Approximate nearest neighbor search using the Hierarchical Navigable Small World (HNSW) algorithm

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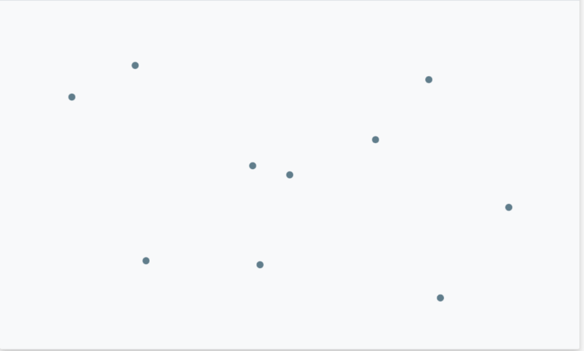
Outline

1. Theoretical foundations
   - Voronoi diagram
   - Delaunay graph
   - Greedy NN search using Delaunay graph
2. HNSW algorithm
   - Navigable small world (NSW)
   - Hierarchical navigable small world (HNSW)
   - Nearest neighbor search using HNSW
3. Performance
   - Search accuracy
   - Build time

Voronoi diagram for a set of points



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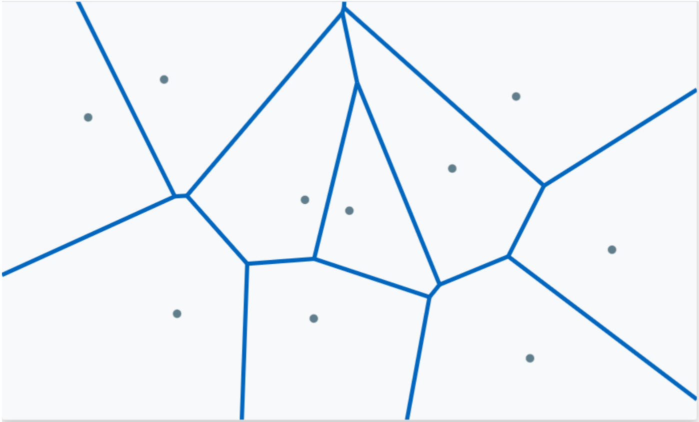
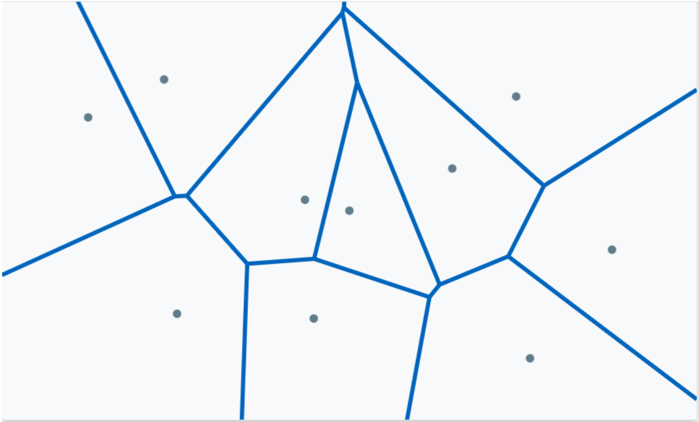


Voronoi diagram to Delaunay graph



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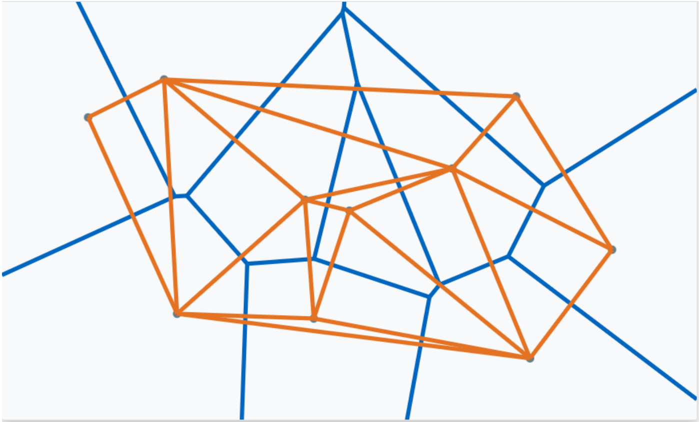
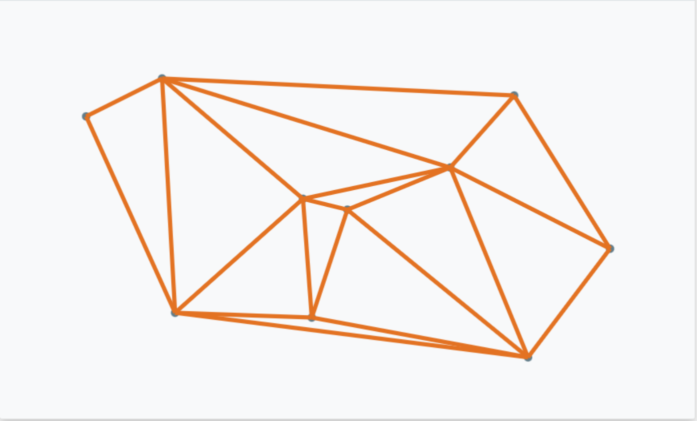


Delaunay graph



Greedy NN search algorithm

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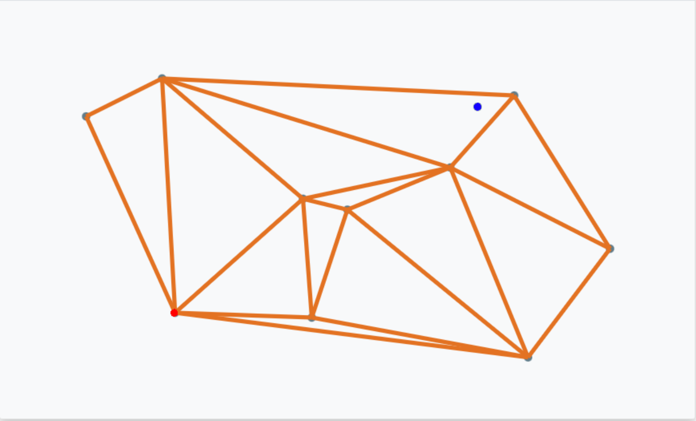
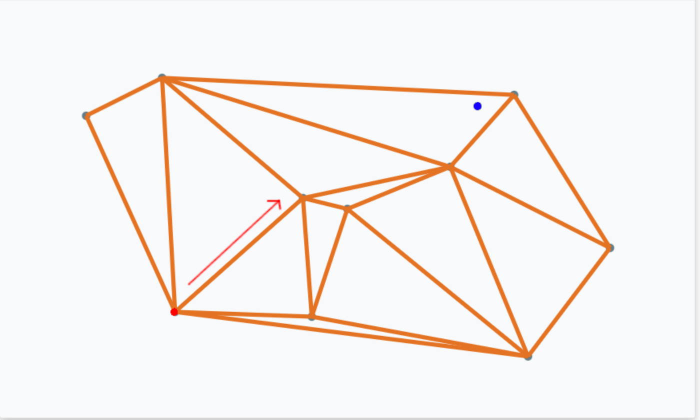
4. Repeat 2 and 3 until no neighbor is closer to query than the current node

Greedy NN search start - Query and entry node



Greedy NN search - iteration



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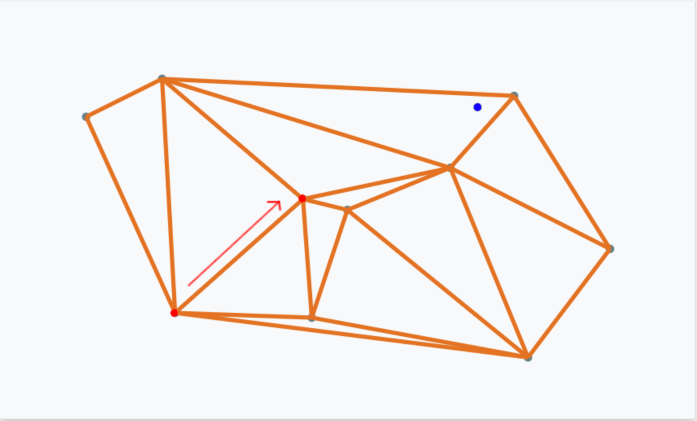

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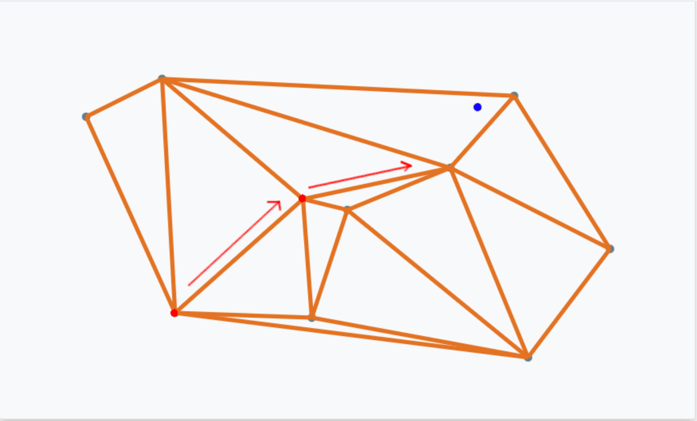

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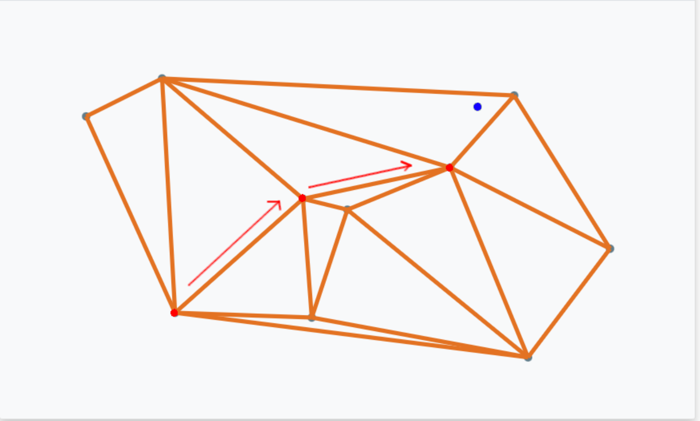
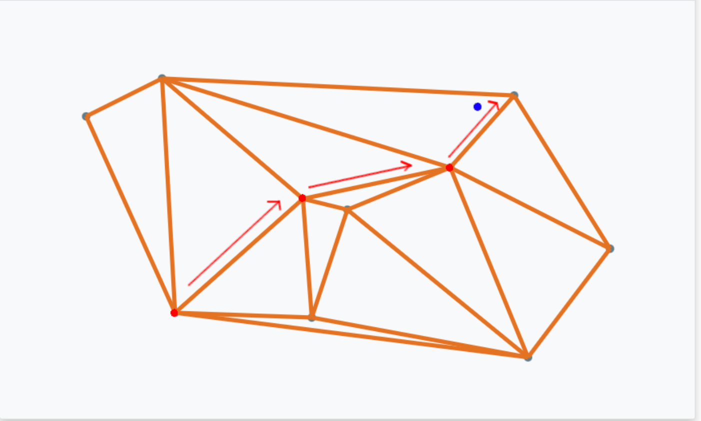
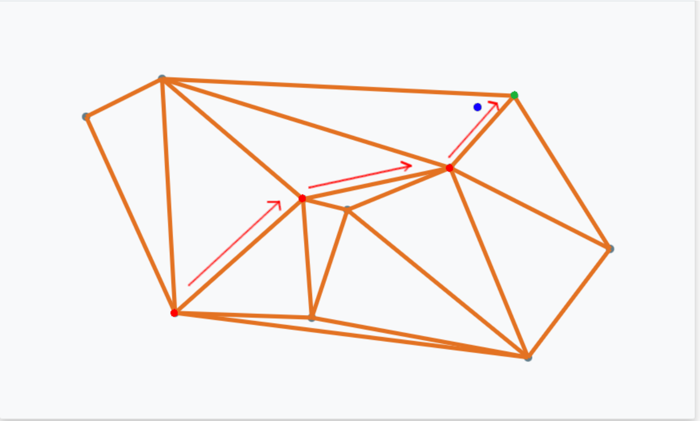


Greedy NN search - iteration



Greedy NN search done!



Drawbacks

- Delaunay graph intractable to construct for large, high-dimensional data sets
- Greedy search might require a lot of steps if graph is large

Navigable small world (NSW) graph

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  - Neighbors of a given node are likely to be neighbors of another (clustering coefficient is high)
- Navigability
  - Greedy search algorithm has logarithmic scalability

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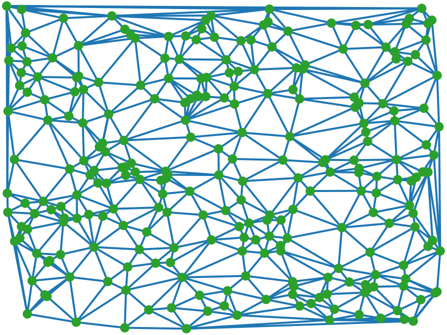
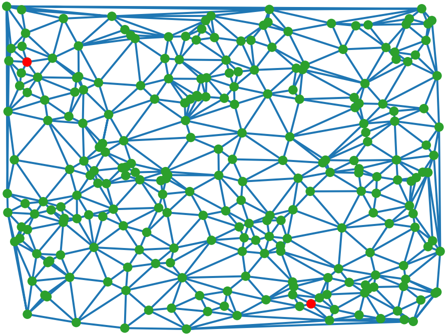
- Logarithmic distance allows us to get anywhere in the graph quickly
- Navigability ensures that the greedy algorithm finds the logaritmic path
- High clustering coefficient lets us zoom in on the actual correct node when we're in the right area

Making Delaunay graph navigable



256 nodes

Making Delaunay graph navigable



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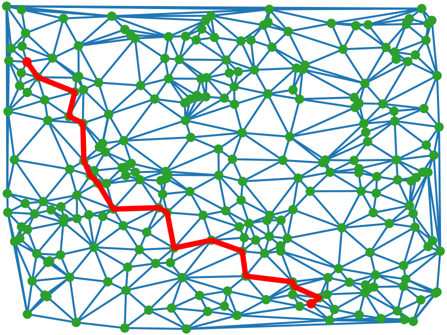
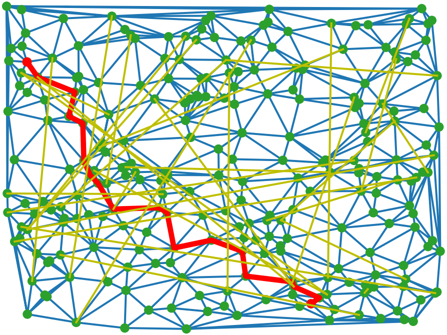


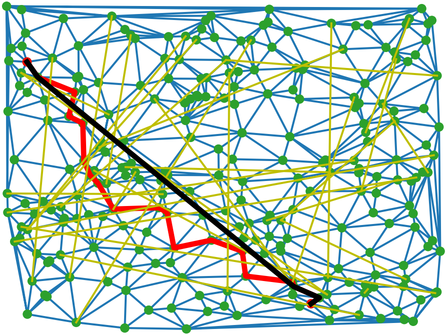
Length of path: 19

Making Delaunay graph navigable



32 random edges added

Making Delaunay graph navigable



Length of path: 5

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- Thus the greedy algorithm doesn't always return the actual nearest neighbor
- Ok since we're doing approximate nearest neighbor search!

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- Goal: Construct a graph that has the Delaunay graph as a subgraph, but also has longer connections to make it navigable

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- Approximation of Delaunay graph is sufficient

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4. Repeat 2 and 3 until all data points have been added

Why does NSW graph creation algorithm work?

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- Adding enough nearest neighbor edges approximates Delaunay graph

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- Adding enough nearest neighbor edges approximates Delaunay graph
- The edges added for the early nodes give long-range connections, enabling navigability

kNN search using NSW graph

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kNN search using NSW graph

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- To improve results we can redo the search m times from different start nodes

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- Algorithm scales polylogarithmically in general (logarithmic scaling in both steps and degrees of nodes)
- Performance degrades on high-dimensional data
- Insertion order must be random

References

- *Efficient and robust approximate nearest neighbor search using Hierarchical Navigable Small World graphs (Malkov et al.* `https://arxiv.org/abs/1603.09320`
- *Approximate nearest neighbor algorithm based on navigable small world graphs (Malkov et al* `https://doi.org/10.1016/j.is.2013.10.006`
- *Voronoi diagrams—a survey of a fundamental geometric data structure (Aurenhammer)* `https://dl.acm.org/doi/10.1145/116873.116880`
- *Hierarchical Navigable Small Worlds (HNSW) (Pinecone blog)* `https://www.pinecone.io/learn/hnsw/`