TD/TP 5
Models For Nonstationary Time Series

5.1 Identify the following as specific ARIMA models. That is, what are p, d, and q and what are the values of the parameters (the ϕ's and θ's)?

a) $Y_t = Y_{t-1} - 0.25Y_{t-2} + e_t - 0.1e_{t-1}$.

b) $Y_t = 2Y_{t-1} - Y_{t-2} + e_t$.

c) $Y_t = 0.5Y_{t-2} + e_t + 0.25e_{t-2}$.

5.2 For each of the ARIMA models below, give the values for $E(\nabla Y_t)$ and $Var(\nabla Y_t)$.

a) $Y_t = 3 + Y_{t-1} + e_t - 0.75e_{t-1}$.

b) $Y_t = 10 + 1.25Y_{t-1} - 0.25Y_{t-2} + e_t - 0.5e_{t-1} + 0.25e_{t-2}$.

c) $Y_t = 5 + 2Y_{t-1} - 1.7Y_{t-2} + 0.7Y_{t-3} + e_t - 0.5e_{t-1} + 0.25e_{t-2}$.

5.3 Suppose that $\{Y_t\}$ is generated according to $Y_t = e_t + ce_{t-1} + ce_{t-2} + ce_{t-3} + ... + ce_0$ for $t > 0$.

a) Find the mean and covariance functions for $\{Y_t\}$. Is $\{Y_t\}$ stationary?

b) Find the mean and covariance functions for $\{\nabla Y_t\}$. Is $\{\nabla Y_t\}$ stationary?

c) Identity $\{Y_t\}$ as a specific ARIMA process.

5.4 Suppose that $Y_t = A + Bt + X_t$, where $\{X_t\}$ is a random walk. First suppose that A and B are constants.

a) Is $\{Y_t\}$ stationary?

b) Is $\{\nabla Y_t\}$ stationary?

Now suppose that A and B are random variables that are independent of the random walk $\{X_t\}$.

c) Is $\{Y_t\}$ stationary?

d) Is $\{\nabla Y_t\}$ stationary?

5.7 Consider two models:

A: $Y_t = 0.9Y_{t-1} + 0.09Y_{t-2} + e_t$.

B: $Y_t = Y_{t-1} + e_t - 0.1e_{t-1}$.

a) Identify each as a specific ARIMA model. That is, what are p, d, and q and what are the values of the parameters, ϕ's and θ's?

b) In what ways are the two models different?

c) In what ways are the two models similar? (Compare ψ-weights and π-weights.)

5.10 Nonstationary ARIMA series can be simulated by first simulating the corresponding stationary ARMA series and then "integrating" it (really partially summing it). Use statistical software to simulate a variety of IMA(1,1) and IMA(2,2) series with a variety of parameter values. Note any stochastic "trends" in the simulated series.

5.11 The data file `winnebago` contains monthly unit sales of recreational vehicles (RVs) from Winnebago, Inc., from November 1966 through February 1972.

a) Display and interpret the time series plot for these data.

b) Now take natural logarithms of the monthly sales figures and display the time series plot of the transformed values. Describe the effect of the logarithms on the behavior of the series.

c) Calculate the fractional relative changes, $(Y_t - Y_{t-1})/Y_{t-1}$, and compare them with the differences of (natural) logarithm, $\nabla \log(Y_t) = \log(Y_t) - \log(Y_{t-1})$. How do they compare for smaller values and for larger values?

5.12 The data file `SP` contains quarterly Standard & Poor's Composite Index stock price values from the first quarter of 1936 through the fourth quarter of 1977.

a) Display and interpret the time series plot for these data.

b) Now take natural logarithms of the quarterly values and display and the time series plot of the transformed values. Describe the effect of the logarithms on the behavior of the series.

c) Calculate the (fractional) relative changes, $(Y_t - Y_{t-1})/Y_{t-1}$, and compare them to the differences of (natural) logarithms, $\nabla \log(Y_t)$. How do they compare for smaller values and for larger values?

5.15 Quarterly earnings per share for the Johnson & Johnson Company are given in the data file named `JJ`. The data cover the years from 1960 through 1980.

a) Display a time series plot of the data. Interpret the interesting features in the plot.

b) Use software to produce a plot similar to Exhibit 5.11, on page 102, and determine the "best" value of λ for a power transformation of these data.

c) Display a time series plot of the transformed values. Does this plot suggest that a stationary model might be appropriate?

d) Display a time series plot of the differences of the transformed values. Does this plot suggest that a stationary model might be appropriate for the differences?

5.16 The file named `gold` contains the daily price of gold (in dollars per troy ounce) for the 252 trading days of year 2005.

a) Display the time series plot of these data. Interpret the plot.

b) Display the time series plot of the differences of the logarithms of these data. Interpret this plot.

c) Calculate and display the sample ACF for the differences of the logarithms of these data and argue that the logarithms appear to follow a random walk model.

d) Display the differences of logs in a histogram and interpret.

e) Display the differences of logs in a quantile-quantile normal plot and interpret.