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Calc 1 : Tutorial 11

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1. If $f(x) = \ln x - 1,\ 1 \leq x \leq 4$, evaluate the Riemann sum with $n = 6$, taking the sample points to be left endpoints. Give your answer correct to six decimal places

$$n = 6, \Delta x = \frac{4-1}{6} = 0.5$$

$$\begin{aligned} R_6 &\approx 0.5\left[(\ln 1 - 1) + (\ln 1.5 - 1) + (\ln 2 - 1) + (\ln 2.5 - 1) + (\ln 3 - 1) + (\ln 3.5 - 1)\right] \\ &= -0.816861 \end{aligned}$$

2. Find an approximation to the integral $\int_0^4 \left(x^2 - 3x\right) dx$ using a Riemann sum with right endpoints and $n = 8$.

$$\int_0^4 \left(x^2 - 3x\right) dx, n = 8, \Delta x = \frac{4-0}{8} = 0.5$$

$$\begin{aligned} \int_0^4 \left(x^2 - 3x\right) dx &\approx 0.5\left(\left((0.5)^2 - 3(0.5)\right) + \left((1)^2 - 3(1)\right) + \left((1.5)^2 - 3(1.5)\right) + \left((2)^2 - 3(2)\right) + \Big((\right. \\ &= -1.5 \end{aligned}$$

3. Use the Midpoint Rule with the given value of n to approximate the integral. Round each answer to **four decimal places**.

   (a) $\int_0^{10} \sin\sqrt{x}dx, n = 5, \Delta x = \frac{10-0}{5} = 2$

      i. For the sake of simplicity, I'm going to create a table

x_{left}	0	2	4	6	8
x_{right}	2	4	6	8	10
x_{mid}	1	3	5	7	9

      ii. Remember to **set your calculator to radians**

$$\begin{aligned} \int_0^{10} \sin\sqrt{x}dx &\approx 2\left(\sin\left(\sqrt{1}\right) + \sin\left(\sqrt{3}\right) + \sin\left(\sqrt{5}\right) + \sin\left(\sqrt{7}\right) + \sin\left(\sqrt{9}\right)\right) \\ &= 6.4643 \end{aligned}$$

(b)

4.

(a) Find $\int_2^5 f(x)\,dx$, given $\int_2^8 f(x)\,dx = 1.7$ and $\int_5^8 f(x)\,dx = 2.5$

$$\begin{aligned}\int_2^8 f(x)\,dx &= \int_2^5 f(x)\,dx + \int_5^8 f(x)\,dx\\ \int_2^5 f(x)\,dx &= \int_2^8 f(x)\,dx - \int_5^8 f(x)\,dx\\ &= 1.7 - 2.5\\ &= -0.8\end{aligned}$$

(b) Find $\int_1^3 f(t)\,dt$, if $\int_0^1 f(t)\,dt = 2$, $\int_0^4 f(t)\,dt = -6$, $\int_3^4 f(t)\,dt = 1$

$$\int_0^4 f(t) = \int_0^1 f(t)\,dt + \int_1^3 f(t)\,dt + \int_3^4 f(t)\,dt$$

5.

(a) $\int_0^2 \frac{1}{\sqrt{1+x^3}} dx, n = 10$

i. Trapezoidal rule

A. Break into subintervals

$$\begin{aligned}\Delta x &= \frac{2-0}{10}\\ &= 0.2\end{aligned}$$

B. Area for a single trapezoid

$$A_i = \frac{\Delta x}{2}\left(f(x_i - 1) + f(x_i)\right)$$

C. Area for the full trapezoids

x	$y = \frac{1}{\sqrt{1+x^3}}$
0	1
0.2	0.99602
0.4	0.96946
0.6	0.90685
0.8	0.81325
1	0.70711
1.2	0.60545
1.4	0.51681
1.6	0.44298
1.8	0.38258
2	0.33333

D. Trapezoidal rule estimate

$$\int_0^2 \frac{1}{\sqrt{1+x^3}} dx \approx \frac{0.2}{2}\left[1+2\left(0.9960+...+0.3826\right)+0.3333\right]$$
$$= 14.01435$$
$$= 14.0144\ (4.d.p.)$$

(b) $\int_0^1 e^{-x^2} dx, n = 10$

i. Trapezoidal rule estimate

ii. Simpson's rule estimate

A. Break into subintervals

$$\Delta x = \frac{1-0}{10}$$
$$= 0.1$$

B. Find area for Simpson's rule

x	$f(x) = e^{-x^2}$	Odd	Even	
0	1			
0.1		0.99005		
0.2			0.96079	
0.3		0.91393		
0.4			0.85214	
0.5		0.77880		
0.6			0.69768	
0.7		0.61263		
0.8			0.52729	
.9		0.44486		
.10	0.36788	3.74027	3.0379	

C. Approximate with Simpson's rule

$$\int_0^3 \frac{1}{1+x^3} dx = \frac{0.1}{3}\left[1+0.36788+4\left(3.74027\right)+2\left(3.0379\right)\right]$$
$$= 0.7468$$

6.

(a) $\int_{\frac{1}{6}}^{\frac{1}{2}} \frac{dx}{(5+6x)^3}$

$$\int_{\frac{1}{6}}^{\frac{1}{2}} \frac{dx}{\left(5+6x\right)^3} = \int_{\frac{1}{6}}^{\frac{1}{2}} \frac{1}{\left(5+6x\right)^3} dx$$
$$= \int_{\frac{1}{6}}^{\frac{1}{2}} \left(5+6x\right)^{-3} dx$$

i. Let $u = 5 + 6x$

$$\frac{du}{dx} = 6$$
$$dx = \frac{du}{6}$$

ii. Find the bounds in terms of u

$$x = \frac{1}{2}$$
$$u = 5 + 6\left(\frac{1}{2}\right)$$
$$= 5 + 3$$
$$= 8$$

$$x = \frac{1}{6}$$
$$u = 5 + 6\left(\frac{1}{6}\right)$$
$$= 6$$

iii. Find the definite integral, in the form of u

$$\int_{\frac{1}{6}}^{\frac{1}{2}} \frac{dx}{(5+6x)^3} = \int_{\frac{1}{6}}^{\frac{1}{2}} (5+6x)^{-3}\, dx$$
$$= \int_6^8 u^{-3} \frac{du}{6}$$
$$= \frac{1}{6} \int_6^8 u^{-3} du$$
$$= \frac{1}{6} \left[\frac{u^{-2}}{-2}\right]_6^8$$
$$= \frac{1}{6} \left(\frac{8^{-2}}{-2} - \frac{6^{-2}}{-2}\right)$$
$$= \frac{7}{6912}$$

(b) $\int_{-2}^{2} \frac{x+6}{\sqrt{x+2}} dx$

$$\int_{-2}^{2} \frac{x+6}{\sqrt{x+2}} dx$$

i. Let $u = \sqrt{x+2}$ (given)

$$\frac{du}{dx} = \frac{1}{2\sqrt{x+2}}$$

$$dx = 2\sqrt{x+2}du$$

$$\begin{aligned} u^2 &= x+2 \\ x &= u^2 - 2 \end{aligned}$$

ii. Find the bounds in terms of u

A. $x = 2$

$$\begin{aligned} u &= \sqrt{2+2} \\ &= 2 \end{aligned}$$

B. $x = -2$

$$\begin{aligned} u &= \sqrt{-2+2} \\ &= 0 \end{aligned}$$

iii. Find the integral

$$\begin{aligned} \int_{-2}^{2} \frac{x+6}{\sqrt{x+2}}dx &= \int_{0}^{2} \frac{x+6}{\sqrt{x+2}} 2\sqrt{x+2}du \\ &= 2\int_0^2 x+6du \\ &= 2\int_0^2 u^2 - 2 + 6du \\ &= 2\int_0^2 u^2 + 4du \\ &= 2\left[\frac{u^3}{3} + 4u\right]_0^2 \\ &= 2\left(\frac{2^3}{3} + 4\left(2\right)\right) \\ &= \frac{64}{3} \end{aligned}$$

(c) $\int_0^3 \sqrt{9-x^2}dx$

i. $x = 3\cos\theta$.

A. Find $d\theta$

$$\begin{aligned} \frac{dx}{d\theta} &= 3\left(-\sin\theta\right) \\ dx &= -3\sin\theta d\theta \end{aligned}$$

B. Find bounds in respect of θ

C. $x = 3$

$$\begin{aligned}\theta &= \cos^{-1}\frac{x}{3}\\ &= \cos^{-1} 1\\ &= 0\end{aligned}$$

D. $x = 0$

$$\begin{aligned}\theta &= \cos^{-1} 0\\ &= \frac{\pi}{2}\end{aligned}$$

E. Find the integral

$$
\begin{aligned}
\int_0^3 \sqrt{9-x^2}dx &= \int_0^3 \sqrt{9-(3\cos\theta)^2}dx \\
&= \int_0^3 \sqrt{9-9\cos^2\theta}dx \\
&= \int_0^3 \sqrt{9\left(1-\cos^2\theta\right)}dx \\
&= \int_0^3 3\sqrt{1-\cos^2\theta}dx \\
&= 3\int_0^3 \sqrt{1-\left(1-\sin^2\theta\right)}dx \\
&= 3\int_0^3 \sqrt{1-\left(1-\sin^2\theta\right)}dx \\
&= 3\int_0^3 \sqrt{\sin^2\theta}dx \\
&= 3\int_0^3 \sin\theta dx \\
&= 3\int_{\frac{\pi}{2}}^0 \sin\theta\left(-3\sin\theta d\theta\right) \\
&= -9\int_{\frac{\pi}{2}}^0 \sin^2\theta d\theta \\
&= -9\int_{\frac{\pi}{2}}^0 \frac{1+\cos\left(2\theta\right)}{2} \\
&= -\frac{9}{2}\int_{\frac{\pi}{2}}^0 1+\cos\left(2\theta\right) \\
&= -\frac{9}{2}\left[\theta-2\sin\left(2\theta\right)\right]_{\frac{\pi}{2}}^0 \\
&= \frac{9}{2}\left(\frac{\pi}{2}\right) \\
&= \frac{9}{4}\pi
\end{aligned}
$$

(d) $\int_4^6 \frac{12}{(x-3)(x+1)}dx$

i. Partial fractions!

$$
\begin{aligned}
\frac{12}{(x-3)(x+1)} &= \frac{A}{(x-3)}+\frac{B}{(x+1)} \\
12 &= A(x+1)+B(x-3)
\end{aligned}
$$

$$A(x+1)+B(x-3)-12=0$$

A. Find A, when $x = 3$

$$\begin{aligned}
A\left(3+1\right) + B\left(3-3\right) - 12 &= 0 \\
4A - 12 &= 0 \\
A &= \frac{12}{4} \\
&= 3
\end{aligned}$$

B. Find B, when $x = -1$

$$\begin{aligned}
A\left(-1+1\right) + B\left(-1-3\right) - 12 &= 0 \\
-4B - 12 &= 0 \\
B &= -\frac{12}{4} \\
&= -3
\end{aligned}$$

C. Find the fractions

$$\begin{aligned}
\frac{12}{\left(x-3\right)\left(x+1\right)} &= \frac{A}{\left(x-3\right)} + \frac{B}{\left(x+1\right)} \\
&= \frac{3}{x-3} - \frac{3}{x+1}
\end{aligned}$$

ii. Find the integration

$$\begin{aligned}
\int_4^6 \frac{12}{\left(x-3\right)\left(x+1\right)} dx &= \int_4^6 \left(\frac{3}{x-3} - \frac{3}{x+1}\right) dx \\
&= 3\int_4^6 \left(\frac{1}{x-3} - \frac{1}{x+1}\right) dx \\
&= 3\left[\int_4^6 \frac{1}{x-3} dx - \int_4^6 \frac{1}{x+1} dx\right] \\
&= 3\left[\left[\ln\left(x-3\right)\right]_4^6 - \left[\ln\left(x+1\right)\right]_4^6\right] \\
&= 3\left[\ln 3 - \ln 1 - \left(\ln 7 - \ln 5\right)\right] \\
&= 3\left[\ln 3 - \ln 1 - \ln 7 + \ln 5\right] \\
&= 3\left[\ln \frac{3 * 5}{7}\right] \\
&= 3\ln\frac{15}{7}
\end{aligned}$$