ECON 219: Problem Set #2

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Problem 1

Consider the National Income Model:

$$
\begin{aligned}
Y &= C + I_0 + G_0 \\
C &= \alpha + \beta(Y - T) \\
T &= \gamma + \delta Y
\end{aligned}
$$

1. Define and interpret each of the components of the model. Identify parameter and variables.
   (a) $Y = C + I_0 + G_0$ Nations income
   (b) $C = \alpha + \beta(Y - T)$ Consumption function
   (c) $T = \gamma + \delta Y$ Tax Function
   (d) I, G investments and government spending are exogenous
   (e) α consumption intercept (minimum consumption when there is no income)
   (f) β Marginal propensity to consume
   (g) γ minimum taxes
   (h) δ tax rate on income
2. Impose reasonable assumptions on the signs and values of the parameters.
   (a) $\alpha > 0$ there is at least some consumption even if there is no income
   (b) $0 < \beta < 1$ hoes holds consume at least something and not all of their income
   (c) $\gamma \geq 0$ there is no negative taxes
   (d) $I_0 > 0$ There is some investments
   (e) $G_0 > 0$ There is some government spending
3. Solve for the equilibrium income. Start from:

$$
\begin{aligned}
Y &= C + I_0 + G_0 \\
&= \alpha + \beta(Y - T) + I_0 + G_0 \\
&= \alpha + \beta(Y - \gamma - \delta Y) + I_0 + G_0 \\
&= \alpha + \beta Y - \beta\gamma - \beta\delta Y + I_0 + G_0 \\
&= Y(1 - \beta + \beta\delta) = \alpha - \beta\gamma + I_0 + G_0 \\
&= \frac{\alpha - \beta\gamma + I_0 + G_0}{1 - \beta(1 - \delta)}
\end{aligned}
$$

4. Obtain an discuss the six comparative-static derivatives. Let:

$$
Y = \frac{\alpha - \beta\gamma + I_0 + G_0}{1 - \beta(1 - \delta)}
$$

   Partial derivatives:

$$\frac{\partial Y}{\partial \alpha} = \frac{1}{1-\beta(1-\delta)} > 0$$
$$\frac{\partial Y}{\partial \gamma} = \frac{-\beta}{1-\beta(1-\delta)} < 0$$
$$\frac{\partial Y}{\partial I_0} = \frac{1}{1-\beta(1-\delta)} > 0$$
$$\frac{\partial Y}{\partial G_0} = \frac{1}{1-\beta(1-\delta)} > 0$$
$$\frac{\partial Y}{\partial \delta} = \frac{\beta(\alpha-\beta\gamma+I_0+G_0)}{[1-\beta(1-\delta)]^2} > 0 \quad \text{(if numerator positive)}$$
$$\frac{\partial Y}{\partial \beta} = \frac{-\gamma(1-\beta(1-\delta)) + (\alpha-\beta\gamma+I_0+G_0)(1-\delta)}{[1-\beta(1-\delta)]^2} \quad \text{(ambiguous)}$$

Summary of signs:

Parameter	**Partial Derivative**	**Sign**
α	$\frac{\partial Y}{\partial \alpha}$	Positive
β	$\frac{\partial Y}{\partial \beta}$	Ambiguous
γ	$\frac{\partial Y}{\partial \gamma}$	Negative
δ	$\frac{\partial Y}{\partial \delta}$	Positive (if numerator > 0)
I_0	$\frac{\partial Y}{\partial I_0}$	Positive
G_0	$\frac{\partial Y}{\partial G_0}$	Positive

Problem 2

Consider the market model:

$$Q_s = Q_d$$
$$Q_d = D(P, Y_0)$$
$$Q_s = S(P)$$

1. Provide an economic interpretation to each of the equations. In you answers, include the assumptions on the signs of the relevant derivatives.

   (a) $Q_s = Q_d$ the quantity supplied is equal to the quantity produced ($Q_s, Q_s \in \mathbb{N}$)

   (b) $Q_d = D(P, Y_0)$ the quantity depended is a function of the price and income ($P, Y_0, D(\cdot) \in \mathbb{R}^+$)

   (c) $Q_s = S(P)$ the quantity supplied is a function of the price ($P, S(\cdot) \in \mathbb{R}^+$)

2. Define the concept of market equilibrium. Provide and explain its mathematical formulation.

   Answer: The market equilibrium is when the P^* is picked such that $D(P^*, Y_0) = S(P^*)$. $Q^* = D(P^*, Y_0) = S(P^*)$ is the equilibrium quantity produced

3. Show that:

$$\frac{dP^*}{dY_0} > 0$$

   Where P^* is the equilibrium price. Provide an economic interpretation for this result.

4. Show that:
$$\frac{dQ^*}{dY_0} > 0$$
Where $Q*$ is the equilibrium price. Provide an economic interpretation for this result.
$$\frac{dS}{dP} \cdot \frac{dP^*}{dY_0} = \frac{dP^*}{dY_0} > 0$$
Answer: An increase in income leads to higher demand, which pushes up both the price and the equilibrium quantity. Suppliers respond to higher prices by increasing supply, leading to a higher Q^*

5. Answer questions c and d using total derivatives.
$$\frac{dS}{dP} \cdot \frac{dP^*}{dY_0} = \frac{\partial D}{\partial P} \cdot \frac{dP^*}{dY_0} + \frac{\partial D}{\partial Y_0}$$
$$\frac{dP^*}{dY_0} = \frac{\frac{\partial D}{\partial Y_0}}{\frac{dS}{dP} - \frac{\partial D}{\partial P}} > 0$$
$$\frac{dQ^*}{dY_0} = \frac{dS}{dP} \cdot \frac{dP^*}{dY_0} > 0$$