Calc 2: Tutorial 6

January 19, 2020

1 2nd order homogeneous linear D.E.

1. Find the general solution of

(a) $y'' - 6y' + 5y = 0$

    i. Rewrite

$$r^2 - 6r + 5 = 0$$

    ii. Factorize

$$(r - 5)(r - 1) = 0$$
$$r = 5, r = 1$$
$$y = C_1 e^{5x} + C_2 e^x$$

(b) $4y'' - 4y' + 1 = 0$

    i. Rewrite

$$4r^2 - 4r + 1 = 0$$

    ii. Factorize

$$(2r - 1)(2r - 1) = 0$$
$$(2r - 1)^2 = 0$$
$$r = \frac{1}{2}$$

    iii. Obtain equation

$$y = C_1 e^{\frac{1}{2}x} + C_2 x e^{\frac{1}{2}x}$$
$$y = e^{\frac{1}{2}x}(C_1 + C_2 x)$$

(c) $4y'' - 2y' + y = 0$

    i. Note:

        A. Complex roots

        B. Use general solution:

$$y = C_1 e^{\alpha x} \sin \beta x + C_2 e^{\alpha x} \cos \beta x$$
$$y = e^{\alpha x}(C_1 \sin \beta x + C_2 \cos \beta x)$$

    ii. Rewrite

$$4r^2 - 2r + 1 = 0$$

iii. Factorize

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2) \pm \sqrt{(-2)^2 - 4\,(4)\,(1)}}{2\,(4)}$$

$$= \frac{2 \pm \sqrt{4 - 16}}{8}$$

$$= \frac{2 \pm \sqrt{-12}}{8}$$

$$= \frac{2 \pm \sqrt{-1}\sqrt{12}}{8}$$

$$= \frac{2 \pm \sqrt{-1} \cdot 2\sqrt{3}}{8}$$

$$= \frac{2 \pm 2i\sqrt{3}}{8}$$

$$= \frac{1 \pm i\sqrt{3}}{4}$$

$$r = \frac{1}{4} \pm \frac{\sqrt{3}}{4}i$$

iv. Obtain equation

$$y = e^{\frac{1}{4}}\left(C_1 \sin \frac{\sqrt{3}}{4}x + C_2 \cos \frac{\sqrt{3}}{4}x\right)$$

(d) $y'' - 2y' + 1 = 0$

    i. Rewrite

$$r^2 - 2r + 1 = 0$$

    ii. Factorize

$$r^2 - 2r + 1 = 0$$
$$(r - 1)(r - 1) = 0$$
$$(r - 1)^2 = 0$$
$$r = 1$$

    iii. Obtain equation

$$y = e^x (C_1 + C_2 x)$$

(e) $2y'' + 7y' - 4 = 0$

    i. Rewrite

$$2r^2 + 7r - 4 = 0$$

    ii. Factorize

$$(2r - 1)(r + 4) = 0$$
$$r = \frac{1}{2}, -4$$

    iii. Obtain equation

$$y = C_1 e^{\frac{1}{2}x} + C_2 e^{-4x}$$

(f) $2y'' - 2y' + 5 = 0$

    i. Rewrite

$$2r^2 - 2r + 5 = 0$$

    ii. Factorize

$$a = 2, b = -2, c = 5$$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$r = \frac{-(-2) \pm \sqrt{(-2)^2 - 4\,(2)\,(5)}}{2\,(2)}$$

$$= \frac{2 \pm \sqrt{4 - 40}}{4}$$

$$= \frac{2 \pm \sqrt{-36}}{4}$$

$$= \frac{2 \pm \sqrt{-1}\sqrt{36}}{4}$$

$$= \frac{2 \pm 6i}{4}$$

$$r = \frac{1}{2} \pm 3i$$

    iii. Obtain equation

$$y = e^{\frac{1}{2}} \left(C_1 \sin 3x + C_2 \cos 3x \right)$$

2. Solve the following I.V.P.:

  (a) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 10y = 0, y = 0$ and $\frac{dy}{dx} = 6$ when $x = 0$

    i. Rewrite

$$r^2 + 2r + 10 = 0$$

    ii. Factorize

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$r = -1 \pm 3i$$

    iii. Obtain equation

$$y = e^{-x} \left(C_1 \sin 3x + C_2 \cos 3x \right)$$

    iv. Find C_1 and C_2

$$0 = e^{-0} \left(C_1 \sin 0 + C_2 \cos 0 \right)$$
$$= C_2$$
$$C_2 = 0$$

$$y' = \frac{d}{dx} \left[e^{-x} \left(C_1 \sin 3x + C_2 \cos 3x \right) \right]$$

$$= \frac{d}{dx} \left[e^{-x} \right] \cdot \left(C_1 \sin 3x + C_2 \cos 3x \right) + e^{-x} \cdot \frac{d}{dx} \left[\left(C_1 \sin 3x + C_2 \cos 3x \right) \right]$$

$$= -e^{-x} \cdot \left(C_1 \sin 3x + C_2 \cos 3x \right) + e^{-x} \left(C_1 \frac{d}{dx} \left[\sin 3x \right] + C_2 \frac{d}{dx} \left[\cos 3x \right] \right)$$

$$= -e^{-x} \left(C_1 \sin 3x + C_2 \cos 3x \right) + e^{-x} \left(3C_1 \cos 3x - 3C_2 \sin 3x \right)$$

      A. Substitute in the values given and C_2

$$6 = -e^0 \left(C_1 \sin 0 + 0 \cos 0 \right) + e^0 \left(3C_1 \cos 0 - 0 \sin 0 \right)$$
$$= 0 + 3C_1\,(1) + 0$$
$$6 = 3C_1$$
$$C_1 = 2$$

    v. Obtain solution

$$y = e^{-x} \left(2 \sin 3x + 0 \cos 3x \right)$$
$$y = 2e^{-x} \sin 3x$$

  (b) $\frac{d^2y}{dx^2} - 9y = 0, y = 2$ and $\frac{dy}{dx} = -1$ when $x = 0$

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i. Rewrite

$$r^2 - 9 = 0$$

ii. Factorize

$$(r - 3)(r + 3) = 0$$
$$r = \pm 3$$

iii. Obtain equation

$$y = C_1 e^{3x} + C_2 e^{-3x}$$

iv. Find C_1 and C_2

$$y = 2, \frac{dy}{dx} = -1, x = 0$$

$$2 = C_1 e^{3(0)} + C_2 e^{-3(0)}$$
$$C_1 + C_2 = 2$$
$$C_2 = 2 - C_1$$

$$y' = \frac{d}{dx}\left[C_1 e^{3x} + C_2 e^{-3x}\right]$$
$$= \frac{d}{dx}\left[C_1 e^{3x}\right] + \frac{d}{dx}\left[C_2 e^{-3x}\right]$$
$$= C_1 \cdot 3e^{3x} + C_2(-3)e^{-3x}$$
$$= C_1 \cdot 3e^{3x} - 3C_2 e^{-3x}$$
$$-1 = C_1 \cdot 3e^0 - 3C_2 e^0$$
$$-1 = 3C_1 - 3C_2$$
$$-1 = 3C_1 - 3(2 - C_1)$$
$$-1 = 3C_1 - 6 + 3C_1$$
$$5 = 6C_1$$
$$C_1 = \frac{5}{6}$$
$$C_1 = \frac{5}{6}$$

A. Substitute in $C_2 = 2 - C_1$

$$C_2 = 2 - \frac{5}{6}$$
$$= \frac{7}{6}$$

v. Find the solution for I.V.P.

$$y = \frac{5}{6}e^{3x} + \frac{7}{6}e^{-3x}$$
$$y = \frac{1}{6}\left(5e^{3x} + 7e^{-3x}\right)$$

(c) $\frac{d^2y}{dx^2} + 16y = 0, y = -10$ and $\frac{dy}{dx} = 3$ when $x = \frac{\pi}{2}$

i. Rewrite

$$r^2 + 16 = 0$$

ii. Factorize

$$r = \frac{-0 \pm \sqrt{-4(16)}}{2}$$
$$= \pm \frac{\sqrt{-1}\sqrt{64}}{2}$$
$$= \pm \frac{8i}{2}$$
$$= \pm 4i$$

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iii. Obtain equation

$$y = e^{0x} \left(C_1 \sin 4 + C_2 \cos 4 \right)$$
$$= C_1 \sin 4x + C_2 \cos 4x$$

iv. Find C_1 and C_2

$$-10 = C_1 \sin 2\pi + C_2 \cos 2\pi$$
$$-10 = C_2$$
$$C_2 = -10$$

$$y' = 4C_1 \cos 4x - 4C_2 \sin 4x$$
$$3 = 4C_1 \cos 4 \left(\frac{\pi}{2} \right) - C_2 \sin 4 \left(\frac{\pi}{2} \right)$$
$$3 = 4C_1 \cos (2\pi) - C_2 \sin 2\pi$$
$$3 = 4C_1 \cos (2\pi)$$
$$C_1 (1) = \frac{3}{4}$$
$$C_1 = \frac{3}{4}$$

v. Find the solution for I.V.P.

$$y = C_1 \sin 3x + C_2 \cos 3x$$
$$y = \frac{3}{4} \sin 3x - 10 \cos 3x$$

(d) $\frac{d^2y}{dx^2} + 14\frac{dy}{dx} + 49y = 0, y = -1$ and $\frac{dy}{dx} = 5$ when $x = -4$

i. Rewrite

$$r^2 + 14r + 49 = 0$$

ii. Factorize

$$r^2 + 14r + 49 = 0$$
$$(r + 7)(r + 7) = 0$$
$$(r + 7)^2 = 0$$
$$r = -7$$

iii. Obtain equation

$$y = e^{-7x} (C_1 + C_2 x)$$
$$y = C_1 e^{-7x} + C_2 e^{-7x} x$$

iv. Find C_1 and C_2
   A. Find the equations

$$-1 = e^{-7(-4)} (C_1 + C_2(-4))$$
$$-1 = e^{28} (C_1 - 4C_2)$$
$$-1 = C_1 e^{28} - 4C_2 e^{28}$$
$$C_1 e^{28} = -1 + 4C_2 e^{28}$$
$$C_1 = -e^{-28} + 4C_2$$

$$y' = \frac{d}{dx} \left[e^{-7x} (C_1 + C_2 x) \right]$$
$$= \frac{d}{dx} \left[C_1 e^{-7x} + C_2 e^{-7x} x \right]$$
$$= -7C_1 e^{-7x} + C_2 \left[-7e^{-7x} \cdot x + e^{-7x} \right]$$
$$y' = -7C_1 e^{-7x} + C_2 e^{-7x} - 7C_2 x e^{-7x}$$

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B. Substitute the values in

$$y = -1 \text{ and } \frac{dy}{dx} = 5 \text{ when } x = -4$$

$$5 = -7C_1 e^{-7(-4)} + C_2 e^{-7(-4)} - 7C_2 (-4) e^{-7(-4)}$$
$$= -7C_1 e^{28} + C_2 e^{28} + 28 C_2 e^{28}$$
$$5 = -7C_1 e^{28} + 29 C e^{28}$$
$$5 = -7 \left(-e^{-28} + 4C_2 \right) e^{28} + 29 C_2 e^{28}$$
$$5 = 7 - 28 C_2 e^{28} + 29 C_2 e^{28}$$
$$-2 = C_2 e^{28}$$
$$C_2 = -\frac{2}{e^{28}}$$

$$C_1 = -e^{-28} + 4C_2$$
$$C_1 = -e^{-28} + 4 \left(-\frac{2}{e^{28}} \right)$$
$$= -e^{-28} - 8e^{-28}$$
$$= -\frac{9}{e^{28}}$$

v. Find the solution for I.V.P.V (Given answer is bolded)

$$y = -\frac{9}{e^{28}} e^{-7x} + -\frac{2}{e^{28}} e^{-7x} x$$
$$= -9e^{-7x-28} - 2xe^{-7x-28}$$
$$\mathbf{y = -9e^{-7(x+4)} - 2xe^{-7(x+4)}}$$
$$= e^{-7x-28} (-2x - 9)$$
$$y = e^{-7(x+4)} (-2x - 9)$$

2 2nd order non-homogeneous linear D.E.

1. Find General Solution for:

(a) $\frac{d^2 y}{dx^2} - 2\frac{dy}{dx} + y = e^{2x}$

   i. Solve the homogeneous D.E. $ay'' + by' + cy = 0$ (find the r)

$$r^2 - 2r + 1 = 0$$
$$(r - 1)(r - 1) = 0$$
$$(r - 1)^2 = 0$$
$$r = 1$$

   ii. Find the complementary function, y_h

$$y_h = e^x (C_1 + C_2 x)$$

   iii. Find the particular solution y_p
   A. Find the derivatives

$$y_p = Ae^{2x}$$
$$y'_p = 2Ae^{2x}$$
$$y''_p = 4Ae^{2x}$$

   B. Plug into the function

$$\frac{d^2 y}{dx^2} - 2\frac{dy}{dx} + y = e^{2x}$$
$$4Ae^{2x} - 2 \left(2Ae^{2x} \right) + Ae^{2x} = e^{2x}$$
$$4Ae^{2x} - 4Ae^{2x} + Ae^{2x} = e^{2x}$$

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C. Find A

$$4Ae^{2x} - 4Ae^{2x} + Ae^{2x} = e^{2x}$$
$$Ae^{2x} = e^{2x}$$
$$A = 1$$

D. Find y_p

$$y_p = e^{2x}$$

iv. Find the non-homogeneous D.E. general solution, $y = y_h + y_p$

$$y = e^x \left(C_1 + C_2 x\right) + e^{2x}$$

(b) $9\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + y = x^2 + 2x + 3$

i. Solve the homogeneous D.E. $ay'' + by' + cy = 0$ (find the r)

$$9r^2 + 6r + 1 = 0$$

$$(3r + 1)^2 = 0$$
$$r = -\frac{1}{3}$$

ii. Find the complementary function, y_h

$$y_h = e^{-\frac{1}{3}x} \left(C_1 + C_2 x\right)$$

iii. Find the particular solution y_p
   A. Find the derivatives

$$y_p = Ax^2 + Bx + C$$
$$y' = 2Ax + B$$
$$y'' = 2A$$

   B. Plug into the function

$$9\left(2A\right) + 6\left(2Ax + B\right) + Ax^2 + Bx + C = x^2 + 2x + 3$$
$$Ax^2 + 12Ax + Bx + 18A + 6B + C = x^2 + 2x + 3$$
$$Ax^2 + \left(12A + B\right)x + 18A + 6B + C = x^2 + 2x + 3$$

   C. Find A, B and C by equating them
   A

$$Ax^2 = x^2$$
$$A = 1$$

   B

$$12A + B = 2$$
$$B = 2 - 12\left(A\right)$$
$$= 2 - 12\left(1\right)$$
$$B = -10$$

   C

$$18A + 6B + C = 3$$
$$18\left(1\right) + 6\left(-10\right) + C = 3$$
$$C = 3 - 18 + 60$$
$$C = 45$$

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D. Find y_p

$$y_p = Ax^2 + Bx + C$$
$$\boldsymbol{y_p = x^2 - 10x + 45}$$

iv. Find the non-homogeneous D.E. general solution, $y = y_h + y_p$

$$y = (C_1 + C_2x)\,e^{-\frac{1}{3}x} + x^2 - 10x + 45$$

(c) $\frac{d^2y}{dx^2} - 4y = 3e^{-2x}$

   i. Solve the homogeneous D.E. $ay'' + by' + cy = 0$ (find the r)

$$\frac{d^2y}{dx^2} - 4y = 0$$
$$r^2 - 4 = 0$$
$$(r-2)(r+2) = 0$$
$$r = \pm 2$$

   ii. Find the complementary function, y_h

$$y_h = e^{2x}C_1 + e^{-2x}C_2$$

   iii. Find the particular solution y_p
   A. Find the derivatives

$$y_p = x\left[Ae^{-2x}\right]$$
$$\boldsymbol{y_p = Axe^{-2x}}$$
$$y_p' = Ae^{-2x} + (-2)\,Axe^{-2x}$$
$$= Ae^{-2x} - 2Axe^{-2x}$$
$$y_p'' = \frac{d}{dx}\left[Ae^{-2x} - 2Axe^{-2x}\right]$$
$$= -2Ae^{-2x} - 2\left(Ae^{-2x} + (-2)\,Axe^{-2x}\right)$$
$$y_p'' = -4Ae^{-2x} + 4Axe^{-2x}$$

   B. Plug into the function

$$\frac{d^2y}{dx^2} - 4y = 3e^{-2x}$$
$$\left(-4Ae^{-2x} + 4Axe^{-2x}\right) - 4\left(Axe^{-2x}\right) = 3e^{-2x}$$

   C. Find A

$$\left(-4Ae^{-2x} + 4Axe^{-2x}\right) - 4\left(Axe^{-2x}\right) = 3e^{-2x}$$
$$-4Ae^{-2x} = 3e^{-2x}$$
$$-4A = 3$$
$$A = -\frac{3}{4}$$

   D. Find y_p

$$y_p = -\frac{3}{4}e^{-2x}x$$

iv. Find the non-homogeneous D.E. general solution, $y = y_h + y_p$

$$y = e^{2x}C_1 + e^{-2x}C_2 - \frac{3}{4}e^{-2x}x$$
$$= e^{2x}C_1 + \left(C_2 - \frac{3}{4}x\right)e^{-2x}$$

(d) $3\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 5y = 9\cos 2x - 23\sin 2x$

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i. Solve the homogeneous D.E. $ay'' + by' + cy = 0$ (find the r)

$$3\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 5y = 0$$
$$3r^2 + 8r + 5 = 0$$
$$(3r + 5)(r + 1) = 0$$
$$r = -\frac{5}{3}, -1$$

ii. Find the complementary function, y_h

$$y_h = e^{-\frac{5}{3}x}C_1 + e^{-x}C_2$$

iii. Find the particular solution y_p

A. Find the derivatives

$$y_p = A\sin(2x) + B\cos(2x)$$
$$y_p' = \frac{d}{dx}[A\sin(2x) + B\cos(2x)]$$
$$= A(2\cos 2x) - 2B\sin(2x)$$
$$y_p' = 2A\cos 2x - 2B\sin(2x)$$
$$y'' = -4A\sin 2x - 4B\cos 2x$$

B. Plug into the function

$$3\frac{d^2y}{dx^2} + 8\frac{dy}{dx} + 5(A\sin(2x) + B\cos(2x)) = 9\cos 2x - 23\sin 2x$$
$$3(-4A\sin 2x - 4B\cos 2x) + 8(2A\cos 2x - 2B\sin(2x)) + 5A\sin(2x) + 5B\cos(2x) = 9\cos 2x - 23\sin 2x$$
$$-12A\sin 2x - 12B\cos 2x + 16A\cos 2x - 16B\sin 2x + 5A\sin 2x + 5B\cos 2x = 9\cos 2x - 23\sin 2x$$
$$-12A\sin 2x + 5A\sin 2x - 16B\sin 2x - 12B\cos 2x + 5B\cos 2x + 16A\cos 2x = 9\cos 2x - 23\sin 2x$$
$$-7A\sin 2x - 16B\sin 2x - 7B\cos 2x + 16A\cos 2x = 9\cos 2x - 23\sin 2x$$
$$(-7B + 16A)\cos 2x + (-7A - 16B)\sin 2x = 9\cos 2x - 23\sin 2x$$

C. Find $A\&B$ (by equating coefficients)

$$-7B + 16A = 9$$
$$16A = 9 + 7B$$
$$A = \frac{9 + 7B}{16}$$

$$-7A - 16B = 23$$
$$-7\left(\frac{9 + 7B}{16}\right) - 16B = 23$$
$$\frac{-63 - 49B}{16} - 16B = 23$$
$$-63 - 49B - 256B = 368$$
$$305B = 305$$
$$B = 1$$

$$A = \frac{9 + 7(1)}{16}$$
$$= \frac{16}{16}$$
$$A = 1$$

D. Find y_p

$$y_p = \sin(2x) + \cos(2x)$$

iv. Find the non-homogeneous D.E. general solution, $y = y_h + y_p$

$$y = e^{-\frac{5}{3}x}C_1 + e^{-x}C_2 + \sin(2x) + \cos(2x)$$

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(e) $y'' + 2y' + 5y = x^2$

    i. Solve the homogeneous D.E. $ay'' + by' + cy = 0$ (find the r)

$$r^2 + 2r + 5 = 0$$
$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$= -1 \pm 2i$$

    ii. Find the complementary function, y_h

$$y_h = e^{-x} \left(C_1 \sin 2x + C_2 \cos 2x \right)$$

    iii. Find the particular solution y_p

        A. Find the derivatives

$$y = Ax^2 + Bx + C$$
$$y' = 2Ax + B$$
$$y'' = 2A$$

        B. Plug into the function

$$2A + 2(2Ax + B) + 5\left(Ax^2 + Bx + C \right) = x^2$$
$$2A + 4Ax + 2B + 5Ax^2 + 5Bx + 5C = x^2$$
$$5Ax^2 + (4A + 5B)x + 2A + 2B + 5C = x^2 + 0x + 0$$

        C. Find A, B, C

$$5A = 1$$
$$A = \frac{1}{5}$$

$$4A + 5B = 0$$
$$4\left(\frac{1}{5} \right) + 5B = 0$$
$$5B = -\frac{4}{5}$$
$$B = -\frac{4}{25}$$

$$2A + 2B + 5C = 0$$
$$2\left(\frac{1}{5} \right) + 2\left(-\frac{4}{25} \right) + 5C = 0$$
$$\frac{2}{5} - \frac{8}{25} + 5C = 0$$
$$5C = -\frac{2}{5} + \frac{8}{25}$$
$$5C = -\frac{2}{25}$$
$$C = -\frac{2}{125}$$

        D. Find y_p

$$y_p = \frac{1}{5}x^2 - \frac{4}{25}x - \frac{2}{125}$$

    iv. Find the non-homogeneous D.E. general solution, $y = y_h + y_p$

$$y = e^{-x} \left(C_1 \sin 2x + C_2 \cos 2x \right) + \frac{1}{5}x^2 - \frac{4}{25}x - \frac{2}{125}$$

(f) $y'' - y' + 9y = 3\sin 3x$

i. Solve the homogeneous D.E. $ay'' + by' + cy = 0$ (find the r)

$$r^2 - r + 9 = 0$$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-1) \pm \sqrt{(-1)^2 - 4(9)}}{2}$$

$$= \frac{1 \pm \sqrt{1 - 36}}{2}$$

$$= \frac{1 \pm \sqrt{-35}}{2}$$

$$= \frac{1 \pm \sqrt{-1}\sqrt{35}}{2}$$

$$= \frac{1 \pm \sqrt{35}i}{2}$$

$$r = \frac{1}{2} \pm \frac{\sqrt{35}}{2}i$$

ii. Find the complementary function, y_h

$$y_h = e^{\frac{1}{2}x} \left(C_1 \sin \frac{\sqrt{35}}{2}x + C_1 \cos \frac{\sqrt{35}}{2}x \right)$$

iii. Find the particular solution y_p
    A. Find the derivatives
        From $3 \sin 3x$, we predict y will be:

$$y = A \sin 3x + B \cos 3x$$
$$y' = 3A \cos 3x - 3B \sin 3x$$
$$y'' = -9A \sin 3x - 9B \cos 3x$$

    B. Plug into the function

$$y'' - y' + 9y = 3 \sin 3x$$
$$(-9A \sin 3x - 9B \cos 3x) - (3A \cos 3x - 3B \sin 3x) + 9(A \sin 3x + B \cos 3x) = 3 \sin 3x$$
$$-9A \sin 3x - 9B \cos 3x - 3A \cos 3x + 3B \sin 3x + 9A \sin 3x + 9B \cos 3x = 3 \sin 3x$$
$$3B \sin 3x - 3A \cos 3x = 3 \sin 3x$$
$$B \sin 3x - A \cos 3x = \sin 3x + 0 \cos 3x$$

    C. Find A, B
$$B \sin 3x - A \cos 3x = \sin 3x + 0 \cos 3x$$

$$B = 1$$
$$A = 0$$

    D. Find y_p

$$y_p = \cos 3x$$

iv. Find the non-homogeneous D.E. general solution, $y = y_h + y_p$

$$y = e^{\frac{1}{2}x} \left(C_1 \sin \frac{\sqrt{35}}{2}x + C_1 \cos \frac{\sqrt{35}}{2}x \right) + \cos 3x$$

(g) $y'' + y' = 2x + 4 + 2e^x$
    i. Solve the homogeneous D.E. $ay'' + by' + cy = 0$ (find the r)

$$y'' + y' = 0$$
$$r^2 + r = 0$$
$$r(r + 1) = 0$$
$$r = 0, -1$$

ii. Find the complementary function, y_h

$$y_h = C_1 e^{0x} + C_2 e^{-1x}$$
$$y_h = C_1 + C_2 e^{-x}$$

iii. Find the particular solution y_p
   A. Find the derivatives (By prediction, find y, then find the rest of derivatives)

$$y = Ax^2 + Bx + Ce^x$$
$$y' = 2Ax + B + Ce^x$$
$$y'' = 2A + Ce^x$$

   B. Plug into the function

$$2A + Ce^x + 2Ax + B + Ce^x = 2x + 4 + 2e^x$$
$$2Ax + 2A + B + 2Ce^x = 2x + 4 + 2e^x$$

   C. Find A, B and C

$$2A = 2$$
$$A = 1$$

$$2A + B = 4$$
$$2(1) + B = 4$$
$$B = 4 - 2$$
$$B = 2$$

$$2C = 2$$
$$C = 1$$

   D. Find y_p
$$y_p = x^2 + 2x + e^x$$

iv. Find the non-homogeneous D.E. general solution, $y = y_h + y_p$

$$y = C_1 + C_2 e^{-x} + x^2 + 2x + e^x$$

(h) $y'' - 3y' - 18y = xe^{4x}$
   i. Solve the homogeneous D.E. $ay'' + by' + cy = 0$ (find the r)

$$y'' - 3y' - 18y = 0$$
$$r^2 - 3r - 18 = 0$$
$$(r - 6)(r + 3) = 0$$
$$r = 6, r = -3$$

   ii. Find the complementary function, y_h
$$y_h = e^{6x} C_1 + e^{-3x} C_2$$

   iii. Find the particular solution y_p
      A. Find the derivatives

$$y = (Bx + C) e^{4x}$$
$$y' = \frac{d}{dx} \left[Bxe^{4x} + Ce^{4x} \right]$$
$$= Be^{4x} + 4Bxe^{4x} + 4Ce^{4x}$$
$$y'' = 4Be^{4x} + 4B \left(e^{4x} + 4xe^{4x} \right) + 16Ce^{4x}$$
$$= 4Be^{4x} + 4Be^{4x} + 16Bxe^{4x} + 16Ce^{4x}$$
$$y'' = 8Be^{4x} + 16Bxe^{4x} + 16Ce^{4x}$$

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B. Plug into the function

$$8Be^{4x} + 16Bxe^{4x} + 16Ce^{4x} - 3\left(Be^{4x} + 4Bxe^{4x} + 4Ce^{4x}\right) - 18\left(Bxe^{4x} + Ce^{4x}\right) = xe^{4x}$$

$$8Be^{4x} + 16Bxe^{4x} + 16Ce^{4x} - 3Be^{4x} - 12Bxe^{4x} - 12Ce^{4x} - 18Bxe^{4x} - 18Ce^{4x} = xe^{4x}$$

$$16Bxe^{4x} - 12Bxe^{4x} - 18Bxe^{4x} - 12Ce^{4x} - 18Ce^{4x} + 16Ce^{4x} + 8Be^{4x} - 3Be^{4x} = xe^{4x}$$

$$16Bxe^{4x} - 30Bxe^{4x} - 14Ce^{4x} + 5Be^{4x} = xe^{4x}$$

$$xe^{4x}\left(-14B\right) - e^{4x}\left(14C - 5B\right) = xe^{4x}$$

C. Find B and C

$$-14B = 1$$

$$B = -\frac{1}{14}$$

$$14C - 5B = 0$$

$$14C = 5\left(-\frac{1}{14}\right)$$

$$C = -\frac{5}{14} * \frac{1}{14}$$

$$= -\frac{5}{196}$$

D. Find y_p

$$y_p = \left(-\frac{1}{14}x - \frac{5}{196}\right)e^{4x}$$

iv. Find the non-homogeneous D.E. general solution, $y = y_h + y_p$

$$y = e^{6x}C_1 + e^{-3x}C_2 + \left(-\frac{1}{14}x - \frac{5}{196}\right)e^{4x}$$

$$= e^{6x}C_1 + e^{-3x}C_2 - \frac{1}{14}x - \frac{5}{196}e^{4x}$$

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