Revision 1: Final

January 30, 2020

1.

(a)

p	q	r	$\sim p$	$\sim p \vee r$	$q \to r$	$(\sim p \vee r) \vee (q \to r)$
0	0	0	1	1	1	1
0	0	1	1	1	1	1
0	1	0	1	1	0	1
0	1	1	1	1	1	1
1	0	0	0	0	1	1
1	0	1	0	1	1	1
1	1	0	0	0	0	1
1	1	1	0	1	1	1

i. Tautology

ii.

A. $PDNF$ of A: $\bar{p}\bar{q}\bar{r} + \bar{p}\bar{q}r + \bar{p}q\bar{r} + \bar{p}qr + p\bar{q}\bar{r} + p\bar{q}r + pq\bar{r} + pqr$

B. PCNF of A: DNE

C. $PDNF$ of $\sim A$: DNE

D. PCNF of $\sim A$:

$$(p+q+r)\,(p+q+\bar{r})\,(p+\bar{q}+r)\,(p+\bar{q}+\bar{r})\,(\bar{p}+q+r)\,(\bar{p}+q+r)\,(\bar{p}+\bar{q}+r)\,(\bar{p}+\bar{q}+\bar{r})$$

(b)

i. Converse

$$\begin{aligned} \sim p \to [\sim p \vee (\sim q \wedge r)] &\equiv p \vee [\sim p \vee (\sim q \wedge r)] \\ &\equiv p + \bar{p} + \bar{q}r \\ &\equiv t \end{aligned}$$

ii. Inverse

$$\begin{aligned} \sim [\sim p \vee (\sim q \wedge r)] \to \sim (\sim p) &\equiv [\sim p \vee (\sim q \wedge r)] \vee p \\ &\equiv \bar{p} + \bar{q}r + p \\ &\equiv t \end{aligned}$$

iii. Contrapositive

$$\begin{aligned}
\sim(\sim p) \to \sim([\sim p \vee (\sim q \wedge r)]) &\equiv \sim p \vee \sim([\sim p \vee (\sim q \wedge r)]) \\
&\equiv \bar{p} + \overline{\bar{p} + \bar{q}r} \\
&\equiv \bar{p} + p(\overline{\bar{q}r}) \\
&\equiv \bar{p} + p(q + \bar{r}) \\
&\equiv (\bar{p} + p)(\bar{p} + q + \bar{r}) \\
&\equiv \bar{p} + q + \bar{r} \\
&\equiv \sim p \vee q \vee \sim r
\end{aligned}$$

(c)

i. $P(x) : x > 0$

ii. $Q(x)$

iii. Answer

$$\mathbb{Z} = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$$

A. Find the range of $P(x)$

$$P(x) = \{1, 2, 3, ...\}$$

B. Find the rnge of $Q(x) = \{... - 3, -1, 1, 3\}$

C. $T(x) = \{..., -3, 0, 3, 6, ...\}$

iv. Part I Answer, There exists a positive integer that is even.

$$\exists x \in \mathbb{Z}\,(P(x) \wedge \sim Q(x))$$

A. When $x = 2$, the integer is both positive and even.

v. Part II Answer, if x is even, then x is not divisible by 3.

$$\forall x \in \mathbb{Z} \ni (\sim Q(x) \to \sim T(x))$$

A. False, counterexample, 6 is even and 6 is divisible by 3.

vi. Part III Answer, if x is odd then x is divisible by 3

A. False, counterexample, 1 is odd and 1 is not divisible by 3.

2. Question 2

(a) Let f be a function from $\mathbb{Z}$ to $\mathbb{Z}$ defined by $f(x) = 5 + 2x^2$, $x \in \mathbb{Z}$. Determine whether the function f is a bijective function. Justify your answer.

i. $f(-1) = f(1)$, BUT $-1 \neq 1$. Therefore, f is NOT a bijective function.

(b) Let $\rho = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 6 & 2 & 5 & 1 & 3 \end{pmatrix}$ be a permutation of the set $A = \{1, 2, 3, 4, 5, 6\}$.

i. Write ρ as a product of disjoint cycles.

$$\rho = (1,4,5) \circ (2,6,3)$$

ii. Write ρ as a product of transpositions.

$$\rho = (1,5) \circ (1,4) \circ (2,3) \circ (2,6)$$

iii. Determine whether ρ is even or odd.

A. ρ is even.

(c) Use the Euclidean algorithm to find the greatest common divisor of 60 and 36. Write the greatest common divisor in the form of $s60 + t36, s, t \in \mathbb{Z}$. Hence find the least common multiple of 60 and 36.

$$\begin{aligned} d &= GCD\,(60, 36) \\ d &= 60s + 36t \\ \ell &= LCM\,(60, 36) \end{aligned}$$

$$\begin{aligned} 60 &= 36\,(1) + 24 \\ 36 &= 24\,(1) + 12 \\ 24 &= 12\,(2) + 0 \end{aligned}$$

$$\begin{aligned} GCD\,(60, 36) &= GCD\,(12, 0) \\ &= 12 \end{aligned}$$

i. LCM

$$\begin{aligned} d\ell &= 60 * 36 \\ \ell &= \frac{60 * 36}{d} \\ &= \frac{60 * 36}{12} \\ &= 180 \end{aligned}$$

(d)

i. Let $x = 2a, a \in \mathbb{Z}$

ii. Then $x + 3 = 2a + 3$

$$\begin{aligned} x + 3 &= 2a + 3 \\ &= 2a + 2 + 1 \\ &= 2\,(a + 1) + 1 \\ x + 3 &= 2b + 1 \\ b &= a + 1 \in \mathbb{Z} \end{aligned}$$

iii. Therefore, $x + 3$ is odd.

3.

(a) Let A = {a, b, c, d, e} and let R = {(a, a), (b, b), (c, c), (d, d), (e, e), (a, b), (b, a), (b, c), (c, d), (d, e), (a, e)} be a relation defined on A.

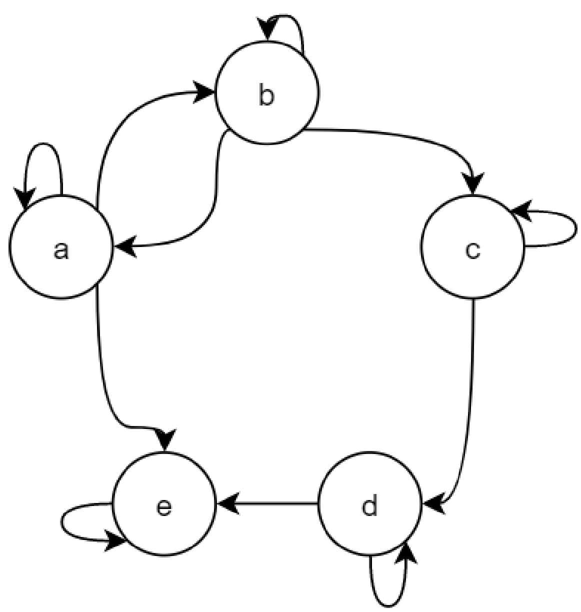
i.

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

ii. Find in-degree and out-degree

Vertex	a	b	c	d	e
In-degree	2	2	2	2	3
Out-degree	3	3	2	2	1

iii.



A.

B. R is reflexive

C. R is not irreflexive since aRa

D. R is not symmetric since bRc but $c\not{R}b$

E. R is not antisymmetric since bRa but $a\not{R}b$

F. R is not transitive since aRb and bRc but $a\not{R}c$

(b) Let $W = \{1, 2, 3, 4\}$ and R be the relation on W where $R = \{(1, 2), (2, 2), (4, 1), (3, 3), (2, 4)\}$. Use Warshall's algorithm to compute the transitive closure of R.

i. W_0

A. Matrix

$$W_0 = \begin{bmatrix} \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & 1 & 0 & 1 \\ \mathbf{0} & 0 & 1 & 0 \\ \mathbf{1} & 0 & 0 & 0 \end{bmatrix}$$

B. Column: $\{4\}$

C. Row:$\{2\}$

D. Add: $(4, 2)$

ii. W_1

A.

$$W_1 = \begin{bmatrix} 0 & \mathbf{1} & 0 & 0 \\ \mathbf{0} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ 0 & \mathbf{0} & 1 & 0 \\ 1 & \mathbf{1} & 0 & 0 \end{bmatrix}$$

B. Column: $\{1, 2, 4\}$

C. Row: $\{2, 4\}$

D. Add: $\{(1, 2), (1, 4), (2, 2), (2, 4), (4, 2), (4, 4)\}$

iii. W_2

$$W_2 = \begin{bmatrix} 0 & 1 & \mathbf{0} & 1 \\ 0 & 1 & \mathbf{0} & 1 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \\ 1 & 1 & \mathbf{0} & 1 \end{bmatrix}$$

A. Row: $\{3\}$

B. Column: $\{3\}$

C. Add: $\{(3, 3)\}$

iv. W_3

$$W_3 = \begin{bmatrix} 0 & 1 & 0 & \mathbf{1} \\ 0 & 1 & 0 & \mathbf{1} \\ 0 & 0 & 1 & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

A. Row: $\{1, 2, 4\}$

B. Column: $\{1, 2, 4\}$

C. Add: $\{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (4, 2), (1, 4), (2, 4), (4, 4)\}$

v. W_4,transitive closure

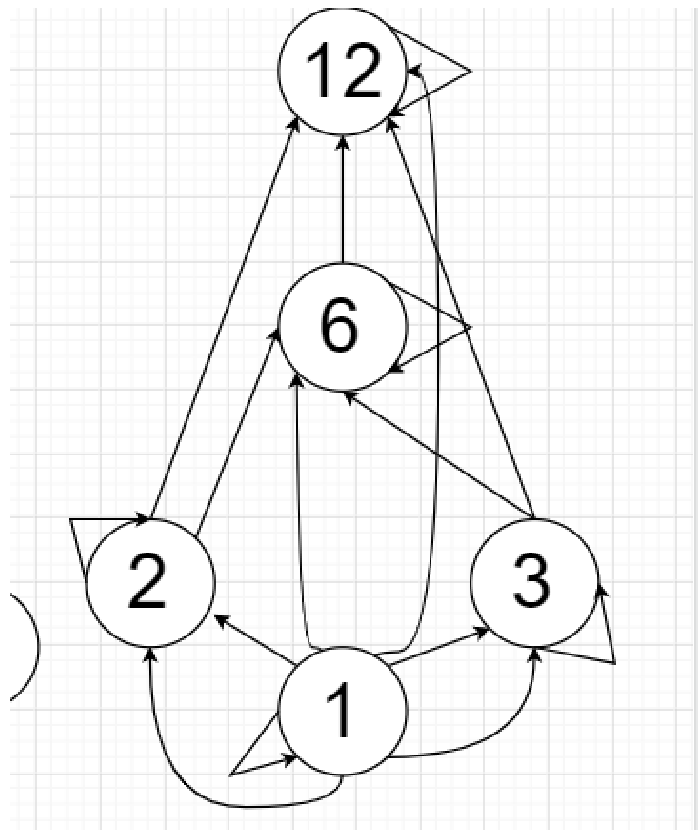
$$M_{R^\infty} = W_4 = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

4.

(a) Let $A = \{1, 2, 3, 6, 12\}$ and let R be the relation on A defined by xRy if and only if x divides y.

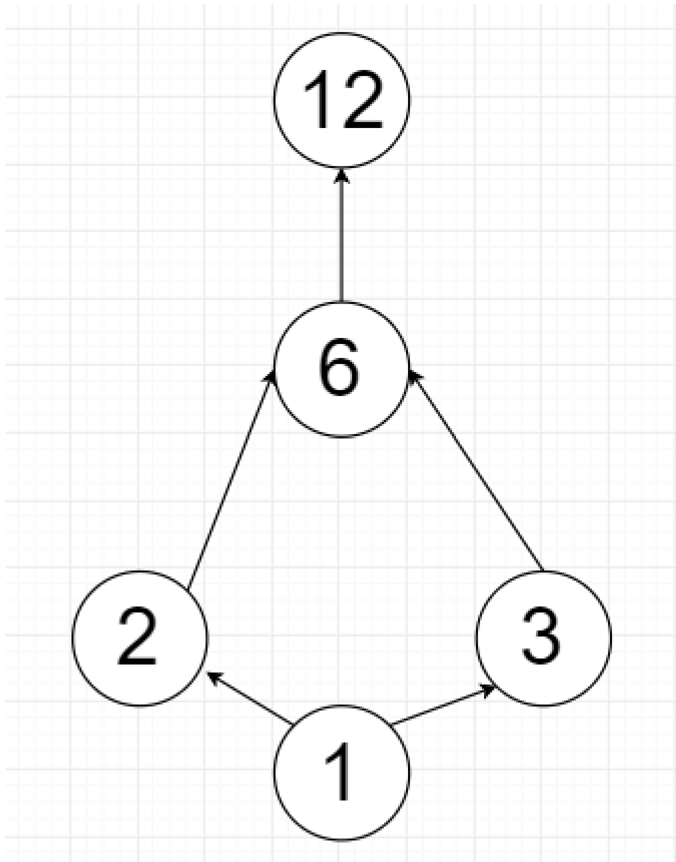
i. Draw the directed graph of the relation R on A.

$R = \{(1,1), (1,2), (1,3), (1,6), (1,12), (2,2), (2,6), (2,12), (3,3), (3,6), (3,12), (6,6), (6,12),$



A.

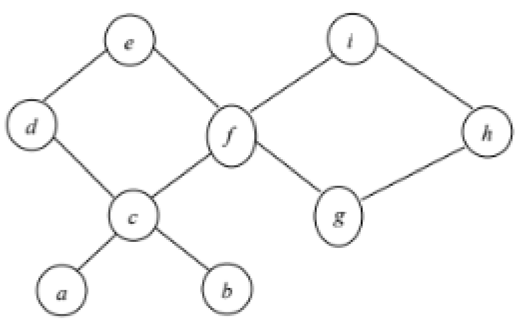
ii. Draw the Hasse diagram of the relation R on A.

A. 

iii. Determine whether R is linearly ordered.

A. R is not linearly ordered

(b) The Hasse diagram for a partial order set, P, is shown below. Find, if exist(s):



i.

ii. the maximal and minimal element(s) of P;

A. Maximal: e, i

B. Minimal: a, b, g

iii. the upper bound(s) and lower bound(s) of $\{c, d, f\}$;

A. Upper bounds: e

B. Lower bounds: c, a, b

iv. the Least Upper Bound and Greatest Lower Bound of $\{c, d, f\}$.

A. LUB: e

B. GLB: c

(c) Let $f(x, y, z) = (x\text{'}y\text{'}z\text{'}) + (x\text{'}yz\text{'}) + (xy\text{'}z\text{'}) + (xyz\text{'}) + (xyz) + (x\text{'}y\text{'}z)$. Draw a Karnaugh map and simplify f(x, y, z) to the simplest form. The Karnaugh map can be constructed in the form given below.

i.

	y'	y'	y	y
x'	1	1	0	1
x	1	0	1	1
	z'	z	z	z'

A. $f(x, y, z) = z' + x'y' + xy$