MATE 5150: Asignacion #3

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Problem 1

Use the proof of Theorem 3.2 to obtain the inverse of each of the following matrices:

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Problem 2

Let A be an $m \times n$ matrix. Prove that if B can be obtained from A by an elementary row [column] operation, then B^T can be obtained from A^T by the corresponding elementary column [row] operation.

Problem 3

Prove that any elementary row [column] operation of type 2 can be obtained by dividing some row [column] by a nonzero scalar.

Problem 4

Find the rank of the following matrix:

$$G = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 2 & 2 & 0 & 2 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

Problem 5

For each of the following matrices, compute the rank and the inverse if exists:

$$F = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Problem 6

For each of the following linear transformations T, determine whether T is invertible, and compute T^{-1} if it exists:

$$T = P_2(\mathbb{R}) \to P_2(\mathbb{R}) \quad \text{defined by} \quad T(f(x)) = (x+1)f'(x)$$

Problem 7

For each of the following linear transformations T, determine whether T is invertible, and compute T^{-1} if it exists:

$$T = \mathbb{R}^3 \to \mathbb{R}^3 \quad \text{defined by} \quad T(a_1, a_2, a_3) = (a_1 + 2a_2 + 3a_3, -a_1 + a_2 + 2a_3, a_1 + a_3)$$

Problem 8

Let $T, U : V \to W$ be linear transformations.

- Prove that $R(T + U) \subseteq R(T) + R(U)$.
- Prove that if W if finite-dimensional, then $rank(T + U) \leq rank(T) + rank(U)$.
- Deduce from (b) that if $rank(A + B) \leq rank(A) + rank(B)$ for any $m \times n$ matrices A and B.