Statistics II T6 - Hypothesis Testing

August 27, 2019

1. The mean weekly sale of the Chocolate Bar in candy stores is 146.3 bars per store with standard deviation of 17.2 bars. After an advertising campaign, the mean weekly sale in 22 stores for a typical week is 153.7 bars with a standard deviation of 16.7 bars. Was the advertising campaign successful? Use $\alpha = 0.05$. **(check with teacher why not T-score)**

   (a) $\mu_b = 146.3, \sigma_b = 17.2, n_a = 22, \bar{x}_a = 153.7, s_a = 16.7, \alpha = 0.05, D = 22 - 1 = 21$

   (b) Hypothesis

      i. $H_0 : \mu_a - \mu_b \leq 0$ The advertising campaign is not successful, the mean weekly sales did not increase

      ii. $H_1 : \mu_a - \mu_b > 0$ The advertising campaign is successful, the mean weekly sales increased

   (c) **Right-tail test**

   (d) Critical value

      i. $\alpha = 0.05$

      ii. $Z_{0.05} = 1.6449$

   (e) Rejection & non-rejection region

      i. Rejection region: $Z > 1.6449$

      ii. Non-rejection region: $Z \leq 1.6449$

   (f) Find the Z-score

$$Z = \frac{\bar{x}_a - \mu_b}{\frac{s_a}{\sqrt{n}}}$$
$$= \frac{153.7 - 146.3}{\frac{16.7}{\sqrt{22}}}$$
$$= 2.078$$

   (g) Conclusion

      i. Since $Z = 2.078 > 1.6449$, we reject H_0 and conclude that, at 5% significance, the advertising campaign is successful, the mean weekly sales increased.

2. The manufacturer of a certain brand of car batteries claims that the mean life of these batteries is 45 months. A consumer protection agency that wants to check this claim took a random sample of 36 such batteries and found that the mean life for this sample is 43.15 months. The lives of all such batteries have a normal distribution with standard deviation of 6.1 months. Test the hypothesis that the mean life of these batteries is less than 45 months at the 2.5% significance level.

   (a) Let subscript-o referes to the original batteries claims, and subscript-s refer to the sampled batteries

   (b) $\mu_o = 45, n_s = 36, \bar{x}_s = 43.15, s_s = 6.1, \alpha = 0.025$

   (c) Hypothesis

       i. $H_0 : \mu_s \geq 45$ The mean life of the batteries is 45 months or more

       ii. $H_0 : \mu_s < 45$ The mean life of batteries is 45 months or less

   (d) **Left-tail test**

   (e) Find critical value (note, since $n > 30$, we can use $Z - score$)

       i. $-Z_{0.025} = -1.96$

   (f) Find rejection/non-rejection range

       i. Rejection range: $Z < -1.96$

       ii. Non-rejection range: $Z > -1.96$

   (g) Find Z-score

$$Z = \frac{\bar{x}_s - \mu}{\frac{S}{\sqrt{n}}}$$
$$= \frac{43.15 - 45}{\frac{6.1}{\sqrt{36}}}$$
$$= -1.820$$

   (h) Conclusion

       i. Since $Z = -1.820 > -1.96$, we failed to reject H_0. Therefore, there is insufficient evidence to show that the mean life of batteries is 45 months or less.

3. The manufacturer of a certain oil-additive claims that the mean net weight of jars of his product is 1 kg. A random sample of size 49 of a large consignment supplied to your company is found to have a mean net weight of 0.99 kg with a standard deviation of 0.02 kg. Test the manufacturer's claim at $\alpha = 0.05$.

   (a) Let subscript-o refers to the claim of the manufacturer. Let subscript s refers to the sampled consignment.

       i. $\mu_o = 1, n = 49, \bar{x}_s = 0.99, s_s = 0.02, \alpha = 0.05$
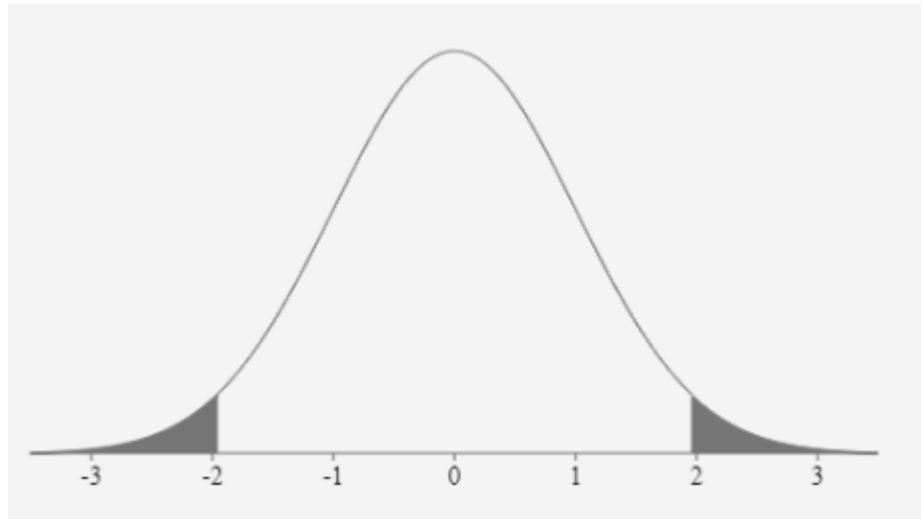
(b) Hypothesis

    i. $H_0 : \mu_s = 1$ The mean net weight of jars of his product is 1kg.

    ii. $H_1 : \mu_s \neq 1$. The mean net weight of jars of his product is NOT 1kg.

(c) **Two-tailed test**

(d) Critical value

    i. $\frac{\alpha}{2} = 0.025$

    ii. $Z_{\frac{\alpha}{2}} = \pm 1.96$

(e) Rejection & non-rejection region

    i.

    ii. Rejection region: $Z < -1.96$ or $Z > 1.96$

    iii. Non-rejection region: $-1.96 < Z < 1.96$

(f) Find your Z-score

    i. $Z = \frac{\bar{X}_s - \mu_o}{\frac{S_s}{\sqrt{n}}}$

$$Z = \frac{0.99 - 1}{\frac{0.02}{\sqrt{49}}}$$

$$= -3.5$$

(g) Conclusion

    i. Since $Z = -3.5 < -1.96$. The event is rare enough for us to reject H_0, and conclude that the mean net weight of jars of his product is NOT 1kg.

4. The mean weight of all babies born at a hospital last year was 3.45 kg. A random sample of 35 babies born at this hospital this year produced the following data in kg.

(a) 3.72 4.13 3.13 2.63 2.90 4.67 5.49 4.13 2.68 3.31 5.08 3.76 2.95 3.22
3.63 4.17 2.59 4.31 3.76 2.86 2.22 3.45 4.58 4.17 3.81 3.40 3.27 3.76
3.27 4.40 2.72 3.67 2.77 3.76 3.04

(b) Test at the 3% significance level whether the mean weight of babies
born at this hospital this year is exceed last year.

(c) **Answer:**

    i. Let subscript-o be the babies born in hospital last year, and let
subscript-n be the babies born at this hospital this year.

        A. $\mu_o = 3.45, n_n = 35, \bar{x}_n = 3.5831, s_n = 0.7487, \alpha = 0.03$

    ii. Hypothesis

        A. $H_o : \mu_n \leq 3.45$ The mean weight of babies born at this
hospital this year is equal or less than last year.

        B. $H_1 : \mu_n > 3.45$ The mean weight of babies born at this
hospital this year is exceed last year.

    iii. **Left-tail**

    iv. Critical value

        A. $Z_{0.03} = 1.8808$

    v. Rejection & non-rejection region

        A. Rejection: $Z \geq 1.8808$

        B. Non-rejection: $Z < 1.8808$

    vi. Find the Z-score

        A.

$$Z = \frac{3.5831 - 3.45}{\frac{0.7487}{\sqrt{35}}}$$
$$= 1.0517$$

    vii. Conclusion: Since $Z = 1.0517 < 1.8808$, failed to reject H_0.

5. A soft-drink vending machine is set to dispense 250 ml per cup. If the
machine is tested 25 times, yielding a mean cup fill of 241.80 ml with a
standard deviation of 6.67 ml, is this evidence at the 1% significance level
that the machine is underfilling?

(a) $\mu = 250ml, n = 25, \bar{x} = 241.80, s = 6.67, \alpha = 0.01$

(b) Hypothesis

    i. $H_0 : \mu \geq 250$, The machine is not underfilling (sentence may not
be required)

    ii. $H_1 : \mu < 250$, The machine is underfilling (sentence may not be
required)

(c) **Left-tail**

(d) Critical value

  i. $-t_{0.01;24} = -2.492$

(e) Rejection range

  i. $T < -2.492$

(f) Check Z-score

  i.

$$T = \frac{241.80 - 250}{\frac{6.67}{\sqrt{25}}}$$
$$= -6.147$$

(g) Conclusion

  i. Since the $T = -6.147 < -2.3263$, H_0 has been rejected. There-
  fore, we can conclude that the machine is underfilling at 1%
  significance level.

6. A computer company that recently introduced a new software product
claims that the mean time it takes to learn how to use this software is not
more than 2 hours for people who are somewhat familiar with computers.
A random sample of 12 such persons was selected. The following data
give the times taken (in hours) by these persons to learn how to use this
software.

 (a) 1.75 2.25 2.40 1.90 1.50 2.75 2.15 2.25 1.80 2.20 3.25 2.60

 (b) Test at the 1% significance level whether the company's claim is
  true. Assume that the times taken by all persons who are some-
  what familiar with computers to learn how to use this software are
  approximately normally distributed.

 (c) $\mu = 2, n = 12, \alpha = 0.01, \bar{x}_n = 2.2333, s = 0.4807, \alpha = 0.01, D.O.F = 11$

 (d) Hypothesis

  i. $H_0 : \mu \geq 2$ (the mean time it takes to learn how to use this
  software is more than or equal to 2 hours)

  ii. $H_1 : \mu < 2$ (the mean time it takes to learn how to use this
  software is not more than 2 hours for people)

 (e) **Left-tail test**

 (f) Critical value

  i. $-t_{0.01;11} = -2.718$

 (g) Rejection region & non-rejection region

  i. Rejection region: $Z < -2.3263$

  ii. Non-rejection region: $Z \geq -2.3263$

 (h) Find the T-score

i. $-T = \frac{\bar{X}_n - \mu_o}{\frac{S}{\sqrt{n}}}$

$$-T = \frac{2.2333 - 2}{\frac{0.4807}{\sqrt{12}}}$$

$$\boldsymbol{T = -1.681}$$

(i) Conclusion

    i. Since $T = -1.681 \geq -2.3263$, we are unable to reject H_0. Therefore, we do not have enough evidence to say that the mean time it takes to learn how to use this software is not more than 2 hours for people.

7. A machine assesses the life of a ball-point pen, by measuring the length of a continuous line drawn using the pen. A random sample of 80 pens of brand X has a mean writing length of 1.21 km. A random sample of 75 pens of brand Y has a mean writing length of 1.25 km. Assuming that the standard deviation of the writing length of a single pen is 0.15 km for both brands, test at the 5% level of significance, whether the mean writing lengths of the two brands differ significantly.

(a) $n_x = 80, \bar{x}_x = 1.21, n_y = 75, \sigma = 0.15, \bar{x}_y = 1.25, \alpha = 0.05$

(b) Hypothesis

    i. $H_0 : \mu_1 - \mu_2 = 0$. The mean writing lengths of the two brands does not differ significantly

    ii. $H_1 : \mu_1 - \mu_2 \neq 0$. The mean writing lengths of the two brands differ significantly.

(c) **Two-tail test**

(d) Find the critical point & the rejection range

    i. Since n for both cases is more than 30, we can use Normal distribution

    ii. Critical point: $Z_{\frac{\alpha}{2}} = Z_{0.025} = \pm 1.96$

    iii. Rejection range: $Z \leq -1.96$ or $Z \geq 1.96$

    iv. Non-rejection range: $-1.96 \leq Z \leq 1.96$

(e) Find the $Z - score$

    i.

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{0.15\sqrt{\frac{1}{80} + \frac{1}{75}}}$$

$$= \frac{(1.21 - 1.25) - 0}{0.15\sqrt{\frac{1}{80} + \frac{1}{75}}}$$

$$= -1.659$$

(f) Conclusion

    i. Since $Z = -1.659, -1.96 \leq Z \leq 1.96$, we failed to reject H_0. Hence, we cannot conclude that the mean writing lengths of the two brands differ significantly.

8. The management of a supermarket wanted to investigate whether the male customers spend less money on average than the female customers. A sample of 35 male customers who shopped at this supermarket showed that they spent an average of RM80 with a standard deviation of RM17.50. Another sample of 40 female customers who shopped at the same supermarket showed that they spent an average of RM96 with a standard deviation of RM14.40. Using the 2% significance level, can you conclude that the mean amount spent by all male customers at this supermarket is less than that spent by all female customers?

$$n_m = 35, \bar{x}_m = 80, S_m = 17.50$$

$$n_f = 40, \bar{x}_f = 96, S_f = 14.40$$

$$\alpha = 0.02$$

(a) Hypothesis

    i. $H_0 : \mu_b - \mu_f \geq 0$ the mean amount spent by all male customers at this supermarket is equal to or more than that spent by all female customers.

    ii. $H_1 : \mu_b - \mu_f < 0$ the mean amount spent by all male customers at this supermarket is less than that spent by all female customers.

(b) **Left-tail test**

(c) Critical value, since both $n > 30$, we can use Normal approximation

    i.

$$-Z_{0.02} = 2.0537$$

$$Z_{0.02} = -2.0537$$

(d) Rejection & non-rejection region

    i. Rejection region: $Z < -2.0537$

    ii. Non-rejection region: $Z \geq -2.0537$

(e) Calculate Z-score

    i. Calculate Z-score (Note: don't pool the variance, only pool on $t - distribution$

$$Z = \frac{(\bar{x}_m - \bar{x}_f) - (\mu_m - \mu_f)}{\sqrt{\frac{S_m^2}{n_m} + \frac{S_f^2}{n_f}}}$$

$$= \frac{(80 - 96) - 0}{\sqrt{\frac{17.50^2}{35} + \frac{14.40^2}{40}}}$$

$$= -4.286$$

(f) Conclusion: Since $Z = -4.286 < -2.0537$, we reject H_0, and conclude that at 2% significance value, the mean amount spent by all male customers at this supermarket is less than that spent by all female customers.

9. The manager of a package courier service believes that packages shipped at the end of the month are heavier than those shipped early in the month. To test this, he weighed a random sample of 20 packages at the beginning of the month. He found that the mean weight was 20.25 kg with standard deviation was 5.84 kg. Ten packages randomly selected at the end of the month had a mean weight of 24.80 kg with standard deviation of 5.67 kg. At the 0.05 significance level, can we conclude that the packages shipped at the end of the month weigh more? Assume equal variances between the packages shipped at the early of the month and at the end of the month.

   (a) $n_b = 20, \bar{x}_b = 20.25, s_b = 5.84$

   (b) $n_a = 10, \bar{x}_a = 24.80, s_a = 5.67$

   (c) $\alpha = 0.05$

   (d) Hypothesis

      i. $H_0 : \mu_b - \mu_a \leq 0$ (packages shipped at the end of the month DO NOT weigh more)

      ii. $H_1 : \mu_b - \mu_a > 0$ (packages shipped at the end of the month weigh more)

   (e) **Right-tail test**

   (f) Since both the n of the data are < 30, and we do not know σ, we must use $T - score$.

   (g) Find the $t - distribution$ value

      i. $t_{0.05;10+20-2} = t_{0.05;28} = 1.701$

   (h) Find the rejection/non-rejection range

      i. Rejection range: $T \leq 1.701$

      ii. Non-rejection range: $T > 1.701$

   (i) Find the T-score

      i. $S_p = \sqrt{\frac{(n_b-1)S_b^2*(n_a-1)S_a^2}{n_b+n_a-2}}$

$$S_p = \sqrt{\frac{(n_b - 1)\,S_b^2 + (n_a - 1)\,S_a^2}{n_b + n_a - 2}}$$
$$= \sqrt{\frac{(20 - 1)\,5.84^2 + (10 - 1)\,5.67^2}{20 + 10 - 2}}$$
$$= 5.7859$$

ii.

$$T = \frac{(20.25 - 24.80) - 0}{5.7859 \cdot \sqrt{\frac{1}{10} + \frac{1}{20}}}$$

$$= -2.030$$

(j) Conclusion

    i. Since the $T - score = -2.030 \leq 1.701$. W reject H_0, hence, at 5% significance level, we conclude that packages shipped at the end of the month weigh more.

10. Computer response time is defined as the length of time a user has to wait for the computer to access information on the disk. Suppose a data center wants to compare the average response times of its two computer disk drives. To test this, independent random samples of 13 response times for disk 1 and 15 response times for disk 2 were selected. The data (recorded in milliseconds) are as follows.

(a) **Disk 1**: 59 73 74 61 92 60 84 33 54 73 47 102 75

(b) **Disk 2**: 71 63 40 34 38 48 60 75 47 41 44 86 53 68 39

(c) Assume that the response times for both disks having same variances/standard deviation $\sigma_1 = \sigma_2$. Test at the 1% significance, whether the mean response time of the two disks differs significantly.

(d)

    i. $\bar{x}_1 = 68.2308, s_1 = 18.6599, n_1 = 13$

    ii. $\bar{x}_2 = 53.8, s_2 = 15.8077, n_2 = 15$

    iii. $\alpha = 0.01$

    iv. State the hypothesis

        A. Claim: the mean response time of the two disks differ significantly. (H_1)

        B. Opposite: the mean response time of the two disks do not differ significantly. (H_0)

    v. $H_0 : \bar{x}_1 - \bar{x}_2 = 0$

    vi. $H_1 : \bar{x}_1 - \bar{x}_2 \neq 0$

    vii. Two tail test. Find the critical point $t_{\frac{\alpha}{2};13+15-2}$

$$t_{0.005;26} = 2.779$$

    viii. The rejection region

        A. $T > 2.797, T < -2.797$

ix. Find the T-score

$$S_p = \sqrt{\frac{(n_1 - 1)\,S_1^2 + (n_2 - 1)\,S_2^2}{n_1 + n_2 - 2}}$$

$$= \sqrt{\frac{12 \cdot 18.6599^2 + 14 \cdot 15.8077^2}{13 + 15 - 2}}$$

$$= 17.1830$$

$$T = \frac{(68.2308 - 53.8) - 0}{17.1830 * \sqrt{\frac{1}{13} + \frac{1}{15}}}$$

$$= 2.2163$$

x. Since T-score is outside the rejection region. We cannot reject H_0, and hence we do not have enough evidence to conclude that at 1% significance, that the mean response time of the two disks differ significantly.

11. A company claims that its 12-week special exercise program significantly reduces weight. A random sample of six persons was selected and these persons were put on this exercise program for 12 weeks. The following table gives the weights (in kg) of those six persons before and after the program.
Before the program 81.6 88.5 80.3 100.2 94.3 90.3
After the program 83.0 84.8 73.0 92.5 89.4 85.7

(a) Test the hypothesis that the 12-week special exercise program significantly reduces weight at $\alpha = 0.01$.

i. Claim: the 12-week special exercise program significantly reduces weight H_1

ii. Oppo: the 12-week special exercise program do not significantly reduces weight H_0

iii. $H_0 : \mu_D \leq 0$

iv. $H_1 : \mu_D > 0$

v. Right-sided test, $t_{0.01;6-1} = 3.365$

vi. Rejection:

A. $T > 3.365$

vii. Note: need to decide on what is $\bar{D}$, in this case, it is $Before - After$

$$\bar{D} = 4.4667$$

$$S_D = 3.2770$$

$$T = \frac{4.4667 - 0}{\frac{3.2770}{\sqrt{6}}}$$

$$= 3.3388$$

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viii. Since $3.3388 < 3.365$, T is not in the rejection region, we failed to reject H_0. Hence, we do not have enough evidence to state that the 12-week special exercise program do not significantly reduces weight.

12. A coin is thrown 1000 times and 546 tails are obtained. Test whether the coin is fair at $\alpha = 0.04$.

$$\hat{p} = \frac{546}{1000} = 0.546$$
$$\hat{q} = 1 - 0.546 = 0.454$$
$$n = 1000$$
$$p = 0.5$$
$$\alpha = 0.04$$

(a) Claim: The coin is fair $H_0 : p = 0.5$

(b) Oppo: The coin is unfair $H_1 : p \neq 0.5$

(c) Two-tail test.
$$Z_{\frac{0.04}{2}} = 2.0537$$

(d) Rejection region

    i. $Z > 2.0537, Z < -2.0537$

(e) Test statistics

$$Z = \frac{0.546 - 0.5}{\sqrt{\frac{0.5*0.5}{1000}}}$$
$$= 2.9093$$

(f) Since $Z = 2.9093 > 2.0537$, we reject H_0 and hence have enough evidence to conclude that at 4% significance level, that the coin is not fair.

13. A manufacturer claimed that at least 95% of the equipment which he supplied to a factory conformed to specifications. An examination of a sample of 200 pieces of equipment revealed that 18 were faulty. Test the manufacturer's claim at a significance level of 0.05.

(a) $p = 0.95$, $\hat{p} = \frac{200-18}{200} = 0.91$, $\hat{q} = 0.09$, $n = 200$, $\alpha = 0.05$

(b) Claim: at least 95% of the equipment which he supplied to a factory conformed to specifications (H_0)

(c) Oppo: Less than 95% of the equipment which he supplied to a factory conformed to specifications (H_1)

(d) $H_0 \geq 0.95$

(e) $H_1 < 0.95$

(f) Right-tail test:

    i. $Z_{0.05} = -1.645$

(g) Rejection region: $Z_{0.05} < -1.645$

(h) Test-statistic:

$$Z = \frac{0.91 - 0.95}{\sqrt{\frac{0.95*0.05}{200}}}$$
$$= -2.596$$

(i) Since $Z = -2.596 < -1.645$, we reject H_0 and DO NOT have enough evidence to conclude that at least 95% of the equipment which he supplied to a factory conformed to specifications.

14. The manager of a supermarket chain wants to determine the difference between the proportion of morning shopper who are men and the proportion of afternoon shoppers who are men. Over a period of 2 weeks, the chain's researchers conduct a systematic random sample survey of 400 morning shoppers, which reveals that 352 are women and 48 are men. During this same period, a systematic random sample of 480 afternoon shoppers reveals that 312 are women and 168 are men. At the 1% significance level, is there a difference between the proportion of men shopper during morning and afternoon period?

(a) Let p be the proportion of shoppers who are male

    i. $n_m = 400, \hat{p}_m = \frac{48}{400} = 0.12, \hat{q}_m = 1 - 0.1364 = 0.88$

    ii. $n_a = 480, \hat{p}_a = \frac{168}{480} = 0.35, \hat{q}_a = 0.65$

    iii. $\hat{p} = \frac{48+168}{400+480} = 0.2454, \hat{q} = 0.7546$

    iv. $\alpha = 0.01$

(b) Claim: there a difference between the proportion of men shopper during morning and afternoon period (H_1)

(c) Opp: there are no difference between the proportion of men shopper during morning and afternoon period (H_0)

(d) $H_0 : p_{m-a} = 0$

(e) $H_1 : p_{m-a} \neq 0$

(f) Two-tail test

(g) Find the critical value

    i. $Z_{\frac{0.01}{2}} = Z_{0.005} = 2.5758$

(h) Rejection region: $Z < -2.5758, Z > 2.5758$

(i) Find test statistics:

$$\hat{p} = \frac{48 + 168}{400 + 480}$$

$$Z = \frac{0.12 - 0.35}{\sqrt{0.2454 * 0.7546 \left(\frac{1}{400} + \frac{1}{480}\right)}}$$

$$= -7.894$$

(j) Since $Z = -7.894 < -2.5758$,and falls in between the rejection region, we reject H_0 and conclude that at 1% significance level, there is enough evidence to suggest that there a difference between the proportion of men shopper during morning and afternoon period.

15. One thousand items from factory B are examined and found to contain 3% defectives. One thousands five hundred similar items from factory C are found to contain only 2% defectives. Can you conclude that the items of factory C are superior to those of factory B? Use $\alpha = 0.05$.

$$n_B = 1000, \hat{p}_B = 0.03, \hat{q}_B = 0.97$$

$$n_C = 1500, \hat{p}_c = 0.02, \hat{q}_C = 0.98$$

(a) Claim: the items of factory C are superior to those of factory B. (H_1)

(b) Opp: the items of factory C are not superior to those of factory B. (H_0)

(c) $H_1 : p_{B-C} > 0$

(d) $H_0 : p_{B-C} \leq 0$

(e) **Right-tail test**

(f) Find the critical value
$$Z_{0.05} = 1.6449$$

(g) Rejection region:
  i. $Z > 1.6449$

(h) Test statistic

$$\hat{p} = \frac{x_B + x_C}{n_B + n_C}$$
$$= \frac{(0.03 * 1000) + (0.02 * 1500)}{1000 + 1500}$$
$$= 0.024$$

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}\hat{q} \cdot \left(\frac{1}{n_B} + \frac{1}{n_C}\right)}}$$
$$= \frac{0.03 - 0.02}{\sqrt{0.024 \cdot 0.976 \cdot \left(\frac{1}{1000} + \frac{1}{1500}\right)}}$$
$$= 1.600$$

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(i) Since $Z = 1.600 < 1.6449$, we failed to reject H_0. Hence there is insufficient evidence to state that the items of factory C are superior to those of factory B.

16. Note:

   (a) While writing the critical value for left-tailed test

   $$-Z_a = ...$$
   $$Z_a = -....$$

      i. DO NOT write:
         A. $-Z_a = -...$
         B. Because it will become $Z_a = ...$

   (b) Accept if only draw/only write the critical value and rejection region part. However, conclusion must be there