

# **ESMA 6787: Exam 2**

Due on December 14, 2025

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## Problem 1: Acceptance of Syllabus

I have read the syllabus, understand its contents, and have no questions.

## Problem 2: Definitions

- (a) **Sample Space:**
- (b) **Kolmgorov Axioms of Probability:**
- (c) **Exponential family:**
- (d) **Convergence in distribution:**
- (e) **Convergence in Probability:**
- (f) **Almost sure convergence (or convergence with probability 1):**
- (g) **Weak law of large numbers:**
- (h) **Strong law of large numbers:**
- (i) **Characteristics functions:**

## Problem 3:

Show that each of the following families of distributions is an exponential family,

- (a) The family of Bernoulli distribution with a unknown value of the parameter  $p$ .
- (b) The family of Poisson distributions with an unknown mean
- (c) The family of negative binomial distributions for which the value of  $r$  is known and the value of  $p$  is unknown.
- (d) The family of normal distributions with an unknown mean and a known variance.
- (e) The family of normal distributions with an unknown variance and a known mean.
- (f) The family of gamma distributions for which the value of  $\alpha$  is unknown and the value of  $\beta$  is known.
- (g) The family of gamma distributions for which the value of  $\alpha$  is known and the value of  $\beta$  is unknown.
- (h) The family of beta distributions for which the value of  $\alpha$  is unknown and the value of  $\beta$  is known.
- (i) The family of beta distributions for which the value of  $\alpha$  is known and the value of  $\beta$  is unknown.