# Approximate nearest neighbor search using the Hierarchical Navigable Small World (HNSW) algorithm

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# Outline

# Voronoi diagram for a set of points

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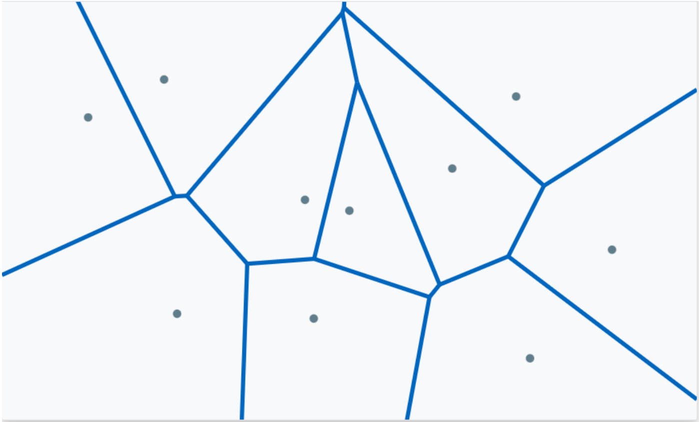
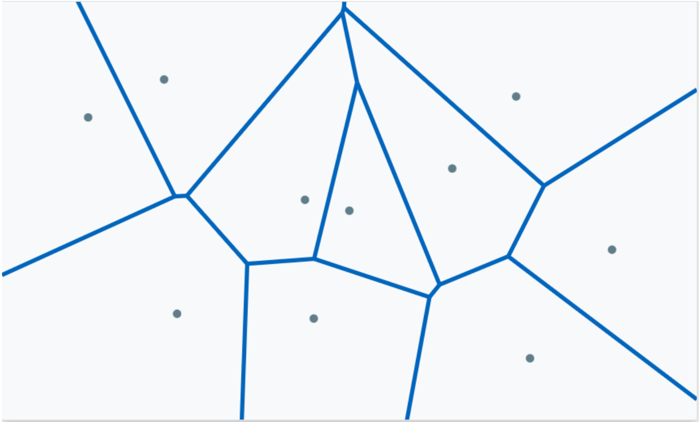


# Voronoi diagram to Delaunay graph



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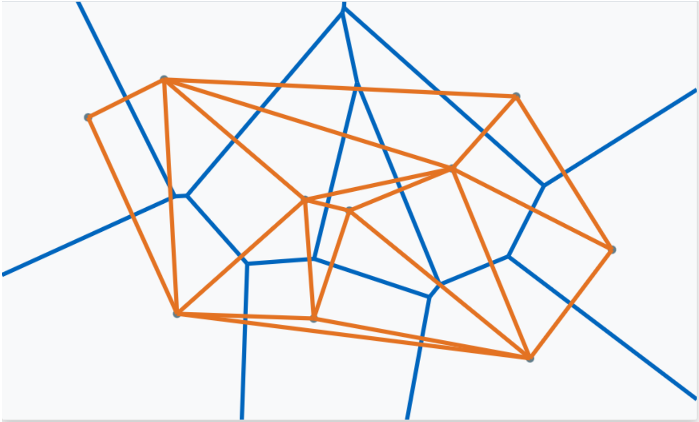
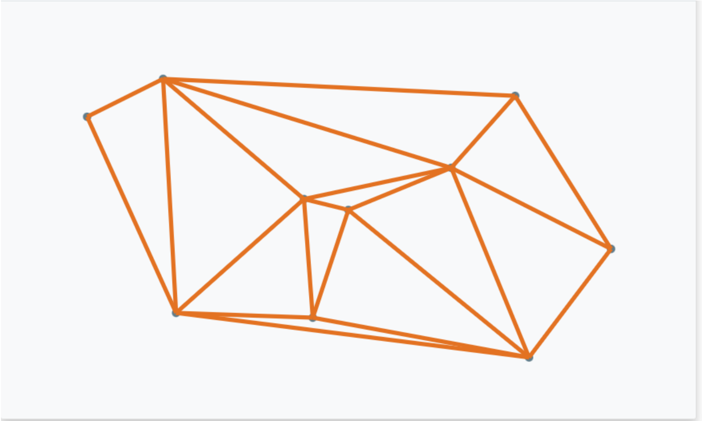
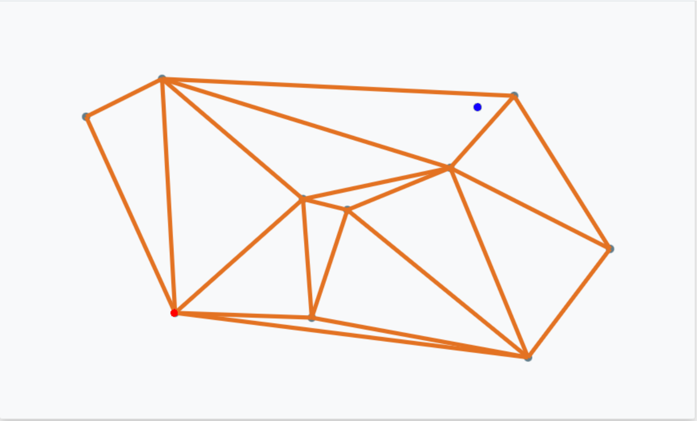


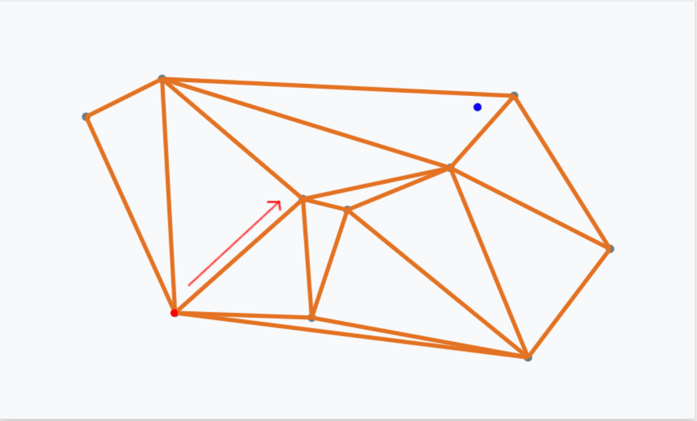
# Delaunay graph



# Greedy NN search start - Query and entry point



# Greedy NN search - iteration



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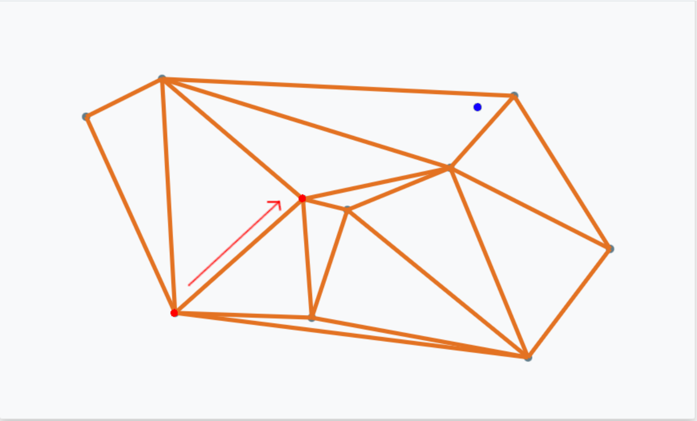

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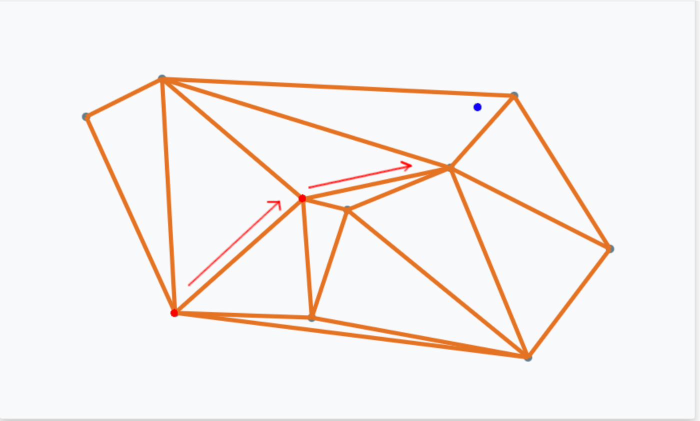

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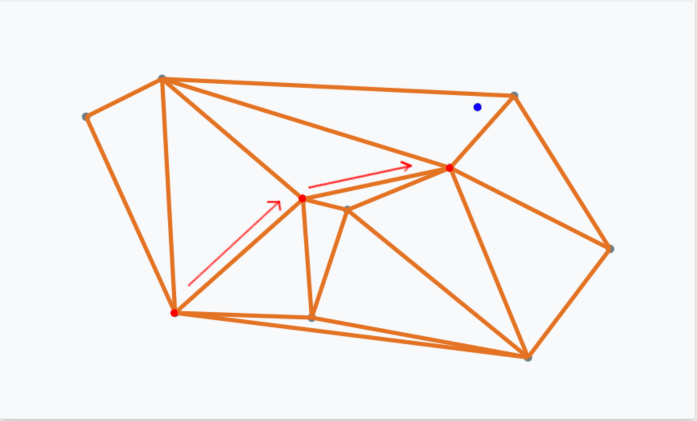
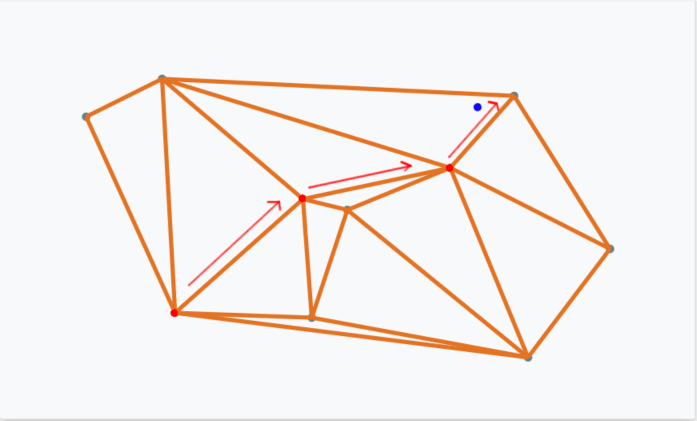
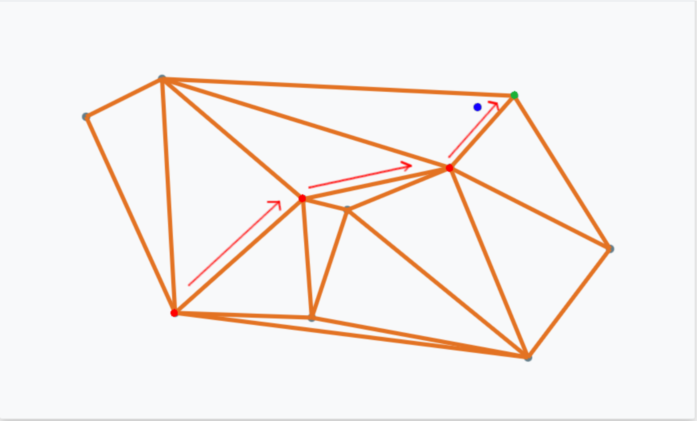


# Greedy NN search - iteration



# Greedy NN search done!



# Drawbacks

- Delaunay graph intractable to construct for large, high-dimensional data sets
- Greedy search might be slow if graph is large

# Navigable small world (NSW) graph

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  - Distance of two random nodes is $log\ N$, where $N$ is the number of nodes in graph
  - Neighbors of a given node are likely to be neighbors of another (clustering coefficient is high)
- Navigability
  - Greedy search algorithm has logarithmic scalability

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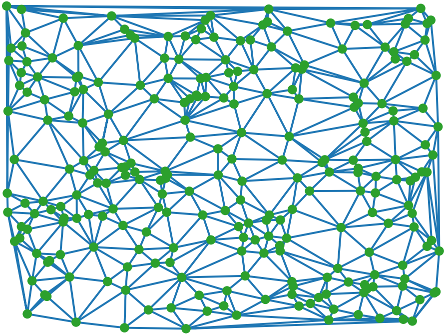
- Navigability ensures that the greedy algorithm finds the logaritmic path
- High clustering coefficient lets us zoom in on the actual correct node when we're in the right area

# Making Delaunay graph navigable



256 nodes

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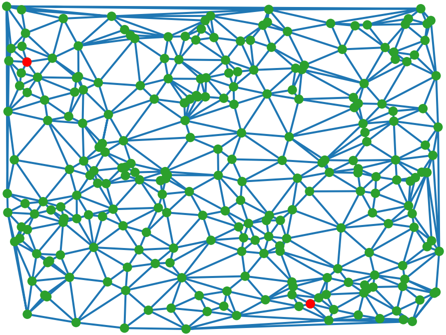

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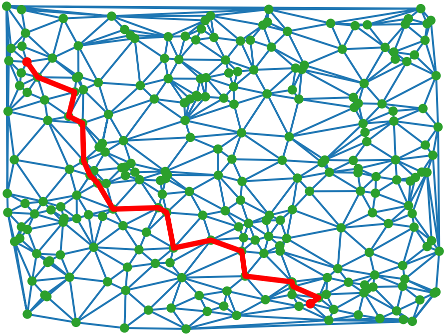
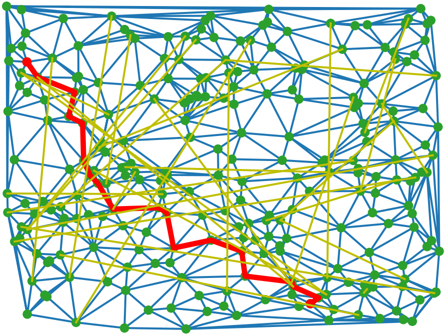


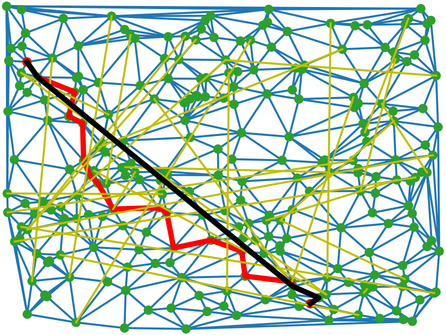
Length of path: 19

# Making Delaunay graph navigable



32 random edges added

# Making Delaunay graph navigable



Length of path: 5

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- Thus the greedy algorithm doesn't always return the actual nearest neighbor
- Ok since we're doing approximate nearest neighbor search!

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- Goal: Construct a graph that has the Delaunay graph as a subgraph, but also has longer connections to make it navigable

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- Approximation of Delaunay graph is sufficient

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2. Add data point to graph
3. Add edges from data point to its $k$ nearest neighbors that are already present in the graph
4. Repeat 2 and 3 until all data points have been added

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- Adding enough nearest neighbor edges approximates Delaunay graph

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- Adding enough nearest neighbor edges approximates Delaunay graph
- The edges added for the early nodes give long-range connections, enabling navigability

# References


- *Efficient and robust approximate nearest neighbor search using Hierarchical Navigable Small World graphs (Malkov et al.* `https://arxiv.org/abs/1603.09320`
- *Approximate nearest neighbor algorithm based on navigable small world graphs (Malkov et al* `https://doi.org/10.1016/j.is.2013.10.006`
- *Voronoi diagrams—a survey of a fundamental geometric data structure (Aurenhammer)* `https://dl.acm.org/doi/10.1145/116873.116880`
- *Hierarchical Navigable Small Worlds (HNSW) (Pinecone blog)* `https://www.pinecone.io/learn/hnsw/`