D.Maths - Tutorial 5

January 13, 2020

1 Negate and simplify each of the following.

1. $\exists x \ni [p(x) \vee q(x)]$

$$\begin{aligned} \sim (\exists x \ni [p(x) \vee q(x)]) &\equiv \forall x \ni \sim [p(x) \vee q(x)] \\ &\equiv \forall x \ni \sim p(x) \wedge \sim q(x) \end{aligned}$$

2. $\forall x, [p(x) \wedge \sim q(x)]$

$$\sim (\forall x, [p(x) \wedge \sim q(x)]) \equiv \exists x, \sim p(x) \vee q(x)$$

3. $\forall x, [p(x) \rightarrow \sim q(x)]$

$$\begin{aligned} \sim (\forall x, [p(x) \rightarrow \sim q(x)]) &\equiv \exists x, \sim (p(x) \rightarrow \sim q(x)) \\ &\equiv \exists x, \sim p(x) \wedge \sim q(x) \end{aligned}$$

4. $\exists x \ni [p(x) \vee q(x) \rightarrow r(x)]$

$$\begin{aligned} \sim (\exists x \ni [p(x) \vee q(x) \rightarrow r(x)]) &\equiv \forall x \ni \sim [p(x) \vee q(x) \rightarrow r(x)] \\ &\equiv \forall x \ni \sim (\sim (p(x) \vee q(x)) \vee r(x)) \\ &\equiv \forall x \ni \sim ((\sim p(x) \wedge \sim q(x)) \vee r(x)) \\ &= \forall x \ni (p(x) \vee q(x)) \wedge \sim r(x) \end{aligned}$$

2 Write the negations for each of the statements below.

1. $\forall x \in R^{nonneg}$,if $x > 3$ and x is even, then $x^2 > 9$
   (a) $\exists x \in R^{nonneg} \ni x > 3 \vee x$ is odd, AND $x^2 \leq 9$
2. $\forall n \in Z$, if n is prime, then n is odd or $n = 2$
   (a) $\exists x \in Z \ni n$ is prime, AND n is NOT odd AND $n \neq 2$

3. $\forall$ integers a, b and c, if $a - b$ is even and $b - c$ is even, then $a - c$ is even

   (a) Calculations (optional)

$$\begin{aligned} (p \wedge q) \to r &\equiv \sim (p \wedge q) \vee r \\ &\equiv \sim p \vee \sim q \vee r \\ \sim ((p \wedge q) \to r) &\equiv p \wedge q \wedge \sim r \end{aligned}$$

   (b) $\exists$ integers a, b and c, such that $a - b$ is even and $b - c$ is even, BUT $a - c$ is NOT even.

      i. Use but here because already have AND earlier, but both also can be used.

4. $\exists x \in R$ such that $x^2 = 2$

   (a) $\forall x \in R, x^2 \neq 2$

5. $\exists x \in Z^+$ such that x is even and prime.

   (a) $\forall x \in Z^+$, x is NOT even OR NOT prime.

6. All even integers have even squares.

   (a) $\exists x \in \mathbb{Z}$ such that x is even, but do not have even squares.

7. No irrational numbers are integers.

   (a) There is at least one real number x , such that x is a rational number and x is an integer.

8. If an integer is divisible by 2, then it is even.

   (a) $\exists x \in \mathbb{R}$ such that x is divisible by 2, and x is not even.

3 Rewrite the statement formally using quantifiers and variables.

1. Everybody trusts somebody.

   (a) $\forall x \in$ people, $\exists y \in$ people, such that x trusts y

   (b) Lazy way: $\forall x, \exists y \in$ people $\ni x$ trusts y

2. Somebody trusts everybody.

   (a) $\exists x \in people, \forall y \in$ people $\ni x$ trusts y.

3. The number of rows in any truth table equals 2^n for some integer n.

   (a) $\forall x \in$ number of rows in any truth table, $\exists y \in$ integer n such that $x = 2^n$ for y

4. Every action has an equal or opposite reaction.
   (a) $\forall x \in actions, \exists$ an action y such that x and y have equal or opposite reaction.
5. There is a prime number between every integer and its double.
   (a) $\exists x \in$ prime number, $\forall y \in$ integer $\ni y < x < 2y$.

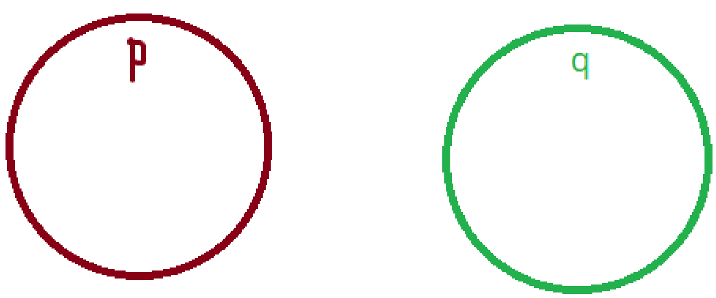
4 For the following statements the universe comprises all nonzero integers. Determine the truth value of each statement.

1. $\exists x \ni \exists y \ni [xy = 1]$
   (a) True
2. $\exists x \ni \forall y, [xy = 1]$
   (a) False
3. $\forall x, \exists y \ni [xy = 1]$
   (a) False
4. $\exists x \ni \exists y \ni [(2x + y = 5) \wedge (x - 3y = -8)]$
   (a) $x = 1, y = 3$.
   (b) True
5. $\exists x \ni \exists y \ni [(3x - y = 7) \wedge (2x + 4y = 3)]$
   (a) Proof
      i. $3x - y = 7$
$$y = 3x - 7$$
      ii. Subsitute into $2x + 4y = 3$
$$\begin{aligned} 2x + 4\left(3x - 7\right) &= 3 \\ 2x + 12x - 28 &= 3 \\ 14x &= 31 \\ x &= \frac{31}{14}/2.2142 \end{aligned}$$
$$\begin{aligned} y &= 3\left(\frac{31}{14}\right) - 7 \\ &= -\frac{5}{14} \end{aligned}$$
   (b) False

5 Rewrite each of the following quantifier statements formally using quantifier and variables, then write a negation for the statement.

1. For every odd integer n, there is an integer k such that $n = 2k + 1$.
   (a) Rewrite: $\forall$odd integer $n, \exists$integer $k, n = 2k + 1$
   (b) Negation: $\exists$ odd integer n, $\forall$ integer k, $n \neq 2k + 1$
2. Any even integer equals twice some other integer.
   (a) For all integers, if x is an even integer, then x is twice some other integer.
   (b) Givenanswer
      i. $\forall$ even integer x, $\exists$ integer $y \ni x = 2y$
      ii. Negate: $\exists$ even integer $x \ni \forall$ integer $y, x \neq 2$
   (c) Personal Answer (ask teacher)
      i. $\forall x, y \in \mathbb{Z}, x$ is an even integer $\rightarrow x$ is twice y.
      ii. $\exists x, y \in \mathbb{Z}, x$ is an even integer AND x is not twice y.

6 Indicate the following argument is valid or invalid by drawing diagrams.

1. No college cafeteria food is good.
   No good food is wasted.
   $\therefore$No college cafeteria food is wasted.
   (a) $p =$ college cafeteria food
   (b) $q =$ good food
   (c) $r =$ wasted food
   (d) No college cafeteria food is good
      i. 

(e) For no good food is wasted the picture is same as above except for the signs

(f) For the conclusion, there is an ambiguity, because college cafeteria food can either be wasted, or not be wasted.

(g) Therefore, the argument is invalid

2. All teachers occasionally make mistakes.
No gods ever make mistakes.
$\therefore$No teachers are gods.

(a) Establish terms

i. $t = teachers$

ii. $m =$ set of those who make mistakes

iii. $g = gods$

(b) Draw out circles

i. For the first one, teachers is inside the set of those who make mistakes

ii. For the second one, gods is outside the set of those who make mistakes

iii. There is only one possible way to draw the third one, which is teachers is outside of the set of those who make mistakes.

(c) The argument is VALID.

3. All polynomial functions are differentiable.
All differentiable functions are continuous.
$\therefore$All polynomial functions are continuous.

(a) Establish the terms

i. $p = polynomial$ functions

ii. $d =$ differentiable functions

iii. $c =$ continuous functions

(b) Draw the set of diagrams

i. For the first set, p-circle is inside d-circle

ii. For the second set $d - circle$ is inside $c - circle$

iii. For the third set, there is only one combination, $p - circle$ inside $d - circle$ inside $c - circle$

(c) Therefore, the argument is VALID.

4. All mathematics lecturers have studied calculus.
Lena is a mathematics lecturer.
$\therefore$Lena had studied calculus.

(a) Establish the terms

i. $m =$ mathematics lecturers

ii. $c =$people who studied calculus

iii. $l =$ Lena

(b) Draw the diagrams

i. For the first set $m - circle$ is inside $c - circle$

ii. For the second set $l - circle$ is inside $m - circle$

iii. For the third set, there is only one possible way to draw, which is $l - circle$ inside $m - circle$ inside $c - circle$

(c) The argument is VALID