

Given  
 an  
 augmented  
 tLDBA  
 $\bar{B}_\varphi =$   
 $(\bar{X}, \bar{x}_{init}, \bar{\Sigma}, \bar{\delta}, \bar{\mathcal{F}})$   
 and  
 a  
 DES  
 $D$ ,  
 a  
 tuple  
 $D^\otimes$   
 $\bar{B}_\varphi^\otimes =$   
 $D^\otimes =$   
 $(S^\otimes, E^\otimes, s_{init}^\otimes, P_T^\otimes, P_E^\otimes, \delta^\otimes, \mathcal{F}^\otimes)$   
 is  
 a  
 product  
 DES,  
 where  
 $S^\otimes =$   
 $S \times$   
 $\bar{X}$   
 is  
 the  
 finite  
 set  
 of  
 states  
 and  
 we  
 represent  
 $s$   
 and  
 $\bar{x}$   
 corresponding  
 with  
 $s^\otimes =$   
 $(s, \bar{x}) \in$   
 $S^\otimes$   
 as  
 and  
 ,  
 respectively;  
 $E^\otimes =$   
 $E \cup$   
 $\{\varepsilon_{\bar{x}'}; \exists \bar{x}' s.t. (\bar{x}, \varepsilon, \bar{x}') \in$   
 $\delta\}$   
 is  
 the  
 finite  
 set  
 of  
 events,  
 where  
 $\varepsilon_{\bar{x}'}$   
 is  
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 represents  
 an

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as

$$P_T^{\otimes}(s^{\otimes'}|s^{\otimes}, e) =$$

$$\{P_T(s'|s, e) \text{ if } (\bar{x}, L((s, e, s')), \bar{x}') \in \bar{\delta}, e \in \mathcal{E}(s) \text{ if } s = s', (\bar{x}, \varepsilon, \bar{x}') \in \delta, e = \varepsilon_{\bar{x}}, 0 \text{ otherwise},$$

where

$$s^{\otimes} =$$

$$(s, (x, v))$$

and

$$s^{\otimes'} =$$

$$(s', (x', v')).$$

$$P_E^{\otimes} :$$

$$E^{\otimes} \times$$

$$S^{\otimes} \times$$

$$2^{E^{\otimes}} \rightarrow$$

$$[0, 1]$$

is

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$$P_E^{\otimes}(e|s^{\otimes}, \pi) =$$

$$P_E(e|s, \pi),$$

$$\delta^{\otimes} =$$

$$\{(s^{\otimes}, e, s^{\otimes'}) \in$$

$$S^{\otimes} \times$$

$$E^{\otimes} \times$$

$$S^{\otimes}; P_T^{\otimes}(s^{\otimes'}|s^{\otimes}, e) >$$

$$0\}$$

is

the

set

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tions,

and

$$\mathcal{F}^{\otimes} =$$

$$\{\bar{F}_1^{\otimes}, \dots, \bar{F}_n^{\otimes}\}$$

is

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tion,

where

$$\bar{F}_i^{\otimes} =$$

$$\{(s, \bar{x}), e, (s', \bar{x}') \in$$

$$\delta^{\otimes}; (\bar{x}, L(s, e, s'), \bar{x}') \in$$

$$\bar{F}_i^{\otimes}\}$$

for

each

$$i \in$$

$$\{1, \dots, n\}.$$

The

two

re-