Approximate nearest neighbor search using the Hierarchical Navigable Small World (HNSW) algorithm

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Outline

Voronoi diagram for a set of points

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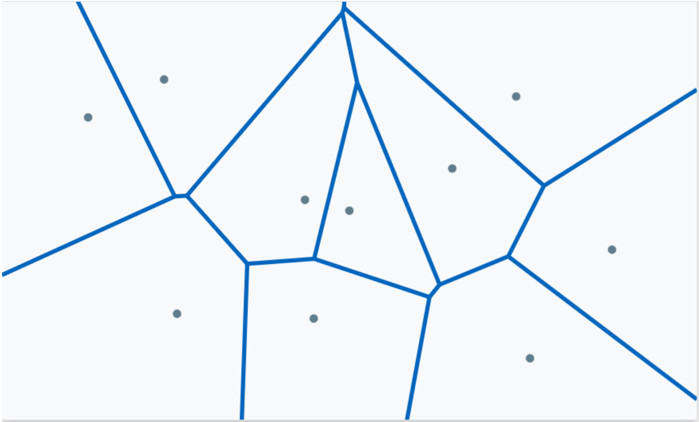
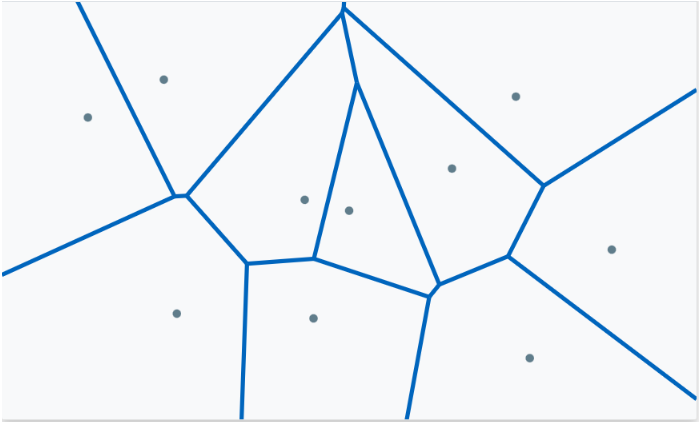


Voronoi diagram to Delaunay graph



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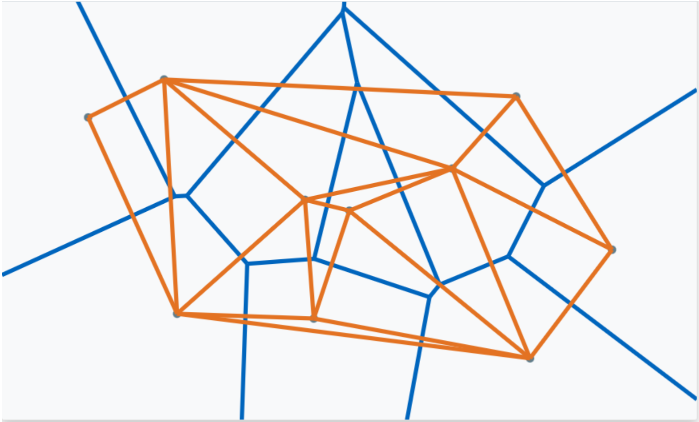
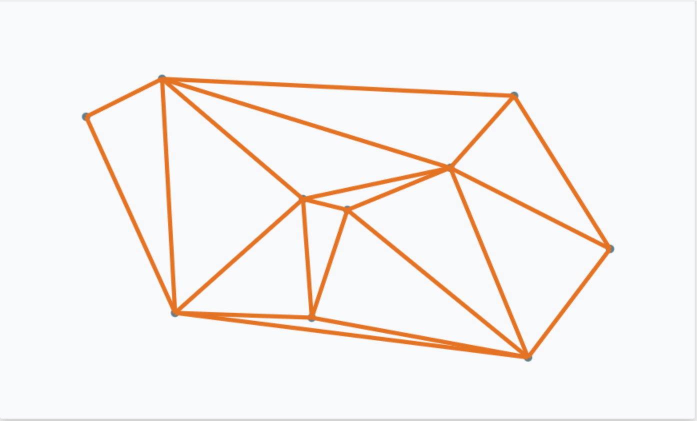


Delaunay graph



Greedy NN search algorithm

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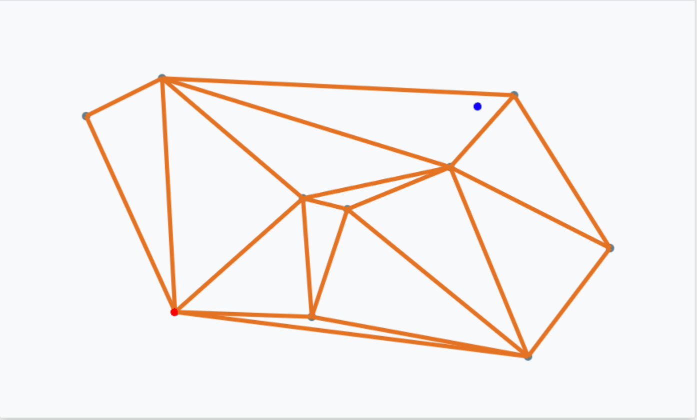
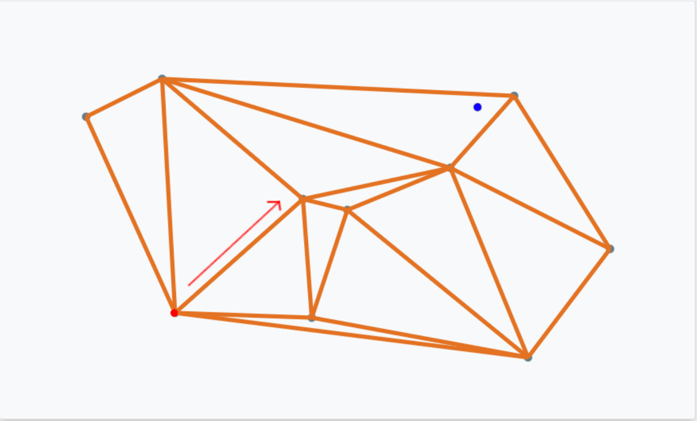
4. Repeat 2 and 3 until no neighbor is closer to query than the current node

Greedy NN search start - Query and entry node



Greedy NN search - iteration



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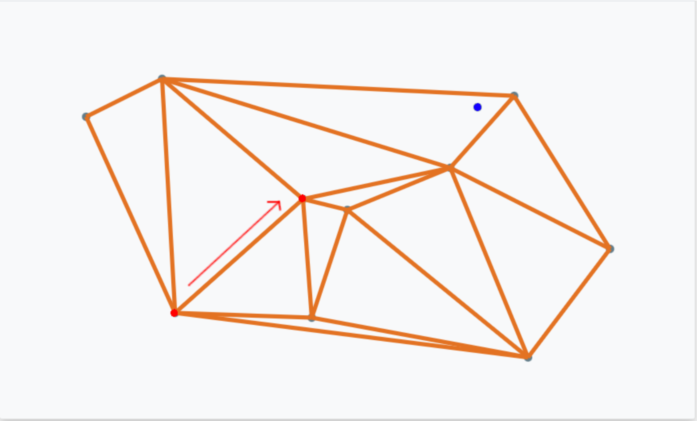

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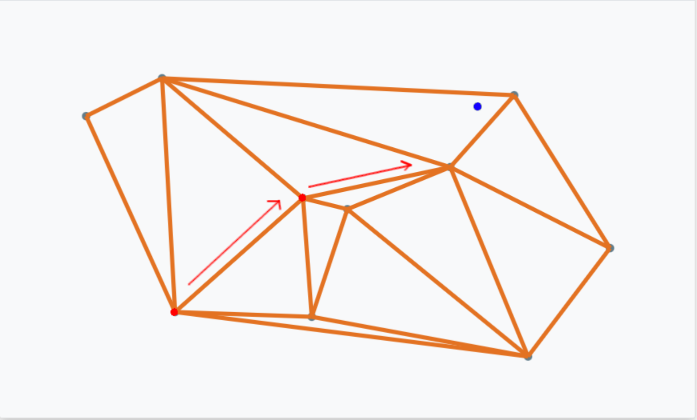

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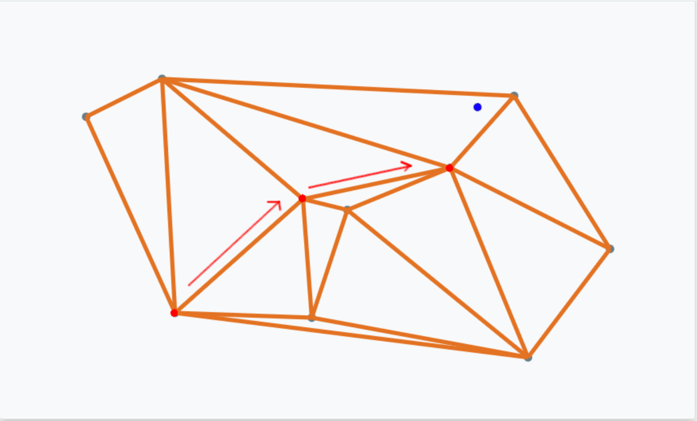

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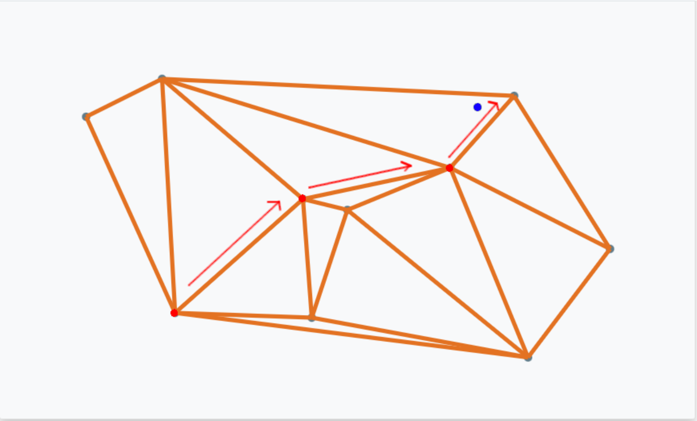
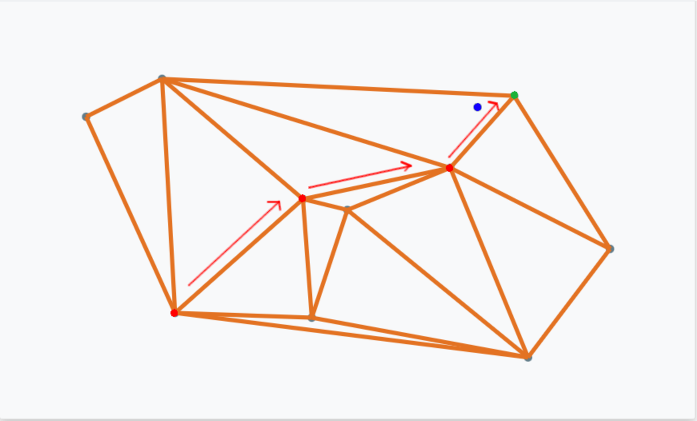


Greedy NN search done!



Drawbacks

- Delaunay graph intractable to construct for large, high-dimensional data sets
- Greedy search might require a lot of steps if graph is large

Navigable small world (NSW) graph

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- Navigability
  - Greedy search algorithm has logarithmic scalability

Why is an NSW useful for nearest neighbor search?

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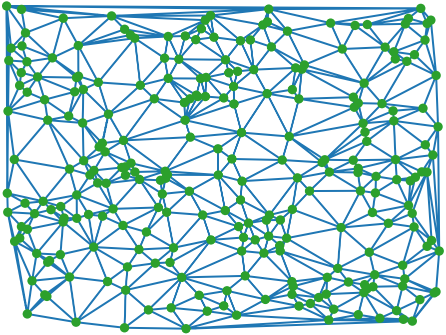
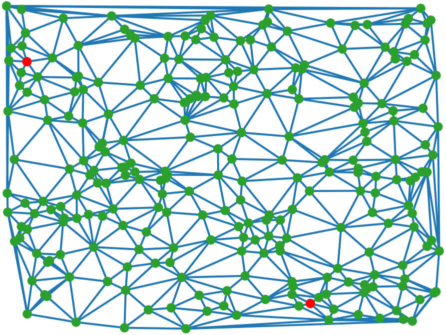
- Navigability ensures that the greedy algorithm finds the logaritmic path
- High clustering coefficient lets us zoom in on the actual correct node when we're in the right area

Making Delaunay graph navigable



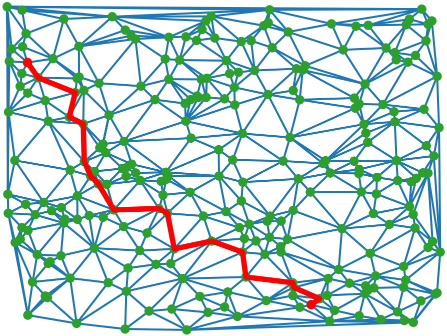
256 nodes

Making Delaunay graph navigable



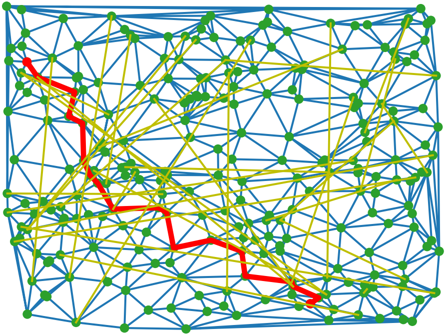
Start and end nodes in red

Making Delaunay graph navigable



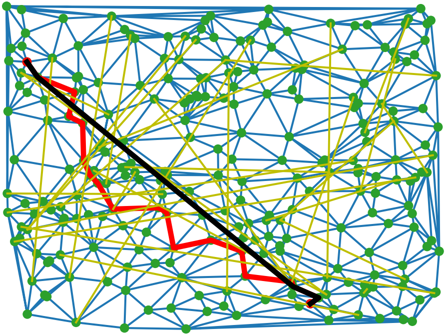
Length of path: 19

Making Delaunay graph navigable



32 random edges added

Making Delaunay graph navigable



Length of path: 5

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- Ok since we're doing approximate nearest neighbor search!

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- Approximation of Delaunay graph is sufficient

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4. Repeat 2 and 3 until all data points have been added

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- Adding enough nearest neighbor edges approximates Delaunay graph
- The edges added for the early nodes give long-range connections, enabling navigability

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- To improve results we can redo the search m times from different start nodes

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5. Repeat until step 2 returns a candidate that's further away than the kth result in the queue

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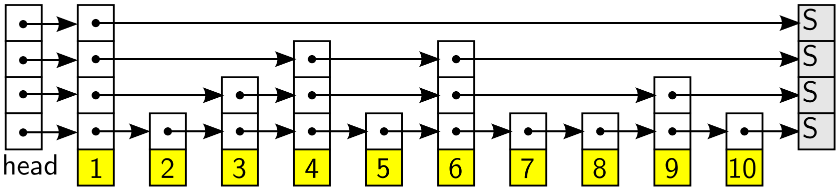
- Performance degrades on high-dimensional data
- Insertion order must be random

Inspiration: Skiplist



https://en.wikipedia.org/wiki/Skip_list

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- NSW enables finding the approximate nearest neighbors
- Skipping allows zooming in to the correct area quickly and reliably
- The zoom-in property is accomplished by a hierarchical construction, like in skiplists

HNSW diagram

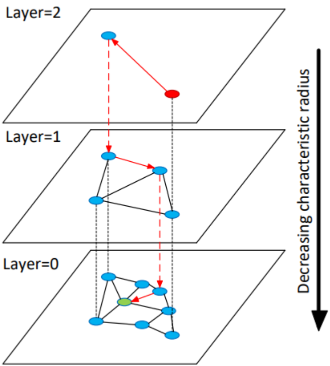

Fig. 1. Illustration of the Hierarchical NSW idea. The search starts from an element from the top layer (shown red). Red arrows show direction of the greedy algorithm from the entry point to the query (shown green).

Efficient and robust approximate nearest neighbor search using Hierarchical Navigable Small World graphs (Malkov et al.) `https://arxiv.org/abs/1603.09320`

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4. Run kNN algorithm on bottom layer (like when using NSW)

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    - After adding nearest neighbors to node, prune connections from neighbors if number exceeds M

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3. *ef* - amount of neighbors to explore in the kNN search during inference (*exploreAdditionalHits* in Vespa)

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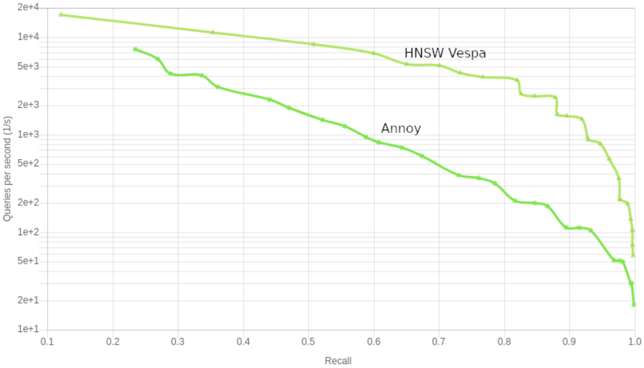
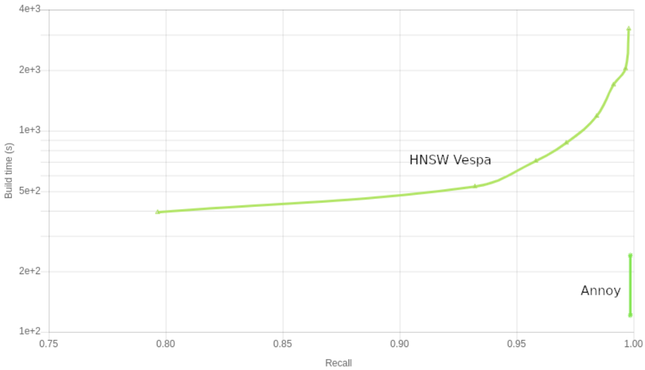
- Logarithmic scalability of search due to hierarchical structure
- Better performance on high-dimensional data
- Insertion order can be anything - randomization happens automatically during index construction

Recall vs queries per second (up and to the right is better)



`https://ann-benchmarks.com/nytimes-256-angular_10_angular.html`

Recall vs build time (down and to the right is better)



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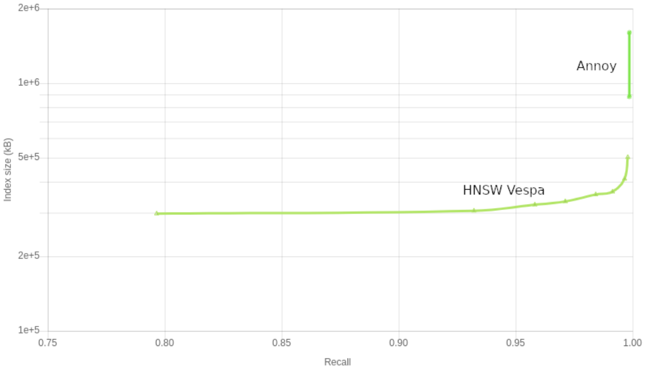
Recall vs index size (down and to the right is better)



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- Performs well on high-dimensional data
- Supports adding new vectors without rebuilding graph from scratch

References

- *Efficient and robust approximate nearest neighbor search using Hierarchical Navigable Small World graphs (Malkov et al.)* `https://arxiv.org/abs/1603.09320`
- *Approximate nearest neighbor algorithm based on navigable small world graphs (Malkov et al.)* `https://doi.org/10.1016/j.is.2013.10.006`
- *Voronoi diagrams—a survey of a fundamental geometric data structure (Aurenhammer)* `https://dl.acm.org/doi/10.1145/116873.116880`
- *Hierarchical Navigable Small Worlds (HNSW) (Pinecone blog)* `https://www.pinecone.io/learn/hnsw/`