Latin Square Designs: [ESMA 6787]

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Contents

1 Introduciotn

1. Row-Collumn Design: units can reasonably be sorted by two characteristics rather thatn one, and the most commonly used of these are Latin Square Designs (LSD)
2. The result is a two-way classification of units based on two protential sources of nuisance variataion, so the unit-to-unit relationships cannot be descirbed independently within rows ignoring columns, or independetly withing columns ignoring rows.
3. In order to ensure treatment-block balance comparable to that dfound in CRBDs,
   (a) Design is a CRBD with respect to rows as blocks, ignoring columns
   (b) Designs is a CRD with respect to columns as block, ignoring rows.
4. This is , in fact, how a Lating Square Design is contructed.

2 Latin Squares design (LSD)

1. THe number of rwo-blocks of units must be t (the number of treatments), since each treatment must appear exactly once in each column-block.
2. The number of colus-blocks of units must be t, since each treatment must appear exactly once in each row-block.
3. THerefore, a LSD must contain a total of t^2 units, t of which must be assigned to each treatment, and result in $N = t^2$ data values for each responce variable.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 2 \\ 3 & 1 & 2 & 2 \\ 8 & 1 & 2 & 1 \end{bmatrix}$$

1. Since each experimetntal unit is consitain in two blocks, randomizaiton is somewhat less straightfoward for LsDs thatn wiht with CBDs.

2. randomizaiton

3 Linear Model for LSD

3.1 ANOVA skeleton for LSD

3.2 Exampel: Operatiors and machine

Consider a factory setting where you are producing a product with 4 operators and 4 machines. We call the columns the operators and the rows the machines. Then you can randomly assign the specific operators to a row and the specific macines to a column. The treatment is one of four protocols for producing the product and our interest is in the average time needed to produce each product. Create a the ANOVA skeleton.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \\ 3 & 4 & 1 & 2 \\ 2 & 3 & 4 & 1 \end{bmatrix}$$

3.3 Model in matix from

Consider the follwing:

$$y = X_1\theta + X_2\tau + \epsilon$$

1. Modle matrix for μ, α, β

$$X_1 = []$$