CSA - Tutorial 2 - Numerical Data Representation

December 7, 2019

1 Signed Number

1. Under what circumstances the Two's Complement is used?
   (a) To obtain the negative representation of a number in binary form.
2. Convert the 8-bit binary number 11010111 into decimal number if the binary number is a(n):
   (a) Unsigned number
      i. $1 + 2 + 4 + 16 + 64 + 128 = 215_{10}$
   (b) Signed number
      i. -41_{10}
3. Differentiate between carry flag and overflow flag. Complete the following table.

Flag	Carry
Definition	When result exceed bit available, disregarding sign
Detect in signed or unsigned number?	Unsigned numbers.
How to detect?	Extra bit generated when negative numbers are added.
Example	$1101_2 + 1110_2 = 11011_2 -> 11100_2$

4. Assuming an 8-bit system is used, show how the following operation is solved through Two's Complement method.

$$-124_{10} - 6_{10}$$

   Verify and comment the answer.
   (a) Find the representation
      i. $-124_{10} = 10000100_2$
      ii. $-6_{10} = 11111010_2$

(b) Do the "addition"

i. $-124 + (-6) = 10000100_2 + 11111010_2 = 101111110_2$ (OF)

(c) Convert back to decimal

$$01111111_2 = 127_{10}$$

(d) Conclusion: Overflow flag is detected because the result is different than the two signs of the operands. The result computed is incorrect

5. Assuming that an 8-bit system is being applied, perform the binary subtraction operation for the following decimal numbers using Two's Complement method.

$$65 - 54$$

Verify your answer by showing the answer in signed decimal value.

(a) Convert to two's complement

i. $65_{10} = 0100\,0001_2$

ii. $-54_{10} = 1100\,1010_2$

(b) Do the "addition"

i. $65_{10} + (-54_{10}) = 00001100_2$

(c) Convert back to decimal

i. $100001011_2 = 11_{10}$

6. Assuming an 8-bit system is used (i.e. the system uses 8 bits to represent an integer). Given the following decimal numbers:

$$-12 + -8$$

(a) Solve the above operation using two's complement method.

i. Convert to binary:

A. $-12_{10} = 1111\,0100_2$

B. $-8_{10} = 1111\,1000_2$

ii. Perform addition

$$1111\,0100 + 1111\,1000 = 1\,1110\,1100_2$$

(b) Verify your answer by showing the answer in signed decimal value.

i. $1110\,1100_2 = -19_{10}$

(c) Justify the validity of the answer obtained.

i. The answer is valid, even carry flag is detected because of 2's complement extra 1 when performing addition and discarded.

(d) Does overflow occur? Justify your answer.

i. No. The sign of the result is same as both of the operands.

7. Assuming an 8-bit system is involve.

   (a) Solve the following operation using Two's Complement method: (PYP-08/14: 5 marks)

   $$(-9_{10}) + (-8_{10})$$

      i. Convert to binary

         A. $11110111_2 + 11111000_2 = 1\,1110\,1111_2$

   (b) Verify your answer by showing the answer in signed decimal value. (PYP-08/14: 1 mark)

      i. Discard carry bit,

      ii. $1110\,1111_2 = -17_{10}$

   (c) Justify the validity of the answer obtained. (PYP-08/14: 4 marks)

      i. The answer is valid, because overflow flag is not detected. The sign of the final result is same as the sign of the two operands.

   (d) Does overflow or/and carry occur?

      i. Carry occured. But overflow did not.

2 Section B: Floating Point Number

1. Perform the following number conversions. Show your conversion steps clearly. If the operation is illogical, explain the reason.

   (a) 30.30_{10} to Binary

      i. Illogical. Because the fraction part do not have a denominator that is a power of two.

   (b) 123.123_5 to Decimal

      i. 38.304_{10}

   (c) $100\,100\,011\,111.11_2$ to Octal

      i. 4437.6_8

2. Perform the following operations. Show your working steps clearly. **If the operation is illogical, explain the reason.**

   (a) Convert $6A.96_{10}$ to hexadecimal number

      i. Illogical. In decimal, there is no A.

   (b) Convert 1807.65_{10} into a hexadecimal number

      i. $70F.A6_{16}$

      ii. Do the front part, then times the back part, the back part is repeating

   (c) Convert 101011.0111_2 into a decimal number

      i. $32 + 8 + 2 + 1 = 43$

ii. $(0 + 0.25 + ...)$
iii. Answer: 43.4375

(d) $111100110011.11000001_2 + 20.5_{10}$. Show your answer in Hex format. (PYP-08/12: 3 marks)

i. Convert the fraction part of the binary figure to decimal.

$$0.11000001 = \frac{1}{2} + \frac{1}{4} + \frac{1}{2^8} = \frac{193}{256}$$

ii. Convert the binary to hex

$$1111\,0011\,0011.1100\,0001_2 = F33.C1_{16}$$

iii. Convert the decimal to binary

A. $20.5 = 10100.1_2$

iv. Convert binary to hex

$$0001\,0100.1000_2 = 14.8_{16}$$

v. Add them together

$$F33.C1_{16} + 14.8_{16} = F48.41_{16}$$

3. Given that:
- An Excess-52 notation is applied.
- The implied decimal point is at the beginning of the mantissa.
- A "5" is used to represent a positive number and a "9" is used to represent a negative number.

(a) Convert −357.24610 to the SEEMMMMM format. (PYP-04/14: 2 marks)

95535725

(b) Convert 55220311 to scientific notation.

0.20311

(c) Convert 95575321 to scientific notation.

$$(-1)^1 \cdot 10^{55-52} \cdot 0.75321 = -0.75321 * 10^3$$

(d) Convert 30.815_{10} to the SEEMMMMM format.

55430815

4. [Popular Question] The following decimal numbers are stored in excess-50 floating point format. A "1"is used to represent a negative sign, and a "5" for positive sign.
Perform the following operations, and present them in standard decimal sign-and-magnitude notation.

(a) $55020311 + 15375321 = 1\,5375300689$

   i. -0.75301 * 10^3

(b) $15176323 * 15486496$

   i. Negative * negative = positive. So 5
   ii. $10^1 * 10^4 = 10^{1+4} = 10^5$, therefore it is 55
   iii. Convert to binary:
      A. 10010101000100011
      B. 10100110111111000
   iv. Multiply binary numbers together
      A. 1100001001111000001011000111101000
   v. Convert back to decimal
      A. 6525311208
   vi. Trim down to 5 digits
      A. 65253
   vii. $5\,55\,65253 = 0.65253 * 10^5$

(c) $55152295 - 15256608$

   i. $a - (-b) = a + b$, therefore, this is an addition, first sign is 5
   ii. Adjust the exponent, perform addition
   iii. SEMMMM: $5\,52\,61838$
      A. $5\,5261838 = 0.61838 * 10^2$

(d) For divide

   i. Change the signs (for the power)
   ii. + becomes minus, - becomes +

5. The floating point decimal numbers below are stored in the form of SEEMMMMM where the exponent is stored in excess-50 with the implied decimal point at the beginning of the mantissa. A 4 in the sign position indicates a positive number and a 3 indicates a negative number:
   45320460
   35520112

   (a) Add these two numbers. Show the result in sign-magnitude notation. (PYP-08/12: 3 marks)
      i. Adjust exponent: $4\,55\,0020460$
      ii. Addition: $3\,55\,20112$
      iii. SEEMMMM: 35519907(40)
      iv. Sign magnitude: $0.19907 * 10^5$
   (b) Multiply these two numbers. Show the result in sign-magnitude notation. (PYP-08/12: 3 marks)
      i. Adjust: $53 + 55 - 50 = 58 = 10^8$

$$-411491.52 = -0.41149 * 10^7$$

ii. SEEMMMM: 3 57 41149

6. Show how the number -5.5_{10} is stored in the computer's storage using IEEE754 32-bit single precision format. You are required to show your conversion steps clearly. (PYP-01/14: 6 marks)

   (a) Convert to binary
   $$101.1$$
   (b) Add exponent
   $$101.1 * 2^0$$
   (c) Position decimal point
   $$1.011 * 2^6$$
   (d) Exponent
   $$127 + 2 = 129$$
   (e) Change 129 to binary
   $$10000001$$
   (f) Sign of mantissa
      i. 1
   (g) Full 32-bit IEEE 754
      i.
   $$110000001011...$$

7. Represent the binary number -10111.01 into IEEE754 single precision format. You are required to show your conversion steps clearly. (PYP-08/15: 5 marks).

   (a) Convert to binary
      i. Already binary
   (b) Add exponent
      i. $10111.01 * 2^0$
   (c) Position decimal point
      i. $1.011101 * 2^4$
   (d) Exponent
      i. $127 + 4 = 131$
   (e) Change exponent to binary
      i. 10000011
   (f) Sign of mantissa
      i. $-$, so 1
   (g) Mantissa in 23 bits

i. 01110100000000000000000

(h) IEEE754 format

$$11000001101110100000000000000000$$

8. Given a decimal number "-30.8125", how this notation can be represented in the IEEE754 single precision notation. You are required to show your working steps.

   (a) Convert to binary

   $$30.8125 = 11110.1101$$

   (b) Add exponent

      i. $10111.01 * 2^0$

   (c) Position decimal point

      i. $1.11101101 * 2^4$

   (d) Exponent

      i. $127 + 4 = 131$

   (e) Change exponent to binary

      i. 10000011

   (f) Sign of mantissa

      i. $-$, so 1

   (g) Mantissa in 23 bits

      i. 11101101000000000000000

   (h) IEEE754 format

   $$1\,1000\,0011\,1110\,1101\ldots$$

9. Given an IEEE754 single precision notation below, show how this notation can be represented in a sign-magnitude notation. You are required to show your working steps.

   $$1\,1000\,0010\,0100\,1000\,0000\,0000\,0000\,000$$

   Assuming that excess-127 is applied.

   (a) First digit is 1, therefore, it is negative

   (b) Convert exponent to decimal

      i. $10000010 = 130$

   (c) Remove excess-127 from exponent

      i. $130 - 127 = 3$

   (d) Convert mantissa to decimal

      i. $0100\,1000 = 1 * 2^{-2} + 1 * 2^{-5} = 0.25 + 0.03125 = 0.28125$

   (e) Write the entire sign-magnitude figure down (without proper decimal place)

   $$N = 1.28125 * 10^3$$