

V00835123

- Assume $(A \bar{\wedge} B) \bar{\wedge} C = A \bar{\wedge} (B \bar{\wedge} C)$

$$\therefore (A \bar{\wedge} B) \bar{\wedge} C \neq A \bar{\wedge} (B \bar{\wedge} C)$$

Hence, the nand operator $\bar{\wedge}$ is non-associative.

- | x | y | $x \wedge y$ | $\overline{x \vee y}$ | $x \oplus y$ | $x \vee y$ | $x \oplus y \oplus (x \vee y)$ |
|---|---|--------------|-----------------------|--------------|------------|--------------------------------|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| 1 | 1 | 1 | 1 | 0 | 1 | 1 |

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- The figure consists of two binary trees. The left tree starts with the root node 11001101. It branches into 1010 and 1011. 1010 branches into 11 and 00, which further branch into 1, 1, 1, 1, and then 0w, 0w, 1, 1, leading to 1+x. 1011 branches into 0001y and 01, which further branch into 00, 01, 01x, 01x, leading to wx. The right tree starts with the root node 11000001. It branches into 00001100z and 0010. 00001100z branches into 0010 and 00, which further branch into 01, 00, 01x, 01x, leading to w+wx. 0010 branches into 0000y and 00, which further branch into 00, 00, 00, 00, leading to 0. Both trees converge to the final polynomial 1+x+wxy+wz+wxz.

5. a) if $f(x_1..0..x_4)=1$ than $f(x_1..1..x_4)=1$

So f is monotone.

b)

0010	x3	
0011		
0110		x1x2
0111	1110	1100
1010	1111	1101
1011		

The full disjunctive normal form is

$$f(x_1, x_2, x_3, x_4) = (\bar{x}_1 \wedge \bar{x}_2 \wedge x_3 \wedge \bar{x}_4) \vee (\bar{x}_1 \wedge \bar{x}_2 \wedge x_3 \wedge x_4) \vee (\bar{x}_1 \wedge x_2 \wedge x_3 \wedge \bar{x}_4) \vee (\bar{x}_1 \wedge x_2 \wedge x_3 \wedge x_4) \vee (x_1 \wedge \bar{x}_2 \wedge x_3 \wedge \bar{x}_4) \vee (x_1 \wedge \bar{x}_2 \wedge x_3 \wedge x_4) \vee (x_1 \wedge x_2 \wedge \bar{x}_3 \wedge \bar{x}_4) \vee (x_1 \wedge x_2 \wedge \bar{x}_3 \wedge x_4) \vee (x_1 \wedge x_2 \wedge x_3 \wedge \bar{x}_4) \vee (x_1 \wedge x_2 \wedge x_3 \wedge x_4)$$

The shortest DFN is

$$f(x_1, x_2, x_3, x_4) = (x_1 \wedge x_2) \vee x_3$$