

Definition 1. The two reward functions $\mathcal{R}_1 : S^\otimes \times 2^{E^\otimes} \rightarrow \mathbb{R}$ and $\mathcal{R}_2 : S^\otimes \times E^\otimes \times S^\otimes \rightarrow \mathbb{R}$ are defined as follows.

$$\mathcal{R}_1(s^\otimes, \pi) = \begin{cases} r_n |\pi| & \text{if } \llbracket s^\otimes \rrbracket_x \notin \text{SinkSet}, \\ 0 & \text{otherwise,} \end{cases} \quad (1)$$

where $|E|$ means number of elements in the set E and r_n is a positive value.

$$\mathcal{R}_2(s^\otimes, e, s^{\otimes'}) = \begin{cases} r_p & \text{if } \exists i \in \{1, \dots, n\}, (s^\otimes, e, s^{\otimes'}) \in \bar{F}_i^\otimes, \\ r_{\text{sink}} & \text{if } \llbracket s^{\otimes'} \rrbracket_x \in \text{SinkSet}, \\ 0 & \text{otherwise,} \end{cases} \quad (2)$$

where r_p and r_{sink} are the positive and negative value, respectively.

For a Markov chain MC_{SV}^\otimes induced by a product MDP D^\otimes with a supervisor SV , let $S_{SV}^\otimes = T_{SV}^\otimes \sqcup R_{SV}^{\otimes 1} \sqcup \dots \sqcup R_{SV}^{\otimes h}$ be the set of states in MC_{SV}^\otimes , where T_{SV}^\otimes is the set of transient states and $R_{SV}^{\otimes i}$ is the recurrent class for each $i \in \{1, \dots, h\}$, and let $R(MC_{SV}^\otimes)$ be the union of all recurrent classes in MC_{SV}^\otimes . Let $\delta_{SV}^{\otimes i}$ be the set of transitions in a recurrent class $R_{SV}^{\otimes i}$, namely $\delta_{SV}^{\otimes i} = \{(s^\otimes, e, s^{\otimes'}) \in \delta^\otimes; s^\otimes \in R_{SV}^{\otimes i}, P_T^\otimes(s^{\otimes'}|s^\otimes, e) > 0, P_E^\otimes(e|s^\otimes, SV(s^\otimes)) > 0\}$, and let $P_{SV}^\otimes : S_{SV}^\otimes \times S_{SV}^\otimes \rightarrow [0, 1]$ such that $P_{SV}^\otimes(s^{\otimes'}|s^\otimes) = \sum_{e \in SV(s^\otimes)} P_T^\otimes(s^{\otimes'}|s^\otimes, e) P_E^\otimes(e|s^\otimes, SV(s^\otimes))$ be the transition probability under SV .

Lemma 1. For any supervisor SV and any recurrent class $R_{SV}^{\otimes i}$ in the Markov chain MC_{SV}^\otimes , MC_{SV}^\otimes satisfies one of the following conditions.

1. $\delta_{SV}^{\otimes i} \cap \bar{F}_j^\otimes \neq \emptyset, \forall j \in \{1, \dots, n\}$,
2. $\delta_{SV}^{\otimes i} \cap \bar{F}_j^\otimes = \emptyset, \forall j \in \{1, \dots, n\}$.

Definition 2. An accepting recurrent class is defined as the recurrent class that has at least one accepting transition in each accepting set \bar{F}_j^\otimes with $j \in \{1, \dots, n\}$. A sink recurrent class is defined as the recurrent class composed of the states S_{sink}^\otimes satisfying $\llbracket s_{\text{sink}}^\otimes \rrbracket_x \in \text{SinkSet}$ for any $s_{\text{sink}}^\otimes \in S_{\text{sink}}^\otimes$.

Theorem 1. Let M^\otimes be the product DES corresponding to a DES M and an LTL formula φ . Let \mathcal{R}_1 be a reward function for control patterns. If there exists a supervisor SV satisfying φ and it satisfies that there is no state $s^\otimes \in S_{SV}^\otimes$ reachable from initial state s_{init}^\otimes such that $\llbracket s^\otimes \rrbracket_x \in \text{SinkSet}$, then there exist a discount factor γ^* , a positive reward $r_p^*(\mathcal{R}_1)$ that satisfies $r_p^*(\mathcal{R}_1) \gg \|\mathcal{R}_1\|_\infty$, and a negative reward $r_{\text{sink}}^*(r_p, \mathcal{R}_1)$ that satisfies $r_{\text{sink}}^*(r_p, \mathcal{R}_1) \ll -(r_p + \|\mathcal{R}_1\|_\infty)$ such that any algorithm that maximizes the expected discounted reward with $\gamma > \gamma^*$, $r_p > r_p^*(\mathcal{R}_1)$, and $r_{\text{sink}} < r_{\text{sink}}^*(r_p, \mathcal{R}_1)$ will find, with probability one, a supervisor satisfying φ and it satisfies that there is no state $s^\otimes \in S_{SV}^\otimes$ reachable from the initial state s_{init}^\otimes such that $\llbracket s^\otimes \rrbracket_x \in \text{SinkSet}$.

Proof. Suppose that there is an algorithm by which an optimal supervisor SV^* is obtained but SV^* does not satisfy the LTL formula φ or there is a state s_{sink}^\otimes reachable from the initial state such that $\llbracket s_{sink}^\otimes \rrbracket_x \in SinkSet$ under SV^* . Then, for any recurrent class $R_{SV^*}^{\otimes i}$ in the Markov chain $MC_{SV^*}^\otimes$ and any accepting set \bar{F}_j^\otimes of the product DES M^\otimes , $\delta_{SV^*}^{\otimes i} \cap \bar{F}_j^\otimes = \emptyset$ holds for the first case by Lemma 1 and there is a sink recurrent class $R_{SV^*}^{\otimes i}$ for the second case. We consider the two cases.

1. Assume that SV^* does not satisfy the LTL formula φ . By the assumption, the reward r_p can be obtained only in transitions from the transient states. We consider the best scenario in the assumption. Let $p^k(s, s')$ be the probability of going to a state s' in k time steps after leaving the state s , let $Post(T_{SV^*}^\otimes)$ be the set of the recurrent states that can be transitioned from states in $T_{SV^*}^\otimes$ by one event occurrence, and let $Pre(R_{SV^*}^\otimes)$ be the set of the transient states that can transition to $R_{SV^*}^\otimes$ by one event occurrence. Let $R_{SV^*}^{\otimes sink}$ be the union of the states s_{sink}^\otimes such that $\llbracket s_{sink}^\otimes \rrbracket_x \in SinkSet$. Recall that $r_{sink} < 0$. Thus, for the initial state s_{init}^\otimes in the set of transient states, it holds that

$$\begin{aligned}
V^{SV^*}(s_{init}^\otimes) &= \sum_{k=0}^{\infty} \sum_{s^\otimes \in S_{SV^*}^\otimes} \gamma^k p^k(s_{init}^\otimes, s^\otimes) \sum_{s'^\otimes \in S_{SV^*}^\otimes} \\
&\quad \sum_{e \in SV(s^\otimes)} P_T^\otimes(s'^\otimes | s^\otimes, e) P_E^\otimes(e | s^\otimes, SV(s^\otimes)) \mathcal{R}(s^\otimes, SV(s^\otimes), e, s'^\otimes) \\
&< \sum_{k=0}^{\infty} \sum_{s^\otimes \in T_{SV^*}^\otimes} \gamma^k p^k(s_{init}^\otimes, s^\otimes) \sum_{s'^\otimes \in T_{SV^*}^\otimes \cup (Post(T_{SV^*}^\otimes) \cap (R(MC_{SV^*}^\otimes) \setminus R_{SV^*}^{\otimes sink}))} P_{SV}^\otimes(s'^\otimes | s^\otimes) r_p \\
&\quad + \sum_{k=0}^{\infty} \sum_{s^\otimes \in Pre(R_{SV^*}^{\otimes sink}) \cup R_{SV^*}^{\otimes sink}} \gamma^k p^k(s_{init}^\otimes, s^\otimes) \sum_{s'^\otimes \in R_{SV^*}^{\otimes sink}} P_{SV}^\otimes(s'^\otimes | s^\otimes) r_{sink} \\
&\quad + \sum_{k=0}^{\infty} \gamma^k \|\mathcal{R}_1\|_\infty \\
&\leq r_p \sum_{k=0}^{\infty} \sum_{s^\otimes \in T_{SV^*}^\otimes} \gamma^k p^k(s_{init}^\otimes, s^\otimes) + \sum_{k=0}^{\infty} \gamma^k \|\mathcal{R}_1\|_\infty,
\end{aligned}$$

where, in the second inequality, the first term on the right hand side represents the assumption r_p can be always obtained in transient states, the second term on the right hand side represents the assumption at least one sink recurrent class exists, the third term on the right hand side represents the assumption the full reward regarding control patterns can always obtained. By the property of the transient states, for any state s^\otimes in $T_{SV^*}^\otimes$, there exists a bounded positive value m such that

$\sum_{k=0}^{\infty} \gamma^k p^k(s_{init}^{\otimes}, s^{\otimes}) < \sum_{k=0}^{\infty} p^k(s_{init}^{\otimes}, s^{\otimes}) < m$ [1]. Therefore, there exists a bounded positive value \bar{m} such that $V^{SV^*}(s_{init}^{\otimes}) < \bar{m} + \frac{1}{1-\gamma} \|\mathcal{R}_1\|_{\infty}$.

2. Assume that SV^* satisfies φ but there is a state s_{sink}^{\otimes} reachable from the initial state such that $\llbracket s_{sink}^{\otimes} \rrbracket_x \in SinkSet$ under SV^* . By the assumption, there is a sink recurrent class $R_{SV^*}^{\otimes i}$ reachable from the initial state. We consider the best scenario in the assumption. In words, we assume that the system obtains the full possible rewards of \mathcal{R}_1 and r_p in all steps. There exist a number $l > 0$, a state $s_{sink}^{\otimes} \in Post(T_{SV^*}^{\otimes}) \cap R_{SV^*}^{\otimes i}$, and a subset of transient states $\{s_1^{\otimes}, \dots, s_{l-1}^{\otimes}\} \subset T_{SV^*}^{\otimes}$ such that $p(s_{init}^{\otimes}, s_1^{\otimes}) > 0$, $p(s_i^{\otimes}, s_{i+1}^{\otimes}) > 0$ for $i \in \{1, \dots, l-2\}$, and $p(s_{l-1}^{\otimes}, s_{sink}^{\otimes}) > 0$ by the property of transient states. By considering only paths that reach the state $s_{sink}^{\otimes} \in R_{SV^*}^{\otimes i}$ in l steps out of all paths reaching sink recurrent classes, we have

$$\begin{aligned} V^{SV^*}(s_{init}^{\otimes}) &< Pr_{SV^*}^{M^{\otimes}}(s_{init}^{\otimes} \models \varphi) \sum_{k=0}^{\infty} \gamma^k (r_p + \|\mathcal{R}_1\|_{\infty}) + \gamma^l p^l(s_{init}^{\otimes}, s_{sink}^{\otimes}) \sum_{k=0}^{\infty} \gamma^k r_{sink} \\ &\quad + Pr_{SV^*}^{M^{\otimes}}(s_{init}^{\otimes} \not\models \varphi) (r_p + \|\mathcal{R}_1\|_{\infty}) \sum_{k=0}^{\infty} \sum_{s^{\otimes} \in T_{\pi^*}^{\otimes}} \gamma^k p^k(s_{init}^{\otimes}, s^{\otimes}) \\ &< \frac{1}{1-\gamma} \{Pr_{SV^*}^{M^{\otimes}}(s_{init}^{\otimes} \models \varphi) (r_p + \|\mathcal{R}_1\|_{\infty}) + \gamma^l p^l(s_{init}^{\otimes}, s_{sink}^{\otimes}) r_{sink}\} + \bar{m}', \end{aligned}$$

where \bar{m}' is a constant such that $\bar{m}' > Pr_{SV^*}^{M^{\otimes}}(s_{init}^{\otimes} \not\models \varphi) (r_p + \|\mathcal{R}_1\|_{\infty}) \sum_{k=0}^{\infty} \sum_{s^{\otimes} \in T_{\pi^*}^{\otimes}} \gamma^k p^k(s_{init}^{\otimes}, s^{\otimes})$.

Therefore, if it holds that $r_{sink} \leq -\frac{Pr_{SV^*}^{M^{\otimes}}(s_{init}^{\otimes} \models \varphi)}{\gamma^l p^l(s_{init}^{\otimes}, s_{sink}^{\otimes})} (r_p + \|\mathcal{R}_1\|_{\infty})$, we then have $V^{SV^*}(s_{init}^{\otimes}) < \bar{m}'$ for any $\gamma \in (0, 1)$.

Let \bar{SV} be a supervisor satisfying φ and it satisfies that there is no state $s^{\otimes} \in S_{SV}^{\otimes}$ reachable from initial state s_{init}^{\otimes} such that $\llbracket s^{\otimes} \rrbracket_x \in SinkSet$. We consider the following two cases.

1. Assume that the initial state s_{init}^{\otimes} is in a recurrent class $R_{SV}^{\otimes i}$ for some $i \in \{1, \dots, h\}$. For any accepting set \bar{F}_j^{\otimes} , $\delta_{SV}^{\otimes i} \cap \bar{F}_j^{\otimes} \neq \emptyset$ holds by the definition of \bar{SV} . The expected discounted reward for s_{init}^{\otimes} is given by

$$V^{\bar{SV}}(s_{init}^{\otimes}) = \mathbb{E}^{SV} \left[\sum_{k=0}^{\infty} \gamma^k \mathcal{R}(s_k, \pi_k, e_k, s_{k+1}) \mid s_0 = s_{init}^{\otimes} \right] \quad (3)$$

For each path $\rho = s_0 \pi_0 e_0 s_1 \dots s_i \pi_i e_i s_{i+1} \dots \in S(2^E ES)^{\omega}$, the stopping time \hat{k} of first returning to the initial state is defined as $\hat{k}(\rho) = \min\{i > 0 \mid s_i = s_0\}$. Recall that the state set S^{\otimes} is finite, hence all of the recurrent classes are positive recurrent [2]. We consider the worst scenario in this case. It holds that

$$\begin{aligned}
V^{\bar{S}V}(s_{init}^{\otimes}) &> \mathbb{E}^{\bar{S}V}[\gamma^{\hat{k}-1}r_p + \gamma^{\hat{k}-2}r_p + \dots + \gamma^{\hat{k}-n}r_p + \gamma^{\hat{k}-1}V^{\bar{S}V}(s_{init}^{\otimes}) | s_0 = s_{init}^{\otimes}] \\
&> \mathbb{E}^{\bar{S}V}[\gamma^{\hat{k}-1}r_p + \gamma^{\hat{k}-1}V^{\bar{S}V}(s_{init}^{\otimes}) | s_0 = s_{init}^{\otimes}] \\
&\geq \gamma^{\mathbb{E}^{\bar{S}V}[\hat{k}-1 | s_0 = s_{init}^{\otimes}]}r_p + \gamma^{\mathbb{E}^{\bar{S}V}[\hat{k}-1 | s_0 = s_{init}^{\otimes}]}V^{\bar{S}V}(s_{init}^{\otimes}).
\end{aligned}$$

Thus,

$$\begin{aligned}
V^{\bar{S}V}(s_{init}^{\otimes}) &> \frac{\gamma^{\mathbb{E}^{\bar{S}V}[\hat{k}-1 | s_0 = s_{init}^{\otimes}]}r_p}{1 - \gamma^{\mathbb{E}^{\bar{S}V}[\hat{k}-1 | s_0 = s_{init}^{\otimes}]}} \\
&> \frac{\gamma^{\hat{K}-1}r_p}{1 - \gamma^{\hat{K}-1}},
\end{aligned}$$

where the second inequality holds since it holds that $\mathbb{E}^{\bar{S}V}[\gamma^{\hat{k}} | s_0 = s_{init}^{\otimes}] \geq \gamma^{\mathbb{E}^{\bar{S}V}[\hat{k} | s_0 = s_{init}^{\otimes}]}$ by Jensen's inequality, \hat{K} is set to $\lceil \mathbb{E}^{\bar{S}V}[\hat{k} | s_0 = s_{init}^{\otimes}] \rceil < \infty$ and its inequality holds by the property of the positive recurrence [3], and the fourth inequality holds since it holds that $\gamma^{\hat{K}} < \gamma^{\mathbb{E}^{\bar{S}V}[\hat{k} | s_0 = s_{init}^{\otimes}]}$ and $\frac{1}{1-\gamma^{\hat{K}}} < \frac{1}{1-\gamma^{\mathbb{E}^{\bar{S}V}[\hat{k} | s_0 = s_{init}^{\otimes}]}}$ for any $\gamma \in (0, 1)$. We set r_p^* , r_{sink}^* , and γ^* to satisfy $\frac{\gamma^{\hat{K}-1}}{1-\gamma^{\hat{K}-1}}r_p^* > \frac{1}{1-\gamma}\|\mathcal{R}_1\|_\infty$ for any $\gamma \in (0, 1)$, $r_{sink}^* \leq -\frac{Pr_{SV}^{M^{\otimes}}(s_{init}^{\otimes} \models \varphi)}{\gamma^l p^l(s_{init}^{\otimes}, s_{sink}^{\otimes})}(r_p^* + \|\mathcal{R}_1\|_\infty)$ for any $\gamma \in (0, 1)$, and $\frac{\gamma^{*\hat{K}-1}}{1-\gamma^{*\hat{K}-1}}r_p^* - \frac{1}{1-\gamma^*}\|\mathcal{R}_1\|_\infty > m$ for any $m > 0$, respectively. Therefore, for any bounded reward function \mathcal{R}_1 , any positive value $r_p > r_p^*$, any negative value $r_{sink} < r_{sink}^*$, any discount factor $\gamma \in (\gamma^*, 1)$, we then have $V^{\bar{S}V}(s_{init}^{\otimes}) > V^{SV^*}(s_{init}^{\otimes})$ since for $m = \max\{\bar{m}, \bar{m}'\}$, by the setting of r_{sink}^* , we have

$$\begin{aligned}
V^{\bar{S}V}(s_{init}^{\otimes}) - V^{SV^*}(s_{init}^{\otimes}) &> \frac{\gamma^{\hat{K}-1}}{1-\gamma^{\hat{K}-1}}r_p - (m + \frac{1}{1-\gamma}\|\mathcal{R}_1\|_\infty) \\
&= (\frac{\gamma^{\hat{K}-1}}{1-\gamma^{\hat{K}-1}}r_p - \frac{1}{1-\gamma}\|\mathcal{R}_1\|_\infty) - m,
\end{aligned}$$

by the settings of γ^* and r_p^* , we have

$$V^{\bar{S}V}(s_{init}^{\otimes}) - V^{SV^*}(s_{init}^{\otimes}) > 0. \quad (4)$$

2. Assume that the initial state s_{init}^{\otimes} is in the set of transient states $T_{SV}^{\otimes}.PM_{SV}^{M^{\otimes}}(s_{init}^{\otimes} \models \varphi) > 0$ holds by the definition of $\bar{S}V$. For an accepting recurrent class $R_{SV}^{\otimes i}$, there exist a number $l' > 0$, a state \hat{s}^{\otimes} in $Post(T_{SV}^{\otimes}) \cap R_{SV}^{\otimes i}$, and a subset of transient states $\{s_1^{\otimes}, \dots, s_{l'-1}^{\otimes}\} \subset T_{SV}^{\otimes}$ such that $p(s_{init}^{\otimes}, s_1^{\otimes}) > 0$, $p(s_i^{\otimes}, s_{i+1}^{\otimes}) > 0$ for $i \in \{1, \dots, l'-2\}$, and $p(s_{l'-1}^{\otimes}, \hat{s}^{\otimes}) > 0$ by the property

of transient states. Hence, it holds that $p^{l'}(s_{init}^{\otimes}, \hat{s}^{\otimes}) > 0$ for the state \hat{s}^{\otimes} . For each path $\rho = s_0 \pi_0 e_0 s_1 \dots s_i \pi_i e_i s_{i+1} \dots \in S(2^E ES)^\omega$ reaching \hat{s}^{\otimes} , the stopping time \hat{k} of first returning to the state \hat{s}^{\otimes} is defined as $\hat{k}(\rho) = \min\{i - j | s_i = \hat{s}^{\otimes}, s_j = \hat{s}^{\otimes}, i > j > 0\}$. Thus, by ignoring positive rewards in T_{SV}^{\otimes} , we have

$$\begin{aligned} V^{SV}(s_{init}^{\otimes}) &= \mathbb{E}^{SV} \left[\sum_{k=0}^{\infty} \gamma^k \mathcal{R}(s_k, \bar{SV}(s_k), e_k, s_{k+1}) | s_0 = s_{init}^{\otimes} \right] \\ &\geq \mathbb{E}^{SV} \left[\gamma^l \sum_{k=0}^{\infty} \gamma^k \mathcal{R}(s_{k+l}, \bar{SV}(s_{k+l}), e_{k+l}, s_{k+l+1}) | s_0 = s_{init}^{\otimes} \right] \\ &> \gamma^{l'} p^{l'}(s_{init}^{\otimes}, \hat{s}^{\otimes}) \mathbb{E}^{SV} [\gamma^{\hat{k}-1} r_p + \gamma^{\hat{k}-1} V^{SV}(\hat{s}^{\otimes}) | s_{l'} = \hat{s}^{\otimes}] \\ &\geq \gamma^{l'} p^{l'}(s_{init}^{\otimes}, \hat{s}^{\otimes}) \{ \gamma^{\mathbb{E}^{SV}[\hat{k}-1 | s_{l'} = \hat{s}^{\otimes}]} r_p + \gamma^{\mathbb{E}^{SV}[\hat{k}-1 | s_{l'} = \hat{s}^{\otimes}]} V^{SV}(\hat{s}^{\otimes}) \}. \end{aligned}$$

As with the case 1, we have

$$\begin{aligned} V^{SV}(s_{init}^{\otimes}) &> \gamma^{l'} p^{l'}(s_{init}^{\otimes}, \hat{s}^{\otimes}) \frac{\gamma^{\mathbb{E}^{SV}[\hat{k}-1 | s_{l'} = \hat{s}^{\otimes}]} r_p}{1 - \gamma^{\mathbb{E}^{SV}[\hat{k}-1 | s_{l'} = \hat{s}^{\otimes}]}} \\ &> \gamma^{l'} p^{l'}(s_{init}^{\otimes}, \hat{s}^{\otimes}) \frac{\gamma^{\hat{K}-1} r_p}{1 - \gamma^{\hat{K}-1}} \end{aligned} \quad (5)$$

where the third inequality holds since it holds that $\mathbb{E}^{SV}[\gamma^{\hat{k}} | s_{l'} = \hat{s}^{\otimes}] \geq \gamma^{\mathbb{E}^{SV}[\hat{k} | s_{l'} = \hat{s}^{\otimes}]}$ by Jensen's inequality, $\hat{K} = \lceil \mathbb{E}^{SV}[\hat{k} | s_{l'} = \hat{s}^{\otimes}] \rceil$, and the fifth inequality holds since it holds that $\gamma^{\hat{K}} < \gamma^{\mathbb{E}^{SV}[\hat{k} | s_{l'} = \hat{s}^{\otimes}]}$ and $\frac{1}{1 - \gamma^{\hat{K}}} < \frac{1}{1 - \gamma^{\mathbb{E}^{SV}[\hat{k} | s_{l'} = \hat{s}^{\otimes}]}}$ for any $\gamma \in (0, 1)$. We set r_p^* , r_{sink}^* , and γ^* to satisfy $\gamma^{l'} p^{l'}(s_{init}^{\otimes}, \hat{s}^{\otimes}) \frac{\gamma^{\hat{K}-1}}{1 - \gamma^{\hat{K}-1}} r_p^* > \frac{1}{1 - \gamma} \|\mathcal{R}_1\|_\infty$ for any $\gamma \in (0, 1)$, $r_{sink}^* \leq -\frac{Pr_{SV^*}^{\otimes}(s_{init}^{\otimes} \models \varphi)}{\gamma^{l'} p^{l'}(s_{init}^{\otimes}, s_{sink}^{\otimes})} (r_p^* + \|\mathcal{R}_1\|_\infty)$ for any $\gamma \in (0, 1)$, and $\gamma^{l'} p^{l'}(s_{init}^{\otimes}, \hat{s}^{\otimes}) \frac{\gamma^{*\hat{K}-1}}{1 - \gamma^{*\hat{K}-1}} r_p^* - \frac{1}{1 - \gamma^*} \|\mathcal{R}_1\|_\infty > m$ for any $m > 0$, respectively. Therefore, for the reward function \mathcal{R}_1 , any positive value $r_p > r_p^*$, any negative value $r_{sink} < r_{sink}^*$, any discount factor $\gamma \in (\gamma^*, 1)$, by the setting of r_{sink}^* , we have

$$\begin{aligned} V^{SV}(s_{init}^{\otimes}) - V^{SV^*}(s_{init}^{\otimes}) &> \gamma^{l'} p^{l'}(s_{init}^{\otimes}, \hat{s}^{\otimes}) \frac{\gamma^{\hat{K}-1}}{1 - \gamma^{\hat{K}-1}} r_p - (m + \frac{1}{1 - \gamma} \|\mathcal{R}_1\|_\infty) \\ &= (\gamma^{l'} p^{l'}(s_{init}^{\otimes}, \hat{s}^{\otimes}) \frac{\gamma^{\hat{K}-1}}{1 - \gamma^{\hat{K}-1}} r_p - \frac{1}{1 - \gamma} \|\mathcal{R}_1\|_\infty) - m, \end{aligned}$$

by the settings of γ^* and r_p^* , we have

$$V^{SV}(s_{init}^{\otimes}) - V^{SV^*}(s_{init}^{\otimes}) > 0 \quad (6)$$

The results contradict the optimality assumption of SV^* \square

References

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