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1. $x^2 \equiv a \pmod 5$

$x=1$: $a=1$

$x=2$: $a=4$

$x=3$: $a=4$

$x=4$: $a=1$

即 $p=5$ 的平方剩余为 1 和 4;

平方非剩余为 2 和 3.

2. $\left(\frac{13}{89}\right) = (-1)^{\frac{13-1}{2}\cdot\frac{89-1}{2}} \cdot \left(\frac{89}{13}\right)$

$= (-1)^{6\cdot 44} \cdot \left(\frac{11}{13}\right) = (-1)^{\frac{11-1}{2}\cdot\frac{13-1}{2}} \cdot \left(\frac{13}{11}\right)$

$= (-1)^{5\times 6}\left(\frac{2}{11}\right) = (-1)^{\frac{11^2-1}{8}} = (-1)^{15} = -1$

即 $\left(\frac{13}{89}\right) = -1$

$13\overline{)89}\ 6$, 78, 11

$11\overline{)130}\ 15$, 110, 20

3. $\begin{cases} 11x^2 \equiv -6 \bmod 13 \\ 11x^2 \equiv -6 \pmod 7 \end{cases} \Rightarrow \begin{cases} -2x^2 \equiv -6 \bmod 13 \\ (2x)^2 \equiv 1 \pmod 7 \end{cases}$

$(2x)^2 \equiv 1 \pmod 7$ $\quad x = \pm 3 \pmod 7$

$x^2 \equiv 3 \bmod 13$ $\quad x = \pm 4 \pmod{13}$ $\quad \left(\frac{3}{13}\right) = (-1)^{\frac{2}{2}\cdot\frac{12}{2}}\left(\frac{1}{3}\right)$

$= 1.$

即有解.

4. $\left(\frac{17}{37}\right) = (-1)^{\frac{16}{2}\cdot\frac{36}{2}} \cdot \left(\frac{37}{17}\right) = \left(\frac{3}{17}\right) = (-1)^{\frac{2}{2}\cdot\frac{16}{2}} \cdot \left(\frac{2}{3}\right) = \left(\frac{2}{3}\right) = (-1)^{\frac{9-1}{8}} = -1$

即 解数为 0.