

Lemma 1. *For a product MDP M^\otimes of an MDP M and an augmented tLDBA \bar{B}_φ corresponding to a given LTL formula and a reward function corresponding to the acceptance condition of M^\otimes , if there exists a finite-memory policy satisfying φ on the MDP M , then there exists a positional policy satisfying φ on M^\otimes .*

Proof. Suppose that there exists a finite-memory policy satisfying φ on M , but there is no positional policy satisfying φ on M^\otimes . By the definition of the augmented tLDBA, a state (x, v) keeps track of previous visits the accepting sets and the accepting transitions of the augmented tLDBA are the first visits to the accepting transitions of the tLDBA. Therefore, the properties of the augmented states and the acceptance condition contradict the assumption. \square