ECON 219: Problem Set #3

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Dr. Sergio Urzua

Alejandro Ouslan

Problem 1

Suppose that a certain wine dealer is in possesion of a particular quantity of wine, which he can either sell at the present time ($i = 0$) for a sum of K dollars of else store for some length of time and then sell at a higher value.

1. Present an expression for the growing value (V) of the wine as a function of time.

   $$V(t) = Ke^{rt}$$

   where:

   - K: initial value of the wine at time $t = 0$,
   - r: appreciation rate,
   - t: time stored,
   - $V(t)$: value after time t.

2. Explain under what assumptions the maximization of profits is the same as maxing the sales renueves (V)

   Let ρ be the discount rate (opportunity cost of capital). The present value of selling at time t is:

   $$\text{PV}(t) = V(t)e^{-\rho t} = Ke^{(r-\rho)t}$$

   Maximizing profit is equivalent to maximizing $V(t)$ only when $\rho = 0$, i.e., there is no discounting.

3. Under the assumption of part b, present the maximization problem of the wine dealer. Assuming storage cost $C(t)$, the profit is:

   $$\Pi(t) = V(t) - C(t) = Ke^{rt} - C(t)$$

   The wine dealer solves:

   $$\max_{t \geq 0} \Pi(t) = Ke^{rt} - C(t)$$

4. Present and intercept the first order condition of the preious problem. Take derivative of $\Pi(t)$:

   $$\frac{d\Pi}{dt} = \frac{dV}{dt} - \frac{dC}{dt} = Kre^{rt} - C'(t)$$

   FOC:

   $$Kre^{rt^*} = C'(t^*)$$

   Interpretation: At the optimum time t^*, marginal benefit equals marginal cost.

5. Present and interpret the second order condition of the wine dealer's problem. The second derivative:

   $$\frac{d^2\Pi}{dt^2} = Kr^2e^{rt} - C''(t)$$

   SOC for a maximum:

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$$Kr^2e^{rt^*} < C''(t^*)$$

Interpretation: Marginal cost increases faster than marginal benefit at t^*.

6. Obtain the optimum length of storage time. From the FOC:

$$Kre^{rt^*} = C'(t^*)$$

Example: If $C(t) = ct$, then $C'(t) = c$. Solve:

$$Kre^{rt^*} = c \Rightarrow e^{rt^*} = \frac{c}{Kr} \Rightarrow t^* = \frac{1}{r}\ln\left(\frac{c}{Kr}\right)$$

7. Confirm that the previous answer defines a maximum and not a minimum. With $C(t) = ct$, $C''(t) = 0$. So second derivative is:

$$\frac{d^2\Pi}{dt^2} = Kr^2e^{rt} > 0$$

In this case, we get a minimum unless a more realistic cost function like $C(t) = ct + dt^2$ is used, where:

$$C'(t) = c + 2dt, \quad C''(t) = 2d$$

Then:

$$\text{SOC: } Kr^2e^{rt^*} < 2d \quad \text{(ensures maximum)}$$

Problem 2

Consider the following firm's renueves function:

$$R_1 = P_{10}Q_1 + P_{20}Q_2$$

where Q_s represent the output level of the two different products the firm produces. The firm's cost is assumed to be:

$$C = Q_1^2 + Q_1Q_1 + 2Q_2^2$$

1. Present a plot of the marginal cost of the firm with refract to its first productita . Interpret you results.

$$MC_1 = \frac{\partial C}{\partial Q_1} = \frac{\partial}{\partial Q_1}(2Q_1^2 + 2Q_2^2) = 4Q_1$$

The marginal cost increases linearly with the quantity of the first good. This indicates increasing marginal cost or decreasing returns to scale for Q_1.

2. Present the profit function of the firm.

$$\pi(Q_1, Q_2) = P_{10}Q_1 + P_{20}Q_2 - (2Q_1^2 + 2Q_2^2)$$

Problem 2 continued on next page. . .

3. Present the firm order condition of the firm's problem. Solve for the optimal level of the production.

$$\frac{\partial \pi}{\partial Q_1} = P_{10} - 4Q_1 = 0 \quad \Rightarrow \quad Q_1^* = \frac{P_{10}}{4}$$
$$\frac{\partial \pi}{\partial Q_2} = P_{20} - 4Q_2 = 0 \quad \Rightarrow \quad Q_2^* = \frac{P_{20}}{4}$$

4. Depict the supply function of both goods.

$$Q_1(P_{10}) = \frac{P_{10}}{4}, \quad Q_2(P_{20}) = \frac{P_{20}}{4}$$

5. Check the second order condition of the optimization problem.

$$\frac{\partial^2 \pi}{\partial Q_1^2} = -4, \quad \frac{\partial^2 \pi}{\partial Q_2^2} = -4, \quad \frac{\partial^2 \pi}{\partial Q_1 \partial Q_2} = 0$$

$$H = \begin{bmatrix} -4 & 0 \\ 0 & -4 \end{bmatrix}$$

6. c and b using total derivatives.

$$\frac{dQ_1}{dP_{10}} = \frac{1}{4}, \quad \frac{dQ_2}{dP_{20}} = \frac{1}{4}$$