Disc. Maths - T2

November 23, 2019

1. Rewrite each statement below in "if – then" form.
   (a) I am on time for lecture if I catch the 7 am bus.
      i. If I catch the 7am bus, then I am on time for lecture.
   (b) David studies hard or he fails the examination.
      i. If David studies hard, then he passes the examination.
   (c) The program is readable only if it is well structured.
      i. If the program is well structured, then it is readable.
   (d) This door will not open unless a security code is entered.
      i. If a security code is entered, then the door will open.
   (e) 2x – 5 = 11 implies x = 8.
      i. If $x = 8$, then $2x - 5 = 11$
   (f) Having two 45° angles is a sufficient condition for this triangle to be a right triangle.
      i. If the triangle is a right triangle, then it has two 45° angles.
   (g) Solving all tutorial's questions is a necessary condition for Alan to pass this subject.
      i. If Alan solve all tutorial questions, then Alan passes the subject.
   (h) To be a citizen in this country, it is sufficient that you were born in this country.
      i. If you were born in this country, then you are a citizen in this country.
   (i) It is necessary to have a valid password to log on to the server.
      i. If you have a valid server, then you can log on to the server.

2. Using truth tables, verify the following,
   (a) $(p \wedge q) \rightarrow r = (p \rightarrow r) \wedge (q \rightarrow r)$

p	q	r	$(p \vee q)$	$(p \vee q) \rightarrow r$	$(p \rightarrow r)$	$(q \rightarrow r)$	$(p \rightarrow r) \wedge (q \rightarrow r)$
0	0	0	0	**1**	1	1	**1**
0	0	1	0	**1**	1	1	**1**
0	1	0	1	**0**	1	0	**0**
0	1	1	1	**1**	1	1	**1**
1	0	0	1	**0**	0	1	**0**
1	0	1	1	**1**	1	1	**1**
1	1	0	1	**0**	0	0	**0**
1	1	1	1	**1**	1	1	**1**

   (b) $(p \wedge (p \rightarrow q)) \rightarrow q = t$

p	q	$p \rightarrow q$	$p \wedge (p \rightarrow q)$	$(p \wedge (p \rightarrow q)) \rightarrow q$	t
0	0	1	0	**1**	**1**
0	1	1	0	**1**	**1**
1	0	0	0	**1**	**1**
1	1	1	1	**1**	**1**

   (c) $(p \rightarrow q) \wedge (\sim p \rightarrow q) \wedge \sim q = c$

p	q	$p \rightarrow q$	$\sim p$	$(\sim p \rightarrow q)$	$(p \rightarrow q) \wedge (\sim p \rightarrow q)$	$\sim q$	$(p \rightarrow q) \wedge (\sim p \rightarrow q) \wedge \sim q$	c
0	0	1	1	0	0	1	**0**	**0**
0	1	1	1	1	1	0	**0**	**0**
1	0	0	0	1	0	1	**0**	**0**
1	1	1	0	1	1	0	**0**	**0**

3. Write negations for each of the following statements.
   (a) If P is a square, then it is a rectangle.
      i. P is a square and is not a rectangle.
   (b) If the sun is shining, then I shall play tennis or swimming this afternoon.
      i. Even if the sun is shining, I shall not play tennis and not swim this afternoon.
   (c) If I am free and I am not tired, then I will go to the supermarket.
      i. Even if I am free and I am not tired, I will not go to the supermarket.
   (d) If $x = 17$ or $x = 8$, then x is prime.
      i. There are cases where $x = 17$ or $x = 8$,and x is not a prime.
4. State the converses, inverses and contrapositives for each of the following implications.
   Converse: $q \to p$
   Inverse: $\sim p \to \sim q$
   Contrapositive: $\sim q \to \sim p$
   (a) If I am late, then I will not take the train to work. $p \to q$
      i. Converse: If I will not take the train to work, then I am late.
      ii. Inverse: If I am not late, then I will take the train to work.
      iii. Contrapositive: If I will take the train to work, then I am not late.
   (b) If I have enough money, then I will buy a car and I will buy a house. $p \to (q \wedge r)$
      i. Converse: If I will buy a car and I will buy a house, then I have enough money.
      ii. Inverse: If I do not have enough money, then I will not buy a car or I will not buy a house.
      iii. Contrapositive: If I will not buy a car or I will not buy a house, then I do not have enough money.
   (c) A positive integer is a prime only if it has no divisors other than 1 and itself.
      i. Converse: If a positive integer has no divisors other than 1 and itself, then it is a prime number.
      ii. Inverse: A positive integer is not a prime only if it has divisors other than 1 and itself.
      iii. Contrapositive: If a positive integer has divisors other than 1 and itself, then it is not a prime number.
   (d) If x is non-negative, then x is positive or x is 0.
      i. Converse: If x is positive or x is 0, then x is non-negative number.
      ii. Inverse: If x is negative, then x is not positive and x is not 0.
      iii. Contrapositive: If x is not positive and x is not 0, then x is not negative.
5. "If Jim studies hard, then he will pass his final examination." Assuming that this statement is true, which of the following must also be true? $p \to q$
   (a) Jim passed his final examination implies he studies hard. $q \to p$
      i. False
   (b) Jim studied hard or he failed his final examination. $p \vee q$
      i. False
   (c) Jim will fail his final examination only if he does not study hard. $\sim q \to \sim p$
      i. True
   (d) Jim will fail his final examination unless he studied hard. $q \wedge p$
      i. False
   (e) A necessary condition for Jim to pass his final examination is that he studied hard. $q \to p$
      i. False
      ii. Note: To understand why: Go here CTRL+F "**If vs. only if**"
   (f) Studying hard is sufficient for Jim to pass his final examination. $p \to q$
      i. True
6. Given $p \to q \equiv \sim p \vee q$ and $p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$.
   Thus, rewrite the following statement form without using $\to$ or $\leftrightarrow$.

(a) $p \wedge \sim q \rightarrow r$

$$\begin{aligned}(p \wedge \sim q) \rightarrow r &= \sim (p \wedge \sim q) \vee r \\ &= \sim p \vee q \vee r\end{aligned}$$

(b) $(p \rightarrow (q \rightarrow r)) \leftrightarrow ((p \wedge q) \rightarrow r)$

$$\begin{aligned}&\equiv (\sim p \vee (q \rightarrow r)) \leftrightarrow (\sim (p \wedge q) \vee r) \\ &\equiv (\sim p \vee \sim q \vee r) \leftrightarrow (\sim (p \wedge q) \vee r) \\ &\equiv (\sim p \vee \sim q \vee r) \leftrightarrow (\sim p \vee \sim q \vee r) \\ &\equiv \sim (\sim p \vee \sim q \vee r) \vee (\sim (p \wedge q) \vee r) \wedge \sim (\sim (p \wedge q) \vee r) \vee (\sim p \vee \sim q \vee r) \\ &\equiv \sim (\sim p \vee \sim q \vee r) \vee (\sim p \vee \sim q \vee r) \wedge \sim (\sim p \vee \sim q \vee r) \vee (\sim p \vee \sim q \vee r) \\ &\equiv \sim (\sim p \vee \sim q \vee r) \vee c \vee (\sim p \vee \sim q \vee r) \\ &\equiv \sim (\sim p \vee \sim q \vee r) \vee (\sim p \vee \sim q \vee r) \vee c \\ &\equiv t \vee c \\ &\equiv t\end{aligned}$$

7. Obtain the PDNF and PCNF of each of the following expressions:

(a) $\sim (p \vee q)$

i. PDNF

$$\begin{aligned}\sim (p \vee q) &= (\sim p \wedge \sim q) \\ &= (\bar{p} \wedge \bar{q})\end{aligned}$$

ii. PCNF (Note, PDNF is all consist of 1 in truth table, PCNF is all consists of 0 is truth table, but with their parameters inverted.)

$$\sim (p \vee q) = (\bar{p} + \bar{q})\,(p + \bar{q})\,(\bar{p} + q)$$

(b) $\sim (p \wedge q)$

i. NOTE: For this question, by using DeMorgan's law, you will find PCNF first (remember, sum-of-products, conjunctive = products)

ii. PCNF

$$\begin{aligned}\sim (p \wedge q) &= \sim p \vee \sim q \\ &= (\bar{p} + \bar{q})\end{aligned}$$

iii. PDNF (Same as part A, in truth table, PCNF is all 0, PDNF is all 1. Take the rest of the terms and invert them)

$$\sim (p \wedge q) = (\bar{p}\bar{q})\,(p\bar{q})\,(\bar{p}q)$$

(c) $\sim (p \rightarrow q)$

i. PDNF

$$\begin{aligned}\sim (p \rightarrow q) &= \sim (\sim p \vee q) \\ &= p \wedge \sim q \\ &= p\bar{q}\end{aligned}$$

ii. PCNF

$$\sim (p \rightarrow q) = (p + q)\,(p + \bar{q})\,(\bar{p} + \bar{q})$$

(d) $\sim (p \leftrightarrow q)$

i. PDNF

$$\begin{aligned}\sim ((\sim p \vee q) \wedge (\sim q \vee p)) &= \sim (\sim p \vee q) \vee \sim (\sim q \vee p) \\ &= (p \wedge \sim q) \vee (q \wedge \sim p) \\ &= p\bar{q} + \bar{p}q\end{aligned}$$

ii. PCNF

A. DeMorgan-the-DeMorgan way (lol)

$$
\begin{aligned}
p\bar{q} + \bar{p}q &= \overline{\overline{p\bar{q} + \bar{p}q}} \\
&= \overline{\overline{p\bar{q}} \cdot \overline{\bar{p}q}} \\
&= \overline{\overline{p\bar{q}} \cdot \overline{\bar{p}q}} \\
&= \overline{(\bar{p} + q) \cdot (p + \bar{q})} \\
&= \overline{\bar{p}p + \bar{p}\bar{q} + pq + q\bar{q}} \\
&= (p + \bar{p})\,(p + q)\,(\bar{p} + \bar{q})\,(\bar{q} + q) \\
&= (p + q)\,(\bar{p} + \bar{q})
\end{aligned}
$$

B. The find-the-terms-not-found-in-PDNF-then-invert-the-parameters way

$$\sim ((\sim p \vee q) \wedge (\sim q \vee p)) = (p + q)\,(\bar{p} + \bar{q})$$

8. Construct a truth table for the expression $A \equiv (p \to q) \wedge (\sim p \vee r)$. Based on the truth table, write the PDNF of A, the PDNF of $\sim A$, the PCNF of A, and the PCNF of ~A.

p	q	r	$(p \to q)$	$\sim p$	$(\sim p \vee r)$	$(p \to q) \wedge (\sim p \vee r)$
0	0	0	1	1	1	1
0	0	1	1	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	0	0	0	0
1	0	1	0	0	1	0
1	1	0	1	0	0	0
1	1	1	1	0	1	1

(a) PDNF of A:

i. $\bar{p}\bar{q}\bar{r} + \bar{p}\bar{q}r + \bar{p}q\bar{r} + \bar{p}qr + pqr$

ii. PCNF of A:

A. $(\bar{p} + q + r)\,(\bar{p} + q + \bar{r})\,(\bar{p} + \bar{q} + r)$

iii. PDNF of $\sim A$:

A. $p\bar{q}\bar{r} + p\bar{q}r + pq\bar{r}$

iv. PCNF of $\sim A$:

A. $(p + q + r)\,(p + q + \bar{r})\,(p + \bar{q} + r)\,(p + \bar{q} + \bar{r})\,(\bar{p} + \bar{q} + \bar{r})$

9. Without constructing truth tables, obtain the PDNF of A, the PDNF of ~A, the PCNF of A, and the PCNF of ~A, (in any order), if the normal forms exist.

(a) $A \equiv q \wedge (p \vee \sim q)$

i. PDNF of A:

$$
\begin{aligned}
A &\equiv q\,(p + \bar{q}) \\
&\equiv pq + q\bar{q} \\
&\equiv pq + c \\
A &\equiv pq
\end{aligned}
$$

ii. PDNF of $\sim A$:

$$A \equiv p\bar{q} + \bar{p}q + \bar{p}\bar{q}$$

iii. PCNF of A:

$$A \equiv (p + q)\,(\bar{p} + q)\,(p + \bar{q})$$

iv. PCNF of $\sim A$:

$$A = (\bar{p} + \bar{q})$$

(b) $A \equiv (\sim p \vee \sim q) \to (p \leftrightarrow \sim q)$

i. PDNF of A:

$$\begin{aligned} A &\equiv \sim(\sim p \vee \sim q) \vee (p \leftrightarrow \sim q) \\ &\equiv (p \wedge q) \vee [(\sim p \vee \sim q) \wedge (q \vee p)] \\ &\equiv pq + [(\bar{p} + \bar{q})(p + q)] \\ &\equiv pq + p\bar{p} + \bar{p}q + p\bar{q} + \bar{q}q \\ &\equiv pq + c + \bar{p}q + p\bar{q} + c \\ &\equiv pq + \bar{p}q + p\bar{q} \end{aligned}$$

ii. PDNF of $\sim A$:

$$\sim A \equiv \bar{p}\bar{q}$$

iii. PCNF of A:

$$A \equiv pq$$

iv. PCNF of $\sim A$:

$$\sim A \equiv \bar{p}q + p\bar{q} + \bar{p}\bar{q}$$

(c) $A \equiv p \rightarrow [p \wedge (q \rightarrow p)]$

i. PDNF of A:

$$\begin{aligned} A &\equiv p \rightarrow [p \wedge (q \rightarrow p)] \\ &\equiv \sim p \vee (p \wedge (q \rightarrow p)) \\ &\equiv \sim p \vee (p \wedge (\sim q \vee p)) \\ &\equiv \bar{p} + p(\bar{q} + p) \\ &\equiv \bar{p} + p\bar{q} + pp \\ &\equiv \bar{p} + p\bar{q} + p \\ &\equiv \bar{p}(q + \bar{q}) + p\bar{q} + p(q + \bar{q}) \\ &\equiv \bar{p}q + \bar{p}\bar{q} + p\bar{q} + pq + p\bar{q} \\ &\equiv pq + p\bar{q} + \bar{p}q + \bar{p}\bar{q} \end{aligned}$$

ii. PDNF of $\sim A$:

$$\sim A \equiv c$$

A. Note: No possible minterms

iii. PCNF of A:

$$A \equiv t$$

A. Note: No possible maxterms

iv. PCNF of $\sim A$:

$$A \equiv (p + q)(p + \bar{q})(\bar{p} + q)(\bar{p} + \bar{q})$$

(d) $A \equiv (q \rightarrow p) \wedge (\sim p \wedge q)$

i. PDNF of A:

$$\begin{aligned} A &\equiv (\sim q \vee p) \wedge (\sim p \wedge q) \\ &\equiv (\bar{q} + p)(\bar{p}q) \\ &\equiv \bar{p}\bar{q}q + p\bar{p}q \\ &\equiv c \text{ (No possible minterms)} \end{aligned}$$

ii. PDNF of $\sim A$:

$$A \equiv pq + \bar{p}q + p\bar{q} + \bar{p}\bar{q}$$

iii. PCNF of A:

$$A \equiv (p + q)(\bar{p} + q)(p + \bar{q})(\bar{p} + \bar{q})$$

iv. PCNF of $\sim A$:

$$A \equiv t \text{ (No possible maxterms)}$$