MATE 5150: Asignacion #1

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Problem 1

Find the equation of the plane through he following pairs of points in space. $P_1(1,1,1)$, $P_2(5,5,5)$, and $P_3(-6,4,2)$.

$$\begin{aligned}
\overrightarrow{AB} &= (5-1, 5-1, 5-1) = (4,4,4) \\
\overrightarrow{AC} &= (-6-1, 4-1, 2-1) = (-7,3,1) \\
x &= (1,1,1) + s(4,4,4) + t(-7,3,1)
\end{aligned}$$

Problem 2

Show thaat the midpoint of the line ssegment joining the points (a,b) and (c,d) is $(\frac{a+c}{2}, \frac{b+d}{2})$.

$$\begin{aligned}
\text{Midpoint} &= \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB}) \\
&= \frac{1}{2}(\begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}) \\
&= \frac{1}{2}(\begin{bmatrix} a+c \\ b+d \end{bmatrix}) \\
&= (\frac{a+c}{2}, \frac{b+d}{2})
\end{aligned}$$

Problem 3

Let $S = \{0,1\}$ and $F = R$. In $\mathcal{F}(\mathcal{S}, \mathcal{R})$, show that $f = g$ and $f+g = h$, where $f(t) = 2t+1$, $g(t) = 1+4t-2t^2$, and $h(t) = 5^t + 1$.

For $f = g$

$$\begin{aligned}
f(0) &= g(0) \\
2t + 1 &= 1 + 4t - 2t^2 \\
1 &= 1 \\
f(1) &= g(1) \\
2t + 1 &= 1 + 4t - 2t^2 \\
3 &= 3
\end{aligned}$$

For $f + g = h$

$$\begin{aligned}
f(0) + g(0) &= h(0) \\
2t + 1 + 1 + 4t - 2t^2 &= 5^t + 1 \\
1 + 1 &= 2 \\
f(1) + g(1) &= h(1) \\
2t + 1 + 1 + 4t - 2t^2 &= 5^t + 1 \\
6 &= 6
\end{aligned}$$

Problem 4

Let V denote the set of ordered pairs of real numbers. If (a_1, a_2) and (b_1, b_2) are elements of V and $c \in R$, define $(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 b_2)$ and $c(a_1, a_2) = (ca_1, a_2)$. Is V a vector space over R with these operations? Justify your answer.

It is not a vector space because it does not hold for the commutative of addition property.

$$0(a_1, a_2) = (0a_1, a_2) = (0, a_2)$$

since a_2 is not unique, and by theorem 1.1, it is not a vector space.

Problem 5

Let $V = \{(a_1, a_2) : a_1, a_2 \in R\}$. For (a_1, a_2) and (b_1, b_2) in V and $c \in R$, define $(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 + b_2)$ and $c(a_1, a_2) = (ca_1, ca_2)$.

$$(a_1, a_2) + (b_1, b_2) = (a_1 + 2b_1, a_2 + 3b_2)$$

Is V a vector space over R with these operations? Justify your answer.
No, It does not hold for the commutative of addition property.

$$\begin{aligned} x + y &= y + x \\ (a_1, a_2) + (b_1, b_2) &= (b_1, b_2) + (a_1, a_2) \\ (a_1 + 2b_1, a_2 + 3b_2) &\neq (b_1 + 2a_1, b_2 + 3a_2) \end{aligned}$$

Problem 6

Prove that $A + A^t$ is symmetric for any matrix A.

$$\begin{aligned} (A + A^t)^t &= A^t + (A^t)^t \\ &= A^t + A \\ &= A + A^t \end{aligned}$$

Problem 7

Let W_1, W_3, and W_4 be as in Excersise 8. Describe $W_1 \cap W_3$, $W_1 \cap W_4$, and $W_3 \cap W_4$, and observe that each is a subspace of R^3.

$$W_1 \cap W_3 = \{(a_1, a_2, a_3) \in R^3 : a_1 \text{ and } a_3 = -a_2\} \cup \{(a_1, a_2, a_3) \in R^3 : 2a_1 - 7a_2 + a_3 = 0\}$$
$$W_1 \cap W_3 = \{(a_1, a_2, a_3) \in R^3 : 6a_2 - 7a_2 - a_2 = 0\}$$
$$W_1 \cap W_3 = \{0\} \text{ and is a subspace of } R^3$$
$$W_1 \cap W_4 = \{(a_1, a_2, a_3) \in R^3 : a_1 \text{ and } a_3 = -a_2\} \cup \{(a_1, a_2, a_3) \in R^3 : a_1 - 4a_2 - a_3 = 0\}$$
$$W_1 \cap W_4 = \{(a_1, a_2, a_3) \in R^3 : 3a_2 - 4a_2 - a_2 = 0\}$$
$$W_1 \cap W_4 = \{0\} \text{ and is a subspace of } R^3$$
$$W_3 \cap W_4 = \{(a_1, a_2, a_3) \in R^3 : 2a_1 - 7a_2 + a_3 = 0\} \cup \{(a_1, a_2, a_3) \in R^3 : a_1 - 4a_2 - a_3 = 0\}$$
$$W_3 \cap W_4 = \{(a_1, a_2, a_3) \in R^3 : a_1 - 3a_2 + 2a_2 = 0\} \text{ and is a subspace of } R^3$$

Problem 8

Let $C^n(R)$ denote the set of all real-valued functions defined on the real line that have a continuous nth derivative. Prove that $C^n(R)$ is a subspace of $\mathcal{F}(R, R)$.

Proof.

$$\begin{aligned} f(x) &= 0 \quad \text{for all} \quad x \in R \\ f'(x) &= 0 \quad \text{for all} \quad x \in R \\ f''(x) &= 0 \quad \text{for all} \quad x \in R \\ &\vdots \\ f^{(n)}(x) &= 0 \quad \text{for all} \quad x \in R \end{aligned}$$

Thus $C^n(R)$ is a subspace of $\mathcal{F}(R, R)$. $\square$