Mid-Term Answer (60% of coursework marks)

August 29, 2019

1.

   (a) $(x-1)^2 - 1 - 7 = (x-1)^2 - 8$ - 1 mark

      i. Sketch these graphs side by side, they each compose of 1 mark

         A. $y = x^2$

         B. $y = (x-1)^2$

         C. $y = (x-1)^2 - 8$

   (b)

      i. $h(x) = -\sqrt{-2x+5}$

$$\begin{aligned} -2x+5 &\geq 0 \\ -2x &\geq -5 \\ x &\leq \frac{5}{2} \end{aligned}$$

         A. Domain: $x \leq \frac{5}{2}$

         B. Range: $h(x) \leq 0$

      ii. $g(x) = 2e^x + 1$

         A. Domain: $x \epsilon \mathbb{R}$

         B. Range: $g(x) > 1$

2.

   (a)

      i. Let $y = \frac{2x}{3x-1}$

$$\begin{aligned} 3xy - y &= 2x \\ 3xy - 2x &= y \\ x &= \frac{y}{3y-2} \\ f^{-1}(x) &= \frac{x}{3x-2}, x \neq \frac{2}{3} \end{aligned}$$

ii. $g\left[f^{-1}(x)\right]=\frac{-\frac{7}{x}}{3x-2}$

A. $g\left[f^{-1}(x)\right]=\frac{-\frac{7}{x}}{3x-2}$

B. $g\circ f(x)=\frac{14-21x}{x}, x\neq\left\{0,\frac{2}{3}\right\}$

(b)

$$\cos t=\frac{x}{2}$$
$$\cos^2 t=\frac{x^2}{4}$$

i.

$$\sin t=y-1$$
$$\sin^2 t=(y-1)^2$$

ii.

$$\sin^2 t+\cos^2 t=1$$

$$y^2-2y+1+\frac{x^2}{4}=1$$
$$y^2-2y+\frac{x^2}{4}=0$$

3.

(a)

$$\lim\frac{\frac{2x^4}{x^4}}{\frac{x^4}{x^4}-\frac{5}{x^4}}=\lim_{x\to\infty}\frac{2}{1-\frac{5}{2^4}}$$
$$=2$$

(b)

$$\lim_{x\to2}\frac{\frac{x-2}{2x}}{2-x}=\lim_{x\to2}-\frac{1}{2x}=-\frac{1}{4}$$

4.

$$\lim_{x\to3}\frac{-x^2-x+12}{x-3}=\lim_{x\to3}\frac{(x-3)(-x-4)}{x-3}$$
$$=\lim_{x\to3}(-x-4)$$
$$=-7$$

(a) Since $f(x)$ is continuous at $x=3$, $f(3)=b=-7$

(b) NOTE: If you plug directly into calculator, the calculator will solve it $= 0$, which is wrong. You only want to factorize it, not solve it $= 0$

$$
\begin{aligned}
-x^2 - x + 12 &= 0 \\
x^2 + x - 12 &= 0 \\
(x-3)(x+4) &= 0
\end{aligned}
$$

5.

(a) Vertical asymptotes

i. Let $9x^2 - 25 = 0$

$$
\begin{aligned}
x^2 &= \frac{25}{4} \\
x &= \pm\frac{5}{2} \\
\therefore x &= \frac{5}{2}, -\frac{5}{2}
\end{aligned}
$$

(b) Horizontal asymtote:

$$
\begin{aligned}
\lim_{x\to\infty} \frac{x+2}{4x^2-25} &= \lim_{x\to\infty} \frac{\frac{1}{x}+\frac{2}{x^2}}{4-\frac{25}{x^2}} \\
&= 0
\end{aligned}
$$

i. $\therefore y = 0$