ECON 219: Midterm #1

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Problem 1

Let $U(x,y) = \sqrt{x} + \sqrt{y}$. Find the following:

1. Marshallian demand function for x and y.

$$\mathcal{L} = \sqrt{x} + \sqrt{y} + \lambda(I - p_x x - p_y y)$$

$$\frac{1}{2\sqrt{x}} = \lambda p_x, \quad \frac{1}{2\sqrt{y}} = \lambda p_y$$

$$\frac{1}{\sqrt{x}p_x} = \frac{1}{\sqrt{y}p_y} \Rightarrow \frac{\sqrt{y}}{\sqrt{x}} = \frac{p_y}{p_x} \Rightarrow \frac{y}{x} = \left(\frac{p_y}{p_x}\right)^2 \Rightarrow y = x\left(\frac{p_y}{p_x}\right)^2$$

$$p_x x + p_y y = I \Rightarrow p_x x + p_y x\left(\frac{p_y}{p_x}\right)^2 = I \Rightarrow x\left(p_x + \frac{p_y^3}{p_x^2}\right) = I$$

$$x^*(p_x, p_y, I) = \frac{I}{p_x + \frac{p_y^3}{p_x^2}} = \frac{Ip_x^2}{p_x^3 + p_y^3}$$

$$y^*(p_x, p_y, I) = x\left(\frac{p_y}{p_x}\right)^2 = \frac{Ip_y^2}{p_x^3 + p_y^3}$$

2. Indirect utility function.

$$v(p_x, p_y, I) = \sqrt{x^*} + \sqrt{y^*} = \sqrt{\frac{Ip_x^2}{p_x^3 + p_y^3}} + \sqrt{\frac{Ip_y^2}{p_x^3 + p_y^3}}$$

$$= \sqrt{\frac{I}{p_x^3 + p_y^3}}(p_x + p_y)$$

3. Expenditure function.

$$u = \sqrt{\frac{I}{p_x^3 + p_y^3}}(p_x + p_y) \Rightarrow \frac{I}{p_x^3 + p_y^3} = \left(\frac{u}{p_x + p_y}\right)^2 \Rightarrow I = (p_x^3 + p_y^3)\left(\frac{u}{p_x + p_y}\right)^2$$

$$e(p_x, p_y, u) = (p_x^3 + p_y^3)\left(\frac{u}{p_x + p_y}\right)^2$$

4. Write the Slutsky equation for good x.

$$\frac{\partial x^*}{\partial p_x} = \frac{\partial h_x}{\partial p_x} - \frac{\partial x^*}{\partial I} \cdot x^*$$

$$x^* = \frac{Ip_x^2}{p_x^3 + p_y^3} \Rightarrow \frac{\partial x^*}{\partial I} = \frac{p_x^2}{p_x^3 + p_y^3}$$

$$\frac{\partial x^*}{\partial p_x} = \frac{\partial h_x}{\partial p_x} - \frac{Ip_x^4}{(p_x^3 + p_y^3)^2}$$

Problem 2

Determine the marginal rate of substitution (MRS) for the following utility functions:

1. $U(x, y) = X^{1/4} + 3y$

$$MU_x = \frac{\partial U}{\partial x} = \frac{1}{4}x^{-3/4}$$

$$MU_y = \frac{\partial U}{\partial y} = 3$$

$$\text{MRS}_{xy} = -\frac{MU_x}{MU_y} = -\frac{\frac{1}{4}x^{-3/4}}{3} = -\frac{1}{12}x^{-3/4}$$

2. $U(x, y) = X^{1/4} + y^{1/4}$

$$MU_x = \frac{\partial U}{\partial x} = \frac{1}{4}x^{-3/4}$$

$$MU_y = \frac{\partial U}{\partial y} = \frac{1}{4}y^{-3/4}$$

$$\text{MRS}_{xy} = -\frac{MU_x}{MU_y} = -\frac{\frac{1}{4}x^{-3/4}}{\frac{1}{4}y^{-3/4}} = -\left(\frac{y}{x}\right)^{3/4}$$

Problem 3

What is the effect of imposing a \$10,000 lump-sum tax on a monopolist? Assume the firm continues to operate after the tax.

1. The firm experiences no change in the profit-maximizing price or quantity or in its profit.
2. **The firm experiences no change in the profit-maximizing price and quantity, but its profit decreases**
3. The firm experiences no change in the profit-maximizing price and quantity, but its profit decreases
4. The firm's profit-maximizing quantity increases, but price does not change.
5. The firm's profit-maximizing price decreases, but quantity does not change.

Problem 4

Consider the production function $q = f(k, l) = l^{0.5}k^{0.1}$. The cost of labor and capital are w and v respectively. Aside from the cost of labor and capital, there there are no cost. Initially assume k is fixed at k_1.

1. Is this function constant return to scale, increasing returns to scale, or decreasing returns to scale? Explain.

$$f(tk, tl) = (tl)^{0.5}(tk)^{0.1} = t^{0.5}l^{0.5} \cdot t^{0.1}k^{0.1} = t^{0.6}f(k, l)$$

Since $t^{0.6} < t$, the production function exhibits:

Decreasing returns to scale

2. Find the short run cost function $C_{SR(q)}$

$$q = l^{0.5}k_1^{0.1} \Rightarrow l = \left(\frac{q}{k_1^{0.1}}\right)^2$$

$$C_{SR}(q) = w \cdot l + v \cdot k_1 = \frac{wq^2}{k_1^{0.2}} + vk_1$$

3. Find short run marginal cost.

$$MC_{SR} = \frac{dC_{SR}}{dq} = \frac{2wq}{k_1^{0.2}}$$

4. Find short run average cost.

$$AC_{SR} = \frac{C_{SR}(q)}{q} = \frac{wq}{k_1^{0.2}} + \frac{vk_1}{q}$$

5. Demostrare that cost is minimized when $AC = MC$.

$$\frac{2wq}{k_1^{0.2}} = \frac{wq}{k_1^{0.2}} + \frac{vk_1}{q} \Rightarrow \frac{wq}{k_1^{0.2}} = \frac{vk_1}{q}$$

$$vk_1 = \frac{wq^2}{k_1^{0.2}}$$

So cost is minimized when $MC = AC$.

Now assume all inputs are variables.

6. Find the long run cost function $C_{LR(q)}$.

$$\min_{k,l} wl + vk \quad \text{subject to} \quad q = l^{0.5}k^{0.1}$$

$$\mathcal{L} = wl + vk + \lambda(q - l^{0.5}k^{0.1})$$

$$\frac{k}{l} = \frac{w}{5v} \Rightarrow k = \frac{w}{5v}l$$

$$q = l^{0.6}\left(\frac{w}{5v}\right)^{0.1} \Rightarrow l = q^{\frac{1}{0.6}}\left(\frac{w}{5v}\right)^{-\frac{1}{6}}$$

$$C_{LR}(q) = wl + vk = \left(\frac{6w}{5}\right)l = \left(\frac{6w}{5}\right)q^{5/3}\left(\frac{w}{5v}\right)^{-1/6}$$

7. Find long run marginal cost.

$$MC_{LR} = \frac{dC_{LR}}{dq} = \frac{5}{3}\left(\frac{6w}{5}\right)\left(\frac{w}{5v}\right)^{-1/6}q^{2/3}$$

8. Use you answer from part (a) to find the elasticity of substitution

$$\sigma = 1$$

Problem 5

A monopolist has a cost function $C(q) = \frac{q^2}{2}$. The monopolist faces a market demand curve given $q = 10 - p$.

1. Find marginal cost. The marginal cost is the derivative of the cost function with respect to quantity q:

$$MC = \frac{dC}{dq} = \frac{d}{dq}\left(\frac{q^2}{2}\right) = q.$$

2. Calculate the profit-maximizing price-quantity combination for the monopolist. Also calculate the monopolist's profits. From the demand curve:

$$p = 10 - q.$$

$$TR = p \times q = (10 - q)q = 10q - q^2.$$

$$TC = \frac{q^2}{2}.$$

$$\pi(q) = TR - TC = 10q - q^2 - \frac{q^2}{2} = 10q - \frac{3q^2}{2}.$$

$$\frac{d\pi}{dq} = 10 - 3q = 0 \implies q^* = \frac{10}{3}.$$

$$p^* = 10 - \frac{10}{3} = \frac{20}{3}.$$

$$\pi^* = 10 \times \frac{10}{3} - \frac{3}{2} \times \left(\frac{10}{3}\right)^2 = \frac{100}{3} - \frac{3}{2} \times \frac{100}{9} = \frac{100}{3} - \frac{50}{3} = \frac{50}{3}.$$

3. What output level would be produced by this industry under perfect competition? Under perfect competition, firms produce where price equals marginal cost:

$$p = MC.$$

$$10 - q = q \implies 2q = 10 \implies q = 5.$$

4. Calculate the consumer surplus obtained in case (c). Show that this exceeds the sum of monopolist's profit and the consumer surplus received in case (b) What is the value of the dead weight loss from the monopolization?

   instruction are not clear clould not calculate it. The surplus does not exceeds consumer surpluse

Problem 6

If the value of th eprice elasticity of demand is 0.2, this means that

1. a 20% decreases in price cause a 1% increase in quantity demanded.
2. **a 5% decrease in price causes a 1% increase in quantity demanded**
3. a 0.2% decrease in price causes a 0.2% increase in quantity demanded.
4. a 100% decrease in price cause a 200% increase in quantity demanded.

Problem 7

Suppose the production possibility frontier for an economy that produces on public and one private good (x) is given by

$$x^2 + 100y^2 = 5000$$

This economy is populated by 100 identical individuals, each with a utility function of the from

$$U(x, y) = \sqrt{xy}$$

Where x_i is the individual's share of private good production (=x/100). Notice that the public good is nonexclusive and that everyone benefits equally from its level of production.

1. If the market for x and y were perfectly competitive, what levels of those goods would be produced? What would the typical individual's utlitty be in this situation? In a perfectly competitive market, the public good is typically underprovided due to the free-rider problem. Thus, individuals will choose:

$$y = 0$$

$$x^2 + 100 \cdot 0^2 = 5000 \implies x = \sqrt{5000} \approx 70.71$$
$$x_i = \frac{x}{100} \approx \frac{70.71}{100} = 0.7071$$
$$U_i = \sqrt{x_i y} = \sqrt{0.7071 \times 0} = 0$$

2. What are the optimal production levels for x and y? What would the typical individual's utility level be? (Hint: The numbers in this problem do not come out evenly, and some approximations should suffice.) The planner maximizes utility subject to the PPF:

$$\max_{x,y} U = \sqrt{\frac{x}{100}y}$$

$$x^2 + 100y^2 = 5000$$

$$\mathcal{L} = \sqrt{\frac{x}{100}y} + \lambda(5000 - x^2 - 100y^2)$$

$$\frac{\partial \mathcal{L}}{\partial x} = \frac{1}{2} \cdot \frac{1}{\sqrt{\frac{x}{100}y}} \cdot \frac{y}{100} - 2\lambda x = 0$$

$$\frac{\partial \mathcal{L}}{\partial y} = \frac{1}{2} \cdot \frac{1}{\sqrt{\frac{x}{100}y}} \cdot \frac{x}{100} - 200\lambda y = 0$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 5000 - x^2 - 100y^2 = 0$$

$$\sqrt{\frac{x}{100}y} = \frac{\sqrt{xy}}{10} \implies \frac{1}{\sqrt{\frac{x}{100}y}} = \frac{10}{\sqrt{xy}}$$

$$\frac{1}{2} \cdot \frac{10}{\sqrt{xy}} \cdot \frac{y}{100} = 2\lambda x \implies \frac{5y}{100\sqrt{xy}} = 2\lambda x \implies \frac{y}{20\sqrt{xy}} = 2\lambda x$$

$$\frac{1}{2} \cdot \frac{10}{\sqrt{xy}} \cdot \frac{x}{100} = 200\lambda y \implies \frac{5x}{100\sqrt{xy}} = 200\lambda y \implies \frac{x}{20\sqrt{xy}} = 200\lambda y$$

$$\frac{\frac{y}{20\sqrt{xy}}}{\frac{x}{20\sqrt{xy}}} = \frac{2\lambda x}{200\lambda y} \implies \frac{y}{x} = \frac{x}{100y}$$

$$100y^2 = x^2 \implies x = 10y$$

$$(10y)^2 + 100y^2 = 5000 \implies 100y^2 + 100y^2 = 5000 \implies 200y^2 = 5000 \implies y^2 = 25 \implies y = 5$$

$$x = 10y = 50$$

$$U_i = \sqrt{\frac{x}{100}y} = \sqrt{\frac{50}{100} \times 5} = \sqrt{2.5} \approx 1.58$$

Problem 8

Which of the following is true of perfect price discrimination?

1. Profit is lower than it would be without discrimination.
2. **Revenue is higher then it would be without discrimination.**
3. (Hint: The numbers in this problem do not come out evenly, and some approximations should suffice.)
4. Profit is zero

Problem 9

If fixed cost at $Q = 100$ is \$130, then

1. fixed cost $Q = 0$ is \$0
2. fixed cost at Q = 0 is less than \$130.
3. fixed cost at Q = 200 is \$260.
4. **fixed cost at Q = 200 is \$130.**
5. it is impossible to calculate fixed costs at any other quantity.