TD01: Probability
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0.1 Basic Concept of Probability

0.1.1 Sample Spaces, Events, and Their Probabilities

1. Construct a sample space for the experiment that consists of rolling a single die. Find the events that correspond to the phrases "an even number is rolled" and "a number greater than two is rolled."

2. Construct a sample space that describes all three-child families according to the genders of the children with respect to birth order.

3. Two fair coins are tossed. Find the probability that the coins match, i.e., either both land heads or both land tails.

4. The breakdown of the student body in a local high school according to race and ethnicity is 51% white, 27% black, 11% Hispanic, 6% Asian, and 5%for all others. A student is randomly selected from this high school. (To select "randomly" means that every student has the same chance of being selected.) Find the probabilities of the following events:

   a. B : student is black

   b. M : the student is minority (that is, not white),

   c. B : the student is not black.

0.1.2 Complements, Intersections, and Unions

5. Two events connected with the experiment of rolling a single die are E: "the number rolled is even" and T: "the number rolled is greater than two." Find the complement of each.

6. Find the probability that at least one heads will appear in five tosses of a fair coin.

7. A single die is rolled.

   a. Suppose the die is fair. Find the probability that the number rolled is both even and greater than two.

   b. Suppose the die has been "loaded" so that$P(1) = 1/12$, $P(6) = 3/12$, and the remaining four outcomes are equally likely with one another. Now find the probability that the number rolled is both even and greater than two.

8. A two-child family is selected at random. Let B denote the event that at least one child is a boy, let D denote the event that the genders of the two children differ, and let M denote the event that the genders of the two children match. Find $B \cup D$ and $B \cup M$.

9. A tutoring service specializes in preparing adults for high school equivalence tests. Among all the students seeking help from the service, 63% need help in mathematics, 34% need help in English, and 27% need help in both mathematics and English. What is the percentage of students who need help in either mathematics or English?

0.1.3 Conditional Probability and Independent Events

10. A fair die is rolled.
    a. Find the probability that the number rolled is a five, given that it is odd.
    b. Find the probability that the number rolled is odd, given that it is a five.

11. Suppose that in an adult population the proportion of people who are both overweight and suffer hypertension is 0.09; the proportion of people who are not overweight but suffer hypertension is 0.11; the proportion of people who are overweight but do not suffer hypertension is 0.02; and the proportion of people who are neither overweight nor suffer hypertension is 0.78. An adult is randomly selected from this population.
    a. Find the probability that the person selected suffers hypertension given that he is overweight.
    b. Find the probability that the selected person suffers hypertension given that he is not overweight.
    c. Compare the two probabilities just found to give an answer to the question as to whether overweight people tend to suffer from hypertension.

12. The specificity of a diagnostic test for a disease is the probability that the test will be negative when administered to a person who does not have the disease. The higher the specificity, the lower the false positive rate.

    Suppose the specificity of a diagnostic procedure to test whether a person has a particular disease is 89%.
    a. A person who does not have the disease is tested for it using this procedure. What is the probability that the test result will be positive?
    b. A person who does not have the disease is tested for it by two independent laboratories using this procedure. What is the probability that both test results will be positive ?

13. The reliability of a system can be enhanced by redundancy, which means building two or more independent devices to do the same job, such as two independent braking systems in an automobile.

    Suppose a particular species of trained dogs has a 90% chance of detecting contraband in airline luggage. If the luggage is checked three times by three different dogs independently of one another, what is the probability that contraband will be detected?

14. A jar contains 10 marbles, 7 black and 3 white. Two marbles are drawn without replacement, which means that the first one is not put back before the second one is drawn.

    a. What is the probability that both marbles are black?

    b. What is the probability that exactly one marble is black?

    c. What is the probability that at least one marble is black?

0.2 Discrete Random Variables

0.2.1 Random Variables

15. Classify each random variable as either discrete or continuous.

    a. The number of arrivals at an emergency room between midnight and $6:00$ a.m.

    b. The weight of a box of cereal labeled "18 ounces."

    c. The number of kernels of popcorn in a 1-pound container.

    d. The number of applicants for a job.

16. Classify each random variable as either discrete or continuous.

    a. The number of boys in a randomly selected three-child family.

    b. The temperature of a cup of coffee served at a restaurant.

    c. The number of no-shows for every 100 reservations made with a commercial airline.

    d. The number of vehicles owned by a randomly selected household.

    e. The average amount spent on electricity each July by a randomly selected household in a certain state.

0.2.2 Probability Distributions for Discrete Random Variables

17. A fair coin is tossed twice. Let X be the number of heads that are observed.

    a. Construct the probability distribution of X.

    b. Find the probability that at least one head is observed.

18. A pair of fair dice are rolled. Let X denote the sum of the number of dots on the top faces.

    a. Construct the probability distribution of X.

    b. Find $P(X \geq 9)$.

    c. Find the probability that X takes an even value.

19. A service organization in a large town organizes a raffle each month. One thousand raffle tickets are sold for \$1 each. Each has an equal chance of winning. The first prize is \$300, the second prize is \$200, and the third prize is \$100. Let X denote the net gain from the purchase of one ticket.

    a. Construct the probability distribution of X.

    b. Find the probability of winning any money in the purchase of one ticket.

    c. Find the expected value of X, and interpret its meaning.

20. A life insurance company will sell a \$200,000 one-year term life insurance policy to an individual in a particular risk group for a premium of \$195. Find the expected value to the company of a single policy if a person in this risk group has a 99.97% chance of surviving one year.

X	-1	0	1	4
$\Pr(X = x)$	0.2	0.5	a	0.1

Compute each of the following quantities.

a. a

b. $\Pr(0)$

c. $\Pr(X > 0)$

d. $\Pr(X \geq 0)$

e. $\Pr(X \leq 0)$

f. The mean μ of X

g. The Variance σ^2 of X

h. The standard deviation σ of X

0.2.3 The Binomial Distribution

21. Seventeen percent of victims of financial fraud know the perpetrator of the fraud personally.

    a. Use the formula to construct the probability distribution for the number X of people in a random sample of five victims of financial fraud who knew the perpetrator personally.

    b. A investigator examines five cases of financial fraud every day. Find the most frequent number of cases each day in which the victim knew the perpetrator.

    c. A investigator examines five cases of financial fraud every day. Find the average number of cases per day in which the victim knew the perpetrator.

    d. Find the mean and standard deviation of the random variable X.

22 A student takes a ten-question true/false exam.

   a. Find the probability that the student gets exactly six of the questions right simply by guessing the answer on every question.

b. Find the probability that the student will obtain a passing grade of 60% or greater simply by guessing.

23. An appliance repairman services five washing machines on site each day. One-third of the service calls require installation of a particular part.

    a. The repairman has only one such part on his truck today. Find the probability that the one part will be enough today, that is, that at most one washing machine he services will require installation of this particular part.

    b. Find the minimum number of such parts he should take with him each day in order that the probability that he have enough for the day's service calls is at least 95%.

0.3 Continuous Random Variables

0.3.1 Uniform Distribution

24. A man arrives at a bus stop at a random time (that is, with no regard for the scheduled service) to catch the next bus. Buses run every 30 minutes without fail, hence the next bus will come any time during the next 30 minutes with evenly distributed probability (a uniform distribution). Find the probability that a bus will come within the next 10 minutes.

25. Buses running a bus line near Desdemona's house run every 15 minutes. Without paying attention to the schedule she walks to the nearest stop to take the bus to town. Find the probability that she waits more than 10 minutes.

0.3.2 Normal Distribution

26. Heights of 25-year-old men in a certain region have mean 69.75 inches and standard deviation 2.59 inches. These heights are approximately normally distributed. Thus the height X of a randomly selected 25-year-old man is a normal random variable with mean $\mu = 69.75$ and standard deviation $\sigma = 2.59$. Sketch a qualitatively accurate graph of the density function for X. Find the probability that a randomly selected 25-year-old man is more than 69.75 inches tall.

27. The amount X of orange juice in a randomly selected half-gallon container varies according to a normal distribution with mean 64 ounces and standard deviation 0.25 ounce.

    a. Sketch the graph of the density function for X.

    b. What proportion of all containers contain less than a half gallon (64 ounces)? Explain.

    c. What is the median amount of orange juice in such containers? Explain.

0.3.3 The Standard Normal Distribution

28. Find the probabilities indicated, where as always Z denotes a standard normal random variable.

    a. $\Pr(Z < 1.48)$.
    b. $\Pr(Z < -0.25)$.
    c. $\Pr(Z > 1.60)$.
    d. $\Pr(Z > -1.02)$.
    e. $\Pr(0.5 < Z < 1.57)$

0.3.4 Probability Computations for General Normal Random Variables

29. Let X be a normal random variable with mean $\mu = 10$ and standard deviation $\sigma = 2.5$. Compute the following probabilities.

    a. $\Pr(X < 14)$
    b. $\Pr(8 < X < 14)$.

30. The lifetimes of the tread of a certain automobile tire are normally distributed with mean $37,500$miles and standard deviation $4,500$ miles. Find the probability that the tread life of a randomly selected tire will be between $30,000$ and $40,000$ miles.

31. Scores on a standardized college entrance examination (CEE) are normally distributed with mean 510 and standard deviation 60. A selective university considers for admission only applicants with CEE scores over 650. Find percentage of all individuals who took the CEE who meet the university's CEE requirement for consideration for admission.

32. The amount X of beverage in a can labeled 12 ounces is normally distributed with mean 12.1 ounces and standard deviation 0.05 ounce. A can is selected at random.

    a. Find the probability that the can contains at least 12 ounces.
    b. Find the probability that the can contains between 11.9 and 12.1 ounces.

0.3.5 Areas of Tails of Distributions

33. Find the value z^* of Z as determined by z-table the value z^* that cuts off a left tail of area 0.0125 in the standard normal distribution. In symbols, find the number z^* such that $P(Z < z^*) = 0.0125$.

34. Find $z_{0.1}$ and $-z_{.01}$, the values of Z that cut off right and left tails of area 0.01 in the standard normal distribution.

35. Find x^* such that $P(X < x^*) = 0.9332$, where X is a normal random variable with mean $\mu = 10$ and standard deviation $\sigma = 2.5$.