Definition 1. The two reward functions $\mathcal{R}_1 : S^{\otimes} \times 2^{E^{\otimes}} \to \mathbb{R}$ and $\mathcal{R}_2 : S^{\otimes} \times E^{\otimes} \times S^{\otimes} \to \mathbb{R}$ are defined as follows.

$$\mathcal{R}_1(s^{\otimes}, \pi) = \begin{cases} r_n |\pi| & \text{if } [\![s^{\otimes}]\!]_q \notin \mathit{SinkSet}, \\ 0 & \text{otherwise,} \end{cases} \tag{1}$$

where $|E|$ means number of elements in the set E and r_n is a positive value.

$$\mathcal{R}_2(s^{\otimes}, e, s^{\otimes\prime}) = \begin{cases} r_p & \text{if } \exists i \in \{1, \dots, n\},\ (s^{\otimes}, e, s^{\otimes\prime}) \in \bar{F}_i^{\otimes}, \\ r_{sink} & \text{if } [\![s^{\otimes\prime}]\!]_q \in \mathit{SinkSet}, \\ 0 & \text{otherwise,} \end{cases} \tag{2}$$

where r_p and r_{sink} are the positive and negative value, respectively.

For a Markov chain $MC_{SV}^{\otimes}$ induced by a product MDP $D^{\otimes}$ with a supervisor SV, let $S_{SV}^{\otimes} = T_{SV}^{\otimes} \sqcup R_{SV}^{\otimes 1} \sqcup \ldots \sqcup R_{SV}^{\otimes h}$ be the set of states in $MC_{SV}^{\otimes}$, where $T_{SV}^{\otimes}$ is the set of transient states and $R_{SV}^{\otimes i}$ is the recurrent class for each $i \in \{1, \dots, h\}$, and let $R(MC_{SV}^{\otimes})$ be the union of all recurrent classes in $MC_{SV}^{\otimes}$. Let $\delta_{SV}^{\otimes i}$ be the set of transtions in a recurrent class $R_{SV}^{\otimes i}$, namely $\delta_{SV}^{\otimes i} = \{(s^{\otimes}, e, s^{\otimes\prime}) \in \delta^{\otimes}; s^{\otimes} \in R_{SV}^{\otimes i},\ P_T^{\otimes}(s^{\otimes\prime}|s^{\otimes}, e) > 0, P_E^{\otimes}(e|s^{\otimes}, SV(s^{\otimes})) > 0\}$, and let $P_{SV}^{\otimes} : S_{SV}^{\otimes} \times S_{SV}^{\otimes} \to [0, 1]$ such that $P_{SV}^{\otimes}(s^{\otimes\prime}|s^{\otimes}) = \sum_{e \in SV(s^{\otimes})} P_T^{\otimes}(s^{\otimes\prime}|s^{\otimes}, e) P_E^{\otimes}(e|s^{\otimes}, SV(s^{\otimes}))$ be the transition probability under SV.

Lemma 1. *For any supervisor SV and any recurrent class $R_{SV}^{\otimes i}$ in the Markov chain $MC_{SV}^{\otimes}$, $MC_{SV}^{\otimes}$ satisfies one of the following conditions.*

1. $\delta_{SV}^{\otimes i} \cap \bar{F}_j^{\otimes} \neq \emptyset$, $\forall j \in \{1, \dots, n\}$,
2. $\delta_{SV}^{\otimes i} \cap \bar{F}_j^{\otimes} = \emptyset$, $\forall j \in \{1, \dots, n\}$.

Definition 2. An accepting recurrent class is defined as the recurrent class that has at least one accepting transition in each accepting set $\bar{F}_j^{\otimes}$ with $j \in \{1, \dots, n\}$. A sink recurrent class is defined as the recurrent class composed of the states $S_{sink}^{\otimes}$ satisfying $[\![s_{sink}^{\otimes}]\!]_q \in \mathit{SinkSet}$ for any $s_{sink}^{\otimes} \in S_{sink}^{\otimes}$.

Theorem 1. *Let $M^{\otimes}$ be the product DES corresponding to a DES M and an LTL formuula φ. Let $\mathcal{R}_1$ be a reward function for control patterns. If there exists a supervisor SV satisfying φ and it satisfies that there is no state $s^{\otimes} \in S_{SV}^{\otimes}$ reachable from initial state $s_{init}^{\otimes}$ such that $[\![s^{\otimes}]\!]_q \in \mathit{SinkSet}$, then there exist a discount factor γ^*, a positive reward $r_p^*(\mathcal{R}_1)$ that satisfies $r_p^*(\mathcal{R}_1) >> ||\mathcal{R}_1||_{\infty}$, and a negative reward $r_{sink}^*(r_p, \mathcal{R}_1)$ that satisfies $r_{sink}^*(r_p, \mathcal{R}_1) << -(r_p + ||\mathcal{R}_1||_{\infty})$ such that any algorithm that maximizes the expected discounted reward with $\gamma > \gamma^*$, $r_p > r_p^*(\mathcal{R}_1)$, and $r_{sink} < r_{sink}^*(r_p^*, \mathcal{R}_1)$ will find, with probability one, a supervisor satisfying φ and it satisfies that there is no state $s^{\otimes} \in S_{SV}^{\otimes}$ reachable from the initial state $s_{init}^{\otimes}$ such that $[\![s^{\otimes}]\!]_q \in \mathit{SinkSet}$.*

Proof. Suppose that there is an algorithym by which an optimal supervisor SV^* is obtained but SV^* does not satisfy the LTL formula φ or there is a state $s^{\otimes}_{sink}$ reachable from the initial state such that $[\![s^{\otimes}_{sink}]\!]_q \in SinkSet$ under SV^*. Then, for any recurrent class $R^{\otimes i}_{SV^*}$ in the Markov chain $MC^{\otimes}_{SV^*}$ and any accepting set $\bar{F}^{\otimes}_j$ of the product DES $M^{\otimes}$, $\delta^{\otimes i}_{SV^*} \cap \bar{F}^{\otimes}_j = \emptyset$ holds for the first case by Lemma 1 and there is a sink recurrent class $R^{\otimes i}_{SV^*}$ for the second case. We consider the two cases.

1. Assume that SV^* does not satisfy the LTL formula φ. By the assumption, the system under the supervisor SV^* can only obtain rewards in the set of transient states and rewards regarding sink states. We consider the best scenario in the assumption. Let $p^k(s,s')$ be the probability of going to a state s' in k time steps after leaving the state s, and let $Post(T^{\otimes}_{SV^*})$ be the set of states in recurrent classes that can be transitioned from states in $T^{\otimes}_{SV^*}$ by one event occurrence. Let $R^{\otimes sink}_{SV^*}$ be the union of the states $s^{\otimes}_{sink}$ such that $[\![s^{\otimes}_{sink}]\!]_q \in SinkSet$. Recall that $r_{sink} < 0$. Thus, for the initial state $s^{\otimes}_{init}$ in the set of transient states, it holds that

$$\begin{aligned} V^{SV^*}(s^{\otimes}_{init}) = & \sum_{k=0}^{\infty} \sum_{s^{\otimes} \in T^{\otimes}_{SV^*}} \gamma^k p^k(s^{\otimes}_{init}, s^{\otimes}) \{ \sum_{s^{\otimes\prime} \in T^{\otimes}_{SV^*} \cup (Post(T^{\otimes}_{\pi^*}) \cap (R(MC^{\otimes}_{SV^*}) \setminus R^{\otimes sink}_{SV^*})} \\ & \sum_{e \in SV(s^{\otimes})} P^{\otimes}_T(s^{\otimes\prime} | s^{\otimes}, e) P^{\otimes}_E(e | s^{\otimes}, SV(s^{\otimes})) \mathcal{R}(s^{\otimes}, SV(s^{\otimes}), e, s^{\otimes\prime}) \\ & + \sum_{s^{\otimes\prime} \in R^{\otimes sink}_{SV}} P^{\otimes}_{SV}(s^{\otimes\prime} | s^{\otimes}) \sum_{l=0}^{\infty} \gamma^l r_{sink} \} \\ < & \; r_p \sum_{k=0}^{\infty} \sum_{s^{\otimes} \in T^{\otimes}_{SV^*}} \gamma^k p^k(s^{\otimes}_{init}, s^{\otimes}) + \sum_{k=0}^{\infty} \gamma^k ||\mathcal{R}_1||_{\infty}. \end{aligned}$$

   By the property of the transient states, for any state $s^{\otimes}$ in $T^{\otimes}_{SV^*}$, there exists a bounded positive value m such that $\sum_{k=0}^{\infty} \gamma^k p^k(s^{\otimes}_{init}, s^{\otimes}) < \sum_{k=0}^{\infty} p^k(s^{\otimes}_{init}, s^{\otimes}) < m$ [1]. Therefore, there exists a bounded positive value $\bar{m}$ such that $V^{SV^*}(s^{\otimes}_{init}) < \bar{m} + \frac{1}{1-\gamma}||\mathcal{R}_1||_{\infty}$.

2. Assume that SV^* satisfies φ but there is a state $s^{\otimes}_{sink}$ reachable from the initial state such that $[\![s^{\otimes}_{sink}]\!]_q \in SinkSet$ under SV^*. By the assumption, there is a sink recurrent class $R^{\otimes i}_{SV^*}$ reachable from the initial state. We consider the best scenario in the assumption. In words, we assume that the system obtains the full possible rewards of $\mathcal{R}_1$ and r_p in all steps. There exist a number $l > 0$, a state $s^{\otimes}_{sink} \in Post(T^{\otimes}_{SV^*}) \cap R^{\otimes i}_{SV^*}$, and a subset of transient states $\{s^{\otimes}_1, \ldots, s^{\otimes}_{l-1}\} \subset T^{\otimes}_{SV^*}$ such that $p(s^{\otimes}_{init}, s^{\otimes}_1) > 0$, $p(s^{\otimes}_i, s^{\otimes}_{i+1}) > 0$ for $i \in \{1, ..., l-2\}$, and $p(s^{\otimes}_{l-1}, s^{\otimes}_{sink}) > 0$ by the property of transient states. By considering only paths that reach the state $s^{\otimes}_{sink} \in R^{\otimes i}_{SV^*}$ in l steps out of all paths reaching sink recurrent classes, we have

$$
\begin{aligned}
V^{SV^*}(s^{\otimes}_{init}) <& Pr^{M^{\otimes}}_{SV^*}(s^{\otimes}_{init} \models \varphi) \sum_{k=0}^{\infty} \gamma^k (r_p + ||\mathcal{R}_1||_{\infty}) + \gamma^l p^l (s^{\otimes}_{init}, s^{\otimes}_{sink}) \sum_{k=0}^{\infty} \gamma^k r_{sink} \\
&+ Pr^{M^{\otimes}}_{SV^*}(s^{\otimes}_{init} \not\models \varphi)(r_p + ||\mathcal{R}_1||_{\infty}) \sum_{k=0}^{\infty} \sum_{s^{\otimes} \in T^{\otimes}_{\pi^*}} \gamma^k p^k (s^{\otimes}_{init}, s^{\otimes}) \\
<& \frac{1}{1-\gamma} \{ Pr^{M^{\otimes}}_{SV^*}(s^{\otimes}_{init} \models \varphi)(r_p + ||\mathcal{R}_1||_{\infty}) + \gamma^l p^l (s^{\otimes}_{init}, s^{\otimes}_{sink}) r_{sink} \} + \bar{m}',
\end{aligned}
$$

where $\bar{m}'$ is a constant such that $\bar{m}' > Pr^{M^{\otimes}}_{SV^*}(s^{\otimes}_{init} \not\models \varphi)(r_p + ||\mathcal{R}_1||_{\infty}) \sum_{k=0}^{\infty} \sum_{s^{\otimes} \in T^{\otimes}_{\pi^*}} \gamma^k p^k (s^{\otimes}_{init}, s^{\otimes})$.

Therefore, if it holds that $r_{sink} \leq -\frac{Pr^{M^{\otimes}}_{SV^*}(s^{\otimes}_{init} \models \varphi)}{\gamma^l p^l (s^{\otimes}_{init}, s^{\otimes}_{sink})}(r_p + ||\mathcal{R}_1||_{\infty})$, we then have $V^{SV^*}(s^{\otimes}_{init}) < \bar{m}'$ for any $\gamma \in (0,1)$.

Let $\bar{SV}$ be a supervisor satisfying φ and it satisfies that there is no state $s^{\otimes} \in S^{\otimes}_{\bar{SV}}$ reachable from initial state $s^{\otimes}_{init}$ such that $[\![s^{\otimes}]\!]_q \in SinkSet$. We consider the following two cases.

1. Assume that the initial state $s^{\otimes}_{init}$ is in a recurrent class $R^{\otimes i}_{\bar{SV}}$ for some $i \in \{1, \ldots, h\}$. For any accepting set $\bar{F}^{\otimes}_j$, $\delta^{\otimes i}_{\bar{SV}} \cap \bar{F}^{\otimes}_j \neq \emptyset$ holds by the definition of $\bar{SV}$. The expected discounted reward for $s^{\otimes}_{init}$ is given by

$$
V^{\bar{SV}}(s^{\otimes}_{init}) = \mathbb{E}^{SV}[\sum_{k=0}^{\infty} \gamma^k \mathcal{R}(s_k, \pi_k, e_k, s_{k+1}) | s_0 = s^{\otimes}_{init}] \tag{3}
$$

For each path $\rho = s_0 \pi_0 e_0 s_1 \ldots s_i \pi_i e_i s_{i+1} \ldots \in S(2^E ES)^{\omega}$, the stopping time $\hat{k}$ of first returning to the initial state is defined as $\hat{k}(\rho) = \min\{i | s_i = s_0\}$. Recall that the state set $S^{\otimes}$ is finite, hence all of the recurrent classes are positive recurrent [2]. We consider the worst scenario in this case. It holds that

$$
\begin{aligned}
V^{\bar{SV}}(s^{\otimes}_{init}) >& \mathbb{E}^{\bar{SV}}[\gamma^{\hat{k}-1} r_p + \gamma^{\hat{k}-2} r_p + \ldots + \gamma^{\hat{k}-n} r_p + \gamma^{\hat{k}-1} V^{\bar{SV}}(s^{\otimes}_{init}) | s_0 = s^{\otimes}_{init}] \\
>& \mathbb{E}^{\bar{SV}}[\gamma^{\hat{k}-1} r_p + \gamma^{\hat{k}-1} V^{\bar{SV}}(s^{\otimes}_{init}) | s_0 = s^{\otimes}_{init}] \\
\geq& \gamma^{\mathbb{E}^{\bar{SV}}[\hat{k}-1 | s_0 = s^{\otimes}_{init}]} r_p + \gamma^{\mathbb{E}^{\bar{SV}}[\hat{k}-1 | s_0 = s^{\otimes}_{init}]} V^{\bar{SV}}(s^{\otimes}_{init}).
\end{aligned}
$$

Thus,

$$
\begin{aligned}
V^{\bar{SV}}(s^{\otimes}_{init}) >& \frac{\gamma^{\mathbb{E}^{\bar{SV}}[\hat{k}-1 | s_0 = s^{\otimes}_{init}]} r_p}{1 - \gamma^{\mathbb{E}^{\bar{SV}}[\hat{k}-1 | s_0 = s^{\otimes}_{init}]}} \\
>& \frac{\gamma^{\hat{K}-1} r_p}{1 - \gamma^{\hat{K}-1}},
\end{aligned}
$$

where the second inequality holds since it holds that $\mathbb{E}^{\bar{SV}}[\gamma^{\hat{k}}|s_0 = s^{\otimes}_{init}] \geq \gamma^{\mathbb{E}^{\bar{SV}}[\hat{k}|s_0=s^{\otimes}_{init}]}$ by Jensen's inequality, $\hat{K} = \lceil \mathbb{E}^{\bar{SV}}[\hat{k}|s_0 = s^{\otimes}_{init}] \rceil < \infty$ and the inequality holds by the property of the positive recurrence [3], and the fourth inequality holds since it holds that $\gamma^{\hat{K}} < \gamma^{\mathbb{E}^{\bar{SV}}[\hat{k}|s_0=s^{\otimes}_{init}]}$ and $\frac{1}{1-\gamma^{\hat{K}}} < \frac{1}{1-\gamma^{\mathbb{E}^{\bar{SV}}[\hat{k}|s_0=s^{\otimes}_{init}]}}$ for any $\gamma \in (0,1)$. We set r^*_p, r^*_{sink}, and γ^* to satisfy $\frac{\gamma^{\hat{K}-1}}{1-\gamma^{\hat{K}-1}} r^*_p > \frac{1}{1-\gamma}||\mathcal{R}_1||_\infty$ for any $\gamma \in (0,1)$, $r^*_{sink} \leq -\frac{Pr^{M^{\otimes}}_{SV^*}(s^{\otimes}_{init} \models \varphi)}{\gamma^l p^l(s^{\otimes}_{init}, s^{\otimes}_{sink})}(r^*_p + ||\mathcal{R}_1||_\infty)$ for any $\gamma \in (0,1)$, and $\frac{\gamma^{*\hat{K}-1}}{1-\gamma^{*\hat{K}-1}} r^*_p - \frac{1}{1-\gamma^*}||\mathcal{R}_1||_\infty > m$ for any $m > 0$, respectively. Therefore, for any bounded reward function $\mathcal{R}_1$, any positive value $r_p > r^*_p$, any negative value $r_{sink} < r^*_{sink}$, any discount factor $\gamma \in (\gamma^*, 1)$, we then have $V^{\bar{SV}}(s^{\otimes}_{init}) > V^{SV^*}(s^{\otimes}_{init})$ since for $m = \max\{\bar{m}, \bar{m}'\}$, by the setting of r^*_{sink}, we have

$$
\begin{aligned}
V^{\bar{SV}}(s^{\otimes}_{init}) - V^{SV^*}(s^{\otimes}_{init}) >& \frac{\gamma^{\hat{K}-1}}{1-\gamma^{\hat{K}-1}} r_p - (m + \frac{1}{1-\gamma}||\mathcal{R}_1||_\infty) \\
=& (\frac{\gamma^{\hat{K}-1}}{1-\gamma^{\hat{K}-1}} r_p - \frac{1}{1-\gamma}||\mathcal{R}_1||_\infty) - m,
\end{aligned}
$$

by the settings of γ^* and r^*_p, we have

$$
V^{\bar{SV}}(s^{\otimes}_{init}) - V^{SV^*}(s^{\otimes}_{init}) > 0. \tag{4}
$$

2. Assume that the initial state $s^{\otimes}_{init}$ is in the set of transient states $T^{\otimes}_{\bar{SV}}$. $P^{M^{\otimes}}_{\bar{SV}}(s^{\otimes}_{init} \models \varphi) > 0$ holds by the definition of $\bar{SV}$. For an accepting recurrent class $R^{\otimes i}_{\bar{SV}}$, there exist a number $l' > 0$, a state $\hat{s}^{\otimes}$ in $Post(T^{\otimes}_{\bar{SV}}) \cap R^{\otimes i}_{\bar{SV}}$, and a subset of transient states $\{s^{\otimes}_1, \ldots, s^{\otimes}_{l'-1}\} \subset T^{\otimes}_{\bar{SV}}$ such that $p(s^{\otimes}_{init}, s^{\otimes}_1) > 0$, $p(s^{\otimes}_i, s^{\otimes}_{i+1}) > 0$ for $i \in \{1, ..., l'-2\}$, and $p(s^{\otimes}_{l'-1}, \hat{s}^{\otimes}) > 0$ by the property of transient states. Hence, it holds that $p^{l'}(s^{\otimes}_{init}, \hat{s}^{\otimes}) > 0$ for the state $\hat{s}^{\otimes}$. For each path $\rho = s_0\pi_0e_0s_1 \ldots s_i\pi_ie_is_{i+1} \ldots \in S(2^E ES)^\omega$ reaching $\hat{s}^{\otimes}$, the stopping time $\hat{k}$ of first returning to the state $\hat{s}^{\otimes}$ is defined as $\hat{k}(\rho) = \min\{i - j | s_i = \hat{s}^{\otimes}, s_j = \hat{s}^{\otimes}, i > j > 0\}$. Thus, by ignoring positive rewards in $T^{\otimes}_{\bar{SV}}$, we have

$$
\begin{aligned}
V^{\bar{SV}}(s^{\otimes}_{init}) =& \mathbb{E}^{SV}[\sum_{k=0}^{\infty} \gamma^k \mathcal{R}(s_k, \bar{SV}(s_k), e_k, s_{k+1}) | s_0 = s^{\otimes}_{init}] \\
\geq& \mathbb{E}^{SV}[\gamma^l \sum_{k=0}^{\infty} \gamma^k \mathcal{R}(s_{k+l}, \bar{SV}(s_{k+l}), e_{k+l}, s_{k+l+1}) | s_0 = s^{\otimes}_{init}] \\
>& \gamma^{l'} p^{l'}(s^{\otimes}_{init}, \hat{s}^{\otimes}) \mathbb{E}^{\bar{SV}}[\gamma^{\hat{k}-1} r_p + \gamma^{\hat{k}-1} V^{\bar{SV}}(\hat{s}^{\otimes}) | s_{l'} = \hat{s}^{\otimes}] \\
\geq& \gamma^{l'} p^{l'}(s^{\otimes}_{init}, \hat{s}^{\otimes}) \{\gamma^{\mathbb{E}^{\bar{SV}}[\hat{k}-1|s_{l'}=\hat{s}^{\otimes}]} r_p + \gamma^{\mathbb{E}^{\bar{SV}}[\hat{k}-1|s_{l'}=\hat{s}^{\otimes}]} V^{\bar{SV}}(\hat{s}^{\otimes})\}.
\end{aligned}
$$

As with the case 1, we have

$$V^{\bar{SV}}(s^{\otimes}_{init}) > \gamma^{l'} p^{l'}(s^{\otimes}_{init}, \hat{s}^{\otimes}) \frac{\gamma^{\mathbb{E}^{\bar{SV}}[\hat{k}-1|s_{l'}=\hat{s}^{\otimes}]} r_p}{1-\gamma^{\mathbb{E}^{\bar{SV}}[\hat{k}-1|s_{l'}=\hat{s}^{\otimes}]}}$$
$$> \gamma^{l'} p^{l'}(s^{\otimes}_{init}, \hat{s}^{\otimes}) \frac{\gamma^{\hat{K}-1} r_p}{1-\gamma^{\hat{K}-1}} \tag{5}$$

where the third inequality holds since it holds that $\mathbb{E}^{\bar{SV}}[\gamma^{\hat{k}}|s_{l'} = \hat{s}^{\otimes}] \geq \gamma^{\mathbb{E}^{\bar{SV}}[\hat{k}|s_{l'}=\hat{s}^{\otimes}]}$ by Jensen's inequality, $\hat{K} = \lceil \mathbb{E}^{\bar{SV}}[\hat{k}|s_{l'} = \hat{s}^{\otimes}] \rceil$, and the fifth inequality holds since it holds that $\gamma^{\hat{K}} < \gamma^{\mathbb{E}^{\bar{SV}}[\hat{k}|s_{l'}=\hat{s}^{\otimes}]}$ and $\frac{1}{1-\gamma^{\hat{K}}} < \frac{1}{1-\gamma^{\mathbb{E}^{\bar{SV}}[\hat{k}|s_{l'}=\hat{s}^{\otimes}]}}$ for any $\gamma \in (0,1)$. We set r^*_p, r^*_{sink}, and γ^* to satisfy $\gamma^{l'} p^{l'}(s^{\otimes}_{init}, \hat{s}^{\otimes}) \frac{\gamma^{\hat{K}-1}}{1-\gamma^{\hat{K}-1}} r^*_p > \frac{1}{1-\gamma}||\mathcal{R}_1||_\infty$ for any $\gamma \in (0,1)$, $r^*_{sink} \leq -\frac{Pr^{M^{\otimes}}_{SV^*}(s^{\otimes}_{init} \models \varphi)}{\gamma^l p^l(s^{\otimes}_{init}, s^{\otimes}_{sink})}(r^*_p + ||\mathcal{R}_1||_\infty)$ for any $\gamma \in (0,1)$, and $\gamma^{l'} p^{l'}(s^{\otimes}_{init}, \hat{s}^{\otimes}) \frac{{\gamma^*}^{\hat{K}-1}}{1-{\gamma^*}^{\hat{K}-1}} r^*_p - \frac{1}{1-\gamma^*}||\mathcal{R}_1||_\infty > m$ for any $m > 0$, respectively. Therefore, for the reward function $\mathcal{R}_1$, any positive value $r_p > r^*_p$, any negative value $r_{sink} < r^*_{sink}$, any discount factor $\gamma \in (\gamma^*, 1)$, by the setting of r^*_{sink}, we have

$$V^{\bar{SV}}(s^{\otimes}_{init}) - V^{SV^*}(s^{\otimes}_{init}) > \gamma^{l'} p^{l'}(s^{\otimes}_{init}, \hat{s}^{\otimes}) \frac{\gamma^{\hat{K}-1}}{1-\gamma^{\hat{K}-1}} r_p - (m + \frac{1}{1-\gamma}||\mathcal{R}_1||_\infty)$$
$$= (\gamma^{l'} p^{l'}(s^{\otimes}_{init}, \hat{s}^{\otimes}) \frac{\gamma^{\hat{K}-1}}{1-\gamma^{\hat{K}-1}} r_p - \frac{1}{1-\gamma}||\mathcal{R}_1||_\infty) - m,$$

by the settings of γ^* and r^*_p, we have

$$V^{\bar{SV}}(s^{\otimes}_{init}) - V^{SV^*}(s^{\otimes}_{init}) > 0 \tag{6}$$

The results contradict the optimality assumption of SV^* □

References

[1] R. Durrett, *Essentials of Stochastic Processes*, 2nd Edition. ser. Springer texts in statistics. New York; London; Springer, 2012.

[2] L. Breuer, "Introduction to Stochastic Processes," [Online]. Available: https://www.kent.ac.uk/smsas/personal/lb209/files/sp07.pdf

[3] S.M. Ross, *Stochastic Processes*, 2nd Edition. University of California, Wiley, 1995.