ESMA 6787: Exam 2

Due on December 14, 2025

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Problem 1: Acceptance of Syllabus

I have read the syllabus, understand its contents, and have no questions.

Problem 2: Definitions

(a) **Sample Space:**
(b) **Kolmgorov Axioms of Probability:**
(c) **Exponential family:**
(d) **Convergence in distribution:**
(e) **Convergence in Probability:**
(f) **Almost sure convergence (or convergence with probability 1):**
(g) **Weak law of large numbers:**
(h) **Strong law of large numbers:**
(i) **Characteristics functions:**

Problem 3:

Show that each of the following families of distributions is an exponential family,
(a) The family of Bernoulli distribution with a unknown value of the parameter p.
(b) The family of Poisson distributions with an unknown mean
(c) The family of negative binomial dstributions for which the value of r is known and the value of p is unknown.
(d) The family of normal distributions with an unknown mean and a known variance.
(e) The family of normal distributions with an unknown variance and a known mean.
(f) The family of gamma distributions for which the value of α is unknown and the value of β is known.
(g) The family of gamma distributions for which the value of α is known and the value of β is unknown.
(h) The family of beta distributions for which the value of α is unknown and the value of β is known.
(i) The family of beta distributions for which the value of α is known and the value of β is unknown.

Problem 4:

Let X be a random variable with a Student's t distribution with p degrees of freedom.
(a) Derive the mean and variance of X.
(b) Show that X^2 has an $F_{1,p}$ distribution.
(c) Let $(f(x|p))$ denote the pdf of X. Show that

$$\lim_{p\to\infty} f(x|p) = \frac{1}{\sqrt{2\pi}} \exp\{-\frac{x^2}{2}\}$$

at each value of x, $-\infty < x < \infty$. This correctly suggests that as $p \to \infty$, X converges in distributions to a $N(0,1)$ random variable (Hint: Use Stirirling's Formula).
(d) Use the results of parts (a) and (b) to argue that, as $p \to \infty$, X converges in distribution to a X_1^2 random variable.
(e) What might you conjecture about the distributional limit, as $p \to \infty$, of $F_{p,q}$

Optional Problem 1:

Suppose that X has the log-normal distribution with parameters μ and σ^2. Find the distribution of $\frac{1}{X}$.

Optional Problem 2:

Suppose $\bar{X}$ is the mean of 100 observations from a population with mean μ and variance $\sigma^2 = 9$. Find limits between which $\bar{X} - \mu$ will lie with probability at least 90%. Use both Chebyshev's inequality and the Central Limit Theorem, and comment on each.