Lemma 1. *For a product MDP $M^{\otimes}$ of an MDP M and an augmented tLDBA $\bar{B}_{\varphi}$ corresponding to a given LTL formula and a reward function corresponding to the acceptance condition of $M^{\otimes}$, if there exists a finite-memoly policy on the MDP M, then there exists a positional policy satisfying the LTL formula on $M^{\otimes}$.*

Proof. Suppose that there exists a finite-memoly policy satisfying φ on M, but there is no positional policy satisfying φ on $M^{\otimes}$. By the definition of an augmented LDBA, a state (x, v) keeps track of previous visits the accepting sets. Therefore, the property of the augmented states contradicts the assumptions. □