Discrete Maths: T3

December 3, 2019

1. Use truth table to determine whether the validity of the following arguments.

   (a) $p \vee q$
   $p \rightarrow \sim q$
   $p \rightarrow r$
   $\therefore r$

   i. Answer

p	q	r	$\boldsymbol{p \vee q}$	$\sim q$	$\boldsymbol{p \rightarrow \sim q}$	$\boldsymbol{p \rightarrow r}$	$\boldsymbol{r}$
0	0	0	0	1	1	1	0
0	0	1	0	1	1	1	1
0	1	0	**1**	0	**1**	**1**	**0**
0	1	1	1	0	1	1	1
1	0	0	1	1	1	0	0
1	0	1	1	1	1	1	1
1	1	0	1	0	0	0	0
1	1	1	1	0	0	1	1

   ii. Since there exists a critical row where the conclusion is NOT true, the argument is invalid.

   (b) $p \vee q$
   $p \wedge \sim q \rightarrow r$
   $q \rightarrow p$
   $\therefore r$

   i. Answer

p	q	r	$\boldsymbol{p \vee q}$	$\sim q$	$p \wedge \sim q$	$\boldsymbol{p \wedge \sim q \rightarrow r}$	$\boldsymbol{q \rightarrow p}$	$\boldsymbol{r}$
0	0	0	0	1	0	1	1	0
0	0	1	0	1	0	1	1	1
0	1	0	**1**	0	0	1	0	**0**
0	1	1	1	0	0	1	0	1
1	0	0	1	1	1	0	1	0
1	0	1	**1**	1	1	**1**	**1**	**1**
1	1	0	**1**	0	0	**1**	**1**	**0**
1	1	1	1	0	0	1	1	1

   ii. Conclusion

A. Since there exists a critical row where the conclusion is NOT true, the argument is invalid.

2. The following are valid arguments. Establish the validity of each by means of the truth table. In each case, determine the critical rows for assessing the validity of the argument and which rows can be ignored.

   (a) $(p \wedge (p \rightarrow q) \wedge r) \rightarrow ((p \vee q) \rightarrow r)$

      i. Arguments:

         A. p

         B. $p \rightarrow q$

         C. r

      ii. Conclusion:

         A. $\therefore ((p \vee q) \rightarrow r)$

      iii. Truth table

$\boldsymbol{p}$	q	$\boldsymbol{r}$	$\boldsymbol{p \rightarrow q}$	$(p \vee q)$	$\boldsymbol{((p \vee q) \rightarrow r)}$
0	0	0	1	0	1
0	0	1	1	0	1
0	1	0	1	1	0
0	1	1	1	1	1
1	0	0	0	0	1
1	0	1	0	0	1
1	1	0	1	1	0
1	1	**1**	**1**	1	**1**

      iv. Critical rows: pqr (the 8th row), the rest of the rows can be ignored.

   (b) $(((p \wedge q) \rightarrow r) \wedge \sim q \wedge (p \rightarrow \sim r)) \rightarrow (\sim p \vee q)$

      i. Arguments:

         A. $(p \wedge q) \rightarrow r$

         B. $\sim q$

         C. $(p \rightarrow \sim r)$

      ii. Conclusion:

         A. $\therefore \sim p \vee q$

      iii. Truth table

p	q	r	$p \wedge q$	$\boldsymbol{(p \wedge q) \rightarrow r}$	$\boldsymbol{\sim q}$	$\sim r$	$\boldsymbol{(p \rightarrow \sim r)}$
0	0	0	0	**1**	**1**	1	**1**
0	0	1	0	**1**	**1**	0	**1**
0	1	0	0	1	0	1	1
0	1	1	0	1	0	0	1
1	0	0	0	**1**	**1**	1	**1**
1	0	1	0	1	1	0	0
1	1	0	1	0	0	1	1
1	1	1	1	1	0	0	0

iv. Critical rows: $\bar{p}\bar{q}\bar{r}, \bar{p}\bar{q}r, p\bar{q}\bar{r}$. 1st, 2nd, and 5th row. The rest can be ignored.

(c) $(p \vee (q \vee r)) \wedge \sim q \rightarrow (p \vee r)$

i. Arguments:

A. $p \vee (q \vee r)$

B. $\sim q$

ii. Conclusion:

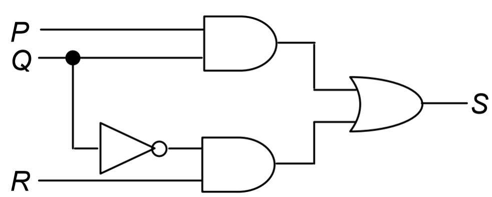
A. $\therefore p \vee r$

iii. Truth table

p	q	r	$(q \vee r)$	$\boldsymbol{p \vee (q \vee r)}$	$\boldsymbol{\sim q}$	$\boldsymbol{p \vee r}$
0	0	0	0	0	1	0
0	0	1	1	**1**	**1**	**1**
0	1	0	1	1	0	0
0	1	1	1	1	0	1
1	0	0	0	**1**	**1**	**1**
1	0	1	1	**1**	**1**	**1**
1	1	0	1	1	0	1
1	1	1	1	1	0	1

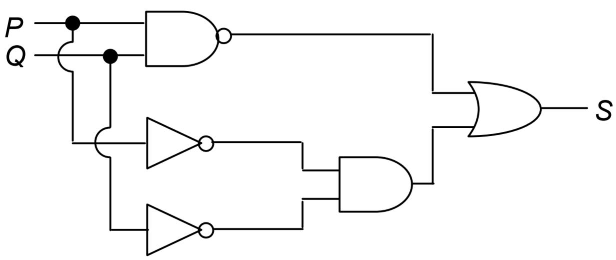
iv. Critical rows: $\bar{p}\bar{q}r, p\bar{q}\bar{r}, p\bar{q}r$. 2nd, 5th, and 6th row. The rest can be safely ignored.

3. Write an input / output table and then find the Boolean expression that corresponds to the circuit below.



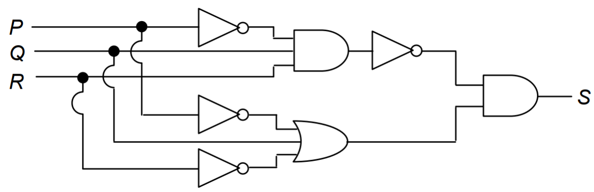
(a)

Input			Extra (Not required)			Output
P	Q	R	$P \wedge Q$	$\sim Q$	$\sim Q \wedge R$	S, $(P \wedge Q) \vee (\sim Q \wedge R)$
0	0	0	0	1	0	0
0	0	1	0	1	1	1
0	1	0	0	0	0	0
0	1	1	0	0	0	0
1	0	0	0	1	0	0
1	0	1	0	1	1	1
1	1	0	1	0	0	1
1	1	1	1	0	0	1



(b)

Input		Extra Calculations				Output, S
P	Q	$P\|Q$	$\sim P$	$\sim Q$	$\sim P \wedge \sim Q$	$(P\|Q) \vee (\sim P \wedge \sim Q)$
0	0	1	1	1	1	1
0	1	1	1	0	0	1
1	0	1	0	1	0	1
1	1	0	0	0	0	0

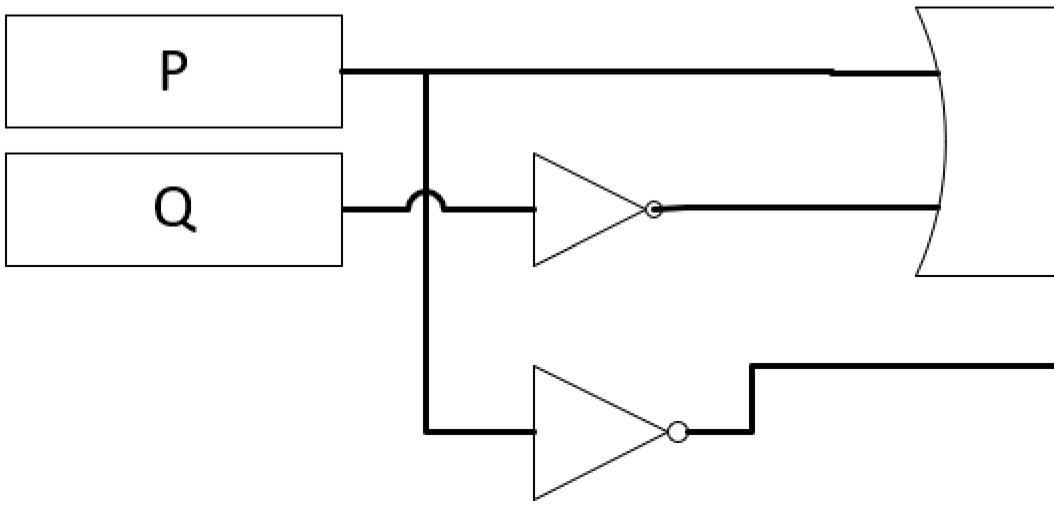


(c)

Input			Extra calculations					Ou
P	Q	R	$\sim P$	$\sim P \wedge Q \wedge R$	$\sim(\sim P \wedge Q \wedge R)$	$\sim R$	$\sim P \vee Q \vee \sim R$	$\sim(\sim P \wedge Q \wedge R)$
0	0	0	1	0	1	1	1	
0	0	1	1	0	1	0	1	
0	1	0	1	0	1	1	1	
0	1	1	1	1	0	0	1	
1	0	0	0	0	1	1	1	
1	0	1	0	0	1	0	0	
1	1	0	0	0	1	1	1	
1	1	1	0	0	1	0	1	

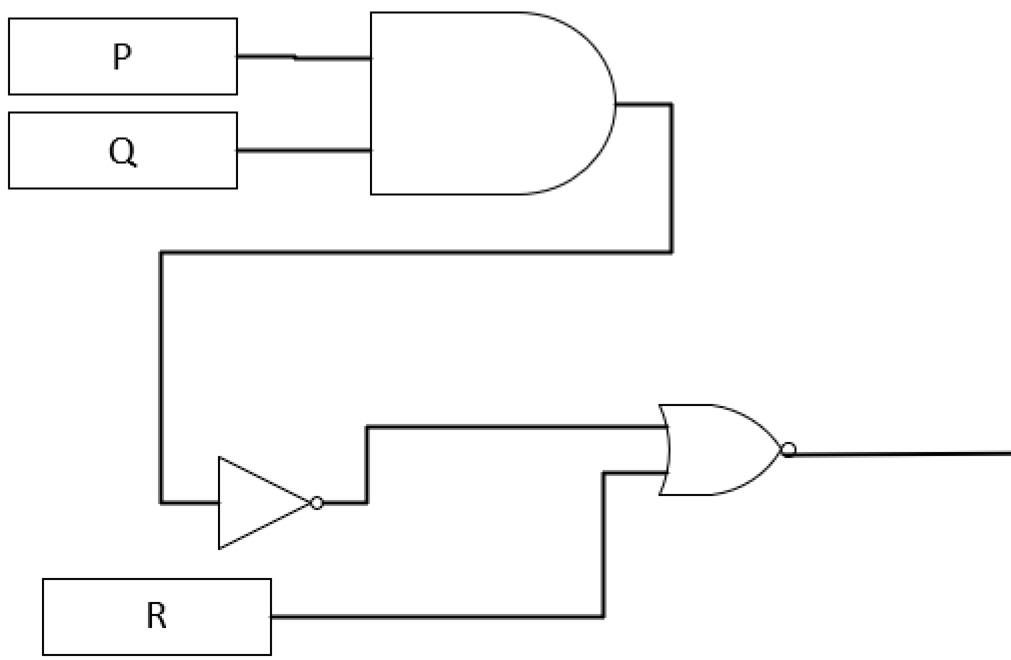
4. Construct circuits for the following Boolean expression.

   (a) $(P \vee \sim Q) \wedge \sim P$



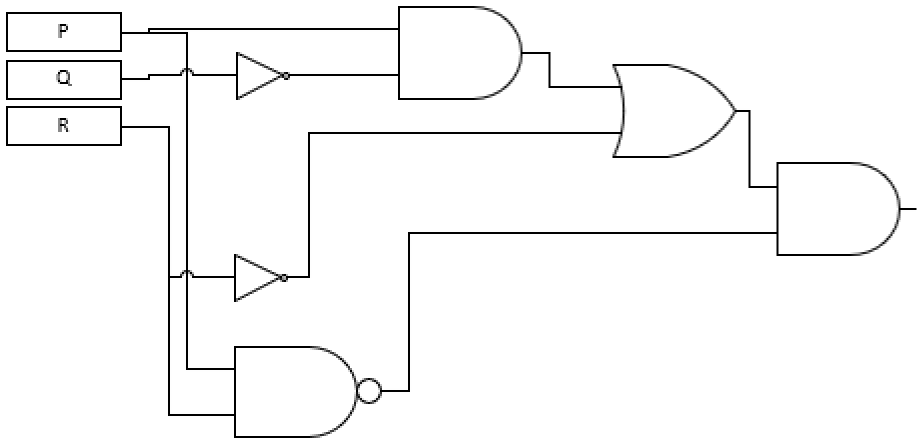
i.

(b) $(\sim (P \wedge Q)) \downarrow R$



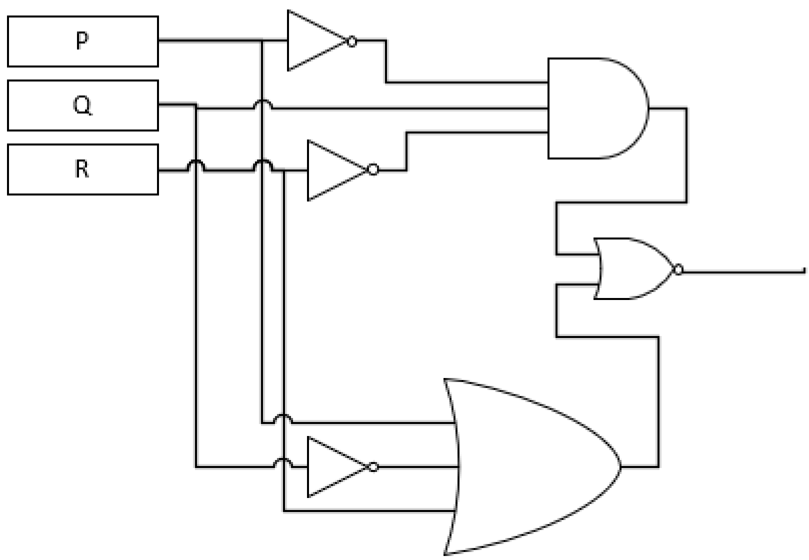
i.

(c) $((P \wedge \sim Q) \vee \sim R) \wedge (P|R)$



i.

(d) $(\sim P \wedge Q \wedge \sim R) \downarrow (R \vee \sim Q \vee P)$



i.

5. For the following truth table, construct a Boolean expression, then construct a circuit for the Boolean expressions.

(a)

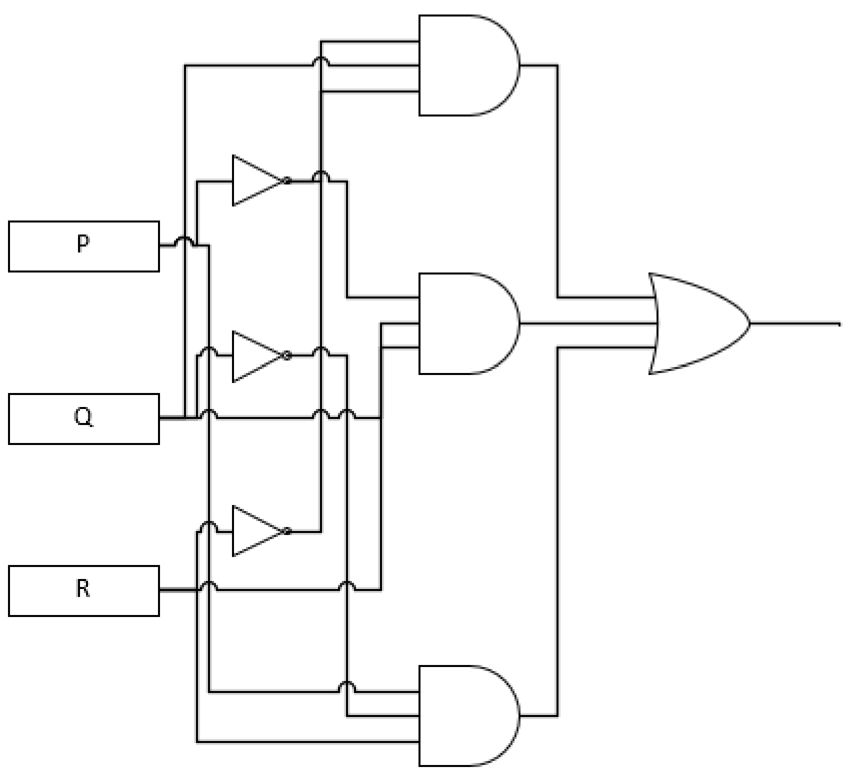
p	q	r	s (output)
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

i. Boolean expression (Laziest is to use minterms): $\bar{p}q\bar{r} + \bar{p}qr + p\bar{q}r$

   A. $s = \bar{p}q\bar{r} + \bar{p}qr + p\bar{q}r = (\sim p \wedge q \wedge \sim r) \vee (\sim p \wedge q \wedge r) \vee (p \wedge \sim q \wedge r)$

ii. Diagram

   A. Of course, lazy on boolean expression, suffer on diagram. Behold the abomination. There are 99% better boolean expressions.

   B. 

(b)

p	q	r	s (output)
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

i. Boolean expression (in this case, since 1 is less, still better to use minterms)

A. $s = \bar{p}\bar{q}\bar{r} + p\bar{q}r + pqr$

ii. Diagram

A. Draw like 5a, but according to the boolean expression in 5b.

(c)

p	q	r	s (output)
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

i. Boolean expression (minterms is still good)

A. $s = \bar{p}q\bar{r} + p\bar{q}\bar{r} + p\bar{q}r$

ii. Diagram

A. Draw like 5a, but according to the boolean expression in 5c.

6. Find the binary notation for the following decimal integer.

(a) $69_{10} = 1000101_2$

(b) $139_{10} = 10001011_2$

(c) $186_{10} = 10111010_2$

(d) $390_{10} = 110000110_2$

7. Represent the integers in decimal notation.

(a) $11001_2 = 25$

(b) $101000_2 = 40$

(c) $11001100_2 = 314$

8. Perform the following arithmetic

(a) $11001_2 + 10001_2 = 101010_2$

(b) $1100111_2 + 11_2 = 1101010_2$

(c) $110011_2 - 1001_2 = 101010_2$

(d) $10000_2 - 1_2 = 1111_2$