ESMA 6787: Homework 1

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Problem 1: Syllabus Acknowledgment

I have read the syllabus, understand its contents,and have no questions.

Problem 2: Definitions in Your Own Words

- **Experiment:** Method to answer a research question, where the researcher controles an environment and only changes one or many treatment holding all other things constant
- **Experimental unit:** unit that will receive the treatment
- **Observational unit:** it is the person, thing or event where are studying.
- **Background variable:** Are all the factor that can affect the study but are not controlled for
- **Independent (predictor) variable:** It is what is applied as treatment
- **Dependent (response) variable:** It is the outcome or what we observe after we apply the treatment
- **Confounded factors:** Are un observe factors that affect both the treatment and the observed factor
- **Experimental error:** It is how far our experiment is from the reality.
- **Randomization:** Treatments are applied randomly
- **Replicate:** recreation of the experiments with the same settings and specification but with different data

Problem 3: Lady Tasting Tea

(a) Units in this experiment: The tea cups

(b) Treatments in this experiment: The order of what was added first to the tea

(c) Randomization method using physical devices: order cups from 1-8 then randomize the order with a program

(d) Adjustments if cups differ in material (porcelain vs china): Assigned cups type randomly to each group

Problem 4: Paper Airplane Experiment

(a) Experimental treatments: The experiments would be each of the 3 recipe, with the simple classic airplane as the controle group

(b) Experimental units and homogeneity: to ensure homogeneity we would use the same type paper, thrown from the same location.

(c) Randomization process: when we get the 20 papers we assign the folding recepte randomly to them

(d) Procedure for applying treatment to unit: when the recepte is chacen use the as much possible machines or tools ensure cons

(e) Measurement process: Mesure the distance the plane flew

Problem 5: Gasoline Mileage Study

(a) Comparison of strengths and weaknesses: Design experiment A is convenient but their are many variables that are not controlled like the brand of car, driving habits, etc. Design B has more strengths given that we use the same care for the same amount of time but a problem that could have the experiment is segregation of the vie vale over time. Design C addresses the concerns of the previouse experiemnt.

(b) Identification of true experiment(s) and justification: Experiment C would say it is the best experiment given that it controles for the most amount of factors buy using the same vehicle.

Problem 6: Baseball League as Experiment

- Treatments and units: The teams and the unites are the games
- Application of treatment to unit: The application of the treatment would be to that a particular team is playing at its expectations of victory is the outcome
- Randomization and replication: Randomizing the matching of the team and ensuring that each team plays each other for the replications
- Possibility and use of blocking: Could be used if there are confounding factors like home-field advantage or team skill differences, but it might not be necessary if teams are assumed to be comparable.

Problem 7: Tomato Fertilizer and Variety

- Experimental setup: We would devide the plot of land into quadrents and each will have a groups with all combinations of Fertilizer and seed type. In addition it will consist of a control group with no Fertilizer and a comon tomato seed.
- Use of replication and randomization: The randomization is applied to which section of the plot of land receives the combination of Fertilizer and seed
- Additional design principles in second season: rotate the treatments, long-term replication, seasonal variation consideration, increased precision in measurement and control of weather variability.

Problem 8: Hand Washing Experiment

(a) Experimental unit: The experimental unit is the individual subject (person) participating in the hand washing experiment

(b) Factors: Wash water temperature, detergent concentration

(c) Response:The response is the bacterial count on the palms of the subjects.

Problem 9: Real-life Application

I would say most economic problem would benefit form experiments designs given that it changes the results from corrolation to causality.

Problem 10: Variance as Quadratic Form

Suppose $y_1, \ldots, y_n \sim N(\mu, \sigma^2)$. Let $y = (y_1, \ldots, y_n)'$ and let $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$. Show that $\frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$ can be written as $y'Ay$ for sum matrix A. Identify the A matrix.

$$\frac{1}{n-1}(y - \bar{y}\mathbf{1})'(y - \bar{y}\mathbf{1})$$

$$y - \bar{y}\mathbf{1} = y - \left(\frac{1}{n}\mathbf{1}'y\right)\mathbf{1} = \left(I - \frac{1}{n}\mathbf{1}\mathbf{1}'\right)y$$

$$\frac{1}{n-1}(y - \bar{y}\mathbf{1})'(y - \bar{y}\mathbf{1}) = \frac{1}{n-1}y'C'Cy$$

$$\frac{1}{n-1}y'Cy$$

$$A = \frac{1}{n-1}\left(I - \frac{1}{n}\mathbf{1}\mathbf{1}'\right)$$

Problem 11: Cell Means Model with Unequal Group Sizes

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(a) Proposed cell means model:

(b) Design matrix X and its rank:

(c) Computation of $X'X$:

(d) OLS estimates as a function of y:

(e) Analysis of Table 1 dataset:

i. OLS estimates:
ii. Projection matrix P_X:
iii. Compute $y'(I - P_X)y$:
iv. Compute $\bar{y}_{\cdot\cdot}$ and $\bar{y}_{i\cdot}$ for all i:
v. Estimability of μ_1:
vi. Estimability of $\mu_2 - \mu_3$:
vii. Estimability of $\mu_1 - \frac{\mu_2+\mu_3}{2}$:

Problem 12: Fixed-Effect Model with Unequal Group Sizes

(a) Proposed fixed-effect model:

(b) Design matrix X and its rank:

(c) Computation of $X'X$:

(d) OLS estimates as a function of y:

(e) Using Table 1 dataset:

i. OLS estimates:
ii. Projection matrix P_X:
iii. Compute $y'(I - P_X)y$:
iv. Compute $\bar{y}_{\cdot\cdot}$ and $\bar{y}_{i\cdot}$ for all i:
v. Estimability of α_1:
vi. Estimability of $\alpha_2 - \alpha_3$:
vii. Estimability of $\alpha_1 - \frac{\alpha_3+\alpha_4}{2}$: