ECON 219: Problem Set #5

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Problem 1

Asume the following smooth production function:

$$Q = Q(K, L)$$

with positive marginal productivities. Let w and r the prices of labor and capital, respectively.

1. Formulate the problem of minimizing costs subject to the technology.
2. Explain under what conditions you might have to consider the xase of corner solution (optimal labor or capital equal to zero). Provide an example.
3. Assuming interior solution present the first order conditions. Provide an economic interpretation to the optimality condition. In you answer, refer to the Langrange multiplier.
4. Provide a graphical representation of the resulting optimal input combination.
5. Present the second order condition. Explain how the strict convexity of the isoquants would ensure a minimum cost. Explain how quasi-concave production function can generate everywhere strictly convex, downard-sloping isoquants.
6. Now, assume $Q = AL^{\alpha}K^{\beta}$. Show that the expansion path (optimal combinations of capital and labor for different isocosts) is characterized by a linear combination.
7. Show the previous result holds for all homogeneous produciton functions.

Problem 2

Consider the following model:

$$Y = X\beta + \epsilon$$

where the standard assumption securing OLS delivers BLUE estimators hold. Assume the error terms is normally distributed with mean 0 and variance σ^2.

1. Present the likeligood function and optimization problem
2. Present the first and second order conditions.
3. Generate a sample of 1000 observations under the following parameterization:

   $$Y = X\beta + \epsilon = \beta_0 + \beta_1 X + \epsilon$$

   where $\beta_0 = 0.5, \quad \beta_1 = -0.75, \quad X \sim (0.5, 2)$, and $\epsilon \sim N(0, 1)$.Present summary statistics
4. Implement the Newton-Raphson algorithm for the MLE problem.
   (a) Report the estimated values for the three parameters.
   (b) Compute the Hessina at the estimated values. How is this connected to the estimators' variance covariance MLE matrix?