MATE 5150: Asignacion #3

Due on Septiembre 26, 2024

Dr. Pedro Vasquez

Alejandro Ouslan

Problem 1

Determine whether T is invertible and justify your answer.

$$T : \mathbb{R}^2 \to \mathbb{R}^3, T(a_1, a_2) = (3a_1 - a_2, a_2, 4a_1)$$

Problem 2

Determine whether T is invertible and justify your answer.

$$T : M_{2\times 2}(\mathbb{R}) \to M_{2\times 2}(\mathbb{R}), T(\begin{bmatrix} a & b \\ c & d \end{bmatrix}) = \begin{bmatrix} a+b & a \\ c & c+d \end{bmatrix}$$

Problem 3

Is the following parirs of vector spaces are isomorphic? Justify your answer.

$$V = \{A \in M_{2\times 2}(\mathbb{R}) : tr(A) = 0\} \text{ and } R^4$$

Problem 4

Let

$$V = \left\{ \begin{bmatrix} a & a+b \\ 0 & c \end{bmatrix} : a, b, c \in F \right\}$$

Construct an isomorphism from V to F^3.

Problem 5

For each of the following pairs of ordered bases β and β' for $\mathbb{R}^2$, find the change of coordinates matrix that changes β-coordinates into β-coordinate

$$\beta = \{(-1, 3), (2, -1)\} \text{ and } \beta' = \{(0, 10), (5, 0)\}$$

Problem 6

For each of the folloing pairs of ordered bases β and β' for $P_2(\mathbb{R})$, find the change of coorinate matrix that changs β'-coordinates into β-coordinates.

$$\beta = \{1, x, x^2\} \text{ and } \beta' = \{a_2x^2 + a_1x + a_0, b_2x^2 + b_1x + b_0, c_2x^2 + c_1x + c_0\}$$

Problem 7

Let T be the linear operator on $\mathbb{R}^2$ defined by

$$T\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2a+b \\ a-3b \end{pmatrix}$$

let β be the standard ordered basis for $\mathbb{R}^2$, and let

$$\beta' = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$$

Use Theorem 2.23 and the fact that

$$\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}^{-1} = \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$

To find $[T]_{\beta'}$.

Problem 8

For each matrix A and ordered basis β, find $[L_A]_\beta$. Also, find and invertible matrix Q such that $[L_A]_\beta = Q^{-1}AQ$.

$$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} \text{ and } \beta = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \end{pmatrix} \right\}$$