ESMA 6787: Exam 2

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Problem 1: Definitions

(a) **Interaction plot:** An interaction plot as a visual tool to determine if there interaction between 2 factors with each other. You determine there is interaction when you see the line intersect with each other.

(b) **Random effect:** When we work with block designs a random effect is the unobserved effects within the blocks, that is to say each group contains its own random effect $N(0, \sigma)$

(c) **Factorial design:** Factorial design is a method to combare multiple factors with each other and determine its effects on the response variable. Example we al looking at the shrinkage of type of cloth and water temperature, in this experiments the type of cloth and water temperature are the factors.

(d) **Intraclass correlation:** This is the correlation that groups can have with each other.

(e) **Split-plot design:** The split-plot are design method where we mesure the effects of factors. This is done by first dividing the entire group in section where each will revive a specific treatment the each group is divided in sub groups that recave the other factor. This results in a multiple big groups that contain a factor and inside those groups there is a sub group that are randomly assigned the factors.

Problem 2: Marketing Problem

A marketing research consultant evaluated the effects of fee schedule, scope of work, and type of supervisory control on the quality of work performed under contract by independent marketing research agencies. The quality of work performed was measured by an index taking into account several characteristics of quality. Four agencies were chosen for each factor level combination and the quality of their work evaluated. See **marketing.txt** files for this dataset.

(a) Create interaction plots of the factors for the fee schedule and scope of the work of the estimated treatment means $\bar{Y}_{ij}$. Does it appear that any interactions are present? Any main effects?

(b) Create interaction plots of the factors fee schedule and type of supervisory control of the estimated treatment means $\bar{Y}_{ijk}$. Do your plots convey the same information as those in part (a)?

(c) Obtain the analysis of variance table.

(d) Test for three-factor interactions: using $\alpha = 0.01$. State the alternatives, division rule, and conclusion. What is the p-value of the test?

(e) Test for two-factor interactions (there are three test). For each test, use $\alpha = 0.01$ and state the alternative, decision rule, and conclusions. What is the p-value of each test?

(f) Test for factor fee schedule main effects; use $\alpha = 0.05$. State the alternatives, decision rule, and conclusion. Hint: You can use the confidence interval.

Problem 3: Calculator

To test the efficiency of its new programmable calculator, a computer company selected at random six engineers who were proficient in the use of both this calculator and an earlier model and asked them to work out two problems on both calculators. One of the problems was statistical in nature, the other was an engineering problem. kthe orcer of the four calculations was randomized independently for each engineer. kthe length of time (in minutes) required to solve each problem was observed. See Table 1

(a) Create interaction plots of the estimated treatment means. Does it appear that treatment interaction effects are present?

(b) Create the skeleton ANOVA. If there arte any random effects mark them.

(c) Obtain the analysis of variance table.

Table 1: The length of time (in minutes) required to solve each problem

Programmer	**Statistical Problem**		**Engineering Problem**	
	New Model	**Earlier Model**	**New Model**	**Earlier Model**
1	3.1	7.5	2.5	5.1
2	3.8	8.1	2.8	5.3
3	3.0	7.6	2.0	4.9
4	3.4	7.8	2.7	5.5
5	3.3	6.9	2.5	5.4
6	3.6	7.8	2.4	4.8

(d) Test Whether or not the two treatment factors interact. State the alternatives, decision rule, and conclusion. What is the p-value of the test?

Problem 4: Automobile Manufacture

An automobile manufacturer wished to study the effects of difference between drivers and differences between cars on gasoline consumption. Four drivers were selected at random; also live cars of the same model with manual transmission were randomly selected from the assembly line. Each driver drove each car twice over a 40-mile test course and the miles per gallon were recorded. See Table 2

Table 2: Miles per gallon for each driver in each car

Driver	**Car 1**	**Car 2**	**Car 3**	**Car 4**	**Car 5**
1	25.3	28.9	24.8	28.4	27.1
1	25.2	30.0	25.1	27.9	26.6
2	33.6	36.7	31.7	35.6	33.7
2	32.9	36.5	31.9	35.0	33.9
3	27.7	30.7	26.9	29.7	29.2
3	28.5	30.4	26.3	30.2	28.9
4	29.2	32.4	27.7	31.8	30.3
4	29.3	32.4	28.9	30.7	29.9

(a) Test whether or not the two factors interact. State the alternative, decision rule, and conclusion. What is the p-value of the test?

(b) Test separately whether or not factor A and factor B main effects are present. For each test, and state the alternatives, decision rule, and conclusion. What is the p-value for each test?

(c) Obtain point estimates of and which factor appears to have the greater effect on gasoline consumption?

(d) Use the Satterhwait procedure to obtain an approximate 95 percent confidence interval. Is your interval estimate reasonably precise? Comment

Problem 5: Derive estimators

Some students (Junior, Ariana, Yeily, etc.) mentioned they wanted to derived some of the equations. Consider the following model $Y_{jik} = \mu + \alpha_i + \beta_j + \mu_{ij} + \epsilon_{ijk}; k = 1, \ldots, K; i = 1, 2$. Further, assume that $\mu_{ij} \sim N(0, \mu_u^2)$ and $\epsilon_{ijk} \sim N(0, \sigma_e^2)$

1. Show that,

$$Var(Y_{ijk}) = \sigma_u^2 + \sigma_e^2$$

2. Show that, $k \neq k'$
$$Cov(Y_{ijk}, Y_{ijk'}) = \sigma_u^2$$

3. Show that, $k \neq k'$
$$Cov(Y_{ijk}, Y_{ijk'}) = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_e^2}$$