Chapter 8: Correlation and Regression Analysis

July 23, 2019

1 Example

2 Example

1.

	X	Y	XY	X^2	Y^2
	3	100	300	9	10000
	5	300	1500	25	90000
	2	90	180	4	8100
	1	30	30	1	900
	3	240	720	9	57600
	4	200	800	16	40000
	1	150	150	1	22500
	3	170	510	9	28900
	1	60	60	1	3600
Sum	**23**	**1340**	**4250**	**75**	**261600**

2. Product Moment Correlation Coefficient

$$\begin{aligned} r &= \frac{n\sum XY - (\sum X)(\sum Y)}{\sqrt{\left[n\sum X^2 - (\sum X)^2\right]\left[n\sum Y^2 - (\sum Y)^2\right]}} \\ &= \frac{9(4250) - (23)(1340)}{\sqrt{\left[9\cdot 75 - (23)^2\right]\left[9\cdot (261600) - (1340)^2\right]}} \\ &= 0.8226 \end{aligned}$$

(a) Very strong correlation

3 Example

	X	Y	XY	X^2	Y^2
	9	496	4464	81	246016
	9.5	465	4417.5	90.25	216225
	10	482	4820	100	232324
	10.5	459	4819.5	110.25	210681
	11	408	4488	121	166464
	11.5	382	4393	132.25	145924
	12	315	3780	144	99225
	12.5	363	4537.5	156.25	131769
	13	309	4017	169	95481
Sum	**99**	**3679**	**39736.5**	**1104**	**1544109**

1.

2. Product Moment Correlation Coefficient

$$r = \frac{9 * 39736.5 - 99 \cdot 3679}{\sqrt{\left[9 * 1104 - (99)^2\right]\left[9 * 1544109 - (3679)^2\right]}}$$
$$= -0.9431$$

3. Conclusion

   (a) The correlation coefficient of $r = -0.9431$ indicates that there is a very high degree of negative correlation between the weekly price and sales. As the weekly price increase, the sales decrease.

4 Example

1. Table

Competitor	A	B	C	D	E	F	G	H	I	J	SUM
X	4	9	2	5	3	10	6	7	8	1	
Y	6	10	2	8	1	9	7	4	5	3	
$d = r_x - r_y$	-2	-1	0	-3	2	1	-1	3	3	-2	
d^2	4	1	0	9	4	1	1	9	9	4	42

2. Spearman's Rank Correlation Coefficient

$$r_s = 1 - \frac{6 * 42}{10\,(100 - 1)} = 0.7435$$

5 Example

1. Table

Q5	X	r_x	Y	r_y	$d = r_x - r_y$	d^2
	1.68	5	3.81	2	3	9
	1.46	3	4.19	3	0	0
	1.57	4	4.87	4	0	0
	13.37	10	22.85	10	0	0
	3.18	8	6.47	6	2	4
	1.95	7	6.48	7	0	0
	1.07	1	2.66	1	0	0
	1.71	6	6.49	8	-2	4
	1.22	2	5.33	5	-3	9
	6.46	9	15.23	9	0	0
						26

2. Spearman's Rank Correlation Coefficient

$$\begin{aligned} r_s &= 1 - \frac{6\sum d^2}{n\left(n^2-1\right)} \\ &= 1 - \frac{6\left(26\right)}{10\left(10^2-1\right)} \\ &= 0.8424 \end{aligned}$$

3. Conclusion

   (a) The result demonstrates relatively high positive correlation. Thus, high rates tend to be paired with high rents and vice versa.

6 Example 6

1. Table

Q6	X	r_x	Y	r_y	$d = r_x - r_y$	d^2	
	30	3.5	30	9	-5.5	30.25	4 same values at 30
	31	6	14	1	5	25	average of ranks 2,3,4,5
	32	7	30	9	-2	4	=(2+3+4+5)/4
	30	3.5	23	4.5	-1	1	3.5
	46	10.5	32	11	-0.5	0.25	
	30	3.5	26	6.5	-3	9	For 31, since the next "rank" after
	19	1	20	2	-1	1	the averages is 6
	35	8	21	3	5	25	
	40	9	23	4.5	4.5	20.25	
	46	10.5	30	9	1.5	2.25	
	57	12	35	12	0	0	
	30	3.5	26	6.5	-3	9	
						127	

2. Spearman's rank correlation coefficient

$$\begin{aligned} r_s &= 1 - \frac{6\sum d^2}{n\left(n^2-1\right)} \\ &= 1 - \frac{6\left(127\right)}{12\left(12^2-1\right)} \\ &= 0.5559 \end{aligned}$$

3. Conclusion

   (a) The results show that there is positive correlation

7 Example