Mathematical Formulae

Trigonometry $\cos^2 A + \sin^2 A = 1$ $\quad 1 + \tan^2 A = \sec^2 A$ $\quad \cot^2 A + 1 = \operatorname{cosec}^2 A$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$\sin P + \sin Q = 2 \sin \frac{1}{2}(P+Q) \cos \frac{1}{2}(P-Q)$

$\sin P - \sin Q = 2 \cos \frac{1}{2}(P+Q) \sin \frac{1}{2}(P-Q)$

$\cos P + \cos Q = 2 \cos \frac{1}{2}(P+Q) \cos \frac{1}{2}(P-Q)$

$\cos P - \cos Q = -2 \sin \frac{1}{2}(P+Q) \sin \frac{1}{2}(P-Q)$

$\sin 3A = 3 \sin A - 4 \sin^3 A$

$\cos 3A = 4\cos^3 A - 3 \cos A$

if $t = \tan \frac{1}{2} A$, then $\sin A = 2t/(1+t^2)$

$\cos A = (1 - t^2)/(1 + t^2)$

Differentiation $\dfrac{d}{dx}(uv) = v\dfrac{du}{dx} + u\dfrac{dv}{dx}$

$\dfrac{d}{dx}\left(\dfrac{u}{v}\right) = \dfrac{1}{v^2}\left(v\dfrac{du}{dx} - u\dfrac{dv}{dx}\right)$

Integration $\int u\,dv = uv - \int v\,du$

$\int \left(\dfrac{f'}{f}\right) dx = \ln f$

y	$\frac{dy}{dx}$	y	$\int y\,dx + c$
x^n	nx^{n-1}	x^n	$\frac{1}{n+1}x^{n+1}$
$\ln x$	$\frac{1}{x}$	$\frac{1}{x}$	$\ln x$
$\ln(ax+b)$	$\frac{a}{ax+b}$	$\frac{1}{ax+b}$	$\frac{1}{a}\ln(ax+b)$
e^{mx}	me^{mx}	e^{mx}	$\frac{1}{m}e^{mx}$
$\sin mx$	$m \cos mx$	$\sin mx$	$-\frac{1}{m}\cos mx$
$\cos mx$	$-m \sin mx$	$\cos mx$	$\frac{1}{m}\sin mx$
$\tan mx$	$m \sec^2 mx$	$\tan mx$	$-\frac{1}{m}\ln \cos mx$
$\cot mx$	$-m \operatorname{cosec}^2 mx$	$\cot mx$	$\frac{1}{m}\ln \sin mx$
$\sec mx$	$m \sec mx \tan mx$	$\sec x$	$\ln(\sec x + \tan x) = \ln \tan\left(\frac{x}{2} + \frac{\pi}{4}\right)$
$cosec\ mx$	$-m \operatorname{cosec} mx \cot mx$	$cosec\ x$	$-\ln(\operatorname{cosec} x + \cot x) = \ln \tan \frac{x}{2}$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$	$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\frac{x}{a}$
$\tan^{-1} x$	$\frac{1}{x^2+1}$	$\frac{1}{x^2+a^2}$	$\frac{1}{a}\tan^{-1}\frac{x}{a}$
$\sinh^{-1}\frac{x}{a}$	$\frac{1}{\sqrt{a^2+x^2}}$	$\frac{1}{x^2-a^2}$	$\frac{1}{2a}\ln\left(\frac{x-a}{x+a}\right), x > a$
$\cosh^{-1}\frac{x}{a}$	$\frac{1}{\sqrt{x^2-a^2}}$	$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\ln\left(\frac{a+x}{a-x}\right), \lvert x\rvert < a$
$\sinh mx$	$m \cosh mx$	$e^{ax}\sin bx$	$\frac{e^{ax}}{a^2+b^2}(a \sin bx - b \cos bx)$
$\cosh mx$	$m \sinh mx$	$e^{ax}\cos bx$	$\frac{e^{ax}}{a^2+b^2}(a \cos bx + b \sin bx)$

Arc length $L = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ $L = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Surface area of revolution $S_x = 2\pi \int_{x_1}^{x_2} y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$ $S_y = 2\pi \int_{y_1}^{y_2} x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

$$S_x = 2\pi \int_{t_1}^{t_2} y(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt \qquad S_y = 2\pi \int_{t_1}^{t_2} x(t)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

Area bounded by parametric equations

$$\boldsymbol{x = f(t), y = g(t), \alpha \le t \le \beta}$$
$$A = \int_{\alpha}^{\beta} g(t)f'(t)dt$$

Integrating factor $\mu = e^{\int P dx}$, if $\frac{dy}{dx} + Py = Q$

Newton-Raphson iteration $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Maclaurin's Series $f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \cdots$

Special Maclaurin's Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \cdots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + \frac{(-1)^n x^{2n}}{(2n)!} + \cdots$$

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots \frac{(-1)^n x^{n+1}}{n+1} + \cdots (-1 < x \le 1)$$

$$(1 + x)^r = 1 + rx + \frac{r(r-1)}{2!}x^2 + \frac{r(r-1)(r-2)}{3!}x^3 + \cdots + \frac{r(r-1)\ldots(r-n+1)}{n!}x^n + \cdots (-1 < x \le 1)$$

Taylor's Series $f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$

Euler's method $y_n = y_{n-1} + hF(x_{n-1}, y_{n-1})$ where h is the step size

Binomial Theorem

$(a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k}b^k$, where $\binom{n}{k} = \frac{n!}{(n-k)!k!}$, n is any positive integer

Binomial Series

$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$, n is any real number and $|x| < 1$