

Self Driving Cars 2023-Fall  
Homework 2  
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1. Suppose we live at a place where days are either sunny, cloudy, or rainy. The weather transition function is a Markov chain with the following transition table:

		tomorrow will be...		
		sunny	cloudy	rainy
today it's...	sunny	.8	.2	0
	cloudy	.4	.4	.2
	rainy	.2	.6	.2

(a) Suppose Day 1 is a sunny day. What is the probability of the following sequence of days: Day2 = cloudy, Day3 = cloudy, Day4 = rainy?

<ANS>

$$1 (\text{Sunny}) * 0.2(\text{sunny} \rightarrow \text{cloudy}) * 0.4(\text{cloudy} \rightarrow \text{cloudy}) * 0.2 (\text{cloudy} \rightarrow \text{rainy}) = 0.016$$

(b) Write a simulator that can randomly generate sequences of “weathers” from this state transition function.

<ANS>

```
1 """
2 @author: OU,TING-WEI @ M.S. in Robotics
3 date : 10-15-2023
4 Self-Driving-Cars HW2 ( NYCU FALL-2023 )
5 """
6 import random
7 import numpy as np
8
9 transition_probabilities = [[0.8, 0.4, 0.2], [0.2, 0.4, 0.6], [0, 0.2, 0.2]]
10
11 def random_generator(init='Sunny', days=10):
12     transition_probabilities = {
13         ('Sunny', 'Sunny'): 0.8,
14         ('Sunny', 'Cloudy'): 0.2,
15         ('Sunny', 'Rainy'): 0,
16         ('Cloudy', 'Sunny'): 0.4,
17         ('Cloudy', 'Cloudy'): 0.4,
18         ('Cloudy', 'Rainy'): 0.2,
19         ('Rainy', 'Sunny'): 0.2,
20         ('Rainy', 'Cloudy'): 0.6,
21         ('Rainy', 'Rainy'): 0.2,
22     }
23
24     sequence = [init]
25
26     for _ in range(int(days) - 1):
27         current_weather = sequence[-1]
28         next_weather = random.choices(['Sunny', 'Cloudy', 'Rainy'],
29                                       [transition_probabilities[(current_weather, 'Sunny')],
30                                       transition_probabilities[(current_weather, 'Cloudy')],
31                                       transition_probabilities[(current_weather, 'Rainy')]])[0]
32         sequence.append(next_weather)
33
34     print("the randomly generate sequence of weather")
35     for day, weather in enumerate(sequence, start=1):
36         if weather == 'Sunny':
37             tomorrow = [0.8, 0.2, 0]
38         if weather == 'Rainy':
39             tomorrow = [0.2, 0.6, 0.2]
40         if weather == 'Cloudy':
41             tomorrow = [0.4, 0.4, 0.2]
42         print(f"day: {day} {weather}, tomorrow will be : ", tomorrow)
```

```

43 def stationary_distribution(init='Sunny', days=10):
44     if init == "Sunny" :
45         state = np.array([1, 0, 0]).T
46     if init == "Cloudy" :
47         state = np.array([0, 1, 0]).T
48     if init == "Rainy" :
49         state = np.array([0, 0, 1]).T
50
51     for i in range(int(days)):
52         state = transition_probabilities @ state
53         print("the probability of tomorrow's weather is : ", state.T)
54
55 def main():
56     init_weather = init_weather = input("today's weather = ")
57     days = input("How many days do you want to predict? ")
58
59     random_generator(init_weather, days)
60
61     print('-----')
62
63     stationary_distribution(init_weather, days)
64
65 if __name__ == '__main__':
66     main()
67

```

< DEMO > Today is cloudy, days = 10

```

the randomly generate sequence of weather :
day: 1 : Cloudy , tomorrow will be : [0.4, 0.4, 0.2]
day: 2 : Cloudy , tomorrow will be : [0.4, 0.4, 0.2]
day: 3 : Cloudy , tomorrow will be : [0.4, 0.4, 0.2]
day: 4 : Cloudy , tomorrow will be : [0.4, 0.4, 0.2]
day: 5 : Cloudy , tomorrow will be : [0.4, 0.4, 0.2]
day: 6 : Rainy , tomorrow will be : [0.2, 0.6, 0.2]
day: 7 : Cloudy , tomorrow will be : [0.4, 0.4, 0.2]
day: 8 : Sunny , tomorrow will be : [0.8, 0.2, 0]
day: 9 : Cloudy , tomorrow will be : [0.4, 0.4, 0.2]
day: 10 : Sunny , tomorrow will be : [0.8, 0.2, 0]

```

< DEMO > Today is Sunny, days = 10

```

the randomly generate sequence of weather :
day: 1 : Sunny , tomorrow will be : [0.8, 0.2, 0]
day: 2 : Sunny , tomorrow will be : [0.8, 0.2, 0]
day: 3 : Sunny , tomorrow will be : [0.8, 0.2, 0]
day: 4 : Sunny , tomorrow will be : [0.8, 0.2, 0]
day: 5 : Sunny , tomorrow will be : [0.8, 0.2, 0]
day: 6 : Cloudy , tomorrow will be : [0.4, 0.4, 0.2]
day: 7 : Rainy , tomorrow will be : [0.2, 0.6, 0.2]
day: 8 : Sunny , tomorrow will be : [0.8, 0.2, 0]
day: 9 : Cloudy , tomorrow will be : [0.4, 0.4, 0.2]
day: 10 : Sunny , tomorrow will be : [0.8, 0.2, 0]

```

< DEMO > Today is Rainy, days = 10

```

the randomly generate sequence of weather :
day: 1 : Rainy , tomorrow will be : [0.2, 0.6, 0.2]
day: 2 : Sunny , tomorrow will be : [0.8, 0.2, 0]
day: 3 : Cloudy , tomorrow will be : [0.4, 0.4, 0.2]
day: 4 : Cloudy , tomorrow will be : [0.4, 0.4, 0.2]
day: 5 : Rainy , tomorrow will be : [0.2, 0.6, 0.2]
day: 6 : Cloudy , tomorrow will be : [0.4, 0.4, 0.2]
day: 7 : Sunny , tomorrow will be : [0.8, 0.2, 0]
day: 8 : Sunny , tomorrow will be : [0.8, 0.2, 0]
day: 9 : Sunny , tomorrow will be : [0.8, 0.2, 0]
day: 10 : Sunny , tomorrow will be : [0.8, 0.2, 0]

```

(c) Use your simulator to determine the stationary distribution of this Markov chain. The stationary distribution measures the probability that a random day will be sunny, cloudy, or rainy.

<ANS>

< DEMO > Today is sunny, days = 10

```
the probability of tomorrow's weather is : [[0.8 0.2 0. ]]
the probability of tomorrow's weather is : [[0.72 0.24 0.04]]
the probability of tomorrow's weather is : [[0.68 0.264 0.056]]
the probability of tomorrow's weather is : [[0.6608 0.2752 0.064 ]]
the probability of tomorrow's weather is : [[0.65152 0.28064 0.06784]]
the probability of tomorrow's weather is : [[0.64704 0.283264 0.069696]]
the probability of tomorrow's weather is : [[0.6448768 0.2845312 0.070592 ]]
the probability of tomorrow's weather is : [[0.64383232 0.28514304 0.07102464]]
the probability of tomorrow's weather is : [[0.643328 0.28543846 0.07123354]]
the probability of tomorrow's weather is : [[0.64308449 0.28558111 0.0713344 ]]
```

< DEMO > Today is Cloudy, days = 10

```
the probability of tomorrow's weather is : [[0.4 0.4 0.2]]
the probability of tomorrow's weather is : [[0.52 0.36 0.12]]
the probability of tomorrow's weather is : [[0.584 0.32 0.096]]
the probability of tomorrow's weather is : [[0.6144 0.3024 0.0832]]
the probability of tomorrow's weather is : [[0.62912 0.29376 0.07712]]
the probability of tomorrow's weather is : [[0.636224 0.2896 0.074176]]
the probability of tomorrow's weather is : [[0.6396544 0.2875904 0.0727552]]
the probability of tomorrow's weather is : [[0.64131072 0.28662016 0.07206912]]
the probability of tomorrow's weather is : [[0.64211046 0.28615168 0.07173786]]
the probability of tomorrow's weather is : [[0.64249661 0.28592548 0.07157791]]
```

< DEMO > Today is Rainy, days = 10

```
the probability of tomorrow's weather is : [[0.2 0.6 0.2]]
the probability of tomorrow's weather is : [[0.44 0.4 0.16]]
the probability of tomorrow's weather is : [[0.544 0.344 0.112]]
the probability of tomorrow's weather is : [[0.5952 0.3136 0.0912]]
the probability of tomorrow's weather is : [[0.61984 0.2992 0.08096]]
the probability of tomorrow's weather is : [[0.631744 0.292224 0.076032]]
the probability of tomorrow's weather is : [[0.6374912 0.2888576 0.0736512]]
the probability of tomorrow's weather is : [[0.64026624 0.287232 0.07250176]]
the probability of tomorrow's weather is : [[0.64160614 0.2864471 0.07194675]]
the probability of tomorrow's weather is : [[0.64225311 0.28606812 0.07167877]]
```