ECON 219: Problem Set #2

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Problem 1

Consider the National Income Model:

$$
\begin{aligned}
Y &= C + I_0 + G_0 \\
C &= \alpha + \beta(Y - T) \\
T &= \gamma + \delta Y
\end{aligned}
$$

1. Define and interpret each of the components of the model. Identify parameter and variables.
   (a) $Y = C + I_0 + G_0$ Nations income
   (b) $C = \alpha + \beta(Y - T)$ Consumtion function
   (c) $T = \gamma + \delta Y$ Tax Function
   (d) I, G investments and goverment spending are exogenous
   (e) α consumption intercept (minimum consumption when there is no income)
   (f) β Marginal propensity to consume
   (g) γ minimum taxes
   (h) δ tax rate on income
2. Impose reasonable assumptions on the signs and values of the parameters.
   (a) $\alpha > 0$ there is at least some consumption even if there is no income
   (b) $0 < \beta < 1$ hoesholds consume at least something and not all of their income
   (c) $\gamma \geq 0$ there is no negitive taxes
   (d) $I_0 > 0$ There is some investments
   (e) $G_0 > 0$ There is some goverment spending
3. Solve for the equilibrium income. Start from:

$$
\begin{aligned}
Y &= C + I_0 + G_0 \\
&= \alpha + \beta(Y - T) + I_0 + G_0 \\
&= \alpha + \beta(Y - \gamma - \delta Y) + I_0 + G_0 \\
&= \alpha + \beta Y - \beta\gamma - \beta\delta Y + I_0 + G_0 \\
&= Y(1 - \beta + \beta\delta) = \alpha - \beta\gamma + I_0 + G_0 \\
&= \frac{\alpha - \beta\gamma + I_0 + G_0}{1 - \beta(1 - \delta)}
\end{aligned}
$$

4. Obtain an discuss the six comparative-static derivatives. Let:

$$
Y = \frac{\alpha - \beta\gamma + I_0 + G_0}{1 - \beta(1 - \delta)}
$$

Partial derivatives:

Problem 1 continued on next page. . .

$$
\begin{aligned}
\frac{\partial Y}{\partial \alpha} &= \frac{1}{1-\beta(1-\delta)} > 0 \\
\frac{\partial Y}{\partial \gamma} &= \frac{-\beta}{1-\beta(1-\delta)} < 0 \\
\frac{\partial Y}{\partial I_0} &= \frac{1}{1-\beta(1-\delta)} > 0 \\
\frac{\partial Y}{\partial G_0} &= \frac{1}{1-\beta(1-\delta)} > 0 \\
\frac{\partial Y}{\partial \delta} &= \frac{\beta(\alpha-\beta\gamma+I_0+G_0)}{[1-\beta(1-\delta)]^2} > 0 \quad \text{(if numerator positive)} \\
\frac{\partial Y}{\partial \beta} &= \frac{-\gamma(1-\beta(1-\delta))+(\alpha-\beta\gamma+I_0+G_0)(1-\delta)}{[1-\beta(1-\delta)]^2} \quad \text{(ambiguous)}
\end{aligned}
$$

Summary of signs:

Parameter	Partial Derivative	Sign
α	$\frac{\partial Y}{\partial \alpha}$	Positive
β	$\frac{\partial Y}{\partial \beta}$	Ambiguous
γ	$\frac{\partial Y}{\partial \gamma}$	Negative
δ	$\frac{\partial Y}{\partial \delta}$	Positive (if numerator > 0)
I_0	$\frac{\partial Y}{\partial I_0}$	Positive
G_0	$\frac{\partial Y}{\partial G_0}$	Positive

Problem 2

Consider the market model:

$$
\begin{aligned}
Q_s &= Q_s \\
Q_s &= D(P, Y_0) \\
Q_d &= S(P)
\end{aligned}
$$

1. Provide an economic interpretation to each of the equations. In you answers, include the assumptions on the signs of the relevant derivatives.
2. Define the concept of market equilibrium. Provide and explain its mathematical formulation.
3. Show that:
$$\frac{dP^*}{dY_0} > 0$$
Where P^* is the equilibrium price. Provide an economic interpretation for this result.
4. Show that:
$$\frac{dQ^*}{dY_0} > 0$$
Where $Q*$ is the equilibrium price. Provide an economic interpretation for this result.
5. Answer questions c and d using total derivatives.