Statistics II C6: Hypothesis Testing

July 7, 2019

1 Example 1

1. For each of the statement given below, identify H_0 and H_1.
   (a) The mean height of females in a country is 156cm.
      i. $H_0 : \mu = 156cm$
      ii. $H_1 : \mu \neq 156cm$
   (b) The mean annual household income is at least \$12,000.
      i. $H_0 : \mu \geq \$12000$
      ii. $H_1 : \mu < \$12000$
   (c) The mean life of a car battery is not more than 40 months.
      i. $H_0 : \mu \leq 40 months$
      ii. $H_1 : \mu > 40 months$
   (d) The mean life of a car battery is above 40 months.
      i. $H_0 : \mu > 40 months$
      ii. $H_1 : \mu \leq 40 months$
   (e) A television executive claims that the majority of teenagers are in favor of sport shows on television **(Something wrong?)**
      i. $H_0 : p > 0.5$
      ii. $H_1 : p \leq 0.5$
   (f) A maximum of 3% of mailing handled by mail order companies will be returned as "address unknown" or "not known at this address".
      i. $H_0 : p \leq 0.03$
      ii. $H_1 : p > 0.03$

2 Example

A company markets car tyres. Their lives are normally distributed with a **mean of 40,000 km** and **standard deviation of 3,000 km**. A change in the production process is believed to result in a better product. A **test sample of 64 new tyres** has a **mean life of 41,200 km**. Can you conclude that the new product is **significantly better** than the current one? ($\alpha = 0.05$)

Answer:

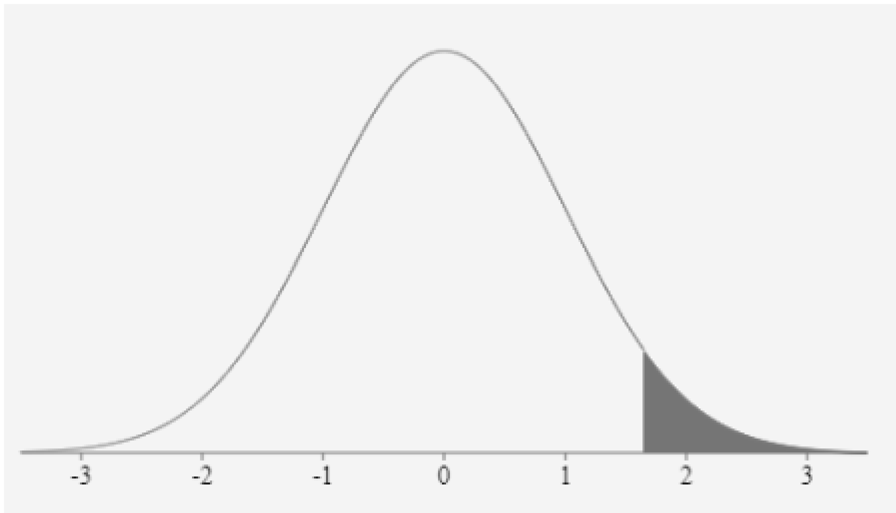
$\mu_o = 40000, \sigma = 3000, n_n = 64, \bar{X}_n = 41200, \alpha = 0.05, D.O.F = 64 - 1 = 63$

1. Identify the specific claim or hypothesis to be tested. State the null and alternative hypothesis.
   (a) Specific claim/hypothesis: The mean life of the new product is significantly better than the current one.
   (b) H_0(null hypothesis) : $\mu_n > \mu_o$
   (c) H_1(alternative hypothesis): $\mu_n \leq \mu_o$
2. Select the distribution (test statistic) to use. (Note: the book has step 2 and 3 inverted)
   (a) Assuming that the standard deviation, σ remains the same for both old and new tyres.
   (b) Since we know the population standard deviation for the new tyres, σ_n, we are going to use **Normal distribution**.
3. Determine the significance level α and the critical value.
   (a) significance level α
      i. $\alpha = 0.05$
   (b) critical value (take note we are only testing the right tail, because we are 95% confident it is better. Testing the left tail is not done because the left tail indicates that it is "worse", as in negative standard deviations away)
      i. $Z_{\frac{\alpha}{2}} = Z_{0.05} = 1.6449$
4. Determine the rejection and non-rejection regions. Set up a decision rule based on the critical value. Draw a distribution curve if necessary.
   (a) Determine the rejection and non-rejection regions.
      i. Non-rejection (null hypothesis is TRUE, take note null hypothesis is the hypothesis we hope to reject, that means we want it to be wrong):
      $$H_0 : \mu_n \leq 40000$$
      (mean life of the new tyres is NOT better than the old tyres)
      ii. Rejection (alternative hypothesis is TRUE):
      $$H_1 : \mu_n > 40000$$
      (mean life of the new tyres is better than the old tyres)

(b) Decision rule

   i. At $\alpha = 0.05$,critical value $= Z_\alpha$=$Z_{0.05} = 1.6449$

(c) Distribution curve

   i. 

5. Calculate the value of the test statistic

$$
\begin{aligned}
Z &= \frac{\bar{X} - \mu_o}{\frac{\sigma}{\sqrt{n}}} \\
&= \frac{41200 - 4000}{\frac{3000}{\sqrt{64}}} \\
&= 3.2
\end{aligned}
$$

6. Make a decision (reject H_0 or fail to reject H_0).

   (a) Since $Z = 3.2 < 1.6449$, the computed value of Z faills in the rejection region. Therefore, we reject $H_0$and accept H_1 at $\alpha = 0.05$. **Hence, we conclude that the new tyres is significantly better than the old tyres.**