AAMS3113 Calculus I - Tutorial 2

July 2, 2019

1. Suppose the graph of $y = f(x)$ is given. Write equations for the graphs that are obtained from the graph of f as follows.

   (a) Shift 3 units upward

      i. $y = f(x) + 3$

   (b)

      i. $y = f(x) - 3$

   (c)

      i. $y = f(x - 3)$

   (d)

      i. $y = f(x + 3)$

   (e)

      i. $y = -f(x)$

   (f)

      i. $y = f(-x)$

   (g)

      i. $y = 3f(x)$

   (h)

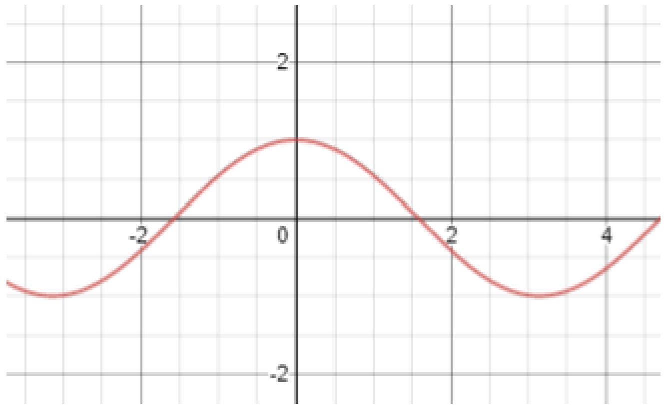
      i. $y = \frac{1}{3}f(x)$

2.

3.

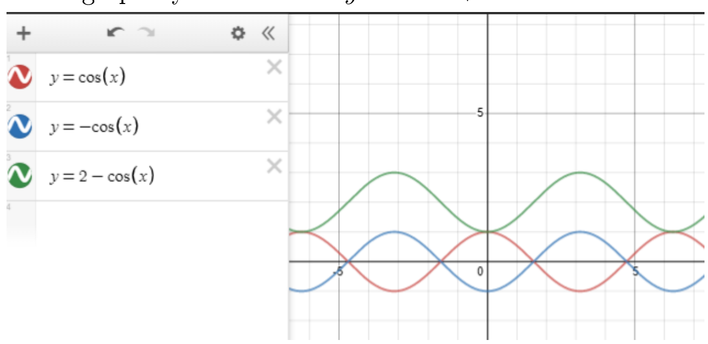
   (a)

      i. Start with $y = cos\, x$



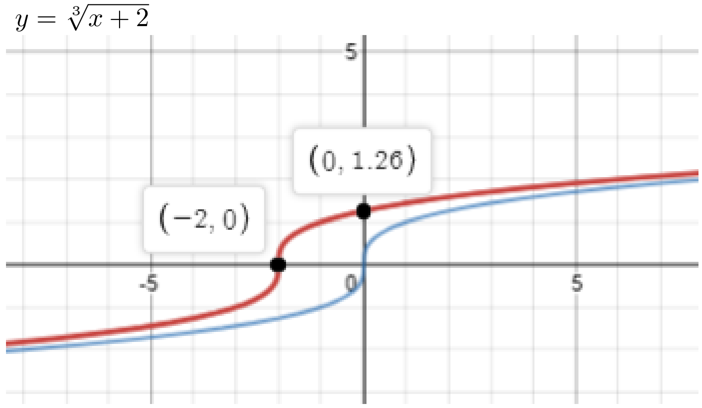
A.

ii. Continue by inversing the equation, $y = -cos\, x$, then finally shift the entire graph by 2 to become $y = -cos + 2$



A.

(b) $y = \sqrt[3]{x+2}$



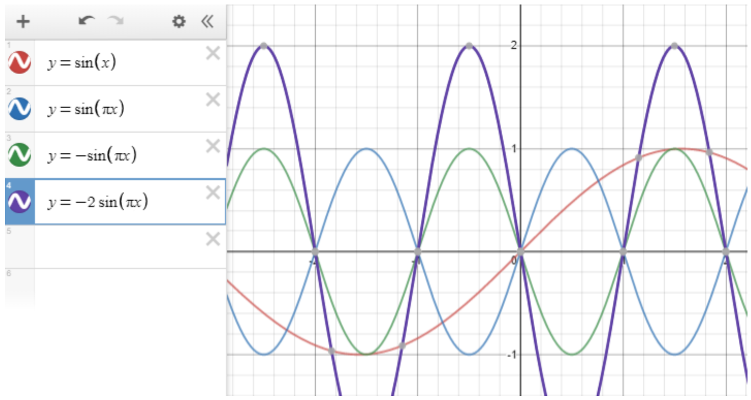
(c) $y = -2 sin \pi x$

i. Start with $y = sin\, x$,

ii. then increase the frequency of the cycles for the entire equation by π, or $y = sin\, \pi x$.

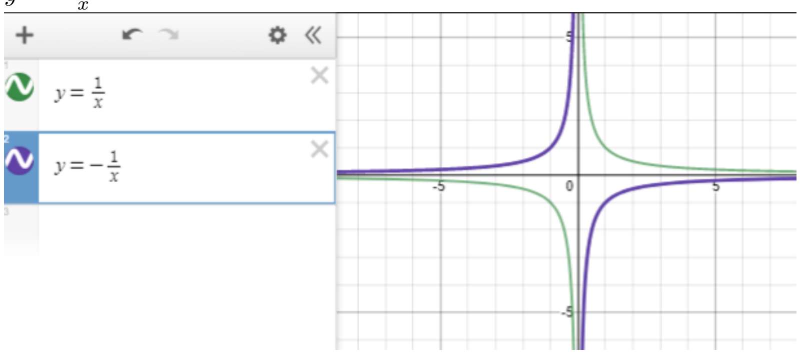
iii. Afterwards, invert the entire graph by multiplying with -1, making it $y = -sin\, \pi x$

iv. Finally, stretch the graph vertically by a factor of 2, to make it $y = -2sin\,\pi x$



v.

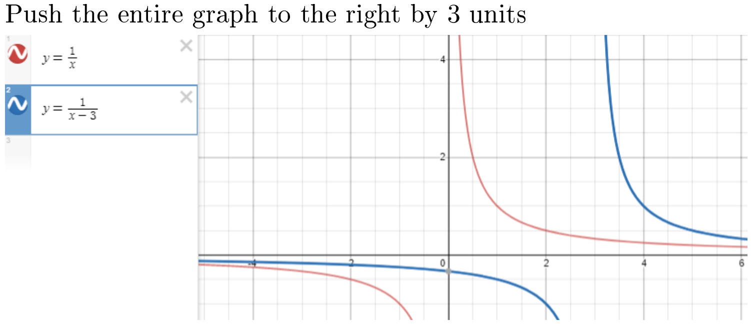
(d) $y = -\frac{1}{x}$



(e)

i. Start with $y = \frac{1}{x}$

ii. Push the entire graph to the right by 3 units



iii.

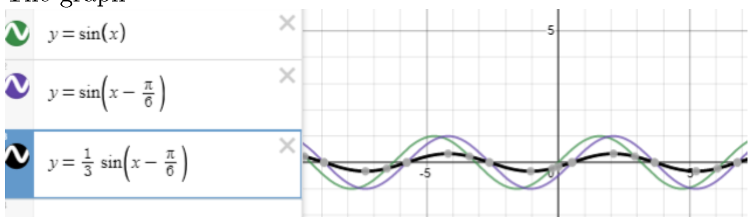
(f)

i. Start with $y = sin\,(x)$

ii. Push the graph forward by $\frac{\pi}{6}$units to make it $y = sin\,\left(x - \frac{\pi}{6}\right)$

iii. Squeeze the graph vertically to transform the graph into $y = \frac{1}{3}\sin\left(x - \frac{\pi}{6}\right)$

iv. The graph

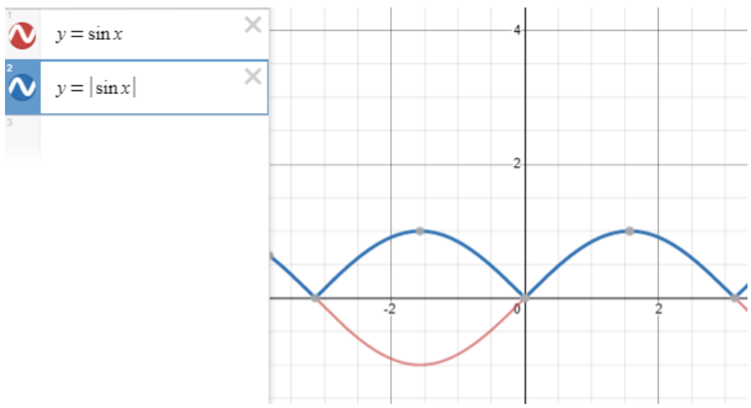


(g)

i. Start with $y = sin\, x$

ii. Take the absolute value of the $f(x)$ by modulus-ing the entire $f(x)$

iii.



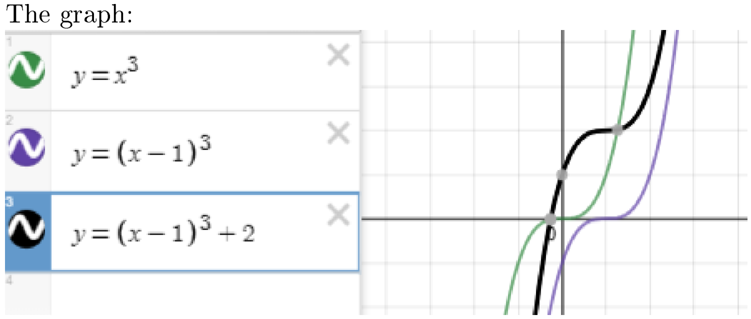
(h)

i. Start with $y = x^3$

ii. Shift the graph forward by 1 units horizontally, to transform the graph into $y = (x-1)^3$

iii. Shift the graph up vertically by 2 units to transform the graph into $y = (x-1)^3 + 2$

iv. The graph:

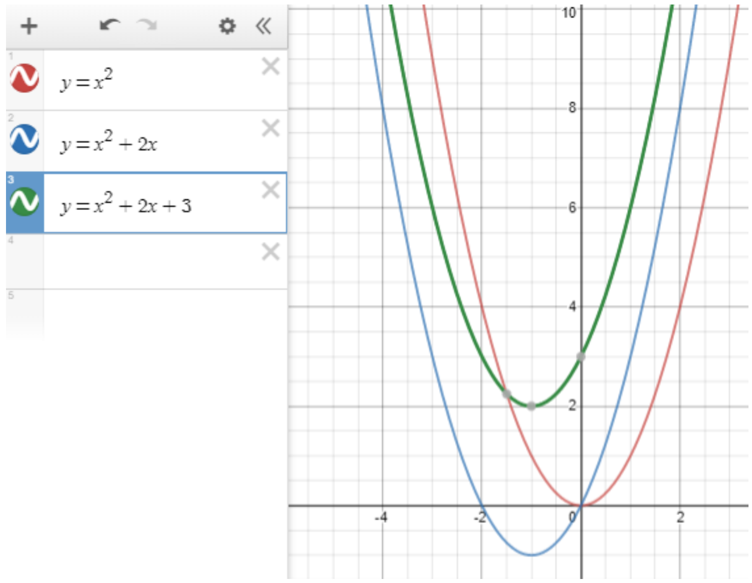


(i)

i. Start with $y = x^2$

ii. Push the entire graph down, left by making the graph cross $x = 0$ and $x = 2$ to make the graph into $y = x^2 + 2x$

iii. Shift the entire graph upwards by 3 units to $y = x^2 + 2x + 3$

iv. 

4. $f(x) = \sqrt{x+1}, g(x) = \sqrt{1-x}$

(a)

$$\begin{aligned} f + g &= \sqrt{x+1} + \sqrt{1-x} \\ D(f+g) &= D(f) \cap D(g) \\ &= -1 \leq x \leq 1 \end{aligned}$$

(b)

$$\begin{aligned} f - g &= \sqrt{x+1} - \sqrt{1-x} \\ D(f-g) &= -1 \leq x \leq 1 \end{aligned}$$

(c)

$$\begin{aligned} fg &= \sqrt{x+1} \cdot \sqrt{1-x} \\ &= \sqrt{(x+1)(1-x)} \\ &= \sqrt{1-x^2} \\ &= [-1, 1] \end{aligned}$$

(d)

$$\begin{aligned}\frac{f}{g} &= \frac{\sqrt{x+1}}{\sqrt{1-x}}\\ &= \sqrt{\frac{x+1}{1-x}}\\ &= -1 \leq x < 1,\end{aligned}$$

5.

(a) $f \circ g$

$$\begin{aligned}D_f &= \mathbb{R} - \{0\}\\ R_f &= \mathbb{R} - \{0\}\end{aligned}$$

$$\begin{aligned}D_g &= \mathbb{R} - \{-2\}\\ &= \mathbb{R} - \{0\}\end{aligned}$$

$$\begin{aligned}f \circ g &= f\left(\frac{x+1}{x+2}\right)\\ &= \left(\frac{x+1}{x+2}\right) + \frac{1}{\left(\frac{x+1}{x+2}\right)}\\ &= \left(\frac{x+1}{x+2}\right) + \left(\frac{x+2}{x+1}\right)\\ &= \frac{(x+1)(x+1) + (x+2)(x+2)}{(x+2)(x+1)}\\ &= \frac{(x+1)^2 + (x+2)^2}{(x+2)(x+1)}\\ &= \frac{x^2 + 2x + 1 + x^2 + 4x + 2}{x^2 + 3x + 2}\\ &= \frac{2x^2 + 6x + 3}{x^2 + 3x + 2}\end{aligned}$$

i. $D_{f \circ g}$
   A. $x \neq -2, -1$
   B. $D_{f \circ g} = \mathbb{R} - \{-2, -1, 0\}$

ii. $R_{f \circ g}$
   A. $R_{f \circ g} = \mathbb{R} - [-2, 2]$

(b) $g \circ f$

$$
\begin{aligned}
g\left(x+\frac{1}{x}\right) &= \frac{x+\frac{1}{x}+1}{x+\frac{1}{x}+2} \\
&= \frac{\frac{x^2+1+x}{x}}{\frac{x^2+1+2x}{x}} \\
&= \frac{x^2+1+x}{x^2+1+2x}
\end{aligned}
$$

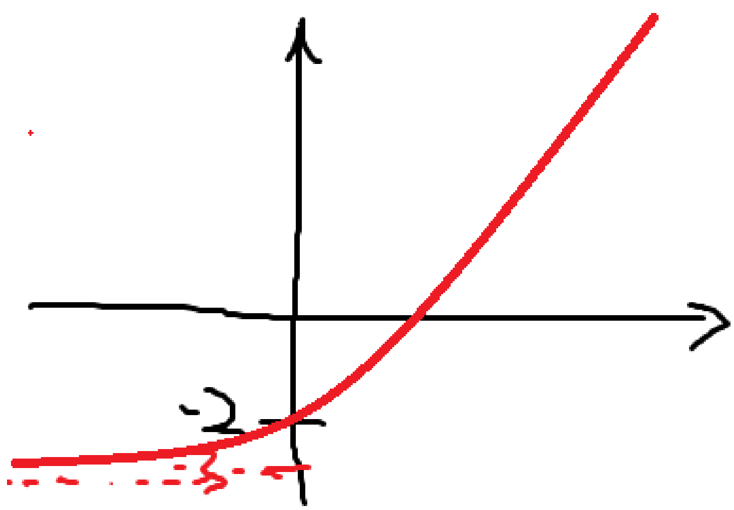
i. $D_{g\circ f} = \mathbb{R} - \{-1, 0\}$, note: 0 is from $D_f$and D_g.

6.

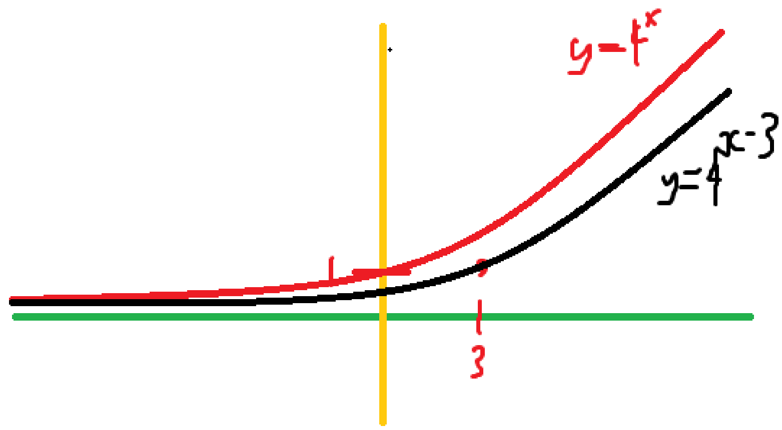
$$
\begin{aligned}
f \circ g \circ h &= f\left(g\left(h\left(x\right)\right)\right) \\
&= f\left(cos\left(\sqrt{x+3}\right)\right) \\
&= \frac{2}{cos\left(\sqrt{x+3}\right)+1}
\end{aligned}
$$

7.

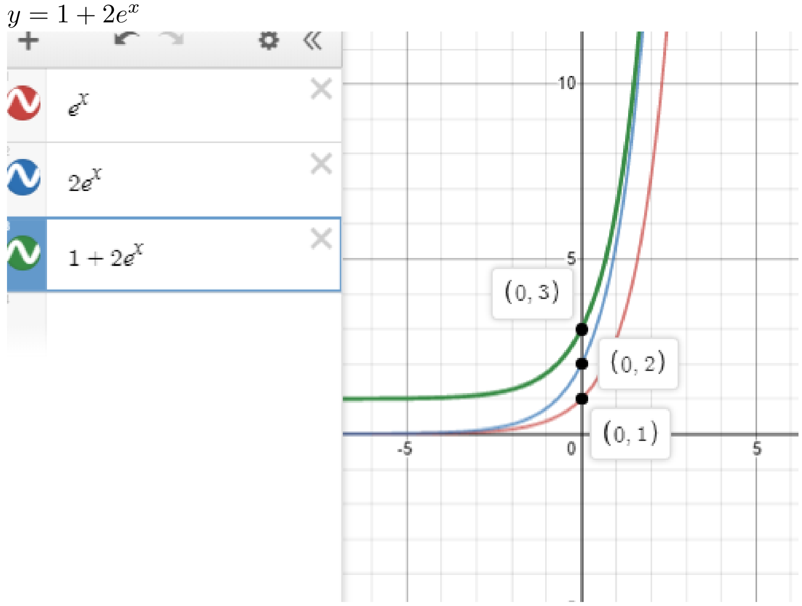
(a) $y = 4^x - 3$



(b) $y = 4^{x-3}$



(c) $y = 1 + 2e^x$



8.

9.

(a) $f(x) = \sqrt{10 - 3x}$, let $y = f(x)$

$$\begin{aligned} y &= 10 - 3x \\ y^2 &= 10 - 3x \\ y^2 - 10 &= -3x \\ y^2 - 10 &= -3x \\ x &= \frac{10 - y^2}{3} \\ f^{-1}(x) &= \frac{10 - x^2}{3} \end{aligned}$$

(b) $y = \ln(x+3)$, let $y = f(x)$

$$\begin{aligned} y &= \ln(x+3) \\ e^y &= x + 3 \\ x &= e^y - 3 \\ f^{-1}(x) &= e^x - 3 \end{aligned}$$

(c) $f(x) = e^{x^3}$, let $y = f(x)$

$$\begin{aligned} y &= e^{x^3} \\ \ln y &= x^3 \\ \sqrt[3]{\ln y} &= x \\ f^{-1}(x) &= \frac{1}{3} \ln y \end{aligned}$$

10.

(a) $log(x+5)$

i. Start with $log(x)$

ii. Shift the graph horizontally, 5 units to the right/"earlier" to find $log(x+5)$

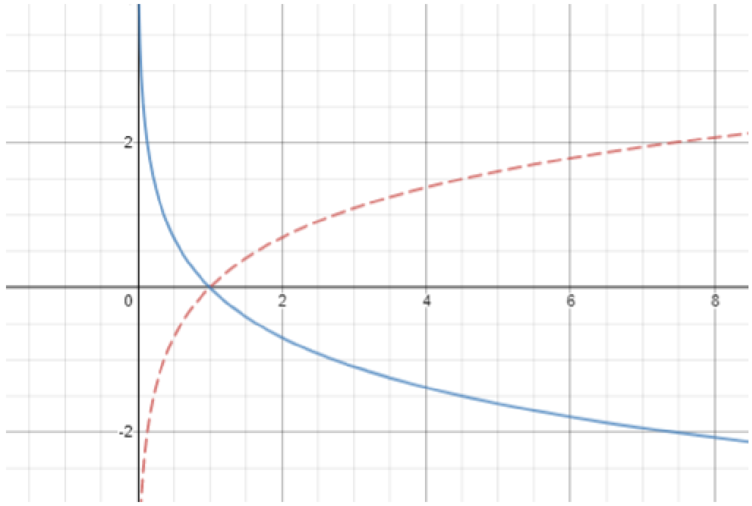
iii. Graph

(b) $y = -\ln(x)$

i. Start with $\ln(x)$

ii. Invert the graph with respect to the y-axis to arrive at $-\ln(x)$

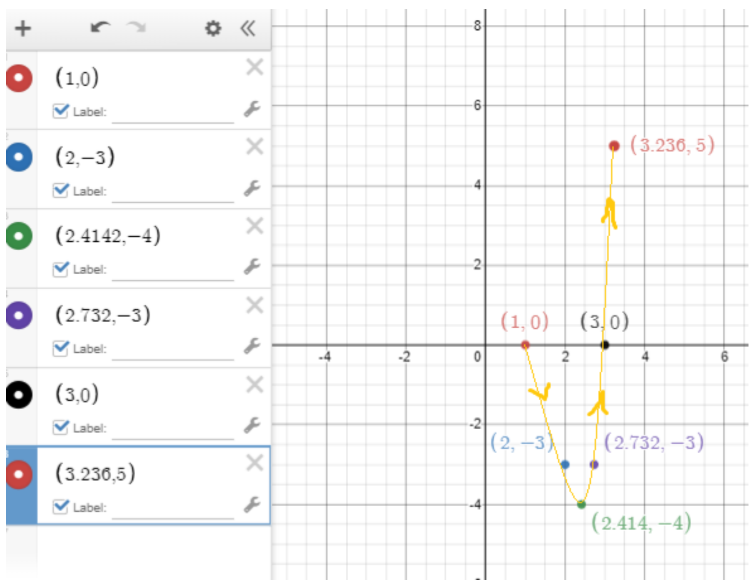
iii. Graph

iv. 

11.

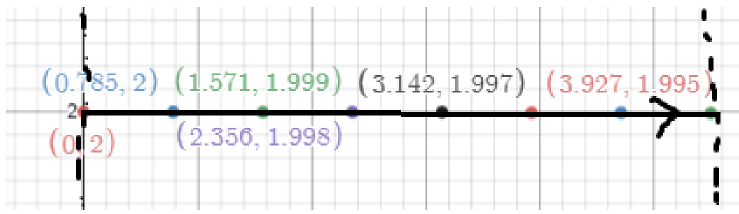
(a)

t	0	1	2	3	4	5
$x = 1 + \sqrt{t}$	1	2	2.4142	2.732	3	3.236
$y = t^2 - 4t$	0	-3	-4	-3	0	5

i. 

(b)

t	0	$\frac{1}{4}\pi$	$\frac{1}{2}\pi$	$\frac{3}{4}\pi$	π	$\frac{5}{4}\pi$	$\frac{3}{2}\pi$	$\frac{7}{4}\pi$	2π
$x = 2\cos t$	0	0.7853	1.5707	2.3561	3.1415	3.9269	4.7123	5.4977	6.2831
$y = t - \cos t$	2	1.9998	1.9992	1.9983	1.9969	1.9953	1.9932	1.9907	1.9879

i. 

12.

(a)

$$\begin{aligned} x &= 2t + 4 \\ x - 4 &= 2t \\ t &= \frac{x-4}{2} \end{aligned}$$

$$\begin{aligned} y &= t - 1 \\ &= \left(\frac{x-4}{2}\right) - 1 \\ &= \frac{x}{2} - 2 - 1 \\ y &= \frac{1}{2}x - 3 \end{aligned}$$

(b)

$$\begin{aligned} x &= 4\cos\theta \\ \theta &= \cos^{-1}\left(\frac{1}{4}x\right) \end{aligned}$$

$$\begin{aligned} y &= 5\sin\theta \\ &= 5\sin\left(cos^{-1}\left(\frac{1}{4}x\right)\right) \\ \boldsymbol{y} &\boldsymbol{= 5\tan\left(\frac{1}{4}x\right)} \end{aligned}$$

(c)

$$\begin{aligned} x &= \ln t \\ t &= e^x \end{aligned}$$

$$\begin{aligned} y &= \sqrt{t} \\ &= \sqrt{e^x} \\ \boldsymbol{y} &\boldsymbol{= e^{\frac{x}{2}}} \end{aligned}$$