

Lecture 2: Introduction + Categorization (Part 1)

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2.1 An Overview of Cognitive Science (CogSci)

2.1.1 Background / Motivation

Purpose: Set the stage for providing visions of cognitive science, introduce sub disciplines, briefly describe key events in cognitive science.

- Artificial Intelligence
- Cognitive Psychology
- Philosophy of Mind
- Cultural Anthropology
- Neuroscience
- Semiotics and Linguistics

Topic involving cognitive science:

- AlphaGo
- Computer Vision
- High Frequency Trading
- Literacy + Grammar

In cognitive science, the problem of the mind is being approached from multiple angles. A **metaphor** that you will often see in cognitive science is that the brain is a machine. The fundamental belief is that if science can figure out how to design and construct a mind, then science can figure out how the mind works. This is one of the reasons why the study of artificial intelligence and machine learning, is so intimately connected to the study of the mind. This approach is characterized by employing **Design Thinking** to create models of the mind.

There are, however, problems with this approach. Many machine learning systems these days are very black box. That is, although we understand the basic principles of how these systems work, they are not fully yet understood.

2.1.2 Metaphors

- "Brains host culture."

2.1.3 Theoretical Constucts and Plausibility

Key writing points: (1) Metaphors and multi aptness (2) Convergence of evidence (works for multiple streams of evidence) example:

2.1.4 What is CogSci?

We don't know what makes us think.

The one thing we can't account for in our scientific explanations is how we produce scientific explanations.

CogSci seeks to develop a common language for describing **cognitive phenomena** that can be understood through multiple disciplines.

There are three 'visions' for cognitive science:

1. Generic Nominalism (Very High Level)

- The weakest definition of CogSci
- In its weakest form, cognitive science is just the sum of the fields mentioned: psychology, artificial intelligence, linguistics, neuroscience, anthropology, and philosophy
- In this vision, CogSci is not it's own discipline. Instead, it is a generic term describing the study of the mind in CogSci's subdisciplines.
- Nominalism means name. Generic corresponds to genre. Cognitive science is a name for a a genre of related topics.
- Under this vision disciplines related to the study of mind would be considered the cognitive sciences. That is, the only requirement for doing cognitive science is to do something related to the study of the mind.
- This vision is generally not accepted in third generation cognitive science

2. Interdisciplinary Ecclecticism (Still High Live)

- A stonger definition than generic nominalism, not the strongest
- Under this vision, different disciplines
- This approach is characterized by drawing from CogSci's sub disciplines to analyze the mind. Instead of holding to a single paradigm or framework of thought, IE seeks to integrate knowledge from all the sub disciplines to gain insight into the mind.
- Analogy: Interfaith dialogue. Suppose that buddhists, catholics, muslims and other people fof different faith are invited to a cocktail party.
- This model, however, is very unstable. Typically this model either devolves into generic nominalism or evolves into synoptic integration.

3. Synoptic Integration (Lowest level)

- The strongest definition of CogSci.
- CogSci, under this model, is a unique discipline. Doing CogSci is deliberate.

2.2 Naturalistic Imperative

One of the goals of science is to 'naturalize' our understanding of the universe. There is an innate human desire to bring our minds in line with all the scientific disciplines.

The Naturalistic Imperative is a term coined by John Vervaeke. It comprises of three parts. Analysis, Formalization, and Mechanization. To understand the naturalistic imperative it is useful to look at previous scientific revolutions:

2.2.1 Analyze

2.2.1.1 A brief introduction to presocratic thinkers

<https://www.youtube.com/watch?v=ZkMAx04jDx0> Video A

Main idea: The Presocratics ushered in the philosophic and scientific mindset that would dramatically alter the course of western civilization.

In general, Ancient Greek philosophers form the intellectual and cultural foundations of western civilization. If one wants to understand western civilization one must understand the works of Socrates, Plato and Aristotle. The concept of rational thought and logos was originated by presocratic thinkers. Talking points: Introduction to the Presocratics: (1) Presocratic philosophers (0) Why study them?

(i) They are the intellectual and cultural foundations of western civilization. If one wants to understand western civilization one must understand Plato and Aristotle.. If one is to understand Plato and Aristotle, one would benefit from understanding the pre-socratics. (ii) Presocratics birthed rational thought... It is here that the logos came about... The fact that rational thought came from here (1) Not actually philosophers, wouldn't call themselves philosophers either.. A better way to describe these group of people would be thinkers (2) End of Presocratic era: The nature of philosophy changed from studying nature to .. Socrates stopped focusing on metaphysical questions, because he felt that studying those questions had no impact on his life. Instead, he started turning his attention to practical morality and political thought. (2) Thales

Thales Not much left on him We know three things

2.2.2 Formalize

2.2.3 Mechanize

- Generic step of the algorithm is to swap a basic variable with a non-basic variable. For now assume that we have selected basic variable x_p and non-basic variable x_q to swap
- x_p can be swapped with x_q if and only if $Y_{pq} \neq 0$ because if Y_{pq} is equal to 0 then column vector Y_q can be represented as linear combination of $m - 1$ basis vectors i.e.

$$Y_q = \sum_{i=1, i \neq p}^m y_{iq} * I_i$$

and hence Y_q cannot be included in basic solution

- Now make q^{th} column as $\begin{bmatrix} 0 & \dots & 0 & 1_p & 0 & \dots & 0 \end{bmatrix}$ where 1_p signifies 1 at p^{th} position. For that divide p^{th} row of matrix $\begin{bmatrix} I & Y \end{bmatrix}$ and matrix $\begin{bmatrix} Y(0) \end{bmatrix}$ by Y_{pq} and apply the row operation $R_i \Rightarrow R_i - Y_{iq} * R_p$

2.3 Determining the Leaving Variable p

- While applying row transformation of $\begin{bmatrix} I & Y \end{bmatrix}$ rows of $\begin{bmatrix} I \end{bmatrix}$ also changes and are given by

$$Y_{i0}' = Y_{i0} - Y_{iq} * Y_{p0}/Y_{i0}$$

Condition $Y_{i0}' \geq 0$ must satisfy otherwise x_q would not be a BFS.

- So choose p such that

$$p \in S = \underset{i}{\operatorname{argmin}} \{Y_{i0}/Y_{iq} | Y_{iq} \geq 0\}$$

- If number of elements in S is > 1 then the would become degenerate. Since non-degeneracy is assumed

$$p = \underset{i}{\operatorname{argmin}} \{Y_{i0}/Y_{iq} | Y_{iq} \geq 0\}$$

2.4 Determining the Entering Variable q

- We Know that

$$\begin{bmatrix} I & Y \end{bmatrix} \begin{bmatrix} x_B \\ x_{NB} \end{bmatrix} = \begin{bmatrix} Y(0) \end{bmatrix}$$

$$x_B = Y_0 - Y x_{NB}$$

$$\text{Where } \begin{bmatrix} x_B \\ x_{NB} \end{bmatrix} \geq 0$$

- Initial Cost:

$$\begin{aligned} c^T \begin{bmatrix} x_B \\ x_{NB} \end{bmatrix} &= c_B^T x_B + c_{NB}^T x_{NB} \\ &= c_B^T x_B \\ &= c_B^T Y_0 \end{aligned}$$

$$\text{since } x_{NB} = 0 \text{ and } I * x_B + Y * x_{NB} = Y_0$$

- Now cost is

$$\begin{aligned} c^T \begin{bmatrix} x_B \\ x_{NB} \end{bmatrix} &= c_B^T x_B + c_{NB}^T x_{NB} \\ &= c_B^T (Y_0 - Y x_{NB}) + c_{NB}^T x_{NB} \\ &= c_B^T Y_0 + (c_{NB} - Y^T c_B)^T x_{NB} \end{aligned}$$

- Now we can choose q such for which $(c_{NB} - Y^T c_B)_q < 0$

- Formalizing the above concept

$$\begin{bmatrix} 1 & 0 & \dots & 0 & Y_{1,m+1} & Y_{1,m+2} & \dots & Y_{1,n} \\ 0 & 1 & \dots & 0 & Y_{2,m+1} & Y_{2,m+2} & \dots & Y_{2,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 1 & Y_{m,m+1} & Y_{m,m+2} & \dots & Y_{m,n} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ \vdots \\ x_m \\ \vdots \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} y_{10} \\ \vdots \\ \vdots \\ y_{m0} \\ \vdots \\ \vdots \\ y_{n0} \end{bmatrix}$$

and

$$(c_{NB} - Y^T c_B)^T x_{NB} = \sum_{j=m+1}^n (c_j - Z_j) * x_j$$

$$\text{Where } Z_j = \sum_{i=1}^m (Y_{i,j} * c_i)$$

- To determine the entering variable choose j such that $(c_j - Z_j) < 0$

2.4.1 Theorem 8.1.

Given a non-degenerate Basic Feasible Solution with objective value Z' . Suppose $c_j - Z_j' < 0$ for some j there is a feasible solution with objective value $< Z'$. Also if variable x_j can be substituted for a variable in the basis for a new BFS, we get new BFS with value $Z_0 < 0$. If this cannot be done then the solution is unbounded.

2.5 Optimality condition

The Basic Feasible Solution is optimal if

$$\forall, \quad c_j - Z_j \geq 0$$

2.6 Some Points to Ponder

- If there does not exist p to replace then we have founded the recession direction and the cost can be reduced to $-\infty$
- In the worst case the Simplex Algorithm might visit all the extreme points. Example - Klee Minty cube

2.7 Duality

Every linear programming problem, referred to as a primal problem, can be converted into a dual problem, which provides an upper bound to the optimal value of the primal problem. The primal problem is:

$$\min_x c^T x$$

$$Ax = b$$

$$x \geq 0$$

$$\text{Where } A \in R^{m \times n}, \text{ Rank}(A) = m$$

with the corresponding symmetric dual problem,

$$\max_y b^T y$$

$$A^T y \leq c$$

$$\text{Where } A \in R^{m \times n}, \text{ Rank}(A) = m$$