Let $\mathcal{SV}^*$ be the set of optimal supervisors. Let $M^{\otimes}_{SV^*}$ denote the product DES $M^{\otimes}$ controlled by the optimal supervisor SV^*.

For a Markov chain $MC^{\otimes}_{SV}$ induced by a product MDP $M^{\otimes}$ with a positional policy π, let $S^{\otimes}_{SV} = T^{\otimes}_{SV} \sqcup R^{\otimes 1}_{SV} \sqcup \ldots \sqcup R^{\otimes h}_{SV}$ be the set of states in $MC^{\otimes}_{SV}$, where $T^{\otimes}_{SV}$ is the set of transient states and $R^{\otimes i}_{SV}$ is the recurrent class for each $i \in \{1, \ldots, h\}$, and let $R(MC^{\otimes}_{SV})$ be the set of all recurrent classes in $MC^{\otimes}_{SV}$. Let $\delta^{\otimes}_{SV,i}$ be the set of transtions in a recurrent class $R^{\otimes i}_{SV}$, namely $\delta^{\otimes}_{SV,i} = \{(s^{\otimes}, \pi, s^{\otimes \prime}) \in \delta^{\otimes}; s^{\otimes} \in R^{\otimes i}_{SV},\ P^{\otimes}(s^{\otimes \prime} | s^{\otimes}, \pi) > 0\}$, and let $P^{\otimes}_{SV} : S^{\otimes}_{SV} \times S^{\otimes}_{SV} \to [0,1]$ be the transition probability under SV.

Theorem 1