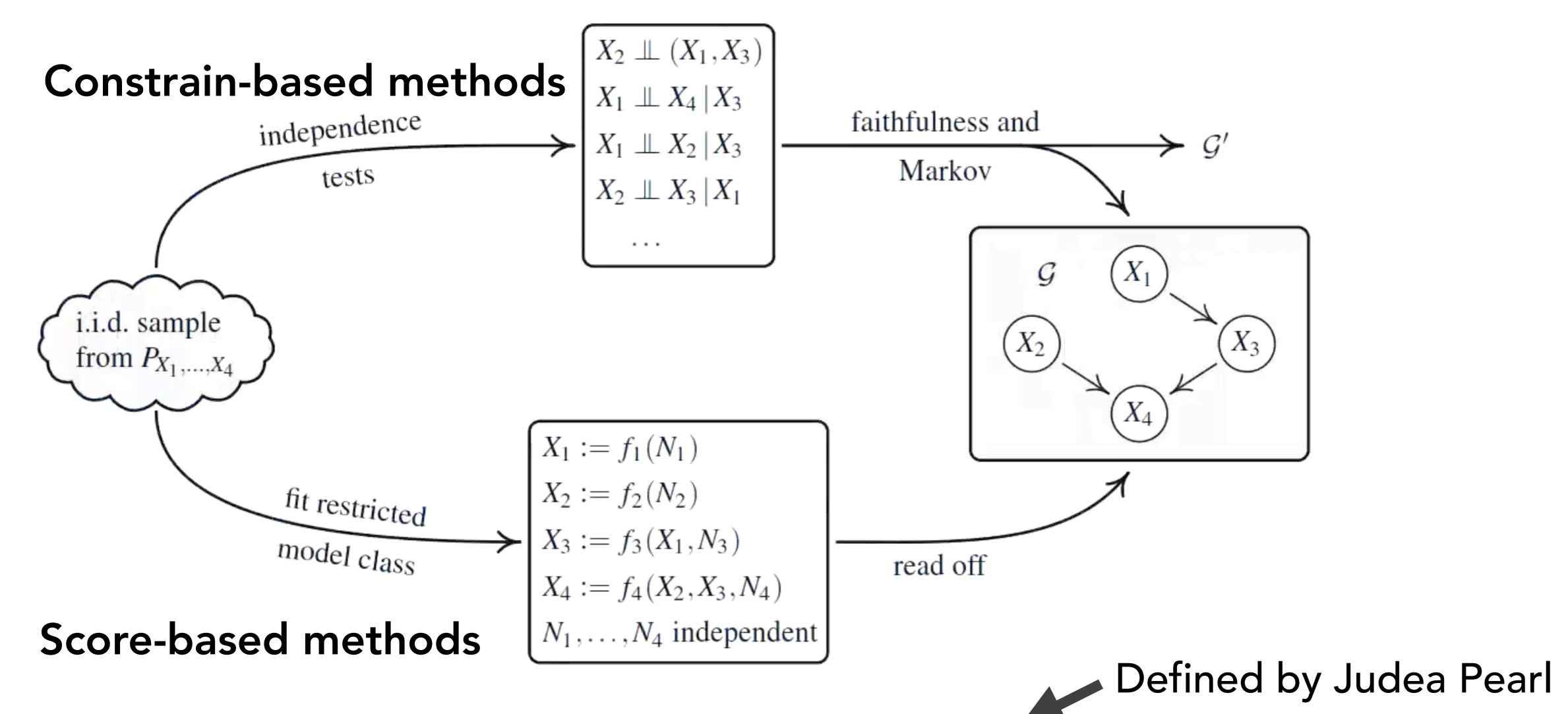
### (Background)

Causal discovery is a fundamental problem in science. Socioeconomic mechanisms come from observational multivariate.

## (Methodology)



Problem: Markov equivalence class(MEC), i.e., causal structure is not unique.

Solution: Identify causal direction by bi-variate methods with further assumptions.

Then, the main types of assumptions:

A prior restriction of the model class

- LiNGAM(Linear Non-Gaussian Acyclic Model)
- ANM(Nonlinear Additive Noise Models)

Independence assumption of cause and mechanism

- IGCI(Information-Geometric Causal Inference)
- Trace condition

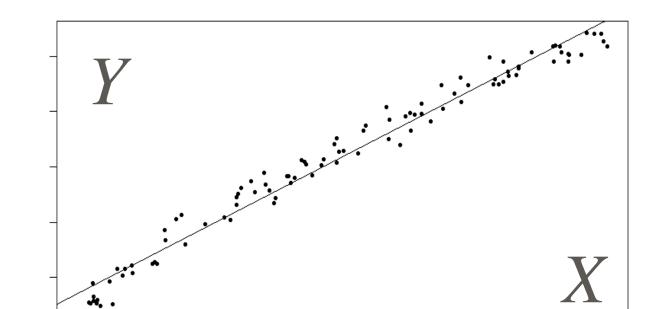
### (LiNGAM)

Assume that  $P_{X,Y}$  admits the linear model  $Y = \alpha X + N_Y$  with continuous random variables  $X, N_Y$  and Y. Then exit  $\beta \in \mathbb{R}$  and a random variable  $N_X$  such that  $X = \beta Y + N_X, N_X \perp \!\!\! \perp Y$ , iif  $N_Y$  and X are Gaussian.

e.g.: To identify 
$$X \to Y, orY \to X$$
 
$$X \to Y, orY \to X$$

Unidentifiable pattern

Joint gaussian noise distribution



Identifiable pattern

Gaussian noise on a Uniform distribution

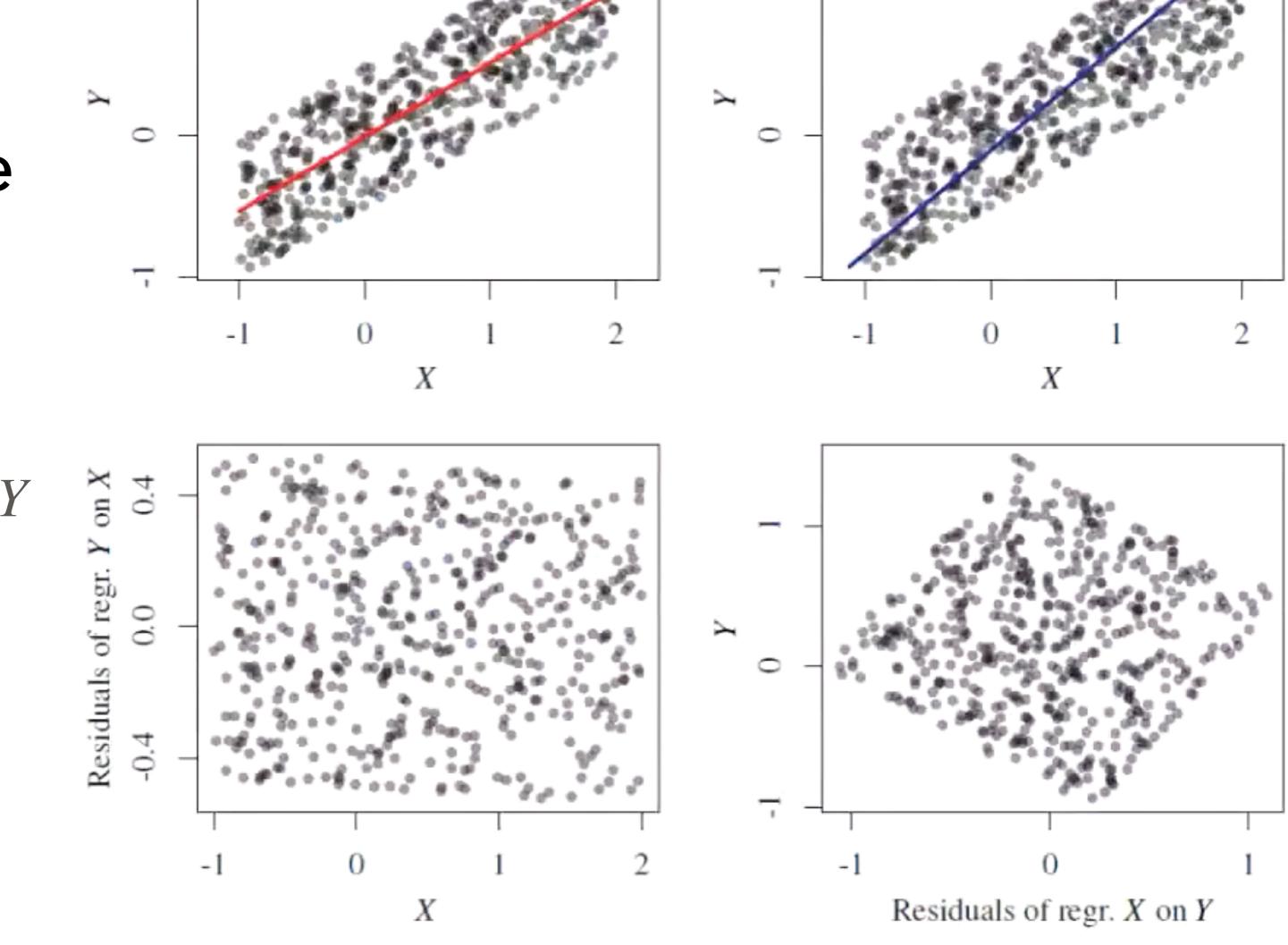
(ANM)

The joint distribution  $P_{X,Y}$  admits an ANM from X to Y if there is a measurable function  $f_Y$  and a noise variable  $N_Y$  such that  $Y = f_Y(X) + N_Y, X \perp \!\!\! \perp N_Y$ 

# \* ANM in practice:

Remark 1: If X and  $N_Y$  are gaussian, we can not identify the causal direction only when  $f_Y$  is linear.

Remark 2: If we know a prior  $P_{X,Y}$  admits an ANM of  $C(\text{cause}) \to E(\text{effect})$ , then generically, there will not be an ANM of  $E \to C$ .



#### (Others, to be continued)

Reference: K. Zhang et al. Nonlinear functional causal models for distinguishing cause from effect. Statistics and Causality: Methods for Applied Empirical Research, pp. 185-202, 2016.