Engine cycle analysis of internal combustion engines

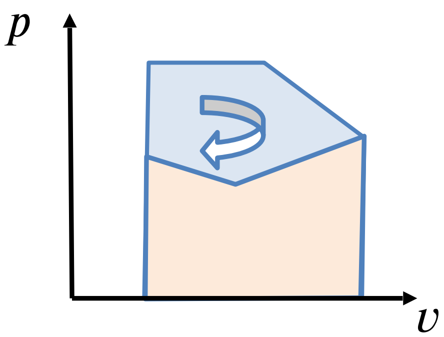
4A13

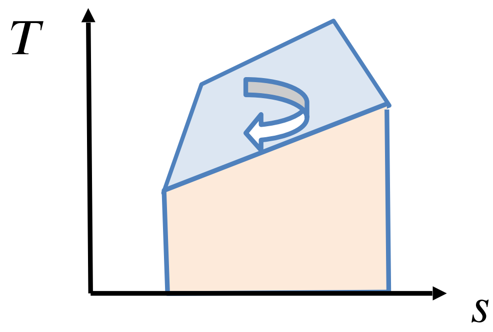
4A13 Internal Combustion Engines

Simone Hochgreb
sh372@cam.ac.uk

Heywood: Ch 5. Ideal Models of Engine Cycles
Stone: Ch. 2: Thermodynamic principles

Cycles





$$w = \oint p\, dv = w_{in} - w_{out}$$

$$w = q$$

$$q = \oint T\, ds = q_{in} - q_{out}$$

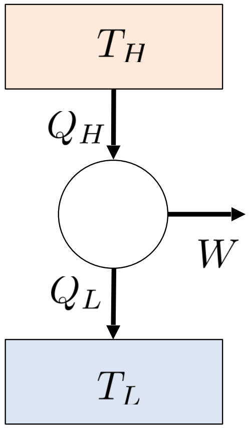
Efficiency:

$$\eta = \frac{w}{q_{in}}$$

Net work

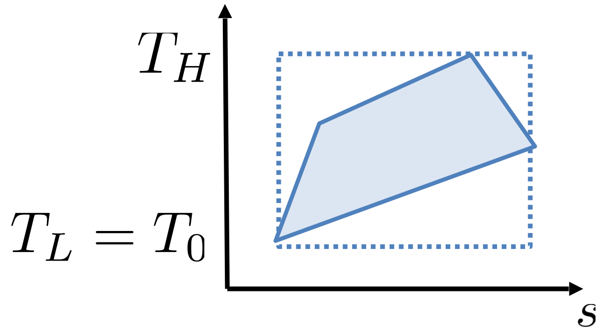
Heat input

Cycle efficiency

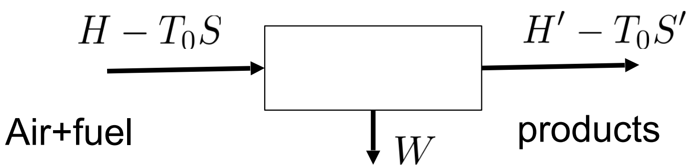


$$\eta_c = 1 - \frac{T_L}{T_H}$$

$$\eta_c = \frac{W}{Q_H}$$



Exergy efficiency

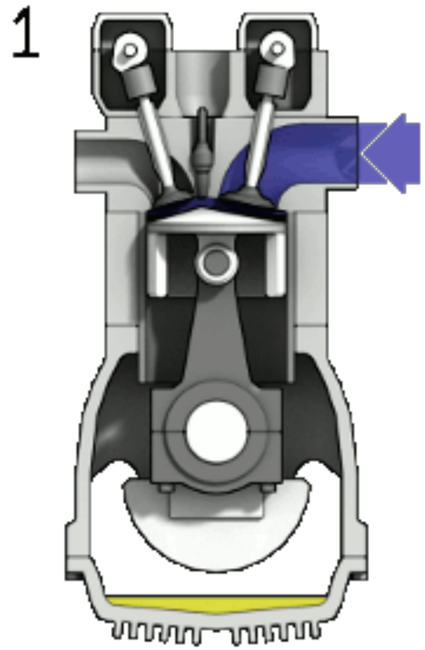


$$\eta_e = \frac{(H - H') - T_0(S - S')}{(H - H_0) - T_0(S - S_0)} \approx \frac{W}{(H - H_0)} = \frac{W}{Q}$$

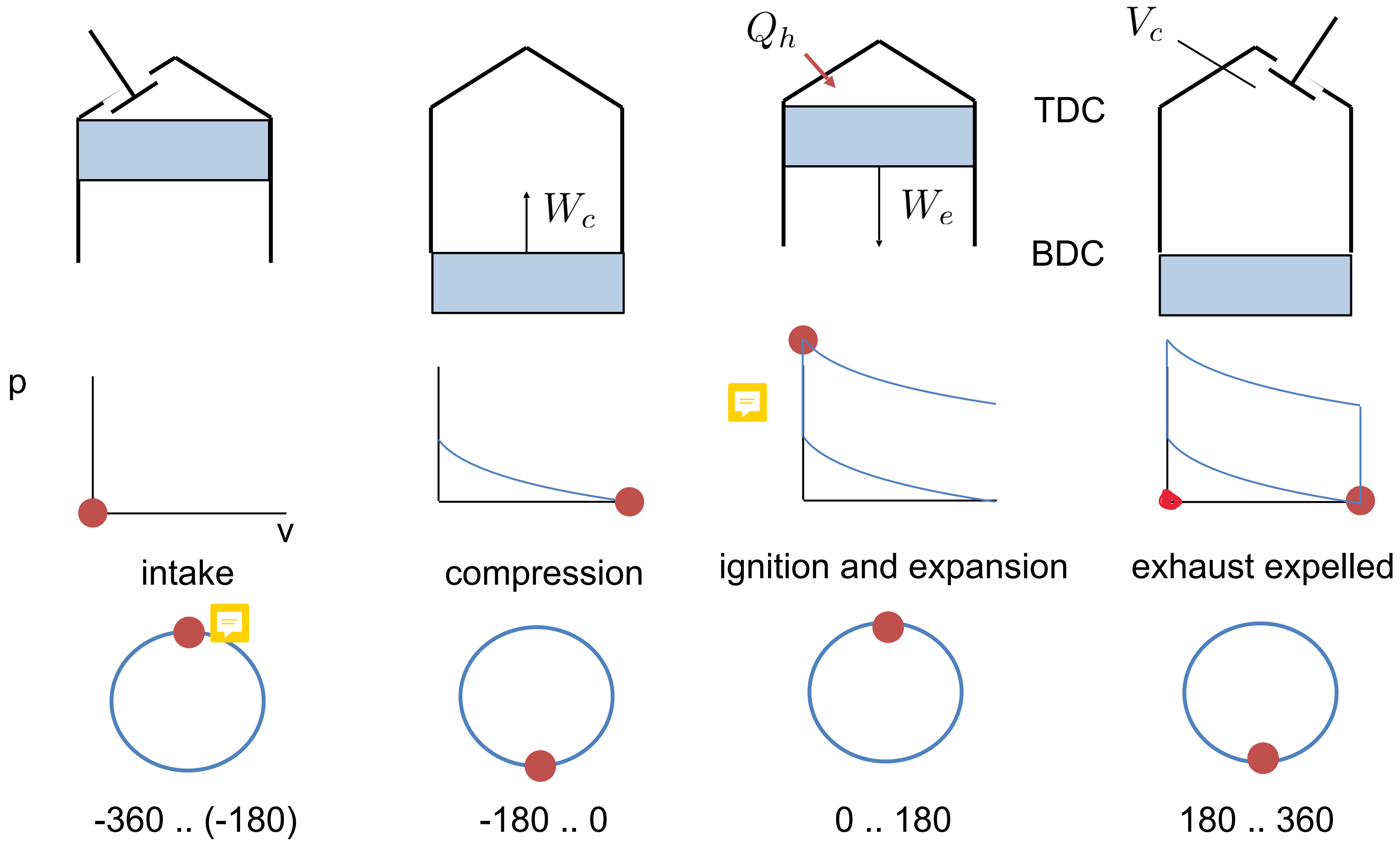
$$(H - H_0) >> T_0(S - S_0)$$

Energy efficiency ~ Exergy efficiency

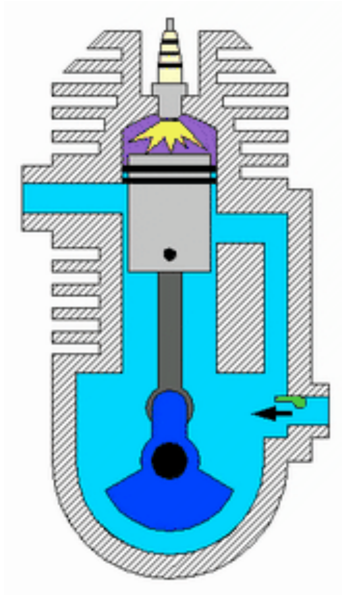
4-stroke SI engine operation



Reciprocating engines: four-stroke

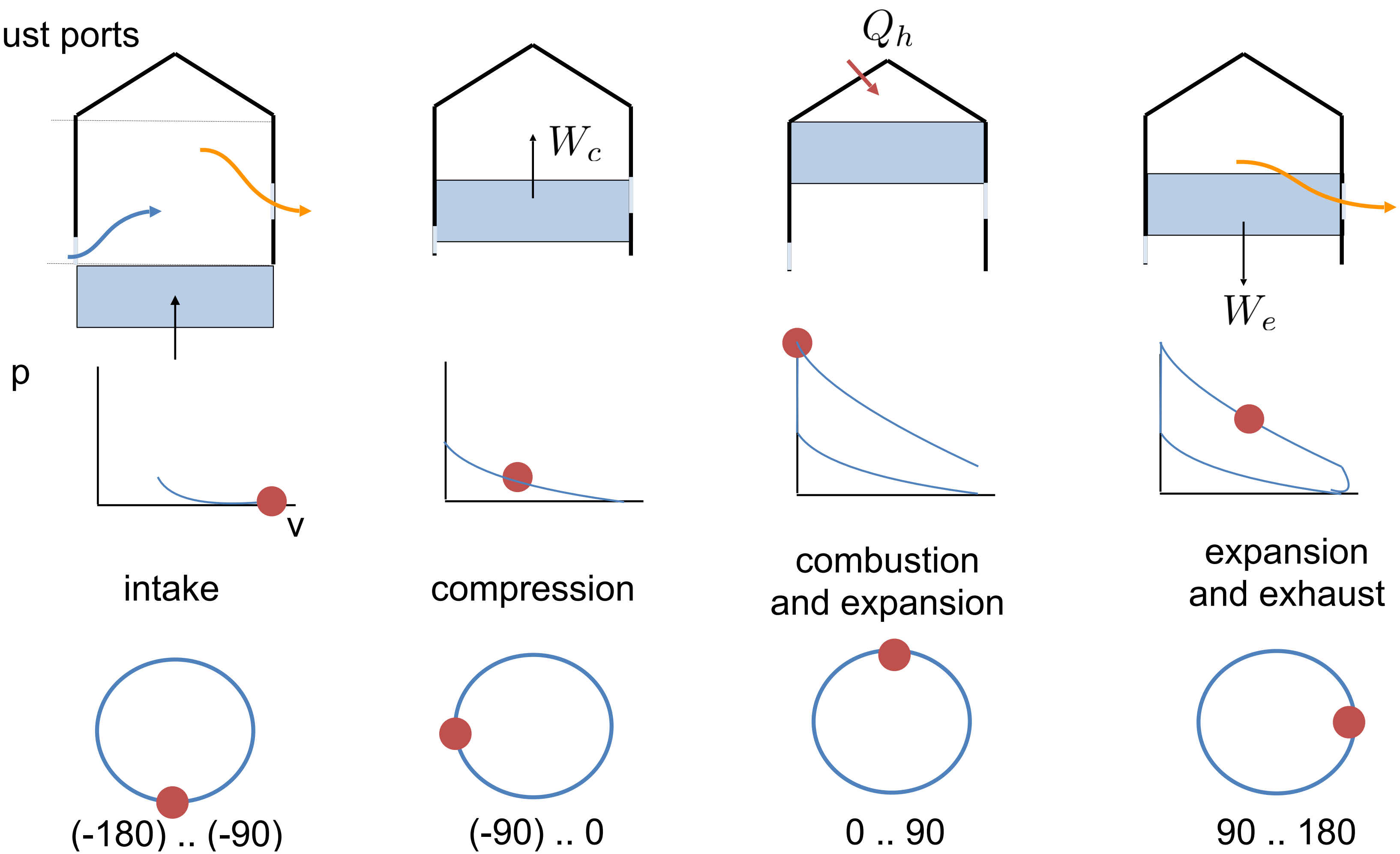


2-stroke engine operation

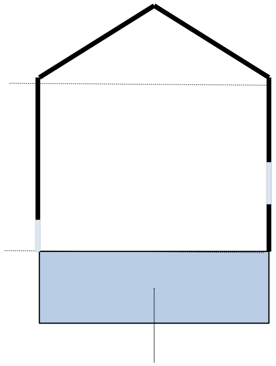


Reciprocating engines: two-stroke

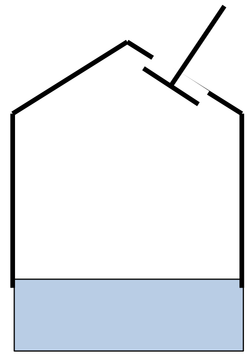
No valves, just ports



Two-stroke vs Four-stroke



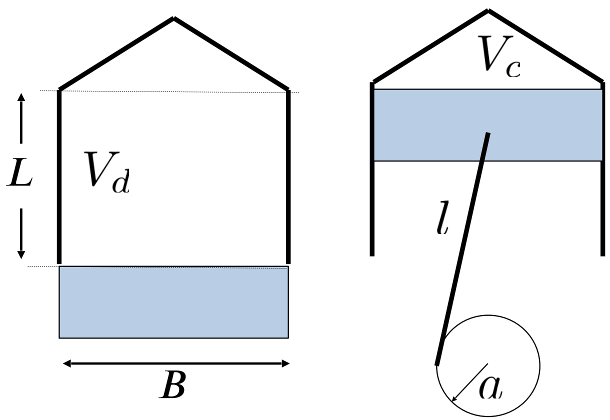
	2	4
Power per stroke	+	-
Power density	+	-
Efficiency	-	+
Scavenging	-	+
Emissions	-	+



Higher power density for 2-stroke (useful for hand-held or portable engines)

Short circuiting of fresh mixture difficult to avoid in 2-stroke: higher emissions and lower combustion efficiency for 2-stroke.

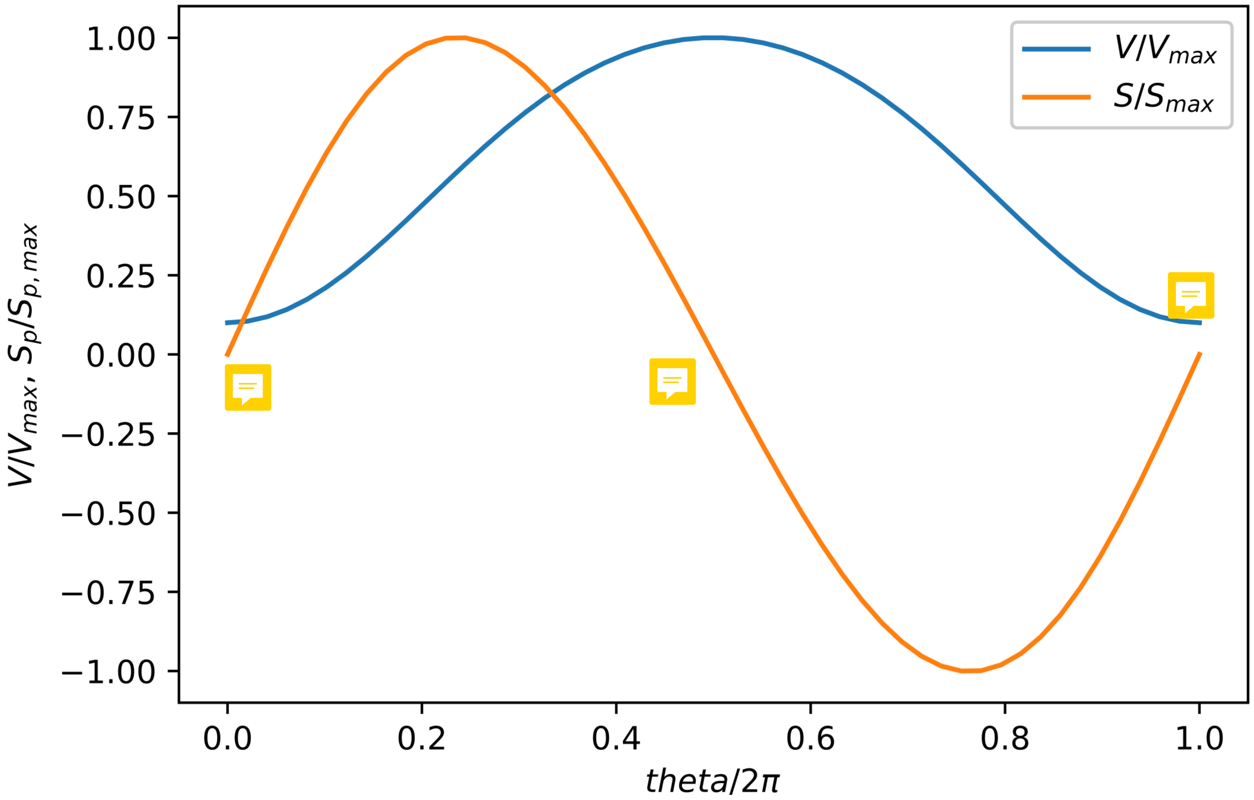
Geometric and kinematic parameters



N	revolutions/time
n_R	revolutions/cycle (1 for 2-stroke or 2 for 4-stroke)

B	bore
L	stroke
V_d	displaced volume
V_c	clearance volume
$r_c = \dfrac{V_d + V_c}{V_c}$	geometric compression ratio
l	crankshaft length
a	crankshaft radius

Volume history and piston speed: four-stroke

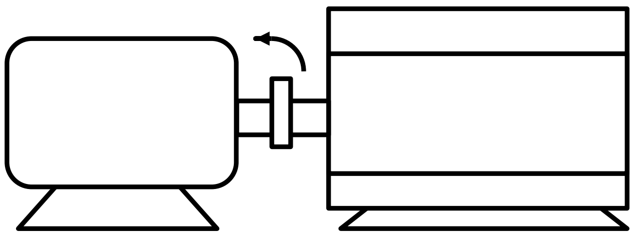


$$\frac{V}{V_{max}} = 1 + \frac{1}{2}(r_c - 1)(\frac{a}{l} + 1 - \cos\theta - \left(\left(\frac{a}{l}\right)^2 - \sin^2\theta\right)^{1/2}$$

$$\frac{S}{S_{max}} = \frac{\pi}{2}\sin\theta\left[1 + \frac{\cos\theta}{\left(\left(\frac{a}{l}\right)^2 - \sin^2\theta\right)^{1/2}}\right]$$

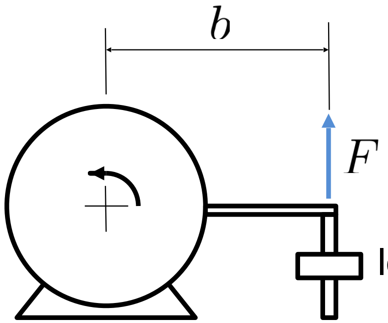
TDC BDC TDC

Torque and power



engine

dynamometer/brake
(cooled electric motor that absorbs or provides power)



torque $T = Fb$

brake power $P_b = 2\pi NT$

net power delivered to powertrain

gross power $P_g = P_b + P_f$

brake friction

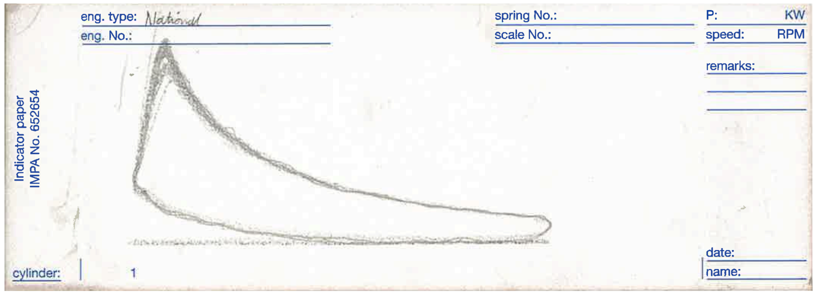
gross power is delivered by the fluid to the mechanical crankshaft, and some power is lost by friction (pistons, crankshaft)

mechanical efficiency $\eta_m = \dfrac{P_b}{P_g} = 1 - \dfrac{P_f}{P_g}$

lost friction power increases, and mechanical efficiency decreases with speed

A purely mechanical p-v indicator diagram

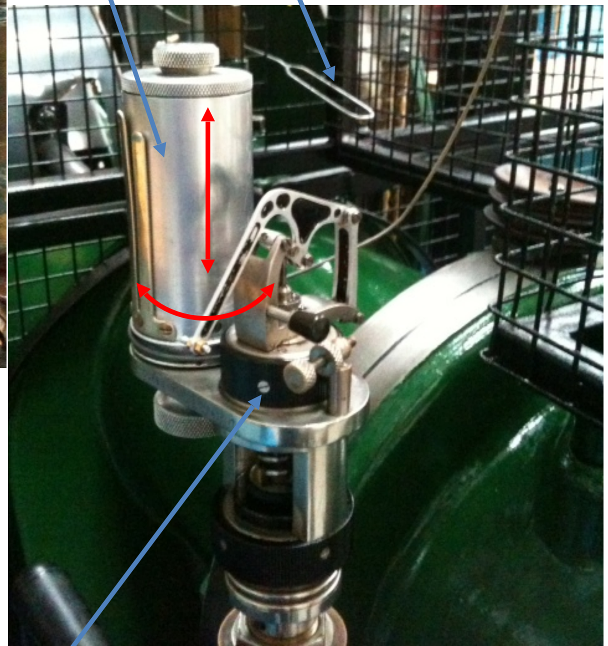
National gas engine (1st year experiment)
CR = 6


Indicator paper
IMPA No. 652654

eng. type: National
eng. No.:
spring No.:
scale No.:
P: KW
speed: RPM
remarks:
date:
name:
cylinder: 1

Crank indicator (to pulley)

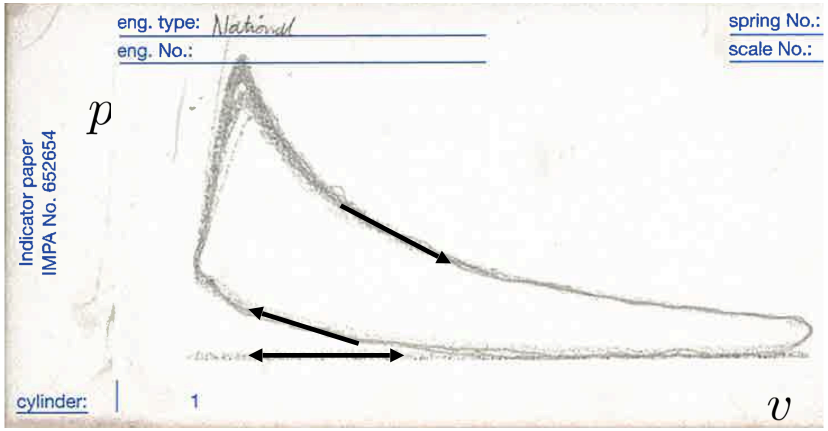
Pressure
paper



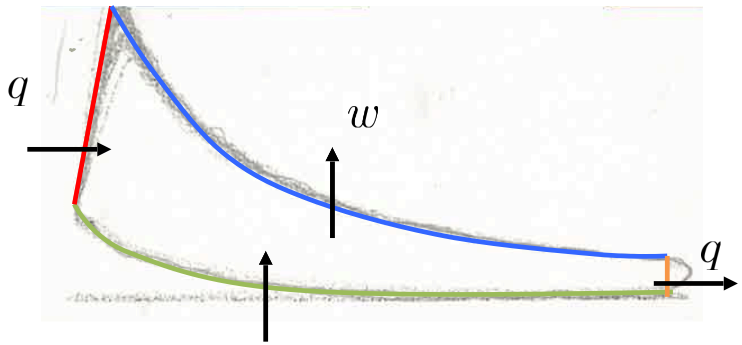
Pressure poppet

Indicator diagram and standard air cycles

Indicator diagram



Equivalent air cycle



National natural gas engine
6:1 compression ratio
Spark ignited
4 stroke
Atmospheric intake

Single working fluid
Constant mass
Constant properties
Ideal gas

Simplification allows analysis of the process

Engine operating parameters

gross indicated work per cycle (J = N.m) $$W_{i,g} = \oint p\,dV$$

net indicated work per cycle (J = N.m) $$W_{i,n} = W_{i,g} + W_p$$

negative when $p_i < p_e$

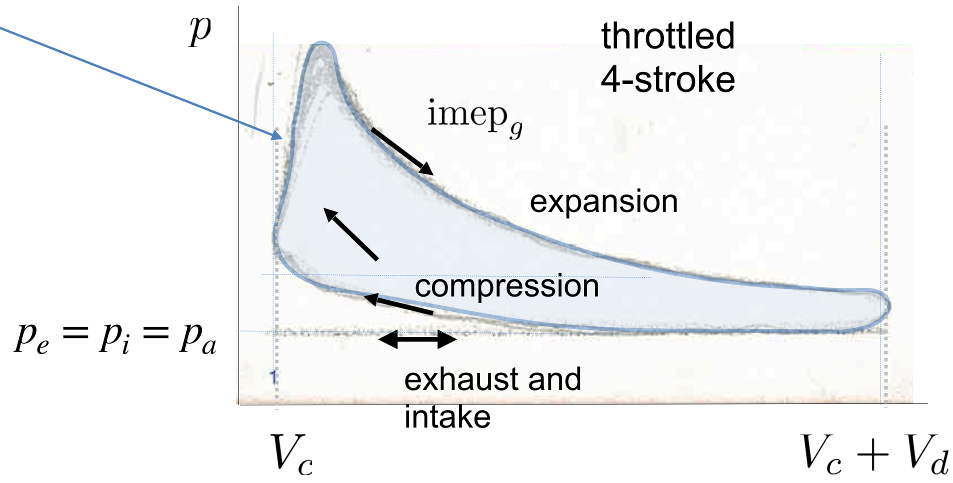
torque (N.m) $$T = \frac{P}{2\pi N} = \frac{W}{2\pi n_R}$$

power (W) $$P = 2\pi N T = \frac{WN}{n_R}$$

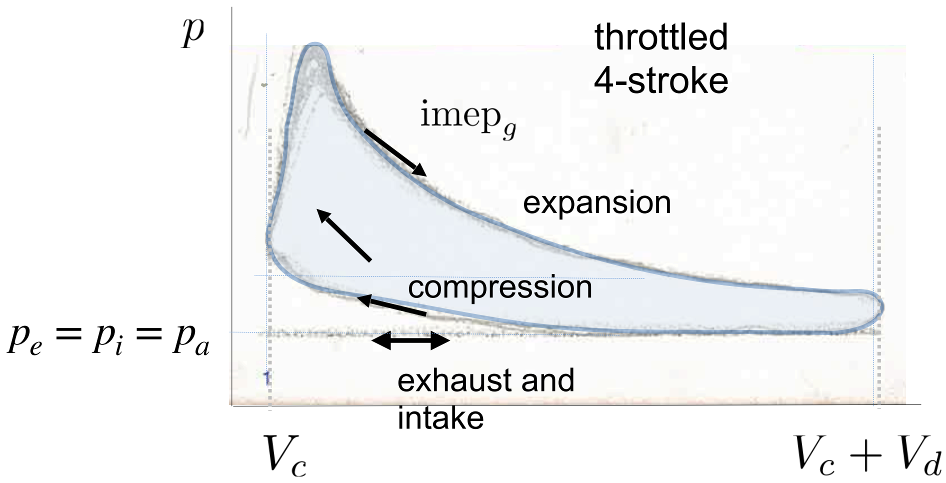
thermal/fuel efficiency (-) $$\eta_f = \frac{W}{m_f q_f} = \frac{P}{\dot{m}_f q_f}$$

specific fuel consumption (sfc, g/kWh) $$\mathrm{sfc} = \frac{1}{\eta_f q_f}$$

volumetric efficiency (-) $$\eta_v = \frac{\dot{m}_a}{\rho_{a,i} V_d}\frac{n_R}{N} = \frac{m_a}{\rho_{a,i} V_d}$$



Performance parameters: load and imep



Work/displaced volume:
indicated **m**ean **e**ffective **p**ressure

$$\text{(i)mep} = \frac{W_i}{V_d}$$

$$\frac{W}{V_d} = \eta_f \left(\frac{m_f q_f}{m} \right) \frac{m}{V_d}$$

Depends on initial state (throttling, turbocharging, residuals)

$$Q^* = \frac{m_f q_f}{m}$$

Heat release per unit mass in the cylinder. Depends on AFR, dilution, volumetric efficiency

$$m = \rho_1 (V_d + V_c)$$

$$\text{imep} = \eta_f Q^* \rho_1 \frac{r_c}{r_c - 1}$$

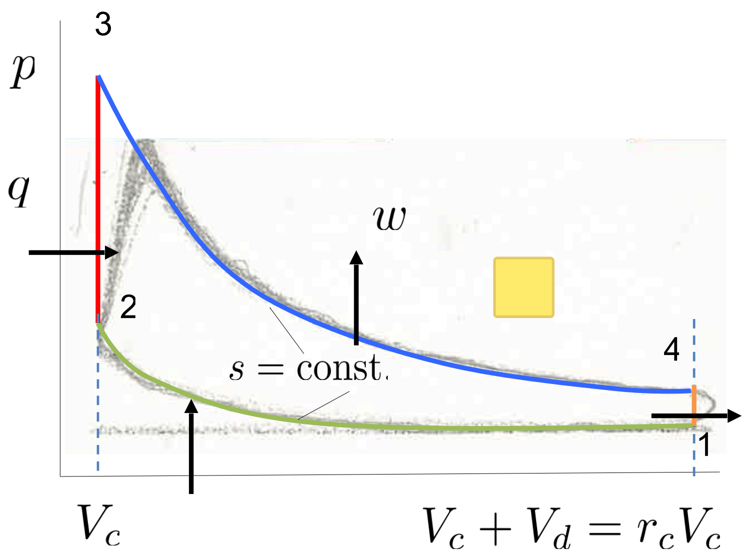
Power density/load (**imep**) depends on efficiency and ability to bring enough mass into the cylinder: displace burned gas and bring fresh mixture

Example

Measurements are made on a 4-stroke engine with a displacement of 1.4 litres. The measured brake torque at 1500 rpm is of 12 N.m. The measured fuel consumption is $\dot{m}_f$= 0.12 x10^{-3} kg/s, and the measured induced air flow is $\dot{m}_f$= 2.0 x10^{-2} kg/s at an atmospheric density of 1.2 kg/m^3. The pumping work is zero, and the estimated mechanical efficiency is 95%. Determine the brake power, gross work per unit cycle, imep, overall efficiency, volumetric efficiency and specific fuel consumption in g/kWh.

Standard Ideal Cycles

Otto (constant-volume) cycle



Equivalent air cycle

1-2: Isentropic compression $p_1 v_1^\gamma = p_2 v_2^\gamma$

2-3: Isochoric heat addition $\dfrac{p_3}{T_3} = \dfrac{p_2}{T_2}$

3-4: Isentropic expansion $p_4 v_4^\gamma = p_3 v_3^\gamma$

4-1: Isochoric heat rejection $\dfrac{p_1}{T_1} = \dfrac{p_4}{T_4}$

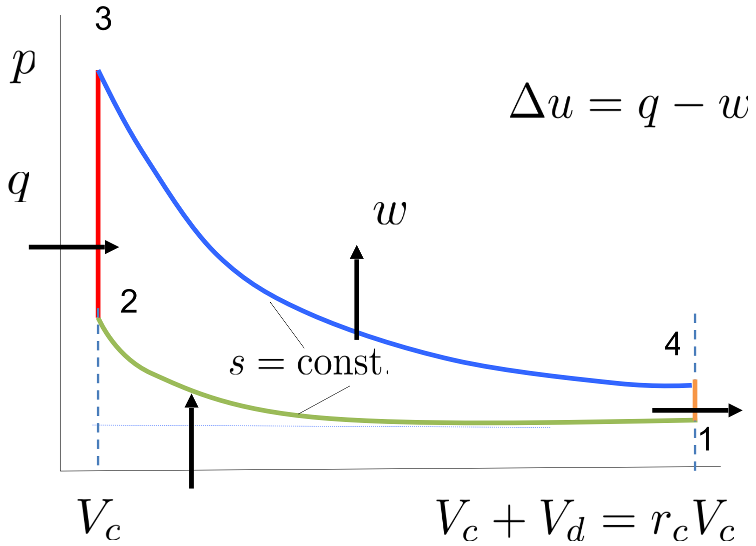
$$p_2 = p_1 (v_1/v_2)^\gamma = p_1 r_c^\gamma$$

$$T_2 = T_1 r_c^{\gamma-1}$$

$$p_3 = T_3/T_2 \; p_2$$

$$T_4 = T_3 \; r_c^{-(\gamma-1)}$$

Otto (constant-volume heat addition) cycle
first law analysis



$$\Delta u = q - w$$

$$q_{12} = 0 \qquad w_{12} = -c_v(T_2 - T_1)$$

$$q_{23} = c_v(T_3 - T_2) \qquad w_{23} = 0$$

$$q_{34} = 0 \qquad w_{34} = c_v(T_3 - T_4)$$

$$q_{41} = -c_v(T_4 - T_1) \qquad w_{41} = 0$$

$$w = \oint \delta w = c_v((T_3 - T_4) - (T_2 - T_1))$$

$$q = \oint \delta q = c_v((T_3 - T_2) - (T_4 - T_1))$$

$$\eta = \frac{w}{q_{23}} = 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

$$\eta = 1 - \frac{T_3 r_c^{-(\gamma-1)} - T_1}{T_3 - T_1 r_c^{\gamma-1}}$$

$$\boxed{\eta = 1 - r_c^{-(\gamma-1)}}$$

$$p_2 = p_1(v_1/v_2)^\gamma = p_1 r_c^\gamma$$

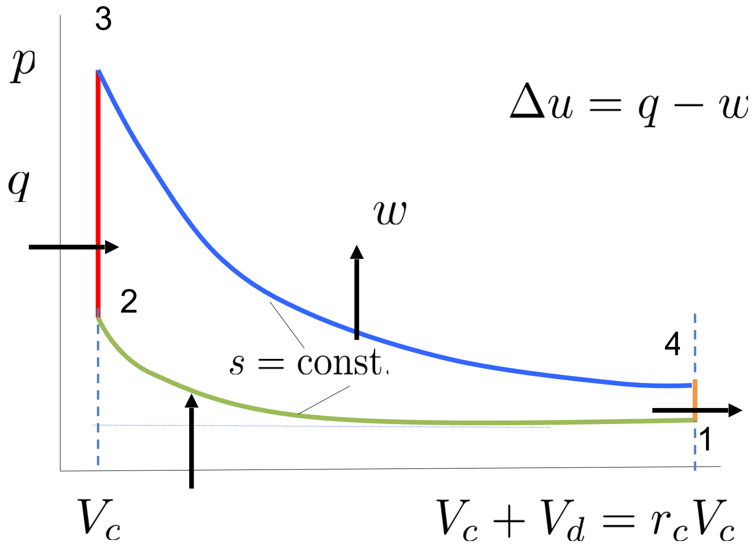
$$T_2 = T_1 r_c^{\gamma-1}$$

$$p_3 = T_3/T_2 \; p_2$$

$$T_4 = T_3 \; r_c^{-(\gamma-1)}$$

Ideal cycle efficiency **only** depends on the compression ratio

Otto (constant-volume heat addition) cycle
first law analysis



$$\Delta u = q - w$$

$$\eta = \eta_f = 1 - r_c^{-(\gamma-1)}$$

$$\frac{\text{imep}}{p_1} = \frac{Q^*}{c_v T_1} \frac{1}{\gamma - 1} \frac{r_c}{r_c - 1} \eta_f$$

$$\frac{\text{imep}}{p_1} = \frac{Q^*}{c_v T_1} \frac{1}{\gamma - 1} \frac{r_c}{r_c - 1} (1 - r_c^{-(\gamma-1)})$$

$$p_2 = p_1 (v_1/v_2)^\gamma = p_1 r_c^\gamma$$

$$T_2 = T_1 r_c^{\gamma-1}$$

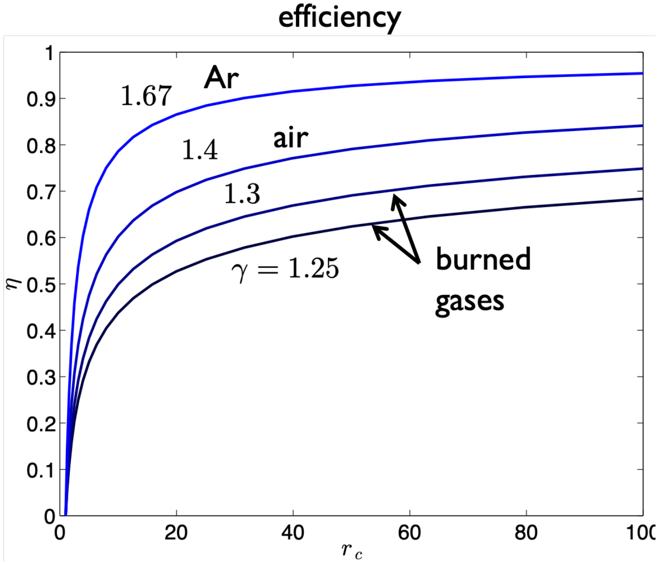
$$p_3 = T_3/T_2 \; p_2$$

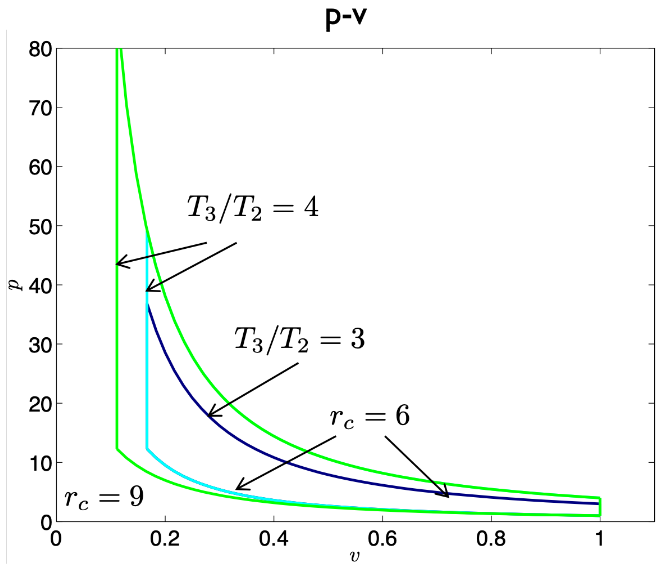
$$T_4 = T_3 \; r_c^{-(\gamma-1)}$$

Ideal imep depends on heat release per unit mass of charge and compression ratio

Efficiency of constant volume cycle

Effect of heat release and r_c





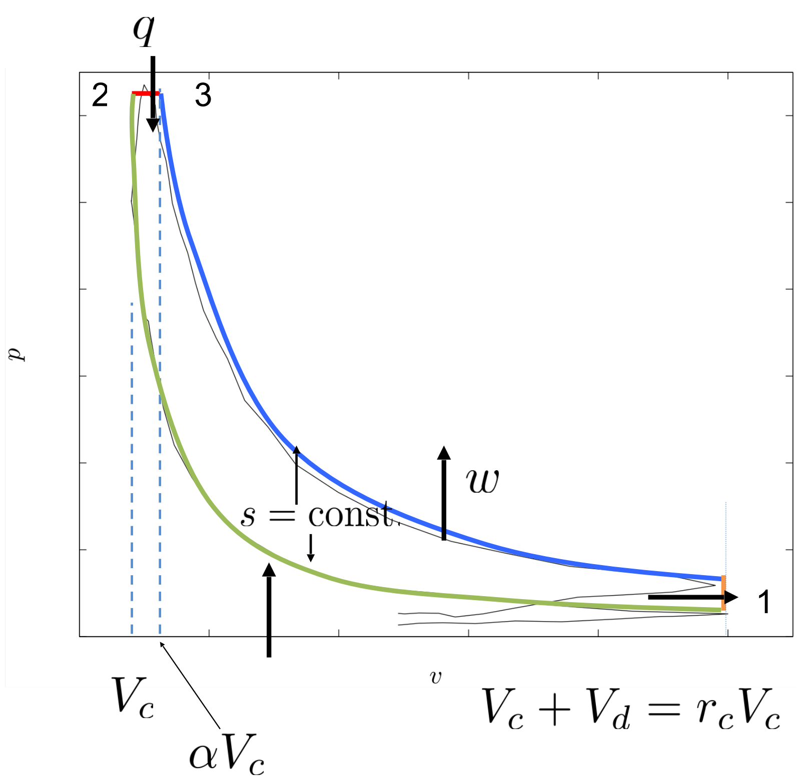
$$\eta_f = 1 - r_c^{-(\gamma-1)}$$

Smaller molecules lead to higher efficiencies

limited by autoignition, NOx

function of gas composition

CI (constant pressure heat addition) cycle



Equivalent air cycle

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4-1: Isochoric heat rejection $\dfrac{p_1}{T_1} = \dfrac{p_4}{T_4}$

$$p_2 = p_1 (v_1/v_2)^\gamma = p_1 r_c^\gamma$$

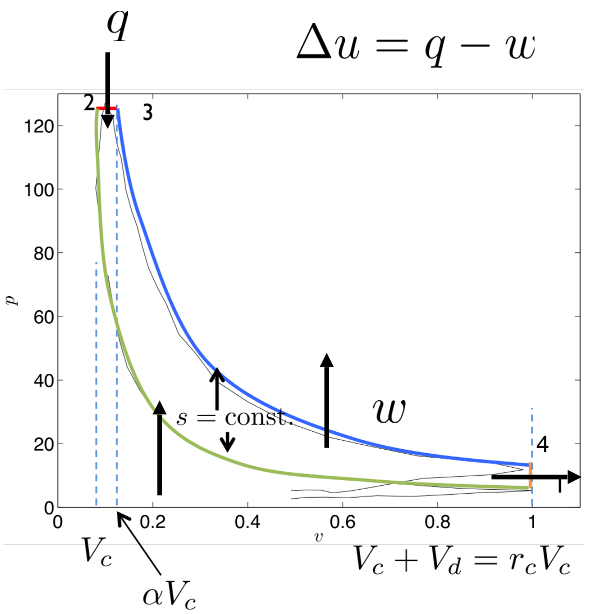
$$T_2 = T_1 r_c^{\gamma-1}$$

$$T_3 = T_2 v_3/v_2 = \alpha T_1 r_c^{\gamma-1}$$

$$T_4 = T_3 \left(\frac{v_4}{v_3}\right)^{\gamma-1} = T_3 \left(\frac{\alpha}{r_c}\right)^{\gamma-1} = \alpha^\gamma T_1$$

CI (constant pressure heat addition) cycle
first law analysis

$$\Delta u = q - w$$



$$p_2 = p_1 (v_1/v_2)^\gamma = p_1 r_c^\gamma$$

$$T_2 = T_1 r_c^{\gamma-1}$$

$$T_3 = T_2 v_3/v_2 = \alpha T_1 r_c^{\gamma-1}$$

$$T_4 = T_3 \left(\frac{v_4}{v_3}\right)^{\gamma-1} = T_3 \left(\frac{\alpha}{r_c}\right)^{\gamma-1} = \alpha^\gamma T_1$$

$$q_{12} = 0 \qquad w_{12} = -c_v (T_2 - T_1)$$

$$q_{23} = c_p (T_3 - T_2) \qquad w_{23} = p_2 (v_3 - v_2)$$

$$q_{34} = 0 \qquad w_{34} = c_v (T_3 - T_4)$$

$$q_{41} = -c_v (T_4 - T_1) \qquad w_{41} = 0$$

$$w = \oint \delta w = -c_v (T_2 - T_1) + p_2 (v_3 - v_2) + c_v (T_3 - T_4)$$

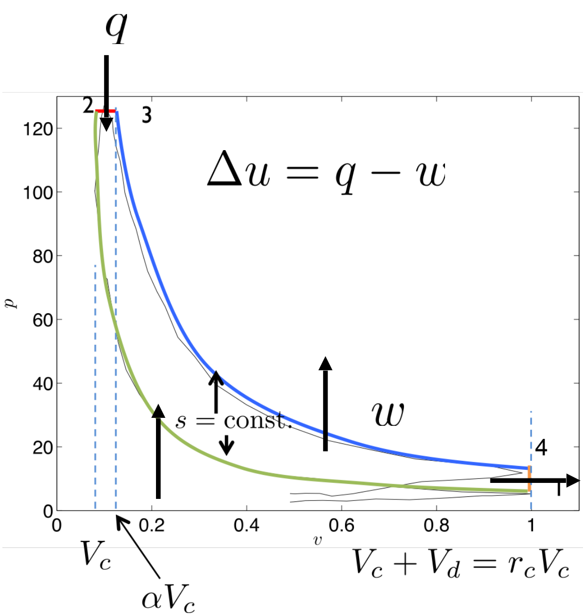
$$q = \oint \delta w = c_p (T_3 - T_2) + c_v (T_4 - T_1)$$

$$\eta = \frac{w}{q_{23}} = \frac{q}{q_{23}} = \frac{q_{23} + q_{41}}{q_{23}} = 1 - \frac{c_v (T_4 - T_1)}{c_p (T_3 - T_2)} = 1 - \frac{1}{\gamma} \frac{(T_4/T_1 - 1)}{T_2/T_1 - T_3/T_1}$$

$$\boxed{\eta = 1 - \underbrace{\frac{1}{r_c^{(\gamma-1)}}}_{\text{Const. v}} \frac{1}{\gamma} \underbrace{\frac{\alpha^\gamma - 1}{\alpha - 1}}_{>1}}$$

Limited pressure cycle efficiency is lower than the constant volume cycle efficiency for the same compression ratio.

CI (constant pressure heat addition) cycle
first law analysis



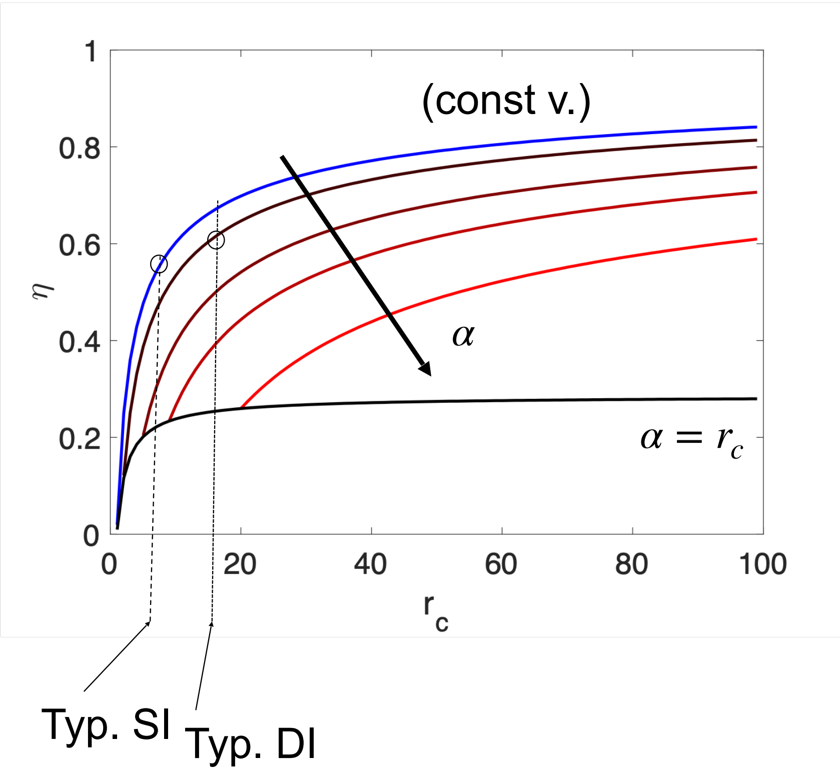
$$\eta_f = 1 - \left(\frac{1}{\gamma} \frac{\alpha^\gamma - 1}{\alpha - 1} \right) r_c^{-(\gamma-1)}$$

$$\frac{Q^*}{c_v T_1} = \gamma(\alpha - 1) r_c^{\gamma - 1}$$

$$\frac{\text{imep}}{p_1} = \frac{Q^*}{c_v T_1} \frac{1}{\gamma - 1} \frac{r_c}{r_c - 1} \eta_f$$

$$\frac{\text{imep}}{p_1} = \frac{1}{(\gamma - 1)} \frac{r_c}{r_c - 1} \left((\alpha - 1) r_c^{\gamma - 1} - \alpha^\gamma - 1 \right)$$

Constant volume vs. constant pressure heat addition



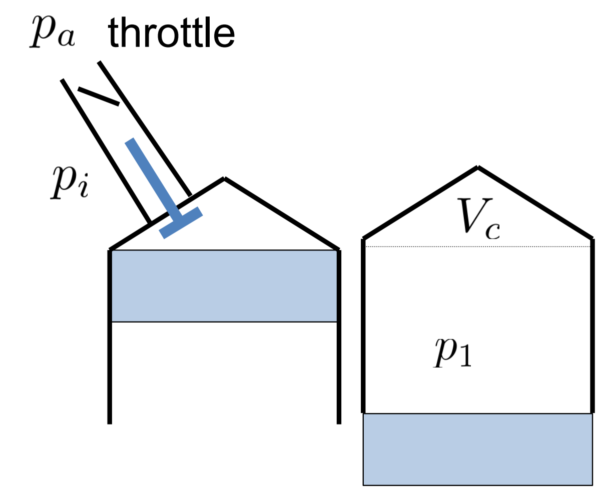
$$\eta_v = 1 - \frac{1}{r_c^{\gamma-1}}$$

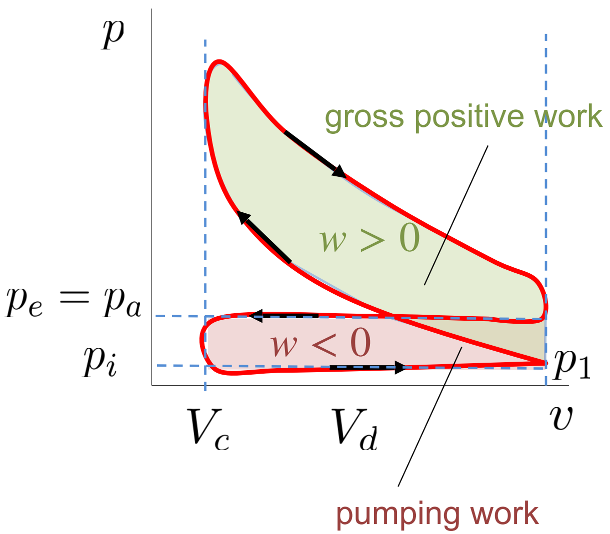
$$\eta_p = 1 - \frac{1}{r_c^{\gamma-1}} \frac{\alpha^\gamma - 1}{\gamma(\alpha - 1)}$$

$$\eta_v > \eta_p$$

For given r_c, constant volume gives higher efficiency, but…
Compression ignition cycles (constant pressure) use higher r_c than spark-ignition engines

Pumping and friction losses in throttled engines: ideal cycle





Pumping **m**ean **e**ffective **p**ressure

$$\text{pmep} = -\frac{W_p}{V_d} = p_e - p_i$$

can be positive (turbo)

$$\text{bmep} = \frac{W_i - W_p - W_f}{V_d} = \text{imep}_\text{g} - \text{pmep} - \text{fmep}$$

brake gross (positive) friction

$$\eta_f = \frac{W_{net}}{m_f q_f} = \frac{P_{net}}{\dot{m}_f q_f}$$

- **Throttling** flow to obtain low intake pressures usual method for controlling mean imep for SI (constant fuel/air ratio): lower pressure leads to lower total mass in cylinder and additional work losses via pumping
- **Efficiency gains** if engine can operate unthrottled, via gas dilution (keeping valves open longer) or fuel-lean
- Load control using **dilution** by exhaust gases can increase efficiency, with a penalty in power density

Example

A 4-stroke engine with compression ratio r_c = 10, bore and stroke $B = L$ = 80 mm is operating with inlet pressure p_i= 0.8 bar, temperature T_i = 300 K and N=2000 rpm. The fuel specific heating value is 45 MJ/kg, and c_v = 800 J/kg, the ratio of specific heats γ = 1.4, the fraction of fuel in the charge is 5%. Estimate the gross efficiency, gross and net imep, and net power based on the standard constant volume air cycle.

$$\eta_f = 1 - r_c^{-(\gamma-1)} = 1 - 10^{-(1.4-1)} = 0.60$$

$$\frac{\text{imep}}{p_1} = \eta_f Q^* \frac{\rho_1}{p_1}\frac{r_c}{r_c - 1} = \eta_f \frac{Q^*}{c_v(\gamma-1)T_1}\frac{r_c}{r_c-1} = (0.6)\frac{(0.05)(45\times10^6\ \text{J/kg})}{(800\ \text{J/kgK})(300\ \text{K})}\frac{1}{0.4}\frac{10}{9} = 15.6$$

$$\text{imep}_\text{g} = (0.8)(15.6) = 12.5\ \text{bar}$$

$$\text{pmep} = (1.0 - 0.8) = 0.2\ \text{bar}$$

$$\text{imep}_\text{n} = 12.5 - 0.2 = 12.3\ \text{bar}$$

$$\eta_n = \frac{\text{imep}_\text{n}}{\text{imep}_\text{g}}\eta_f = 0.59$$

$$P_n = \frac{\text{imep}_\text{n}\ V_d N}{n_R} = (12.3\times10^5\text{N/m}^2)\frac{(\pi(0.08)^2(0.08)\ m^3)}{4}\frac{2000}{(60)(2)}\text{s}^{-1} = 8.2\ \text{kW}$$

Quiz

https://www.vle.cam.ac.uk/mod/quiz/view.php?id=11751692

Summary

- First law analysis: air cycles
  - Constant volume heat addition
  - Constant pressure heat addition
  - Limited pressure cycles

- Efficiency depends on:
  - Compression ratio
  - Gas properties
  - Pumping losses
  - Friction losses