1.

(a) 1 - the rest = 0.31

(b)

i. $P(X=1) = \mathbf{0.13}$

ii. $P(X \leq 1)$

$$\begin{aligned} P(X \leq 1) &= P(X=0) + P(X=1) \\ &= 0.03 + 0.13 \\ \boldsymbol{P(X \leq 1)} &\boldsymbol{= 0.16} \end{aligned}$$

iii. $P(X \geq 3)$

$$\begin{aligned} P(X \geq 3) &= 0.31 + 0.19 + 0.12 \\ \boldsymbol{P(X \geq 3)} &\boldsymbol{= 0.62} \end{aligned}$$

iv. $P(X < 3)$

$$\begin{aligned} P(X < 3) &= 0.03 + 0.13 + 0.22 \\ \boldsymbol{P(X < 3)} &\boldsymbol{= 0.38} \end{aligned}$$

v. $P(X > 3)$

$$\begin{aligned} P(X > 3) &= 0.19 + 0.12 \\ \boldsymbol{P(X > 3)} &\boldsymbol{= 0.31} \end{aligned}$$

vi. $P(2 \leq X \leq 4)$

$$\begin{aligned} P(2 \leq X \leq 4) &= 0.22 + 0.31 + 0.19 \\ &= 0.72 \end{aligned}$$

(c)

$$F(x) = \begin{cases} 0 & x < 0 \\ 0.03 & 0 \leq x < 1 \\ 0.16 & 1 \leq x < 2 \\ 0.38 & 2 \leq x < 3 \\ 0.69 & 3 \leq x < 4 \\ 0.88 & 4 \leq x < 5 \\ 1 & x \geq 5 \end{cases}$$

2.

(a) $P(X=4) = 0.14$

(b) $P(X \geq 3) = 0.18 + 0.14 + 0.12 + 0.06 = 0.5$

(c) $P(X < 3) = 0.12 + 0.16 + 0.22 = 0.5$

(d) $P(3 \leq x \leq 5) = 0.18 + 0.14 + 0.12 = 0.44$

3.

(a)

i.

Number of order received per day	2	3	4	5	6
Probability	0.12	0.21	0.34	0.19	0.14

(b)

i. $P(X = 3) = 0.21$

ii. $P(X \geq 3) = 1 - 0.12 = 0.88$

iii. $P(2 \leq X \leq 4) = 0.12 + 0.21 + 0.34 = 0.67$

iv. $P(X < 4) = 0.33$

4.

(a)

$$\begin{aligned}
1 &= \int_2^5 c(1+x)\,dx \\
1 &= c\int_2^5 (1+x)\,dx \\
1 &= c\left[x + \frac{x^2}{2}\right]_2^5 \\
1 &= c\left(5 + \frac{5^2}{2} - 2 - \frac{2^2}{2}\right) \\
1 &= c\left(5 + \frac{25}{2} - 2 - 2\right) \\
1 &= c\left(\frac{27}{2}\right) \\
c &= \frac{2}{27}
\end{aligned}$$

(b)

i.

$$\begin{aligned}
P(X < 4) &= \frac{2}{27}\int_2^4 (1+x)\,dx \\
&= \frac{2}{27}\left[x + \frac{x^2}{2}\right]_2^4 \\
&= \frac{2}{27}\left[4 + \frac{16}{2} - 2 - 2\right] \\
&= \frac{2}{27}[8] \\
&= \frac{16}{27}
\end{aligned}$$

ii.

$$\begin{aligned}
P(3 \leq X < 4) &= \frac{2}{27}\int_3^4 (1+x)\,dx \\
&= \frac{2}{27}\left[x + \frac{x^2}{2}\right]_3^4 dx \\
&= \frac{2}{27}\left[4 + \frac{4^2}{2} - 3 - \frac{3^2}{2}\right] \\
&= \frac{2}{27}\left[4 + \frac{16}{2} - 3 - \frac{9}{2}\right] \\
&= \frac{2}{27}\left[4 + 8 - 3 - \frac{9}{2}\right] \\
&= \frac{1}{3}
\end{aligned}$$

iii.

$$F(x) = \begin{cases} 0 & 2 < x \\ & 2 \leq x \leq 4 \\ 1 & x > 4 \end{cases}$$

$$\begin{aligned}
F(x) &= \frac{2}{27}\int_2^x (1+x)\,dx \\
&= \frac{2}{27}\left[x + \frac{x^2}{2}\right]_2^x \\
&= \frac{2}{27}\left(x + \frac{x^2}{2} - 2 + 2\right) \\
&= \frac{2}{27}x + \frac{x^2}{27}
\end{aligned}$$

(c)

$$F(x) = \begin{cases} 0 & x < 2 \\ \frac{2}{27}\left(x + \frac{x^2}{2} - 4\right) & 2 \leq x < 5 \\ 1 & x \geq 5 \end{cases}$$

$$\begin{aligned}
\int_2^x f(x) &= \frac{2}{27}\int_2^x (1+x)\,dx \\
&= \frac{2}{27}\left[x + \frac{x^2}{2}\right]_2^x \\
&= \frac{2}{27}\left(x + \frac{x^2}{2} - \left(2 + \frac{2^2}{2}\right)\right) \\
&= \frac{2}{27}\left(x + \frac{x^2}{2} - 4\right)
\end{aligned}$$

5.

(a)

$$\begin{aligned}
\int_0^2 f(x)\,dx &= \int_0^1 x\,dx + \int_1^2 2 - x\,dx \\
&= \left[\frac{x^2}{2}\right]_0^1 + \left[2x - \frac{x^2}{2}\right]_1^2 \\
&= \frac{1}{2} + \left(2(2) - \frac{2^2}{2} - 2(1) + \frac{1}{2}\right) \\
&= \frac{1}{2} + \left(4 - 2 - 2 + \frac{1}{2}\right) \\
&= 1
\end{aligned}$$

(b)

$$\begin{aligned}
P(X < 1.2) &= \int_0^{1.2} f(x)\,dx \\
&= \int_0^1 x\,dx + \int_1^{1.2} (2 - x)\,dx \\
&= \frac{1}{2} + \left[2x - \frac{x^2}{2}\right]_1^{1.2} \\
&= \frac{1}{2} + \left[2(1.2) - \frac{(1.2)^2}{2} - \left(2(1) - \frac{1}{2}\right)\right] \\
&= \frac{1}{2} + [0.18] \\
&= 0.68
\end{aligned}$$

(c)

$$F(x) = \begin{cases} 0 & x \le 0 \\ \frac{x^2}{2} & 0 < x < 1 \\ 2x - \frac{x^2}{2} - 1 & 1 \le x < 2 \\ 1 & x \ge 2 \end{cases}$$

i. Between $0 < x \leq 2:$

$$
\begin{aligned}
F(x) &= \int_0^2 f(x)\,dx \\
&= \int_0^1 x\,dx + \int_1^x 2 - x\,dx \\
&= \left[\frac{x^2}{2}\right]_0^1 + \left[2x - \frac{x^2}{2}\right]_1^x \\
&= \frac{1}{2} + \left(2x - \frac{x^2}{2}\right) - \left(2 - \frac{1}{2}\right) \\
&= \frac{1}{2} + \left(2x - \frac{x^2}{2}\right) - \frac{3}{2} \\
&= \left(2x - \frac{x^2}{2}\right) - 1 \\
&= 2x - \frac{x^2}{2} - 1
\end{aligned}
$$

6.

(a) Mean

$$
\begin{aligned}
\mu &= \sum x f(x) \\
&= 0 * \frac{8}{27} + 1 * \frac{4}{9} + 2 * \frac{2}{9} + 3 * \frac{1}{27} \\
&= 1
\end{aligned}
$$

(b) Variance

$$
\begin{aligned}
\sigma^2 &= \sum x^2 f(x) - \mu^2 \\
&= \frac{5}{3} - 1 \\
&= \frac{2}{3}
\end{aligned}
$$

7.

(a)

$$
\begin{aligned}
\sigma &= \sqrt{\sigma^2} \\
&= 3.0414
\end{aligned}
$$

8. By investing in a particular stock, a person can make a profit in 1 year of RM4,000 with probability of 0.3 or take a loss of RM1,000 with probability 0.7. What is this person's expected gain?

(a) Calculation

$$\begin{aligned}\mu &= 0.3 * 4000 + 0.7 * -1000\\ &= RM500\end{aligned}$$

(b) Conclusion

i. $\therefore$The expected gain is RM500.

9.

(a)

$$\begin{aligned}\frac{8}{52}(3-e)+\frac{8}{52}(5-e)+\frac{36}{52}(-e) &= 0\\ \frac{24}{52}-\frac{8}{52}e+\frac{40}{52}-\frac{8}{52}e-\frac{36}{52}e &= 0\\ \frac{64}{52}-\frac{52}{52}e &= 0\\ \frac{64}{52}-e &= 0\\ \frac{64}{52} &= e\\ \frac{64}{52} &= e\\ e &= \frac{16}{13}\end{aligned}$$

Therefore, she should pay $\$\frac{16}{5}$per game assuming that the game is fair.

10.

(a) Mean

$$\begin{aligned}\mu = E\left[X=x\right] &= \int_1^2 2x\left(x-1\right)dx\\ &= \int_1^2 2x\left(x-1\right)dx\\ &= \left[\frac{2x^3}{3}-x^2\right]_1^2\\ &= \left[\frac{16}{3}-4-\left(\frac{2}{3}-1\right)\right]\\ &= \frac{5}{3}litres\end{aligned}$$

(b) Variance

$$\begin{aligned}
\sigma^2 &= E\left(X^2\right) - \mu^2 \\
&= \int x^2 f\left(x\right) dx - \mu^2 \\
&= \int_1^2 2x^2\left(x-1\right) dx - \left(\frac{5}{3}\right)^2 \\
&= \int_1^2 2x^3 - 2x^2 dx - \left(\frac{5}{3}\right)^2 \\
&= \left[\frac{1}{2}x^4 - \frac{2}{3}x^3\right]_1^2 - \frac{25}{9} \\
&= \frac{1}{2}\left(2\right)^4 - \frac{2}{3}\left(2\right)^3 - \left(\frac{1}{2}\left(1\right)^4 - \frac{2}{3}\left(1\right)^3\right) - \frac{25}{9} \\
&= \frac{1}{18} litres^2
\end{aligned}$$

11.

(a) Expected proportion: $\frac{8}{15}$

(b) Calculation

$$\begin{aligned}
\mu &= \frac{2}{5}\int_0^1 x\left(x+2\right) dx \\
&= \frac{2}{5}\int_0^1 x^2 + 2x dx \\
&= \frac{2}{5}\left[\frac{x^3}{3} + \frac{2x^2}{2}\right]_0^1 \\
&= \frac{2}{5}\left[\frac{1}{3} + 1\right] \\
&= \frac{2}{5}\left(\frac{4}{3}\right) \\
&= \frac{8}{15}
\end{aligned}$$

$$\begin{aligned}
\sigma^2 &= \frac{2}{5}\int_0^1 x^3 + 2x^2 dx - \mu^2 \\
&= \frac{2}{5}\left[\frac{x^4}{4} + \frac{2x^3}{3}\right]_0^1 - \left(\frac{8}{15}\right)^2 \\
&= \frac{2}{5}\left(\frac{1}{4} + \frac{2}{3}\right) - \left(\frac{8}{15}\right)^2 \\
&= \frac{37}{450}
\end{aligned}$$

12.

(a) Mean

$$
\begin{aligned}
\mu &= \int_0^1 x\,(x)\,dx + \int_1^2 x\,(2-x)\,dx \\
&= 1 \\
&= 100 hours
\end{aligned}
$$

(b) Variance

$$
\begin{aligned}
\sigma^2 &= \int_0^1 x^2\,(x)\,dx + \int_1^2 x^2\,(2-x)\,dx - (1)^2 \\
&= \frac{1}{6}\,(variance\,of\,X)
\end{aligned}
$$