ECON 219: Problem Set #4

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Problem 1

Consider a monopolistic firm operating in three differenc markets. Its revenue and cost function:

$$R = R_1(Q_1) + R_2(Q_2) + R_3(Q_3)$$
$$C = C(Q) \text{ Where } Q = Q_1 + Q_2 + Q_3.$$

1. Define the profit maximization problem of the firm.

$$\Pi = R(Q_1, Q_2, Q_3) - C(Q_1 + Q_2 + Q_3)$$

Substituting the expressions for revenue and cost:

$$\Pi = (R_1(Q_1) + R_2(Q_2) + R_3(Q_3)) - C(Q_1 + Q_2 + Q_3)$$

Thus, the firm's profit maximization problem is:

$$\max_{Q_1,Q_2,Q_3} (R_1(Q_1) + R_2(Q_2) + R_3(Q_3) - C(Q_1 + Q_2 + Q_3))$$

2. Present the first-order condition (set of equations).

$$\frac{\partial \Pi}{\partial Q_1} = \frac{dR_1(Q_1)}{dQ_1} - \frac{dC(Q)}{dQ_1} = 0$$

$$\frac{\partial \Pi}{\partial Q_2} = \frac{dR_2(Q_2)}{dQ_2} - \frac{dC(Q)}{dQ_2} = 0$$

$$\frac{\partial \Pi}{\partial Q_3} = \frac{dR_3(Q_3)}{dQ_3} - \frac{dC(Q)}{dQ_3} = 0$$

3. Provide an economic interpretation to the firs order condition. Specifically, connect marginal revenues and demand elasticities to explain under what condition the frim will change a higher price.

$$MR_1(Q_1) = MC(Q)$$
$$MR_2(Q_2) = MC(Q)$$
$$MR_3(Q_3) = MC(Q)$$

Where: - $MR_i(Q_i) = \frac{dR_i}{dQ_i}$ is the marginal revenue in market i. - $MC(Q) = \frac{dC}{dQ}$ is the marginal cost of production, where $Q = Q_1 + Q_2 + Q_3$.

This implys that the firm will set the marginal revenue equal to marginal cost for each market.

4. Present the Hessian of the firm's objective function.

$$\Pi(Q_1, Q_2, Q_3) = R_1(Q_1) + R_2(Q_2) + R_3(Q_3) - C(Q_1 + Q_2 + Q_3)$$

The Hessian matrix H is given by:

$$H = \begin{bmatrix} \frac{\partial^2 \Pi}{\partial Q_1^2} & \frac{\partial^2 \Pi}{\partial Q_1 \partial Q_2} & \frac{\partial^2 \Pi}{\partial Q_1 \partial Q_3} \\ \frac{\partial^2 \Pi}{\partial Q_2 \partial Q_1} & \frac{\partial^2 \Pi}{\partial Q_2^2} & \frac{\partial^2 \Pi}{\partial Q_2 \partial Q_3} \\ \frac{\partial^2 \Pi}{\partial Q_3 \partial Q_1} & \frac{\partial^2 \Pi}{\partial Q_3 \partial Q_2} & \frac{\partial^2 \Pi}{\partial Q_3^2} \end{bmatrix}$$

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Where the second derivatives are:

$$\frac{\partial^2 \Pi}{\partial Q_i^2} = \frac{d^2 R_i(Q_i)}{dQ_i^2} - \frac{d^2 C(Q)}{dQ_i^2}$$

$$\frac{\partial^2 \Pi}{\partial Q_i \partial Q_j} = \frac{d^2 R_i(Q_i)}{dQ_i dQ_j} - \frac{d^2 C(Q)}{dQ_i dQ_j}$$

5. Assume each of the revenue function is concave and convex cost. Would this structure secure the second- order condition? Explain.
   (a) **Concave Revenue Functions:** If the revenue functions $R_1(Q_1)$, $R_2(Q_2)$, and $R_3(Q_3)$ are concave, their second derivatives will be non-positive. This implies that the marginal revenue functions are decreasing, which is necessary for profit maximization.
   (b) **Convex Cost Function:** If the cost function $C(Q)$ is convex, then its second derivative will be non-decreasing (positive), meaning the marginal cost increases with total output.

Problem 2

Let $U = U(x, y)$ be the utility function of the agent, x and y repesents the good. Assume positive marginal utilities. Let P_X, P_y be the associated prices and I income.

1. State the agent's utility maximization problem.

$$\max_{x,y} U(x, y)$$

   subject to the budget constraint:

$$P_x x + P_y y = I$$

   where $x, y \geq 0$ and $I, P_x, P_y > 0$.

2. Present the Langrangian funciton.

$$\mathcal{L}(x, y, \lambda) = U(x, y) + \lambda (I - P_x x - P_y y)$$

3. Derive and interpret the first order condition. In your analysis, you must include the Langrge multiplier.

$$\frac{\partial \mathcal{L}}{\partial x} = \frac{\partial U(x, y)}{\partial x} - \lambda P_x = 0$$

$$\frac{\partial U(x, y)}{\partial x} = \lambda P_x$$

$$\frac{\partial \mathcal{L}}{\partial y} = \frac{\partial U(x, y)}{\partial y} - \lambda P_y = 0$$

$$\frac{\partial U(x, y)}{\partial y} = \lambda P_y$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = I - P_x x - P_y y = 0$$

$$P_x x + P_y y = I$$

4. Present a graphical interpretation of the optimality condition (include indifferent curves and budget sets).

$$P_x x + P_y y = I$$
$$\text{MRS} = \frac{\partial U(x,y)/\partial x}{\partial U(x,y)/\partial y} = \frac{P_x}{P_y}$$

This is the optimality condition.

5. Present the second- order condition and descrive the condition under which one could secure a maximum.

6. Solve the previous questions but assuming:

   (a) $U(x,y) = x^a y^b$ where $a + b < 1$

$$\mathcal{L}(x,y,\lambda) = x^a y^b + \lambda(I - P_x x - P_y y)$$
$$a x^{a-1} y^b = \lambda P_x \quad \text{(FOC for } x\text{)}$$
$$b x^a y^{b-1} = \lambda P_y \quad \text{(FOC for } y\text{)}$$
$$\frac{a}{b} = \frac{P_x}{P_y}$$

   (b) $U(x,y) = ax + by$

$$\mathcal{L}(x,y,\lambda) = ax + by + \lambda(I - P_x x - P_y y)$$
$$a = \lambda P_x \quad \text{(FOC for } x\text{)}$$
$$b = \lambda P_y \quad \text{(FOC for } y\text{)}$$
$$\lambda = \frac{a}{P_x} = \frac{b}{P_y}$$

   (c) $U(x,y) = Min(x,y)$

$$\mathcal{L}(x,y,\lambda) = \min(x,y) + \lambda(I - P_x x - P_y y)$$
$$P_x x + P_y x = I$$