Examen :

(1)

Ex 1 :

1- l'onde plane : surface d'onde = plan

$\Rightarrow$ Amplitude constante

monochromatique = sinusoïdal

2- l'onde plane monochromatique n'∄ réellement

l'onde plane monochromatique = modèle parfait

l'onde réelle $= \sum$ onde plane

$=$ superposition d'onde planes

eq de Maxwell et l'eq de propagation sont linéaire

Exp $\Delta \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \vec{0}$

$\Delta(\vec{E}_1 + \vec{E}_2) = \Delta \vec{E}_1 + \Delta \vec{E}_2$

$\frac{\partial^2}{\partial t^2}(\vec{E}_1 + \vec{E}_2) = \frac{\partial \vec{E}_1}{\partial t^2} + \frac{\partial^2 \vec{E}_2}{\partial t^2}$

3 -

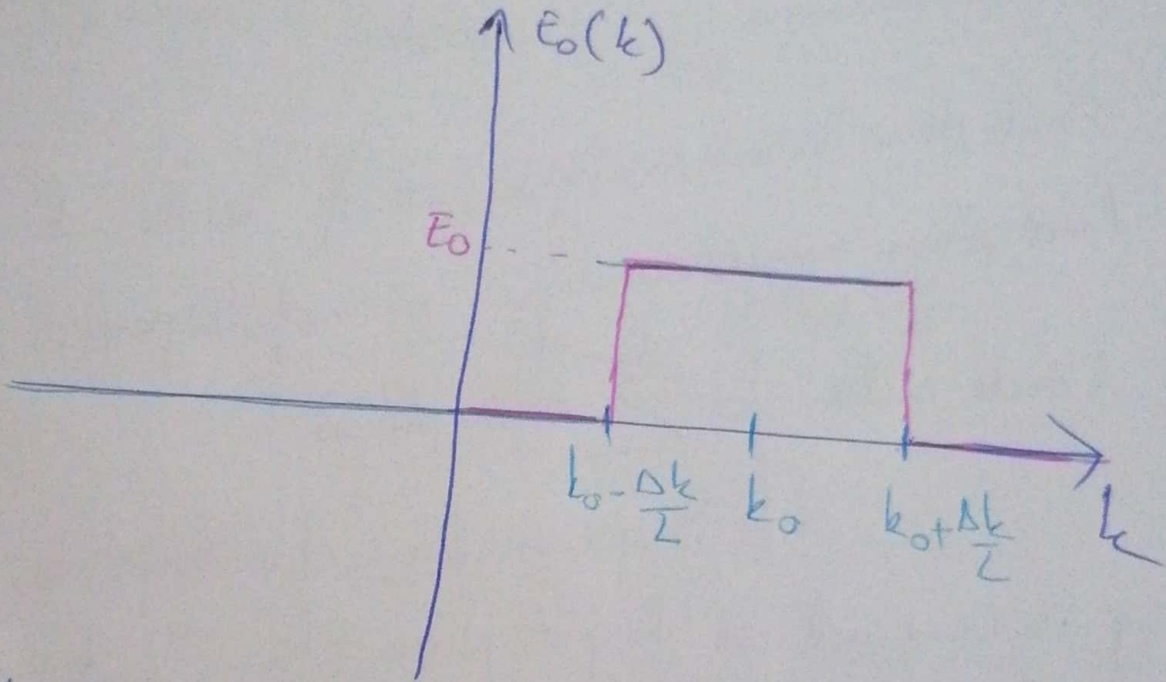
$$E_0(k) = \begin{cases} E_0 & \text{si } |k-k_0| < \frac{\Delta k}{2} \\ 0 & \text{si } |k-k_0| > \frac{\Delta k}{2} \end{cases}$$

$$|k-k_0| < \frac{\Delta k}{2} \Rightarrow -\frac{\Delta k}{2} < k-k_0 < \frac{\Delta k}{2}$$

Si $k_0 - \frac{\Delta k}{2} < k < k_0 + \frac{\Delta k}{2}$

$$E_0(k) = E_0$$

Sinon $E_0(k) = 0$



4 -

$$\vec{E} = E_0(k) \exp i(\omega_0 t - kz)\, \vec{u}_x$$

$$\vec{E}_{tot} = \int_0^{\infty} E_0(k) \exp i(\omega_0 t - kz)\, dk\, \vec{u}_x$$

$$= \int_{k_0 - \frac{\Delta k}{2}}^{k_0 + \frac{\Delta k}{2}} E_0 \exp i(\omega_0 t - kz)\, dk\, \vec{u}_x$$

(2)

$$\vec{E}_{tot} = \int_{k_0-\frac{\Delta k}{2}}^{k_0+\frac{\Delta k}{2}} E_0 \exp i\omega_0 t \cdot \exp -ikz \, dk \, \hat{u}_x$$

$$= E_0 \exp i\omega_0 t \int_{k_0-\frac{\Delta k}{2}}^{k_0+\frac{\Delta k}{2}} \exp -ikz \, dk \, \hat{u}_x$$

$$= E_0 \exp i\omega_0 t \left[\frac{1}{-iz} \exp -ikz \right]_{k_0-\frac{\Delta k}{2}}^{k_0+\frac{\Delta k}{2}}$$

$$= \frac{E_0}{-iz} \exp i\omega_0 t \left[\exp -iz\left(k_0+\frac{\Delta k}{2}\right) - \exp -i\left(k_0-\frac{\Delta k}{2}\right)z \right] \hat{u}_x$$

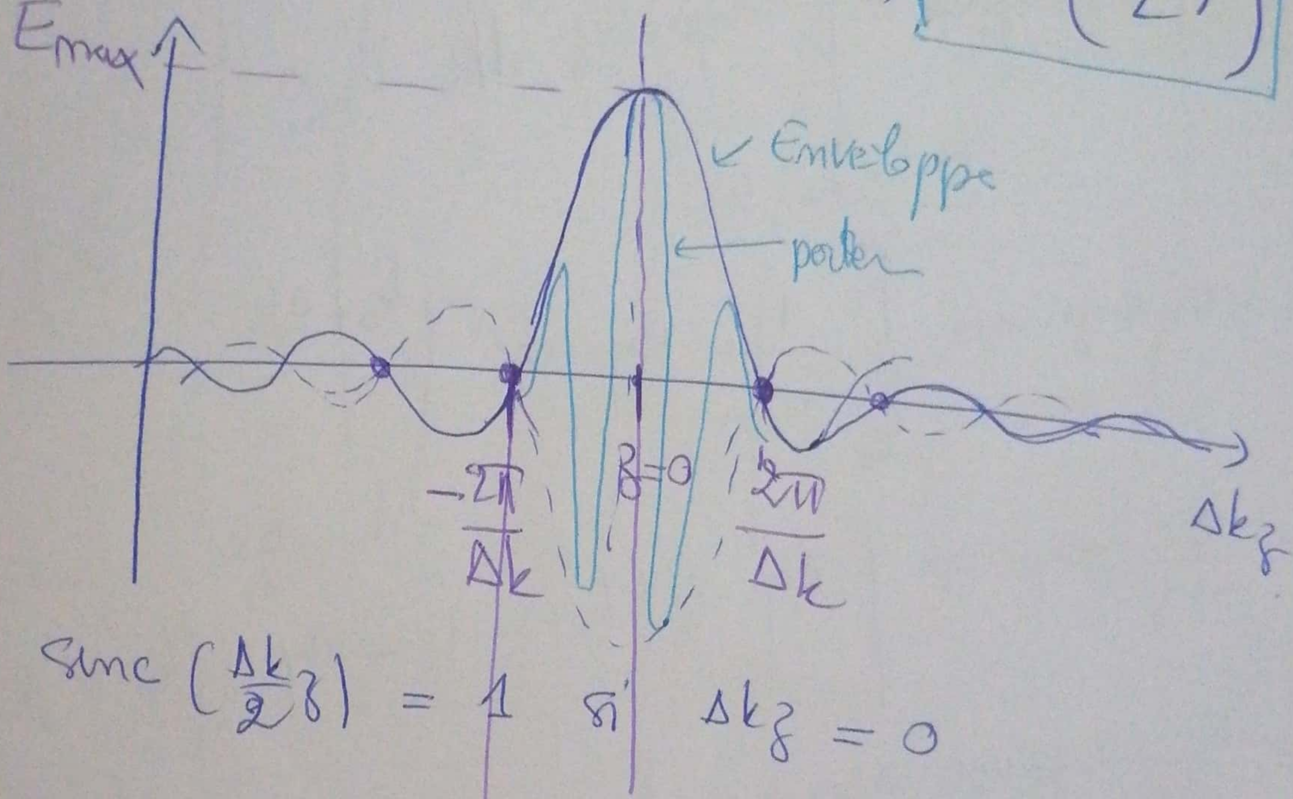
$$= \frac{E_0}{-iz} \exp i\omega_0 t \, \exp -ik_0 z \underbrace{\left[\exp -iz\frac{\Delta k}{2} - \exp i\frac{\Delta k}{2}z \right]}_{-2i\sin\left(\frac{\Delta k}{2}z\right)} \hat{u}_x$$

$$\sin x = \frac{\exp ix - \exp -ix}{2i}$$

$$= \frac{2E_0 \Delta k}{\Delta k z} \exp i(\omega_0 t - k_0 z) \sin\left(\frac{\Delta k}{2}z\right)$$

$$\frac{\sin\left(\frac{\Delta k}{2} z\right)}{\frac{\Delta k}{2} z} = \operatorname{sinc}\left(\frac{\Delta k}{2} z\right)$$

$$\hat{E}_{tot} = E_0 \, \Delta k \, \exp i(\omega_0 t - k_0 z) \boxed{\operatorname{sinc}\left(\frac{\Delta k}{2} z\right)} \vec{u}_x$$



$\operatorname{sinc}\left(\frac{\Delta k}{2} z\right) = 1$ si $\Delta k z = 0$

$$\operatorname{sinc}\left(\frac{\Delta k z}{2}\right) = 0 \Rightarrow \sin\left(\frac{\Delta k}{2} z\right) = 0$$

$$\frac{\Delta k}{2} z = m\pi$$

$$z = \frac{2m\pi}{\Delta k} \quad m \in \mathbb{Z}^*$$

6- $\vec{B}(n,t) = ?$ 3

$\vec{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\vec{\nabla} \wedge \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$$\begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \wedge \begin{pmatrix} E \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{\partial E}{\partial z} \\ -\frac{\partial E}{\partial y} = 0 \\ 0 \end{pmatrix}$$

$$= \Big($$

$\frac{\partial E}{\partial z} = ?$

$$\mathrm{sinc}\left(\frac{\Delta k z}{2}\right) = \frac{\sin\left(\frac{\Delta k z}{2}\right)}{\frac{\Delta k z}{2}}$$

$$\left(\mathrm{sinc}\left(\frac{\Delta k z}{2}\right)\right)' = \frac{\left(\frac{\Delta k}{2}\right)^2 \cos\left(\frac{\Delta k}{2} z\right) z - \frac{\Delta k}{2} \sin\left(\frac{\Delta k z}{2}\right)}{\left(\frac{\Delta k}{2} z\right)^2}$$

7- $\vec{k} = k_0 \vec{e}_z$

$\vec{E} = E\,\vec{u}_x$

$\vec{E} \perp \vec{k}$ l'onde est transversale électrique

$\vec{B} = B\,\vec{u}_y \perp \vec{k}$: transversale magnétique

TEM

8- $V\varphi = ?$ $\quad \varphi(z,t) = \omega_0 t - k_0 z$

$V\varphi = \frac{dz}{dt}$ lorsque $d\varphi = 0$

$\omega_0 dt - k_0 dz = 0$

$\omega_0 dt = k_0 dz$

$V\varphi = \frac{dz}{dt} = \frac{\omega_0}{k_0}$

Bilan énergétique

Ex 1:

1) $\vec{E}_1 = E_0 \exp i(\omega t - (k\cos\alpha x + k\sin\alpha y))\,\vec{u}_z$

2) onde plane : l'eq des surfaces d'onde

$\vec{k}_1 \cdot \vec{r} = cte$

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$\vec{k}_1 \cdot \vec{r} = k \cos\alpha x + k \sin\alpha y = cte$

eq. d'un plan

$\Rightarrow$ onde plane

3) l'onde est progressive de la forme $f\left(t - \frac{d}{c}\right)$

eq de dispersion $k = \frac{\omega}{c}$

4) harmonique = sinusoïdal

5) polarisation rectiligne suivant $\vec{u}_z$

6) $\vec{k} \cdot \vec{r} = cte \Rightarrow \boxed{k \cos\alpha x + k \sin\alpha y = cte}$

eq cart d'un plan d'onde

7) $\vec{B} = \frac{\vec{k} \wedge \vec{E}}{\omega} = \begin{pmatrix} k_x \\ k_y \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 0 \\ E \end{pmatrix}$

$k_x = k \cos\alpha$

$k_y = k \sin\alpha$

$= \begin{pmatrix} \frac{k_y}{\omega} E \\ -\frac{k_x}{\omega} E \\ 0 \end{pmatrix}$

$\vec{k} \perp \vec{E}$: TE
$\vec{k} \perp \vec{B}$: TM

TEM

4) $\vec{\Pi}$: vecteur de poynting

$$\vec{\Pi} = \frac{\vec{E} \wedge \vec{B}}{\mu_0}$$

$\vec{E}$ et $\vec{B}$ en mot réelle $\vec{E} = \text{Réel}(\underline{\vec{E}})$

$$\begin{cases} \vec{E} = E_0 \cos(\omega t - \vec{k}_1 \vec{r}) \vec{u}_z \\ \vec{B} = \frac{k \sin\alpha E_0}{\omega} \cos(\omega t - \vec{k}_1 \cdot \vec{r}) \vec{u}_x + \frac{k \cos\alpha E_0}{\omega} \end{cases}$$

$$\cos(\omega t - \vec{k}_1 \cdot \vec{r}) \vec{u}_y$$

$$\vec{\Pi} = \frac{1}{\mu_0} \begin{pmatrix} 0 \\ 0 \\ E \end{pmatrix} \wedge \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{E B_y}{\mu_0} \\ \frac{E B_x}{\mu_0} \\ 0 \end{pmatrix}$$

(5)

$$\langle \cos^2(\omega t - \vec{k}\cdot\vec{r})\rangle_t = \frac{1}{2} = \langle \sin^2(\omega t - \vec{k}_1\cdot\vec{r})\rangle_t$$

$$\langle \cos(\omega t - \vec{k}\,\vec{r})\rangle_t = 0 = \langle \sin(\omega t - \vec{k}_1\cdot\vec{r})\rangle$$

$$\langle \sin(\text{———})\cdot\cos(\text{———})\rangle = 0$$

$$\langle\vec{\Pi}\rangle = \frac{E_0^2}{2\mu_0}\frac{k}{\omega}\left(\cos\alpha\,\hat{u}_x + \sin\alpha\,\hat{u}_y\right)$$

$$k = \frac{\omega}{C} \quad (\text{onde plane})$$

$$\langle\vec{\Pi}\rangle = \frac{E_0^2}{2\mu_0}\frac{1\times C}{C\times C}\left(\cos\alpha\,\hat{u}_x + \sin\alpha\,\hat{u}_y\right)$$

$$= \frac{E_0^2}{2\mu_0 C^2}\,C\left(\cos\alpha\,\hat{u}_x + \sin\alpha\,\hat{u}_y\right)$$

$$\frac{1}{\mu_0 C^2} = \varepsilon_0$$

$$\langle\vec{\Pi}\rangle = \frac{\varepsilon_0 E_0^2}{2}\,C\left(\cos\alpha\,\hat{u}_x + \sin\alpha\,\hat{u}_y\right)$$

$$\vec{\pi}_s \begin{pmatrix} \frac{k\cos\alpha}{\mu_0\omega} E_0^2 \cos^2(\omega t - \vec{k}_1\cdot\vec{r}) \\ \frac{k\sin\alpha}{\mu_0\omega} E_0^2 \cos^2(\omega t - \vec{k}_1\cdot\vec{r}) \\ 0 \end{pmatrix}$$

$\vec{\pi}$: le vecteur densité de courant d'énergie

~~la~~ ~~loi~~ renseigne sur la direction de propagation de l'énergie de l'onde

$$\|\vec{\pi}\| = \frac{dP}{dS} = \text{puissance surfacique} = \frac{\text{Watt}}{m^2}$$

π en $\text{Watt}\cdot m^{-2}$

9-2)

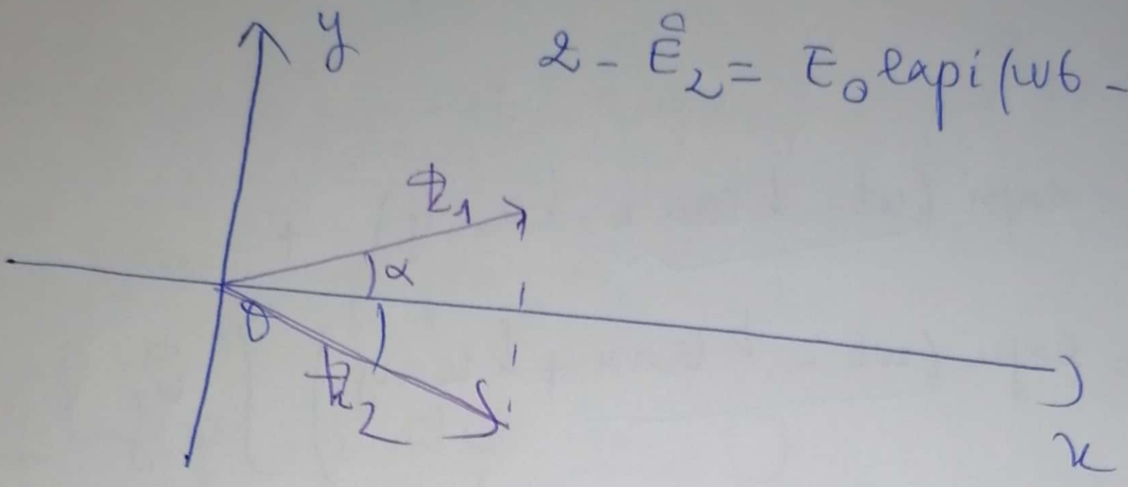
$$\boxed{\vec{\pi}} = \langle \pi_x \rangle \hat{u}_x + \langle \pi_y \rangle \hat{u}_y$$

$$= \frac{k\cos\alpha}{2\mu_0\omega} E_0^2 \hat{u}_x + \frac{k\sin\alpha\, E_0^2}{2\mu_0\omega} \hat{u}_y$$

$$= \frac{E_0^2}{2\mu_0\omega} \left[k\cos\alpha\, \hat{u}_x + k\sin\alpha\, \hat{u}_y \right) = \boxed{\frac{E_0^2}{2\mu_0\omega} \vec{k}_1}$$

⑥

Partie II



2- $\hat{E}_2 = E_0 \exp i(\omega t - \vec{k}_2 \cdot \vec{r})\, \vec{u}_z$

$\vec{k}_1 = k_1 (\cos\alpha\, \vec{u}_x + \sin\alpha\, \vec{u}_y)$; $k_1 = \dfrac{\omega}{c}$

$\vec{k}_2 = k_2 (\cos\alpha\, \vec{u}_x - \sin\alpha\, \vec{u}_y)$; $k_2 = \dfrac{\omega}{c}$

$$\hat{B}_2 = \frac{\vec{k}_2 \wedge \hat{E}_2}{\omega} = \begin{pmatrix} \frac{\cancel{\omega}}{\cancel{\omega} c}\cos\alpha \\ -\frac{\cancel{\omega}}{\cancel{\omega} c}\sin\alpha \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 0 \\ E_2 \end{pmatrix}$$

$$= \begin{pmatrix} -\frac{\cancel{\omega}}{\cancel{\omega} c}\sin\alpha\, E_2 \\ -\frac{\cancel{\omega}}{\cancel{\omega} c}\cos\alpha\, E_2 \\ 0 \end{pmatrix}$$

Partie III :

$$\hat{E} = \hat{E}_1 + \hat{E}_2$$

$$= \Big[E_0 \exp i\,(\underbrace{wt - k\cos\alpha\, x}_{} - k\cos\alpha\, y) +$$

$$E_0 \exp i\,(\underbrace{wt - k\cos\alpha\, x}_{} + k\cos\alpha\, y)\Big]\, \vec{u}_z$$

$$= E_0 \exp i\,(wt - k\cos\alpha\, x)\Big[\exp - ik\cos\alpha\, y + \exp ik\cos\alpha\, y\Big]\, \hat{u}_z$$

$$\exp ix + \exp -ix = 2\cos x$$

$$\hat{E} = 2E_0 \cos(k\cos\alpha\, y)\, \exp i(wt - k\cos\alpha\, x)\, \hat{u}_z$$

$$\hat{B} = \hat{B}_1 + \hat{B}_2$$

$k = \frac{w}{c}$

$$= \begin{pmatrix} \frac{k\sin\alpha}{w} E_1 \\ -\frac{k\cos\alpha}{w} E_1 \\ 0 \end{pmatrix} + \begin{pmatrix} -\frac{\sin\alpha}{c} E_2 \\ -\frac{\cos\alpha\, E_2}{c} \\ 0 \end{pmatrix}$$

5) L'onde n'est pas plane

$\varphi(x,t) = \omega t - k\cos\alpha\, x$

plan equiphase $x = cte$

6) $V_\varphi \Rightarrow d\varphi = 0$

$\omega dt - k\cos\alpha\, dx = 0$

$$V_\varphi = \frac{dx}{dt} = \frac{\omega}{k\cos\alpha} = \frac{c}{\cos\alpha}$$

7) L'onde n'est plane TE n'est pas transverse magnetique, polarisée rectilignement suivant $\vec{u}_z$

l'onde est progressive suivant (ox) (car x est couplé avec le temps t) et elle est stationnaire suivant (oy) car (y n'est pas couplé avec le temps)

$$\vec{B} = \begin{pmatrix} \frac{\sin\alpha}{c}(E_1 - E_2) \\ -\frac{\cos\alpha}{c}(E_1 + E_2) \\ 0 \end{pmatrix} = \cancel{\begin{pmatrix} \frac{-2i\sin\alpha\sin(k\cos\alpha y)}{c} \exp i \end{pmatrix}}$$

$$= \begin{pmatrix} \frac{-2i\sin\alpha}{c} E_0 \sin(k\cos\alpha y) \exp i(\omega t - k\cos\alpha x) \\ -\frac{\cos\alpha}{c} 2E_0 \cos(k\cos\alpha y) \exp i(\text{———}) \\ 0 \end{pmatrix}$$

3) direct de propagat $\Rightarrow \vec{e}_x$

$\vec{E} = E\,\vec{U}_z$

$\vec{E}$ $\perp$ direct de prop

$\Rightarrow$ TE

$$\vec{B} = \begin{pmatrix} B_x \\ B_y \\ 0 \end{pmatrix} = \begin{pmatrix} B_{\parallel} \\ B_{\perp} \\ 0 \end{pmatrix} = \begin{pmatrix} B\ \text{longitud} \\ B\ \text{trans} \\ 0 \end{pmatrix}$$

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Rq : $\hat{E}_1 = E_0 \exp i(\omega t - kx)\, \hat{u}_z$

$\hat{E}_2 = E_0 \exp i(\omega t + kx)\, \hat{u}_z$

$\hat{E} = \hat{E}_1 + \hat{E}_2 = E_0 \exp i\omega t \left[\exp -ikx + \exp ikx\right] \hat{u}_z$

$= 2E_0 \cos(kz) \exp i\omega t\, \hat{u}_z$

en mot réelle

$\hat{E} = 2E_0 \cos(kz) \cos(\omega t)\, \hat{u}_z$

Onde stationnaire suivant z

8- $u_e = \frac{1}{2} \varepsilon_0 \langle \|\hat{E}\|^2 \rangle$

$= \frac{1}{2} \varepsilon_0 \left(2E_0 \cos(k \cos\alpha\, y)\right)^2 \times \frac{1}{2}$

$= \frac{\cancel{4} E_0^2}{\cancel{4}} \varepsilon_0 \cos^2(k \cos\alpha\, y)$

9- $u_m = \frac{\langle \|\vec{B}\|^2 \rangle}{2\mu_0} = \left\langle \frac{B_{\parallel}^2 + B_{\perp}^2}{2\mu_0} \right\rangle$

en mot réelle

$$B \begin{pmatrix} \frac{2E_0 \sin\alpha}{c} \sin(k\cos\alpha y)\sin(\_\_\_) \\ -\frac{2E_0\cos\alpha}{c} \cos(k\cos\alpha y)\cos(\_\_\_) \\ 0 \end{pmatrix}$$

$\exp ix = \cos x + i\sin x$ $\quad$ Réel $(\exp ix) = \cos x$

$i\exp ix = i\cos x - \sin x$ $\quad$ Réel $(i\exp ix) = -\sin x$

$$U_m = \frac{1}{2\mu_0}\frac{1}{2}\left[\frac{4E_0^2}{c^2}\sin^2\alpha\sin^2(k\cos\alpha y) + \frac{4E_0^2}{c^2}\cos^2\alpha\cos^2(k\cos\alpha y)\right]$$

12)

$$\vec{\Pi} = \frac{\vec{E}\wedge\vec{B}}{\mu_0} = \frac{1}{\mu_0}\begin{pmatrix}0\\0\\E\end{pmatrix}\wedge\begin{pmatrix} \\ \\ .\end{pmatrix}$$

$S_1 M = D\left(1 - \frac{ax}{2D^2}\right)$

$(1+X)^\alpha \simeq 1 + \alpha X$

$S_2 M$ de m̂

$S_2 M = D\left(1 + \frac{ax}{2D^2}\right)$

$\delta(M) = S_2 M - S_1 M \simeq \boxed{\frac{ax}{D}}$

I_{max} f.b $\delta = m\lambda$

$\frac{ax}{D} = m\lambda$

$x_m = \frac{m\lambda D}{a}$

$i = x_{m+1} - x_m = \frac{\lambda D}{a}$ interfrange

I_{min} f.sombre $\delta = \left(m+\frac{1}{2}\right)\lambda$

$x_m = \left(m+\frac{1}{2}\right)\frac{\lambda D}{a}$

Interf il faut $\omega_1 = \omega_2 = \omega$

$E_1 = E_1 \cos(\omega t - \varphi_1(n))$

$$\boxed{I(n) = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\varphi}$$

$\Delta\varphi = \varphi_2(n) - \varphi_1(n)$

$= \frac{2\pi}{\lambda}\delta(n)$

I_{max} si $\cos\Delta\varphi = 1$

f. brillante $\Delta\varphi = 2m\pi$

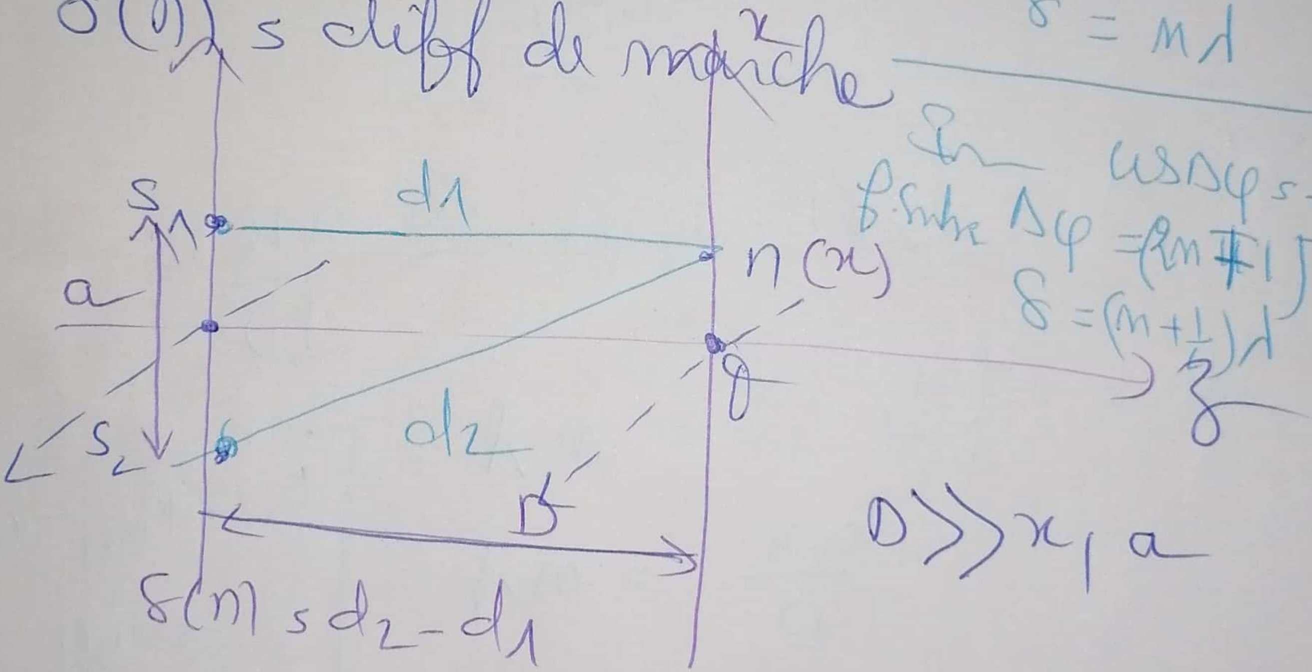
$\delta = m\lambda$

I_{min} $\cos\Delta\varphi = -1$

f. sombre $\Delta\varphi = (2m+1)\pi$

$\delta = (m + \frac{1}{2})\lambda$

$\delta(n) =$ diff de marche



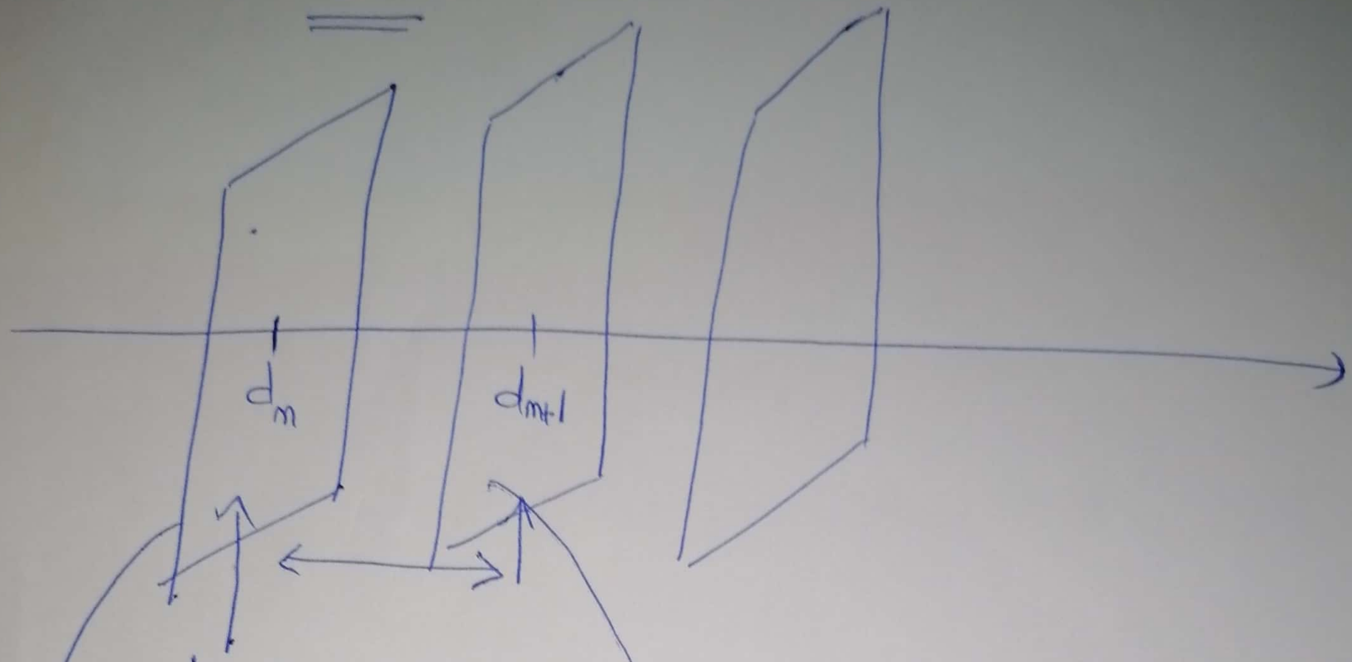
$D \gg x, a$

$\delta(n) = d_2 - d_1$

$S_1 n = d_1 = S_1 n = \sqrt{(x_n - x_{S_1})^2 + (y_n - y_{S_1})^2 + (z_n - z_{S_1})^2}$

$= \sqrt{(x - \frac{a}{2})^2 + (-D)^2} = D\sqrt{1 + \cancel{\frac{x^2}{D^2}} + \cancel{\frac{a^2}{D^2}} - \frac{2ax}{2D^2}}$

à $t = t_1$



$\vec{k}.\vec{OM} = cte$

plan d'onde $\frac{k\sqrt{2}}{2} z + \frac{k\sqrt{2}}{2} y = cte$

$$\frac{k\sqrt{2}}{2}(z+y) = cte$$

$$\varphi_n = \frac{2\pi}{\lambda} d_n$$

$$\varphi_{n+1} = \frac{2\pi}{\lambda} d_{n+1}$$

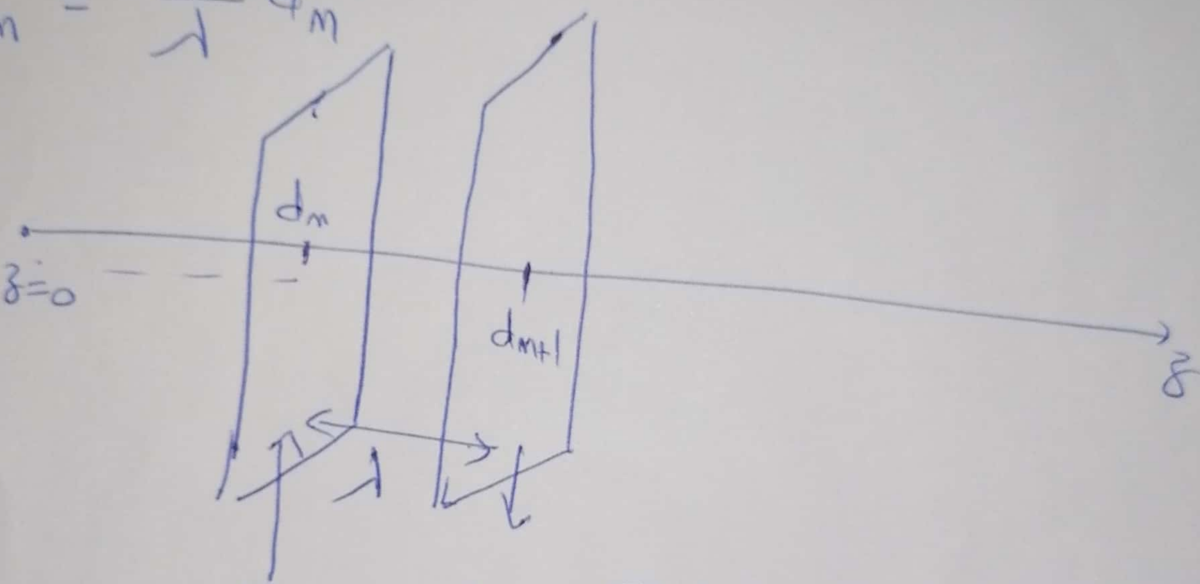
Surface equiphase

$\vec{k}.\vec{OM} = cte$: eq d'un plan d'onde

$k \frac{\sqrt{2}}{2} \underbrace{(y+z)}_{d} = cte$

$\frac{2\pi}{\lambda} d = cte$

$\varphi_m = \frac{2\pi}{\lambda} d_m$



$\varphi_m = \frac{2\pi}{\lambda} d_m \quad \varphi_{m+1} = \frac{2\pi}{\lambda} d_{m+1}$

$\varphi_{m+1} - \varphi_m = \frac{2\pi}{\lambda}(d_{m+1} - d_m) = 2\pi$

$d_{m+1} - d_m = \lambda$

$$\vec{B} = \begin{pmatrix} -\frac{\partial E}{\partial z} = +\alpha E_0 \sin(\alpha z) \sin(\omega t - kx) \\ 0 \\ \frac{\partial E}{\partial x} = -k E_0 \cos(\alpha z) \cos(\omega t - kx) \end{pmatrix}$$

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3- eq de dispersion

On injecte le $\vec{E}$ dans l'eq de propagation

l'onde se propage dans le vide :

l'eq de D'Alembert $\boxed{\Delta \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = \vec{0}}$

$\Delta \vec{E} = ?$

Onde est plane

$\Delta \vec{E} = -k^2 \vec{E}$

Onde n'est plane

$\Delta \vec{E} \neq -k^2 \vec{E}$

$$\Delta \vec{E} = \left(\frac{\partial^2 E}{\partial x^2} + \frac{\partial E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} \right) \vec{u}_y$$

$$\Delta \vec{E} = \begin{pmatrix} \frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} \\ \frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2} \\ \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2} \end{pmatrix}$$

$$\Delta\vec{E} = \left(\frac{\partial^2 E}{\partial x^2} + \underset{\substack{\shortparallel \\ 0}}{\frac{\partial^2 E}{\partial y^2}} + \frac{\partial^2 E}{\partial z^2}\right)\vec{u}_y$$

#

$$\frac{\partial E}{\partial x} = -k E_0 \cos(\alpha z)\cos(\omega t - kx)$$

$$\frac{\partial^2 E}{\partial x^2} = -k^2\left(E_0 \cos(\alpha z)\sin(\omega t - kx)\right)$$

$$\frac{\partial^2 E}{\partial z^2} = -\alpha^2\left(E_0 \cos(\alpha z)\sin(\omega t - kx)\right)$$

$$\Delta\vec{E} = -(\alpha^2 + k^2)\, E_0 \cos(\alpha z)\sin(\omega t - kx)\,\vec{u}_y$$

$$\frac{\partial^2 \vec{E}}{\partial t^2} = ? \qquad \frac{\partial^2 \hat{\vec{E}}}{\partial t^2} = -\omega^2 \hat{\vec{E}}$$

$$\Rightarrow \left[-(\alpha^2 + k^2) - \frac{1}{c^2}(-\omega^2)\right]\hat{\vec{E}} = \vec{0}$$

$$\hat{\vec{E}} \neq 0 \qquad -(\alpha^2 + k^2) + \frac{\omega^2}{c^2} = 0$$

$$\Rightarrow \alpha^2 + k^2 = \frac{\omega^2}{c^2}$$

$$\boxed{k^2 = \frac{\omega^2}{c^2} - \alpha^2}$$

$k(\omega)$ n'est pas linéaire

$\Rightarrow$ il y a dispersion

Onde plane

$k = \frac{\omega}{c}$

linéaire

milieu (vide)

non dispersif

Onde n'est pas plane

$k^2 = \frac{\omega^2}{c^2} - \alpha^2$; $\vec{E} = E_0 \cos(\alpha y)\sin(\omega t - kx)\,\vec{e}_y$

n'est pas linéaire

milieu (vide)

dispersif

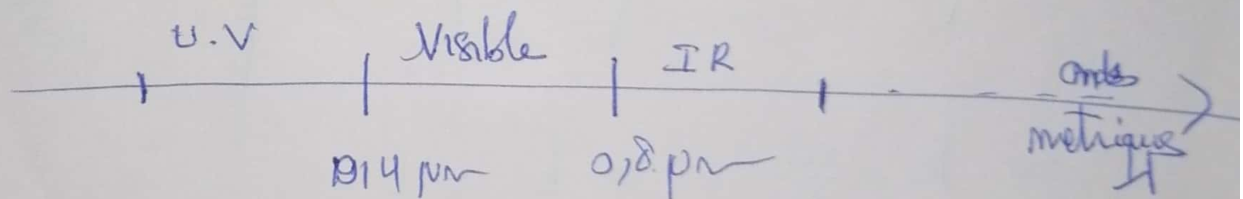
Ex3:

1- $f = 10^{10}$ Hz $\lambda = c.T = \frac{c}{f} = \frac{3\,10^8}{10^{10}}$

$= 3\,10^{-2}$ m

$= 3$ cm

les ondes métriques



2.1) $\vec{k} = k\vec{u} = k\frac{\sqrt{2}}{2}(\vec{u}_y + \vec{u}_z)$

2.1) $\vec{E} = \vec{E}_0 \exp i(\omega t - \vec{k}.\vec{OM})$

$E_0\vec{u}_x$ (polarisée rectiligne)

$\vec{k}$ → direction de propagation (Ox) de l'onde

$\vec{E}_0$ → " de polarisation de l'onde

$$\vec{E} = E_0 \exp i\left(\omega t - k\frac{\sqrt{2}}{2}y - k\frac{\sqrt{2}}{2}z\right) \vec{u}_x$$

$$\vec{k}\cdot\vec{OM} = k\begin{pmatrix} 0 \\ \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{pmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = k\frac{\sqrt{2}}{2}y + \frac{k\sqrt{2}}{2}z$$

2.2) Onde plane $\Rightarrow \vec{B} = \dfrac{\vec{k}\wedge\vec{E}}{\omega}$

$$\vec{B} = \frac{k}{\omega}\begin{pmatrix} 0 \\ \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{pmatrix} \wedge \begin{pmatrix} E \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{k}{\omega}\frac{\sqrt{2}}{2}E \\ -E\frac{k\sqrt{2}}{\omega 2} \end{pmatrix}$$

or $k = \dfrac{\omega}{c} \Rightarrow \dfrac{k}{\omega} = \dfrac{1}{c}$

$$\vec{B} = \begin{pmatrix} 0 \\ \frac{\sqrt{2}}{2c}E_0 \exp i\left(\underline{\qquad\qquad}\right) \\ -\frac{E_0\sqrt{2}}{2c}\exp i\left(\underline{\qquad\qquad}\right) \end{pmatrix}$$

(13)

2.3) plan equiphases / $\varphi(M,t) = cte$ à t fixé

$\varphi(M,t) = \omega t - k\frac{\sqrt{2}}{2}y - k\frac{\sqrt{2}}{2}z$

$\varphi = cte \Rightarrow d\varphi = 0$

$d\varphi = \omega dt - \left(k\frac{\sqrt{2}}{2}dy + k\frac{\sqrt{2}}{2}dz\right) = 0$

$= \omega dt - d\left(\vec{k}\cdot\vec{OM}\right) = 0$

à t = cte t fixe $dt = 0$

$\vec{k}\cdot\vec{OM} = cte$

autrement surf equiph $\varphi = cte$

$d\varphi = 0$ $\quad \vec{grad}\,\varphi \cdot \vec{dl}$

$df = \vec{grad}\,f \cdot \vec{dl}$

à $t = cte$ une surface equiphase

$\varphi(M,t) = \varphi(M) = k\frac{\sqrt{2}}{2}y + k\frac{\sqrt{2}}{2}z$

$= k\frac{\sqrt{2}}{2}(y+z) = cte$

$\boxed{k\frac{\sqrt{2}}{2}y + k\frac{\sqrt{2}}{2}z = cte}$ eq d'un plan

$ax + by + cz = cte$: eq d'un plan

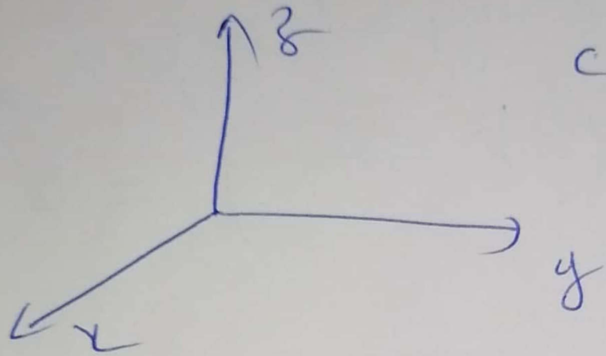
les surfaces equiphases = des plans = les plans d'ondes

cas particulier

$\vec{E} = E_0 \cos(\omega t - kz)\,\vec{u}_x$

surf. equi-ph $kz = cte \Rightarrow z = cte$

c'est un plan



2.4)

$\varphi(M,t) = \omega t - \frac{k\sqrt{2}}{2} y - k\frac{\sqrt{2}}{2} z$

$\varphi_m = \omega t_m - k\left(\frac{\sqrt{2}}{2} y_m + \frac{\sqrt{2}}{2} z_m\right)$

$= \frac{2\pi}{T} t_m - \frac{2\pi}{\lambda} d_m$

$\varphi_{m+1} = \frac{2\pi}{T} t_{m+1} - \frac{2\pi}{\lambda} d_{m+1}$

$\varphi_{m+1} - \varphi_m = \cancel{\frac{2\pi}{T}(t_{m+1} - t_m)} - \frac{2\pi}{\lambda}(d_{m+1} - d_m)$

$= 2\pi$

Ex2 :

Amp variable

$\vec{E} = E_0 \cos(\alpha z) \sin(\omega t - kx)\vec{u}_y$

1- $= f(z)\ g\left(t - \frac{x}{c}\right)\vec{u}_y$

- l'onde n'est pas plane car son amplitude $f(z)$ suivant (Ox)
- progressive ✓ car de la forme $g\left(t - \frac{x}{c}\right)$
- harmonique : oui (sinusoïdal en fct du temps)

2- l'onde est plane

$\vec{B} = \frac{\vec{k} \wedge \vec{E}}{\omega}$

onde n'est pas plane

Eq de M.F :

$\vec{rot}\,\vec{E} = -\frac{\partial \vec{B}}{\partial t}$

$\vec{rot}\,\vec{E} = \vec{\nabla} \wedge \vec{E}$ (le syst de coord cartésien)

$$\vec{rot}\,\vec{E} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \wedge \begin{pmatrix} 0 \\ E \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{\partial E}{\partial z} \\ 0 \\ \frac{\partial E}{\partial x} \end{pmatrix}$$