Running Lecture Outline: [Course Code]

[Full Name]

Academic Year 2019-2020

Contents

1   Fall 2019

1.1   Fri, Sept 6: Phenomenology of Microscopic Physics

- Newtonian mechanics (i.e., $\mathbf{F} = m\mathbf{a}$) is an excellent theory; it applies to the vast majority of human-scale (and even interplanetary-scale) physics.

- Apart from relativistic effects at very high velocities (special relativity) or in very strong gravitational fields (general relativity), Newtonian mechanics accurately describes a huge range of phenomena, but around the end of the Nineteenth Century people became aware of some physical effects for which there is no sensible Newtonian explanation.

- Examples include:

  - the **double slit experiment** (done with light by Thomas Young in 1801, and with electrons by Tonomura in 1986)
  - the photoelectric effect (analyzed by Einstein in 1905 — in fact his Nobel-winning work)
  - the "quantum Venn diagram" puzzle, involving the overlaps of three polarizing filters
  - the stability of the hydrogen atom (i.e., the fact that the electron doesn't lose energy and spiral inward toward the proton).

  Remark 1. *How now, brown cow?*

  Definition 1. *The* Feynman kernel *is given by*

  $$K(x_b, t_b; x_a, t_a) = \int_{x(t_a)=x_a}^{x(t_b)=x_b} e^{(i/\hbar)S[x(t)]} \ \mathscr{D}x(t).$$

1.2   Mon, Sept 9: Review of Newtonian Mechanics

- A *Newtonian trajectory* $\mathbf{x}(t)$ ($t \in \mathbb{R}$) is given by solutions of the second order ODE

  $$m\,\ddot{\mathbf{x}}(t) = \mathbf{F}(\mathbf{x}(t)),$$

  where $m > 0$ is a basic parameter associated with a given Newtonian particle, called its *mass*.

- The force field $\mathbf{F}(\mathbf{x})$ — which we take to be static (i.e., not intrinsically dependent on time) for simplicity — is said to be *conservative* if there is a *potential function* $V(\mathbf{x})$ such that

$$\mathbf{F}(\mathbf{x}) = -\nabla V(\mathbf{x}).$$

Here, '∇' denotes the *gradient operator*,

$$\nabla V = \left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z} \right).$$

- For a conservative force field, we can find a *conserved quantity* along the Newtonian trajectories, namely the *total mechanical energy*.

$$E = H(\mathbf{x}, \mathbf{p}) := \frac{1}{2m} \mathbf{p}^2 + V(\mathbf{x}).$$

Here, $\mathbf{p}^2 := \mathbf{p} \cdot \mathbf{p} = \|\mathbf{p}\|^2$, and $\mathbf{p} := m\mathbf{v} := m\dot{\mathbf{x}}$ is the *momentum*.

1.3 Tue, Sept 10: Alternative Formulations of Newtonian Mechanics

- The **Hamiltonian formulation**:

$$\dot{\mathbf{x}} = \frac{\partial H}{\partial \mathbf{p}}, \qquad \dot{\mathbf{p}} = -\frac{\partial H}{\partial \mathbf{x}}.$$

- The **Lagrangian formulation**:

$$\delta S[\mathbf{x}(t)] = 0,$$

where the *action* on the time interval $[t_a, t_b]$ is given by

$$S[\mathbf{x}(t)] := \int_{t_a}^{t_b} \left[\frac{m}{2} \dot{\mathbf{x}}(t)^2 - V(\mathbf{x}(t)) \right] dt.$$

- Etc.

2 Spring 2020