MATE 5150: Asignacion #3

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Problem 1

For each of the following homogeneous systems of linear equations, find the dimensions of and a basis for the solution space.

$$\begin{aligned} x_1 + 2x_2 + x_3 + x_4 &= 0 \\ x_2 - x_2 + x_3 &= 0 \end{aligned}$$

Problem 2

Using the results of Exercise 2, find all solutions to the following system of linear equations.

$$\begin{aligned} &x_1 + 2x_2 + x_3 + x_4 &&= 0 \\ &x_2 - x_2 + x_3 &&= 0 \end{aligned}$$

Problem 3

Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be defined by $T(a, b, c) = (a + b, 2a - c)$. Determine $T^{-1}(1, 11)$.

Problem 4

Determine which of the following systems of liner equations has a solution.

$$\begin{aligned} &x_1 + x_2 + 3x_3 - x_3 &&= 0 \\ &x_1 + x_2 + x_3 + x_4 &&= 1 \\ &x_1 - 2x_2 + x_3 - x_4 &&= 1 \\ &4x_1 + x_2 + 8x_3 - x_4 &&= 0 \end{aligned}$$

Problem 5

A certain economy consists of two sectors: goods and services. Suppose that 60% of all goods and 30% of all services are used in the production of goods. What proportion of the total economic output is used in the production of goods?

Problem 6

Use Gaussian elimination to solve the following systems of linear equations.

$$\begin{aligned} 2x_1 - 2x_2 - x_3 + 6x_4 - 2x_5 &= 1 \\ x_1 - x_2 + x_3 + 2x_4 - x_5 &= 2 \\ 4x_1 - 4x_2 + 5x_3 + 7x_4 - x_5 &= 6 \end{aligned}$$

Problem 7

Let the reduce row echolon form of A be

$$\begin{bmatrix} 1 & 0 & 2 & 0 & -2 \\ 0 & 1 & -5 & 0 & -3 \\ 0 & 0 & 0 & 1 & 6 \end{bmatrix}$$

Determine A if the first, second , and forth columns of A are

$$\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}, \text{ and } \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$$

Problem 8

Let W be the subspace of $M_{2\times 2}(\mathbb{R})$ consisting of the symmetric 2×2 matrices. Then set

$$S = \left\{ \begin{bmatrix} 0 & -1 \\ -1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 1 & 9 \end{bmatrix}, \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}, \begin{bmatrix} -1 & -2 \\ 2 & -1 \end{bmatrix} \right\}$$

generates W. Find a subset of S that is a basis for W.

Problem 9

Let V be a in Exercise 12

$$\begin{aligned} x_1 - x_2 + 2x_4 - 3x_5 + x_6 &= 0 \\ 2x_1 - x_2 - x_3 + 3x_4 - 4x_5 + 4x_6 &= 0 \end{aligned}$$

1. Show that $S = \{(0,1,0,1,1,0), (0,2,1,1,0,0)\}$ is a linerly independent subset of V.
2. Extend S to a basis for V.