

# **ESMA 6787: Exam 2**

Due on December 14, 2025

*Israel Almodovar*

**Alejandro Ouslan**

## Problem 1: Definitions

- (a) **Interaction plot:** An interaction plot as a visual tool to determine if there is interaction between 2 factors with each other. You determine there is interaction when you see the lines intersect with each other.
- (b) **Random effect:** When we work with block designs a random effect is the unobserved effects within the blocks, that is to say each group contains its own random effect  $N(0, \sigma)$
- (c) **Factorial design:** Factorial design is a method to compare multiple factors with each other and determine its effects on the response variable. Example we are looking at the shrinkage of type of cloth and water temperature, in this experiments the type of cloth and water temperature are the factors.
- (d) **Intraclass correlation:** This is the correlation that groups can have with each other.
- (e) **Split-plot design:** The split-plot are design method where we measure the effects of factors. This is done by first dividing the entire group in sections where each will receive a specific treatment. The each group is divided in sub groups that receive the other factor. This results in a multiple big groups that contain a factor and inside those groups there is a sub group that are randomly assigned the factors.

## Problem 2: Marketing Problem

A marketing research consultant evaluated the effects of fee schedule, scope of work, and type of supervisory control on the quality of work performed under contract by independent marketing research agencies. The quality of work performed was measured by an index taking into account several characteristics of quality. Four agencies were chosen for each factor level combination and the quality of their work evaluated. See **marketing.txt** files for this dataset.

- (a) Create interaction plots of the factors for the fee schedule and scope of the work of the estimated treatment means  $\bar{Y}_{ij}$ . Does it appear that any interactions are present? Any main effects?
- Answer:** There appears to be no interaction between schedule. The main effect appears to be that as the fee of the fee increases the quality of work decrease.

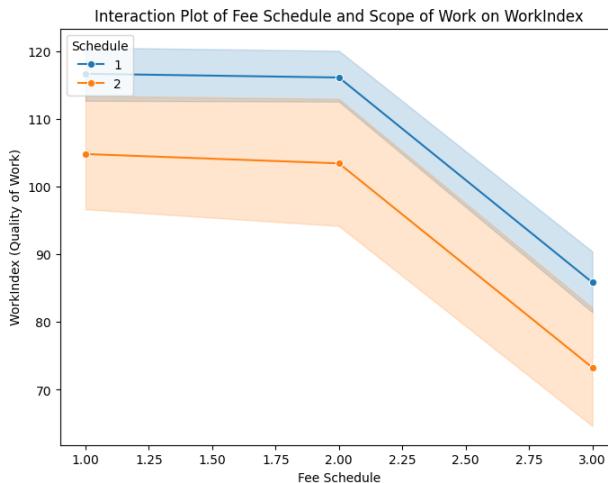


Figure 1: Interaction Graph

- (b) Create interaction plots of the factors fee schedule and type of supervisory control of the estimated treatment means  $\bar{Y}_{ijk}$ . Do your plots convey the same information as those in part (a)?
- Answer:** There appears to be no interaction between supervisory. The main effect appears to be that as the fee of the fee increases the quality of work decrease. This seems to be the same effect of part a

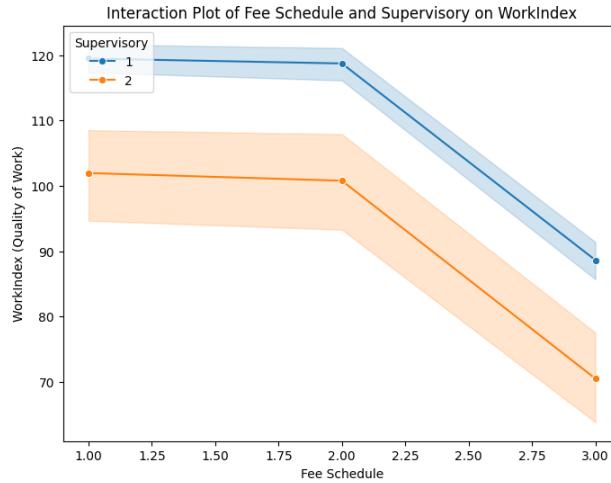


Figure 2: Interaction Graph

(c) Obtain the analysis of variance table.

Table 1: ANOVA for the Effect of Fee and Schedule on Work Index

Source	df	Sum of Squares	Mean Square	F-statistic	p-value
C(Fee)	2.0	10044.27	5022.14	45.09	$3.50 \times 10^{-11}$
C(Schedule)	1.0	1833.98	1833.98	16.47	$2.11 \times 10^{-4}$
C(Fee):C(Schedule)	2.0	1.60	0.80	0.01	$9.93 \times 10^{-1}$
Residual	42.0	4678.04	111.38	-	-

- (d) Test for three-factor interactions: using  $\alpha = 0.01$ . State the alternatives, division rule, and conclusion. What is the p-value of the test?
- $H_0$ : There is no three way interaction
  - $H_a$ : There is three way interaction

Table 2: ANOVA for the Effect of Fee, Schedule, and Supervisory on Work Index (Part 1)

Source	df	Sum of Squares	Mean Square
C(Fee)	2.0	10044.27	5022.14
C(Schedule)	1.0	1833.98	1833.98
C(Supervisory)	1.0	3832.40	3832.40
C(Fee):C(Schedule)	2.0	1.60	0.80
C(Fee):C(Supervisory)	2.0	0.79	0.39
C(Schedule):C(Supervisory)	1.0	574.78	574.78
C(Fee):C(Schedule):C(Supervisory)	2.0	3.94	1.97
Residual	36.0	266.14	7.39

Table 3: ANOVA for the Effect of Fee, Schedule, and Supervisory on Work Index (Part 2)

Source	F-statistic	p-value
C(Fee)	679.34	$2.59 \times 10^{-29}$
C(Schedule)	248.08	$1.00 \times 10^{-17}$
C(Supervisory)	518.40	$5.74 \times 10^{-23}$
C(Fee):C(Schedule)	0.11	$8.98 \times 10^{-1}$
C(Fee):C(Supervisory)	0.05	$9.48 \times 10^{-1}$
C(Schedule):C(Supervisory)	77.75	$1.60 \times 10^{-10}$
C(Fee):C(Schedule):C(Supervisory)	0.27	$7.67 \times 10^{-1}$
Residual	-	-

**Answer:** Given that C(Fee):C(Schedule):C(Supervisory) as a p-value of 0.67 we don't have enough evidence to reject the null hypothesis thus we conclude there is not sufficient evidence to conclude that there is a three way interaction

- (e) Test for factor fee schedule main effects; use  $\alpha = 0.05$ . State the alternatives, decision rule, and conclusion. Hint: You can use the confidence interval.
- (a)  $H_0$ : The fee has no impact on work productivity.
  - (b)  $H_a$ : The fee has an impact on work productivity.
  - (c)  $H_0$ : The schedule has no impact on work productivity.
  - (d)  $H_a$ : The schedule has an impact on work productivity.

Table 4: ANOVA for the Main Effects of Fee and Schedule on Work Index

Source	df	Sum of Squares	Mean Square	F-statistic	p-value
C(Fee)	2.0	10044.27	5022.14	47.22	$1.12 \times 10^{-11}$
C(Schedule)	1.0	1833.98	1833.98	17.24	$1.49 \times 10^{-4}$
Residual	44.0	4679.65	106.36	-	-

Table 5: Confidence Intervals for the Factors in the ANOVA Model

Factor	Lower Bound	Upper Bound
Intercept	110.91	122.91
C(Fee)[T.2]	-8.31	6.39
C(Fee)[T.3]	-38.50	-23.81
C(Schedule)[T.2]	-18.36	-6.36

**Answer:** Looking at the p-values both the fee and the schedule and the fee have a negative impact of the work productivity. Looking at the confidence intervals the fee 3 seems to have the negative impact while level 2 does not seem to have an impact given that it does not include 0.

### Problem 3: Calculator

To test the efficiency of its new programmable calculator, a computer company selected at random six engineers who were proficient in the use of both this calculator and an earlier model and asked them to work out two problems on both calculators. One of the problems was statistical in nature, the other was an engineering problem. The error of the four calculations was randomized independently for each engineer. The length of time (in minutes) required to solve each problem was observed. See Table 1

Table 6: The length of time (in minutes) required to solve each problem

Programmer	Statistical Problem		Engineering Problem	
	New Model	Earlier Model	New Model	Earlier Model
1	3.1	7.5	2.5	5.1
2	3.8	8.1	2.8	5.3
3	3.0	7.6	2.0	4.9
4	3.4	7.8	2.7	5.5
5	3.3	6.9	2.5	5.4
6	3.6	7.8	2.4	4.8

- (a) Create interaction plots of the estimated treatment means. Does it appear that treatment interaction effects are present?

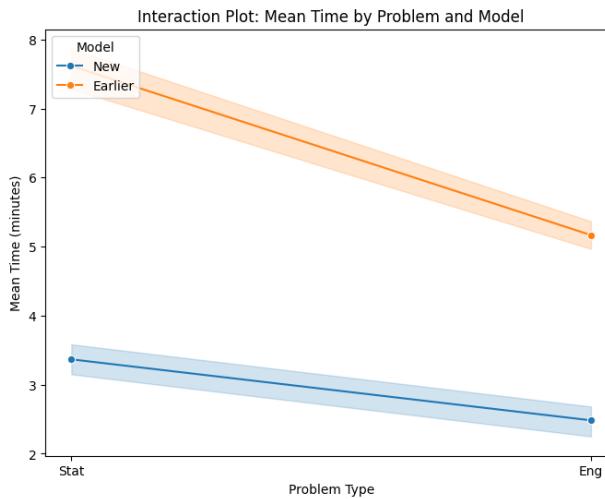


Figure 3: Interaction Graph

**Answer:** There appear to be no interaction between model type and problem. There appear to be a reduction in the completion time from a stats problem to a engineering problem. The overall effect is that the new model reduced the completion time.

- (b) Create the skeleton ANOVA. If there are any random effects mark them.

Table 7: Skeleton ANOVA Table for Mixed Effects Model

Source of Variation	df	SS	MS	F
<b>Fixed Effects</b>				
Model (C(Model))	1	$SS_M$	$MS_M$	$F_M$
Problem (C(Problem))	1	$SS_{Problem}$	$MS_P$	$F_P$
Model $\times$ Problem (C(Model):C(Problem))	1	$SS_{M \times P}$	$MS_{M \times P}$	$F_{M \times P}$
<b>Random Effects (R)</b>				
Programmer (random)	$b - 1 = 5$	$SS_{Prog}$	$MS_{Prog}$	—
Residual/Error (within Programmer)	$ab - a - b + 1 = 18$	$SS_R$	$MS_R$	—
Total	$N - 1 = 23$	$SS_{Total}$	—	—

- (c) Obtain the analysis of variance table.

Table 8: Mixed Linear Model Fixed Effects Results

Effect	Coef.	Std. Err.	z-value	p-value	95% CI
Intercept	5.167	0.131	39.402	0.000	[4.910, 5.424]
C(Model)[T.New]	-2.683	0.150	-17.911	0.000	[-2.977, -2.390]
C(Problem)[T.Stat]	2.450	0.150	16.354	0.000	[2.156, 2.744]
C(Model)[T.New]:C(Problem)[T.Stat]	-1.567	0.212	-7.394	0.000	[-1.982, -1.151]

- (d) Test Whether or not the two treatment factors interact. State the alternatives, decision rule, and conclusion. What is the p-value of the test?

(a)  $H_0$ : There is no interaction

(b)  $H_a$ : There is interaction

**Answer:** Given the previous table we see that the p value of 0.00 and it is less than the critical value 0.05 we reject the null hypothesis and conclude that the time to solve the problem depends on the type of problem

## Problem 4: Automobile Manufacture

An automobile manufacturer wished to study the effects of difference between drivers and differences between cars on gasoline consumption. Four drivers were selected at random; also five cars of the same model with manual transmission were randomly selected from the assembly line. Each driver drove each car twice over a 40-mile test course and the miles per gallon were recorded. See Table 2

Table 9: Miles per gallon for each driver in each car

Driver	Car 1	Car 2	Car 3	Car 4	Car 5
1	25.3	28.9	24.8	28.4	27.1
1	25.2	30.0	25.1	27.9	26.6
2	33.6	36.7	31.7	35.6	33.7
2	32.9	36.5	31.9	35.0	33.9
3	27.7	30.7	26.9	29.7	29.2
3	28.5	30.4	26.3	30.2	28.9
4	29.2	32.4	27.7	31.8	30.3
4	29.3	32.4	28.9	30.7	29.9

- (a) Test whether or not the two factors interact. State the alternative, decision rule, and conclusion. What is the p-value of the test?
- (b) Test separately whether or not factor A and factor B main effects are present. For each test, and state the alternatives, decision rule, and conclusion. What is the p-value for each test?
- (c) Obtain point estimates of and which factor appears to have the greater effect on gasoline consumption?
- (d) Use the Satterhwait procedure to obtain an approximate 95 percent confidence interval. Is your interval estimate reasonably precise? Comment

## Problem 5: Derive estimators

Some students (Junior, Ariana, Yeily, etc.) mentioned they wanted to derived some of the equations. Consider the following model  $Y_{jik} = \mu + \alpha_i + \beta_j + \mu_{ij} + \epsilon_{ijk}; k = 1, \dots, K; i = 1, 2$ . Further, assume that

$\mu_{ij} \sim N(0, \sigma_u^2)$  and  $\epsilon_{ijk} \sim N(0, \sigma_e^2)$

1. Show that,

$$\text{Var}(Y_{ijk}) = \sigma_u^2 + \sigma_e^2$$

2. Show that,  $k \neq k'$

$$\text{Cov}(Y_{ijk}, Y_{ijk'}) = \sigma_u^2$$

3. Show that,  $k \neq k'$

$$\text{Cov}(Y_{ijk}, Y_{ijk'}) = \frac{\sigma_u^2}{\sigma_u^2 + \sigma_e^2}$$