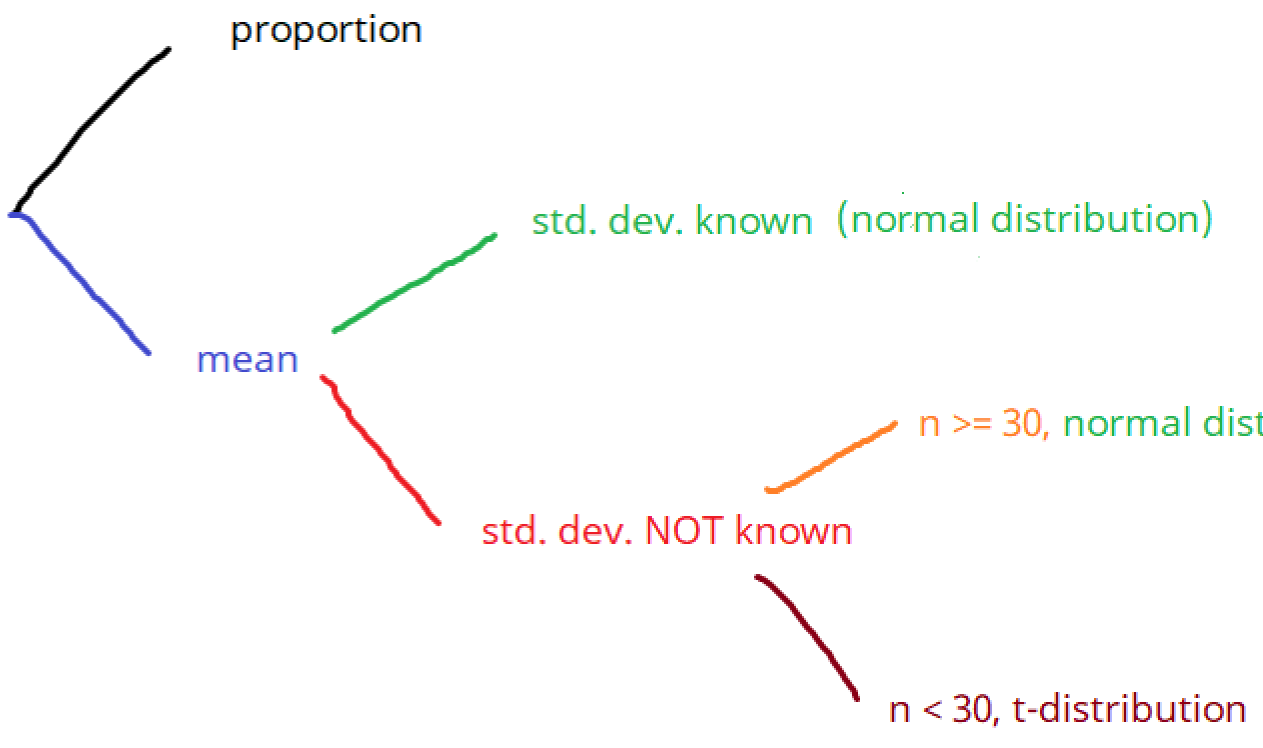
STAT II - T5: Estimation of Mean and Proportion

August 27, 2019

1 Notes (in tutorial)



2 Answers

1. An electrical firm manufactures light bulbs that have a length of life that is approximately normally distributed with a standard deviation of 40 hours. If a random sample of 30 bulbs has an average life of 780 hours, find a 96% confidence interval for the population mean of all bulbs produced by this firm.

   (a) **Answer**

      i. Check if it satisfy the conditions, where it is a random sample, σ given, and it is either normal or satisfy Central Limit Theorem. YES!

      ii. Find the critical value, $Z_{\frac{\alpha}{2}}$

      $$Z_{\frac{\alpha}{2}} = 2.0537$$

      iii. Find the margin of error, E

      $$Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} = 2.0537 * \frac{40}{\sqrt{30}}$$
      $$= 14.998$$

      iv. Find the confidence interval

      $$\mu - E < \mu < \mu + E$$

      $$780 - 14.998 < \mu < 780 + 14.998$$

      $$765.002 < \mu < 794.998$$

      v. Interpretation

         A. I am not sure what is the average life of all the light bulbs. But I am 96% confident that it is between 765.002 hours and 794.998 hours.

2. A sample of 50 college students showed mean height of 167.16 cm and standard deviation of 6.86 cm.

   (a) Estimate the mean height of all college students.

      i. **Answer (Point estimate)**

      $$167.16cm$$

   (b) Construct a 98% confidence interval for the mean height of all college students.

      i. **Answer**

         A. Check if it satisfy the conditions, where it is a random sample, σ given, and it is either normal or satisfy Central Limit Theorem. YES!

B. Find the critical value, $Z_{\frac{\alpha}{2}}$

$$Z_{\frac{\alpha}{2}} = 2.3263$$

C. Find the margin of error, E

$$\sigma = 6.86, n = 50, \bar{X} = 167.16$$

$$\begin{aligned} Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} &= 2.3263 * \frac{6.86}{\sqrt{50}} \\ &= 2.257 \end{aligned}$$

D. Find the confidence interval

$$\mu - E < \mu < \mu + E$$

$$167.16 - 2.257 < \mu < 167.16 + 2.257$$

$$164.903cm < \mu < 169.417cm$$

E. Interpretation

F. I am not sure what the mean height all the college student is, but I am 98% sure that it falls in between 164.903cm and 169.417cm

3. A **random sample** of eight cigarettes of a certain brand has average nicotine content of 18.6 milligrams and a **standard deviation of 2.4 milligrams**. Construct a 99% confidence interval for the true average nicotine content of this particular brand of cigarettes, **assuming an approximately normal distribution**.

   (a) **Answer**

      i. Check if it satisfy the conditions, where it is a random sample, σ given, and it is either normal or satisfy Central Limit Theorem. YES!

      ii. Find the critical value, $Z_{\frac{\alpha}{2}}$, from the Normal Percentile table, where $\frac{a}{2} = 0.005$

$$Z_{\frac{\alpha}{2}} = 2.5758$$

      iii. Find the margin of error, E

$$\sigma = 2.4, n = 8, \bar{X} = 18.6$$

$$\begin{aligned} Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}} &= 2.5758 * \frac{2.4}{\sqrt{8}} \\ &= 2.186 \end{aligned}$$

iv. Find the confidence interval

$$\mu - E < \mu < \mu + E$$

$$18.6 - 2.186 < \mu < 18.6 + 2.186$$

$$16.414mg < \mu < 20.786mg$$

v. Interpretation

A. I am not sure what is the average nicotine content of all the cigarettes of a certain brand. But I am 99% sure, that it falls within the range of 16.414mg and 20.786mg.

4. A machine is producing metal pieces that are cylindrical in shape. A **sample** of pieces is taken and the diameters are 1.01, 0.97, 1.03, 1.04, 0.99, 0.98, 0.99, 1.01 and 1.03 inches. Find a 99% confidence interval for the mean diameter of all pieces from this machine, **assuming a normal distribution.**

(a) Note: **Trick question.** This kind will frequently come and bite you in the exams, so be very careful (I got bitten and lost around 14/100%). The question says that the population is approximately normal distribution, but didn't mention that the sample is normally distributed. Therefore, you still **cannot use Z-score.**

(b) **Answer**

i. Check if it satisfy the conditions, where it is a random sample (since not stated, assuming random), σ given, and it is either normal or satisfy Central Limit Theorem. **Everything except σ is not given, therefore, we are using $t - distribution$.**

ii. Find the critical value, $t_{\frac{a}{2};n-1}$, from the t-distribution table, where $\frac{a}{2} = 0.005$

$$\begin{aligned} t_{\frac{a}{2};9-1} &= t_{0.005;8} \\ &= 3.355 \end{aligned}$$

iii. Find the margin of error, E

$$n = 9$$

$$\begin{aligned} \bar{X} &= \frac{\sum x}{N} \\ &= \frac{9.05}{9} \\ &= 1.0056 \end{aligned}$$

$$
\begin{aligned}
S &= \sqrt{\frac{\sum X^2 - \frac{(\sum \bar{X})^2}{N}}{N-1}} \\
&= \sqrt{\frac{\sum X^2 - \frac{(\sum \bar{X})^2}{N}}{N-1}} \\
&= \sqrt{\frac{9.1051 - \frac{9.05^2}{9}}{8}} \\
&= 0.0246
\end{aligned}
$$

$$
\begin{aligned}
E &= t_{\frac{\alpha}{2};n-1}\frac{S}{\sqrt{n}} \\
&= 3.355 * \frac{0.0246}{\sqrt{9}} \\
&= 0.0275
\end{aligned}
$$

iv. Find the confidence interval

$$\mu - E < \mu < \mu + E$$

$$1.0056 - 0.0275 < \mu < 1.0056 + 0.0275$$

$$\boldsymbol{0.9781 inch < \mu < 1.0331 inch}$$

v. Interpretation

A. I am not sure what is the average diameter of all the cylindrical metal pieces produced by the machine. But I am 99% sure, that it falls within the range of 0.9781 inches, and 1.0331 inches.

5. According to a survey of 1500 adults conducted by the National Sleep Foundation, 1125 of them said that they have symptoms of sleep problems such as frequent walking during the night or snoring.

(a) What is the **point estimate** of the population proportion?

i. **Answer**

$$
\begin{aligned}
\hat{P} &= \frac{1125}{1500} \\
\boldsymbol{\hat{P}} &= \mathbf{0.75}
\end{aligned}
$$

(b) Find a 97% confidence interval for the percentage of all adults who have such symptoms.

i. **Answer**

A. Find $\hat{p}, \hat{q}, n$

$$\boldsymbol{\hat{p} = 0.75,}$$
$$\hat{q} = 1 - \hat{p}$$
$$\boldsymbol{\hat{q} = 0.25}$$
$$\boldsymbol{n = 1500}$$

B. Find the critical value, $Z_{\frac{\alpha}{2}}$, from the Normal Percentile table, where $\frac{a}{2} = 0.015$

$$Z_{\frac{\alpha}{2}} = 2.1701$$

C. Find the margin of error, E

$$\begin{aligned} E &= Z_{\frac{\alpha}{2}} * \sqrt{\frac{\hat{p}\hat{q}}{n}} \\ &= 2.1701 * \sqrt{\frac{0.75 * 0.25}{1500}} \\ &= 0.0243 \end{aligned}$$

D. Find the confidence interval

$$\hat{p} - E < p < \hat{p} + E$$
$$0.75 - 0.0243 < \mu < 0.75 + 0.0243$$

$$0.7257 < \mu < 0.7743$$
$$\mathbf{72.57\% < \boldsymbol{\mu} < 77.43\%}$$

E. Interpretation

F. Layman Explanation: I am not sure what is the average proportion of adults having sleep problems. But I am 97% sure, that it falls within the range of 72.57%, and 77.43%.

G. NOTE: The above is NOT the correct way to describe (according to our lecturer). It should be described as: **A 97% confidence interval is 72.57% < p < 77.43%**.

6. A large corporation is concerned about the declining quality of medical services provided by their group health insurance. A **random sample** of 100 office visits by employees of this corporation to primary care physicians **last year** found that the doctors spent an **average of 19 minutes** with each patient. **This year a random sample of 108** such visits showed that doctors spent an **average of 15.5 minutes** with each patient. Assume that the **population standard deviations are 2.7 and 2.1 minutes** respectively. Construct a **94% confidence interval** for the **difference between the two population means** for these two years.

(a) **Answer**

i. Check if it matches all conditions

A. RANDOM sample. $CHECK!$

B. σ/ population standard deviation is given. $CHECK!, \sigma_1 = 2.7, \sigma_2 = 2.1$

C. $n > 30$ or it is a normal distribution. $CHECK!, n_1 = 100, n_2 = 108$

ii. Find the critical value, $Z_{\frac{\alpha}{2}}$,for $\frac{a}{2} = 0.03$

$$Z_{\frac{\alpha}{2}} = 1.8808$$

iii. Find the margin of error

$$\begin{aligned} E &= Z_{\frac{\alpha}{2}} * \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \\ &= 1.8808 * \sqrt{\frac{2.7^2}{100} + \frac{2.1^2}{108}} \\ &= 0.6343 \end{aligned}$$

iv. Find the confidence interval

$$\left(\bar{X}_1 - \bar{X}_2\right) - E < (\mu_1 - \mu_2) < \left(\bar{X}_1 - \bar{X}_2\right) + E$$
$$(19 - 15.5) - 0.6343 < (\mu_1 - \mu_2) < (19 - 15.5) + 0.6343$$
$$2.8657 < (\mu_1 - \mu_2) < 4.1343$$

v. Interpretation

A. I am not 100% sure what is the difference between the two population means for these two years. But I am 97% sure, that it falls within the range of 2.8657 minutes, and 4.1343 minutes.

7. To compare the starting salaries of graduates majoring in education and accounting, **random samples of 50** recent graduates jin each major were selected and the following information was obtained:

Major	Mean	Standard Deviation
Education	RM 2,555.40	RM 222.50
Accounting	RM 2,334.80	RM 237.50

(a) Find a point estimate for the difference in the average starting salaries of all graduates majoring in education and accounting.

$$\begin{aligned} \bar{X}_1 - \bar{X}_2 &= 2555.40 - 2334.80 \\ &= \boldsymbol{RM 220.6} \end{aligned}$$

(b) Construct a 95% confidence interval for $\mu_1 - \mu_2$, the difference in the average starting salaries.

i. **Answer**

A. Check if it matches all conditions

B. RANDOM sample. $CHECK!$

C. σ/ population standard deviation is given. $CHECK!, \sigma_1 = 222.50, \sigma_2 = 237.50$

D. $n > 30$ or it is a normal distribution. $CHECK!, n_1 = 50, n_2 = 50$

E. Find the critical value, $Z_{\frac{\alpha}{2}}$,for $\frac{\alpha}{2} = 0.025$

$$Z_{\frac{\alpha}{2}} = 1.9600$$

F. Find the margin of error

$$E = Z_{\frac{\alpha}{2}} * \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$
$$= 1.9600 * \sqrt{\frac{222.50^2}{50} + \frac{237.50^2}{50}}$$
$$\boldsymbol{E = 90.208}$$

G. Find the confidence interval

$$\left(\bar{X}_1 - \bar{X}_2\right) - E < (\mu_1 - \mu_2) < \left(\bar{X}_1 - \bar{X}_2\right) + E$$

$$220.6 - 90.208 < (\mu_1 - \mu_2) < 220.6 + 90.208$$

$$\boldsymbol{RM130.392 < (\mu_1 - \mu_2) < RM310.808}$$

H. Interpretation

I. I am not 100% sure what is the difference between the two population means for starting salaries of graduates majoring in education and accounting. But I am 95% sure, that it falls within the range of RM130.392, and RM310.808.

8. A sample of 14 cans of Brand A diet soda gave the mean number of calories of 23 per can with a standard deviation of 3 calories. Another sample of 16 cans of Brand B diet soda gave the mean number of calories of 25 per can with a standard deviation of 4 calories. Assume that the calories per can of diet soda are **normally distributed** for both brands with **equal standard deviations**. Find the 98% confidence interval for the d**ifference in the mean number of calories for these two brands of diet soda**.

(a) **Answer**

i. Check the conditions

A. RANDOM sample. $CHECK$!

B. σ/ population standard deviation is given. $CHECK$!,NOT given, therefore, it is $t - distribution$. But mentioned that **both** population standard deviation is the same. Therefore, we also need to calculate S_p

ii. Find the critical value, $t_{\frac{a}{2};n_A+n_B-2}$,for $\frac{a}{2} = 0.01$

$$\begin{aligned} t_{0.01;14+16-2}, &= t_{0.01;28} \\ &= 2.467 \end{aligned}$$

iii. Find the pooled variance, σ_p^2

$$\begin{aligned} S_A^2 &= 3^2 = 9 \\ S_B^2 &= 4^2 = 16 \end{aligned}$$

$$\begin{aligned} S_p &= \sqrt{\frac{(n_A - 1)\, S_A^2 + (n_B - 1)\, S_B^2}{n_A + n_B - 2}} \\ &= \sqrt{\frac{(14-1)\, 9 + (16-1)\, 16}{14 + 16 - 2}} \\ &= \sqrt{\boldsymbol{12.75 calories^2}} \\ &= 3.571 calories \end{aligned}$$

iv. Find the margin of error

$$\begin{aligned} E &= t_{\frac{a}{2};n_A+n_B-2} * S_p * \sqrt{\frac{1}{n_A} + \frac{1}{n_B}} \\ &= 2.467 * 3.571 * \sqrt{\frac{1}{14} + \frac{1}{16}} \\ \boldsymbol{E} &\boldsymbol{= 3.224 calories} \end{aligned}$$

v. Find the confidence interval

$$\left(\bar{X}_A - \bar{X}_B\right) - E < (\mu_A - \mu_B) < \left(\bar{X}_A - \bar{X}_B\right) + E$$

$$(23 - 25) - 3.224 < (\mu_A - \mu_B) < (23 - 25) + 3.224$$

$$\boldsymbol{-5.224 calories < (\mu_A - \mu_B) < 1.224 calories}$$

vi. Interpretation

A. I am not 100% sure what is the difference between the two population means for **number of calories**. But I am 98% sure, that it falls within the range of -5.224calories, and 1.224calories.

9. Ten soldiers were selected at random from each of two companies to participate in a rifle-shooting competition. Their scores are listed in the table below.

Company 1	72	29	62	60	68	59	61	73	38	48
Company 2	75	43	63	63	61	72	73	82	47	43

Assume that the scores for both company having **equal variances**. Construct a 90% confidence interval for the **difference between the mean scores** for the two companies.

(a) **Answer**

i. Check the conditions

A. RANDOM sample. $CHECK$!

B. σ/ population standard deviation is given. $CHECK$!,**NOT given**, therefore, it is $t - distribution$. But mentioned that **both** population's variance is the same, but are unknown. Therefore, we also need to calculate S_p.

ii. Find the critical value, $t_{\frac{a}{2};n_A+n_B-2}$,for $\frac{a}{2} = 0.05$

$$t_{0.05;10+10-2}, = t_{0.05;18}$$
$$t_{0.05;10+10-2} = 1.734$$

iii. Find the pooled variance, σ_p^2

$$S_A^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1} = 14.461^2 = 209.12$$
$$S_B^2 = \frac{\sum x^2 - \frac{(\sum x)^2}{n}}{n-1} = 13.903^2 = 193.29$$

$$S_p = \sqrt{\frac{(n_A - 1)\, S_A^2 + (n_B - 1)\, S_B^2}{n_A + n_B - 2}}$$
$$= \sqrt{\frac{9 * 209.12 + 9 * 193.29}{18}}$$
$$\boldsymbol{S_p = 14.185 calories}$$

(b) Find the margin of error

$$E = t_{0.05;10+10-2} * S_p * \sqrt{\frac{1}{n_A} + \frac{1}{n_B}}$$
$$= 1.734 * 14.185 * \sqrt{\frac{1}{10} + \frac{1}{10}}$$
$$\boldsymbol{E = 11.00 calories}$$

i. Find the confidence interval

$$\bar{X}_A = \frac{\sum x}{n} = 57$$
$$\bar{X}_B = \frac{\sum x}{n} = 62.2$$

$$(57 - 62.2) - E < (\mu_A - \mu_B) < (57 - 62.2) + E$$
$$(57 - 62.2) - 11.00 < (\mu_A - \mu_B) < (57 - 62.2) + 11.00$$
$$\boldsymbol{-16.2 < (\mu_A - \mu_B) < 5.8}$$

ii. Interpretation

A. I am not 100% sure what is the difference between the two population means for **score**. But I am 90% sure, that it falls within the range of -16.2 marks, and 5.8 marks.

10. A researcher wanted to find the effect of a special diet on systolic blood pressure. He selected a sample of seven adults and put them on this dietary plan for three months. The following table gives the systolic blood pressures of these seven adults before and after the completion of this plan.
Before **210 180 195 220 231 199 224**
After **193 186 186 223 220 183 233**
Construct a 95% confidence interval for the **mean reduction** in the systolic blood pressures due to this special dietary plan for all adults. Assume that the population of paired differences is **approximately normally distributed.**

(a) **Answer**

i. Check the conditions

A. RANDOM SAMPLE

B. POP. STD DEV not given

C. T-distribution, so no need to worry about n

D. Is it before - answer or vice versa? $Before - After$, reason: Because reduction is considered "good", and a "downslope".

ii. Find the critical value, $t_{\frac{a}{2};n-1}$,for $\frac{a}{2} = 0.025$

$$t_{0.025;6} = 2.447$$

iii. Find the margin of error

$$E = t_{\frac{a}{2};n-1} * \frac{S_D}{\sqrt{n}}$$

A. Find the S_D,

Before, X1	After, X2	Paired Difference, D = X1-X2	D^2
210	193	17	
180	186	-6	
195	186	9	
220	223	-3	
231	220	11	
199	183	16	
224	233	-9	
SUM		35	

$$S_D = \sqrt{\frac{\sum D_i^2 - \frac{(\sum D_i)^2}{n}}{n-1}}$$
$$= 10.786$$

B.

$$E = 2.447 * \frac{10.786}{\sqrt{7}}$$
$$= 9.976$$

iv. Find the confidence interval

A. $\bar{D} = 5$

$$\bar{D} - E < \mu_{\bar{D}} < \bar{D} + E$$

$$5 - 9.976 < \mu_D < 5 + 9.976$$
$$-4.976 < \mu_D < 14.976$$

v. Inference

A. The 95% confidence interval for $\mu_1 - \mu_2 = -4.976 < \mu_D < 14.976$

11. A certain geneticist is interested in the proportion of males and females in the population that have a certain minor blood disorder. In a random sample of 1000 males 250 are found to be afflicted, whereas 275 of 1000 females tested appear to have the disorder.

(a) Estimate the difference in the true proportion of males and females that have the blood disorder.

i. Estimated difference:

A. $\frac{250}{1000} - \frac{275}{1000} = -\frac{25}{1000}$

(b) Compute a 95% confidence interval for the difference between the proportion of males and females that have the blood disorder.

i. 95% C.I.

A. $\hat{p}_m = 0.25, \hat{q}_m = 0.75, \hat{p}_f = 0.275, \hat{q}_f = 0.725, n = 1000$

B. Calculate critical value

C. $Z_{\frac{0.05}{2}} = Z_{0.025} = 1.9600$

D. Calculate margin of error

$$E = 1.9600 * \sqrt{\frac{(0.25)\,(0.75)}{1000} + \frac{(0.275)\,(1 - 0.275)}{1000}}$$
$$= 0.03855$$

E. Find the confidence interval

F. $\hat{p}_m - \hat{p}_f = -\frac{25}{1000}$

$$-\frac{25}{1000} - 0.03855 < p_m - p_f < -\frac{25}{1000} + 0.03855$$
$$-\frac{25}{1000} - 0.03855 < p_m - p_f < -\frac{25}{1000} + 0.03855$$
$$-0.06355 < p_m - p_f < 0.01355$$

12. A marketing researcher wants to find a 95% confidence interval for the mean amount that visitors to a theme park spend per person per day. He knows that the standard deviation of the amounts spent per person per day by all visitors to this park is RM11. How large a sample should the researcher select so that the estimate will be within RM2 of the population mean?

$$\alpha = 0.05, \sigma = 11$$

   (a) Confidence interval

$$\bar{x} - 2 < \mu < \bar{x} + 2$$

   (b) Find the critical value

$$\alpha = 0.05$$

$$\begin{aligned} Z_{\frac{0.05}{2}} &= Z_{0.025} \\ &= 1.9600 \end{aligned}$$

   (c) Find the margin of error, which corresponds to 2

$$\begin{aligned} E &= 2 \\ 1.9600 * \frac{11}{\sqrt{n}} &= 2 \\ \frac{11}{\sqrt{n}} &= \frac{2}{1.9600} \\ n &= \left(\frac{1.9600}{2} * 11\right)^2 \\ &= 116.20 \\ &\approx 117 \end{aligned}$$

   (d) Notes

      i. We never round down n, because if we do, then the margin of error will exceed 2, the smaller the margin of error, the better.

13. Johnny's Pizza guarantees all pizza deliveries within 30 minutes of the placement of orders. A researcher wants to estimate the proportion of all pizzas delivered within 30 minutes by Johnny's. The 99% confidence interval for the population proportion has a margin of error to within 0.02.

   (a) What is the most conservative estimate of the sample size?

      i. The most conservation estimate is when $\hat{p}$ and $\hat{q}$are both $= 0.5$, which is the largest yield possible for standard error.

ii. $p = 0.5, q = 0.5$

iii. Critical value at $Z_{\frac{0.01}{2}} = Z_{0.005} = 2.5758$

iv.

$$
\begin{aligned}
0.02 &= 2.5758 * \sqrt{\frac{(0.5)(0.5)}{n}} \\
\frac{0.02}{2.5758} &= \sqrt{\frac{(0.5)(0.5)}{n}} \\
\frac{\left(\frac{0.02}{2.5758}\right)^2}{0.25} &= \frac{1}{n} \\
n &= \frac{0.25}{\left(\frac{0.02}{2.5758}\right)^2} \\
&= 4146.71 \\
\boldsymbol{n} &\boldsymbol{\approx} \mathbf{4147}
\end{aligned}
$$

(b) Assume that a preliminary study has shown that 93% of all Johnny's pizzas are delivered within 30 minutes. How large should the sample size?

i. Let p be the proportions of pizza delivered successfully

ii.

$$p = 0.93, q = 0.07$$

$$
\begin{aligned}
0.02 &= 2.5758 * \sqrt{\frac{(0.93)(0.07)}{n}} \\
n &= \frac{(0.93)(0.07)}{\left(\frac{0.02}{2.5758}\right)^2} \\
&= 1079.80 \\
\boldsymbol{n} &\boldsymbol{\approx} \mathbf{1080}
\end{aligned}
$$

14. A small insurance company has 1861 life insurance policyholders. A random sample of 100 life insurance policyholders showed that the mean premium they pay on their life insurance policies is RM685 per year with a standard deviation of RM71. Assuming that the life insurance policy premiums for all life insurance policyholders have a normal distribution, construct a 92% confidence interval for the population mean.

$$N = 1861, n = 100, \bar{X} = 685, S = 71, \alpha = 0.08$$

$$Z_{\frac{\alpha}{2}} = Z_{0.04} = 2.6521$$

$$\begin{aligned}E &= Z_{0.04} * \frac{S}{\sqrt{n}} * \sqrt{\frac{N-n}{N-1}} \\ &= 1.7507 * \frac{71}{\sqrt{100}} * \sqrt{\frac{1861-100}{1861-1}} \\ &= 12.09\end{aligned}$$

92% confidence interval

$$\bar{X} - E < \mu < \bar{X} + E$$
$$685 - 12.09 < \mu < 685 + 12.09$$

$$672.91 < \mu < 697.09$$

15. In a large metropolitan area in which a total of 800 gasoline service stations are located, a random sample of 36 gasoline service stations, 20 of the stations carry a particular nationally advertised brand of oil. Find a 95% confidence interval estimate for

    (a) the proportion of all stations in the area which carry the oil,

$$N = 800, n = 36, x = 20, \alpha = 0.05, \hat{p} = \frac{20}{36} = \frac{5}{9}.\hat{q} = \frac{4}{9}$$

$$\begin{aligned}Z_{\frac{\alpha}{2}} &= Z_{0.025} \\ &= 1.96\end{aligned}$$

$$\begin{aligned}E &= Z_{\frac{\alpha}{2}} * \sqrt{\frac{\hat{p}\hat{q}}{n}} * \sqrt{\frac{N-n}{N-1}} \\ &= 1.96 * \sqrt{\frac{\frac{5}{9} * \frac{4}{9}}{36}} * \sqrt{\frac{800-36}{800-1}} \\ &= 0.1587\end{aligned}$$

$$\hat{p} - E < p < \hat{p} + E$$

$$\frac{5}{9} - 0.1587 < p < \frac{5}{9} + 0.1587$$
$$0.397 < p < 0.714$$

(b) the total number of service stations in the area which carry the oil.

$$\begin{gathered}0.397 * 800 < Service\,Stations < 0.714 * 800 \\ 317.6 < Service\,Stations < 571.2 \\ 318 < Service\,Stations < 572\end{gathered}$$