ECON 121: Problem Set #7

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Problem 1

A representative consumer has a utility function $U(x, y) = xy$. A representative firm makes good x and has a production function $x = f(k, l) = (kl)^{0.25}$ and an unavoidable fixed cost equal to A. There are 100 consumers and, initially, 100 firms. Prices are $w = v = P_y = 1$, and P_x is determined in a competitive market. Representative consumer income is $I = 2$. In the short run, the number of firms is fixed, and capital is fixed at $k_1 = 1$.

1. Find the representative individual's Marshallian demand for good x.

$$U(x, y) = xy$$

$$P_x x + P_y y = I,$$

$$P_x x + y = 2.$$

$$\max_{x,y} \; xy \quad \text{subject to} \quad P_x x + y = 2.$$

$$\mathcal{L} = xy + \lambda(2 - P_x x - y)$$

$$\frac{\partial \mathcal{L}}{\partial x} = y - \lambda P_x = 0 \quad \Rightarrow \quad \lambda = \frac{y}{P_x}$$

$$\frac{\partial \mathcal{L}}{\partial y} = x - \lambda = 0 \quad \Rightarrow \quad \lambda = x$$

$$x = \frac{y}{P_x} \quad \Rightarrow \quad y = P_x x$$

$$P_x x + y = 2 \Rightarrow P_x x + P_x x = 2 \Rightarrow 2P_x x = 2 \Rightarrow x = \frac{1}{P_x}$$

$$x(P_x, I) = \frac{1}{P_x}$$

2. Find the representative firm's supply for good x.

$$x = f(k, l) = (kl)^{0.25}.$$

$$k = k_1 = 1.$$

$$\pi = P_x x - wl - A,$$

$$\pi = P_x l^{0.25} - l - A.$$

$$\frac{d\pi}{dl} = P_x \cdot 0.25\, l^{-0.75} - 1 = 0.$$

$$0.25 P_x l^{-0.75} = 1 \implies l^{-0.75} = \frac{4}{P_x} \implies l^{0.75} = \frac{P_x}{4}.$$

$$l = \left(\frac{P_x}{4}\right)^{\frac{4}{3}}.$$

$$x = l^{0.25} = \left[\left(\frac{P_x}{4}\right)^{\frac{4}{3}}\right]^{0.25} = \left(\frac{P_x}{4}\right)^{\frac{1}{3}}.$$

$$x(P_x) = \left(\frac{P_x}{4}\right)^{\frac{1}{3}}.$$

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3. Find total demand and total supply for good x.

$$x(P_x, I) = \frac{1}{P_x}.$$

$$X_d = 100 \cdot x(P_x, I) = 100 \cdot \frac{1}{P_x}.$$

$$x(P_x) = \left(\frac{P_x}{4}\right)^{\frac{1}{3}}.$$

$$X_s = 100 \cdot x(P_x) = 100 \cdot \left(\frac{P_x}{4}\right)^{\frac{1}{3}}.$$

4. Solve for the equilibrium price and quantity of good x.

$$X_d = X_s.$$

$$X_d = 100 \cdot \frac{1}{P_x}, \quad X_s = 100 \cdot \left(\frac{P_x}{4}\right)^{\frac{1}{3}}.$$

$$100 \cdot \frac{1}{P_x} = 100 \cdot \left(\frac{P_x}{4}\right)^{\frac{1}{3}}.$$

$$\frac{1}{P_x} = \left(\frac{P_x}{4}\right)^{\frac{1}{3}}.$$

$$\left(\frac{1}{P_x}\right)^3 = \frac{P_x}{4}.$$

$$\frac{1}{P_x^3} = \frac{P_x}{4}.$$

$$1 = \frac{P_x^4}{4},$$

$$P_x^4 = 4,$$

$$P_x = \sqrt[4]{4} = 2^{1/2} = \sqrt{2}.$$

$$P_x = \sqrt{2}.$$

$$x = \frac{1}{P_x} = \frac{1}{\sqrt{2}}.$$

$$x = \frac{1}{\sqrt{2}}.$$

5. Find total producer surplus in the short run.

$$\pi = P_x x - wl - A.$$

$$\pi = P_x \left(\frac{P_x}{4}\right)^{\frac{1}{3}} - l - A.$$

$$TR = P_x x = P_x \left(\frac{P_x}{4}\right)^{\frac{1}{3}}.$$

$$TVC = wl = l.$$

$$TVC = \left(\frac{P_x}{4}\right)^{\frac{4}{3}}.$$

$$PS = TR - TVC - A.$$

$$PS_{\text{total}} = 100 \cdot PS.$$

In the long run, the number of firms is M (determined endogenously), and k is a variable.

1. First, assume M stays at 100, and find equilibrium price and quantity, re-doing whatever parts are necessary now that capital isn't fixed.

$$x = f(k,l) = (kl)^{0.25}.$$
$$\pi = P_x x - wl - A.$$
$$\pi = P_x (kl)^{0.25} - wl - A.$$
$$\frac{d\pi}{dl} = P_x \cdot 0.25 \cdot (kl)^{-0.75} \cdot k - w = 0.$$
$$P_x \cdot 0.25 \cdot kl^{-0.75} = w,$$
$$l^{-0.75} = \frac{4w}{P_x k},$$
$$l = \left(\frac{P_x k}{4w}\right)^{\frac{4}{3}}.$$
$$x = (kl)^{0.25}.$$
$$x = \left[k\left(\frac{P_x k}{4w}\right)^{\frac{4}{3}}\right]^{0.25}.$$
$$x = k^{0.25} \cdot \left(\frac{P_x k}{4w}\right)^{\frac{1}{3}}.$$
$$x = k^{0.25} \cdot \left(\frac{P_x}{4w}\right)^{\frac{1}{3}} \cdot k^{\frac{1}{3}} = k^{\frac{1}{2}} \cdot \left(\frac{P_x}{4w}\right)^{\frac{1}{3}}.$$
$$X_s = 100 \cdot x = 100 \cdot k^{\frac{1}{2}} \cdot \left(\frac{P_x}{4w}\right)^{\frac{1}{3}}.$$
$$X_d = 100 \cdot \frac{1}{P_x}.$$
$$X_d = X_s.$$
$$100 \cdot \frac{1}{P_x} = 100 \cdot k^{\frac{1}{2}} \cdot \left(\frac{P_x}{4w}\right)^{\frac{1}{3}}.$$
$$\frac{1}{P_x} = k^{\frac{1}{2}} \cdot \left(\frac{P_x}{4w}\right)^{\frac{1}{3}}.$$
$$\left(\frac{1}{P_x}\right)^3 = k^{\frac{3}{2}} \cdot \frac{P_x}{4w}.$$
$$\frac{1}{P_x^3} = k^{\frac{3}{2}} \cdot \frac{P_x}{4w},$$
$$\frac{1}{P_x^4} = \frac{k^{\frac{3}{2}}}{4w},$$
$$P_x^4 = 4wk^{\frac{3}{2}},$$
$$P_x = \sqrt[4]{4wk^{\frac{3}{2}}}.$$
$$P_x = \sqrt[4]{4wk^{\frac{3}{2}}}.$$
$$x = \frac{1}{P_x} = \frac{1}{\sqrt[4]{4wk^{\frac{3}{2}}}}.$$
$$x = \frac{1}{\sqrt[4]{4wk^{\frac{3}{2}}}}.$$

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2. Suppose $A = 2$. Based on economic profit, will M increase or decrease in the long run?

$$\pi_{\text{economic}} = \text{Total Revenue} - \text{Total Costs}.$$

$$TR = P_x x,$$

$$TC = wl + A.$$

$$\pi_{\text{economic}} = P_x x - wl - A.$$

$$TR = P_x \cdot k^{\frac{1}{2}} \cdot \left(\frac{P_x}{4w}\right)^{\frac{1}{3}}.$$

$$TC = w \cdot \left(\frac{P_x k}{4w}\right)^{\frac{4}{3}} + A.$$

$$\pi_{\text{economic}} = P_x \cdot k^{\frac{1}{2}} \cdot \left(\frac{P_x}{4w}\right)^{\frac{1}{3}} - w \cdot \left(\frac{P_x k}{4w}\right)^{\frac{4}{3}} - A.$$

$$\pi_{\text{economic}} = 0.$$

$$P_x x = wl + A.$$

3. Give $A = 1$, find the long run M, X, and P_x. (Hint: re-do long run supply in terms of M instead of 100, find new equilibrium in terms of M, and then use the profit condition).

$$x = k^{\frac{1}{2}} \cdot \left(\frac{P_x}{4w}\right)^{\frac{1}{3}}.$$

$$X_s = M \cdot x = M \cdot k^{\frac{1}{2}} \cdot \left(\frac{P_x}{4w}\right)^{\frac{1}{3}}.$$

$$\pi_{\text{economic}} = 0 \quad \Rightarrow \quad P_x x = wl + A.$$

$$P_x x = wl + 1.$$

$$P_x \cdot 0.25 \cdot kl^{-0.75} = w.$$

$$l = \left(\frac{P_x k}{4w}\right)^{\frac{4}{3}}.$$

$$P_x x = w \cdot \left(\frac{P_x k}{4w}\right)^{\frac{4}{3}} + 1.$$

$$P_x \cdot k^{\frac{1}{2}} \cdot \left(\frac{P_x}{4w}\right)^{\frac{1}{3}} = w \cdot \left(\frac{P_x k}{4w}\right)^{\frac{4}{3}} + 1.$$

$$X_d = 100 \cdot \frac{1}{P_x}.$$

$$X_s = X_d \quad \Rightarrow \quad M \cdot k^{\frac{1}{2}} \cdot \left(\frac{P_x}{4w}\right)^{\frac{1}{3}} = 100 \cdot \frac{1}{P_x}.$$

$$M \cdot k^{\frac{1}{2}} \cdot \left(\frac{P_x}{4w}\right)^{\frac{1}{3}} = 100 \cdot \frac{1}{P_x}.$$

$$X_s = M \cdot x.$$

Problem 2

Suppose we are in the long run equilibrium determined by part(h). Note that this long run equilibrium represents a particular short run equilibrium where no firms has an incentive to change k, and no firm has an incentive to enter or exit.

1. Suppose representative consumer income increases form 2 to 4. Find the new short run equilibrium price and quantity where k is fixed at the value determined by part h.

$$x(P_x, I) = \frac{I}{P_x}.$$

$$x(P_x, 4) = \frac{4}{P_x}.$$

$$X_d = 100 \cdot \frac{4}{P_x} = \frac{400}{P_x}.$$

$$x(P_x) = k^{\frac{1}{2}} \cdot \left(\frac{P_x}{4w}\right)^{\frac{1}{3}}.$$

$$X_s = 100 \cdot k^{\frac{1}{2}} \cdot \left(\frac{P_x}{4}\right)^{\frac{1}{3}}.$$

$$\frac{400}{P_x} = 100 \cdot k^{\frac{1}{2}} \cdot \left(\frac{P_x}{4}\right)^{\frac{1}{3}}.$$

$$\frac{4}{P_x} = k^{\frac{1}{2}} \cdot \left(\frac{P_x}{4}\right)^{\frac{1}{3}}.$$

$$\left(\frac{4}{P_x}\right)^3 = k^{\frac{3}{2}} \cdot \frac{P_x}{4}.$$

$$\frac{64}{P_x^3} = k^{\frac{3}{2}} \cdot \frac{P_x}{4}.$$

$$\frac{64}{P_x^4} = \frac{k^{\frac{3}{2}}}{4},$$

$$P_x^4 = 256 \cdot k^{\frac{3}{2}},$$

$$P_x = \sqrt[4]{256 \cdot k^{\frac{3}{2}}}.$$

$$P_x = \sqrt[4]{256} = 4.$$

$$X_d = \frac{400}{4} = 100.$$

2. Following (i), find the new long run equilibrium price and quantity where k and M are variable.

$$x(P_x, 4) = \frac{4}{P_x}.$$

$$X_d = 100 \cdot \frac{4}{P_x} = \frac{400}{P_x}.$$

$$x(P_x) = k^{\frac{1}{2}} \cdot \left(\frac{P_x}{4w}\right)^{\frac{1}{3}}.$$

$$X_s = M \cdot k^{\frac{1}{2}} \cdot \left(\frac{P_x}{4}\right)^{\frac{1}{3}}.$$

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$$P_x \cdot x = w \cdot l + A.$$

$$x = k^{\frac{1}{2}} \cdot \left(\frac{P_x}{4}\right)^{\frac{1}{3}}.$$

$$TR = P_x \cdot x = P_x \cdot k^{\frac{1}{2}} \cdot \left(\frac{P_x}{4}\right)^{\frac{1}{3}}.$$

$$TC = w \cdot l + A.$$

$$P_x \cdot k^{\frac{1}{2}} \cdot \left(\frac{P_x}{4}\right)^{\frac{1}{3}} = w \cdot l + A.$$

3. Income is 2(so, back to part h). Suppose the government taxes sellers \$1 per unit. what is the new long run equilibrium quantity?

   A \$1 per unit tax on sellers will affect the cost structure of the firms. The total cost function becomes:

$$TC = w \cdot l + A + 1 \cdot x.$$