Tutorial 12

January 28, 2020

1. Apply the rules of Boolean algebra, verify the following.

   (a) $y \wedge (x \vee (x' \wedge (y \vee y'))) = y$

   i. Start from LHS

$$
\begin{aligned}
y\left(x+\left(\bar{x}\cdot\left(y+\bar{y}\right)\right)\right) &= y\left(x+\left(\bar{x}\cdot t\right)\right) \text{ Inverse law}\\
&= y\left(x+\bar{x}\right)\text{Identity law}\\
&= y\left(t\right)\text{Inverse law}\\
&= y \text{ Identity law}\\
&= RHS
\end{aligned}
$$

   (b) $((x \wedge y') \wedge (z \vee (x \wedge y')))' = x' \vee y$

   i. Start from LHS

$$
\begin{aligned}
\left(\left(x\bar{y}\right)\left(z+xy'\right)\right)' &= \overline{\left(x\bar{y}\right)}+\overline{\left(z+xy'\right)}\text{DeMorgan's Law}\\
&= \bar{x}+y+\bar{z}\cdot\overline{\left(xy'\right)}\text{DeMorgan's Law}\\
&= \bar{x}+y+\bar{z}\cdot\left(\bar{x}+y\right)\text{ DeMorgan's Law}\\
&= \bar{x}+y+\bar{z}\bar{x}+\bar{z}y\text{Distributive law}\\
&= \bar{x}+\bar{z}\bar{x}+y+\bar{z}y\text{Associative law}\\
&= \bar{x}\left(1+\bar{z}\right)+y\left(1+\bar{z}\right)\text{ Distributive law}\\
&= \bar{x}+y\text{Null law}\\
&= x'\vee y\\
&= RHS
\end{aligned}
$$

2. Simplify the following Boolean functions.

   (a) $f(x,y) = (x \wedge y') \vee (x' \wedge y) \vee (x \wedge y)$

$$
\begin{aligned}
f\left(x,y\right) &= \left(x\wedge y'\right)\vee\left(x'\vee x\right)\wedge y\\
&= \left(x\wedge y'\right)\vee t\wedge y\\
&= \left(x\wedge y'\right)\vee y\\
&= \left(x\vee y\right)\wedge 1\\
f\left(x,y\right) &= \left(x\vee y\right)
\end{aligned}
$$

(b) $f(x,y,z) = (x' \wedge y' \wedge z') \vee (x \wedge y' \wedge z') \vee (x' \wedge y \wedge z') \vee (x \wedge y \wedge z')$

$$\begin{aligned} f(x,y,z) &= (x' \wedge y' \wedge z') \vee (x \wedge y' \wedge z') \vee (x' \wedge y \wedge z') \vee (x \wedge y \wedge z') \\ &= (x'y'z') + (xy'z') + (x'yz') + (xyz') \\ &= (x' + x)\, y'z' + (x' + x)\, yz' \\ &= y'z' + yz' \\ &= z'(y' + y) \\ f(x,y,z) &= z' \end{aligned}$$

(c) $f(x,y,z) = (x \wedge y) \vee \left[x \wedge (y \wedge z)'\right]$

$$\begin{aligned} (x \wedge y) \vee \left[x \wedge (y \wedge z)'\right] &= (x \wedge y) \vee [x \wedge (y' \vee z')] \\ &= (x \wedge y) \vee (x \wedge y') \vee (x \wedge z') \\ &= x \wedge (y \vee y') \vee (x \wedge z') \\ &= x \wedge 1 \vee (x \wedge z') \\ &= x \vee (x \wedge z') \\ &= x \wedge (1 \vee z') \\ (x \wedge y) \vee \left[x \wedge (y \wedge z)'\right] &= x \\ f(x,y,z) &= x \end{aligned}$$

3. Find the disjunctive normal form of the Boolean function $f = f(x,y,z)$ with the given truth table. Then, simplify the expression by constructing the Karnaugh Map

(a)

x	y	z	$f(x,y,z)$
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

i. Find PDNF (all ones):

$$f(x,y,z) = \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + x\bar{y}\bar{z} + xy\bar{z}$$

ii. Construct karnaugh map

A.

	y'	y'	y	y
x'	**1**	**1**	0	0
x	**1**	0	0	**1**
	z'	z	z	z'

$$\bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + x\bar{y}\bar{z} + xy\bar{z} = \boldsymbol{x'y' + xz'}$$

(b)

x	y	z	$f(x,y,z)$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

i. Find PDNF

A. $\bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}y\bar{z} + \bar{x}yz$

ii. Construct Karnaugh Map

A.

	y'	y'	y	y
x'	**1**	**1**	**1**	**1**
x	0	0	**1**	1
	z'	z	z	z'

B. Conclusion: $\bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}y\bar{z} + \bar{x}yz \equiv \boldsymbol{x' + yz}$

4. In the following questions, Karnaugh maps of functions are given, write the simplified Boolean expression for these functions.

(a)

	y'	y
x'	**1**	0
x	0	**1**

i. $f(x,y) = (x' \wedge y') \vee (x \wedge y)$

(b)

	y'	y
x'	**1**	**1**
x	**1**	0

i. $f(x,y) = x \wedge y$

(c)

	y'	y'	y	y
x'	**1**	**1**	**1**	**1**
x	**1**	0	0	**1**
	z'	z	z	z'

i. $x' + z'$

(d)

	y'	y'	y	y
x'	**1**	**1**	0	**1**
x	0	**1**	0	**1**
	z'	z	z	z'

i. $x'y' + yz + yz'$

(e)

	y'	y'	y	y
x'	**1**	**1**	**1**	**1**
x	0	0	**1**	0
	z'	z	z	z'

i. $x' + yz$

(f)

	y'	y'	y	y
x'	0	**1**	0	**1**
x	**1**	**1**	0	**1**
	z'	z	z	z'

i. $xy' + yz' + y'z$

5. Draw a Karnaugh map for the Boolean expression whose disjunctive normal forms are as follow. Hence find a simplified version of the expression.

   (a) $f(x, y, z) = (x\text{'} \wedge y\text{'} \wedge z) \vee (x\text{'} \wedge y \wedge z) \vee (x \wedge y \wedge z\text{'}) \vee (x \wedge y \wedge z)$

   i.

	y'	y'	y	y
x'	0	**1**	**1**	0
x	0	0	**1**	**1**
	z'	z	z	z'

   A. $x'z + xy$

   (b) $f(x, y, z, w) = (x \wedge y \wedge z \wedge w) \vee (x \wedge y \wedge z \wedge w\text{'}) \vee (x\text{'} \wedge y \wedge z \wedge v \wedge w) \vee (x\text{'} \wedge y \wedge z \wedge w\text{'}) \vee (x \wedge y\text{'} \wedge z\text{'} \wedge w\text{'}) \vee (x\text{'} \wedge y\text{'} \wedge z\text{'} \wedge w\text{'})$

   i.

	z'	z'	z	z	
x'	**1**	0	0	0	y'
x'	0	0	**1**	**1**	y
x	0	0	**1**	**1**	y
x	**1**	0	0	0	y'
	w'	w	w	w'	

   A. $y'z'w' + yz$

6. Find the disjunctive normal form of the Boolean function $f(x, y, z)$ with the following truth table and then draw a Karnaugh map to find a simplified version of $f(x, y, z)$.

   (a)

x	y	z	$f(x, y, z)$
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

   i. Find PDNF

   A. $\bar{x}\bar{y}\bar{z} + \bar{x}y\bar{z} + x\bar{y}z + xyz$

   ii.

	y'	y'	y	y
x'	**1**	0	0	**1**
x	0	**1**	**1**	0
	z'	z	z	z'

iii. $x'z' + xz$

7. Construct a truth table for the Boolean expression $(x \wedge (y' \vee z)) \vee (x' \wedge (y \vee z'))$ and hence determine its disjunctive normal form. Draw a Karnaugh map and hence find a simplified version of $f(x, y, z)$.

(a)

x	y	z	$y' \vee z$	$y \vee z'$	$x \wedge (y' \vee z)$	$x' \wedge (y' \vee z')$	$x \wedge (y' \vee z) \vee x' \wedge (y' \vee z')$
0	0	0	1	1	0	1	1
0	0	1	1	0	0	0	0
0	1	0	0	1	0	1	1
0	1	1	1	1	0	1	1
1	0	0	1	1	1	0	1
1	0	1	1	0	0	0	0
1	1	0	0	1	1	0	1
1	1	1	1	1	1	0	1

i. $x'y'z' + x'y'z' + x'yz + xy'z' + xyz' + xyz$

(b)

	y'	y'	y	y
x'	**1**	0	**1**	**1**
x	**1**	**1**	**1**	0
	z'	z	z	z'

i. $y'z' + xz + x'y$