ESMA 6000: Asignacion #1

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Israel Almodovar

Alejandro Ouslan

Problem 1

Using your own words define the following concepts:

- **Sample Space:** The sample space is the set of all possible outcomes of an experiment. It is to say everythig that could happen in the experiment
- **Countable Sets (give an example):** A countable set is a set that can be put in a one-to-one correspondence with the set of natural numbers $\mathbb{N}$. An example of a countable set is the set would be the set of possible lottery numbers given that we can express or quantify the set of numbers that can be drawn.
- **σ-algebra $\mathcal{B}$:** σ-algebra is a collection of combinations of sets or events of the sample space.
- **The following triplet $(\mathcal{S}, \mathcal{B}, P)$:** The triplet $(\mathcal{S}, \mathcal{B}, P)$ is given the sample space $\mathcal{S}$, the event/set of events σ-algebra $\mathcal{B}$ has the probability measure P.

Problem 2

For each of the following experiments, describe the sample space.

- Toss a coin four times.
  The sample space is the set of all possible outcomes of the experiment. An example of the events that are contain in the sample space are $\{HHHH, HHHT, \ldots, TTTT\}$.
- Count the number of insects-damaged leaves on a plant.
  The sample space is the set of all possible counts of insects-damaged leaves on a plant. An example asumming that there are 10 plants the sample space would be $\{0, 1, 2, \ldots, 10\}$.
- Measure the lifetime (in hours) of a particular brand of lights bulbs. The sample space is the set of all possible lifetimes (in hours) of a particular brand of lights. An example given the brand *generic-x* and we looked at 5 bulbs the sample space would be $\{0, \ldots, 1000, \ldots, \infty\}$.
- Record the weights of 10-day-old babies.
  The sample space is the set of all possible weights of 10-day-old babies. An example given the weights of 10-day-old babies in grams the sample space would be $\{0, \ldots, 1000, \ldots, \infty\}$.
- Observe the proportion of defective in a shipment of electreonic components.
  The sample space is the set of all possible proportions of defective in a shipment of electronic components. An example given the proportion of defective in a shipment of electronic components the sample space would be $\{0, \ldots, 1\}$.

Problem 3

Supose that $A \subset B$. Show that $B^c \subset A^c$

Proof.

□

Problem 4

Let S be a sample space. Show that the collection $\mathcal{B} = \{\emptyset, S\}$ is a σ-algebra.

Proof.

□

Problem 5

Let $\Omega = \{1, 2, 3\}$ be the sample space. Let $\mathcal{B} = \{\{1\}, \{2, 3\}, \emptyset, \Omega\}$. be a collection of S.

- Verify that $\mathcal{B}_\infty$ and $\mathcal{B}_\in$ are σ-algebras.
- Verify that $\mathcal{B}_\infty \cap \mathcal{B}_\in$ is a σ-algebra.
- Verify that $\mathcal{B}_\infty \cup \mathcal{B}_\in$ is not a σ-algebra.
- Discuss your results from (b) and (c).

Problem 6

One ball is to be selected from a box containing red, white, blue, yellow, and green balls. If the probability that the selected ball will be red is $\frac{1}{5}$, and the probability that it will be white is $\frac{2}{5}$, what is the probability that it will be blue, yellow, or green?

Problem 7

If $P(A) = \frac{1}{3}$ and $P(B^c) = \frac{1}{2}$, can A and B be disjoint? Explain.

Problem 8

Prove that every two events A and B, the probability that exactly one of the two events will occur is given by the expression

$$P(A) + P(B) - 2P(A \cap B)$$

Problem 9

For events A and B, find formulas for the probabilities of the following events in terms of the quantities $P(A)$, $P(B)$, and $P(A \cap B)$:

- either A or B or both occur.
- either A or B but not both occur.
- at least one of A or B.
- at most one of A or B.