1. The initial-value problem

   $y' = 1 + (y/t) + (y/t)^2$, $1 \le t \le 2$, $y(1) = 0$ has the exact solution $y(t) = t\tan(\ln t)$.

a. Use Euler's method with $h = 0.1$ to approximate the solution, and

compare it with the actual values of y.

b. Use Taylor's method of order 2 with $h = 0.1$ to approximate the solution, and compare it with the actual values of y.

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>> hw5
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t	Euler	Taylor 2	Exact	Euler Error	Taylor Error	Euler %Err	Taylor %Err
1.00	0.00000	0.00000	0.00000	0.00000e+00	0.00000e+00	0.00	0.00
1.10	0.10000	0.10500	0.10516	5.15982e-03	1.59816e-04	4.91	0.15
1.20	0.20992	0.22092	0.22124	1.13254e-02	3.23615e-04	5.12	0.15
1.30	0.33047	0.34861	0.34912	1.86506e-02	5.08747e-04	5.34	0.15
1.40	0.46235	0.48895	0.48968	2.73281e-02	7.27910e-04	5.58	0.15
1.50	0.60629	0.64288	0.64388	3.75899e-02	9.92550e-04	5.84	0.15
1.60	0.76304	0.81144	0.81275	4.97113e-02	1.31457e-03	6.12	0.16
1.70	0.93348	0.99579	0.99749	6.40191e-02	1.70740e-03	6.42	0.17
1.80	1.11854	1.19725	1.19944	8.09019e-02	2.18692e-03	6.74	0.18
1.90	1.31929	1.41734	1.42012	1.00823e-01	2.77230e-03	7.10	0.20
2.00	1.53694	1.65779	1.66128	1.24338e-01	3.48709e-03	7.48	0.21

Euler 和 Taylor2 是跑的值，Exact 是準確值

Error 和%Err 分別是誤差值跟百分誤差

2. The system of initial-value problems

$$u_1' = 9u_1 + 24u_2 + 5\cos t - \frac{1}{3}\sin t, \quad u_1(0) = \frac{4}{3},$$

$$u_2' = -24u_1 - 52u_2 - 9\cos t + \frac{1}{3}\sin t, \quad u_2(0) = \frac{2}{3},$$

has the unique solution

$$u_1 = 2e^{-3t} - e^{-39t} + \frac{1}{3}\cos t, \quad u_2 = -e^{-3t} + 2e^{-39t} - \frac{1}{3}\cos t.$$

Try $h = 0.05$ and $h = 0.1$ in Runge-Kutta method, and compare their results with the exact value.

```
=== Results for h = 0.05 ===

    t    u1 approx     u1 exact     u1 error    u2 approx     u2 exact     u2 error
 0.00     1.333333     1.333333     0.00e+00     0.666667     0.666667     1.11e-16
 0.05     1.721880     1.912059     1.90e-01    -0.499599    -0.909077     4.09e-01
 0.10     1.726915     1.793063     6.61e-02    -0.832598    -1.032002     1.99e-01
 0.15     1.617161     1.601967     1.52e-02    -0.890373    -0.961459     7.11e-02
 0.20     1.481687     1.423902     5.78e-02    -0.861042    -0.874681     1.36e-02
 0.25     1.348945     1.267646     8.13e-02    -0.807505    -0.795221     1.23e-02
 0.30     1.227063     1.131577     9.55e-02    -0.750341    -0.724999     2.53e-02
 0.35     1.117478     1.012999     1.04e-01    -0.695886    -0.663060     3.28e-02
 0.40     1.019525     0.909409     1.10e-01    -0.645732    -0.608214     3.75e-02
 0.45     0.931977     0.818630     1.13e-01    -0.599934    -0.559389     4.05e-02
 0.50     0.853541     0.738788     1.15e-01    -0.558092    -0.515658     4.24e-02
 0.55     0.783017     0.668275     1.15e-01    -0.519706    -0.476225     4.35e-02
 0.60     0.719337     0.605710     1.14e-01    -0.484290    -0.440411     4.39e-02
 0.65     0.661560     0.549909     1.12e-01    -0.451407    -0.407635     4.38e-02
 0.70     0.608868     0.499860     1.09e-01    -0.420673    -0.377404     4.33e-02
 0.75     0.560547     0.454695     1.06e-01    -0.391754    -0.349296     4.25e-02
 0.80     0.515980     0.413671     1.02e-01    -0.364365    -0.322954     4.14e-02
 0.85     0.474633     0.376158     9.85e-02    -0.338259    -0.298076     4.02e-02
 0.90     0.436043     0.341614     9.44e-02    -0.313226    -0.274409     3.88e-02
 0.95     0.399812     0.309583     9.02e-02    -0.289089    -0.251739     3.74e-02
 1.00     0.365600     0.279675     8.59e-02    -0.265698    -0.229888     3.58e-02
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=== Results for h = 0.1 ===

    t    u1 approx     u1 exact     u1 error    u2 approx     u2 exact     u2 error
 0.00     1.333333     1.333333     0.00e+00     0.666667     0.666667     1.11e-16
 0.10    -3.052437     1.793063     4.85e+00     8.989305    -1.032002     1.00e+01
 0.20   -23.847795     1.423902     2.53e+01    51.192704    -0.874681     5.21e+01
 0.30  -130.165202     1.131577     1.31e+02   269.269193    -0.724999     2.70e+02
 0.40  -680.231485     0.909409     6.81e+02  1399.368584    -0.608214     1.40e+03
 0.50 -3531.299585     0.738788     3.53e+03  7258.241839    -0.515658     7.26e+03
 0.60 -18312.795052     0.605710     1.83e+04 37634.955483    -0.440411     3.76e+04
 0.70 -94951.331907     0.499860     9.50e+04 195131.871735    -0.377404     1.95e+05
 0.80 -492306.465639     0.413671     4.92e+05 1011721.872078    -0.322954     1.01e+06
 0.90 -2552513.623867     0.341614     2.55e+06 5245578.826590    -0.274409     5.25e+06
 1.00 -13234278.789168     0.279675     1.32e+07 27197287.206587    -0.229888     2.72e+07
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