ECOG 219: Problem Set #1

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Problem 1

If the domain of the function $y = 5 + = x$ is the set $\{x | 1 \geq x \geq 9\}$, find the range of the function and express it as a set.

$$\begin{aligned} DOM &= \{x | 1 \geq x \geq 9\} \\ RANGE &= \{x | 32 \geq x \geq 8\} \end{aligned}$$

Problem 2

In the theory of the firm, economists consider the total cost C to be a function of the output level $Q : C = f(Q)$.

1. According to the definition of a function, should each cost figure be associated with a unique level of output?
   Answer: Not necessary since there could be multiple outputs (Q) that could be map to the same C but each Q can only map to one C.

2. Should each level of output determine a unique cost figure?
   Answer: Yes, given that each Q can only map to one C.

Problem 3

If an output level Q_1 can be produced at a cost of C_1 , then it must also be possible (by being less efficient) to produce Q_1 at a cost of $C_1 + \$1$, or $C_1 + \$2$, and so on. Thus it would seem that output Q does not uniquely determine total cost C. If so, to write $C = f(Q)$ would violate the definition of a function. How, in spite of the this reasoning, would you justify the use of the function $C = f(Q)$?
Answer: The function would still be valid since adding some cost constant $\forall a \in \mathbb{R}^+$ would only displace the cost function upwards by a units. A cost increase of a units could be described in the following method:

$$\begin{aligned} C &= f(Q) \\ C + a &= f(Q) + a; \forall a \in \mathbb{R}^+ \end{aligned}$$

Problem 4

Show that $X^{m/n} = \sqrt[n]{[x^m]} = (\sqrt[n]{[x]})^m$.Specify the rules applied in each step.

$$\begin{aligned} x^{m/n} &= x^{m\frac{1}{n}} \\ &= (x^m)^{\frac{1}{n}} \\ &= \sqrt[n]{[x^m]} \end{aligned}$$

$$\begin{aligned} x^{m/n} &= x^{m\frac{1}{n}} \\ &= x^{\frac{1}{n}m} \\ &= (x^{\frac{1}{n}})^m \\ &= (\sqrt[n]{[x]})^m \end{aligned}$$

Problem 5

Prove Rule VI and Rule VII:

1. **Rule VI:** $(x^m)^n = x^{m \cdot n}$

   Proof. To prove that $(x^m)^n = x^{m \cdot n}$ is true we use the induction method. First, we prove that:

$$P_1 : (x^m)^1 \Rightarrow x^m = x^m$$

$$P_2 : (x^m)^2 \Rightarrow x^m \cdot x^m \Rightarrow x^{m+m} = x^{m \cdot 2}$$
$$P_n : (x^m)^n \Rightarrow x_1^m \cdot x_2^m \cdot ... \cdot x_n^m \Rightarrow x^{m+m+...+m} = x^{m \cdot n}$$

   Then we prove that P_1 hold for all P_{n+1}, that is:

$$P_{n+1} : (x^m)^n \Rightarrow x_1^m \cdot x_2^m \cdot ... \cdot x_n^m \cdot x_{n+1}^m$$
$$\Rightarrow x^{m \cdot n} \cdot x^m$$
$$\Rightarrow x^{m\dot{n}+m} = x^{m \cdot (n+1)}$$

   Therefore P_{n+1} holds for all positive integers n since $P_1 \implies P_{n+1}$. □

2. **Rule VII:** $x^m \cdot y^m = (x \cdot y)^m$

   Proof. To prove that $x^m \cdot y^m = (x \cdot y)^m$ is true we use the induction method. First, we prove that:

$$P_1 : x^1 \cdot y^1 \Rightarrow x \cdot y = x \cdot y$$

$$P_2 : x^2 \cdot y^2 \Rightarrow x \cdot y \cdot x \cdot y \Rightarrow (x \cdot y)^2 = (x \cdot y)^2$$
$$P_n : x^m \cdot y^m \Rightarrow x_1 \cdot y_1 \cdot x_2 \cdot y_2 \cdot ... \cdot x_n \cdot y_n \Rightarrow (x \cdot y)^m$$

   Then we prove that P_1 hold for all P_{n+1}, that is:

$$P_{n+1} : x^{m+1} \cdot y^{m+1} \Rightarrow x_1 \cdot y_1 \cdot x_2 \cdot y_2 \cdot ... \cdot x_n \cdot y_n \cdot x_{m+1} \cdot y_{m+1}$$
$$\Rightarrow (x \cdot y)^m \cdot (x \cdot y)_{m+1}$$
$$\Rightarrow (x \cdot y)^{m+1}$$

   Therefore P_{n+1} holds for all positive integers n since $P_1 \implies P_{n+1}$. □

Problem 6

Given the market model

$$Q_d = Q_s$$
$$Q_d = 21 - 3P$$
$$Q_s = -4 + 8P$$

Problem 6 continued on next page. . .

Find P^* and P^* by elimination of variables.

$$\begin{aligned} Q_d &= Q_s \\ Q_d &= 21 - 3P \\ Q_s &= -4 + 8P \\ 21 - 3P &= 3P + 8P \\ 25 &= 11P \\ P^* &= \frac{25}{11} \\ Q^* &= \frac{156}{11} \end{aligned}$$

Using the formulas:

$$\begin{aligned} P^* &= \frac{a+c}{b+d} \\ &= \frac{21+4}{3+8} \\ &= \frac{25}{11} \\ Q^* &= \frac{ad-bc}{b+d} \\ &= \frac{21 \cdot 8 - 3 \cdot 4}{3+8} \\ &= \frac{156}{11} \end{aligned}$$

Problem 7

Find the equilibrio solution for each of the following models:

1.

$$\begin{aligned} Q_d &= Q_s \\ Q_d &= 3 - P^2 \\ Q_s &= 6P - 4 \end{aligned}$$

Answer:

$$\begin{aligned} Q_d &= Q_s \\ Q_d &= 3 - P^2 \\ Q_s &= 6P - 4 \\ 3 - p^2 &= 6p - 4 \\ p^2 6p + 7 &= 0 \\ (p+7)(p-1) &= 0 \\ P &= 1 \\ Q* &= 2 \end{aligned}$$

Problem 7 continued on next page. . .

2.

$$\begin{aligned}
Q_d &= Q_s \\
Q_d &= 8 - P^2 \\
Q_s &= P^2 - 2 \\
P^2 - 2 &= 8 - p^2 \\
2(P^2 - 5) &= 0 \\
P^* &= \sqrt{5} \\
Q* &= 3
\end{aligned}$$

Problem 8

The market equilibrium condition, $Q_d = Q_s$, is often expressed in an equivalent alternative from, $Q_d - Q_s = 0$, which has the economic interpretation "excess demand is zero". Does (3.7) represent this latter version of the equilibrium condition? If not, supply an appropriate economic interpretation for (3.7).

$$\begin{aligned}
Q_d &= Q_s \\
Q_d - Q_s &= 0 \\
4 - p^2 - 4p + 1 &= 0 \\
p^2 + 4p - 5 &= 0
\end{aligned}$$

Problem 9

The demand and supply function of a two-commodity market model are as follows:

$$\begin{aligned}
Q_{d_1} &= 18 - 3P_1 + P_2 \\
Q_{d_2} &= 12 + P_1 - 2P_2 \\
Q_{s_1} &= -2 + 4P_1 \\
Q_{s_2} &= -2 + 3P_2 \\
-7p_1 + P_2 &= -20 \\
P_1 + 5P_2 &= -14 \\
\begin{bmatrix} -7 & 1 \\ 1 & -5 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} &= \begin{bmatrix} -20 \\ -14 \end{bmatrix} \\
\left[\begin{array}{cc|c} -7 & 1 & -20 \\ 1 & -5 & -14 \end{array}\right]& \\
\left[\begin{array}{cc|c} 1 & -5 & -14 \\ 0 & 1 & \frac{59}{17} \end{array}\right]& \\
\left[\begin{array}{cc|c} 1 & 0 & \frac{57}{17} \\ 0 & 1 & \frac{59}{19} \end{array}\right]&
\end{aligned}$$

Problem 10

Let the national-income model be:

$$\begin{aligned} Y &= C + I_0 + G \\ C &= a + b(Y - T_0) \\ G &= gY \end{aligned}$$

1. Identify the endogenous variables.

   Answer: Y, C, G are endogenous.

2. Give the economic meaning of the parameter g.

   Answer:

3. Find the equilibrium national income.

   Answer:

4. What restriction on the parameters is needed for a solution to exist?

   Answer:

Problem 11

Rewrite the market model (3.12) in the format of (4.1) with the variables arranged in the following order: Q_{d1} , Q_{s1} , Q_{d2} , Q_{s2} , P_1 , P_2 . Write out the coefficient matrix, the variable vector, and the constant vector.

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -a_1 & -a_2 \\ 0 & 1 & 0 & 0 & -b_1 & -b_2 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -\alpha_1 & -\alpha_2 \\ 0 & 0 & 0 & 1 & \beta_1 & -\beta_2 \end{bmatrix} \begin{bmatrix} Q_{d1} \\ Q_{s1} \\ Q_{d2} \\ Q_{s2} \\ P_1 \\ P_2 \end{bmatrix} \begin{bmatrix} 0 \\ a_0 \\ b_0 \\ 0 \\ \alpha_1 \\ \beta_1 \end{bmatrix}$$

Problem 12

Can the market model (3.6) be rewritten in the format of (4.1)? Why?
Answer: No, because there is a quadratic equation, only linear equations can be written in the format of matrices.

Problem 13

Having sold n items of merchandise t quantities $Q_1, \cdots, Q_n$ and prices $P_1, \cdots, P_n$, howo would you express the total revenue in (a) $\sum$ notation and (b) vector notation?

- $$\sum_{i=1}^{n} P_i Q_i$$

- $$\begin{bmatrix} P_1 \\ \vdots \\ P_n \end{bmatrix} \begin{bmatrix} Q_1 & \dots & Q_n \end{bmatrix}$$

Problem 14

The triangular inequality is written with the weak inequality sign $\geq$, rather than the strict inequality sign $<$. Under what circumstances would the "=" part of the inequality apply?
Answer: It would apply when one is a liner representation of the other

Problem 15

Name some situations or contexts where the notion of a weighted or unweighted sum of square may be relevant.
Answer: It could be useful you have outliers.

Problem 16

Show that the diagonal matrix

$$\begin{bmatrix} a_{11} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & a_{ij} \end{bmatrix}$$

can be idempotent only if each diagonal element is either 1 or 0. How many different numerical idempotent diagonal matrices of dimension $n \times n$ can be constructed altogether from such a matrix?
Answer: 2^n

Problem 17

Given $A = \begin{bmatrix} 0 & 4 \\ -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -8 \\ 0 & 1 \end{bmatrix}$, and $C = \begin{bmatrix} 1 & 0 & 9 \\ 6 & 1 & 1 \end{bmatrix}$, find A', B' and C'

$$A' = \begin{bmatrix} 0 & -1 \\ 4 & 3 \end{bmatrix}$$

$$B' = \begin{bmatrix} 3 & 0 \\ -8 & 1 \end{bmatrix}$$

$$C' = \begin{bmatrix} 1 & 6 \\ 0 & 1 \\ 9 & 1 \end{bmatrix}$$

Problem 18

Use the matrices given in Prob. 1 to verify that

Problem 18 continued on next page...

1. $(A+B)' = A' + B'$

$$(A+B)' = A' + B'$$
$$\left(\begin{bmatrix} 0 & 4 \\ -1 & 3 \end{bmatrix} + \begin{bmatrix} 3 & -8 \\ 0 & 1 \end{bmatrix}\right)' = \begin{bmatrix} 0 & 4 \\ -1 & 3 \end{bmatrix}' + \begin{bmatrix} 3 & -8 \\ 0 & 1 \end{bmatrix}'$$
$$\left(\begin{bmatrix} 3 & -4 \\ -1 & 4 \end{bmatrix}\right)' = \begin{bmatrix} 0 & -1 \\ 4 & 3 \end{bmatrix} + \begin{bmatrix} 3 & 0 \\ -8 & 1 \end{bmatrix}$$
$$\begin{bmatrix} 3 & -1 \\ -4 & 4 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ -4 & 4 \end{bmatrix}$$

Problem 19

Let $A = I - X(X'X)^{-}X'$

1. Must A be square? Must$(X'X)$ be square? Must X be Square? **Answer:** A must be square, $(X'X)$ has to be also square and X does not need to be square
2. Show that matrix A is idempotent.

Problem 20

Consider the situation of a mass layoff (i.e., a factory shuts down) where 1,200 people become unemployed and now begin a job search. In this case there are two states: employed (E) and unemployed (u) with an initial vector

$$X_0' = \begin{bmatrix} E & U \end{bmatrix} = \begin{bmatrix} 0 & 1,200 \end{bmatrix}$$

Suppose that in any given period an unemployed person will find a job with probability. .7 and will therefore remanan unemployed with a probability of .3. Additionally, persons who find themselves employed with probability of .1 (and will have a .9 probability of remaining employed).

1. Set up the Markov transition matrix for this problem.

$$P = \begin{bmatrix} 0.9 & 0.1 \\ 0.7 & 0.3 \end{bmatrix}$$

2. What will be the number of unemployed people after (i) 2 periods; (ii) 3 periods; (iii) 5 periods; (iv) 10 periods?

   (a) $P_1 = \begin{bmatrix} 0 & 1200 \end{bmatrix} \begin{bmatrix} 0.9 & 0.1 \\ 0.7 & 0.3 \end{bmatrix} = \begin{bmatrix} 840 & 360 \end{bmatrix}$

   (b) $P_2 = \begin{bmatrix} 840 & 360 \end{bmatrix} \begin{bmatrix} 0.9 & 0.1 \\ 0.7 & 0.3 \end{bmatrix} = \begin{bmatrix} 1008 & 192 \end{bmatrix}$

   (c) $P_3 = \begin{bmatrix} 1008 & 192 \end{bmatrix} \begin{bmatrix} 0.9 & 0.1 \\ 0.7 & 0.3 \end{bmatrix} = \begin{bmatrix} 1041.6 & 151.68 \end{bmatrix}$

   (d) $P_4 = \begin{bmatrix} 1041.6 & 151.68 \end{bmatrix} \begin{bmatrix} 0.9 & 0.1 \\ 0.7 & 0.3 \end{bmatrix} = \begin{bmatrix} 1058.32 & 150.336 \end{bmatrix}$

Problem 20 continued on next page. . .

3. what is the steady-state level of unemployment?

$$\begin{cases} 0.9E + 0.7U = E \\ 0.1E + 0.3U = U \\ E + U = 1 \end{cases} \quad U = \frac{1}{8} E = \frac{7}{8}$$

150 people

Problem 21

The textbook case labor supply model uses the utility function $U(c, l)$ where "c" is consumption and "I" is leisure.

1. What are the standard assumption imposed on $U(., .)$
   (a) More is better
   (b) diminishing marginal utility
   (c) convexity
2. Justify the alternative model $U(c_1, c_2, l)$ where "c_1" and c_2 represent the consumption level of two different goods. What assumptions would you impose on $U(\cdot)$

   Answer: The alternative model where we incorporate multiple consumption is more characteristic of the real world given that we have more than one option of consumption. A representation of the model would be to pick c_1 as a specific consumption and leaf c_2 as everything else. Overall the assumptions would stay the same.