Causal Discovery form Observational Multivariate

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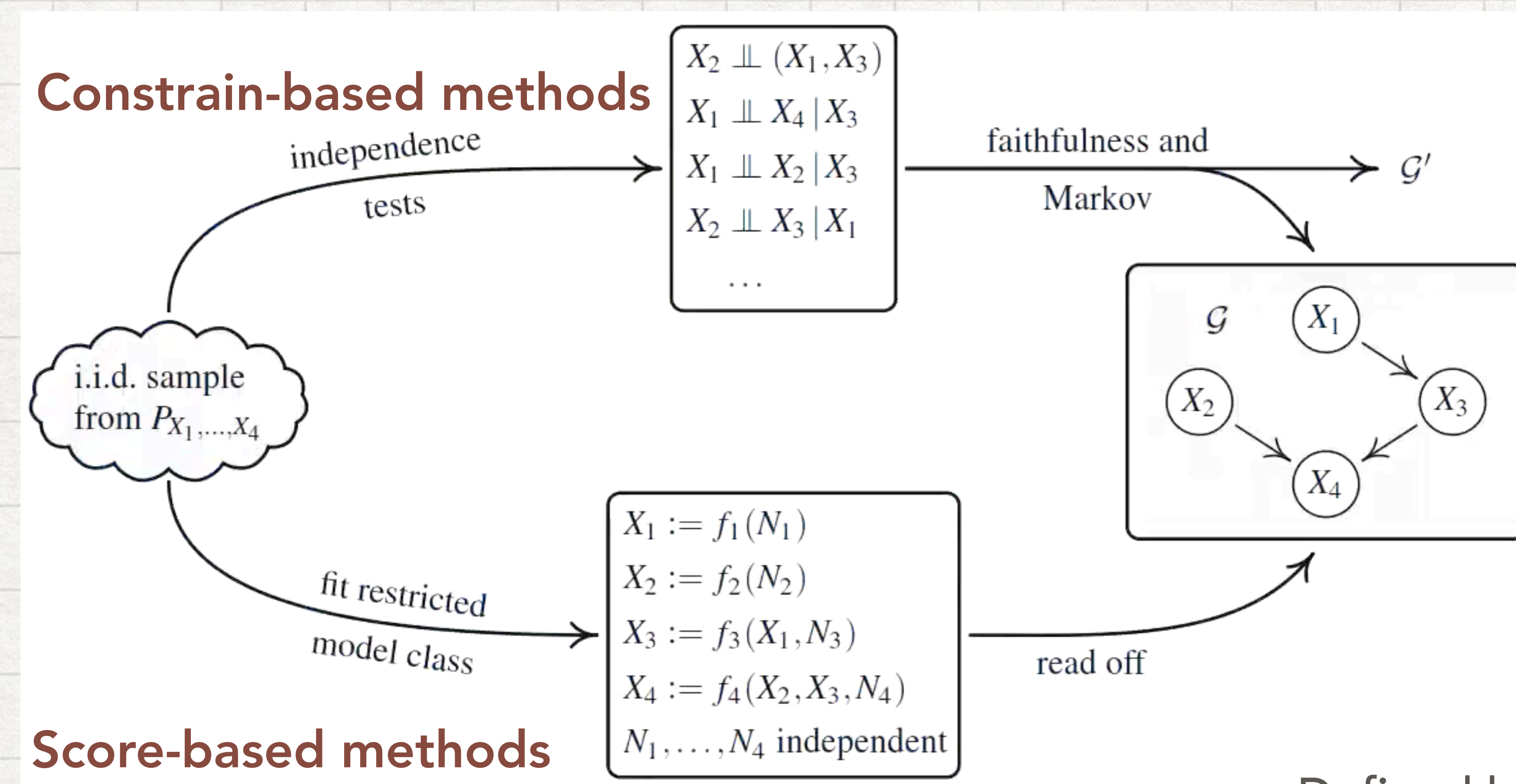Ou DENG (dengou@toki.waseda.jp)
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(Motivation)

Causal discovery is a fundamental problem in science.

Socioeconomic mechanisms come from observational multivariate.

(Methodology)

Constrain-based methods

$X_2 \perp\!\!\!\perp (X_1, X_3)$
$X_1 \perp\!\!\!\perp X_4 \mid X_3$
$X_1 \perp\!\!\!\perp X_2 \mid X_3$
$X_2 \perp\!\!\!\perp X_3 \mid X_1$
...

independence tests → faithfulness and Markov → $\mathcal{G}'$

i.i.d. sample from $P_{X_1,\dots,X_4}$

$X_1 := f_1(N_1)$
$X_2 := f_2(N_2)$
$X_3 := f_3(X_1, N_3)$
$X_4 := f_4(X_2, X_3, N_4)$
$N_1, \dots, N_4$ independent

fit restricted model class   read off

Score-based methods

Defined by Judea Pearl

<u>Problem</u>: Markov equivalence class(MEC), i.e., causal structure is not unique.

<u>Solution</u>: Identify causal direction by bi-variate methods with further assumptions.

Then, the <u>main types of assumptions</u>:
A prior restriction of the model class
- LiNGAM(Linear Non-Gaussian Acyclic Model)
- ANM(Nonlinear Additive Noise Models)
Independence assumption of cause and mechanism
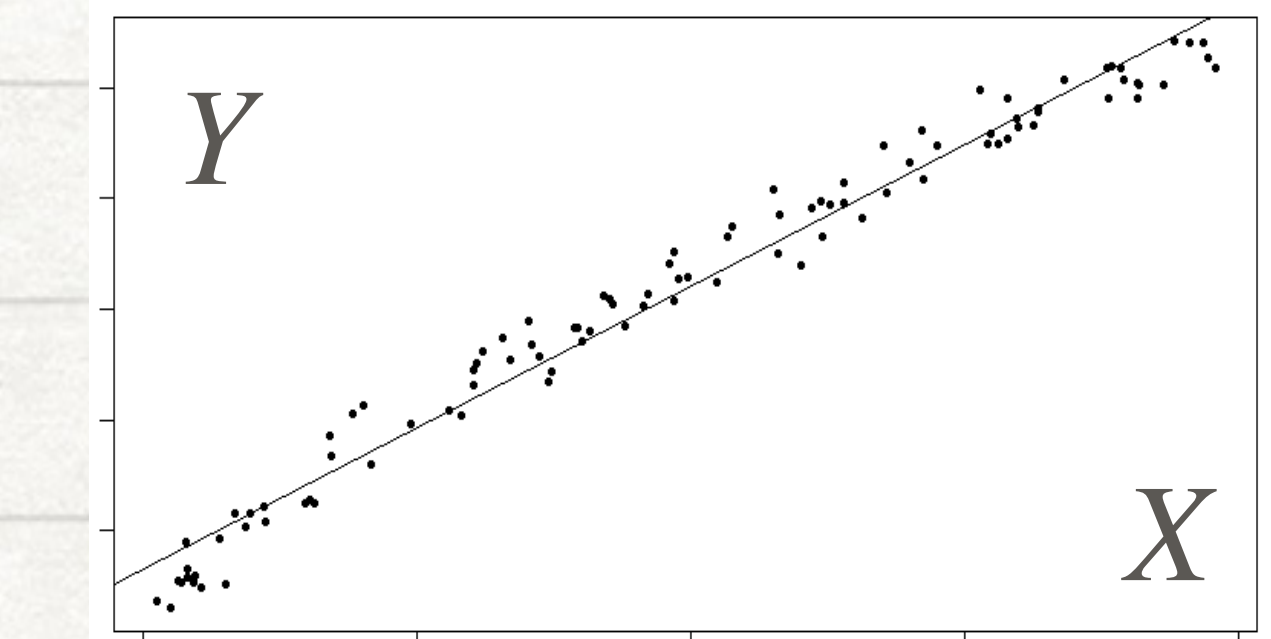- IGCI(Information-Geometric Causal Inference)
- Trace condition

(LiNGAM)

(Darmois-Skitovitch Therom) Assume that $P_{X,Y}$ admits the linear model $Y = \alpha X + N_Y$ with continuous random variables X, N_Y and Y. Then exit $\beta \in \mathbb{R}$ and a random variable N_X such that $X = \beta Y + N_X$, $N_X \perp\!\!\!\perp Y$, iif N_Y and X are Gaussian.

e.g.: To identify
$X \rightarrow Y, or Y \rightarrow X$

Unidentifiable pattern
Joint gaussian noise distribution

Identifiable pattern
Gaussian noise on a Uniform distribution

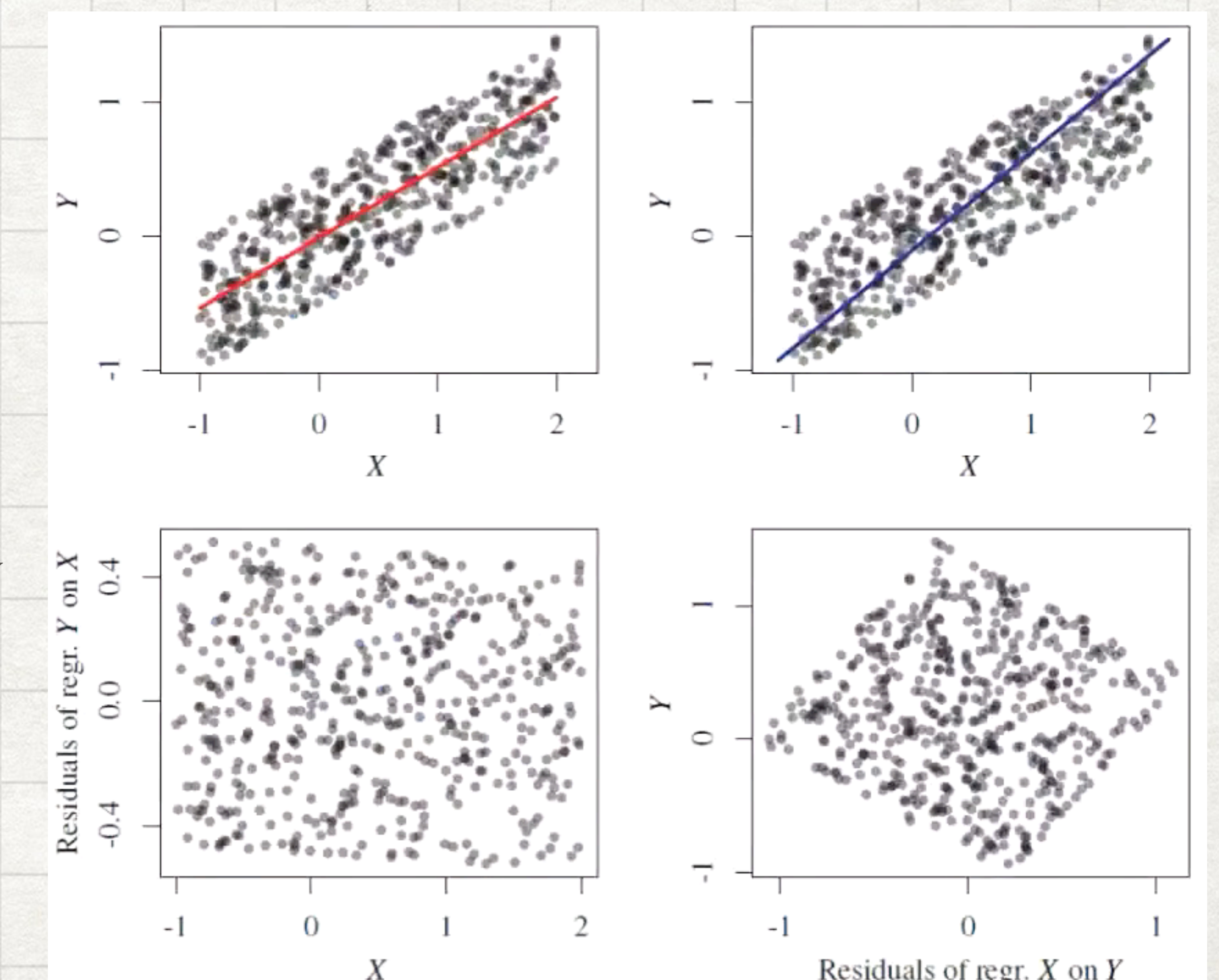## (ANM)

The joint distribution $P_{X,Y}$ admits an ANM from X to Y if there is a measurable function f_Y and a noise variable N_Y such that $Y = f_Y(X) + N_Y$, $X \perp\!\!\!\perp N_Y$

* ANM in practice:
Remark 1: If X and N_Y are gaussian, we can not identify the causal direction only when f_Y is linear.
Remark 2: If we know a prior $P_{X,Y}$ admits an ANM of C(cause) $\rightarrow E$(effect), then generically, there will not be an ANM of $E \rightarrow C$.

(Others, to be continued)

Reference: K. Zhang et al. Nonlinear functional causal models for distinguishing cause from effect. Statistics and Causality: Methods for Applied Empirical Research, pp. 185-202, 2016.

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