Formula Sheet

Derivations

Period of an Orbit

$$\begin{aligned}
F_g &= \frac{mv^2}{R} \\
\frac{GmM_E}{R^2} &= \frac{mv^2}{R} \\
\frac{GM_E}{R} &= v^2 \\
\frac{GM_E}{R} &= \left(\frac{2\pi R}{T}\right)^2 \\
\frac{GM_E}{R} &= \frac{4\pi^2 R^2}{T^2} \\
T^2 &= \frac{4\pi^2 R^3}{GM_E} \\
T &= \boxed{\sqrt{\frac{4\pi^2 R^3}{GM_E}}}
\end{aligned}$$

Speed of Satellite in Orbit

$$\begin{aligned}
\sum F_{\text{inwards}} &= F_g = m_{\text{satellite}} \cdot a_c \\
\frac{m_{\text{satellite}} \cdot v_t^2}{r} &= \frac{G \cdot m_E \cdot m_{\text{satellite}}}{r^2} \\
v_t &= \boxed{\sqrt{\frac{G \cdot m_E}{r}} = \sqrt{\frac{G \cdot m_E}{r_E + \text{altitude}}}}
\end{aligned}$$

Conical Pendulum

$$\begin{aligned}
F_{T_y} &= F_T \cos\theta = mg \\
F_{T_{in}} &= F_T \sin\theta = \frac{mv_t^2}{r}
\end{aligned}$$

$$\begin{aligned}
F_T &= \frac{mg}{\cos\theta} \Longrightarrow F_{T_{in}} = \left(\frac{mg}{\cos\theta}\right)\sin\theta \\
m\left(\frac{2\pi r}{T^2 \cdot r}\right) &\Longrightarrow g\tan\theta = \frac{4\pi^2 r}{T^2} \\
\sin\theta = \frac{r}{L} &\Longrightarrow r = L\sin\theta \\
g\frac{\sin\theta}{\cos\theta} = \frac{4\pi^2 L\sin\theta}{T^2} &\Longrightarrow T = \boxed{\sqrt{\frac{4\pi^2 L\cos\theta}{g}}}
\end{aligned}$$

Ballistic Pendulum

$$\begin{aligned}
h &= L(1-\cos\theta) \\
v_{sys} &= \frac{mv_b}{m_b + m_p} \\
\frac{1}{2}(m_b + m_p)v_{sys}^2 &= (m_p + m_b)gh \\
\frac{1}{2}(m_b + m_p)\frac{m_b^2 v_1^2}{(m_b + m_p)^2} &= (m_b + m_p)gh \\
v_b &= \sqrt{2gh}\left(1 + \frac{m_p}{m_b}\right) \\
v_b &= \boxed{\sqrt{2gL(1-\cos\theta)}\left(1 + \frac{m_p}{m_b}\right)}
\end{aligned}$$

Velocity to Go in a Vertical/Horizontal Loop

$$\begin{aligned}
F_C = \{t\}/\{F_N\} - mg &\Longrightarrow F_C = mg \quad \text{(tension/normal force is zero)} \\
\frac{mv^2}{r} = mg &\Longrightarrow v = \boxed{\sqrt{rg}}
\end{aligned}$$

Minimum μ_s for Flat Circular Track

$$\frac{v^2}{r} = \frac{\sum F_r}{m} \qquad \text{(radial direction)}$$

$$\frac{v^2}{r} = \frac{\mu m g}{m}$$

$$\mu = \boxed{\frac{v^2}{rg}}$$

Angle of Ramp with No Friction

$$F_N \sin\theta = \frac{mv^2}{r} \Longrightarrow \sin\theta = \frac{mv^2}{F_N \cdot r} \qquad \text{(general equation)}$$

$$F_N \cos\theta = mg \Longrightarrow F_N = \frac{mg}{\cos\theta} \qquad \text{(solve for } F_N\text{)}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{mv^2}{mg \cdot r} \Longrightarrow \boxed{\theta = \tan^{-1}\left(\frac{v^2}{rg}\right)}$$

Speed to Remain on Banked Curve

$$a_y = 0 = \frac{\sum F_y}{m} = \frac{F_N \cos\theta - mg}{m}$$

$$F_N = \frac{mg}{\cos\theta} \qquad (\mathrm{m} \neq 0)$$

$$a_x = \frac{v^2}{r} = \frac{\sum F_x}{m} = \frac{F_N \sin\theta}{m}$$

$$\frac{v^2}{r} = \frac{mg\sin\theta}{m\cos\theta}$$

$$v^2 = \frac{L\cos\theta \cdot mg\sin\theta}{m\cos\theta} \qquad (r = L\cos\theta)$$

$$v = \boxed{\sqrt{gL\sin\theta}} \qquad (\mathrm{L} = \text{diagonal length})$$

Max Velocity with a Breaking Tension

$$T_{\max} - mg = \frac{mv^2}{r}$$

$$v = \boxed{\sqrt{\frac{r}{m}(T_{\max} - mg)}}$$

Falling to Earth Time

T_E = period of revolution

a = semimajor axis

$2t$ = period with no orbital velocity

$a/2$ = major axis with no orbital velocity

$$\frac{(2t)^2}{T_E^2} = \frac{\left(\frac{a}{2}\right)^3}{a^3}$$

$$t^2 = \frac{1}{2}\frac{T_E^2}{16}$$

$$t = \boxed{\frac{T_E}{4\sqrt{2}}}$$

$$\text{for Earth: } t = \frac{365.26}{4\sqrt{2}} = \underline{\underline{64.57 \text{ days}}}$$

Geosynchronous Orbit

$$
\begin{aligned}
F_g &= \frac{GmM}{(R+h)^2} && \text{(law of gravitation)} \\
F_c &= \frac{mv^2}{R+h} && \text{(centripetal force)} \\
v = \omega(R+h) &= \frac{2\pi}{T}(R+h) && \text{(tangential speed)} \\
\frac{GmM}{(R+h)^2} &= \frac{4\pi^2 m\,(R+h)}{T^2} && \text{(substitution)} \\
G\frac{MT^2}{4\pi^2} &= (R+h)^3 && \text{(isolation)} \\
h &= \boxed{\left(\sqrt[3]{\frac{GMT^2}{4\pi^2}}\right) - R} && \text{(answer)}
\end{aligned}
$$

Formula Sheet

One Object

$$
\begin{aligned}
d &= \frac{v_0^2}{2g\mu_s} && \text{(min. braking distance)} \\
v_0 &= \sqrt{2\mu_k g d} && \text{(derived from above)}
\end{aligned}
$$

Two Objects Colliding

$$
\begin{aligned}
d &= \frac{v_0^2 m^2}{2g\mu_k(m+M)^2} && \text{(min. braking distance)} \\
v_0 &= \frac{M+m}{m}\sqrt{2\mu_k g d} && \text{(derived from above)}
\end{aligned}
$$

Energy

$$
\begin{aligned}
W_{\text{nonconservative}} &= \Delta\text{KE} + \Delta\text{PE} && \text{(friction)} \\
W_{\text{net}} = \Delta\text{KE} &= \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 && \text{(work-energy principle)} \\
PE_G = mgy \quad | \quad PE_{el} &= \frac{1}{2}kx^2 && \text{(potential energy)}
\end{aligned}
$$

$$
\begin{aligned}
v &= \sqrt{2gh\left(1 - \frac{\mu_s}{\tan\theta}\right)} && \text{(velocity of block w/friction on ramp)} \\
k &= m\left(\frac{a_{\max}}{v_0}\right)^2 && \text{(spring constant w/}a_{\max}\text{ horizontally)} \\
F_{\tan} &= \frac{mg\sin\theta_{\text{ramp}} \cdot d_{\text{one rev}}}{2\pi r_{\text{pedal}}} && \text{(cycling up a ramp)}
\end{aligned}
$$

A spring (mass m, constant k, displacement x_0, initial velocity v_0) :

$$
\begin{aligned}
v_{\max} &= \sqrt{v_0^2 + \frac{k}{m}x_0^2} && \text{(maximum speed)} \\
x_{\max} &= \sqrt{x_0^2 + \frac{m}{k}v_0^2} && \text{(maximum stretch)}
\end{aligned}
$$

Momentum and SHO

$$v_A' = \left(\frac{m_A - m_B}{m_A + m_B}\right) v_A \qquad \text{(elastic)}$$

$$v_B' = \left(\frac{2m_A}{m_A + m_B}\right) v_A \qquad \text{(elastic)}$$

$$h_A = \frac{v_A^2}{2g} \qquad \text{(height - level ground)}$$

$$KE = \frac{p^2}{2m} = \frac{1}{2}mv^2 \qquad \text{(KE formulas)}$$

$$\text{Impulse} = m\Delta v = \Delta p = F\Delta t = \int F dt \qquad \text{(impulse)}$$

$$KE\ (J) = F\Delta t = \int F\ dt = \Delta p \qquad \text{(interpretations)}$$

$$x_{CM} = \frac{m_A x_A + m_B x_B + \cdots}{m_A + m_B + \cdots} \qquad \text{(center of mass)}$$

$$F_{\text{net}} = Ma_{CM}$$

$$F = ma_{net} = m(\sqrt{a_c^2 + a_{lin}^2}) \qquad \text{(rotational + linear)}$$

$$\theta = \tan^{-1}\frac{a_{\tan}}{a_c} \qquad \text{(angle of rotational force)}$$

Angular Equations for Constant Angular Acceleration

$$\omega = \omega_0 + \alpha t \qquad \text{(constant } \alpha\text{, 1)}$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \qquad \text{(constant } \alpha\text{, 2)}$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta \qquad \text{(constant } \alpha\text{, 3)}$$

$$\overline{\omega} = \frac{\omega + \omega_0}{2} \qquad \text{(constant } \alpha\text{, 4)}$$

Torques

$$|\tau| = rF\sin\theta$$

$$\tau = mr^2\alpha \Rightarrow \text{Inertia} = \sum mr^2$$

$$\text{rotational KE} = \frac{1}{2}I\omega^2$$

$$KE = \frac{1}{2}Mv_{CM}^2 + \frac{1}{2}I_{CM}\omega^2$$

$$L = I\omega$$

```
while college.admit_status() == False {
    for tests in ap_test {
        if (ap_test.score() == 5) {
            commit: "../body/processes/breathe.exe"
        } else {
            commit: "../body/processes/death.exe"
        }
    }
}
```