Tutorial 2 - Discrete Probability Distribution

August 27, 2019

1.

(a)

$$P(x; 25) = \frac{1}{25}$$

(b)

$$\begin{aligned}\mu &= \frac{1}{25} * (1 + 2 + 3... + 25) \\ &= \frac{1}{25} * \frac{25 * (25 + 1)}{2} \\ &= \frac{325}{25} \\ &= 13\end{aligned}$$

$$\sigma^2 = 52$$

(c)

$$\begin{aligned}Prime &= \{2, 3, 5, 7, 11, 13, 17, 19, 23\} \\ &= \frac{9}{25}\end{aligned}$$

2. (Geometric) Let X be the number of applicants interviewed until the first applicant with advanced ytsininh in vompuyrt ptohtsmminh. $X \sim Geometric(p = 0.2)$

(a)

i. $P(X = 5) = g(5; 0.2)$

$$\begin{aligned}g(5; 0.2) &= (0.8)^4 (0.2) \\ &= 0.08192\end{aligned}$$

ii. $P(X < 5) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4)$

$$\begin{aligned}P(X = 5) &= 0.2 + 0.8 * 0.2 + 0.8^2 * 0.2 + 0.8^3 * 0.2 \\ &= 0.5904\end{aligned}$$

(b)

i. Mean

$$\mu = \frac{1}{0.2}$$
$$= 5 tries$$

ii. Median

$$\sigma^2 = \frac{1-p}{p^2}$$
$$= \frac{1-0.2}{0.2^2}$$
$$= 20 tries^2$$

3.

(a)

$$P\left(X \geq 3\right) = P\left(X = 3\right) + P\left(X = 4\right)$$
$$=^4 C_3 * 0.4^3 * 0.6 + 0.4^4$$
$$= 0.1792$$

(b)

$$P\left(X = 0\right) =^4 C_0 * 0.6^4$$
$$= 0.1296$$

(c)

$$P\left(X = 1, 2\right) = P\left(X = 1\right) + P\left(X = 2\right)$$
$$=^4 C_1 * 0.4^1 * 0.6^3 +^4 C_2 * 0.4^2 * 0.6^2$$
$$= 0.6912$$

4. $p = 0.5$

(a)

$$P\left(X \leq 2\right) = P\left(X = 0\right) + P\left(X = 1\right) + P\left(X = 2\right)$$
$$= \binom{10}{0} 0.5^{10} + \binom{10}{1} 0.5^9 * 0.5 + \binom{10}{2} 0.5^8 * 0.5^2$$
$$= 0.0546875$$

(b)

$$P\left(X > 3\right) = 1 - P\left(X = 0\right) - P\left(X = 1\right) - P\left(X = 2\right) - P\left(X = 3\right)$$
$$= 1 - 0.0546875 - \binom{10}{3} 0.5^{10}$$
$$= 0.9453125 - 0.1171875$$
$$= 0.828125$$

(c)

$$\begin{aligned}P\left(4<x<8\right) &= P\left(X=5\right)+P\left(X=6\right)+P\left(X=7\right)\\ &= \binom{10}{5}0.5^{10}+\binom{10}{6}0.5^{10}+\binom{10}{7}0.5^{10}\\ &= 252*0.5^{10}+210*0.5^{10}+120*0.5^{10}\\ &= 0.5684\end{aligned}$$

5.

(a)

$$p=0.3,q=0.7,n=20$$

$$\begin{aligned}P\left(3\leq X\leq 6\right) &= P\left(X=3\right)+P\left(X=4\right)+P\left(X=5\right)+P\left(X=6\right)\\ &= \binom{20}{3}\cdot 0.3^{3}\cdot 0.7^{17}+\binom{20}{4}\cdot 0.3^{4}\cdot 0.7^{16}+\binom{20}{5}\cdot 0.3^{5}\cdot 0.7^{15}+\binom{20}{6}\cdot 0.3^{6}\cdot 0.7\\ &= 0.57253\end{aligned}$$

(b)

$$\begin{aligned}\mu &= np\\ &= 20*0.3\\ &= 6customers\end{aligned}$$

$$\begin{aligned}\sigma^{2} &= np\left(1-p\right)\\ &= 6*q\\ &= 4.2customers^{2}\\ \sigma &= 2.0494customers\end{aligned}$$

(c)

$$\begin{aligned}P\left(X=6\right) &= \binom{20}{6}*0.3^{6}*0.7^{14}\\ &= 0.1916\end{aligned}$$

6. $\mu=2,\sigma^{2}=1.6$

(a)

$$\begin{aligned}2 &= np\\ 1.6 &= np\left(1-p\right)\\ &= 2\left(1-p\right)\\ 0.8 &= 1-p\\ -p &= -0.2\\ p &= 0.2\end{aligned}$$

(b)

$$2 = np$$
$$2 = 0.2n$$
$$n = 10$$

$$P(X = 6) = \binom{10}{6} * 0.2^6 * 0.8^4$$
$$= 210 * 0.2^6 * 0.8^4$$
$$= 0.005505$$

7.

(a) Let X be the number of marriages that occur in June $X \sim P_o(5), x = 0, 1, 2, 3..$

$$P(X < 3) = P_o(X = 0) + P_0(X = 1) + ... + P_o(X = 2)$$
$$= e^{-5}\left(\frac{\lambda^0}{0!} + \frac{\lambda^1}{1!} + \frac{\lambda^2}{2!}\right), where\, \lambda = 5$$
$$= 0.1246$$

(b) Let X be the number of marriages that occur in July and August $X \sim P_o(10), x = 0, 1, 2...$

$$P(X = 10) = e^{-10}\frac{\lambda^{10}}{10!}$$
$$= 0.1251$$

(c) Let X be the number of marriages that occur between the 3 months $X \sim P_o(15), x = 0, 1, 2...$

$$P(14 \leq X \leq 18) = P_o(X = 14) + P_0(X = 15) + ... + P_o(X = 18)$$
$$= e^{-15}\left(\frac{15^{14}}{14!} + \frac{15^{15}}{15!} + \frac{15^{16}}{16!} + \frac{15^{17}}{17!} + \frac{15^{18}}{18!}\right)$$
$$= 0.45625\, \boldsymbol{Double\, check : true\, answer = 0.4561}$$

8. (in test, won't tell you Poisson, but will tell you week/duration)

$$\lambda = 2$$

(a) Let X be the number of claims received per week such that $X \sim P_o(2), x = 0, 1, 2, 3....$

$$P(X > 3) = 1 - P(X = 0) - P(X = 1) - P(X = 2) - P(X = 3)$$
$$= 1 - e^{-2}\left(1 + \frac{2^1}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!}\right)$$
$$= \mathbf{0.1429}$$

(b) Let X be the number of claims received per fortnight such that $X \sim P_o(4), x = 0, 1, 2, 3...$

$$\begin{aligned} P(X \geq 2) &= 1 - P(0) - P(1) \\ &= 1 - e^{-4}(1+4) \\ &= \mathbf{0.9084} \end{aligned}$$

(c) Let X be the number of claims received per 5 days such that $X \sim P_o\left(\frac{2}{5}\right), x = 0, 1, 2, 3...$

$$\begin{aligned} P(X = 0) &= e^{-\frac{2}{5}}(1) \\ &= \mathbf{0.6703} \end{aligned}$$

9.

(a) Let X_1 be the number of cars arriving in a particular 5 minutes interval $X_1 \sim P_o(2.5), x = 1, 2, 3...$

i.

$$\begin{aligned} P(0) &= P_o(X = 0) \\ &= e^{-2.5}\left(\frac{2.5^0}{0!}\right) \\ &= 0.08208 \end{aligned}$$

ii. Let X_2 be the number of cars arriving in a particular 10 minutes interval $X_2 \sim P_o(5), x = 1, 2, 3...$

$$\begin{aligned} P(X < 3) &= P_o(X = 0) + P(X = 1) + P(X = 2) \\ &= 0.1247 \end{aligned}$$

(b) Let X_3 be the number of cars arriving in a particular 15 minutes interval $X_3 \sim P_o(7.5), x = 1, 2, 3...$

$$\begin{aligned} \mu &= \lambda \\ &= 7.5 \end{aligned}$$

$$\begin{aligned} \sigma^2 &= \lambda \\ &= 7.5 \\ \sigma &= 2.7386 \end{aligned}$$

10. $\lambda = 5$

(a) Let X be the number of vegetables a salad contains in a tossed salad. $X \sim P_o(5), x = 0, 1, 2, 3, ...$ **Note: Salad can contain no vegetables (Darn it)**

$$\begin{aligned}
P(X > 5) &= 1 - P(X = 1) - P(X = 2) - P(X = 3) - P(X = 4) - P(X = 5) \\
&= 1 - e^{-5}\left(\lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \frac{\lambda^4}{4!} + \frac{\lambda^5}{5!}\right) \\
&= 1 - e^{-5}\left(1 + 5 + \frac{5^2}{2} + \frac{5^3}{6} + \frac{5^4}{24} + \frac{5^5}{120}\right) \\
&= 0.3840
\end{aligned}$$

(b) Binomial (depends on Poisson) $X_1$is the number of days out of 4 days which the number of vegetables contained in a tossed salad is 3. $X \sim B(4, 0.3840)$

$$\begin{aligned}
P(X_1 = 3) &= ^4C_3(0.3840)^3(1 - 0.3840)^1 \\
&= 0.1395
\end{aligned}$$

(c) Geometric (depends on Poisson). $X_2$is the numbers of days until the first day with a tossed salad containing more than 5 vegetables

$$\begin{aligned}
P(X_2 = 5) &= (1 - 0.3840)^4(0.3840) \\
&= 0.0553
\end{aligned}$$

11. $p = 0.01, n = 100$. Since $n > 50$ and $np = 1 < 5$,Let X be the number of faulty light bults in a box $X \sim P_o(1), x = 1, 2, 3...$

(a)

$$\begin{aligned}
P(X = 0) &= \frac{e^{-1}\lambda^0}{0!} \\
&= 0.3679
\end{aligned}$$

(b)

$$\begin{aligned}
P(X = 2) &= \frac{e^{-1}\lambda^2}{2!} \\
&= 0.1839
\end{aligned}$$

(c)

$$\begin{aligned}
P(X > 3) &= 1 - P(X \leq 3) \\
&= 1 - P(0) - P(1) - P(2) - P(3) \\
&= 1 - 0.3679 - \frac{e^{-1}}{1!} - 0.1839 - \frac{e^{-1}}{3!} \\
&= 0.019007
\end{aligned}$$

12.

$$
\begin{aligned}
p &= 0.005\,(Defective)\\
n &= 200
\end{aligned}
$$

$$
\begin{aligned}
np &= 1 < 5\\
n &> 50
\end{aligned}
$$

$$
\begin{aligned}
P\left(X \leq 2\right) &= P\left(X = 0\right) + P\left(X = 1\right) + P\left(X = 2\right)\\
&= e^{-1}\left(1 + 1 + \frac{1}{2}\right)\\
&= 0.9197
\end{aligned}
$$