Tutorial 3 - Continuous Probability Distribution

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1. Since the thickness is uniformly distributed, let X be the thickness of photoresist where $X \sim U\left(0.2050, 0.2150\right)$

   (a) $P\left(X > 0.2125\right)$

$$f\left(x\right) = \begin{cases} \frac{1}{0.01} & 0.2050 \leq x \leq 0.2150 \\ 0 & otherwise \end{cases}$$
$$c = 100$$

$$\begin{aligned} P\left(X > 0.2125\right) &= 1 - P\left(X < 0.2125\right) \\ &= 1 - \int_{0.2050}^{0.2125} 100\,dx \\ &= 1 - 100\left(0.2125 - 0.2050\right) \\ &= 0.25 \end{aligned}$$

   (b) $P\left(X > a\right) = 0.1$

$$\begin{aligned} \int_{a}^{0.2150} 100\,dx &= 0.1 \\ 100\left(0.2150 - a\right) &= 0.1 \\ 21.5 - 100a &= 0.1 \\ 100a &= 21.4 \\ a &= 0.2140\mu m \end{aligned}$$

   (c)

$$\begin{aligned} \mu &= \frac{0.2050 + 0.2150}{2} \\ &= 0.21\mu m \end{aligned}$$

$$
\begin{aligned}
\sigma &= \sqrt{\frac{(b-a)^2}{12}} \\
&= \sqrt{\frac{(0.2150-0.2050)^2}{12}} \\
&= 0.002886\mu m
\end{aligned}
$$

2. Let X be the lifetime of a mechanical assembly in a vibration test with $X \sim E\left(400\right)$
   (a) $P\left(X < 100\right)$

$$
\begin{aligned}
P\left(X < 100\right) &= \int_0^{100} \frac{1}{400} e^{-\frac{x}{400}}\, dx \\
&= 1 - \frac{1}{e^{\frac{1}{4}}} \\
&= 0.2212
\end{aligned}
$$

   (b) $P\left(X > 500\right)$

$$
\begin{aligned}
P\left(X > 500\right) &= 1 - P\left(X < 500\right) \\
&= 1 - \int_0^{500} \frac{1}{400} e^{-\frac{x}{400}}\, dx \\
&= \frac{1}{e^{\frac{5}{4}}} \\
&= 0.2865
\end{aligned}
$$

   (c)

$$
\begin{aligned}
\sigma &= \sqrt{\mu^2} \\
&= \sqrt{400^2} \\
&= 400 hours
\end{aligned}
$$

3. (Important notes: if don't know what to write the formulas, can just write the number for Z scores)
   (a) Important note 2 (lol): Always ignore past 2 decimal places in tables, no need to do interpolation, too small to matter
   (b) P(Z < -0.6)

$$X \sim N\left(1, 0\right)$$

$$P\left(Z > 0.6\right) = 0.27425$$

   (c) $P\left(Z > -1.28\right)$

$$
\begin{aligned}
1 - P\left(Z > 1.28\right) &= 1 - 0.1003 \\
&= 0.8997
\end{aligned}
$$

(d) $P(0.81 < Z < 1.94)$

$$P\left(Z < 1.94\right) - P\left(Z < 0.81\right) = 0.18278$$

(e)

$$\begin{aligned} P(-0.68 < Z < 0) &= 0.5 - P\left(Z > 0.68\right) \\ &= 0.5 - 0.2483 \\ &= 0.2517 \end{aligned}$$

(f) $P(-0.46 < Z < 2.21)$

$$1 - P\left(Z > 0.46\right) - P\left(Z > 2.21\right) = 0.66365$$

(g) $P\left(0.81 < Z < 1.94\right)$

$$\begin{aligned} P\left(Z > 0.81\right) - P\left(Z > 1.94\right) &= 0.209 - 0.0262 \\ &= 0.1828 \end{aligned}$$

4.

(a)

$$\begin{aligned} P\left(-1.2 \leq Z \leq K\right) &= 0.523 \\ P\left(Z \leq -1.2\right) + P\left(Z \geq K\right) &= 1 - 0.523 \\ P\left(Z \leq -1.2\right) + P\left(Z \geq K\right) &= 0.477 \\ P\left(Z \geq K\right) &= 0.477 - 0.11507 \\ &= 0.36193 \\ K &= 0.35 \end{aligned}$$

(b)

$$\begin{aligned} P\left(K \leq Z \leq 1.8\right) &= 0.355 \\ P\left(Z > 1.8\right) + P\left(Z < K\right) &= 1 - 0.355 \\ P\left(Z > 1.8\right) + P\left(Z < K\right) &= 0.645 \\ P\left(Z < K\right) &= 0.645 - 0.03593 \\ &= 0.60907 \\ P\left(Z > K\right) &= 0.39093 \\ K &= 0.27 \end{aligned}$$

(c)

i. Calculuation

$$\begin{aligned} P\left(Z \leq -0.8\right) + P\left(Z > K\right) &= 0.616 \\ P\left(Z > K\right) &= 0.616 - 0.21186 \\ &= 0.40414 \\ K &= 0.24 \end{aligned}$$

(d)

$$\begin{aligned}
P\left(Z \leq K\right) &= 4 \cdot P\left(Z \leq K\right) \\
5 \cdot P\left(Z \leq K\right) &= 1 \\
P\left(Z \leq K\right) &= \frac{1}{5} \\
\frac{1}{5} &= P\left(Z \geq -K\right) \\
-K &= 0.8 \\
K &= -0.84
\end{aligned}$$

5. $\mu = 20.02, \sigma = 0.05$

(a)

$$\begin{aligned}
P\left(X < 19.9\right) &= P\left(Z < \frac{19.9 - 20.02}{0.05}\right) \\
&= P\left(Z < -2.4\right) \\
&= 8.1974 * 10^{-3} \\
Percentage &= 8.1974 * 10^{-3} * 100\% \\
&= 0.82\% \, or \, 0.0082
\end{aligned}$$

(b)

$$\begin{aligned}
P\left(X > 20.1\right) &= P\left(Z > \frac{20.1 - 20.02}{0.05}\right) \\
&= P\left(Z > 1.6\right) \\
&= 0.054799 \\
Percentage &= 5.4799\% \, or \, 0.0054779
\end{aligned}$$

6. $\mu_a = 1000, \sigma_a = 100, \ \mu_b = 900, \sigma_b = 50$

(a) Let X be the breaking strength of the rope with $X \sim N\left(1000, 10000\right)$

$$\begin{aligned}
P\left(X < 750\right) &= P\left(Z < \frac{750 - 1000}{100}\right) \\
&= P\left(Z < -2.5\right)
\end{aligned}$$

(b) Let X_2 be the breaking strength of th rope with $X_2 \sim N\left(900, 2500\right)$

$$\begin{aligned}
P\left(X_2 < 750\right) &= P\left(Z < \frac{750 - 900}{50}\right) \\
&= P\left(Z < -3\right)
\end{aligned}$$

(c) Conclusion. All things considered, the company should pick supplier B, because the ropes that are lower than 750kg of breaking strength are less

7. $\mu = 5000, \sigma = 1000$

   (a)

   i.

$$\begin{aligned} P(5500 \leq X \leq 6500) &= P\left(\frac{5500-5000}{1000} \leq Z \leq \frac{6500-5000}{1000}\right) \\ &= P\left(\frac{1}{2} \leq Z \leq \frac{3}{2}\right) \\ &= 0.24173 \end{aligned}$$

   ii.

$$\begin{aligned} P(X < 5000) &= P(Z < 0) \\ &= 0.5 \end{aligned}$$

   (b)

$$\begin{aligned} P(X < 7500) &= P\left(Z < \frac{7500-5000}{1000}\right) \\ &= P\left(Z < \frac{2500}{1000}\right) \\ &= P\left(Z < \frac{5}{2}\right) \\ &= 0.99379 \end{aligned}$$

8. $\mu = 500, \sigma = 20$

   (a) $n = 2000$, let X be the packets weight with $X \sim N(500, 400)$

   i.

$$\begin{aligned} P(X > 520) &= P\left(Z > \frac{520-500}{20}\right) \\ &= P(Z > 1) \\ &= 0.15866 \end{aligned}$$

   A. $2000 * 0.15866 \approx 317\, packets$

   ii.

$$\begin{aligned} P(X < 470) &= P\left(Z < \frac{470-500}{20}\right) \\ &= P\left(Z < -\frac{3}{2}\right) \\ &= 0.066807 \end{aligned}$$

A. $2000 * 0.066807 = 133.614 \approx 134 packets$

iii.

$$\begin{aligned} P\left(520 \leq X \leq 530\right) &= P\left(\frac{520-500}{20} \leq Z \leq \frac{530-500}{20}\right) \\ &= P\left(1 \leq Z \leq 1.5\right) \\ &= 0.09185 \end{aligned}$$

A. $2000 * 0.09185 = 183.7 \approx 184 packets$

9. $\mu = 45 minutes, \sigma = 8 minutes$

(a)

$$\begin{aligned} P\left(X > 50\right) &= P\left(Z > \frac{50-45}{8}\right) \\ &= P\left(Z > \frac{5}{8}\right) \\ &= 0.26599 \end{aligned}$$

(b)

$$\begin{aligned} P\left(40 \leq X \leq 52\right) &= P\left(\frac{40-45}{8} \leq X \leq \frac{52-45}{8}\right) \\ &= P\left(-\frac{5}{8} \leq Z \leq \frac{7}{8}\right) \\ &= 0.54322 \end{aligned}$$

(c)

$$\begin{aligned} P\left(X \leq a\right) &= 0.9 \\ P\left(Z \leq \frac{a-45}{8}\right) &= 0.9 \\ P\left(Z > \frac{a-45}{8}\right) &= 0.1 \\ \frac{a-45}{8} &= 1.2815 \\ a &= 55.252 minutes \end{aligned}$$

(d)

$$\begin{aligned} P(X \leq a) &= 0.3 \\ P\left(Z \leq \frac{a-45}{8}\right) &= 0.3 \\ -P\left(Z \geq \frac{a-45}{8}\right) &= 0.3 \\ \frac{a-45}{8} &= -0.52466 \\ a &= 40.8027 minutes \end{aligned}$$

10. $p = 0.1, n = 500$

    (a) X are the males out of 500 males who suffer from a certain disease with $X \sim B(500, 0.1)$

        i. Cannot use Poisson because $np < 5$

        ii. To approximate with normal, $np > 5$, $nq > 5$. The rule is plus or minus to make it slightly bigger, dont need to remember details.

$$\begin{aligned} P(X > 60) &= P(X > 60.5) \\ &= P\left(Z > \frac{60.5 - 50}{\sqrt{45}}\right) \\ &= P\left(Z > \frac{10.5}{\sqrt{45}}\right) \\ &= P(Z > 1.57) \\ P(Z > 1.57) &= \mathbf{0.058763} \end{aligned}$$

    (b) QUESTION: HOW TO KNOW IF WE SHOULD TAKE SAMPLE DISTRIBUTION?

        i. Answer: Sample distribution is only when we take sample from a population. In this case, we are only using Normal to approximate another form of probability distribution, and not calculating, and therefore, we do not need to.

11. $p = 0.1$, $n = 1000, np = 100, \mu = 100, \sigma^2 = npq = 1000 * 0.1 * 0.9 = 90$

    (a) Let X be the number of chocolates produced with mis-shapes such that $X \sim N(100, 100)$

$$\begin{aligned} P(X < 80) &= P(X < 79.5) \, \text{make it slightly bigger} \\ & P\left(Z < \frac{79.5 - 100}{\sqrt{90}}\right) \\ &= P(Z < -2.16) \\ &= P(Z > 2.16) \\ &= \mathbf{0.01539} \end{aligned}$$

(b) $P\left(90 < X < 115\right)$

$$\begin{aligned} P\left(90 < X < 115\right) &= P\left(89.5 < X < 115.5\right) \\ &= P\left(\frac{89.5-100}{\sqrt{90}} < X < \frac{115.5-100}{\sqrt{90}}\right) \\ &= P\left(-1.11 < Z < 1.63\right) \\ & 1-0.1335-0.0516 \\ &= \mathbf{0.8149} \end{aligned}$$

(c) P($X \geq 120$)

$$\begin{aligned} P\left(X \geq 120\right) &= P\left(X > 119.5\right) \\ &= P\left(Z > \frac{119.5-100}{\sqrt{90}}\right) \\ &= P\left(Z > 2.06\right) \\ &= \mathbf{0.0197} \end{aligned}$$

12. X is the number of bacteria on a plate viewed under a microscope where $X \sim P_o\left(\lambda = 60\right)$

 (a) Since $\lambda > 20$, and n is unobtainable using $X \sim N\left(60, 60\right)$ as approximation

 i. $P\left(55 \leq X \leq 75\right)$

$$\begin{aligned} P\left(55 < X < 75\right) &= P\left(55.5 < X < 74.5\right) \\ &= P\left(-0.5809 < Z < 1.8719\right) \\ &= 0.68874 \end{aligned}$$

 (b) $P\left(X < 38\right)$

$$\begin{aligned} P\left(X < 38\right) &= P\left(X < 37.5\right) \\ &= P\left(Z < \frac{37.5-60}{\sqrt{60}}\right) \\ &= P\left(Z < -2.905\right) \\ &= P\left(Z > 2.905\right) \\ &= 0.00185 \end{aligned}$$

 i. Plates $= 0.00185 * 2000 \approx 4$

13. Since a minute is an interval, and only the average rate is given. Let X be the customers arrivig at a department store in any particular minute, where $X \sim P_o\left(18.6\right)$. Since $\lambda$is big (> 10), and n cannot be easily obtained, we can approximate it with $X \sim N\left(18.6, 4.3128^2\right)$

(a)

$$
\begin{aligned}
P(X \leq 25) &= P(X < 25.5) \\
&= P\left(Z < \frac{25.5 - 18.6}{4.3128}\right) \\
&= P(Z < 1.5998) \\
P(X \leq 25) &= \mathbf{0.94518}
\end{aligned}
$$