ESMA 6787: Homework 1

Alejandro M. Ouslan

Due Date: September 4, 2025

Problem 1: Syllabus Acknowledgment

Problem 2: Definitions in Your Own Words

- **Experiment:**
- **Experimental unit:**
- **Observational unit:**
- **Background variable:**
- **Independent (predictor) variable:**
- **Dependent (response) variable:**
- **Confounded factors:**
- **Experimental error:**
- **Randomization:**
- **Replicate:**

Problem 3: Lady Tasting Tea

(a) Units in this experiment:

(b) Treatments in this experiment:

(c) Randomization method using physical devices:

(d) Adjustments if cups differ in material (porcelain vs china):

Problem 4: Paper Airplane Experiment

(a) Experimental treatments:

(b) Experimental units and homogeneity:

(c) Randomization process:

(d) Procedure for applying treatment to unit:

(e) Measurement process:

Problem 5: Gasoline Mileage Study

(a) Comparison of strengths and weaknesses:

(b) Identification of true experiment(s) and justification:

Problem 6: Baseball League as Experiment

- Treatments and units:
- Application of treatment to unit:
- Randomization and replication:
- Possibility and use of blocking:

Problem 7: Tomato Fertilizer and Variety

- Experimental setup:
- Use of replication and randomization:
- Additional design principles in second season:

Problem 8: Hand Washing Experiment

(a) Experimental unit:

(b) Factors:

(c) Response:

Problem 9: Real-life Application

Problem 10: Variance as Quadratic Form

Problem 11: Cell Means Model with Unequal Group Sizes

(a) Proposed cell means model:

(b) Design matrix X and its rank:

(c) Computation of $X'X$:

(d) OLS estimates as a function of y:

(e) Analysis of Table 1 dataset:

i. OLS estimates:
ii. Projection matrix P_X:
iii. Compute $y'(I - P_X)y$:
iv. Compute $\bar{y}_{\cdot\cdot}$ and $\bar{y}_{i\cdot}$ for all i:
v. Estimability of μ_1:
vi. Estimability of $\mu_2 - \mu_3$:
vii. Estimability of $\mu_1 - \frac{\mu_2+\mu_3}{2}$:

Problem 12: Fixed-Effect Model with Unequal Group Sizes

(a) Proposed fixed-effect model:

(b) Design matrix X and its rank:

(c) Computation of $X'X$:

(d) OLS estimates as a function of y:

(e) Using Table 1 dataset:

i. OLS estimates:

ii. Projection matrix P_X:

iii. Compute $y'(I - P_X)y$:

iv. Compute $\bar{y}_{..}$ and $\bar{y}_{i\cdot}$ for all i:

v. Estimability of α_1:

vi. Estimability of $\alpha_2 - \alpha_3$:

vii. Estimability of $\alpha_1 - \frac{\alpha_3+\alpha_4}{2}$: