ECOG 219: Problem Set #1

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Problem 1

If the domain of the function $y = 5+ = x$ is the set $\{x|1 \geq x \geq 9\}$, find the range of the function and express it as a set.

$$\begin{aligned} DOM &= \{x|1 \geq x \geq 9\} \\ RANGE &= \{x|32 \geq x \geq 8\} \end{aligned}$$

Problem 2

In the theory of the firm, economists consider the total cost C to be a function of the output level $Q : C = f(Q)$.

1. According to the definition of a function, should each cost figure be associated with a unique level of output?
   Answer: Not nevesary since there could be multiple outputs (Q) that could be map to the same C but each Q can only map to one C.

2. Should each level of output determine a unique cost figure?
   Answer: Yes, given that each Q can only map to one C.

Problem 3

If an output level Q_1 can be produced at a cost of C_1 , then it must also be possible (by being less effcient) to produce Q_1 at a cost of C_1 + \$1, or C_1 + \$2, and so on. Thus it would seem that output Q does not uniquely determine total cost C. If so, to write $C = f(Q)$ would violate the deffnition of a function. How, in spite of the this reasoning, would you justify the use of the function $C = f(Q)$?
Answer: The function would still be valid since adding some cost constant $\forall a \in \mathbb{R}^+$ would only displace the cost function upwards by a units. A cost increas of a units could be described in the following method:

$$\begin{aligned} C &= f(Q) \\ C + a &= f(Q) + a; \forall a \in \mathbb{R}^+ \end{aligned}$$

Problem 4

Show that $X^{m/n} = \sqrt[n]{[x^m]} = (\sqrt[n]{[x]})^m$.Spevify the rules applied in each step.

$$\begin{aligned} x^{m/n} &= x^{m\frac{1}{n}} \\ &= (x^m)^{\frac{1}{n}} \\ &= \sqrt[n]{[x^m]} \end{aligned}$$

$$\begin{aligned} x^{m/n} &= x^{m\frac{1}{n}} \\ &= x^{\frac{1}{n}m} \\ &= (x^{\frac{1}{n}})^m \\ &= (\sqrt[n]{[x]})^m \end{aligned}$$

Problem 5

Prove Rule VI and Rule VII:

1. **Rule VI:** $(x^m)^n = x^{m \cdot n}$

   Proof. To prove that $(x^m)^n = x^{m \cdot n}$ is true we use the induction method. First, we prove that:

$$P_1 : (x^m)^1 \Rightarrow x^m = x^m$$

$$P_2 : (x^m)^2 \Rightarrow x^m \cdot x^m \Rightarrow x^{m+m} = x^{m \cdot 2}$$

$$P_n : (x^m)^n \Rightarrow x_1^m \cdot x_2^m \cdot ... \cdot x_n^m \Rightarrow x^{m+m+...+m} = x^{m \cdot n}$$

   Then we prove that P_1 hold for all P_{n+1}, that is:

$$P_{n+1} : (x^m)^n \Rightarrow x_1^m \cdot x_2^m \cdot ... \cdot x_n^m \cdot x_{n+1}^m$$

$$\Rightarrow x^{m \cdot n} \cdot x^m$$

$$\Rightarrow x^{m\dot{n}+m} = x^{m \cdot (n+1)}$$

   Therefore P_{n+1} holds for all positive integers n since $P_1 \implies P_{n+1}$. □

2. **Rule VII:** $x^m \cdot y^m = (x \cdot y)^m$

   Proof. To prove that $x^m \cdot y^m = (x \cdot y)^m$ is true we use the induction method. First, we prove that:

$$P_1 : x^1 \cdot y^1 \Rightarrow x \cdot y = x \cdot y$$

$$P_2 : x^2 \cdot y^2 \Rightarrow x \cdot y \cdot x \cdot y \Rightarrow (x \cdot y)^2 = (x \cdot y)^2$$

$$P_n : x^m \cdot y^m \Rightarrow x_1 \cdot y_1 \cdot x_2 \cdot y_2 \cdot ... \cdot x_n \cdot y_n \Rightarrow (x \cdot y)^m$$

   Then we prove that P_1 hold for all P_{n+1}, that is:

$$P_{n+1} : x^{m+1} \cdot y^{m+1} \Rightarrow x_1 \cdot y_1 \cdot x_2 \cdot y_2 \cdot ... \cdot x_n \cdot y_n \cdot x_{m+1} \cdot y_{m+1}$$

$$\Rightarrow (x \cdot y)^m \cdot (x \cdot y)_{m+1}$$

$$\Rightarrow (x \cdot y)^{m+1}$$

   Therefore P_{n+1} holds for all positive integers n since $P_1 \implies P_{n+1}$. □

Problem 6

Given the market model

$$Q_d = Q_s$$
$$Q_d = 21 - 3P$$
$$Q_s = -4 + 8P$$

Problem 6 continued on next page. . .

Find P^* and P^* by elimination of variables.

$$\begin{aligned}
Q_d &= Q_s \\
Q_d &= 21 - 3P \\
Q_s &= -4 + 8P \\
21 - 3P &= 3P + 8P \\
25 &= 11P \\
P^* &= \frac{25}{11} \\
Q^* &= \frac{156}{11}
\end{aligned}$$

Using the formaulas:

$$\begin{aligned}
P^* &= \frac{a+c}{b+d} \\
&= \frac{21+4}{3+8} \\
&= \frac{25}{11} \\
Q^* &= \frac{ad-bc}{b+d} \\
&= \frac{21 \cdot 8 - 3 \cdot 4}{3+8} \\
&= \frac{156}{11}
\end{aligned}$$

Problem 7

Find the equilibrion solution for each of the following models:

1.

$$\begin{aligned}
Q_d &= Q_s \\
Q_d &= 3 - P^2 \\
Q_s &= 6P - 4
\end{aligned}$$

Answer:

$$\begin{aligned}
Q_d &= Q_s \\
Q_d &= 3 - P^2 \\
Q_s &= 6P - 4 \\
3 - p^2 &= 6p - 4 \\
p^2 6p + 7 &= 0 \\
(p+7)(p-1) &= 0 \\
P &= 1 \\
Q* &= 2
\end{aligned}$$

Problem 7 continued on next page. . .

2.

$$
\begin{aligned}
Q_d &= Q_s \\
Q_d &= 8 - P^2 \\
Q_s &= P^2 - 2 \\
P^2 - 2 &= 8 - p^2 \\
2(P^2 - 5) &= 0 \\
P^* &= \sqrt{5} \\
Q* &= 3
\end{aligned}
$$

Problem 8

The market equilibrium condition, $Q_d = Q_s$, is often expressed in an equivalent alternative from, $Q_d - Q_s = 0$, which has the economiv interpretation "excess demand is zero". Does (3.7) represent this latter version of the equilibrium condition? if not, supply an appropriate economic interpretation for (3.7).

$$
\begin{aligned}
Q_d &= Q_s \\
Q_d - Q_s &= 0 \\
4 - p^2 - 4p + 1 &= 0 \\
p^2 + 4p - 5 &= 0
\end{aligned}
$$

Problem 9

The demand and supply function of a two-commodity market model are as follows:

$$
\begin{aligned}
Q_{d_1} &= 18 - 3P_1 + P_2 \\
Q_{d_2} &= 12 + P_1 - 2P_2 \\
Q_{s_1} &= -2 + 4P_1 \\
Q_{s_2} &= -2 + 3P_2 \\
-7p_1 + P_2 &= -20 \\
P_1 + 5P_2 &= -14 \\
\begin{bmatrix} -7 & 1 \\ 1 & -5 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \end{bmatrix} &= \begin{bmatrix} -20 \\ -14 \end{bmatrix} \\
&\left[\begin{array}{cc|c} -7 & 1 & -20 \\ 1 & -5 & -14 \end{array}\right] \\
&\left[\begin{array}{cc|c} 1 & -5 & -14 \\ 0 & 1 & \frac{59}{17} \end{array}\right] \\
&\left[\begin{array}{cc|c} 1 & 0 & \frac{57}{17} \\ 0 & 1 & \frac{59}{19} \end{array}\right]
\end{aligned}
$$

Problem 10

Let the national-income model be:

$$
\begin{aligned}
Y &= C + I_0 + G \\
C &= a + b(Y - T_0) \\
G &= gY
\end{aligned}
$$

1. Identify the endogenous variables.

   Answer: Y, C, G are endogenous.

2. Give the economic meaning of the parameter g.

   Answer:

3. Find the equilibrium national income.

   Answer:

4. What restriction on the parameters is needed for a solution to exist?

   Answer:

Problem 11

Rewrite the market model (3.12) in the format of (4.1) with the variables arranged in the following order: Q_{d1} , Q_{s1} , Q_{d2} , Q_{s2} , P_1 , P_2 . Write out the coefficient matrix, the variable vector, and the constant vector.

$$
\begin{bmatrix}
1 & -1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & -a_1 & -a_2 \\
0 & 1 & 0 & 0 & -b_1 & -b_2 \\
0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 & -\alpha_1 & -\alpha_2 \\
0 & 0 & 0 & 1 & \beta_1 & -\beta_2
\end{bmatrix}
\begin{bmatrix}
Q_{d1} \\ Q_{s1} \\ Q_{d2} \\ Q_{s2} \\ P_1 \\ P_2
\end{bmatrix}
\begin{bmatrix}
0 \\ a_0 \\ b_0 \\ 0 \\ \alpha_1 \\ \beta_1
\end{bmatrix}
$$

Problem 12

Problem 13

Problem 14

Problem 15

Problem 16

Problem 17

Problem 18

Problem 19

Problem 20