

# **ECON 121: Final**

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## Problem 1

What is a mechanism? Who is the mechanism designer?

## Problem 2

Describe the concept of rationalizability.

## Problem 3

What is a dominant strategy?

## Problem 4

Solve the following game for its Nash Equilibrium/Equilibria.

	Crime	Action	Thriller	Comedy	Doc.
Crime	(10, 10)	(4, 3)	(4, 2)	(4, 1)	(4, -10)
Action	(3, 4)	(9, 9)	(3, 2)	(3, 1)	(3, -10)
Thriller	(2, 4)	(2, 3)	(8, 8)	(2, 1)	(2, -10)
Comedy	(1, 4)	(1, 3)	(1, 2)	(5, 5)	(1, -10)
Doc.	(-10, 4)	(-10, 3)	(-10, 2)	(-10, 1)	(-10, 10)

## Problem 5

You are in charge of regulating a local utility that runs effectively as a monopoly. The utility just recently had a potential entrant, and in response it lowered its prices substantially (and temporarily). The potential entrant was scared away before entering at all. If, when you ask the utility to lower its prices, it complains its costs are high, why are you not convinced?

## Problem 6

Two firms (1 and 2) compete on price with product differentiation such that demand for firm  $i$  is  $q_i = 10 - p_i + p_j$ . They incur costs  $c_i q_i$ , where  $c_i$  is “low enough” to encourage market participation. All parameters are positive, and costs ( $c_1 c_2$ ) are firm dependent.

- Find the prices  $p_1 p_2$  that occur in equilibrium in the static game.
- For part (b), assume  $c_2 = 30$ . Suppose firm 1 invests in a technology that determines  $c_1$  paying an investment cost equal to  $K - 20c_1$ , where  $K$  is some constant, and then the firms simultaneously compete on price. Firm 1’s technology investment occurs first, viewed by both firms. What “taxonomy” strategy will firm 1 employ? Why? Hint: What does lowering  $c_1$  do to the behavior of firm 2? Find the unique SPNE prices and  $c_1$ .
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## Problem 7

Consider the following two stage games, the Prisoner's Dilemma (PD, left) and the Revenge game (R, right). When relevant, the discount factor is  $\delta, 0 < \delta < 1$ .

	$m$	$f$
$M$	$(4, 4)$	$(-1, 5)$
$F$	$(5, -1)$	$(1, 1)$

	$l$	$g$
$L$	$(0, 0)$	$(-4, -1)$
$G$	$(-1, -4)$	$(-3, -3)$

- Take both games separately as static games. Solve for all pure strategy Nash equilibria (NE) for PD, and then do the same for R.
- Suppose the players play a three-stage game in the following order: R, PD, PD. Find the two pure strategy subgame perfect Nash equilibria (SPNE).
- Suppose the players play a two-stage game in the following order: PD, R. Find a pure strategy SPNE that supports the play of (M,m) in the first round for high enough  $\delta$ . Find the lowest  $\delta$  that supports the SPNE you found in part (c).

## Problem 8

Consider a symmetric independent private values setting with two buyers. The seller owns an indivisible object which she is commonly known to value at zero. Suppose that  $v_i \in \{1, 2, 3\}$  for  $i = 1, 2$ , and that each realization is equally likely (i.e., occurs with the probability  $1/3$ ). Suppose also that  $S_i = NO, 1, 2, 3$  for  $i = 1, 2$ . That is a bidder may choose not to participate or he may bid one of his three possible valuations. Finally, suppose that ties in bidding are broken randomly without bias.

- Suppose the seller announces a second-price sealed bid auction with reserve price of 1. Find a symmetric Bayesian Nash Equilibrium (BNE),  $b^*(v)$ , in which the bidders play weakly dominant strategies. How much expected revenue is generated in this BNE?
- Suppose the seller announces a second-price sealed bid auction with reserve price of 1. Find an asymmetric BNE,  $(b_1(v_1), b_2(v_2))$ .
- Suppose the seller announces a first-price sealed-bid auction with reserve price of 1. Find the symmetric BNE,  $b^{**}(v)$  which generates the highest expected revenue. How much revenue is generated in the BNE?
- Suppose the seller announces an all-pay auction with a reserve price of 1. Find a symmetric BNE,  $b^{***}(v)$ . How much expected revenue is generated in this BNE?

## Problem 9

Let  $U(x, y) = \sqrt{x} + \sqrt{y}$

- Marshallian demand functions for  $x$  and  $y$ .
- Indirect utility function.
- Compensated (Hicksian) demand functions for  $x$  and  $y$ .
- Expenditure function.

5. Write the Slutsky equation for good  $x$ .
6. Explain, in words, how you could have derived Marshallian demand in a different way.
7. Explain, in words, how you could have derived Hicksian demand in a different way.

## Problem 10

A seller sells a good to a prospective buyers. The buyer values the good at  $\theta q$ , Where  $\theta$  is his (privately known) marginal utility of quality and  $q$  is the good's quality. It is common knowledge that  $\theta$  is high ( $\theta = 2$ ) with a probability of  $\frac{1}{4}$  and  $\theta$  is low ( $\theta = 1$ ) with probability of  $\frac{3}{4}$ . The monopolist incurs a cost based on quality  $c(q) = \frac{q}{2}$  so that his profits is  $q - \frac{q^2}{2}$ . The buyer can reject an offer (not buy anything) and get a payoff of 0, or he can buy a good and get a payoff of  $\theta q - p$ . The seller offer a menu consisting of  $\{p_1, q_1, p_2, q_2\}$ , where the subscript means the price and quality is meant for the seller of that type ( $\theta$ ), and the buyer picks which good she wants.

- (a) Suppose, for part (a) only, the seller observe  $\theta$  directly and can offer a single type of good based on the buyer's type  $\{p_i, q_i\}$ ,  $i \in [1, 2]$ . Construct the optimal price/quality combination when  $\theta = 1$  and when  $\theta = 2$ .
- (b) Suppose the seller ask the buyer what his type is, assume he answers honestly, and offer and contract like in part (a). Who will lie, and why?
- (c) Construct the optimal contract where everyone buys their appropriate good.
- (d) Whose good has an inefficient level of quality? Is it too high or low? Why does the seller do this?