ECON 121: Problem Set #7

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Problem 1

Consider a competitive producer with a production function of $l^{0.4}k^{0.1}$, labor price of w and capital price of 1 (not v, the number one), and an output price of p. First suppose capital in the short run is fixed at k_1.

1. Find the short run cost function.

$$f(l,k) = l^{0.4}k^{0.1}$$
$$q = l^{0.4}k_1^{0.1} \Rightarrow l^{0.4} = \frac{q}{k_1^{0.1}} \Rightarrow l = \left(\frac{q}{k_1^{0.1}}\right)^{\frac{1}{0.4}} = \left(\frac{q}{k_1^{0.1}}\right)^{2.5}$$
$$C(q) = wl + 1 \cdot k_1$$
$$C(q) = w\left(\frac{q}{k_1^{0.1}}\right)^{2.5} + k_1$$
$$C(q) = wq^{2.5}k_1^{-0.25} + k_1$$

2. Use the cost function to find the profit maximizing quantity.

$$C(q) = wq^{2.5}k_1^{-0.25} + k_1$$
$$\pi(q) = pq - C(q) = pq - \left(wq^{2.5}k_1^{-0.25} + k_1\right)$$
$$\frac{d\pi}{dq} = p - \frac{d}{dq}\left(wq^{2.5}k_1^{-0.25}\right) = 0$$
$$\frac{d}{dq}\left(wq^{2.5}k_1^{-0.25}\right) = w \cdot 2.5q^{1.5}k_1^{-0.25}$$
$$p = 2.5wq^{1.5}k_1^{-0.25}$$
$$q^{1.5} = \frac{p}{2.5wk_1^{-0.25}} = \frac{pk_1^{0.25}}{2.5w} \Rightarrow q = \left(\frac{pk_1^{0.25}}{2.5w}\right)^{\frac{2}{3}}$$

3. Find the firm's unconditional demand for labor.

$$q = l^{0.4}k_1^{0.1} \Rightarrow l = \left(\frac{q}{k_1^{0.1}}\right)^{\frac{1}{0.4}} = \left(\frac{q}{k_1^{0.1}}\right)^{2.5}$$
$$q^* = \left(\frac{pk_1^{0.25}}{2.5w}\right)^{\frac{2}{3}}$$
$$l^* = \left(\frac{1}{k_1^{0.1}} \cdot \left(\frac{pk_1^{0.25}}{2.5w}\right)^{\frac{2}{3}}\right)^{2.5}$$
$$l^* = \left(\left(\frac{pk_1^{0.25}}{2.5w}\right)^{\frac{2}{3}} \cdot k_1^{-0.1}\right)^{2.5} = \left(\frac{p^{2/3}k_1^{0.25\cdot\frac{2}{3}}}{(2.5w)^{2/3}k_1^{0.1}}\right)^{2.5}$$
$$= \left(\frac{p^{2/3}}{(2.5w)^{2/3}} \cdot k_1^{\frac{0.5}{3}-0.1}\right)^{2.5} = \left(\frac{p^{2/3}}{(2.5w)^{2/3}} \cdot k_1^{\frac{1}{6}-\frac{1}{10}}\right)^{2.5}$$
$$\frac{1}{6} - \frac{1}{10} = \frac{5-3}{30} = \frac{2}{30} = \frac{1}{15}$$
$$l^* = \left(\frac{p^{2/3}}{(2.5w)^{2/3}} \cdot k_1^{1/15}\right)^{2.5} = \frac{p^{5/3}}{(2.5w)^{5/3}} \cdot k_1^{\frac{2.5}{15}} = \frac{p^{5/3}}{(2.5w)^{5/3}} \cdot k_1^{1/6}$$

Problem 1 continued on next page. . .

4. Find the profit function.

$$q^* = \left(\frac{pk_1^{0.25}}{2.5w}\right)^{\frac{2}{3}}, \quad C(q^*) = wq^{2.5}k_1^{-0.25} + k_1$$

$$pq^* = p\left(\frac{pk_1^{0.25}}{2.5w}\right)^{\frac{2}{3}} = p^{\frac{5}{3}} \cdot \left(\frac{k_1^{0.25}}{2.5w}\right)^{\frac{2}{3}}$$

$$q^{2.5} = \left(\left(\frac{pk_1^{0.25}}{2.5w}\right)^{\frac{2}{3}}\right)^{2.5} = \left(\frac{pk_1^{0.25}}{2.5w}\right)^{\frac{5}{3}}$$

$$C(q^*) = w\left(\frac{pk_1^{0.25}}{2.5w}\right)^{\frac{5}{3}} k_1^{-0.25} + k_1 = w \cdot \frac{p^{5/3}k_1^{(0.25)(5/3)}}{(2.5w)^{5/3}} \cdot k_1^{-0.25} + k_1$$

$$= \frac{p^{5/3}}{(2.5)^{5/3}w^{2/3}} \cdot k_1^{\frac{5}{12}-\frac{1}{4}} + k_1$$

$$= \frac{p^{5/3}}{(2.5)^{5/3}w^{2/3}} \cdot k_1^{\frac{5}{12}-\frac{3}{12}} + k_1 = \frac{p^{5/3}}{(2.5)^{5/3}w^{2/3}} \cdot k_1^{1/6} + k_1$$

$$\pi(p, w, k_1) = pq^* - C(q^*) = \frac{p^{5/3}}{(2.5)^{2/3}w^{2/3}}k_1^{1/6} - \left(\frac{p^{5/3}}{(2.5)^{5/3}w^{2/3}}k_1^{1/6} + k_1\right)$$

$$= \left(\frac{p^{5/3}}{(2.5)^{2/3}w^{2/3}} - \frac{p^{5/3}}{(2.5)^{5/3}w^{2/3}}\right)k_1^{1/6} - k_1$$

$$= \frac{p^{5/3}k_1^{1/6}}{w^{2/3}}\left(\frac{1}{(2.5)^{2/3}} - \frac{1}{(2.5)^{5/3}}\right) - k_1$$

Now suppose capital is variable (long run)

1. Repeat parts (a) though (d). The profit function is extra credit.
2. Extra credit. Capital remains unconstrained, but the firm has some market power such that $p = 10-2q$. Find the new optimal quantity and unconditional demand for labor.