Ginwidth=

Calc II - Tutorial PYQ

January 15, 2020

1 2015/16

1. Given the power series expansion of the function $\frac{1}{1-x} = 1 + x + x^2 + ..., |x| < 1$. Show that the first three non-zero coefficients in the power series expansion about $x = 0$ of $\frac{1}{\cos x}$ is given by $1 + \frac{x^2}{2} + \frac{5x^4}{24} + ...$

   (a) **Way 1: Traditional listing out and writing it down.**

      i. List down the terms

$$\begin{aligned}
f &= \frac{1}{\cos x} = \sec x \\
f' &= \tan x \sec x \\
f'' &= \tan^2 x \sec x + \sec^3 x \\
&= \left(\sec^2 - 1\right) \sec x + \sec^3 x \\
&= \sec^3 - \sec x + \sec^3 x \\
&= 2\sec^3 x - \sec x \\
&= 2f^3 - f \\
f''' &= 6f^2 f' - f' \\
f'''' &= \left(12ff'\left(f'\right) - 6f^2 f''\right) - f'' \text{Product rule here}
\end{aligned}$$

      ii. Write down the values

$$\begin{aligned}
f\left(0\right) &= 1 \\
f'\left(0\right) &= 0 \\
f''\left(0\right) &= 2\left(1\right)^3 - 1 \\
&= 1 \\
f'''\left(0\right) &= 6f^2 f' - f' \\
&= 0 \\
f''''\left(0\right) &= 0 + 6\left(1\right)\left(1\right) - 1 \\
&= 5
\end{aligned}$$

iii. Write down the final series (using MacLaurin Polynomial)

$$\begin{aligned}\frac{1}{\cos x} &= f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + ... + \frac{f^n(0)}{n!}x^n \\ &= 1 + \frac{x^2}{2} + \frac{5x^4}{24} + ...\end{aligned}$$

(b) **Way 2: Utilizing MacLaurin Series**

2. Determine the Maclaurin series for $f(x) = e^{-4x}$ in ascending powers of x up to and including the term in x^2. Then use the series to estimate the value of $f(x)$ when $x = 0.1$. Give your answer correct to 2 decimal places.

(a) Find the sequence

i. $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + ...$

ii. $e^{-4x} = 1 + \frac{(-4x)}{1!} + \frac{(-4x)^2}{2!} + ... = 1 - 4x + 8x^2 + ...$

(b) When $x = 0.1$

$$\begin{aligned}e^{-4(0.1)} &= e^{-0.4} \\ &= 1 + \frac{(-4(0.1))}{1!} + \frac{(-4(0.1))^2}{2!} + ... \\ &\approx 0.6008 \\ &= 0.60(2\,d.p.)\end{aligned}$$

2 2016/17

1. Determine the Maclaurin series for $f(x) = xe^{3x}$ in ascending powers of x up to and including the term x^4. Then use the series to estimate the value of $f(x)$ when $x = 0.05$. Give your answer correct to 4 decimal places.

(a) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} ...$

(b) $e^{3x} = 1 + \frac{3x}{1!} + \frac{(3x)^2}{2!} + \frac{(3x)^3}{3!} + \frac{(3x)^4}{4!} ...$

(c) $xe^{3x} = x + \frac{3x^2}{1!} + \frac{(3x)^2}{2!} + \frac{(3x)^3}{3!} + \frac{(3x)^4}{4!} ...$ (Note: x doesn't change as n increases, therefore, easiest way is just to multiple in)

(d) Estimate $x = 0.05$

$$\begin{aligned}f(0.05) &= 0.05 + 3(0.05)^2 + \frac{9}{2}(0.05)^3 + \frac{9}{2}(0.05)^4 + ... \\ &= 0.0581\end{aligned}$$

2. Use series to approximate the definite integral of $\int_0^{0.05} xe^{3x}dx$ correct to 4 decimal places.

(a) $e^{3x} = 1 + \frac{3x}{1!} + \frac{(3x)^2}{2!} + \frac{(3x)^3}{3!} + \frac{(3x)^4}{4!} ..$

(b) $xe^{3x} = x + \frac{3x^2}{1!} + \frac{(3x)^2}{2!} + \frac{(3x)^3}{3!} + \frac{(3x)^4}{4!}...$

$$\begin{aligned}\int_0^{0.05} xe^{3x} &= \int_0^{0.05} x + \frac{3x^2}{1!} + \frac{(3x)^2}{2!} + \frac{(3x)^3}{3!} + \frac{(3x)^4}{4!} dx \\ &= 0.0014\end{aligned}$$