Mid-Term Test PYQ Practice

December 24, 2019

1 Question 1

Integration using calculator is allowed. Leave answer in 3 d.p.

1. Part A

   (a)

   $$\begin{aligned} L &= \int_0^{\pi} \sqrt{1 + (\cos(2x))^2} dx \\ &= \int_0^{\pi} \sqrt{1 + 2\cos(2x)} dx \\ &= 5.270 \end{aligned}$$

   (b)

   i. Intersection points

   $$\begin{aligned} \cos(2x) &= 0 \\ \cos(2x) &= 0 \\ 2x &= \frac{1}{2}\pi, \frac{3}{2}\pi \\ x &= \frac{1}{4}\pi, \frac{3}{4}\pi \end{aligned}$$

   ii. Find the area

   $$\begin{aligned} A &= \int_0^{\frac{1}{4}\pi} \cos 2x dx - \int_{\frac{1}{4}\pi}^{\frac{3}{4}\pi} \cos 2x dx + \int_{\frac{3}{4}\pi}^{\pi} \cos 2x dx \\ &= 2 units \end{aligned}$$

2.

   $$\begin{aligned} x &= 2\sin t \\ \frac{dx}{dt} &= 2\cos t \\ \frac{dy}{dt} &= -5\sin t \end{aligned}$$

(a) $A = \int_0^{\frac{\pi}{2}} 5\cos t \cdot 2\cos t dt = \frac{5}{2}\pi$units

(b)

$$\begin{aligned} S_x &= 2\pi \int_0^{\frac{\pi}{2}} (5\cos t)\sqrt{4\cos^2 t + 25\sin^2 t dt} \\ &= \frac{5\pi\left(4\ln\left(\frac{5+\sqrt{21}}{2}\right) + 5\sqrt{21}\right)}{\sqrt{21}} \\ &= 100.022unit^2 \end{aligned}$$

3.

$$\begin{aligned} -x^2 + 8x + 4000 &= 0.5x^2 - 3x + 120 \\ -1.5x^2 + 11x + 3880 &= 0 \\ x &= 54.658, -47.32(ignored) \end{aligned}$$

$$\bar{x} = 54.658$$

$$p(54.658) = RM1449.77$$

$$\begin{aligned} CS &= \int_0^{54.628} -x^2 + 8x + 4000 - 1449.77dx \\ &= RM96910.21 \end{aligned}$$

(a) Mean value

$$\begin{aligned} \frac{1}{2-1}\int_1^2 \cos(\ln x)\, dx &= \int_1^2 \cos(\ln x)\, dx \\ &= 0.9082 \end{aligned}$$

2 Question 2

2.1 2a

1.

n	x_n	y'	y'
0	0	0	0
1	.1	$0 + 0.1(0 - 3(0))$	0
2	.2		.01
3	.3		0.027
4	.4		0.0489
5	.5		0.07423

(a) $y(0.5) \approx 0.074$

2.

$$\begin{aligned} \frac{d}{dx}[y] &= -3ce^{-3x} + \frac{1}{3} \\ y' &= \frac{1}{3} - 3ce^{-3x} \end{aligned}$$

$$\begin{aligned} \frac{1}{9} &= ce^{-3x} + \frac{x}{3} - y \\ \frac{1}{3} &= 3ce^{-3x} + x - 3y \end{aligned}$$

$$\begin{aligned} y' &= 3ce^{-3x} + x - 3y - 3ce^{-3x} \\ y' &= x - 3y \end{aligned}$$

$$\begin{aligned} 0 &= ce^{-3(0)} + \frac{0}{3} - \frac{1}{9} \\ &= c - \frac{1}{9} \\ c &= \frac{1}{9} \end{aligned}$$

3.

$$\begin{aligned} \frac{dy}{dx} &= x - 3y \\ \frac{dy}{dx} + 3y &= x \end{aligned}$$

$$\begin{aligned} \mu &= e^{\int 3dx} \\ &= e^{3x} \end{aligned}$$

$$\begin{aligned} e^{3x}\left[\frac{dy}{dx} + 3y\right] &= e^{3x}[x] \\ e^{3x}\frac{dy}{dx} + 3e^{3x}y &= xe^{3x} \\ \int \frac{d}{dx}\left[e^{3x}y\right] dx &= \int xe^{3x}dx \\ e^{3x}y &= \int xe^{3x}dx \\ e^{3x}y &= \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + c \\ y &= \frac{1}{3}x - \frac{1}{9} + \frac{c}{e^{3x}} \end{aligned}$$

(a) Let $u = x, v' = e^{3x}dx$

$$\begin{aligned} du &= dx \\ v &= \int e^{3x}dx \\ v &= \frac{1}{3}e^{3x} \end{aligned}$$

$$\begin{aligned} \int xe^{3x}dx &= x\left(\frac{1}{3}e^{3x}\right) - \int \frac{1}{3}e^{3x}du \\ &= \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + c \end{aligned}$$

3 (Another) Question 2

1.

n	X_n	$y = y_{n-1} + 0.1\,(y')$	y
0	0	1	1
1	0.1	$1 + 0.1\,(0 + 1 - 0 \cdot y)$	1.1
2	0.2		1.209
3	0.3		1.326
4	0.4		1.4488
5	0.5		1.5757

$y\,(0.5) \approx 1.576(3dp)$

2.

$$\frac{dy}{dx} + xy = 4x$$

(a)

$$\begin{aligned} \frac{dy}{dx} &= 4x - xy \\ &= x\,(4 - y) \\ \int \frac{1}{4-y}dy &= \int xdx \\ -\ln(4-y) &= \frac{x^2}{2} + c \\ \ln(4-y) &= -\frac{x^2}{2} + c \\ 4 - y &= e^{-\frac{x^2}{2}+c} \\ -y &= e^{-\frac{x^2}{2}+c} - 4 \\ y &= 4 - e^{-\frac{x^2}{2}+c} \end{aligned}$$

i. When $y = 3, x = 0$

$$\begin{aligned} 3 &= 4 - e^{0+c} \\ &= 4 - e^{c} \\ -1 &= -e^{c} \\ e^{c} &= 1 \\ c &= 0 \end{aligned}$$

$$\begin{aligned} y &= 4 - e^{-\frac{x^2}{2}} \\ &= 4 - \frac{1}{\sqrt{e^{x^2}}} \end{aligned}$$

3.

$$\begin{aligned} \frac{dy}{dx} &= \frac{y}{x\left(x+1\right)} \\ \int \frac{1}{y} dy &= \int \frac{1}{x\left(x+1\right)} dx \end{aligned}$$

$$\begin{aligned} \frac{1}{x\left(x+1\right)} &= \frac{A}{x} + \frac{B}{\left(x+1\right)} \\ 1 &= A\left(x+1\right) + Bx \end{aligned}$$

(a) When $x = -1$

$$\begin{aligned} 1 &= A\left(-1+1\right) + B\left(-1\right) \\ 1 &= -B \\ B &= -1 \end{aligned}$$

(b) When $x = 0$

$$A = 1$$

$$\begin{aligned} \int \frac{1}{y} dy &= \int \frac{1}{x\left(x+1\right)} dx \\ \ln y &= \int \frac{1}{x} dx - \int \frac{1}{x+1} dx \\ &= \ln x - \ln\left(x+1\right) + c \\ \ln y &= \ln \frac{x}{x+1} \cdot e^{c} \\ y &= \frac{e^{c} x}{x+1} \end{aligned}$$

$$y = 1, x = 1$$

$$
\begin{aligned}
1 &= \frac{e^c}{2} \\
2 &= e^c \\
c &= \ln 2
\end{aligned}
$$

$$
\begin{aligned}
y &= \frac{e^{\ln 2} x}{x+1} \\
y &= \frac{2x}{x+1}
\end{aligned}
$$