probability

1) $$\frac{\binom{15}{8}}{15^8}$$

$\binom{15}{8} \longrightarrow$ 8 distinct students

$15^8 \longrightarrow$ no constraint, num of all possibilities

2) constraints: $\_ \; \_ \; \_ \; \_ \; \_$

- all digits unique
- first two digits odd
- last digit odd

total / without constraints $= \binom{100000}{8}$

with constrants $= \binom{5}{2} \times 2! \times \binom{5}{1} \times \binom{7}{2} \times 2!$

$= 4200$

answer: $$\frac{\binom{4200}{5}\binom{95800}{3}}{\binom{100000}{8}}$$

3)

A:

1a: 2 dices 4 or above: $\left(\frac{1}{2}\right)^3 \times \binom{3}{2} = \frac{3}{8}$

1b: 3 dices 4 or above: $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$

$P(A) = \frac{1}{8} + \frac{3}{8} = \frac{4}{8} = \frac{1}{2}$

B:

$p(B) = \left(\frac{1}{6}\right)^3 \times 6 = \frac{1}{36}$

$P(A \cap B) = P(\text{all } 4) + P(\text{all } 5) + P(\text{all } 6)$

$= \left(\frac{1}{6}\right)^3 \times 3 = \frac{1}{72}$

$P(A) \times P(B) = \frac{1}{2} \times \frac{1}{36} = \frac{1}{72}$

$P(A) \times P(B) = P(A \cap B)$

Thus they are independent

4) $$P(\text{flush}) = \frac{\binom{13}{5}\binom{4}{1}}{\binom{52}{5}}$$

x = number of hands until flush

$$E(x) = \frac{1}{P(\text{flush})}$$

5) A = win with superstar
B = superstar is healthy $\quad P(B) = 0.75$
C = win without superstar
D = win 4 out of 5

$$P(D|B) = \binom{5}{4} \times P(A)^4 \times (1 - P(A))$$
$$= (0.7)^4 \times (0.3) = 0.36$$

$$P(D|B') = \binom{5}{4} \times P(C) \times (1 - P(C)) = 5 \times 0.5^4 \times 0.5$$
$$= 0.156$$

$$P(B|D) = \frac{P(D|B)\,P(B)}{P(D|B)\,P(B) + P(D|B')\,P(B')}$$

$$= \frac{0.36 \times 0.75}{0.36 \times 0.75 + 0.156 \times 0.25} = 0.87$$