COG250H1: Introduction to Cognitive Science **Fall 2018**

Lecture 4: Prototype Theory + Micro Theory

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4.1 Categorization

What does categorizaiton do for us? Makes the basic claim of being able to classify objects.

4.1.1 Smith on Categorizaiton

Claim: Similarity judgements drive categorizaiton Expereiment: 1. Suppose that an observer is shown two profile pictures. One of a cat and one of a dog. 2. Suppose that ask the observer if they are similar (expect a yes) 3. Are they the same (expect a no)

4.1.2 On Salience

Salience is not a feature of the world. It is determined by the observer.

4.1.3 Classical Theory Of Concepts

Problem: Locate things in terms of the object to object relation the natural world.

Critiques: - Essences - Family of resemblance

4.1.4 Prototype Theory

Proponents: Elanor Rosch

- What is a prototype? A prototype is the central or representative member of a category.
-

- coined by Elanor Rosch - created as a seponse to the classical Theory

4.1.5 What is CogSci?

We don't know what makes us think.

The one thing we can't account for in our scientific explanations is how we produce scientific explanations.

Cogsci seeks to develop a common language for describing **cognitive phenomena** that can be understood through multiple disciplines.

There are three models of cognitive science:

1. Generic Nominalism
   - The weakest definition of CogSci
   - In this vision, CogSci is not it's own discipline. Instead, it is a generic term describing the study of mind in CogSci's subdisciplines.
   - This vision is generally not accepted in third generation cognitive science
2. Interdisciplinary Ecclecticsm
   - A stonger definition than generic nominalism, not the strongest
   - This approach is characterized by drawing from CogSci's sub disciplines to analyze the mind. Instead of holding to a single paradigm or framework of thought, IE seeks to integrate knowledge from all the sub disciplines to gain insight into the mind.
   - Analogy: Interfaith dialgoue
   - This model, however, is very unstable. Typically this model either devolves into generic nominalism or evolves into synoptic integration.
3. Synoptic Integration
   - The strongest definition of CogSci.
   - CogSci, under this model, is a unique discipline. Doing CogSci is deliberate.

4.2 Naturalistic Imperative

On of the goals of science is to 'naturalize' our understanding of the universe. There is a innate human desire to bring our minds in line with all the scientific disciplines.

The Naturalistic Imperative is a term coined by John Vervaeke. It comprises of three parts. Analysis, Formalization, and Mechanization. To understand the naturalistic imperative it is useful to look at previous scientific revolutions:

4.2.1 Analyze

4.2.1.1 A brief introduction to presocratic thinkers

https://www.youtube.com/watch?v=ZkMAx04jDx0Video A

Main idea: The Presocratics ushered in the philosophic and scientific mindset that would dramatically alter the course of western civilization.

In general, Ancient greek philosophers form the intellectual and cultural foundations of western civilization. If one wants to understand western civilization one must understand the works of Socrates, Plato and Aristotle. The concept of rational thought and logos was originated by presocratic thinkers. Talking points: Introduction to the Presocratics: (1) Presocratic philosophers (0) Why study them?

(i) They are the intellectual and cultural foundations of western civilization If one wants to understand western civilization one must understand Plato and Aristotle.. If one is to understand Plato and Aristotle, one would benefit from understanding the pre socratics. (ii) Presocratics birthed rational thought... It is here that the logos came about... The fact that rational thought came from here (1) Not actually philosophers, wouldn't call themselves philosophers either.. A better way to describe these group of ople would be thinkers (2) End of Presocratic era: The nature of philosophy changed from studying nature to .. Socrates stopped focusing on metaphysical questions, becuase he felt that studying those questions had no impact on his life. Instead, he started turned his attention to practical morality and political thought. (2) Thales

Thales Not much left on him We know three things

4.2.2 Formalize

4.2.3 Mechanize

- Generic step of the algorithm is to swap a basic variable with a non basic variable. For now assume that we have selected basic variable x_p and non-basic variable x_q to swap
- x_p can be swapped with x_q if and only if $Y_{pq} \neq 0$ because if Y_{pq} is equal to 0 then column vector Y_q can be represented as linear combination of m $-$ 1 basis vectors i.e.

$$Y_q = \sum_{i=1 \;\; i \neq p}^{m} y_{iq} * I_i$$

  and hence Y_q cannot be included in basic solution
- Now make q^{th} column as $\begin{bmatrix} 0 & ... & 0 & 1_p & 0 & ... & 0 \end{bmatrix}$ where 1_p signifies 1 at p^{th} position. For that dividep_{th}row of matrix$\begin{bmatrix} I & Y \end{bmatrix}$ and matrix $\begin{bmatrix} Y(0) \end{bmatrix}$ by Y_{pq} and apply the row operation $R_i \Rightarrow R_i - Y_{iq} * R_p$

4.3 Determining the Leaving Variable p

- While applying row transformation of $\begin{bmatrix} I & Y \end{bmatrix}$ rows of $\begin{bmatrix} I \end{bmatrix}$ also changes and are given by

$$Y_{i0}' = Y_{i0} - Y_{iq} * Y_{p0}/Y_{i0}$$

  Condition $Y_{i0}' \geq 0$ must satisfy otherwise x_q would not be a BFS.
- So choose p such that

$$p \in S = \underset{i}{argmin}\{Y_{i0}/Y_{iq} | Y_{iq} \geq 0\}$$

- If number of elements in S is $>$ 1 then the would become degenerate. Since non-degeneracy is assumed

$$p = \underset{i}{argmin}\{Y_{i0}/Y_{iq} | Y_{iq} \geq 0\}$$

4.4 Determining the Entering Variable q

- We Know that

$$\begin{bmatrix} I & Y \end{bmatrix} \begin{bmatrix} x_B \\ x_{NB} \end{bmatrix} = \begin{bmatrix} Y(0) \end{bmatrix}$$

$$x_B = Y_0 - Yx_{NB}$$

$$Where \begin{bmatrix} x_B \\ x_{NB} \end{bmatrix} \geq 0$$

- Initial Cost:

$$c^T \begin{bmatrix} x_B \\ x_{NB} \end{bmatrix} = c_B^T x_B + c_{NB}^T x_{NB}$$
$$= c_B^T x_B$$
$$= c_B^T Y_0$$
$$since \;\; x_{NB} = 0 \;\; and \;\; I * x_B + Y * x_{NB} = Y_0$$

- Now cost is

$$c^T \begin{bmatrix} x_B \\ x_{NB} \end{bmatrix} = c_B^T x_B + c_{NB}^T x_{NB}$$
$$= c_B^T (Y_0 - Yx_{NB}) + c_{NB}^T x_{NB}$$
$$= c_B^T Y_0 + (c_{NB-Y^T c_B})^T x_{NB}$$

- Now we can choose q such for which $(c_{NB} - Y^T c_B)_q < 0$
- Formalizing the above concept

$$\begin{bmatrix} 1 & 0 & ... & 0 & Y_{1,m+1} & Y_{1,m+2} & ... & Y_{1,n} \\ 0 & 1 & ... & 0 & Y_{2,m+1} & Y_{2,m+2} & ... & Y_{2,n} \\ . & . & . & . & . & . & ... & . \\ . & . & . & . & . & . & ... & . \\ . & . & . & . & . & . & ... & . \\ 0 & 0 & ... & 1 & Y_{m,m+1} & Y_{m,m+2} & ... & Y_{m,n} \end{bmatrix} \begin{bmatrix} x_1 \\ . \\ . \\ . \\ x_m \\ . \\ . \\ . \\ x_n \end{bmatrix} = \begin{bmatrix} y_{10} \\ . \\ . \\ . \\ y_{m0} \\ . \\ . \\ . \\ y_{n0} \end{bmatrix}$$

and

$$(c_{NB} - Y^T c_B)^T x_{NB} = \sum_{j=m+1}^{n} (c_j - Z_j) * x_j$$

$$Where \;\; Z_j = \sum_{i=1}^{m} (Y_{i,j} * c_i)$$

- To determine the entering variable choose j such that $(c_j - Z_j) < 0$

4.4.1 Theorem 8.1.

Given a non-degenerate Basic Feasible Solution with objective value Z' . Suppose $c_j - Z_j{}' < 0$ for some j there is a feasible solution with objective value $< Z'$. Also if variable x_j can be substituted for a variable in the basis for a new BFS, we get new BFS with value $Z_0 < 0$. If this cannot be done then the solution is unbounded.

4.5 Optimality condition

The Basic Feasible Solution is optimal if

$$\forall, \quad c_j - Z_j \geq 0$$

4.6 Some Points to Ponder

- f there does not exist p to replace then we have founded the recession direction and the cost can be reduced to $-\infty$

- In the worst case the Simplex Algorithm might visit all the extreme points. Example - Klee Minty cube

4.7 Duality

Every linear programming problem, referred to as a primal problem, can be converted into a dual problem, which provides an upper bound to the optimal value of the primal problem. The primal problem is:

$$\min_{x} c^T x$$

$$Ax = b$$

$$x \geq 0$$

$$Where \;\; A \in R^{m \times n},\; Rank(A) = m$$

with the corresponding symmetric dual problem,

$$\max_{y} b^T y$$

$$A^T y \leq c$$

$$Where \;\; A \in R^{m \times n},\; Rank(A) = m$$