COG250H1: Introduction to Cognitive Science **Fall 2018**

Lecture 2: Categorization (Part 1)

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2.1 An Overview of Cognitive Science (CogSci)

2.1.1 Motivation

Key writing points: (1) Black box approach (2) Examples of cogsci in real life (Machine learning, Stock Crash,) As machine learning is picking up (quickly), also hitting a problem that many of these problems are ”black boxes” (not understanding how it actually works, or how it works under the hood)

2.1.2 What is CogSci?

Cogsci seeks to develop a common language for describing **cognitive phenomena** that can be understood through multiple disciplines.

There are three models of cognitive science:

1. Generic Nominalism
    - The weakest definition of CogSci
    - In this vision, CogSci is not it’s own discipline. Instead, it is a generic term describing the study of mind in CogSci’s subdisciplines.
    - This vision is generally not accepted in third generation cognitive science
2. Interdisciplinary Ecclecticsm
    - The weakest definition of CogSci
    - In this vision, CogSci is not it’s own discipline. Instead, it is a generic term describing the study of mind in CogSci’s subdisciplines.
    - This vision is generally not accepted in third generation cognitive science
3. Synoptic Integration
    - The weakest definition of CogSci
    - In this vision, CogSci is not it’s own discipline. Instead, it is a generic term describing the study of mind in CogSci’s subdisciplines.
    - This vision is generally not accepted in third generation cognitive science

2.2 Basic Steps of Algorithm

- Generic step of the algorithm is to swap a basic variable with a non basic variable. For now assume that we have selected basic variable x_p and non-basic variable x_q to swap
- x_p can be swapped with x_q if and only if $Y_{pq} \neq 0$ because if Y_{pq} is equal to 0 then column vector Y_q can be represented as linear combination of m $-$ 1 basis vectors i.e.

$$Y_q = \sum_{i=1 \ i \neq p}^{m} y_{iq} * I_i$$

  and hence Y_q cannot be included in basic solution
- Now make q^{th} column as $\begin{bmatrix} 0 & ... & 0 & 1_p & 0 & ... & 0 \end{bmatrix}$ where 1_p signifies 1 at p^{th} position. For that dividep_{th}row of matrix$\begin{bmatrix} I & Y \end{bmatrix}$ and matrix $\begin{bmatrix} Y(0) \end{bmatrix}$ by Y_{pq} and apply the row operation $R_i \Rightarrow R_i - Y_{iq} * R_p$

2.3 Determining the Leaving Variable p

- While applying row transformation of $\begin{bmatrix} I & Y \end{bmatrix}$ rows of $\begin{bmatrix} I \end{bmatrix}$ also changes and are given by

$$Y_{i0}' = Y_{i0} - Y_{iq} * Y_{p0}/Y_{i0}$$

  Condition $Y_{i0}' \geq 0$ must satisfy otherwise x_q would not be a BFS.
- So choose p such that

$$p \in S = \underset{i}{argmin}\{Y_{i0}/Y_{iq} | Y_{iq} \geq 0\}$$

- If number of elements in S is $>$ 1 then the would become degenerate. Since non-degeneracy is assumed

$$p = \underset{i}{argmin}\{Y_{i0}/Y_{iq} | Y_{iq} \geq 0\}$$

2.4 Determining the Entering Variable q

- We Know that

$$\begin{bmatrix} I & Y \end{bmatrix} \begin{bmatrix} x_B \\ x_{NB} \end{bmatrix} = \begin{bmatrix} Y(0) \end{bmatrix}$$

$$x_B = Y_0 - Y x_{NB}$$

$$Where \begin{bmatrix} x_B \\ x_{NB} \end{bmatrix} \geq 0$$

- Initial Cost:

$$c^T \begin{bmatrix} x_B \\ x_{NB} \end{bmatrix} = c_B^T x_B + c_{NB}^T x_{NB}$$

$$= c_B^T x_B$$

$$= c_B^T Y_0$$

$$since \ \ x_{NB} = 0 \ \ and \ \ I * x_B + Y * x_{NB} = Y_0$$

- Now cost is

$$c^T \begin{bmatrix} x_B \\ x_{NB} \end{bmatrix} = c_B^T x_B + c_{NB}^T x_{NB}$$
$$= c_B^T (Y_0 - Y x_{NB}) + c_{NB}^T x_{NB}$$
$$= c_B^T Y_0 + (c_{NB-Y^T c_B})^T x_{NB}$$

- Now we can choose q such for which $(c_{NB} - Y^T c_B)_q < 0$
- Formalizing the above concept

$$\begin{bmatrix} 1 & 0 & ... & 0 & Y_{1,m+1} & Y_{1,m+2} & ... & Y_{1,n} \\ 0 & 1 & ... & 0 & Y_{2,m+1} & Y_{2,m+2} & ... & Y_{2,n} \\ . & . & . & . & . & . & ... & . \\ . & . & . & . & . & . & ... & . \\ . & . & . & . & . & . & ... & . \\ 0 & 0 & ... & 1 & Y_{m,m+1} & Y_{m,m+2} & ... & Y_{m,n} \end{bmatrix} \begin{bmatrix} x_1 \\ . \\ . \\ . \\ x_m \\ . \\ . \\ . \\ x_n \end{bmatrix} = \begin{bmatrix} y_{10} \\ . \\ . \\ . \\ y_{m0} \\ . \\ . \\ . \\ y_{n0} \end{bmatrix}$$

and

$$(c_{NB} - Y^T c_B)^T x_{NB} = \sum_{j=m+1}^{n} (c_j - Z_j) * x_j$$
$$Where \ \ Z_j = \sum_{i=1}^{m} (Y_{i,j} * c_i)$$

- To determine the entering variable choose j such that $(c_j - Z_j) < 0$

2.4.1 Theorem 8.1.

Given a non-degenerate Basic Feasible Solution with objective value Z' . Suppose $c_j - Z_j{}' < 0$ for some j there is a feasible solution with objective value $< Z'$. Also if variable x_j can be substituted for a variable in the basis for a new BFS, we get new BFS with value $Z_0 < 0$. If this cannot be done then the solution is unbounded.

2.5 Optimality condition

The Basic Feasible Solution is optimal if

$$\forall, \quad c_j - Z_j \geq 0$$

2.6 Some Points to Ponder

- f there does not exist p to replace then we have founded the recession direction and the cost can be reduced to $-\infty$
- In the worst case the Simplex Algorithm might visit all the extreme points. Example - Klee Minty cube

2.7 Duality

Every linear programming problem, referred to as a primal problem, can be converted into a dual problem, which provides an upper bound to the optimal value of the primal problem. The primal problem is:

$$\min_x c^T x$$

$$Ax = b$$

$$x \geq 0$$

$$Where \;\; A \in R^{m \times n}, \; Rank(A) = m$$

with the corresponding symmetric dual problem,

$$\max_y b^T y$$

$$A^T y \leq c$$

$$Where \;\; A \in R^{m \times n}, \; Rank(A) = m$$