ESMA 6661: The Delta Method

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Spring 2026

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1 Taylor Expancions

1.1 Definition Taylor Expansion

1.2 Taylor Expansion of an Estimator

Let $T_1, \ldots, T_k$ be a random variables with means $\theta_1, \ldots, \theta_k$ and define $T = (T_1, \ldots, T_k)$ and $\theta = (\theta_1, \ldots, \theta_k)$. Suppose there is a differentiable function $g(T)$. We are interested in approximate and estimate for the variance. Define $g(\theta) = g'(t)$ and $t_1 = \theta_1, \ldots, t_k = \theta_k$.

$$E[g(t)] \approx g(\theta) + g'(\theta)E[(T - \theta)] = g(\theta)$$
$$E[g(t)] \approx g(\theta)$$

For the variance

$$Var(g(t)) \approx [g'(\theta)]^2 Var(T)$$

This tells us that:

1. $g(T) \sim (g(\theta), [g'(\theta)]^2 Var(T))$

2. I dont know the distribution but i can guess the mena and the var

1.3 Example: Approximate variance for the odds

Let $X_1, \ldots, X_n \sim Bernoulli(p)$. Define the odds as $\frac{p}{1-p}$, consider the estimator $\hat{p}$.

$$x_i = \begin{cases} 1; & \text{With probability } p \\ 0; & \text{with probability } 1 - p \end{cases}$$
$$\frac{p}{1-p} = g(p)$$
$$\text{Consider } \hat{p}\frac{1}{n}\sum_{i=1}^{n} x_i = \bar{n}; \text{ By CLT} \bar{X} \sim N(p, \frac{p(1-p)}{n})$$
$$g(\hat{p}) = \frac{\hat{p}}{1-\hat{p}} = \frac{\bar{x}}{1-\bar{x}} \sim F$$