1. Code with detailed explanations (30%)

i. Part1 (kernel k-means 5%, normalized cut 5%, ratio cut 5%)

```
def load_image(image_path):
    img = np.array(Image.open(image_path).convert('RGB'))
    h, w, _ = img.shape
    return img, h, w
```

kernel k-means

$$\mathbf{k}(x, x') = e^{-\gamma_s \|S(x) - S(x')\|^2} \times e^{-\gamma_c \|C(x) - C(x')\|^2}$$

- $S(x)$: spatial coordinate of pixel x (e.g., row, column)
- $C(x)$: color (RGB) value of pixel x
- γ_s: controls the influence of spatial distance
- γ_c: controls the influence of color difference

```
def compute_kernel_matrix(img, gamma_s, gamma_c):
    h, w, c = img.shape
    N = h * w

    # Create spatial coordinate vectors: S(x)
    spatial = np.array([[i // w, i % w] for i in range(N)])

    # Flatten the image to get color vectors: C(x)
    rgb = img.reshape((-1, 3))

    # Compute pairwise squared distances in spatial domain: ||S(x) - S(x')||^2
    spatial_dists = squareform(pdist(spatial, 'sqeuclidean'))

    # Compute pairwise squared distances in color domain: ||C(x) - C(x')||^2
    rgb_dists = squareform(pdist(rgb, 'sqeuclidean'))

    # Combine both distance matrices using Gaussian kernel
    # k(x, x') = exp(-γ_s * spatial_dist) * exp(-γ_c * color_dist)
    K = np.exp(-gamma_s * spatial_dists) * np.exp(-gamma_c * rgb_dists)

    return K
```

The squared distance between a sample x_j and the centroid μ_k^ϕ of cluster k in the feature space is:

$$\left\|\phi(x_j) - \mu_k^\phi\right\|^2 = \left\|\phi(x_j) - \frac{1}{|C_k|}\sum_{n=1}^{N} \alpha_{kn}\phi(x_n)\right\|^2$$

Expanding this using kernel functions:

$$= \underbrace{\mathbf{k}(x_j, x_j)}_{self similarity} - \frac{2}{|C_k|}\sum_n \alpha_{kn}\mathbf{k}(x_j, x_n) + \frac{1}{|C_k|^2}\sum_p\sum_q \alpha_{kp}\alpha_{kq}\mathbf{k}(x_p, x_q)$$

- $\phi(x)$: The high-dimensional mapping of x
- μ_k^ϕ: The centroid of cluster k in the feature space
- α_{kn}: 1 if $x_n \in C_k$; otherwise 0
- $\mathbf{k}(x_i, x_j)$: The kernel function

```python
def kernel_kmeans(K, k, max_iter=100, tol=1e-3, init_method='kmeans++'):
    N = K.shape[0]
    if init_method == 'random':
        np.random.seed(42)
        labels = np.random.randint(0, k, size=N)
    elif init_method == 'kmeans++':
        np.random.seed(42)
        centers = [np.random.randint(0, N)]
        for _ in range(1, k):
            D = np.array([1 - np.max(K[i, centers]) for i in range(N)])
            probs = D ** 2 / np.sum(D ** 2)
            next_center = np.random.choice(N, p=probs)
            centers.append(next_center)
        labels = np.zeros(N, dtype=int)
        for i in range(N):
            labels[i] = np.argmax([K[i, c] for c in centers])
    else:
        raise ValueError("init_method must be 'random' or 'kmeans++'")

    snapshots = []

    for it in range(max_iter):
        cluster_indices = [np.where(labels == c)[0] for c in range(k)]
        intra_K = np.zeros(k)
        for c in range(k):
            idx = cluster_indices[c]
            if len(idx) > 0:
                intra_K[c] = np.sum(K[np.ix_(idx, idx)]) / (len(idx) ** 2)

        new_labels = np.zeros(N, dtype=int)
        for i in range(N):
            best_c = 0
            min_dist = float('inf')
            for c in range(k):
                idx = cluster_indices[c]
                if len(idx) == 0:
                    continue
                term1 = K[i, i]
                term2 = -2 * np.sum(K[i, idx]) / len(idx)
                term3 = intra_K[c]
                dist = term1 + term2 + term3
                if dist < min_dist:
                    min_dist = dist
                    best_c = c
            new_labels[i] = best_c

        snapshots.append(new_labels.copy())
        changed = np.sum(new_labels != labels)
        print(f"Iteration {it + 1}: {changed} points changed.")

        if changed < tol * N:
            print("Converged.")
            break

        labels = new_labels

    return labels, snapshots
```

▪ **Step 1: Compute intra-cluster kernel terms**

For each cluster c:

- Find all indices i assigned to cluster c.
- Compute:

$$\text{intra\_K}[c] = \frac{1}{|C_c|^2} \sum_{i,j \in C_c} K[i,j]$$

- This represents the squared norm of the centroid μ_c^ϕ in feature space.

▪ Step 2: Assign new cluster labels

For each data point xix_ixi, compute its distance to each cluster centroid using:

$$\text{dist}(x_i, C_c) = K[i,i] - \frac{2}{|C_c|} \sum_{j \in C_c} K[i,j] + \text{intra\_K}[c]$$

- Assign point to the cluster with the **minimum kernel-based distance**.

▪ Step 3: Check convergence

- Count how many points changed labels.
- If `changed < tol * N`, stop early (converged).

```
def kernel_kmeans_pipeline(image_path, n_clusters, gamma_s, gamma_c, init_method='kmeans++'):
    img, h, w = load_image(image_path)
    K = compute_kernel_matrix(img, gamma_s, gamma_c)
    labels, snapshots = kernel_kmeans(K, n_clusters, init_method=init_method)
    file_prefix = f"{os.path.splitext(os.path.basename(image_path))
[0]}_kernel_{init_method}_{n_clusters}clusters"
    save_gif_and_png(snapshots, h, w, n_clusters, file_prefix)
```

Executes the full kernel k-means clustering pipeline on an image. Includes feature extraction, kernel computation, clustering, and visualization

spectral clustering

Spectral Clustering is a graph-based clustering algorithm that uses the eigenvectors of the graph Laplacian to embed the data into a lower-dimensional space, where K-Means can then be applied.

```
def construct_laplacian(W, method='normalized'):
    D = np.diag(np.sum(W, axis=1))
    if method == 'ratio':
        return D - W
    elif method == 'normalized':
        D_inv_sqrt = np.diag(1.0 / np.sqrt(np.diag(D)))
        return np.identity(W.shape[0]) - D_inv_sqrt @ W @ D_inv_sqrt
    else:
        raise ValueError("method must be 'normalized' or 'ratio'")
```

- Constructs the **graph Laplacian matrix** used in Spectral Clustering.
- `W` is the **similarity (adjacency) matrix**, typically derived from a kernel.
- Two common variants: - **Ratio cut (unnormalized):**

$$L = D - W$$

- Normalized cut (symmetric):

$$L_{\text{sym}} = I - D^{-1/2} W D^{-1/2}$$

Where:

- D: Degree matrix, a diagonal matrix with $D_{ii} = \sum_j W_{ij}$
- I: Identity matrix

```
def spectral_embedding(L, n_clusters, skip_first=True):
    eigvals, eigvecs = eigh(L)
    start_idx = 1 if skip_first else 0
    U = eigvecs[:, start_idx:start_idx + n_clusters]
    return U / np.linalg.norm(U, axis=1, keepdims=True)
```

- Performs **eigen-decomposition** of the Laplacian matrix L to obtain the **embedding matrix** U.
- This embedding captures the geometry of the data graph.
- The first eigenvector is usually skipped (`skip_first=True`) because: - For normalized Laplacian: the first eigenvalue is 0 and corresponds to the trivial constant eigenvector. Output:
- $U \in \mathbb{R}^{N \times k}$ where each row is the projection of a point into the **spectral space**. Why normalize the rows?
- Row-wise normalization ensures that clustering is based on **direction** (angular similarity) in the spectral space.

```
def kmeans(X, k, max_iter=100, tol=1e-4, init_method='kmeans++'):
    N = X.shape[0]
    centers = initialize_kmeans(X, k, init_method)
    labels = np.zeros(N, dtype=int)
    snapshots = []

    for it in range(max_iter):
        dists = np.linalg.norm(X[:, None, :] - centers[None, :, :], axis=2)
        new_labels = np.argmin(dists, axis=1)
        snapshots.append(new_labels.copy())

        if np.sum(new_labels != labels) < tol * N:
            print(f"Converged at iteration {it + 1}")
            break

        labels = new_labels
        for i in range(k):
            if np.any(labels == i):
                centers[i] = X[labels == i].mean(axis=0)

    return labels, snapshots
```

- Applies the **K-Means algorithm** to points in the **spectral embedding space**.
- Used after computing eigenvectors of the Laplacian.
- Initialization supports:
  - `'random'` : randomly choose initial centroids
  - `'kmeans++'` : probabilistic spread-out selection of centroids
- Tracks label changes across iterations for convergence.

```
def spectral_clustering_pipeline(image_path, n_clusters, gamma_s, gamma_c, method='normalized',
init_method='kmeans++'):
    img, h, w = load_image(image_path)
    K = compute_kernel_matrix(img, gamma_s, gamma_c)
    L = construct_laplacian(K, method)
    U = spectral_embedding(L, n_clusters)
    labels, snapshots = kmeans(U, n_clusters, init_method=init_method)
    file_prefix = f"{os.path.splitext(os.path.basename(image_path))
[0]}_spectral_{method}_{init_method}_{n_clusters}clusters"
```

```
    save_gif_and_png(snapshots, h, w, n_clusters, file_prefix)
    visualize_eigenspace(U, labels, file_prefix)
```

1. **Load image** and extract its RGB data and shape.
2. **Compute the kernel matrix** K using both spatial and color similarities.
3. **Construct the graph Laplacian** (`L`) using ratio or normalized method.
4. **Compute eigenvectors** of the Laplacian and extract the top `n_clusters` ones as embedding matrix `U` .
5. **Apply K-Means** in the eigenspace to cluster pixels.
6. **Save results** as a `.gif` (animated clustering) and `.png` (final output).
7. **Visualize the eigenspace** (2D or 3D scatter plot of eigenvectors with cluster colors)

Part4: Experiments on the coordinates in the eigenspace

visualize the coordinates of each data point (e.g., image pixel) in that eigenspace, colored by its final cluster assignment.

```
def visualize_eigenspace(U, labels, file_prefix, output_dir='./eigenspace_vis'):
    os.makedirs(output_dir, exist_ok=True)
    k = U.shape[1]
    fig_path = os.path.join(output_dir, f"{file_prefix}_eigenspace.png")
    if k == 2:
        plt.figure(figsize=(8, 6))
        for c in np.unique(labels):
            plt.scatter(U[labels == c, 0], U[labels == c, 1], label=f"Cluster {c}", s=5)
        plt.xlabel("Eigenvector 1")
        plt.ylabel("Eigenvector 2")
        plt.title("Eigenspace Visualization (2D)")
        plt.legend()
        plt.grid(True)
        plt.savefig(fig_path)
        plt.close()
        print(f"Eigenspace plot saved to: {fig_path}")
    elif k == 3:
        fig = plt.figure(figsize=(10, 8))
        ax = fig.add_subplot(111, projection='3d')
        for c in np.unique(labels):
            ax.scatter(U[labels == c, 0], U[labels == c, 1], U[labels == c, 2], label=f"Cluster {c}",
s=5)
        ax.set_title("Eigenspace Visualization (3D)")
        ax.set_xlabel("Eigenvector 1")
        ax.set_ylabel("Eigenvector 2")
        ax.set_zlabel("Eigenvector 3")
        ax.legend()
        plt.savefig(fig_path)
        plt.close()
        print(f"Eigenspace 3D plot saved to: {fig_path}")
    else:
        print(f"[Warning] Eigenspace visualization only supported for k=2 or 3 (got k={k}). Skipped.")
```

- `U` : Spectral embedding matrix (shape: $N \times k$), where each row is the coordinate of a data point in eigenspace.
- `labels` : Cluster assignments for each point.
- `file_prefix` : Output file naming base.
- `output_dir` : Directory to save the image.

Output

- Saves a `.png` image showing:
  - **2D scatter plot** (if $k = 2$)
  - **3D scatter plot** (if $k = 3$)
- Each point is colored by its cluster label.

```
def spectral_clustering_pipeline(...):
    ...
    save_gif_and_png(snapshots, h, w, n_clusters, file_prefix)
    visualize_eigenspace(U, labels, file_prefix)
```

```
def labels_to_rgb(labels, n_clusters):
    colormap = plt.get_cmap("tab10", n_clusters)
    colors = (colormap(np.arange(n_clusters))[:, :3] * 255).astype(np.uint8)
    return colors[labels]
```

```
def save_gif_and_png(snapshots, h, w, n_clusters, file_prefix, output_dir="./output"):
    os.makedirs(output_dir, exist_ok=True)
    gif_frames = []
    for labels in snapshots:
        rgb_img = labels_to_rgb(labels, n_clusters).reshape(h, w, 3)
        gif_frames.append(rgb_img)

    gif_path = os.path.join(output_dir, f"{file_prefix}.gif")
    imageio.mimsave(gif_path, gif_frames, format='GIF', duration=0.5)
    print(f"Saved GIF to {gif_path}")

    png_path = os.path.join(output_dir, f"{file_prefix}_final.png")
    final_frame = gif_frames[-1]
    Image.fromarray(final_frame).save(png_path)
    print(f"Saved final PNG to {png_path}")
```

- Converts each label map (from `snapshots`) to an RGB image.
- Creates an animated `.gif` of the entire clustering process.
- Saves the final frame (last iteration) as a `.png`.

```
def run_all_configs(image_path, gamma_s, gamma_c, n_clusters_list, init_methods, modes,
laplacian_methods):
    for mode in modes:
        for init_method in init_methods:
            for n_clusters in n_clusters_list:
                if mode == 'kernel':
                    print(f"\n[Kernel K-Means] clusters={n_clusters}, init={init_method}")
                    try:
                        kernel_kmeans_pipeline(
                            image_path=image_path,
                            n_clusters=n_clusters,
```

```
                            gamma_s=gamma_s,
                            gamma_c=gamma_c,
                            init_method=init_method
                        )
                    except Exception as e:
                        print(f"Error in kernel_kmeans_pipeline: {e}")

                elif mode == 'spectral':
                    for lap_method in laplacian_methods:
                        print(f"\n[Spectral Clustering] clusters={n_clusters}, init={init_method},
laplacian={lap_method}")
                        try:
                            spectral_clustering_pipeline(
                                image_path=image_path,
                                n_clusters=n_clusters,
                                gamma_s=gamma_s,
                                gamma_c=gamma_c,
                                method=lap_method,
                                init_method=init_method
                            )
                        except Exception as e:
                            print(f"Error in spectral_clustering_pipeline: {e}")

                else:
                    print(f"Unknown mode: {mode}")
```

Runs **batch experiments** for all combinations of clustering mode, number of clusters, initialization methods, and Laplacian types.

Parameters:

- `image_path`: Path to the input image.
- `gamma_s`, `gamma_c`: Kernel parameters for spatial and color similarity.
- `n_clusters_list`: List of values for number of clusters (e.g., `[2, 3, 4]`).
- `init_methods`: List of initialization strategies (e.g., `['random', 'kmeans++']`).
- `modes`: `['kernel', 'spectral']` for selecting clustering algorithms.
- `laplacian_methods`: Applicable only if mode is `'spectral'` (e.g., `['ratio', 'normalized']`). Features:
- Each configuration will:
  - Print progress and configuration info.
  - Attempt to run the full pipeline and handle exceptions gracefully.
- Output includes `.gif`, `.png`, and optional eigenspace plots.

2. Experiments settings and results(30%) & discussion (20%)

Part1+Part2 + Part3

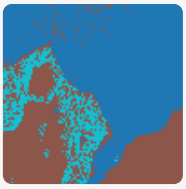
TA Can See the GIF images version in this link

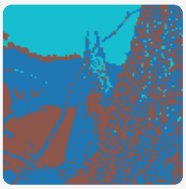
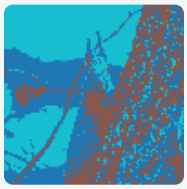
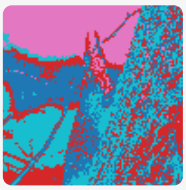
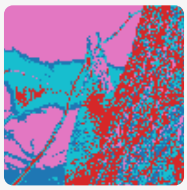
```
if __name__ == "__main__":
    gamma_s = 0.001
    gamma_c = 0.001
    n_clusters_list = [2, 3, 4] # Part2: Try more clusters (5%)
    init_methods = ['random', 'kmeans++'] # Part3: Try different initializations. (5%)
    modes = ['kernel', 'spectral']
    laplacian_methods = ['ratio', 'normalized']

    for i in range(1, 3):
        image_path = f"./image{i}.png"
        run_all_configs(
            image_path=image_path,
            gamma_s=gamma_s,
            gamma_c=gamma_c,
            n_clusters_list=n_clusters_list,
            init_methods=init_methods,
            modes=modes,
            laplacian_methods=laplacian_methods
        )
```

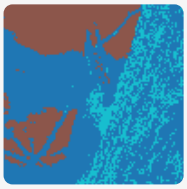
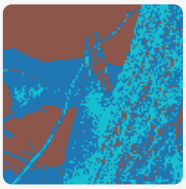
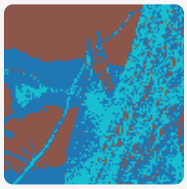
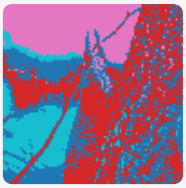
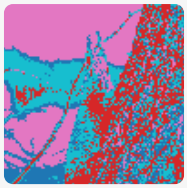
Image1

	kernel k-means method	spectral clustering (normalized)	spectral clustering (ratio)
	random	random	random
2 clusters			
3 clusters			
4 clusters			

	kernel k-means method	spectral clustering (normalized)	spectral clustering (ratio)
	k-means++	k-means++	k-means++

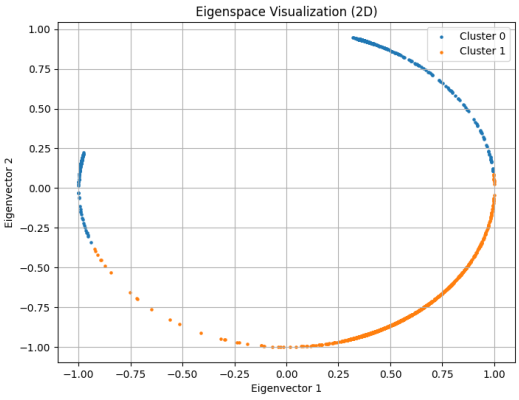
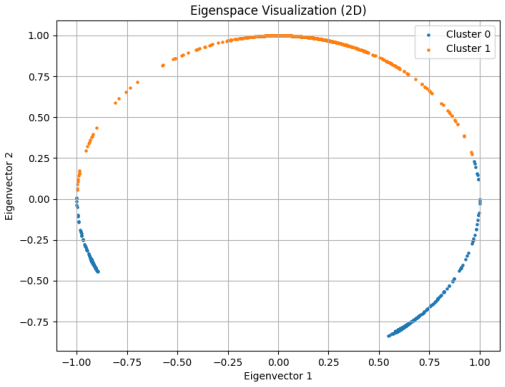
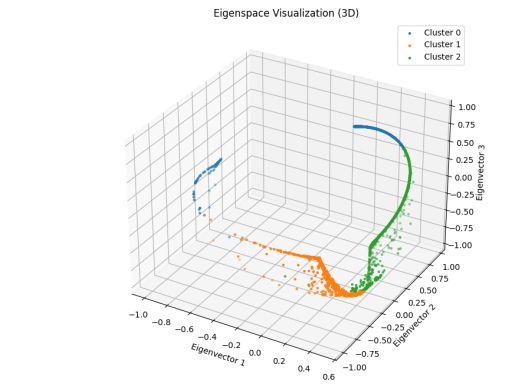
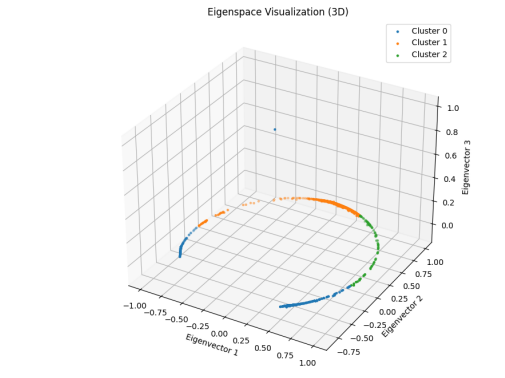
	kernel k-means method	spectral clustering (normalized)	spectral clustering (ratio)
2 clusters			
3 clusters			
4 clusters			

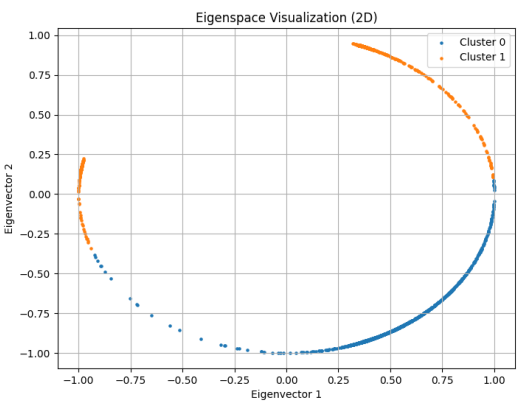
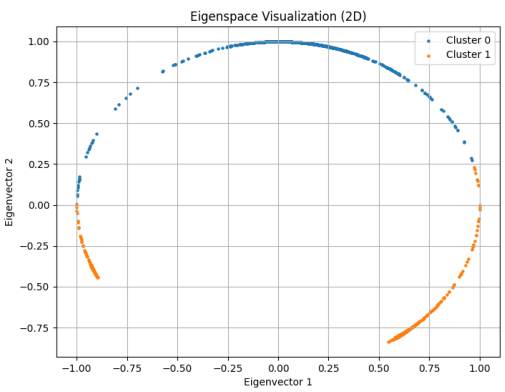
	kernel k-means method	spectral clustering (normalized)	spectral clustering (ratio)
	random	random	random
2 clusters			
3 clusters			
4 clusters			

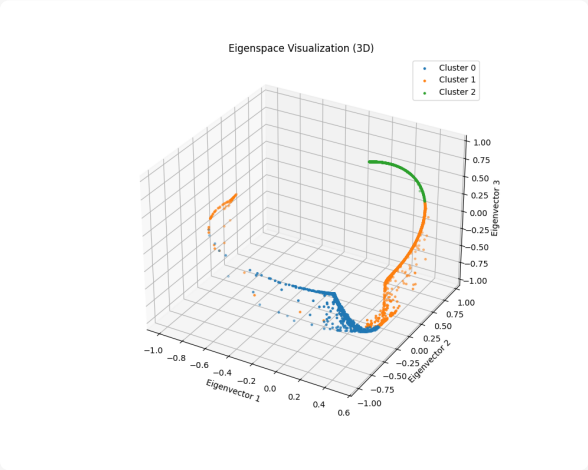
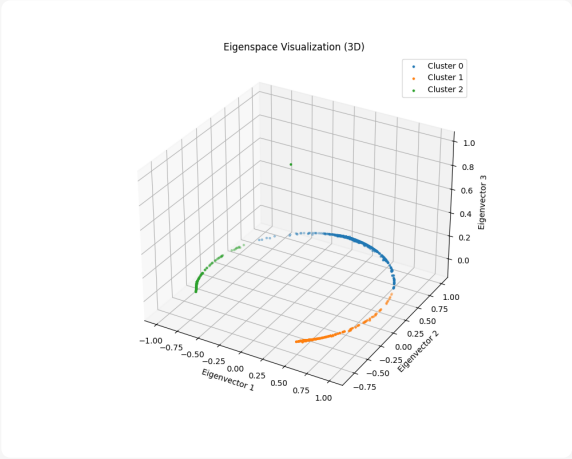
	kernel k-means method	spectral clustering (normalized)	spectral clustering (ratio)
	k-means++	k-means++	k-means++
2 clusters			
3 clusters			
4 clusters			

Part4

To observe how well-separated the clusters are in the spectral embedding space (formed by the Laplacian's eigenvectors). Better separability in eigenspace often leads to more reliable clustering.

	spectral clustering (normalized)	spectral clustering (ratio)
	random	random
2 clusters		
3 clusters		

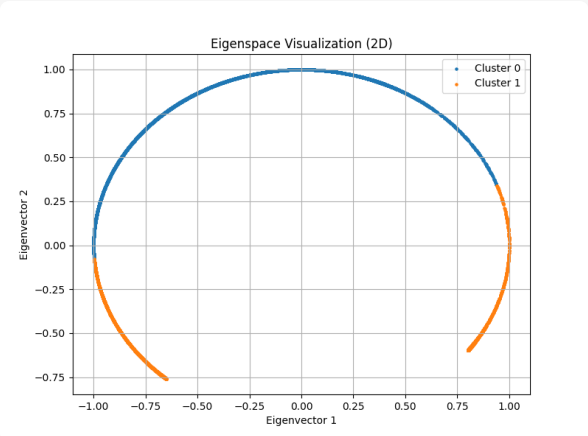
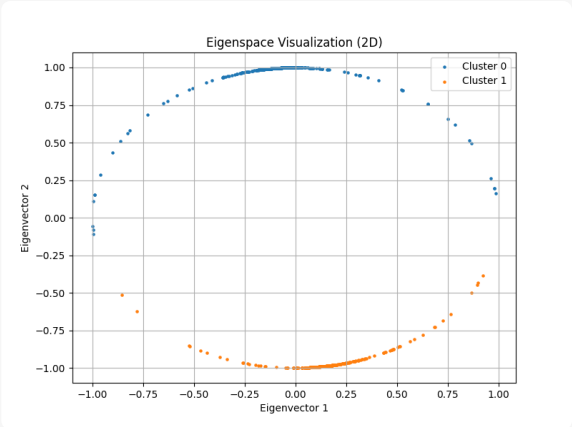
	spectral clustering (normalized)	spectral clustering (ratio)
	kmeans++	kmeans++
2 clusters		

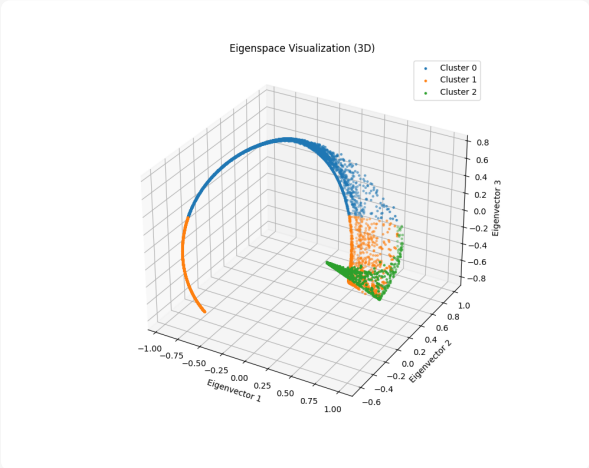
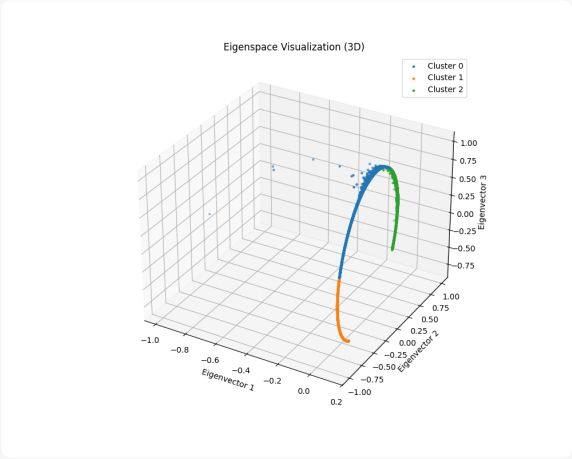
	spectral clustering (normalized)	spectral clustering (ratio)
3 clusters		

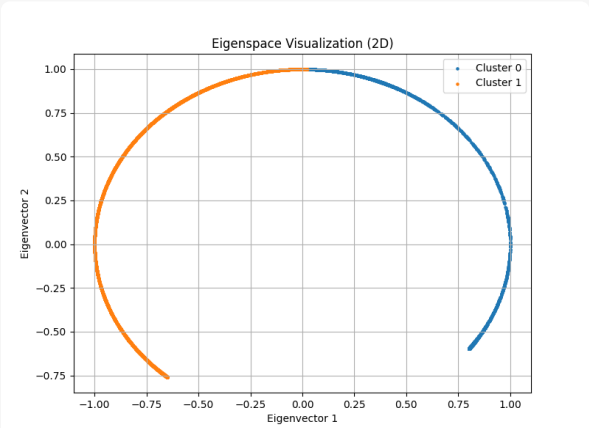
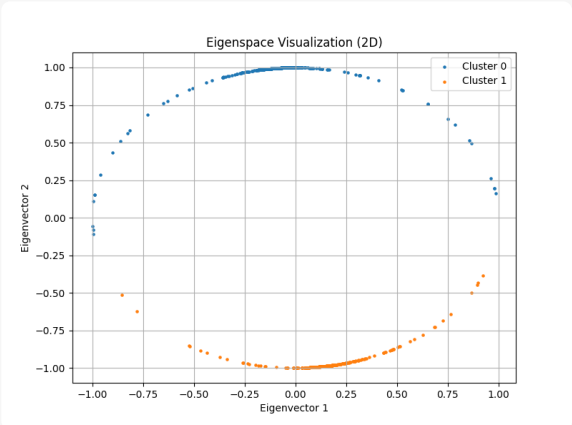
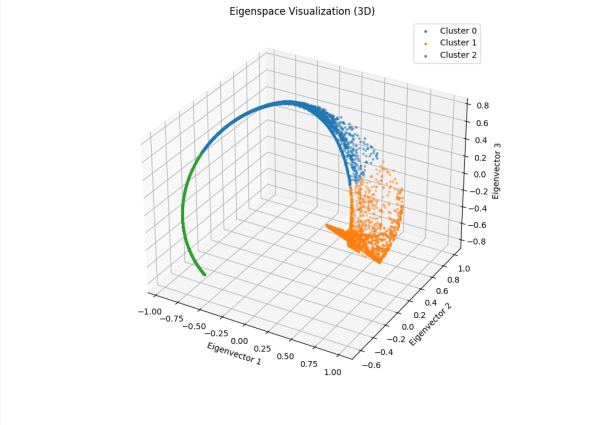
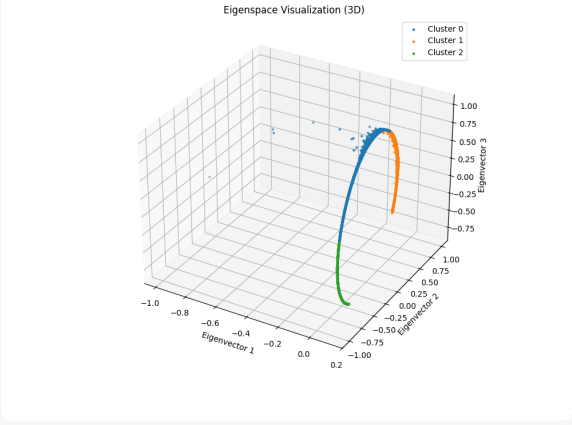
Aspect	Spectral (Normalized)	Spectral (Ratio)
2 Clusters	Strong structure, circular arc	Same; slightly tighter clusters
3 Clusters	Smooth curved embedding, good	More compact but slightly sparse
k-means++ vs Random	Improves clarity	Greatly improves density & balance

- The **eigenspace structure of Image1** is **strongly curved**, which makes spectral clustering ideal.
- **Normalized Laplacian** provides smoother embeddings.
- **Ratio Laplacian** benefits more from **k-means++**, especially at higher cluster counts.

image2

	spectral clustering (normalized)	spectral clustering (ratio)
	random	random
2 clusters		

	spectral clustering (normalized)	spectral clustering (ratio)
3 clusters		

	spectral clustering (normalized)	spectral clustering (ratio)
	kmeans++	kmeans++
2 clusters		
3 clusters		

Setting	Normalized Laplacian	Ratio Laplacian
2 Clusters	Arc-shaped, clean boundary	Similar arc, more sensitive to init
3 Clusters	Hook-shaped 3D arc, moderate overlap	Good shape but less balanced w/ random
k-means++ Effect	Refines boundaries, improves clarity	Dramatically improves compactness & separation

- Image2's spectral embeddings **form clear low-dimensional manifolds**.
- **Spectral Clustering (Ratio)** can match or exceed normalized performance when paired with `k-means++`.
- Choosing proper initialization is crucial when embeddings are **sparse or uneven**.

3. Observations and discussion (20%)

Compare the performance between different clustering methods. (8%)

image1

2 Clusters

Random Initialization

- **Kernel K-Means**: Clusters appear noisy, some irregularities near boundaries.
- **Spectral Clustering (Normalized/Ratio)**: Cleaner segmentation; objects are more clearly separated, especially the background and main object. **K-Means++ Initialization**
- Improved consistency across all methods.
- **Spectral Clustering (Normalized)** maintains the most visually stable and coherent segmentation.
- **Kernel K-Means** becomes noticeably better, with reduced noise and clearer regions.

Spectral Clustering (especially normalized) effectively captures global structures even with just two clusters. `k-means++` helps eliminate instability from random starts.

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3 Clusters

Random Initialization

- **Kernel K-Means**: More fragmentation observed. One region appears over-segmented.
- **Spectral Clustering (Normalized)**: Provides distinguishable separation of three regions.
- **Spectral Clustering (Ratio)**: Similar to normalized, though slightly less defined.

K-Means++ Initialization

- Clear performance improvement in all methods.
- **Spectral Clustering (Normalized)** shows the sharpest boundaries and consistent cluster shapes.
- **Kernel K-Means** is more balanced than in the random case.

With more clusters, initialization plays a bigger role. Spectral Clustering retains robustness and interpretability.

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4 Clusters

Random Initialization

- All methods show some fragmentation or over-segmentation.
- **Spectral (Normalized)** still outperforms the rest, though boundary clarity is slightly reduced.
- **Kernel K-Means** has the most scattered small regions.

K-Means++ Initialization

- **Spectral Clustering (Normalized)** shows a clear and meaningful 4-region split.
- **Spectral (Ratio)** is still acceptable but slightly noisier.
- **Kernel K-Means** improves compared to random, but cluster shapes are less smooth.

When the number of clusters increases, Spectral Clustering with normalization remains the most reliable. Kernel K-Means becomes less stable and more sensitive to initialization.

image2

2 Clusters

Random Initialization

- **Kernel K-Means**:
  - Clean split with large uniform regions.
  - Some subtle noise on boundaries.
- **Spectral Clustering (Normalized & Ratio)**:
  - More sensitive to edge textures and finer details.
  - Resulting segmentation includes scattered fragments.
  - Ratio method appears slightly smoother than normalized.

K-Means++ Initialization

- **Kernel K-Means**:
  - Still clean and consistent; slightly more balanced than with random.
- **Spectral Clustering**:
  - Still noisy in fine-textured regions, but more stable than random.
  - Both normalized and ratio produce very similar results.

Kernel K-Means is more spatially coherent under 2-cluster scenarios, while Spectral Clustering is more sensitive to local texture details. Initialization method does not heavily affect outcome at 2 clusters, but `k-means++` offers slightly cleaner results.

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3 Clusters

Random Initialization

- **Kernel K-Means**:
  - Good separation of major areas, but some mixed boundaries.
- **Spectral Clustering**:
  - Segment boundaries align more with texture transitions.
  - Noticeable patchiness, especially in normalized version.

K-Means++ Initialization

- **Kernel K-Means**:
  - Reduced patch noise, better-defined cluster shapes.
- **Spectral Clustering**:
  - Less chaotic than random; smoother transitions.
  - Normalized and ratio still differ slightly in region compactness.

With more clusters, the effect of initialization becomes more noticeable. Spectral methods capture more nuanced structures but are prone to over-segmentation without proper tuning.

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4 Clusters

Random Initialization

- **Kernel K-Means**:
  - Cluster regions look natural, but smaller details may be under-represented.
- **Spectral Clustering**:

- Both normalized and ratio highlight very fine-grained structures.
- More scattered pixel clusters, which might be undesirable for object-level segmentation.

K-Means++ Initialization

- **Kernel K-Means**:
  - Significantly improved — clearer separation and compact clusters.
- **Spectral Clustering**:
  - Ratio version becomes more structured and readable.
  - Normalized still prone to noise in textured regions, but better than random.

With higher cluster counts, `k-means++` is essential to stabilize the results. Spectral Clustering (Ratio) becomes more visually coherent than Normalized in high-cluster settings.

Compare the execution time of different settings. (8%)

image1

	kernel k-means method	spectral clustering (normalized)	spectral clustering (ratio)
	random	random	random
2 clusters	40.37 sec	312.72 sec	173.45 sec
3 clusters	30.43 sec	176.54 sec	200.60 sec
4 clusters	84.31 sec	209.88 sec	197.34 sec

	kernel k-means method	spectral clustering (normalized)	spectral clustering (ratio)
	k-means++	k-means++	k-means++
2 clusters	45.47 sec	208.94 sec	150.79 sec
3 clusters	41.60 sec	210.32 sec	173.40 sec
4 clusters	79.79 sec	218.64 sec	152.71 sec

Observations

1. **General Trends**

- **Kernel K-Means** is significantly faster across all settings.
- **Spectral Clustering (Normalized)** is the slowest method, particularly for `2 clusters` and `k-means++`.
- **Spectral Clustering (Ratio)** lies between the two in terms of runtime, generally faster than normalized but slower than kernel k-means.

2. **Effect of Initialization Method**

- `k-means++` **initialization** increases runtime slightly for **Kernel K-Means**, likely due to the additional computation of better-initialized centers.
- Surprisingly, for **Spectral Clustering**, `k-means++` slightly **reduces or stabilizes runtime** compared to random — likely because better initialization reduces k-means iterations after spectral embedding.

3. **Effect of Cluster Number**

- For all methods, increasing the number of clusters does **not always linearly increase runtime**.

- **Kernel K-Means**: runtime fluctuates, e.g., `3 clusters` takes **less time** than `2 clusters` in the `random` setting — likely due to early convergence.
- **Spectral Clustering (Normalized)**: runtime increases with more clusters, especially in `k-means++`.
- **Spectral Clustering (Ratio)**: similar pattern but with smaller increases.

- **Kernel K-Means** is clearly the most efficient method, making it suitable for time-sensitive applications or larger images.
- **Spectral Clustering** methods, while slower, may offer better segmentation quality — a trade-off between accuracy and speed.
- Choosing `k-means++` may slightly increase cost for Kernel K-Means but improves convergence in Spectral Clustering.
- **Spectral Clustering (Ratio)** provides a good middle ground in terms of both **runtime** and **segmentation smoothness**.

image2

	kernel k-means method	spectral clustering (normalized)	spectral clustering (ratio)
	random	random	random
2 clusters	19.10 sec	203.68 sec	176.47 sec
3 clusters	19.68 sec	206.87 sec	153.53 sec
4 clusters	20.88 sec	207.15 sec	164.27

	kernel k-means method	spectral clustering (normalized)	spectral clustering (ratio)
	k-means++	k-means++	k-means++
2 clusters	15.05 sec	209.93sec	166.47 sec
3 clusters	44.16 sec	196.51 sec	167.22 sec
4 clusters	36.29 sec	196.04 sec	170.13 sec

Observations

1. **Overall Performance**

- **Kernel K-Means** is significantly faster than both spectral clustering variants in all configurations.
- **Spectral Clustering (Normalized)** has the highest runtime consistently (~196–210 seconds).
- **Spectral Clustering (Ratio)** performs slightly faster than normalized in most settings, especially under `random`.

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2. **Effect of Initialization Method**

Kernel K-Means:

- **Unusual behavior**:
  - `k-means++` is **faster** than `random` in `2 clusters`, but **slower** for `3 and 4 clusters`.
  - This may be due to variability in early convergence behavior depending on initialization.

Spectral Clustering:

- The choice between `random` and `k-means++` **has minimal impact** on execution time.
  - This is expected, since most of the time is spent computing the Laplacian and its eigenvectors, not in k-means.

3. **Effect of Cluster Number**

Kernel K-Means:

- Runtime increases slightly with more clusters under `random`.
- Under `k-means++`, runtime spikes for `3 clusters` (44.16 sec) and drops again for `4 clusters` — possibly due to convergence complexity at `k=3`.

Spectral Clustering:

- Both **Normalized** and **Ratio** variants maintain **stable runtime** across different cluster numbers.
- This shows that the **eigen-decomposition dominates** runtime and is not strongly affected by `k`.

- **Kernel K-Means** is extremely efficient on Image2, completing most runs under **45 seconds**.
- **Spectral Clustering** is significantly slower, taking **~3 to 10× longer**, regardless of initialization or cluster count.
- **Spectral (Ratio)** remains a viable trade-off between runtime and structure-aware segmentation.
- `k-means++` helps Kernel K-Means only in specific settings; for spectral methods, it has **negligible effect on time**.

Anything you want to discuss. (4%)

	kernel k-means method	spectral clustering (normalized)	spectral clustering (ratio)
	random	random	random
2 clusters	15	4	4
3 clusters	11	9	23
4 clusters	38	12	19

	kernel k-means method	spectral clustering (normalized)	spectral clustering (ratio)
	k-means++	k-means++	k-means++
2 clusters	22	4	4
3 clusters	14	12	2
4 clusters	33	7	4

- **Spectral Clustering** (both normalized and ratio) consistently converges **much faster** (mostly within 2–12 iterations) than **Kernel K-Means**, which often requires **10–40+ iterations**.
- **Normalized Spectral Clustering** is especially stable and predictable, showing **almost constant convergence** around 4–12 iterations regardless of `k`.
- **Kernel K-Means** is **more sensitive** to:
  - the number of clusters (`k`)
  - the initialization method (`k-means++` generally helps but not always)

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image2

	kernel k-means method	spectral clustering (normalized)	spectral clustering (ratio)
	random	random	random
2 clusters	11	8	3
3 clusters	13	13	9
4 clusters	17	20	62

	kernel k-means method	spectral clustering (normalized)	spectral clustering (ratio)
	k-means++	k-means++	k-means++
2 clusters	8	21	2
3 clusters	41	9	8
4 clusters	34	10	10

- **Spectral Clustering (Ratio)** shows **extreme sensitivity** to cluster number and initialization:
  - It converges **very quickly** in some cases (e.g., 2 clusters, `k-means++` : 2 iterations),
  - But can take **very long** in others (e.g., 4 clusters, `random` : 62 iterations).
- **Spectral Clustering (Normalized)** is generally more **stable** in convergence speed, mostly staying below 20 iterations.
- **Kernel K-Means** remains **variable**:
  - `k-means++` sometimes reduces iterations (e.g., 2 clusters: from 11 → 8),
  - But may also **increase** iteration count unexpectedly (e.g., 3 clusters: 13 → 41).