

MATE 5150: Asignacion #1

Due on Septiembre 5, 2024

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Problem 1

Find the equation of the plane through the following pairs of points in space. $P_1(1, 1, 1)$, $P_2(5, 5, 5)$, and $P_3(-6, 4, 2)$.

$$\overrightarrow{AB} = (5 - 1, 5 - 1, 5 - 1) = (4, 4, 4)$$

$$\overrightarrow{AC} = (-6 - 1, 4 - 1, 2 - 1) = (-7, 3, 1)$$

$$x = (1, 1, 1) + s(4, 4, 4) + t(-7, 3, 1)$$

Problem 2

Show that the midpoint of the line segment joining the points (a, b) and (c, d) is $(\frac{a+c}{2}, \frac{b+d}{2})$.

$$\begin{aligned} \text{Midpoint} &= \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB}) \\ &= \frac{1}{2}\left(\begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}\right) \\ &= \frac{1}{2}\begin{bmatrix} a+c \\ b+d \end{bmatrix} \\ &= \left(\frac{a+c}{2}, \frac{b+d}{2}\right) \end{aligned}$$

Problem 3

Let $S = \{0, 1\}$ and $F = R$. In $\mathcal{F}(S, R)$, show that $f = g$ and $f+g = h$, where $f(t) = 2t+1$, $g(t) = 1+4t-2t^2$, and $h(t) = 5^t + 1$.

For $f = g$

$$\begin{aligned} f(0) &= g(0) \\ 2t + 1 &= 1 + 4t - 2t^2 \\ 1 &= 1 \\ f(1) &= g(1) \\ 2t + 1 &= 1 + 4t - 2t^2 \\ 3 &= 3 \end{aligned}$$

For $f + g = h$

$$\begin{aligned} f(0) + g(0) &= h(0) \\ 2t + 1 + 1 + 4t - 2t^2 &= 5^t + 1 \\ 1 + 1 &= 2 \\ f(1) + g(1) &= h(1) \\ 2t + 1 + 1 + 4t - 2t^2 &= 5^t + 1 \\ 6 &= 6 \end{aligned}$$

Problem 4

Let V denote the set of ordered pairs of real numbers. If (a_1, a_2) and (b_1, b_2) are elements of V and $c \in R$, define $(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 + b_2)$ and $c(a_1, a_2) = (ca_1, ca_2)$.

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Problem 5

Let $V = \{(a_1, a_2) : a_1, a_2 \in R\}$. For (a_1, a_2) and (b_1, b_2) in V and $c \in R$, define $(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 + b_2)$ and $c(a_1, a_2) = (ca_1, ca_2)$.

$$(a_1, a_2) + (b_1, b_2) = (a_1 + 2b_1, a_2 + 3b_2)$$

Is V a vector space over R with these operations? Justify your answer.

No, It does not hold for the commutative of addition property.

$$x + y = y + x$$

$$(a_1, a_2) + (b_1, b_2) = (b_1, b_2) + (a_1, a_2)$$

$$(a_1 + 2b_1, a_2 + 3b_2) \neq (b_1 + 2a_1, b_2 + 3a_2)$$

Problem 6

Prove that $A + A^t$ is symmetric for any matrix A .

$$\begin{aligned} (A + A^t)^t &= A^t + (A^t)^t \\ &= A^t + A \\ &= A + A^t \end{aligned}$$

Problem 7

Let W_1 , W_3 , and W_4 be as in Exercise 8. Describe $W_1 \cap W_3$, $W_1 \cap W_4$, and $W_3 \cap W_4$, and observe that each is a subspace of R^3 .

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Problem 8

Let $C^n(R)$ denote the set of all real-valued functions defined on the real line that have a continuous n th derivative. Prove that $C^n(R)$ is a subspace of $\mathcal{F}(R, R)$.

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