MATE 5150: Asignacion #7

Due on Noviembre 14, 2024

Dr. Pedro Vasquez

Alejandro Ouslan

Problem 1

Compute the determinants of the following matrices in $M_{2x2}(C)$.

$$\begin{bmatrix} 2i & 3 \\ 4 & 6i \end{bmatrix}$$

$$\begin{aligned} |C| &= 2i \cdot 6i - 3 \cdot 4 \\ |C| &= 12i^2 - 12 \\ |C| &= -12 - 12 \\ |C| &= -24 \end{aligned}$$

Problem 2

The classical adjoint of a $2x2$ matrix $A \in M_{2x2}(F)$ is the matrix

$$C = \begin{bmatrix} A_{22} & -A_{12} \\ -A_{21} & A_{11} \end{bmatrix}$$

Prove that

- $CA = AC = [det(A)]I$

$$\begin{aligned} CA &= \begin{bmatrix} A_{22} & -A_{12} \\ -A_{21} & A_{11} \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} A_{22}A_{11} - A_{12}A_{21} & 0 \\ 0 & -A_{21}A_{12} + A_{11}A_{22} \end{bmatrix} \\ AC &= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} A_{22} & -A_{12} \\ -A_{21} & A_{11} \end{bmatrix} = \begin{bmatrix} A_{11}A_{22} - A_{12}A_{21} & 0 \\ 0 & -A_{21}A_{12} + A_{11}A_{22} \end{bmatrix} \\ \det(A) &= (A_{22}A_{11} - A_{12}A_{21}) \cdot (-A_{21}A_{12} + A_{11}A_{22}) \\ \det(A)I &= \begin{bmatrix} A_{22}A_{11} - A_{12}A_{21} & 0 \\ 0 & -A_{21}A_{12} + A_{11}A_{22} \end{bmatrix} \end{aligned}$$

- $det(C) = det(A)$

$$\begin{aligned} det(C) &= A_{22}A_{11} - A_{12}A_{21} \\ det(A) &= A_{11}A_{22} - A_{12}A_{21} \end{aligned}$$

- The classical adjoint of A^T is C^T

$$\begin{aligned} A^T &= \begin{bmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{bmatrix} \\ C^T &= \begin{bmatrix} A_{22} & -A_{21} \\ -A_{12} & A_{11} \end{bmatrix} \end{aligned}$$

- If A is invertible, then $A^{-1} = [det(A)]^{-1}C$

$$\begin{aligned} A^{-1} &= \frac{1}{det(A)} \begin{bmatrix} A_{22} & -A_{12} \\ -A_{21} & A_{11} \end{bmatrix} \\ A^{-1} &= \begin{bmatrix} \frac{A_{22}}{det(A)} & \frac{-A_{12}}{det(A)} \\ \frac{-A_{21}}{det(A)} & \frac{A_{11}}{det(A)} \end{bmatrix} \end{aligned}$$

Problem 3

Find the value of k such that satisfies the following equation.

$$det \begin{bmatrix} b_1+c_1 & b_2+c_2 & b_3+c_3 \\ a_1+c_1 & a_2+c_2 & a_3+c_3 \\ a_1+b_1 & a_2+b_2 & a_3+b_3 \end{bmatrix} = k \cdot det \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

$$det \begin{bmatrix} b_1+c_1 & b_2+c_2 & b_3+c_3 \\ a_1+c_1 & a_2+c_2 & a_3+c_3 \\ a_1+b_1 & a_2+b_2 & a_3+b_3 \end{bmatrix} =$$

Problem 4

Evaluate the determinant of the given matrix by cofactor expansion along the indicated row.

$$\begin{bmatrix} 1 & -1 & 2 & -1 \\ -3 & 4 & 1 & -1 \\ 2 & -5 & -3 & 8 \\ -2 & 6 & -4 & 1 \end{bmatrix}$$

along the fourth row.

Problem 5

Compute $det(E_i)$ if E_i is an elementary matrix of type i.

Problem 6

Use Cramer's rule to solve the given system of linear equations.

$$\begin{aligned} x_1 - x_2 + 4x_3 &= -4 \\ -8x_1 + 3x_2 + x_3 &= 8 \\ 2x_1 + x_2 + x_3 &= 0 \end{aligned}$$

Problem 7

A matrix $M \in M_{nxn}(F)$ is called nipotent if, some positive interrger k, $M^k = 0$, where 0 is the nxn zero matrix. Prove that if M is nilpotent, then $det(M) = 0$.

Problem 8

Use determinants to prove that if $A, B \in M_{nxn}(F)$ are sucht that $AB = I$, then A is invertible (and hence $B = A^{-1}$).

Problem 9

Let $A \in M_{nxn}(F)$ have the form

$$A = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & a_0 \\ -1 & 0 & 0 & \cdots & 0 & a_1 \\ 0 & -1 & 0 & \cdots & 0 & a_2 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -1 & a_{n-1} \end{bmatrix}$$

Compute $det(A + tI)$ where I is the nxn identity matrix.

Problem 10