```
Augmentation
of
tLDG-
BAs
and
Syn-
the-
\sin
Method
      We
in-
tro-
duce
an
au-
tom a-
ton
aug-
mented
with
bi-
nary
vec-
tors.
The
au-
toma-
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\operatorname{can}
ex-
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rep-
re-
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whether
tran-
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tions
in
each
ac-
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set
oc-
\operatorname{cur}
at
least
once,
and
en-
sure
tran-
si-
{\rm tions}
in
{\rm each}
ac-
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set
oc-
cur
in-
finitely
of-
ten.

Let V = \{(v_1, \dots, v_n)^T : v_i \in \{0, 1\}, i \in \{1, \dots, n\}\}
be
a
```

 $_{\rm of}^{\rm set}$ 

```
re-
  spec-
  tively.
  In
  or-
  der
  to
  aug-
ment
  tLDBA
  B_{\varphi}, we
  in-
  tro-
  {\rm duce}
  three
  func-
  tions
  visitf:
 \delta \rightarrow V, \\ reset: V \rightarrow V, \\ and Maximum Maximum
  Max:
 V \times V \to V
  as
  fol-
  lows.
  \quad \text{For} \quad
  any
  e \in \delta
 visitf(e) = (v_1, \dots, v_n)^T, where
 \mathbf{v}_i = \\ \{1if \ e \in F_i, 0otherwise. \\ \text{For } \end{cases}
  any
  v \in V, re-
  set(v)
 any
any v, u \in V, Max(v, u) = (l_1, \dots, l_n)^T, where
  l_i =
  \max\{v_i, u_i\}
  for
  any
  i \in \{1, \dots, n\}.
                                   Each
  vec-
  \operatorname{tor}
  v
  is \\
  {\rm called}
  a
  mem-
  ory
  vec-
  \operatorname{tor}
  and
  rep-
```

re-

```
\begin{array}{l} \mathbf{P}^{\otimes}(s^{\otimes\prime}|s^{\otimes},a) = \\ \{1if\ (s\!=\!s') \wedge (v\!=\!v') \wedge (x,\varepsilon_{x'},x')\!\in\!\bar{\delta}, 0otherwise, \end{array}
s^{\otimes} =
(s,(x,v))
and
s^{\otimes} =
(s',(x',v')).
           [Reward
as-
\operatorname{sign-}
ments]
The
re-
ward
func-
tion
\begin{array}{l} \mathcal{R}: \\ S^{\otimes} \times \\ A^{\otimes} \times \\ S^{\otimes} \rightarrow \end{array}
\tilde{R}_{\geq 0}
is -
de-
fined
as R(s^{\otimes}, a, s^{\otimes'}) = \{r_p \ if \ \exists i \in \{1, \dots, n\}, \ (s^{\otimes}, a, s^{\otimes'}) \in \bar{F}_i^{\otimes}, 0 \ otherwise, where
r_p is
\mathbf{a}
pos-
{\rm tive}
value.
           Under
the
prod-
uct
_{M^{\otimes}}^{\mathrm{MDP}}
and
the
re-
ward
func-
{\rm tion}
\mathcal{R}, \\ \mathrm{which}
is
based
on
the
ac-
cep-
tance
con-
di-
tion
of M^{\otimes},
we
show
that
if
there
ex-
ists
\mathbf{a}
po-
si-
```

tional policy

```
state
and
the
ac-
tion
space is
spec-
i-
fied
with
\mathcal{A}(s) = \{Right, Left, Up, Down\} for
state
s \neq
s_4
and
A(s_4) = \{to\_s_0, to\_s_1, to\_s_2, \}
to_{-s_3}, to_{-s_5}, to_{-s_6}, to_{-s_7}, to_{-s_8}\}, where
to\_s_i
means
at-
tempt-
ing
to
go
to
\quad \text{the} \quad
state

        for

i \in \{0, 1, 2, 3, 5, 6, 7, 8\}. The
robot
moves
in
the
in-
tended
di-
rec-
{\rm tion}
with
prob-
a-
bil-
ity
0.9
and
it
stays
in
the
same
state
with
prob-
a-
bil-
ity
0.1
if
it
is
in
the
state
\frac{s_4}{\ln}
the
states
```

other