MATE 5150: Asignacion #1

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Problem 1

Solve the following systems of linear equations by the method introduced introduces in this section.

$$\begin{array}{lllll} x_1 & +2x_2 & +2x_3 & & = 2 \\ x_1 & & +8x_3 & +5x_4 & = -6 \\ x_1 & +x_2 & +5x_3 & +5x_4 & = 3 \end{array}$$

Problem 2

In each part, determine whether the given vector is in the span of S.

$$(2, -1, 1, -3) \quad \text{where} \quad S = \{x^3 + x^2 + x + 1, x^2 + x + 1, x + 1\}$$

Problem 3

Show that the vectors $(1, 1, 0)$, $(1, 0, 1)$, and $(0, 1, 1)$ generate $\mathbb{F}^3$

Problem 4

Let S_1 and S_2 be subsets of a vector space V. Prove that $span(S_1 \cap S_2) \subseteq span(S_1) \cap span(S_2)$. Give an example in which $span(S_1 \cap S_2)$ and $span(S_1) \cap span(S_2)$ are equal and one in which they are not equal.

Problem 5

Determine whether the following sets are linearly dependent or linearly independent.

$$\{x^3 - x, 2x^2 + 4, -2x^3 + 3x^2 + 2x + 6\}$$

Problem 6

In $M_{m\times n}(\mathbb{F})$, let E_{ij} denote the matrix whose only nonzero entry is a 1 in the ith row and jth column. Prove that $\{E_{ij} : 1 \leq i \leq m, 1 \leq j \leq n\}$ is linearly independent.

Problem 7

Let $f, g, \in \mathcal{F}(R, R)$ be the function defined by $f(x) = e^{rt}$ and $g(x) = e^{st}$, where $r \neq s$. Prove that f and g are linearly independent in $\mathcal{F}(R, R)$.

Problem 8

Determine which of the following sets are bases for $\mathbb{R}^3$.

$$\{(1,-3,1),(-3,1,3),(-2,-10,2)\}$$

Problem 9

The vectors $u_1 = (2,-3,1)$, $u_2 = (1,4,-2)$, $u_3 = (-8,12,-4)$, $u_4 = (1,37,-17)$ and $u_5 = (-3,-5,8)$ generate $\mathbb{R}^3$. Find a subset of the set $\{u_1,u_2,u_3,u_4,u_5\}$ that is a basis for $\mathbb{R}^3$.

Problem 10

The vectors $u_1 = (1,1,1,1)$, $u_2 = (0,1,1,1)$, $u_3 = (0,0,1,1)$, and $u_4 = (0,0,0,1)$ form a basis for $\mathbb{R}^4$. Find the unique representation of an arbitrary vector $v = (a_1,a_2,a_3,a_4)$ $\mathbb{R}^4$ as a linear combination of u_1,u_2,u_3,u_4.

Problem 11

In each part, use the Lagrange interpolation formula to construct the polynomial of smallest degree whose graph contains the following points. $(-2,3),(-1,-6),(1,0),(3,-2)$.

Problem 12

The set of all upper triangular $n\times n$ matrices is a subspace of W of $M_{n\times n}(\mathbb{F})$. Find a basis for W. What is the dimension of W?