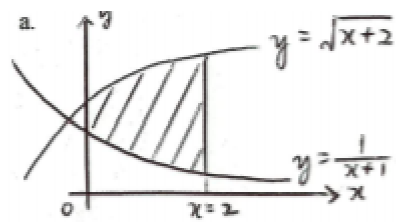
# Calc 2 - Tutorial 1

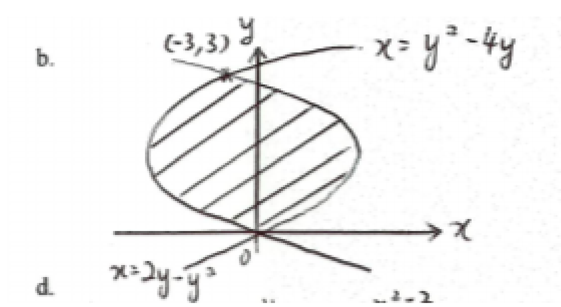
October 14, 2019

1. Find area of shaded region

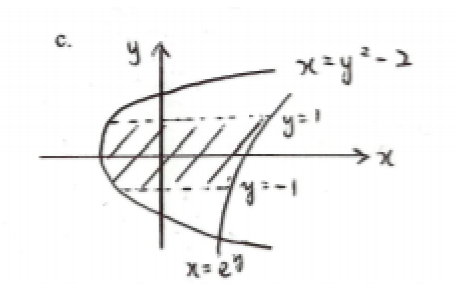
   (a) 


      i. Integrate in between

$$\int_0^2 \sqrt{x+2} - \frac{1}{x+1} dx = \frac{16}{3} - \frac{4\sqrt{2}}{3} - \ln(3)$$
$$= 2.3491$$

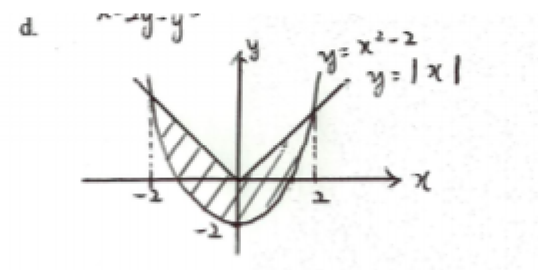
   (b) 


$$\begin{aligned} A &= \int_0^3 \left(2y - y^2\right) - \left(y^2 - 4y\right) dy \\ &= \int_0^3 2y - 2y^2 + 4y dy \\ &= \int_0^3 6y - 2y^2 dy \\ &= 27 - 18 \\ &= 9 \end{aligned}$$

(c) 


i. Note: One of them is $x = e^y$, not $x = 2^y$

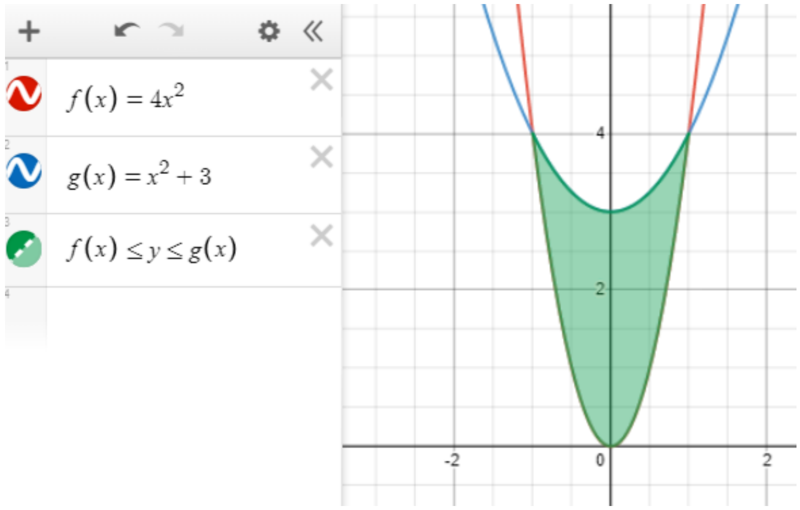
$$
\begin{aligned}
A &= \int_{-1}^{1} e^y - \left(y^2 - 2\right) dy \\
&= \int_{-1}^{1} e^y dy - \int_{-1}^{1} y^2 - 2dy \\
&= \left[\frac{e^y}{\ln(e)}\right]_{-1}^{1} + \frac{10}{3} \\
&= e - \frac{1}{e} + \frac{10}{3}
\end{aligned}
$$

(d) 


$$
\begin{aligned}
A &= \int_{-2}^{2} x^2 - 2 - |x| \, dx \\
&= \int_{-2}^{0} (-x) - \left(x^2 - 2\right) dx + \int_{0}^{2} x - \left(x^2 - 2\right) \, dx \\
&= \frac{20}{3}
\end{aligned}
$$

2. Sketch the region enclosed by the curves. Find the enclosed area region.

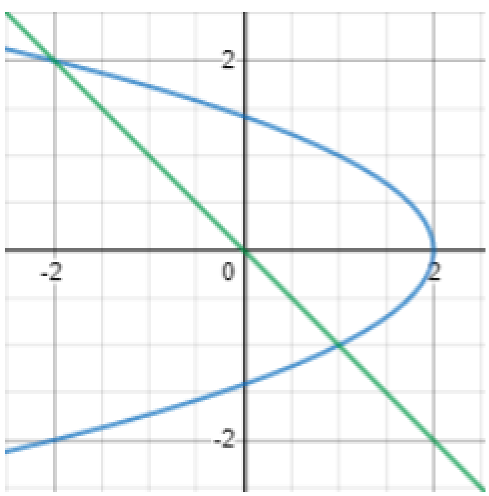
(a) $y = 4x^2, y = x^2 + 3$



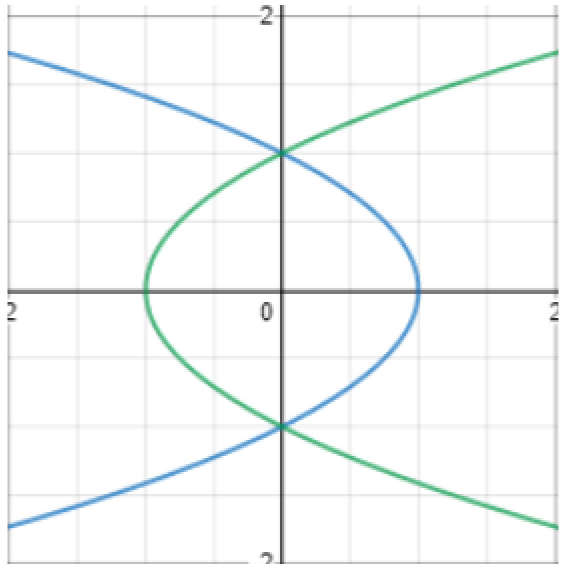

$$A = \int_{-1}^{1} (x^2+3) - (4x^2)\,dx$$
$$= 4$$

(b) $x + y^2 = 2, x + y = 0$

$$x = 2 - y^2$$
$$y = -x$$



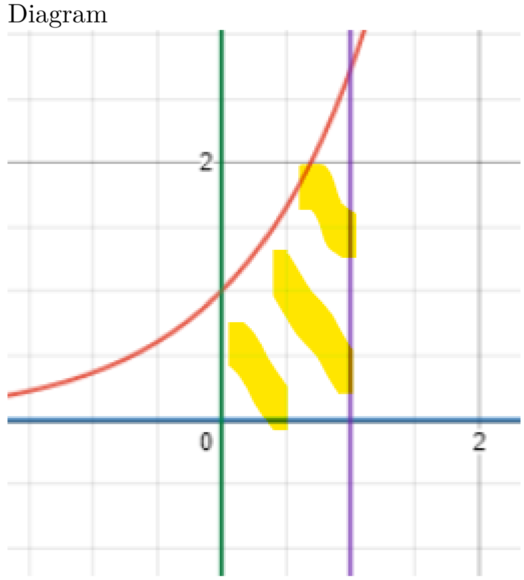
(c) $x = 1 - y^2, x = y^2 - 1$

i. 


3. Find the volume of the solid generated by rotating the region bounded by the curves, about the specific axis. Sketch the region, the solid, and a typical disk or a “washer”.

   (a) $y = e^x, y = 0, x = 0, x = 1, about\, x - axis$

      i. Diagram



ii. Integration

$$
\begin{aligned}
V &= \int_0^1 \pi \left[f\left(x\right)\right]^2 dx \\
&= \int_0^1 \pi \left(e^x\right)^2 dx \\
V &= \pi \int_0^1 \left(e^x\right)^2 dx \\
&= \pi \int_0^1 e^x e^x dx
\end{aligned}
$$

iii. $u - substitution$

$$
\begin{aligned}
u &= e^x \\
du &= e^x dx
\end{aligned}
$$

iv. Find endpoints

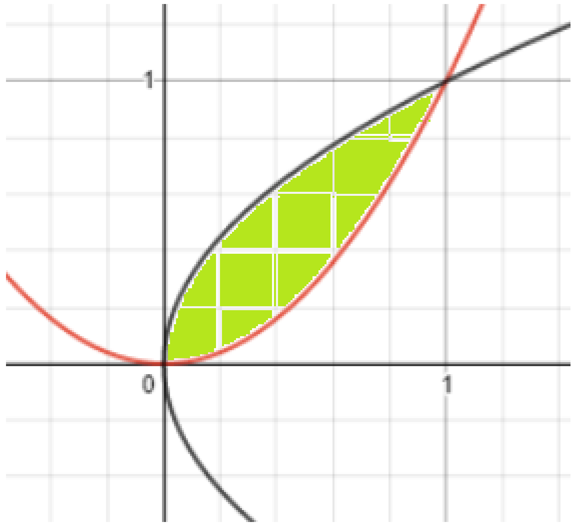
$$
\begin{aligned}
u|_{x=0} &= e^0 \\
u|_{x=0} &= 1 \\
\\
u|_{x=1} &= e^1 \\
&= e
\end{aligned}
$$

v. Substitute back

$$
\begin{aligned}
V &= \pi \int_1^e udu \\
&= \frac{\pi}{2}\left(e^2 - 1\right)
\end{aligned}
$$

(b) $y = x^2, y^2 = x$,about $x - axis$

i. Diagram




ii. Intersection

$$\begin{aligned}
y &= x^2 \\
y &= \sqrt{x} \\
x^2 &= \sqrt{x} \\
x^2 - \sqrt{x} &= 0 \\
x^4 - x &= 0 \\
x\left(x^3 - 1\right) &= 0 \\
x &= 0, 1
\end{aligned}$$

iii. Integration

$$\begin{aligned}
V &= \pi \int_0^1 \left(\sqrt{x}\right)^2 - \left(x^2\right)^2 dx \\
&= \pi \int_0^1 x - x^4 dx \\
&= \frac{3\pi}{10}
\end{aligned}$$

(c) $y^2 = x, x = 2y$; about $x - axis$

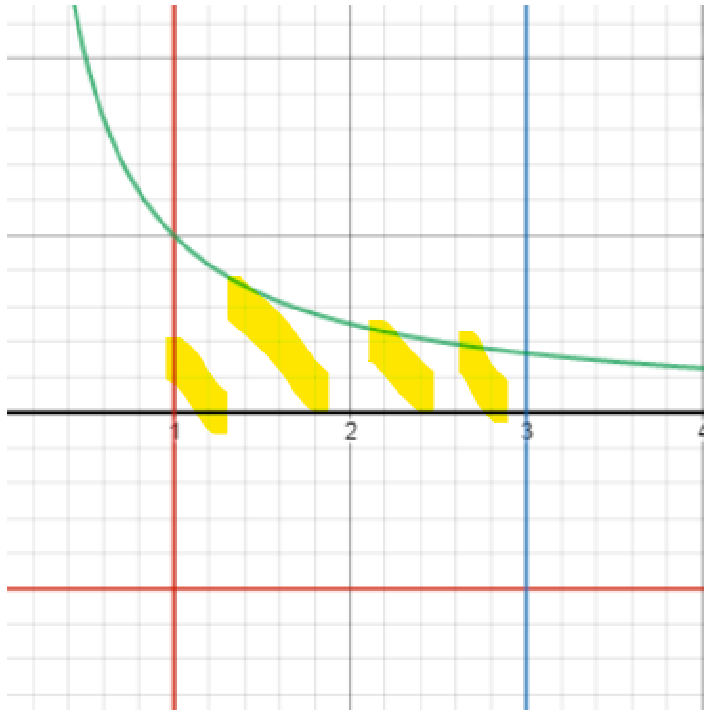
i. Image

ii. Integration

$$\pi \int_0^4 \left(x^{\frac{1}{2}}\right)^2 - \left(\frac{x}{2}\right)^2 dx = \frac{8\pi}{3}$$

(d) $y = \frac{1}{x}, y = 0, x = 1, x = 3, about\, y = -1$

i. Image

A. 


ii. Find the intersections

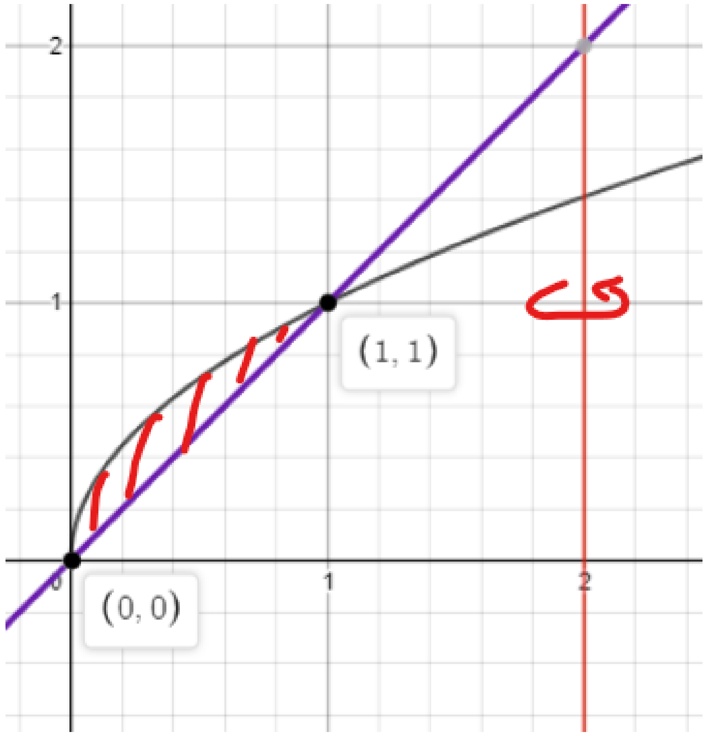
A. $x = 1, x = 3$

iii. Find the integration

$$\begin{aligned} V &= \pi \int_1^3 \left(\frac{1}{x} - (-1)\right)^2 - (0 - (-1))\, dx \\ &= \pi \int_1^3 \left(\frac{1}{x} - (-1)\right)^2 - 1dx \\ &= 2\pi \left(\frac{1}{3} + \ln(3)\right) \end{aligned}$$

(e) $y = x, y = \sqrt{x}$; about $x = 2$

i. Image

A. 


ii. This time, its rotating with respect to $x = 2$ (some kind of y-axis)

iii. The intersection points are $y = 0, y = 1$

iv. The equations

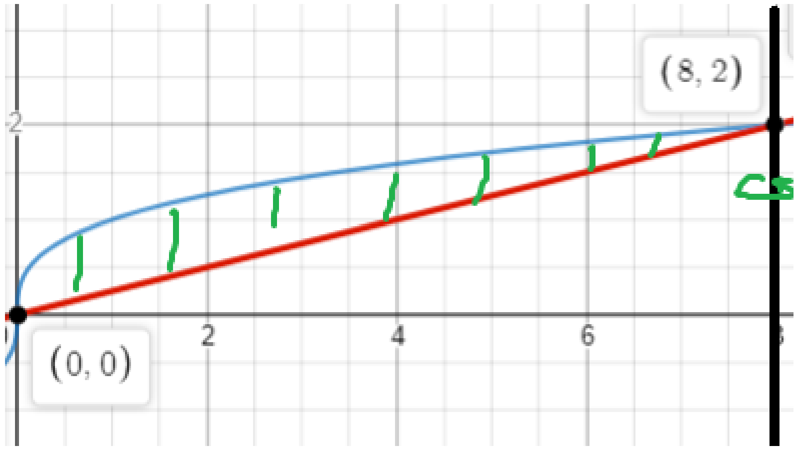
$$x = y$$
$$x = y^2$$

v. Find the integration

$$\begin{aligned} V &= \pi \int_0^1 \left(y^2 - 2\right)^2 - (y-2)^2 \, dy \\ &= \frac{8\pi}{15} \end{aligned}$$

4. The region enclosed by the curve $x = 4y$ and $y = \sqrt[3]{x}$ in the first quadrant about the line $x = 8$. FInd the volume generated.

   (a) Diagram




(b) Figure out the equations

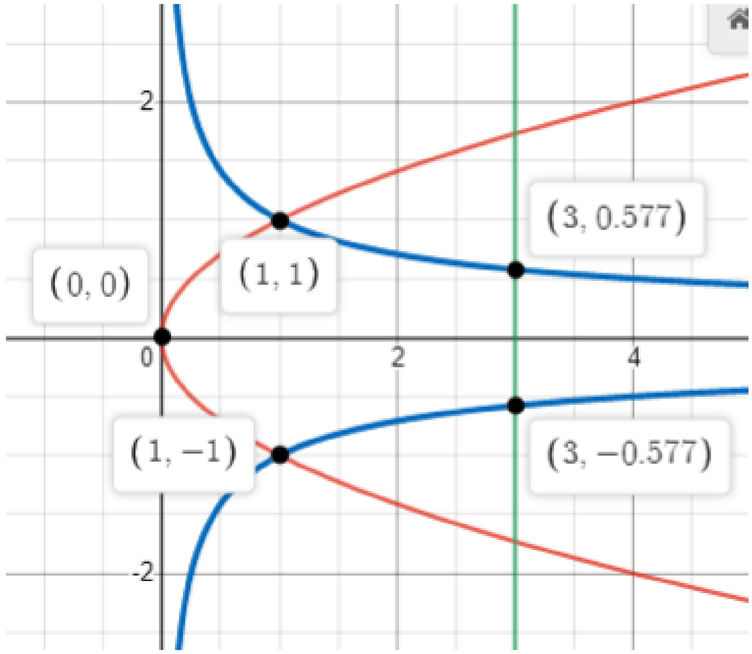
$$x = 4y$$
$$x = y^3$$

(c) Integrate

$$V = \pi \int_0^2 \left(y^3 - 8\right)^2 - (4y - 8)^2 \, dy$$
$$V = \frac{832\pi}{21}$$

5. Sketch the region bounded by the curves $y^2 = x$, $y^2 = \frac{1}{x}$, and the line $x = 3$. Find the volume generated when the region is rotated about the $x - axis$.

(a) Diagram

i. 


(b) Because its a mirror, we only need to calculate the top side. Therefore, the intersection points are $x = 1, x = 3$

(c) Integration

$$
\begin{aligned}
V &= \int_0^1 \pi \left(\sqrt{x}\right)^2 dx + \int_1^3 \pi \left(\frac{1}{x}\right) dx \\
&= \pi \int_0^1 x dx + \pi \int_1^3 \left(\frac{1}{x}\right) dx \\
&= \frac{\pi}{2} + \pi \ln(3) \\
V &= \pi \left(\frac{1}{2} + \ln 3\right)
\end{aligned}
$$