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D.M. T4

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1. Determine the truth value of the following universal statements. If a statement is false, suggest a counterexample for the statement.

   (a) $\forall x \in \{1, 2, 3, 5, 11\}, x$ is prime.

      i. False. 1 is not a prime number because it only have 1 divisor.

   (b) $\forall x \in \{0, 2, 6, 12, 36, 48, 52\}, x$ is non-negative AND even.

      i. True

   (c) $\forall x \in Z$, the square of x is positive.

      i. False, 0 is neither positive nor negative.

   (d) $\forall a \in Z, \frac{(a-1)}{a}$ is not integer.

      i. False, $a = 1$

      $$\frac{(1-1)}{1} = \frac{0}{1} = 0$$

      ii. 0 is an integer

   (e) $\forall x, y \in R, \sqrt{x+y} = \sqrt{x} + \sqrt{y}$

      i. False, $x = 1, y = 1$

      $$\begin{aligned} RHS &= \sqrt{1+1} \\ &= \sqrt{2} \end{aligned}$$

      $$\begin{aligned} LHS &= \sqrt{1} + \sqrt{1} \\ &= 1 + 1 \\ &= 2 \\ &\neq RHS \end{aligned}$$

   (f) $\forall$ prime x, $x^3$is odd.

      i. False, 2 is a prime number, $2^3 = 8$ is even.

2. Let $D = \{-48, -14, -8, 0, 1, 3, 16, 23, 26, 32, 36\}$. Determine which of the following statements are true and which are false. Provide counterexamples for those statements that are false.

(a) $\forall x \in D$, if x is odd, then $x > 0$

i. True

(b) $\forall x \in D$,if x is less than 0, then x is even

i. True

(c) $\forall x \in D$, if x is even then $x < 0$.

i. False, $x = 26$

(d) $\forall x \in D$, if the ones digit of x is 2, then the tens digit is 3 or 4.

i. True

(e) $\forall x \in D$, if the ones digit of x is 6, then the tens digit is 1 or 2.

i. False, $x = 36$

3. Rewrite each of the following statements in the two forms "$\forall x$, if-then" and "∀___x, ___" (without using if-then).

(a) The sum of any even integer is even.

i. $\forall x, y \in Z$, if x and y are even, then $x + y$ is even.

ii. $\forall$ even x, y, $x + y$ is even.

(b) All polynomial functions are continuous.

i. $\forall x \in$ functions, if x is polynomial, then x is continuous.

ii. $\forall$ polynomial functions x, x is continuous.

(c) No integer is a factor of 4.

i. $\forall x \in \mathbb{R}$, if x is integer, then x is not a factor of 4.

ii. $\forall$ integer x, x is not a factor of 4.

4. Determine the truth value of the following existential statements. Prove or disprove the statements.

(a) $\exists x \in \{1, 2, 3, 5, 11\}$ such that x is prime and even.

(b) $\exists x \in \{2, 4, 8, 16, 32\}$ such that x is not divisible by 2

(c) $\exists x \in Z^-$, such that x equals its square.

(d) $\exists x \in Z^+$such that $4x^2 - 1 = 0$.

5. Consider the following statement

$$\exists x \in R \text{ such that} x^2 = 2$$

Which of the following are equivalent ways of expressing this statement?

(a) If x is a real number, then $x^2 = 2$.

i. Not equivalent. $3^2 \neq 2$

(b) Some real number has square 2.

i. Equivalent.$\left(\sqrt{2}\right)^2 = 2$

(c) Some real numbers have square 2.

i. Equivalent. $\left(-\sqrt{2}\right)^2 = 2$ and $\left(\sqrt{2}\right)^2 = 2$

(d) The number x has square 2, for some real number x.

i. Equivaent.$\left(\sqrt{2}\right)^2 = 2$

(e) The square of each real number is 2.

i. Not equivalent. $0^2 \neq 2$

(f) There is at least one real number whose square is 2.

i. Equivalent $\left(\sqrt{2}\right)^2 = 2$

6. Rewrite the following statements in the two forms "∃_x such that _" and "$\exists x$ such that ___ and ___".

   (a) Some exercises have answers.

      i. $\exists$ answers x such that x have answers.
      ii. $\exists x$ such that x is exercise and x has answers.

   (b) Some questions are easy.

      i. $\exists$ questions x such that x is easy
      ii. $\exists x$ such that x is a question and x is easy.

   (c) There exists an even integer divisible by 4.

      i. $\exists$ even integer x such that x is divisible by 4.
      ii. $\exists x$ such that x is an even integer and x is divisible by 4.

   (d) Some people are rich but unhappy.

      i. $\exists$ people x such that x is rich but x is unhappy.
      ii. $\exists x$ such that x is a rich people and x is unhappy.