Discrete Math - T1

October 16, 2019

1. Determine whether each of the following sentences is a statement.
   (a) y + 3 is a positive integer
      i. NO
   (b) 128 = 26
      i. YES
   (c) x = 26
      i. NO
   (d) Is 2 a positive number?
      i. NO

2. Give the negation of the following statement.
   (a) 2 + 7 ≤ 11
      i. $2 + 7 > 11$
   (b) $2 + 1 = 3$
      i. $2 + 1 \neq 3$
   (c) 2 is an even integer and 8 is an odd integer.
      i. 2 is NOT an even integer OR 8 is NOT an odd number
   (d) Today is Wednesday
      i. Today is NOT Wednesday

3. Let p, q, r be the propositions
   p : You have a flu.
   q : You miss the final exam.
   r : You pass the course
   (a) $p \vee q \vee r$
      i. You have a flu OR you miss the final exam OR you pass the course.
   (b) $(p \wedge q) \vee (\sim q \wedge r)$

i. You have a flu AND you miss the final exam OR you DO NOT miss the final exam AND you pass the course.

(c) $\sim p \wedge \sim q \wedge r$

i. You DO NOT have a flu, AND you DO NOT miss the final exam, AND you pass the course.

4. Let h : " John is healthy." w : "John is wealthy." s : "John is wise." Use the indicated letters and logical connectors to represent the following compound statements.

(a) John is healthy and wealthy.

i. $h \wedge w$

(b) John is healthy and not wise.

i. $h \wedge \sim s$

(c) John is healthy and wealthy but not wise.

i. $h \wedge w \wedge \sim s$

(d) John is not wealthy but he is healthy and wise.

i. $\sim w \wedge h \wedge s$

(e) John is either wealthy or healthy, or both.

i. $w \vee h$

(f) John is wealthy or he is healthy but not wealthy and healthy.

i. $w \veebar h$

(g) John is neither healthy nor wealthy.

i. $\sim h \wedge \sim w$

(h) John is neither healthy, wealthy, nor wise.

i. $\sim h \wedge \sim w \wedge \sim s$

5. Determine the truth or falsity of each of the following statement.

(a) $2 > 3$ and 3 is a positive integer.

i. False AND True $\equiv False$

(b) $2 < 3$ or 3 is not a positive integer.

i. True OR False $\equiv True$

(c) 2 is a prime but 3 is not a prime.

i. $T \wedge F \equiv False$

(d) It is not true that 2 is not a prime or 3 is prime.

i. $\sim (F \vee T) \equiv False$

(e) It is false that 2 is prime or multiple of 4 (Ask later: $\sim T \vee F$)

i. $\sim (T \wedge F) \equiv True.$

6. Find the truth value of each proposition if p and r are true and q is false.

$$\begin{aligned} p &= T \\ r &= T \\ q &= F \end{aligned}$$

(a) $\sim p \wedge (q \vee r)$

$$\begin{aligned} \sim T \wedge (F \vee T) &\equiv \sim T \wedge T \\ &\equiv F \end{aligned}$$

(b) $p \wedge (\sim (q \vee \sim r))$

$$\begin{aligned} p \wedge (\sim (q \vee \sim r)) &\equiv T \wedge (\sim (F \vee \sim T)) \\ &\equiv T \wedge (\sim F) \\ &\equiv T \wedge T \\ &\equiv T \end{aligned}$$

(c) $(r \wedge \sim q) \vee (p \wedge r)$

$$\begin{aligned} (r \wedge \sim q) \vee (p \wedge r) &\equiv (T \wedge \sim F) \vee (T \wedge T) \\ &\equiv (T \wedge T) \vee T \\ &\equiv T \vee T \\ &\equiv T \end{aligned}$$

(d) $(q \wedge r) \wedge (p \wedge \sim r)$

$$\begin{aligned} (q \wedge r) \wedge (p \wedge \sim r) &\equiv (F \wedge T) \wedge (T \wedge \sim T) \\ &\equiv F \wedge (T \wedge F) \\ &\equiv F \wedge F \\ &\equiv F \end{aligned}$$

7. Construct a truth table for the following compound statements.

(a) $(p \veebar q) \veebar (q \veebar r)$

i.

p	q	r	$(p \veebar q)$	$(q \veebar r)$	$(p \veebar q) \veebar (q \veebar r)$
0	0	0	0	0	0
0	0	1	0	1	1
0	1	0	1	1	0
0	1	1	1	0	1
1	0	0	1	0	1
1	0	1	1	1	0
1	1	0	0	1	1
1	1	1	0	0	0

(b) $(p \downarrow q) \wedge (q \downarrow r)$

i.

p	q	r	$(p \downarrow q)$	$(q \downarrow r)$	$(p \downarrow q) \wedge (q \downarrow r)$
0	0	0	1	1	1
0	0	1	1	0	0
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	0	1	0
1	0	1	0	0	0
1	1	0	0	0	0
1	1	1	0	0	0

(c) $(p|q) \wedge r$

i.

p	q	r	$(p\|q)$	$(p\|q) \wedge r$
0	0	0	1	0
0	0	1	1	1
0	1	0	1	0
0	1	1	1	1
1	0	0	1	0
1	0	1	1	1
1	1	0	0	0
1	1	1	0	0

(d) $(p|q) \vee (p|r)$

i.

p	q	r	$(p\|q)$	$(p\|r)$	$(p\|q) \vee (p\|r)$
0	0	0	1	1	1
0	0	1	1	1	1
0	1	0	1	1	1
0	1	1	1	1	1
1	0	0	1	1	1
1	0	1	1	0	1
1	1	0	0	1	1
1	1	1	0	0	0

8. Determine which of the pairs of statement forms are logically equivalent. Justify your answers using truth tables.
Note: The ideal solution is to place both the statements in one table. But we do not have enough space.

(a) $\sim (p \wedge (\sim (p \wedge q)))$ and $p \wedge (\sim q \wedge \sim p)$

i.

p	q	$(p \wedge q)$	$\sim (p \wedge q)$	$p \wedge (\sim (p \wedge q))$	$\sim (p \wedge (\sim (p \wedge q)))$
0	0	0	1	0	**1**
0	1	0	1	0	**1**
1	0	0	1	1	**0**
1	1	1	0	0	**1**

ii.

p	q	$\sim q$	$\sim p$	$\sim q \wedge \sim p$	$p \wedge (\sim q \wedge \sim p)$
0	0	1	1	1	**0**
0	1	0	1	0	**0**
1	0	1	0	0	**0**
1	1	0	0	0	**0**

iii. $\therefore \sim (p \wedge (\sim (p \wedge q))) \not\equiv p \wedge (\sim q \wedge \sim p)$

(b) $(p \downarrow p) \downarrow (q \downarrow q)$ and $p \wedge q$

i.

p	q	$(p \downarrow p)$	$(q \downarrow q)$	$(p \downarrow p) \downarrow (q \downarrow q)$	$p \wedge q$
0	0	1	1	**0**	**0**
0	1	1	0	**0**	**0**
1	0	0	1	**0**	**0**
1	1	0	0	**1**	**1**

ii. $\therefore (p \downarrow p) \downarrow (q \downarrow q) \equiv p \wedge q$

(c) $(p \veebar q) \wedge r$ and $(p \wedge r) \veebar (q \wedge r)$

i.

p	q	r	$(p \veebar q)$	$(p \veebar q) \wedge r$	$(p \wedge r)$	$(q \wedge r)$	$(p \wedge r) \veebar (q \wedge r)$
0	0	0	0	**0**	0	0	**0**
0	0	1	0	**0**	0	0	**0**
0	1	0	1	**0**	0	0	**0**
0	1	1	1	**1**	0	1	**1**
1	0	0	1	**0**	0	0	**0**
1	0	1	1	**1**	1	0	**1**
1	1	0	0	**0**	0	0	**0**
1	1	1	0	**0**	1	1	**0**

ii. $(p \veebar q) \wedge r \equiv (p \wedge r) \veebar (q \wedge r)$

9. Use truth table to determine each of the statement forms below is a tautology, contradiction or contingency.

(a) $((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q$

i.

p	q	r	$\sim p$	$(\sim p \wedge q)$	$(q \wedge r)$	$((\sim p \wedge q) \wedge (q \wedge r))$	$\sim q$	$((\sim p \wedge q) \wedge (q \wedge r)) \wedge$
0	0	0	1	0	0	0	1	**0**
0	0	1	1	0	0	0	1	**0**
0	1	0	1	1	0	0	0	**0**
0	1	1	1	1	1	1	0	**0**
1	0	0	0	0	0	0	1	**0**
1	0	1	0	0	0	0	1	**0**
1	1	0	0	0	0	0	0	**0**
1	1	1	0	0	1	0	0	**0**

ii. $\therefore ((\sim p \wedge q) \wedge (q \wedge r)) \wedge \sim q \equiv c$ (contradiction)

(b) $(\sim p \vee q) \vee (p \wedge \sim q)$

i.

p	q	$\sim p$	$(\sim p \vee q)$	$\sim q$	$(p \wedge \sim q)$	$(\sim p \vee q) \vee (p \wedge \sim q)$
0	0	1	1	1	0	1
0	1	1	1	0	0	1
1	0	0	0	1	1	1
1	1	0	1	0	0	1

ii. $\therefore (\sim p \vee q) \vee (p \wedge \sim q) \equiv t$ (Tautology)

10. Simplify the following statements using law of logical equivalences.

(a) $(p \vee q) \wedge \sim (\sim p \wedge q)$

$$
\begin{aligned}
(p \vee q) \wedge \sim (\sim p \wedge q) &\equiv (p \vee q) \wedge (p \vee \sim q) \\
&\equiv (p + q)(p + \bar{q}) \\
&\equiv pp + pq + pq + p\bar{q} \\
&\equiv p + pq + p\bar{q} \\
&\equiv p(1 + q + \bar{q}) \\
&\equiv p(1) \\
&\equiv p
\end{aligned}
$$

(b) $((p \vee q) \wedge (p \vee \sim q)) \vee q$

$$
\begin{aligned}
((p \vee q) \wedge (p \vee \sim q)) \vee q &\equiv ((p + q)(p + \bar{q})) + q \\
&\equiv (pp + pq + p\bar{q} + q\bar{q}) + q \\
&\equiv (p(1 + q + \bar{q}) + 1) + q \\
&\equiv (p(1)) + q \\
&\equiv p + q \\
&\equiv p \vee q
\end{aligned}
$$

11. Use the law of logical equivalences to verify the following logical equivalences.

(a) $p \vee q \vee (\sim p \wedge \sim q \wedge r) \equiv p \vee q \vee r$

i. LHS (Using a lot of OR distributive law)

$$
\begin{aligned}
p + q + \bar{p}\bar{q}r &\equiv p + \bar{p}\bar{q}r + q \\
&\equiv p + \bar{p}(\bar{q}r) + q \\
&\equiv (p + \bar{p})(p + \bar{q}r) + q \\
&\equiv 1(p + \bar{q}r) + q \\
&\equiv p + (q + \bar{q}r) \\
&\equiv p + ((q + \bar{q}) \cdot (q + r)) \\
&\equiv p + (q + r) \\
&\equiv p + q + r \\
&\equiv RHS
\end{aligned}
$$

(b) $\sim (p \downarrow q) \equiv (\sim p | \sim q)$

i. LHS

$$\begin{aligned} \sim (p \downarrow q) &\equiv \sim (\sim (p \vee q)) \\ &\equiv p \vee q \end{aligned}$$

ii. RHS

$$\begin{aligned} (\sim p | \sim q) &\equiv \sim (\sim p \wedge \sim q) \\ &\equiv \sim (\sim p \wedge \sim q) \\ &\equiv (p \vee q) \text{ (DeMorgan's)} \\ &\equiv LHS \end{aligned}$$