ECON 219: Problem Set #5

Due on May 29, 2025

Dr. Sergio Urzua

Alejandro Ouslan

Problem 1

Suppose that a firm's fixed production function is given by

$$q = \min(5k, 10l)$$

1. Calculate the firm's long-run total, average, and marginal cost function.

$$q = \min(5k, 10l) \Rightarrow 5k = 10l \Rightarrow k = 2l$$

$$q = 10l \Rightarrow l = \frac{q}{10}, \quad k = \frac{q}{5}$$

$$C(q) = vk + wl = v \cdot \frac{q}{5} + w \cdot \frac{q}{10} = \left(\frac{v}{5} + \frac{w}{10}\right) q$$

Therefore:

$$\text{Total Cost (TC)} : C(q) = \left(\frac{v}{5} + \frac{w}{10}\right) q$$
$$\text{Average Cost (AC)} : AC(q) = \frac{C(q)}{q} = \frac{v}{5} + \frac{w}{10}$$
$$\text{Marginal Cost (MC)} : MC(q) = \frac{dC}{dq} = \frac{v}{5} + \frac{w}{10}$$

2. Suppose that k is fixed at 10 in the short run. Calculate the firm's short run total, average, and marginal cost function.

$$q = \min(5 \cdot 10, 10l) = \min(50, 10l) \Rightarrow q \leq 50, \quad l \geq \frac{q}{10}$$

$$C_{\text{SR}}(q) = 10v + \frac{w}{10} q$$

Hence:

$$\text{Short-Run Total Cost (TC)} : C_{\text{SR}}(q) = 10v + \frac{w}{10} q$$
$$\text{Short-Run Average Cost (AC)} : AC_{\text{SR}}(q) = \frac{10v}{q} + \frac{w}{10}$$
$$\text{Short-Run Marginal Cost (MC)} : MC_{\text{SR}}(q) = \frac{dC_{\text{SR}}}{dq} = \frac{w}{10}$$

3. Suppose $v = 1$ and $w = 3$. Calculate this firm's long-run and short-run average and marginal cost curves. *Long Run:*

$$C(q) = \left(\frac{1}{5} + \frac{3}{10}\right) q = 0.5q$$
$$\text{LR TC}(q) = 0.5q$$
$$\text{LR AC}(q) = 0.5$$
$$\text{LR MC}(q) = 0.5$$

Short Run:

$$C_{\text{SR}}(q) = 10 \cdot 1 + \frac{3}{10} q = 10 + 0.3q, \quad \text{for } q \leq 50$$
$$\text{SR TC}(q) = 10 + 0.3q$$
$$\text{SR AC}(q) = \frac{10}{q} + 0.3$$
$$\text{SR MC}(q) = 0.3$$

Problem 2

An enterprising entrepreneur purchases two factories to produce widgets. Each factory produces identical products, and each has a production function given by

$$q = \sqrt{k_i l_i}, i = 1, 2$$

The factories differ, however, in the amount of capital equipment each has. In particular, factory 1 has $k_1 = 25$, whereas factory 2 has $k_2 = 100$. Rental rates for k and l are given $w = v = \$1$.

1. If the entrepreneur wishes to minimize short-run total costs of widget production, how should output be allocated between the two factories?
2. Given that output is optimally allocated between the two factories, calculate the short-run total, average, and marginal cost curves. What is the marginal cost of the 100th widget? The 125th widget? The 200th widget?
3. How should the entrepreneur allocate widget production between the two factories in the long run? Calculate the long-run total, average, and marginal cost curves for widget production
4. How would your answer to part (c) change if both factories exhibited diminishing returns to scale?

Problem 3

Consider the production function $q = f(k, l) = l^{0.8}k^{0.2}$. The cost of labor and capital are w and v respectively. Aside from the cost of labor and capital, there are no costs. Initially assume k is fixed at k_1.

1. Find the short run cost function $C_{SR}(q)$.
2. Find short run marginal cost.
3. Find the short run average cost.
4. Find short run average cost.
5. Demostrare that cost is minimized when $AC = MC$.

   now asume all inputs are variable.
6. Find the long run cost function $C_{LR}(q)$.
7. Find the long run marginal Cost.
8. Find the long run average cost.