ESMA 6787: Homework 1

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Problem 1: Syllabus Acknowledgment

I have read the syllabus, understand its contents, and have no questions.

Problem 2: Definitions in Your Own Words

- Experiment: Method to answer a research question, where the researcher controles an environment and only changes one or many treatment holding all other things constant
- Experimental unit: unit that will receive the treatment
- Observational unit: it is the person, thing or event where are studying.
- Background variable: Are all the factor that can affect the study but are not controlled for
- Independent (predictor) variable: It is what is applied as treatment
- Dependent (response) variable: It is the outcome or what we observe after we apply the treatment
- Confounded factors: Are un observe factors that affect both the treatment and the observed factor
- Experimental error: It is how far our experiment is from the reality.
- Randomization: Treatments are applied randomly
- Replicate: recreation of the experiments with the same settings and specification but with different data

Problem 3: Lady Tasting Tea

- (a) Units in this experiment:
- (b) Treatments in this experiment:
- (c) Randomization method using physical devices:
- (d) Adjustments if cups differ in material (porcelain vs china):

Problem 4: Paper Airplane Experiment

- (a) Experimental treatments:
- (b) Experimental units and homogeneity:
- (c) Randomization process:
- (d) Procedure for applying treatment to unit:
- (e) Measurement process:

Problem 5: Gasoline Mileage Study

- (a) Comparison of strengths and weaknesses:
- (b) Identification of true experiment(s) and justification:

Problem 6: Baseball League as Experiment

- Treatments and units:
- Application of treatment to unit:
- Randomization and replication:
- Possibility and use of blocking:

Problem 7: Tomato Fertilizer and Variety

- Experimental setup:
- \bullet Use of replication and randomization:
- Additional design principles in second season:

Problem 8: Hand Washing Experiment

- (a) Experimental unit:
- (b) Factors:
- (c) Response:

Problem 9: Real-life Application

Problem 10: Variance as Quadratic Form

Problem 11: Cell Means Model with Unequal Group Sizes

- (a) Proposed cell means model:
- (b) Design matrix X and its rank:
- (c) Computation of X'X:
- (d) OLS estimates as a function of y:
- (e) Analysis of Table 1 dataset:
 - i. OLS estimates:
 - ii. Projection matrix P_X :
 - iii. Compute $y'(I P_X)y$:
 - iv. Compute $\bar{y}_{..}$ and \bar{y}_{i} . for all i:
 - v. Estimability of μ_1 :
 - vi. Estimability of $\mu_2 \mu_3$:
 - vii. Estimability of $\mu_1 \frac{\mu_2 + \mu_3}{2}$:

Problem 12: Fixed-Effect Model with Unequal Group Sizes

- (a) Proposed fixed-effect model:
- (b) Design matrix X and its rank:
- (c) Computation of X'X:
- (d) OLS estimates as a function of y:
- (e) Using Table 1 dataset:
 - i. OLS estimates:
 - ii. Projection matrix P_X :
 - iii. Compute $y'(I P_X)y$:
 - iv. Compute $\bar{y}_{\cdot \cdot}$ and \bar{y}_{i} for all i:
 - v. Estimability of α_1 :
 - vi. Estimability of $\alpha_2 \alpha_3$:
 - vii. Estimability of $\alpha_1 \frac{\alpha_3 + \alpha_4}{2}$: