Discrete Maths C1

October 8, 2019

1 Notes

1. If there are = signs, 100% of the time they are actually ≡ signs. But ≡ signs are much more tedious to type that = signs (5 keystrokes vs 1 keystroke). Therefore, I tend to use them interchangably. But in your answers, you should write ≡ instead of =.

1.1 Example 11

$$\begin{aligned}\sim(\sim((p \vee q) \wedge r) \vee \sim q) &\equiv \sim((p \vee q) \wedge r) \vee \sim\sim q \\ &\equiv ((p \vee q) \wedge r) \vee q \\ &\equiv (p \vee q) \wedge r \vee q \\ &\equiv q \wedge r\end{aligned}$$

1.2 Example 18

$$\begin{aligned}s &= (p \wedge \sim q) \vee (r \wedge t) \\ s^d &= (p \vee \sim q) \wedge (r \vee c)\end{aligned}$$

1.3 Example 20 (Disjunctive: Sums of products)

$$\begin{aligned}p \wedge (p \rightarrow q) &\equiv p \wedge (\sim p \vee q) \\ &\equiv \boldsymbol{p \wedge \sim p \vee p \wedge q} \texttt{(Can stop here)} \\ &\equiv c \vee p \wedge q \\ p \wedge (p \rightarrow q) &\equiv p \wedge q\end{aligned}$$

1. Eliminate, distribute, then $p \wedge \sim p = c$, then $c \vee a = a$.
   (a) Notes:
   $$p\bar{p} + pq$$

1.4 Example 21 (Simplified workings)

$$
\begin{aligned}
\sim[(p \vee q) \leftrightarrow (p \wedge q)] &= \sim[((p \vee q) \rightarrow (p \wedge q)) \wedge ((p \wedge q) \rightarrow (p \vee q))] \\
&= \sim[(\sim(p \vee q) \vee (p \wedge q)) \wedge (\sim(p \wedge q) \vee (p \vee q))] \\
&= \sim(\sim(p \vee q) \vee (p \wedge q)) \vee \sim(\sim(p \wedge q) \vee (p \vee q)) \\
&= [(p \vee q) \wedge \sim(p \wedge q)] \vee [(p \wedge q) \wedge \sim(p \vee q)] \\
&= (p+q)\,(\overline{pq}) + (pq)\,(\overline{p+q}) \\
&= p(\overline{pq}) + q(\overline{pq}) + p(\overline{p+q}) + q(\overline{p+q}) \\
&= p\,(\bar{p}+\bar{q}) + q\,(\bar{p}+\bar{q}) + p\,(\bar{p}\bar{q}) + q\,(\bar{p}\bar{q}) \\
&= c + p\bar{q} + \bar{p}q + c + c + c \\
&= p\bar{q} + \bar{p}q
\end{aligned}
$$

1.5 Example 22 Conjunctive normal form

1. $p \wedge (p \rightarrow q)$

$$
\begin{aligned}
p \wedge (p \rightarrow q) &= p \wedge (\sim p \vee q) \\
&= p\,(\bar{p}q) \text{ Can stop here} \\
&= p\bar{p} + pq \\
&= c + pq \\
&= pq
\end{aligned}
$$

$\sim(p \vee q) \leftrightarrow (p \wedge q)$

$$
\begin{aligned}
\sim(p \vee q) \leftrightarrow (p \wedge q) &= [\sim(p \vee q) \rightarrow (p \wedge q)] \wedge [(p \wedge q) \rightarrow \sim(p \vee q)] \\
&= [(p \vee q) \vee (p \wedge q)] \wedge [\sim(p \wedge q) \vee \sim(p \vee q)] \\
&= [(p \vee q) \vee (p \wedge q)] \wedge [(\sim p \vee \sim q) \vee (\sim p \wedge \sim q)] \\
&= (p+q+pq)\,(\bar{p}+\bar{q}+\bar{p}\bar{q}) \\
&= (p+q)\,(\bar{p}+\bar{q})
\end{aligned}
$$

1.6 Example 23 (CNF + t)

$$
\begin{aligned}
q \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q) &= q \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q) \\
&= q + p\bar{q} + \bar{p}\bar{q} \\
&= q + \bar{q}\,(p+\bar{p}) \\
&= q + \bar{q}\,(1) \\
&= q + \bar{q} \\
&= 1 \\
&= t
\end{aligned}
$$

1.7 Extra - Minterms and Maxterms

1. $p \vee (q \wedge r) = p + (qr)$

   (a) Minterms:

$$
\begin{aligned}
F &= p + qr \\
&= p\,(q + \bar{q})\,(r + \bar{r}) + (p + \bar{p})\,qr \\
&= (pq + p\bar{q})\,(r + \bar{r}) + pqr + \bar{p}qr \\
&= pqr + p\bar{q}r + pq\bar{r} + p\bar{q}\bar{r} + \cancel{pqr} + \bar{p}qr \\
&= pqr + p\bar{q}r + pq\bar{r} + p\bar{q}\bar{r} + \cancel{pqr} + \bar{p}qr
\end{aligned}
$$

   (b) Or even DeMorgan the DeMorgan still perfectly acceptable

$$
\begin{aligned}
p \vee (q \wedge r) &= p + (qr) \\
&= \overline{\overline{p + (qr)}} \\
&= \overline{\bar{p} \cdot \overline{qr}} \\
&= \overline{\bar{p} \cdot (\bar{q} + \bar{r})} \\
&= \overline{\bar{p}\bar{q} + \bar{p}\bar{r}} \\
&= \overline{\bar{p}\bar{q} + \bar{p}\bar{r}} \\
&= \overline{\bar{p}\bar{q}} \cdot \overline{\bar{p}\bar{r}} \\
\boldsymbol{p \vee (q \wedge r)} &\boldsymbol{= (p + q) \cdot (p + r)}
\end{aligned}
$$

1.8 Example 24 (Principal Disjunctive normal forms/ sum of products Canonical form)

1.
$$p \to q \equiv \bar{p}\bar{q} + \bar{p}q + pq$$

2.
$$p \vee q \equiv \bar{p}q + p\bar{q} + pq$$

3.
$$\sim (p \wedge q) \equiv p\bar{q} + \bar{p}q + \bar{p}\bar{q}$$

1.9 Example 26

1. $p \vee (p \wedge q) \Leftrightarrow p$

   (a) Easiest way

$$
\begin{aligned}
LHS : p \vee (p \wedge q) &= p + pq \\
&= p\,(1 + q)\,; 1 + q = t/1 \\
&= p \Leftrightarrow RHS
\end{aligned}
$$

(b) Alternatively

$$\begin{aligned} LHS : p \vee (p \wedge q) &= p\left(q+\bar{q}\right)+pq \\ &= pq + p\bar{q} + pq \\ &= p\bar{q} + pq \end{aligned}$$

$$\begin{aligned} RHS : p =& p\left(q+\bar{q}\right) \\ =& pq + p\bar{q} \\ =& p\bar{q} + pq \\ =& LHS \end{aligned}$$

$$LHS \Leftrightarrow RHS$$

2. $p \vee (\sim p \wedge q) \Leftrightarrow p \vee q$

(a) Solution

$$\begin{aligned} LHS : p \vee (\sim p \wedge q) =& p + (\bar{p}q) \\ =& p\left(q+\bar{q}\right) + (\bar{p}q) \\ =& pq + p\bar{q} + \bar{p}q \end{aligned}$$

$$\begin{aligned} RHS : p \vee q =& p + q \\ =& p\left(q+\bar{q}\right) + q\left(p+\bar{p}\right) \\ =& pq + p\bar{q} + \cancel{qp} + \bar{p}q \\ =& pq + p\bar{q} + \bar{p}q \\ =& LHS \end{aligned}$$

$$LHS \Leftrightarrow RHS$$

1.10 Example 27

$$\begin{aligned} p \rightarrow [(p \rightarrow q) \wedge \sim (\sim q \vee \sim p)] &\equiv \sim p \vee [(\sim p \vee q) \wedge \sim (\sim q \vee \sim p)] \\ &\equiv \sim p \vee [(\sim p \vee q) \wedge \sim (\sim q \vee \sim p)] \\ &\equiv \sim p \vee [(\sim p \vee q) \wedge (q \wedge p)] \\ &\equiv \bar{p} + [(\bar{p}+q)(pq)] \\ &\equiv \bar{p} + \bar{p}p + \bar{p}q + pq + qq \\ &\equiv \bar{p} + \bar{p}q + pq + q \\ &\equiv \bar{p}\left(q+\bar{q}\right) + \bar{p}q + pq + q\left(p+\bar{p}\right) \\ &\equiv \bar{p}q + \bar{p}\bar{q} + \cancel{\bar{p}q} + pq + \cancel{pq} + \cancel{\bar{p}q} \\ &\equiv pq + \bar{p}q + \bar{p}\bar{q} \end{aligned}$$

Example 28 Let A represent $(\sim p \to r) \wedge (q \leftrightarrow p)$. Obtain the pcnf of A, and of ~A. Deduce the pdnf of A.

$$A \equiv (\sim p \to r) \wedge (q \leftrightarrow p)$$

1. Obtain PCNF of A

$$\begin{aligned} A &\equiv (p \vee r) \wedge ((q \to p) \wedge (p \to q)) \\ &\equiv (p \vee r) \wedge ((\sim q \vee p) \wedge (\sim p \vee q)) \\ &\equiv (p + r)\,((\bar{q} + p)\,(\bar{p} + q)) \\ &\equiv (p + r)\,(\bar{q} + p)\,(\bar{p} + q) \end{aligned}$$

   (a) PCNF of A

$$\begin{aligned} PCNF_A &\equiv (p + q\bar{q} + r)\,(p + \bar{q} + r\bar{r})\,(\bar{p} + q + r\bar{r}) \\ &\equiv (p + q + r)\,(p + \bar{q} + r)\,\cancel{(p + \bar{q} + r)}\,(p + \bar{q} + \bar{r})\,(\bar{p} + q + r)\cdot \\ &\quad (\bar{p} + q + \bar{r}) \\ &\equiv (p + q + r)\,(p + \bar{q} + r)\,(p + \bar{q} + \bar{r})\,(\bar{p} + q + r)\,(\bar{p} + q + \bar{r}) \end{aligned}$$

      i. Note: for this part, $(p + q\bar{q} + r) \equiv (p + q + r)\,(p + \bar{q} + r)$

$$\begin{aligned} (p + q + r)\,(p + \bar{q} + r) &\equiv pp + pq + pr + qp + q\bar{q} + qr \\ &\quad + pr + \bar{q}r + rr \\ &\equiv p + \cancel{pq} + \cancel{pr} + \cancel{pq} + q\bar{q} + qr + \cancel{pr} + \bar{q}r + r \\ &\equiv p + q\bar{q} + qr + \bar{q}r + r \\ &\text{(Note: if } q \text{ is false, } qr \text{ is false. If } r \text{ is false, } \bar{q}r \text{ is false)} \\ &\equiv p + q\bar{q} + r \end{aligned}$$

   (b) PCNF of $\sim A$, according to the lecture notes, it is stated that the principal disjunctive (conjunctive) normal form of ~A will consist of the **disjunctive (conjunctive) of the remaining minterms (maxterms) which do not appear in the principal disjunctive (conjunctive) normal form of A**.

      i. Remember your Analysis & Design of IS? There's a total combinations formula for boolean expressions where

$$Total\,combinations = 2^n combinations$$

where n is the number of combinations

      ii. There should be $2^3 = 8$ combinations. Or $8 - 5 = 3$ remaining combinations

$$\sim A_{PCNF} \equiv (p + q + \bar{r})\,(\bar{p} + \bar{q} + r)\,(\bar{p} + \bar{q} + \bar{r})$$

Example 29

1. PDNF of A
$$A_{PDNF} \equiv \bar{p}\bar{q}r + pq\bar{r} + pqr$$

2. PCNF of A
$$A_{PCNF} \equiv (\bar{p} + \bar{q} + \bar{r})\,(p + \bar{q} + r)\,(p + \bar{q} + \bar{r})\,(\bar{p} + q + r)\,(p + q + \bar{r})$$

3. To understand minterms and maxterms:

   (a) **Minterms**: If one term is true, the entire expression is true. So the 'minimal' terms are required for true. Therefore, we form the equations from terms that 'evaluates to true'. Because if we get 1 true, even if everything else is false, we get true.

   (b) **Maxterms**: All terms must be true, for the entire expression to be true. Hence the 'max' terms are required. Therefore, we form the equations from terms that 'evaluates to false'. Because we must get all true (they are all false), to get true.

$$(p\bar{q}\bar{r}) + (p\bar{q}r) + (pqr)$$

Example 38

$$\begin{aligned}
[(P\wedge \sim Q) \vee (P \wedge Q)] \wedge Q\,(LHS) &\equiv \left(P\bar{Q} + PQ\right) Q \\
&\equiv \left(P\bar{Q}Q + PQQ\right) \\
&\equiv (P + PQ) \\
&\equiv P\,(1 + Q) \\
&\equiv PQ
\end{aligned}$$