Université de Lille

FACULTÉ DES SCIENCES ET TECHNOLOGIES

University of Lille 1

Faculty of Science and Technology

Adaptative Integrator Backstepping Controller for Quadrotor Stabilization

Bibliographic Project

Electrical Engineering and Sustainable Development

Master Program

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Abstract

The great interest in flying robotics has encouraged many research efforts to improve its control strategies. This project carries out modelling and control of a class of flying robotic systems namely Quadrotors.

This project presents a first literature review regarding the quadrotors and their application in several fields. Afterwards the development of mathematical model is carried out in order to describe the dynamics of the quadrotor using Euler-Newton equations and state space representation is obtained. Then a preliminary Integrator Backstepping controller is used to regulate both altitude and attitude of the quadrotor when its mass is claimed to be known. A new enhanced Backstepping controller namely Adaptive Integrator Backstepping Controller comes to the rescue to ensure stability and good performances under the complete unknowingness of the quadrotor's mass.

Due to its inherent highly nonlinear dynamics, variable coupling, underactuation property and model uncertainties. Quadrotor's control is a difficult task that, requires to solve, an advanced set of nonlinear controllers at the time linear controllers are not a suitable choice due the limited and poor performance they have shown specifically for this type of systems. This project presents the development of Integrator Backstepping controller where all quadrotor's parameters are known. For this purpose, the quadrotor system is divided into two subsystems, the first one represents the controllable variables (altitude and attitude) and the other one depicts the underactuated degrees of freedom. The aim of this controller is to ensure the asymptotic stabilization of the position and attitude of the quadrotor.

Another approach this project will treat is that quadrotor's mass is unknown. In such case, zero steady state error is achieved using high-gain control. This strategy however has one limitation due to actuators saturation in addition to high energy consumption because of high control effort. To solve this problematic, it is convenient to determine the estimation of the mass at each time by introducing an adaptation law. Herein defined adaptive integrator backstepping control where the backstepping controller is combined with Lyapunov-based adaptation law. A good choice of Lyapunov function and adaptation law will ensure the stability of the origin in the presence of unknown parameter that itself is estimated. Finally, the derivation of the control law combined with an adaptation law will empower to overcome the limitation of integrator backstepping controller deployment only.

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1. Chapter 1

1.1 Introduction

Over the past century, VTOL (vertical take-off and landing) aircraft have undergone significant advancements and have become a focal point for researchers across various fields. This is due to the unique capabilities and potential uses of these machines, which surpass those of traditional fixed-wing Unmanned aerial vehicles (UAV). Researchers and engineers have been drawn to VTOL planes for a variety of reasons, including:

1. The minimal space required for take-off and landing

2. Their greater portability in comparison to fixed-wing aircraft

3. Their suitability for aerial photography and videography, particularly for small VTOL planes.

1.2 Historical Background

Throughout history, a variety of VTOL machines have been developed using different propulsion techniques. Some examples include:

1. Rotary wing propulsion which are helicopters that use rotary wing propulsion to achieve VTOL capabilities. This involves the use of one or more rotors that rotate to generate lift and propulsion

2. Hybrid propulsion that uses a combination of jet and rotary wing propulsion to achieve vertical take-off and landing capabilities.

3. Multi rotors also known as quadrotors, use multiple rotors to achieve VTOL capabilities. They are becoming increasingly popular due to their simplicity and versatility.

From a technical perspective, VTOL UAVs are classified as underactuated mechanical systems. Underactuation occurs when a system has fewer actuators than degrees of freedom. This poses significant limitations on the control system and adds complexity to the modelling of the system.

1.3 Underactuated systems

Underactuated systems are systems who has fewer independent control actuators than the degrees of freedom to be controlled which obliges to the user to choose between degrees of freedom to manipulate (control) while the rest is remained uncontrollable. In real life many mechanical systems are underactuated systems such as: translational oscillator with rotational actuator [1], underwater vehicles [2], mobile robots [3]. This property of certain mechanical systems raises many other difficulties such as coupling and nonlinearity. Such shortcoming makes some nonlinear control techniques such as feedback linearization uncapable to control this type of systems, because it requires one-to-one correspondence between control inputs and system's states. Furthermore, inherent undesirable phenomenon such as high relative degree and nonminimum phase behaviour are also manifested in underactuated systems. Nevertheless, underactuated systems present other advantages including tolerance for actuators' failure [4].

1.4 Working principle

The quadrotor vehicle operates on the basis of variable torques and thrusts. Each propeller usually consists of a brushless DC motor and rotor with fixed pitch. The quadrotor has 4 motors arranged in pairs along the horizontal and vertical axes such that the forward pair rotating clockwise and the horizontal pair rotating counter-clockwise. Based on this design, the manipulation of reaction torques from pairs of motors being opposed to each other creates all the direction a quadrotor can go along. Hence the control of each rotor's torque creates roll, pitch, yaw and hovering motions.

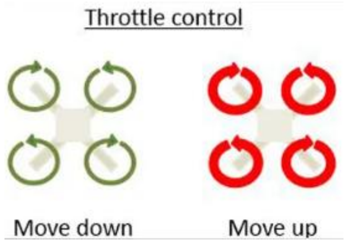

Figure 1: Quadrotor hovering

To realize the hovering of quadrotor, all motors are set to the same high rotational speed (red) and as a result each rotor generates its own thrust that lifts the quadrotor. To move down, rotors' speeds are slowed down(green).

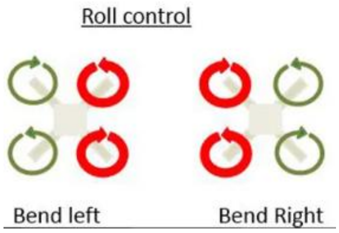

Figure 2: Roll motion of quadrotor

The horizontal movement is achieved by making the vehicle roll or pitch. For roll motion, the thrust difference between the pairs of left and right rotors makes the quadrotor to bend in a given direction.

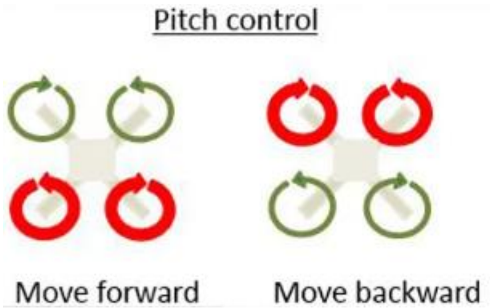

Figure 3: Pitch motion of quadrotor

Regarding pitch motion, quadrotor is pushed in a forwarding direction by attributing high rotational speed to rear propellers while front propellers are slowed down. Analogously, front propellers are commanded to high rotational speed hence backward direction is realized.

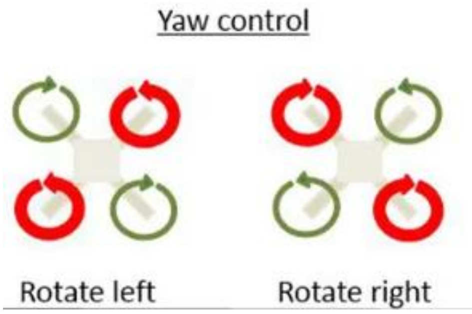

Figure 4: Yaw motion of quadrotor

As depicted in the picture above, a quadrotor spinning motion is achieved by varying the speeds of the pairs of the motors to create a non-zero net counter torque.

2. Chapter 2

2.1 Literature review and research statement

1.2.1 Literature review

With the democratization of drones and their tremendous accessibility, these devices are more and more used in a wide range of fields such as search and rescue, surveillance, traffic monitoring, firefighting, weather monitoring and photography [5]. A quadrotor is an unmanned aerial vehicle system that is flying and can be controlled remotely or fly autonomously using programs in its embedded system. However, the control of quadrotors to do their assigned missions is not an easy task, due to their complexities associated with the dynamic model of the system such as nonlinearities, coupled dynamic as well as high number of state variables. These facts are due to the underactuated nature of the quadrotor where its degrees of freedom (6) outnumber the control inputs (4) which are thrusts generated by the motors. Another difficulty that faces the development of control systems for quadrotors is the uncertainty of physical parameters and external disturbances. In light of this, many linear controllers have been deployed in the literature, such as PID (proportional, integral and derivative control) and linear quadratic regulators (LQR) are applied in [6], nevertheless these linear controllers are synthesized upon linear models of the system or linearized ones which make them effective in certain limited region of operation. In addition to this, linear controllers are not a good choice when it comes to MIMO systems (multi-input multi-output) systems where coupling between states is existing which makes controller's gains tuning a complex procedure. To widen the range of operation and domain of controllable flight, nonlinear control technique must be used. Several nonlinear controllers are used in the literature such as sliding mode control [7], feedback linearization [8], backstepping [9] or a combination of them [10] as well as other nonlinear control techniques.

1.2.2 Research Statement

The former evoked techniques used have shown good results for both stabilization and tracking problems, nonetheless due to parametric uncertainties some of these techniques fails to guarantee the sought-after performances or even to instability. In the extreme case where a finite number of parameters are unknown, the control law synthesizing is impossible because of the dependency between control signal and physical parameters of the system. Herein adaptive control is able to solve this problematic by estimating the unknown parameters which makes therefore control law derivation possible. The bibliographic project treats this problem by developing firstly a mathematical model of the quadrotor using Euler-Newton modelling approach. Under some assumptions of modelling, the dynamical model is derived that is a nonlinear and underactuated system. Secondly, a state space representation is conducted comprises of twelve state variables, four inputs and four outputs namely altitude, and attitude. Integrator backstepping is developed then two cases are going to be undertook, the first is that quadrotor mass is known therefore the procedure of integrator backstepping is applied. In the second case, the mass of quadrotor is going to be assumed unknown, at that moment, adaptation law based on Lyapunov function is proposed to ensure the stability of the system.

3. Chapter 3

3.1 State space representation of the quadrotor

Before proceeding to controller derivation, the system of equations should be expressed according to the state space representation of a nonlinear system:

$$\begin{cases} \dot{\eta} = f(\eta) + g(\eta)u(t) & (3.1) \\ \zeta = h(\eta) & (3.2) \end{cases}$$

Where:

η: State variable vector

u: System input vector

ζ: System output vector

f, g and h are smooth real-valued functions.

The state variables are chosen as follows:

$$[x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}]^T = [x, \dot{x}, y, \dot{y}, z, \dot{z}, \phi, \dot{\phi}, \theta, \dot{\theta}, \psi, \dot{\psi}]^T$$

The state space representation of the model of quadrotor is given by:

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= (sinx_7 sinx_{11} + cosx_{11} cosx_7 sinx_9)\frac{U1}{m} - \frac{kx}{m} x_2|x_2| \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= (cosx_7 sinx_9 sinx_{11} - cosx_{11} sinx_7)\frac{U1}{m} - \frac{ky}{m} x_4|x_4| \\
\dot{x}_5 &= x_6 \\
\dot{x}_6 &= -g + (cosx_7 cosx_9)\frac{U1}{m} - \frac{kz}{m} x_6|x_6| \\
\dot{x}_7 &= x_8 \\
\dot{x}_8 &= \frac{J_y - J_z}{J_x} x_{10} x_{12} - \frac{J_r}{J_x}\omega x_{10} - \frac{k_\phi}{J_x} {x_8}^2 + \frac{1}{J_x} U2 \\
\dot{x}_9 &= x_{10} \\
\dot{x}_{10} &= \frac{J_z - J_x}{J_y} x_{12} x_8 + \frac{J_r}{J_y}\omega x_8 - \frac{k_\theta}{J_y} {x_{10}}^2 + \frac{1}{J_y} U3 \\
\dot{x}_{11} &= x_{12} \\
\dot{x}_{12} &= \frac{J_x - J_y}{J_z} x_8 x_{10} - \frac{k_\psi}{J_z} {x_{12}}^2 + \frac{1}{J_z} U4
\end{aligned} \quad (3.3)$$

3.2 Integrator backstepping control of the quadrotor

The aim of this section is to apply integrator backstepping controller to stabilize the altitude and attitude of the quadrotor.

2.3.1 Presentation of the integrator backstepping technique

Let’s consider a special case of backstepping technique, called integrator backstepping:

$$\begin{cases} \dot{\eta} = f(\eta) + g(\eta)\xi & (3.4) \\ \dot{\xi} = u & (3.5) \end{cases}$$

Where η and ξ are real-valued states and u is the control input. The functions f, g are smooth. With $f(0)$=0. The goal is to design a state feedback control law to stabilize the origin ($\eta = 0$, $\xi = 0$). The functions f and g are assumed to be known. Suppose that (3.4) can be stabilized using a smooth state feedback control law $\xi = \rho(\eta)$ with $\rho(0) = 0$ so that the origin of

$$\dot{\eta} = f(\eta) + g(\eta)\rho(\eta) \tag{3.6}$$

Is asymptotically stable. We suppose also that we know a smooth, positive definite Lyapunov function $V(\eta)$ that satisfies the inequality:

$$\frac{\partial V}{\partial \eta}[f(\eta) + g(\eta)\rho(\eta)] \leq -W(\eta) \tag{3.7}$$

With $W(\eta)$ is positive definite. By adding and subtracting $g(\eta)\rho(\eta)$ on (3.4), we obtain the representation

$$\begin{cases} \dot{\eta} = f(\eta) + g(\eta)\rho(\eta) + g(\eta)[\xi - \rho(\eta)] \\ \dot{\xi} = u \end{cases} \tag{3.8}$$

The change of variables:

$$z = \xi - \rho(\eta) \tag{3.9}$$

Results in the system:

$$\begin{cases} \dot{\eta} = f(\eta) + g(\eta)\rho(\eta) + g(\eta)[\xi - \rho(\eta)] \\ \dot{z} = u - \dot{\rho} \end{cases} \tag{3.10}$$

The derivative $\dot{\rho}$ can be computed since f, g and ρ are known. If we take $v = u - \dot{\rho}$ the system becomes:

$$\begin{cases} \dot{\eta} = f(\eta) + g(\eta)\rho(\eta) + g(\eta)[\xi - \rho(\eta)] \\ \dot{z} = v \end{cases} \tag{3.11}$$

Which is similar to the system we started from (3.4), the difference resides on a new component at the right-hand side that must be stabilized to the origin to stabilize the overall system. Using

$$V_C(\eta) = V(\eta) + \frac{1}{2}z^2 \tag{3.12}$$

as a Lyapunov function candidate, we obtain:

$$\begin{aligned} \dot{V}_C &= \frac{\partial V}{\partial \eta}[f(\eta) + g(\eta)\rho(\eta)] + \frac{\partial V}{\partial \eta}g(\eta)z + zv \\ &\leq -W(\eta) + \frac{\partial V}{\partial \eta}g(\eta)z + zv \end{aligned} \tag{3.13}$$

Choosing

$$v = -\frac{\partial V}{\partial \eta}g(\eta) - kz,\ k > 0 \tag{3.14}$$

Leads to:

$$\dot{V}_C \leq -W(\eta) - kz^2$$

Which shows that the origin ($\eta = 0$, $\xi = 0$) is asymptotically stable. Finally, the real control input u is given by:

$$u = -\frac{\partial \rho}{\partial \eta}[f(\eta) + g(\eta)\xi] - \frac{\partial V}{\partial \eta}g(\eta) - k[\xi - \rho(\eta)] \tag{3.15}$$

2.3.2 Attitude Stabilization using Integrator Backstepping Control

- Altitude control

Step 1: Define the stabilization error of the altitude

$$\varepsilon_5 = x_{5ref} - x_5 \tag{3.16}$$

$$\dot{\varepsilon}_5 = \dot{x}_{5ref} - x_6 \tag{3.17}$$

Let's consider the Lyapunov candidate function:

$$V(\varepsilon_5) = \frac{1}{2}{\varepsilon_5}^2 \tag{3.18}$$

Time differentiation of this function leads to:

$$V(\dot{\varepsilon_5}) = \varepsilon_5\dot{\varepsilon}_5 = \varepsilon_5(\dot{x}_{5ref} - x_6) \tag{3.19}$$

The stabilization of ε_5 can be obtained by introducing a virtual control input

$$x_6 = k_5\varepsilon_5 + \dot{x}_{5ref}, k_5 > 0 \tag{3.20}$$

In this case (3.19) becomes:

$$\dot{V}(\varepsilon_5) = \ -k_5{\varepsilon_5}^2 \tag{3.21}$$

Which implies that the origin $\varepsilon_5 = 0$ is globally asymptotically stable.

Step 2: Let's consider the error between the virtual control input and state variable x_6:

$$\varepsilon_6 = x_6 - (k_5\varepsilon_5 + \dot{x}_{5ref}) \tag{3.22}$$

Let's define the augmented Lyapunov function:

$$V(\varepsilon_5, \varepsilon_6) = \frac{1}{2}({\varepsilon_5}^2 + {\varepsilon_6}^2) \tag{3.23}$$

Time differentiation of leads to:

$$\begin{aligned}\dot{V}(\varepsilon_5, \varepsilon_6) = \varepsilon_5\dot{\varepsilon}_5 + \varepsilon_6\dot{\varepsilon}_6 &= \varepsilon_5(-k_5\varepsilon_5 - \varepsilon_6) + \varepsilon_6(-g + (cosx_7cosx_9)\tfrac{U1}{m} - \tfrac{kz}{m}x_6|x_6| - k_5\dot{\varepsilon}_5 - \ddot{x}_{5ref})\\ &= -k_5{\varepsilon_5}^2 + \varepsilon_6(-\varepsilon_5 - g + (cosx_7cosx_9)\tfrac{U1}{m} - \tfrac{kz}{m}x_6|x_6| - k_5\dot{\varepsilon}_5 - \ddot{x}_{5ref})\end{aligned} \tag{3.24}$$

The ensure the convergence of ε_6 the control input is chosen such that:

$$U1 = \frac{m}{cosx_7cosx_9}(-k_6\varepsilon_6 + \varepsilon_5 + g + \tfrac{kz}{m}x_6|x_6| + k_5\dot{\varepsilon}_5 + \ddot{x}_{5ref}) \text{ with } k_6 > 0 \tag{3.25}$$

Replacing (3.25) in (3.24) we obtain:

$$\dot{V}(\varepsilon_5, \varepsilon_6) = -k_5{\varepsilon_5}^2 - k_6{\varepsilon_6}^2 < 0 \tag{3.26}$$

According to Lyapunov theory, the stabilization of the altitude is achieved.

- Attitude control

The same procedure of design regarding altitude is pursued to derive the control inputs stabilizing roll, pitch and yaw. The control inputs are therefore:

$$U2 = \ J_x(-k_8\varepsilon_8 + \varepsilon_7 - \frac{J_y - J_z}{J_x}x_{10}x_{12} + \frac{J_r}{J_x}\omega x_{10} + \frac{k_\phi}{J_x}{x_8}^2 + k_7\dot{\varepsilon}_7 + \ddot{x}_{7ref})$$

(3.27)

$$U3 = J_y(-k_{10}\varepsilon_{10} + \varepsilon_9 - \frac{J_z - J_x}{J_y}x_8x_{12} - \frac{J_r}{J_y}\omega x_8 + \frac{k_\theta}{J_x}{x_{10}}^2 + k_9\dot{\varepsilon}_9 + \ddot{x}_{9ref}) \quad (3.28)$$

$$U4 = J_z(-k_{12}\varepsilon_{12} + \varepsilon_{11} - \frac{J_x - J_y}{J_x}x_8x_{10} + \frac{k_\psi}{J_z}{x_{12}}^2 + k_{11}\dot{\varepsilon}_{11} + \ddot{x}_{11ref}) \quad (3.29)$$

With : $\varepsilon_7 = x_{7ref} - x_7$,$\varepsilon_8 = x_8 - (k_7\varepsilon_7 + \dot{x}_{7ref})$, $\varepsilon_9 = x_{9ref} - x_9$, $\varepsilon_{10} = x_{10} - (k_9\varepsilon_9 + \dot{x}_{9ref})$

$\varepsilon_{11} = x_{11ref} - x_{11}$ and $\varepsilon_{12} = x_{12} - (k_{11}\varepsilon_{11} + \dot{x}_{11ref})$,

$k_7, k_8, k_9, k_{10}, k_{11}, k_{12}$ are some positive constants.

3.3 Adaptive integrator backstepping Control for Quadrotor Stabilization

In the previous section, physical parameters are considered to be known. Nevertheless, it is not the case every time since physical parameters are subjected to uncertainties for many reasons (identification error, changes in the environment, parametric variations of the system…). In this project, the quadrotor's mass is assumed to be unknown but constant. To overcome this problematic, adaptive control can be used in order to estimate the unknown mass so the integrator backstepping controller can be used. In the literature, there are many techniques such as Model reference adaptive control (MRAC), Lyapunov-based adaptive control. This project applies the second technique.

The adaptive integrator backstepping controller offers an iterative and systematic method, that allows, for a class of nonlinear system for any order, to construct recursively the three parts to build the Lyapunov-based adaptive control:

- Control law: Permits to satisfy the desired specifications of the controlled system
- Adaptation law: determines the dynamic of estimation of unknown parameters. It must guarantee their convergence to their real values respectively, without comprising the stability and predefined specifications.
- Lyapunov function: Allows to make adequate choice of the previous two laws and guarantees convergence and stability of the adaptive structure at each time.

Since the variable that is concerned by either the unknown parameter and stabilization target is altitude, we consider the former equation (3.25), we assume that the quadrotor's mass is constant.

If m is known then the equation (3.25) is satisfied.

If the mass is unknown, the controller described by the equation cannot be realized, the mass can be replaced then by its equivalent (certainty equivalence principle), we replace m by $\widehat{m}$, the control signal is therefore:

$$U1 = \frac{\widehat{m}}{cosx_7cosx_9}(-k_6\varepsilon_6 + \varepsilon_5 + g + \frac{kz}{\widehat{m}}x_6|x_6| + k_5\dot{\varepsilon}_5 + \ddot{x}_{5ref}) \quad (3.30)$$

If we replace in (3.3) we will find:

$$\dot{x}_6 = -\left(1 - \frac{\widehat{m}}{m}\right)\left(g + \varepsilon_5 - k_6\varepsilon_6 + k_5\dot{\varepsilon}_5 + \ddot{x}_{5ref}\right) + \left(\varepsilon_5 - k_6\varepsilon_6 + k_5\dot{\varepsilon}_5 + \ddot{x}_{5ref}\right) \quad (3.31)$$

Let's define the mass error:

$$m\tilde{\theta} = m - \widehat{m} \quad (3.32)$$

The equation, becomes:

$$\dot{V}(\varepsilon_5,\varepsilon_6) = \varepsilon_5(-k_5\varepsilon_5 - \varepsilon_6) + \varepsilon_6(-\tilde{\theta}(g + \varepsilon_5 - k_6\varepsilon_6 + k_5\dot{\varepsilon}_5 + \ddot{x}_{5ref}) + \varepsilon_5 - k_6\varepsilon_6 + k_5\dot{\varepsilon}_5 + \ddot{x}_{5ref} - k_5\dot{\varepsilon}_5 - \ddot{x}_{5ref})$$

$$\dot{V}(\varepsilon_5,\varepsilon_6) = -k_5{\varepsilon_5}^2 - k_5{\varepsilon_6}^2 - \varepsilon_6(g + \varepsilon_5 - k_6\varepsilon_6 + k_5\dot{\varepsilon}_5 + \ddot{x}_{5ref})\tilde{\theta} \quad (3.33)$$

Given that the sign of $\tilde{\theta}$ is undefined, no conclusion could be drawn regarding the stability of the system. For the sake of knowing more about this stability, we construct a dynamic controller, by augmenting (3.23) with an updating law for the estimation of $\hat{m}$. A good choice of this law, to ensure the stability of the whole system, is to be determined. We define then a new Lyapunov function by adding, to the initial function given by (3.23), a quadratic term of the estimation error $\tilde{\theta}$:

$$V(\varepsilon_5,\varepsilon_6,\tilde{\theta}) = \frac{1}{2}({\varepsilon_5}^2 + {\varepsilon_6}^2) + \frac{1}{2\gamma}m\tilde{\theta}^2 \quad (3.34)$$

Where, γ is a positive scalar that represents the adaptation gain. The derivative of this function becomes:

$$\dot{V}(\varepsilon_5,\varepsilon_6,\tilde{\theta}) = \dot{V}(\varepsilon_5,\varepsilon_6) + \frac{1}{\gamma}m\tilde{\theta}\dot{\tilde{\theta}} \quad (3.35)$$

$$\dot{V}(\varepsilon_5,\varepsilon_6,\tilde{\theta}) = -k_5{\varepsilon_5}^2 - k_5{\varepsilon_6}^2 - \varepsilon_6(g + \varepsilon_5 - k_6\varepsilon_6 + k_5\dot{\varepsilon}_5 + \ddot{x}_{5ref})\tilde{\theta} + \frac{1}{\gamma}m\tilde{\theta}\dot{\tilde{\theta}}$$

$$\dot{V}(\varepsilon_5,\varepsilon_6,\tilde{\theta}) = -k_5{\varepsilon_5}^2 - k_5{\varepsilon_6}^2 + \frac{1}{\gamma}\tilde{\theta}\left(m\dot{\tilde{\theta}} - \gamma(g + \varepsilon_5 - k_6\varepsilon_6 + k_5\dot{\varepsilon}_5 + \ddot{x}_{5ref})\varepsilon_6\right)$$

$$\dot{V}(\varepsilon_5,\varepsilon_6,\tilde{\theta}) = -k_5{\varepsilon_5}^2 - k_5{\varepsilon_6}^2 + \frac{1}{\gamma}\tilde{\theta}\left(m\dot{\tilde{\theta}} - \tau\right) \quad (3.36)$$

Where,

$$\tau = \gamma(g + \varepsilon_5 - k_6\varepsilon_6 + k_5\dot{\varepsilon}_5 + \ddot{x}_{5ref})\varepsilon_6 \quad (3.37)$$

The derivative remains so far undefined. However, the degree of freedom, offered by the free choice of the updating dynamic law, allows to choose it in a way to cancel the second term of the equation. The choice:

$$m\dot{\tilde{\theta}} = -\dot{\hat{m}} = \tau \quad (3.38)$$

Permits to obtain

$$\dot{V}(\varepsilon_5,\varepsilon_6,\tilde{\theta}) = -k_5{\varepsilon_5}^2 - k_5{\varepsilon_6}^2 < 0 \quad (3.39)$$

Finally, the system given by (3.31) with the control law (3.30), and the updating law (3.38) is:

$$\dot{x}_6 = -\left(1 - \frac{\hat{m}}{m}\right)(g + \varepsilon_5 - k_6\varepsilon_6 + k_5\dot{\varepsilon}_5 + \ddot{x}_{5ref}) + (\varepsilon_5 - k_6\varepsilon_6 + k_5\dot{\varepsilon}_5 + \ddot{x}_{5ref}) \quad (3.40)$$

$$\dot{\hat{m}} = -\gamma(g + \varepsilon_5 - k_6\varepsilon_6 + k_5\dot{\varepsilon}_5 + \ddot{x}_{5ref})\varepsilon_6 \quad (3.41)$$

3.4 Simulation results

This section is dedicated to present the results of application of adaptive integrator backstepping controller to stabilize the altitude and attitude of the quadrotor.

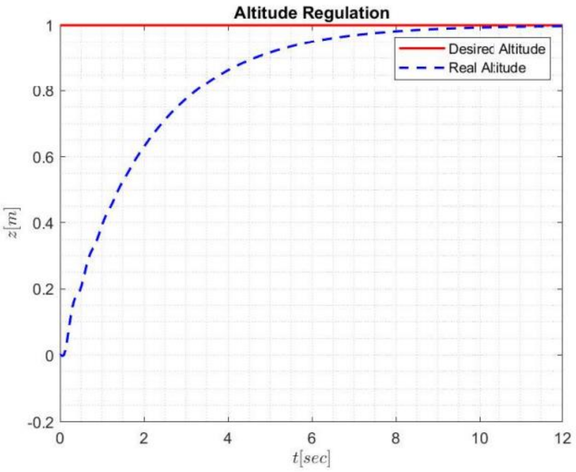

Figure 6 : Comparison between desired and actual altitude

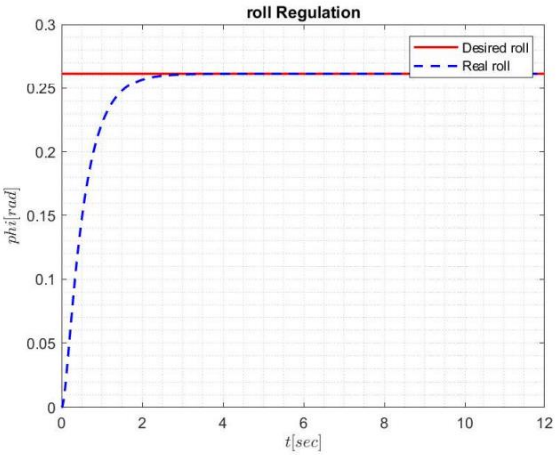

Figure 5: Comparison between desired and actual roll

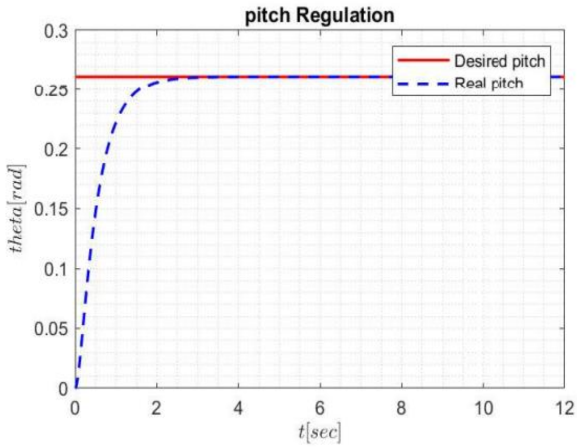

Figure 8: Comparison between desired and actual pitch

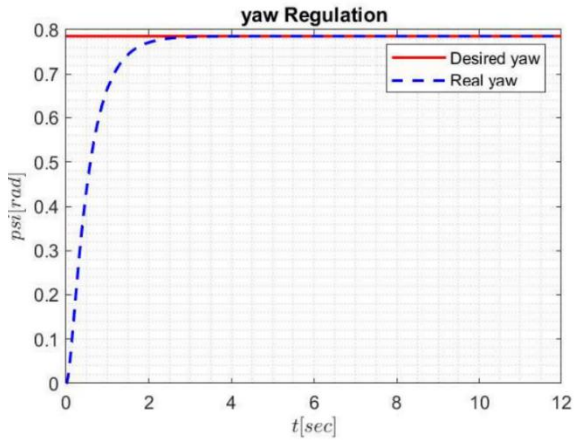

Figure 7: Comparison between desired and actual yaw

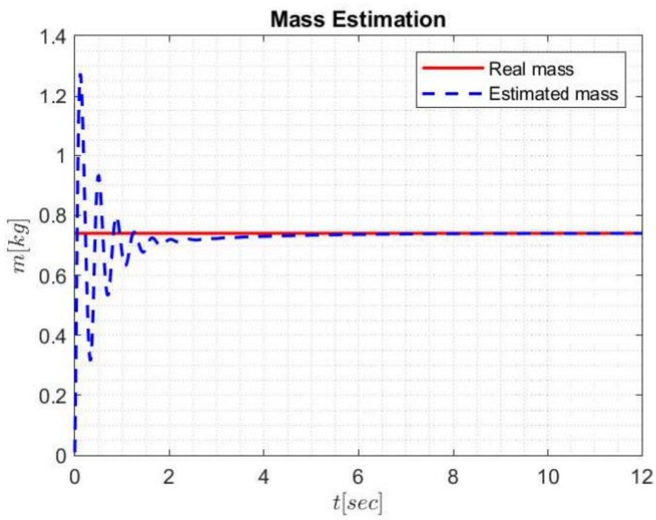

Figure 9:Comparison between the real and estimated mass

From the figures above, we observe that adaptive backstepping controller has succeeded to drive the altitude and attitude to their references. The specifications have been fixed to obtain a satisfying performance and are directly related to controller gains. The higher controller's gains are, the faster the response of the system would be. Furthermore, we remark regarding the altitude precisely that its response is quite a bite lower. This is due to the direct coupling between the mass and the altitude. Since the mass is updating at each time and the controller uses the estimate of the mass as if it were the real value (certainty equivalence principle) the altitude evolves slowly according with the dynamic estimation of the mass. That dynamic of estimation can be accelerated by choosing a high gain adaptation law hence the mass estimation converges rapidly to its real value which in turn accelerated the vanishing of altitude error.

It is worth to know that the stability and performances are obtainable even without having the real estimate of the mass always using the adaptation law. This is explained by the fact that the adaptation law was derived to ensure the stability at a desired equilibrium point regardless of whether the mass was accurately estimated or not. The adaptation law makes therefore the controller robust against parametric uncertainties under the assumption are constant or at worst slowly time-varying.

3.5 Conclusion

This project has approached the application of adaptive integrator backstepping controller to stabilize the altitude and attitude of the quadrotor. It has been shown that this controller has given good results even with an underactuated coupled nonlinear MIMO system. Nevertheless, this controller has many deficiencies such as complexities when the system model has high number of cascades, the application of this technique leads to explosive expressions. Regarding adaptive control, the adaptation loop can lead to instability when combined to the original controller. In addition to the adaptation gain that, if chosen too high, spells poor transient performances and sometimes to instability. Another drawback is that the unknown parameter is supposed constant or slowly time-varying which is not true in some case for example, the air density that changes according to altitude.

External disturbances were not included which are present in practice due to weather conditions. This case is treated under the assumption that the disturbances are bounded. In the literature, there are many techniques such as Lyapunov synthesis, nonlinear damping and other techniques that robustifies the initial controller (Backstepping for instance) and ensure the desired performances.

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4. Appendix

4.1 Mathematical modelling of the Quadrotor

As previously discussed above, the quadrotor is an underactuated and presents high nonlinearities and dynamic coupling.

The Newton-Euler method is used to model the dynamics of a quadrotor. The model predicts how the forces and torques produced by the quadrotor's four propellers will impact its motion.

1.4.1 Assumptions and Constraints

The basic quadrotor structure used for the model development is depicted above showing the Euler angles earlier spoken of (roll, pitch, yaw) =(ϕ, θ, ψ), coordinate frame {B} and global coordinate frame{E}.

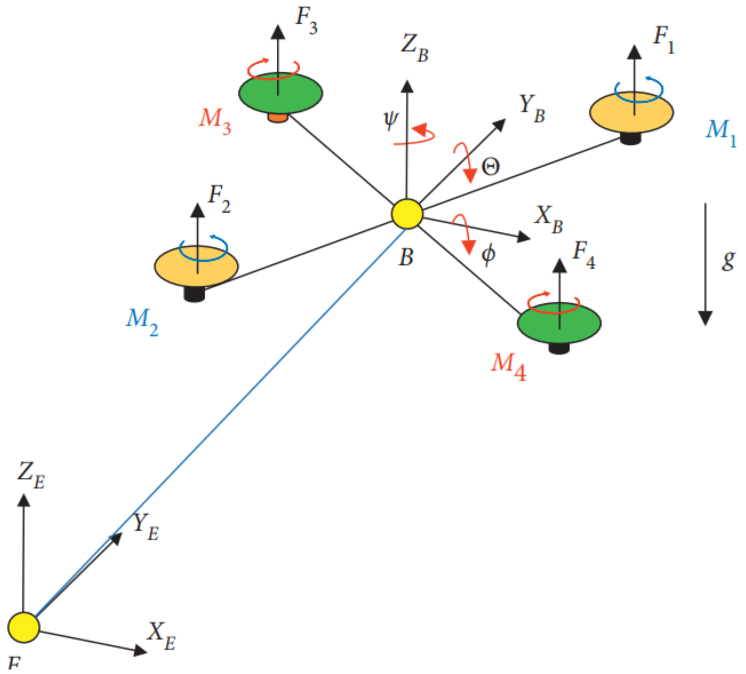

Figure 10: Euler angles in body frame

The assumptions under which the model will be based upon are the following:

1. The quadrotor structure is rigid and symmetrical and the center of gravity and body-fixed frame origin are assumed to be aligned.

2. Thrust and drag of each motor is proportional to the square of the motor angular velocity.

3. The propellers are considered to be rigid and blade flapping is neglected

4. Propellers' weight is negligible and propellers are identical

5. The earth is flat and non-rotating.

6. External disturbances are negligible.

7. Ground effect is neglected.

The following list defines the variables of the figure 5:

X_E: The global (East) position of the quadrotor (m).
Y_E: The global (North) position of the quadrotor (m).

Z_E: The global (Up) position of the quadrotor (m).

X_B: The local (East) position of the quadrotor (m)

Y_B: The local (North) position of the quadrotor (m)

Z_B: The local (up) position of the quadrotor (m)

ϕ : Roll angle along X_B (radians)

θ : pitch angle along Y_B (radians)

ψ : pitch angle along Z_B (radians)

1.4.2 Kinematic Description

In order to describe the position and attitude of the body-fixed frame with respect to the earth frame, we define the position and attitude as follows:

$$P = [x\ y\ z]^T \tag{4.1}$$

$$Q = [\phi\ \theta\ \psi]^T \tag{4.2}$$

P is the linear position and contains the coordinates of the body-fixed frame with respect earth frame, and Q is the angular position of the body-fixed frame with respect to the earth frame. Before any further development and it is necessary to adopt a transformation between earth frame and body-fixed frame. This transformation is called rotational matrix which is a mathematical representation to parametrize the rotation of a rigid body in the space.

- Translational Kinematic:

The state variables for velocity are expressed in the body frame but the state variables for position are in the earth frame. Using Euler angles, we define the three rotational matrices:

$$R(\psi) = \begin{pmatrix} \cos(\psi) & \sin(\psi) & 0 \\ -\sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{4.3}$$

$$R(\theta) = \begin{pmatrix} \cos(\theta) & 0 & -\sin(\theta) \\ 0 & 1 & 0 \\ \sin(\theta) & 0 & \cos(\theta) \end{pmatrix} \tag{4.4}$$

$$R(\phi) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi) & \sin(\phi) \\ 0 & -\sin(\phi) & \cos(\phi) \end{pmatrix} \tag{4.5}$$

The global rotation matrix between the earth frame and body-fixed frame is expressed as follows:

$$R_E^B = R(\phi)R(\theta)R(\psi) \tag{4.6}$$

Which expresses the transformation from the initial frame E(Earth) to the second frame B(Body).

The expression of rotational transformation is given below:

$$R_E^B = \begin{pmatrix} \cos(\psi)\cos(\theta) & \sin(\psi)\cos(\theta) & -\sin(\theta) \\ \cos(\psi)\sin(\phi)\sin(\theta) - \cos(\phi)\sin(\psi) & \sin(\psi)\sin(\phi)\sin(\theta) + \cos(\theta)\cos(\psi) & \cos(\theta)\sin(\phi) \\ \cos(\psi)\cos(\phi)\sin(\theta) + \sin(\phi)\sin(\psi) & \cos(\psi)\sin(\phi)\sin(\theta) - \sin(\phi)\cos(\psi) & \cos(\theta)\cos(\phi) \end{pmatrix} \tag{4.7}$$

Any vector in the earth frame can be expressed in the body frame using this transformation, we write

$$X_B = R_E^B X_E \tag{4.8}$$

For the other way around:

$$X_E = (R_E^B)^{-1} X_B = (R_E^B)^T X_B = (R_B^E) X_B \tag{4.9}$$

With:

$$R_B^E = \begin{pmatrix} \cos(\psi)\cos(\theta) & \sin(\psi)\cos(\theta) & -\sin(\theta) \\ \cos(\psi)\sin(\phi)\sin(\theta) - \cos(\phi)\sin(\psi) & \sin(\psi)\sin(\phi)\sin(\theta) + \cos(\theta)\cos(\psi) & \cos(\theta)\sin(\phi) \\ \cos(\psi)\cos(\phi)\sin(\theta) + \sin(\phi)\sin(\psi) & \cos(\psi)\sin(\phi)\sin(\theta) - \sin(\phi)\cos(\psi) & \cos(\theta)\cos(\phi) \end{pmatrix}^T \tag{4.10}$$

- Rotational Kinematic:

The angular velocities in the body frame are noted $\omega_b = [\mathrm{p, q, r}]^T$, we use the rotation matrix derived above to express them as a function of time derivative of Euler angles:

$$\omega_b = \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} + R(\phi) \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + R(\phi)\, R(\theta) \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} \tag{4.11}$$

$$\omega_b = \begin{pmatrix} 1 & 0 & -\sin(\theta) \\ 0 & \cos(\phi) & \cos(\theta)\sin(\phi) \\ 0 & -\sin(\phi) & \cos(\theta)\cos(\phi) \end{pmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \tag{4.12}$$

Hence:

$$\omega_e = \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{pmatrix} 1 & \tan(\theta)\sin(\phi) & \tan(\theta)\cos(\phi) \\ 0 & \cos(\phi) & -\sin(\phi) \\ 0 & \dfrac{\sin(\phi)}{\cos(\phi)} & \dfrac{\cos(\phi)}{\cos(\theta)} \end{pmatrix} \omega_b \tag{4.13}$$

Before moving to dynamic motion of the quadrotor, motor dynamics are assumed to be neglected.

1.4.3 Dynamic description

- Translational dynamics:

The dynamic motion of quadrotor is expressed using Euler-Newton formulas as follow:

$$m\ddot{X}_G(t) = -F_{G-E}(t) + F_{T-B}(t) + F_{d-E}(t) \tag{4.14}$$

With:

F_G: Gravitational force, acceleration of gravity acting on the center of gravity of the airframe(N)

F_T: Thrust force, force acting on quadrotor and generate by the four motors(N)

F_d: Drag force, aerodynamic friction acting on the system(N)

$\ddot{X}_G$: Acceleration of the quadrotor at the gravity center(m/s²)

m: Mass of the quadrotor(kg)

Both gravitational and drag forces are expressed in the earth frame, that is why thrust force should be brought to the earth frame, hence:

$$F_{T-E} = R_B^E F_{T-B} \tag{4.15}$$

Forces expression :

$$F_G = [0,0,-mg]^T \tag{4.16}$$

$$F_{T-B} = \left[0,0,\sum_{i=1}^{4} F_i\right]^T \tag{4.17}$$

$$F_d = \begin{pmatrix} kx & 0 & 0 \\ 0 & \mathrm{ky} & 0 \\ 0 & 0 & \mathrm{kz} \end{pmatrix} \begin{bmatrix} \dot{x}|\dot{x}| \\ \dot{y}|\dot{y}| \\ \dot{z}|\dot{z}| \end{bmatrix} \tag{4.18}$$

g: Gravitation acceleration (m/s²)

F_i: Thrust force generated by each motor(N)

kx : Drag coefficient for x axis (N.s²/m²)

- Rotational dynamics:

The rotational equations of motion are defined in the body-fixed frame in order to compute the rotations about the quadrotor's center and not the center of the earth frame. Using Euler-Newton equation, the rotational dynamics are given below:

$$J_b \dot{\omega}_e(t) = -\big(\omega_e(t) \wedge J_b \omega_e(t)\big) + M_T(t) + M_d(t) + M_G(t) \tag{4.19}$$

With:

$$\omega_e = [\dot{\phi}, \dot{\theta}, \dot{\psi}]^T$$

$$M_T = \left[0,0,\sum_{i=1}^{4} l.F_i\right]^T \tag{4.20}$$

$$M_d = \begin{pmatrix} k_\phi & 0 & 0 \\ 0 & k_\theta & 0 \\ 0 & 0 & k_\psi \end{pmatrix} \begin{bmatrix} \dot{\phi}^2 \\ \dot{\theta}^2 \\ \dot{\psi}^2 \end{bmatrix} \tag{4.21}$$

$$M_G = \begin{bmatrix} J_r \omega \dot{\theta} \\ -J_r \omega \dot{\phi} \\ 0 \end{bmatrix} \tag{4.22}$$

$$J_b = \begin{pmatrix} J_x & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{pmatrix} \tag{4.23}$$

J_b: Inertial moment of the body.

l : quadrotor arm length(m).

M_T: Torques provided by the rotors.

M_d: Torque resulting from aerodynamic friction.

M_G : Torque due to gyroscopic effect.

ω : residual angular velocity due to gyroscopic effect.

J_x,J_y,J_z : Inertia around x, y and z respectively.

k_ϕ, k_θ, k_ψ : Coefficients of aerodynamic friction around roll, pitch and yaw respectively.

Therefore, after simplification, the dynamical model of the quadrotor is given:

$$\ddot{x} = (\sin(\phi)\sin(\psi) + \sin(\theta)\cos(\phi)\cos(\psi))\frac{U1}{m} - \frac{kx}{m}\dot{x}|\dot{x}| \quad (4.24)$$

$$\ddot{y} = (\cos(\phi)\sin(\psi)\sin(\theta) - \sin(\phi)\cos(\psi))\frac{U1}{m} - \frac{ky}{m}\dot{y}|\dot{y}| \quad (4.25)$$

$$\ddot{z} = -g + \cos(\phi)\cos(\theta)\frac{U1}{m} - \frac{kz}{m}\dot{z}|\dot{z}| \quad (4.26)$$

$$\ddot{\phi} = \frac{J_y - J_z}{J_x}\dot{\theta}\dot{\psi} - \frac{J_r}{J_x}\omega\dot{\theta} - \frac{k_\phi}{J_x}\dot{\phi}^2 + \frac{1}{J_x}U2 \quad (4.27)$$

$$\ddot{\theta} = \frac{J_z - J_x}{J_y}\dot{\phi}\dot{\psi} + \frac{J_r}{J_y}\omega\dot{\phi} - \frac{k_\theta}{J_y}\dot{\theta}^2 + \frac{1}{J_y}U3 \quad (4.28)$$

$$\ddot{\psi} = \frac{J_x - J_y}{J_z}\dot{\phi}\dot{\theta} - \frac{k_\psi}{J_z}\dot{\psi}^2 + \frac{1}{J_z}U4 \quad (4.29)$$

With:

$$U1 = \left[0,0,\sum_{i=1}^{4} l.F_i\right]^T \quad (4.30)$$

$$[U2, U3, \mathrm{U4}] = \left[\sum l.F_{roll}, \sum l.F_{pitch}, \sum l.F_{yaw}\right]^T \quad (4.31)$$

Numerical values of the physical parameters of the system

Parameter	Value
J_x(Kg.m²)	0.004
J_y(Kg.m²)	0.004
J_z(Kg.m²)	0.0084
J_r (Kg.m²)	2.8385e-5
g (m/s²)	9.81
l (m)	0.225
k_x(N/m²/s²)	0.001
k_y(N/m²/s²)	0.001
k_z(N/m²/s²)	0.001
k_ϕ(N/m/s)	0.01
k_θ(N/m/s)	0.01
k_ψ(N/m/s)	0.01
M(kg)	0.74

Numerical values of the Controller gains

Gains	Value
k_1	2
k_2	10
k_3	2
k_4	10
k_5	2
k_6	10
k_7	0.5
k_8	5
γ	2