

MATE 5150: Asignacion #3

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Problem 1

For each of the following homogeneous systems of linear equations, find the dimensions of and a basis for the solution space.

$$\begin{aligned}
 x_1 + 2x_2 + x_3 + x_4 &= 0 \\
 x_2 - x_3 + x_4 &= 0 \\
 \begin{bmatrix} 1 & 2 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 & 0 \end{bmatrix} \\
 x_2 &= x_3 - x_4 \\
 x_1 &= -x_3 - x_4 \\
 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} &= \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix} x_3 + \begin{bmatrix} -1 \\ -1 \\ 0 \\ 1 \end{bmatrix} x_4
 \end{aligned}$$

The rank of the matrix is 2, and dimension of the solution space is 2. A basis for the solution space is $\{(1, 1, 1, 0), (1, -1, 0, 1)\}$.

Problem 2

Using the results of Exercise 2, find all solutions to the following system of linear equations.

$$\begin{aligned}
 x_1 + 2x_2 + x_3 + x_4 &= 0 \\
 x_2 - x_3 + x_4 &= 0 \\
 K &= \left\{ t_1 \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \end{bmatrix} + t_2 \begin{bmatrix} -1 \\ -1 \\ 0 \\ 1 \end{bmatrix} : t_1, t_2 \in \mathbb{R} \right\}
 \end{aligned}$$

Problem 3

Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be defined by $T(a, b, c) = (a + b, 2a - c)$. Determine $T^{-1}(1, 11)$.

$$\begin{aligned}
 T(a, b, c) &= (a + b, 2a - c) \\
 1 &= a + b \quad 11 = 2a - c \\
 b &= 1 - a \quad c = 2a - 11 \\
 (a, b, c) &= (a, 1 - a, 2a - 11) \\
 T^{-1}(1, 11) &= \{(a, 1 - a, 2a - 11) : a \in \mathbb{R}\}
 \end{aligned}$$

Problem 4

Determine which of the following systems of linear equations has a solution.

$$\begin{aligned}
 x_1 + x_2 + 3x_3 - x_4 &= 0 \\
 x_1 + x_2 + x_3 + x_4 &= 1 \\
 x_1 - 2x_2 + x_3 - x_4 &= 1 \\
 4x_1 + x_2 + 8x_3 - x_4 &= 0
 \end{aligned}$$

$$A = \begin{bmatrix} 1 & 1 & 3 & -1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & -2 & 1 & -1 & 1 \\ 4 & 1 & 8 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & \frac{25}{6} \\ 0 & 1 & 0 & 0 & \frac{4}{3} \\ 0 & 0 & 1 & 0 & -\frac{5}{2} \\ 0 & 0 & 0 & 1 & -2 \end{bmatrix}$$

$$s = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \frac{25}{6} \\ \frac{4}{3} \\ -\frac{5}{2} \\ -2 \end{bmatrix}$$

Problem 5

u A certain economy consists of two sectors: goods and services. Suppose that 60% of all goods and 30% of all services are used in the production of goods. What proportion of the total economic output is used in the production of goods?

$$P = \frac{.6G + .3S}{G + S} = \frac{.6G + .3S}{T_2}$$

$$P = \frac{.9G}{T_2} = .45, \text{ where } G = 1, S = 1, T_2 = 2$$

Problem 6

Use Gaussian elimination to solve the following systems of linear equations.

$$\begin{aligned} 2x_1 - 2x_2 - x_3 + 6x_4 - 2x_5 &= 1 \\ x_1 - x_2 + x_3 + 2x_4 - x_5 &= 2 \\ 4x_1 - 4x_2 + 5x_3 + 7x_4 - x_5 &= 6 \end{aligned}$$

$$A = \begin{bmatrix} 2 & -2 & -1 & 6 & -2 & 1 \\ 1 & -1 & 1 & 2 & -1 & 2 \\ 4 & -4 & 5 & 7 & -1 & 6 \end{bmatrix}$$

Problem 7

Let the reduce row echolon form of A be

$$\begin{bmatrix} 1 & 0 & 2 & 0 & -2 \\ 0 & 1 & -5 & 0 & -3 \\ 0 & 0 & 0 & 1 & 6 \end{bmatrix}$$

Determine A if the first, second, and forth columns of A are

$$\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \text{ and } \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$$

Problem 8

Let W be the subspace of $M_{2 \times 2}(\mathbb{R})$ consisting of the symmetric 2×2 matrices. Then set

$$S = \left\{ \begin{bmatrix} 0 & -1 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 1 & 9 \end{bmatrix}, \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}, \begin{bmatrix} -1 & -2 \\ 2 & -1 \end{bmatrix} \right\}$$

generates W . Find a subset of S that is a basis for W .

Problem 9

Let V be a in Exercise 12

$$\begin{aligned} x_1 - x_2 + 2x_4 - 3x_5 + x_6 &= 0 \\ 2x_1 - x_2 - x_3 + 3x_4 - 4x_5 + 4x_6 &= 0 \end{aligned}$$

1. Show that $S = \{(0, 1, 0, 1, 1, 0), (0, 2, 1, 1, 0, 0)\}$ is a linearly independent subset of V .
2. Extend S to a basis for V .