Statistics 2, Chapter 1: Random Variables & Probability Distributions

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Random Variable

- A variable whose value is determined by the outcome of a random experiment.
- Can be discrete or continuous.

Discrete Random Variable

- A random variable that assumes countable values.

Continuous Random Variable

- A random variable that can assume any value contained in one or more intervals.

1 Example

Three cards are selected without replacement from a deck of 52 cards. A random variable may be defined as

$$X = \text{number of aces obtained}$$

1. Then X can assume the values 0, 1, 2 or 3.
2. Since X can assume only 4 values, it is a discrete random variable.

2 Example

3 Example

Get on a scale and weigh yourself. Let the random variable, X be defined as your weigh in kilogram.

- Then X could be any of the infinitely many values between 40kg and 90kg,
- i.e. $40 < X < 90$. $\Rightarrow$ X is continuous random variable.

4 Discrete Probability Distribution

4.1 Definition

4.2 Example

(a) No, because they do not add up to 1
(b) Yes
(c) No, a probability distribution should not have negative probability values (values have to fall between 0 and 1)

5 Example

Toss 2 fair dice.

- Let X denotes the sum of the spots on the 2 dice.
- Find the probability distribution of the r. v. X.

		Second Die					
	$X_1 + X_2$	1	2	3	4	5	6
	1	2	3	4	5	6	7
First Die	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

Probability Distribution

x	2	3	4	5	6	7	8	9	10	11	12
$P[X = x]$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

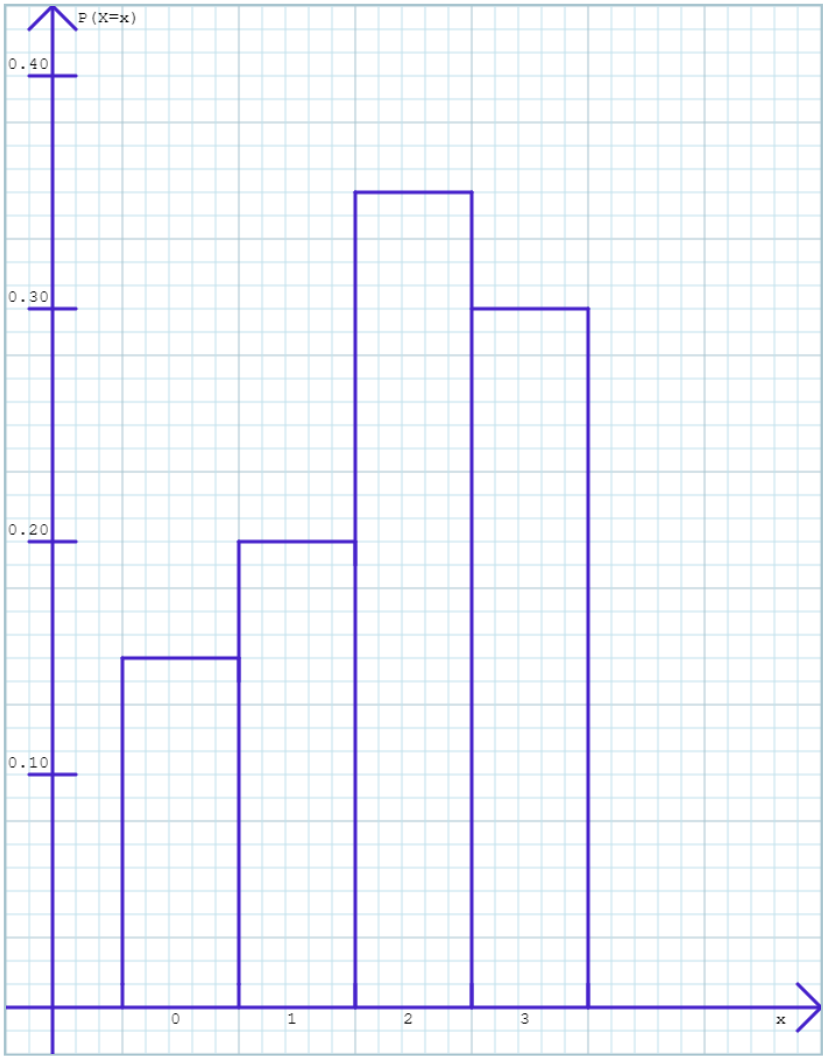
Sum $= \frac{36}{36} = 1$

6 Example

Heads	0	1	2	3
P($Heads$)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

7 Example

1.



(a)

(b)

i. $P(X = 2) = 0.35$

ii. $P(0 \leq X \leq 2)$

$$\begin{aligned} P(0 \leq X \leq 2) &= 0.15 + 0.20 + 0.35 \\ &= 0.7 \end{aligned}$$

iii. $P(X > 1)$

$$\begin{aligned} P(X > 1) &= 0.35 + 0.30 \\ &= 0.65 \end{aligned}$$

iv. $P(X \leq 1)$

$$\begin{aligned} P(X \leq 1) &= P(X = 1) + P(X = 2) \\ &= 0.35 \end{aligned}$$

8 Example

9 Example