MATE 5150: Exam 01 Review

Alejandro Ouslan

Academic Year 2024-2025

Contents

1 Determine if T is a Vector Space

1.1 Conditions

1. $x + y = y + x$
2. $x + (y + z) = (x + y) + z$
3. $\exists 0 \in V \quad \text{s.t.} \quad x + 0 = x$
4. $\exists 0 \in V \quad \text{s.t.} \quad x + y = 0$
5. $1x = x$
6. $a(bx) = (ab)x$
7. $a(x + y) = ax + ay$
8. $(a + b)x = ax + bx$

1.2 Summery of the Conditions

1. $0x = 0$
2. $(-a)x = -ax = a(-x)$
3. $a0 = 0$

1.3 Example

Let $S = \{(a_1, a_2) : a_1, a_2 \in \mathbb{R}\}$ For $(a_1, a_2), (b_1, b_2) \in S$ and $c \in \mathbb{R}$, define

$$(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 - b_2) \quad \text{and} \quad c(a_1, a_2) = (ca_1, ca_2)$$

Property 1: (fails to be a vector space)

$$\begin{aligned}(a_1, a_2) + (b_1, b_2) &= (b_1, b_2) + (a_1, a_2) \\ (a_1 + b_1, a_2 - b_2) &\neq (b_1 + a_1, b_2 - a_2)\end{aligned}$$

2 Determine if T is in the Span of S

3 Proove that the pare of vectors is a basis for vector space V

1. Proove that the set S is linearly independent.
2. Proove that the set S spans V.

4 Generate a polynomial of Lagrange Interpolation of degree 3

$$P(x) = \sum_{i=0}^{n} y_i \mathscr{L}_i(x)$$

$$\mathscr{L}_i(x) = \prod_{\substack{0 \le j \le n \\ j \neq i}} \frac{x - x_j}{x_i - x_j} = \frac{(x - x_0) \cdots (x - x_i) \cdots (x - x_n)}{(x_i - x_0) \cdots (x_i - x_{i-1}) \cdots (x_i - x_n)}$$

4.1 Example for $n = 3$

$$\{(x_0, f(x_0)), (x_1, f(x_1)), (x_2, f(x_2)), (x_3, f(x_3))\}$$

$$\begin{aligned}P(x) = &\frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)} f(x_0) + \\ &+ \frac{(x - x_0)(x - x_2)(x - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)} f(x_1) + \\ &+ \frac{(x - x_0)(x - x_1)(x - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)} f(x_2) +\end{aligned}$$

5 Determine if T is bijective

1. Proove that T is linear.
2. Find the kernel of T.
3. Find the Rank of T.
4. Determine if it is 1-1.

6 Change of Basis

7 Answer of Given Questions

In R^2, let L be the line $y = mx$, where $m \neq 0$. Find an expression for $T(x, y)$, where

1. T is the reflection of R^2 about L.

$$M_x = \frac{x_1 + x_2}{2} \quad \text{and} \quad M_y = \frac{y_1 + y_2}{2}$$
$$AB = \frac{y_2 - y_1}{x_2 - x_1} = -1/m$$

2. T is the projection on L along the line perpe qqndicular to L. (See the definition of projection in the exercises of Section 2.1.)