MATE 5150: Asignacion #1

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Problem 1

Find the equation of the plane through he following pairs of points in space. $P_1(1,1,1)$, $P_2(5,5,5)$, and $P_3(-6,4,2)$.

$$\begin{aligned}\overrightarrow{AB} &= (5-1, 5-1, 5-1) = (4,4,4)\\ \overrightarrow{AC} &= (-6-1, 4-1, 2-1) = (-7,3,1)\\ x &= (1,1,1) + s(4,4,4) + t(-7,3,1)\end{aligned}$$

Problem 2

Show thaat the midpoint of the line ssegment joining the points (a,b) and (c,d) is $(\frac{a+c}{2}, \frac{b+d}{2})$.

$$\begin{aligned}\text{Midpoint} &= \frac{1}{2}(\overrightarrow{OA} + \overrightarrow{OB})\\ &= \frac{1}{2}(\begin{bmatrix} a \\ b \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix})\\ &= \frac{1}{2}(\begin{bmatrix} a+c \\ b+d \end{bmatrix})\\ &= (\frac{a+c}{2}, \frac{b+d}{2})\end{aligned}$$

Problem 3

Let $S = \{0,1\}$ and $F = R$. In $\mathcal{F}(\mathcal{S}, \mathcal{R})$, show that $f = g$ and $f+g = h$, where $f(t) = 2t+1$, $g(t) = 1+4t-2t^2$, and $h(t) = 5^t + 1$.

$$d$$

Problem 4

Let V denote the set of ordered pairsa of real numbers. If (a_1, a_2) and (b_1, b_2) are elements of V and $c \in R$, define $(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 b_2)$ and $c(a_1, a_2) = (ca_1, ca_2)$.

$$d$$

Problem 5

Let $V = \{(a_1, a_2) : a_1, a_2 \in R\}$. For (a_1, a_2) and (b_1, b_2) in V and $c \in R$, defin $(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 + b_2)$ and $c(a_1, a_2) = (ca_1, ca_2)$. Is V a vector space over R with these operations? Justify your answer.

$$d$$

Problem 6

Prove that $A + A^t$ is symmetric for any matrix A.

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Problem 7

Let W_1, W_3, and W_4 be as in Excersise 8. Describe $W_1 \cap W_3$, $W_1 \cap W_4$, and $W_3 \cap W_4$, and observe that each is a subspace of R^3.

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Problem 8

Let $C^n(R)$ denote the set of all real-valued functions defined on the real line that have a continuous nth derivative. Prove that $C^n(R)$ is a subspace of $\mathcal{F}(R, R)$.

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