Chapter 7:
Laminar non-premixed flames and droplet combustion

Objectives

The objectives of this Chapter are:
- To discuss the structure of non-premixed flames.
- To present the rudiments of a theory for non-premixed flames.
- To develop an expression for the time needed to evaporate a fuel droplet.
- To discuss practical applications and other aspects of non-premixed flames.

Introduction

Many applications of combustion involve supply of fuel and air in separate streams. Although combustion will occur only where fuel and air meet, supplying them separately is convenient and safe. Flames where the fuel and oxidiser are initially separate are called *non-premixed flames* and form a very large percentage of combustion practice. Diesel engines, gas turbines, most industrial burners, and coal combustion are just few of the examples. In this Chapter, we will discuss the structure of such flames and how they can be mathematically analysed.

Many fuels are liquids and are sprayed as fine droplets into the combustion chamber. Since combustion will occur in the gaseous phase, the phenomenon of *evaporation* will be an important component of the overall process. The paradigm problem of droplet evaporation is described here and the results will help us understand certain aspects of current combustion technology, as well as provide realistic estimates of the *evaporation time*.

Non-premixed flames

Basic description, assumptions, formulation

Consider a <u>non</u>-uniform mixture of fuel and air as depicted in Fig. 7-1. Fuel is discharged from a slot into an oxidising environment that could be stagnant or with flow. The slot is infinitely long in the direction normal to the page. A flame will exist somewhere at the interface between the two reactants and, essentially, separates the fuel and air. There is no flame propagation. Instead, the flame is fixed and the reactants move towards the flame by the action of diffusion and (possibly) advection. Here we will present the governing equations for a two-dimensional steady flame, but the techniques and concepts to be developed apply equally well to more complex situations (e.g. three-dimensional, unsteady). We will not solve these equations (it is too tough!), but instead we will <u>explore</u> them to discover certain relationships that will prove extremely powerful. We assume:
- constant pressure and adiabatic flow (no heat loss),
- steady flame (no transient terms)
- one-step global chemistry (from Chapters 4, 5, and 6),
- constant λ and c_p and unity Lewis number, i.e. $\rho D = \dfrac{\lambda}{c_p}$.

The conservation equations for species and temperature are *partial differential equations* in two variables (*x* and *y*) and can be written as:

$$\rho U \frac{\partial Y_{fu}}{\partial x} + \rho V \frac{\partial Y_{fu}}{\partial y} = \frac{\partial}{\partial x}\left(\rho D \frac{\partial Y_{fu}}{\partial x}\right) + \frac{\partial}{\partial y}\left(\rho D \frac{\partial Y_{fu}}{\partial y}\right) - \dot{w}_{fu} \tag{7.1}$$

$$\rho U \frac{\partial Y_{ox}}{\partial x} + \rho V \frac{\partial Y_{ox}}{\partial y} = \frac{\partial}{\partial x}\left(\rho D \frac{\partial Y_{ox}}{\partial x}\right) + \frac{\partial}{\partial y}\left(\rho D \frac{\partial Y_{ox}}{\partial y}\right) - S\dot{w}_{fu} \tag{7.2}$$

$$\rho c_p U \frac{\partial T}{\partial x} + \rho c_p V \frac{\partial T}{\partial y} = \frac{\partial}{\partial x}\left(\lambda \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right) + Q\dot{w}_{fu} \tag{7.3}$$

The one-step chemistry model gives for the reaction rate:

$$\dot{w}_{fu} = A \; \frac{MW_{fu}}{MW_{fu} MW_{ox}} \; \rho^2 \; Y_{fu} \; Y_{ox} \; \exp\left(-E/R^0 T\right) \tag{7.4}$$
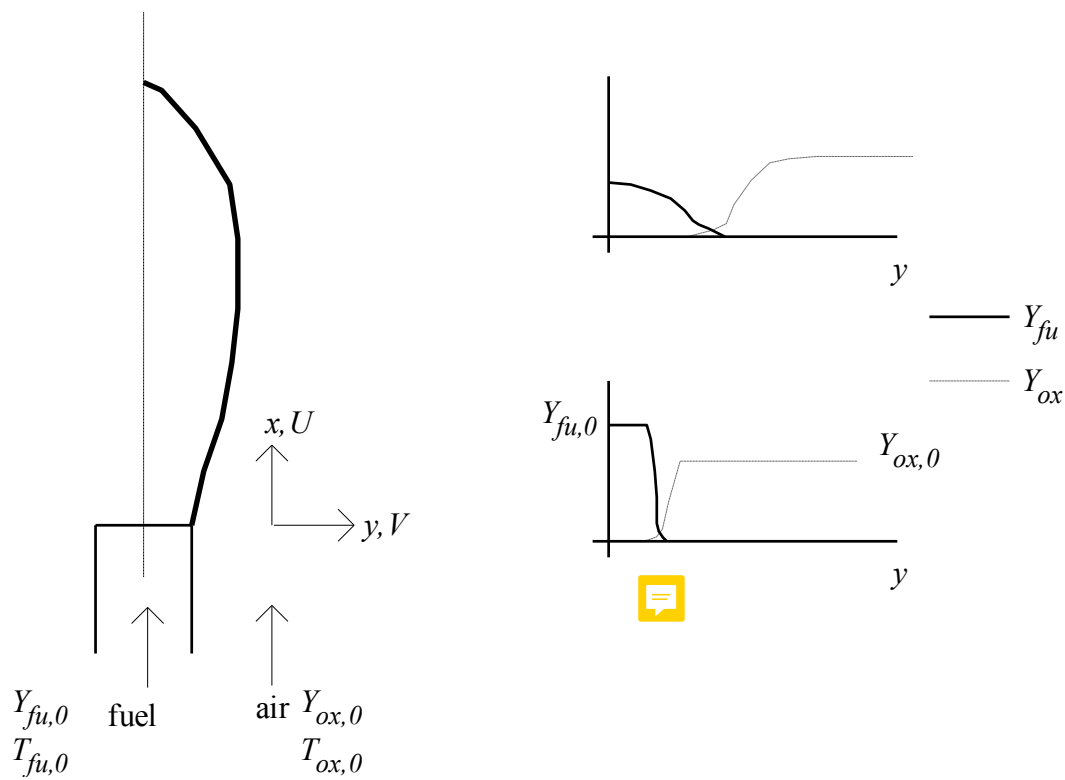

Figure 7-1. Typical profiles species mass fractions for a jet non-premixed flame close to the nozzle and further downstream. A flame zone of finite thickness is shown (fuel and oxygen overlap).
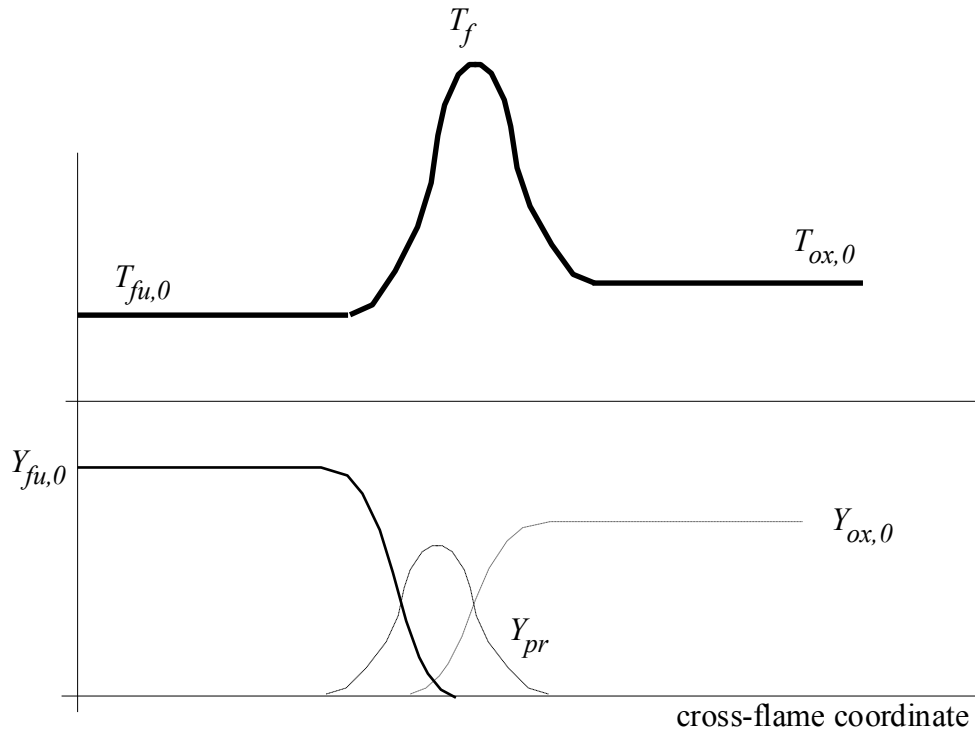
Figure 7-2. Typical profiles of temperature and mass fractions across a non-premixed flame. The overlap between fuel and oxidant is infinitesimally small in the flame sheet model, but shown of finite width here.

Recall that S is the oxygen to fuel ratio at stoichiometry and is given by $S = \nu\, MW_{ox}/MW_{fu}$, where ν are the kmols of oxygen needed to completely oxidize the fuel. The density can be found from the equation of state:

$$\rho = \frac{P\,\overline{MW}}{R^0 T} \tag{7.5}$$

The boundary conditions are given below.

$$
\begin{array}{llll}
\text{At} & y = 0, & \partial/\partial y = 0 & , \quad \text{for all quantities} \\
\text{At} & y = \infty, & Y_{fu} = 0, \quad Y_{ox} = Y_{ox,0}, \quad T = T_{ox,0} \\
\text{At} & x = 0, & Y_{fu} = Y_{fu,0}, \quad Y_{ox} = 0, \quad T = T_{fu,0} \\
\text{At} & x = \infty, & \partial/\partial x = 0 & , \quad \text{for all quantities}
\end{array} \tag{7.6}
$$

The velocity field U and V could be calculated by Fluid Mechanical considerations. Hence, our conservation equations form a well-posed closed system and hence can be solved. A numerical treatment is necessary because it is almost impossible to obtain analytical solutions. To obtain such solutions is a very active research field.

A very powerful method to understand what is going on in the flame, <u>without</u> resort to any solution, can be developed through the use of conserved scalars, introduced in Chapter 4 and also used in Chapters 5 and 6. We present this below, which forms a crucial piece of combustion science.

The mixture fraction – mathematical formulation

Multiplying Eq. (7.1) by Q and adding to (7.3) gives:

$$\rho U \frac{\partial\left(Y_{fu}Q + c_pT\right)}{\partial x} + \rho V \frac{\partial\left(Y_{fu}Q + c_pT\right)}{\partial y} = \frac{\partial}{\partial x}\left[\rho D \frac{\partial\left(Y_{fu}Q + c_pT\right)}{\partial x}\right] + \frac{\partial}{\partial y}\left[\rho D \frac{\partial\left(Y_{fu}Q + c_pT\right)}{\partial y}\right] \quad (7.7)$$

Multiplying Eq. (7.2) by Q and adding to S times (7.3) gives:

$$\rho U \frac{\partial\left(Y_{ox}Q + Sc_pT\right)}{\partial x} + \rho V \frac{\partial\left(Y_{ox}Q + Sc_pT\right)}{\partial y} = \frac{\partial}{\partial x}\left[\rho D \frac{\partial\left(Y_{ox}Q + Sc_pT\right)}{\partial x}\right] + \frac{\partial}{\partial y}\left[\rho D \frac{\partial\left(Y_{ox}Q + Sc_pT\right)}{\partial y}\right] \quad (7.8)$$

Dividing Eq. (7.2) by S and subtracting from (7.1) gives:

$$\rho U \frac{\partial\left(Y_{fu} - Y_{ox}/S\right)}{\partial x} + \rho V \frac{\partial\left(Y_{fu} - Y_{ox}/S\right)}{\partial y} = \frac{\partial}{\partial x}\left[\rho D \frac{\partial\left(Y_{fu} - Y_{ox}/S\right)}{\partial x}\right] + \frac{\partial}{\partial y}\left[\rho D \frac{\partial\left(Y_{fu} - Y_{ox}/S\right)}{\partial y}\right] \quad (7.9)$$

By manipulating the conservation equations, we can therefore generate a series of scalar variables for which the conservation equation does not have a chemical source term. These are called *conserved scalars* or *coupling functions* or *Schvab-Zel'dovich coupling functions*.
If there are two feeds into the system, we can define a further conserved scalar, based on any of the above, but usually we choose the scalar in Eq. (7.9). Denote the fuel stream by 1 and the air stream by 2. Then we define the *mixture fraction* by:

$$\xi = \frac{\left(Y_{fu} - Y_{ox}/S\right) - \left(Y_{fu} - Y_{ox}/S\right)_2}{\left(Y_{fu} - Y_{ox}/S\right)_1 - \left(Y_{fu} - Y_{ox}/S\right)_2} \quad (7.10)$$

So $\xi = 1$ at the fuel stream and $\xi = 0$ at the air stream. If there is no oxygen in the fuel stream and no fuel in the air stream (as would usually – but not always – be the case), the mixture fraction becomes:

$$\xi = \frac{Y_{fu} - Y_{ox}/S + Y_{ox,0}/S}{Y_{fu,0} + Y_{ox,0}/S} \quad (7.11)$$

It is easy to show that the governing equation for the mixture fraction is

$$\rho U \frac{\partial \xi}{\partial x} + \rho V \frac{\partial \xi}{\partial y} = \frac{\partial}{\partial x}\left(\rho D \frac{\partial \xi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\rho D \frac{\partial \xi}{\partial y}\right) \quad (7.12)$$

subject to the following boundary conditions for the jet flame geometry:

At $\quad y = 0, \quad \partial\xi/\partial y = 0$,
At $\quad y = \infty, \quad \xi = 0$
At $\quad x = 0, \quad \xi = 1$
At $\quad x = \infty, \quad \partial\xi/\partial x = 0 \qquad\qquad (7.13)$

Therefore if we solve Eq. (7.12), we obtain the value of the coupling function $Y_{fu} - Y_{ox}/S$ for all points in the flame through Eq. (7.11). Note that our job is not finished yet, since to find the mass fractions of fuel and oxygen individually, we need some additional information relating these to the mixture fraction. This is discussed later. The important point is that:

- through the concept of mixture fraction, we have reduced the need to solve conservation equations for reactive scalars (Y_{fu}, Y_{ox}, T) to the need to solve an equation for a single conserved scalar (ξ);
- the equation for ξ has no chemical source term, hence it is independent of the progress of the reaction (and more easily solved!).

The mixture fraction – physical significance

The mixture fraction signifies the mass fraction of mixture that originated in the fuel feed. If there were no chemical reaction, ξ would be equal to $Y_{fu}/Y_{fu,0}$, or simply equal to the fuel mass fraction if $Y_{fu,0}=1$. By employing the mixture fraction, we relate what goes on in the flame to would go on in the absence of chemical reaction, i.e. in an inert mixing problem. For the jet flame, for example, lines of constant mixture fraction look like the bold line in Fig. 7-1. High ξ is found close to the nozzle, while ξ decreases with downstream and radial distance to become zero at the air stream.

The mixture fraction is an extremely powerful concept and dominates theoretical descriptions of non-premixed flames. The main reason, apart from the reduction in the complexity of the problem to be solved, is that the mass fractions have simple dependencies on the mixture fraction, as we shall see next.

How are the mass fractions and the temperature related to the mixture fraction?

1. Very fast chemistry – the flame sheet model

Assume that the reaction is very fast. This implies that as soon as reactants approach the reaction zone, they are consumed. Hence, no fuel penetrates the flame towards the air stream and no air penetrates towards the fuel stream. Fuel and air co-exist only in a very thin zone (the reaction zone). If the reaction is very fast, the thickness of this zone is very small, and at the limit of infinite speed of reaction, we have the so-called *flame sheet* model or *infinitely-fast chemistry* model.

We know from Chapters 5 and 6 that the reaction time (τ_{chem}, Eq. 6.29) is not zero, but finite. However, in many applications of non-premixed flames, the timescale of the reaction is very much shorter than the timescale of the flow or, in other words, the rate at which reactants are supplied to the flame (by diffusion) is usually much slower than the rate at which these reactants can be consumed (by the flame). This justifies why no fuel can penetrate the reaction zone into the oxygen stream and vice versa.

2. Reaction zone location – the stoichiometric mixture fraction

In the flame sheet model, the reaction zone will be located where the temperature will have a maximum (due to the Arrhenius term, Eq. 7.4). Consider the temperature achieved by complete combustion of various fuel/oxygen mixtures. As we have seen before (e.g. Chapter 1), the temperature of the products will be highest if the reactants are at stoichiometric proportions. This will happen when $Y_{fu}=Y_{ox}/S$. Hence, from Eq. (7.11), the *stoichiometric mixture fraction* is given by:

$$\xi_{st} = \frac{Y_{ox,0}/S}{Y_{fu,0} + Y_{ox,0}/S} \tag{7.14}$$

For example, for methane-air combustion, $S = 2 \cdot 31.999/16.043 = 3.989$. For a jet of pure methane ($Y_{fu,0}=1$) into air ($Y_{ox,0}=0.233$), Eq. (7.14) gives that $\xi_{st}=0.0552$. If the air or the fuel is diluted by an inert, the stoichiometric mixture fraction will shift to a lower or higher value respectively. The flame sheet model states that reaction occurs only at ξ_{st} and is zero elsewhere. Hence calculating ξ_{st} from Eq. (7.14) gives the flame location.

3. State relationships and the flame temperature

Since the reaction is zero everywhere apart from an infinitesimally thin region, we may consider the two regions left ($\xi < \xi_{st}$) and right ($\xi > \xi_{st}$) of the flame separately. In each of the regions, we have inert flow. The result is depicted in Fig. 7-3. The fuel and oxygen mass fractions are related to the mass fractions by piece-wise straight lines, whose equations are:
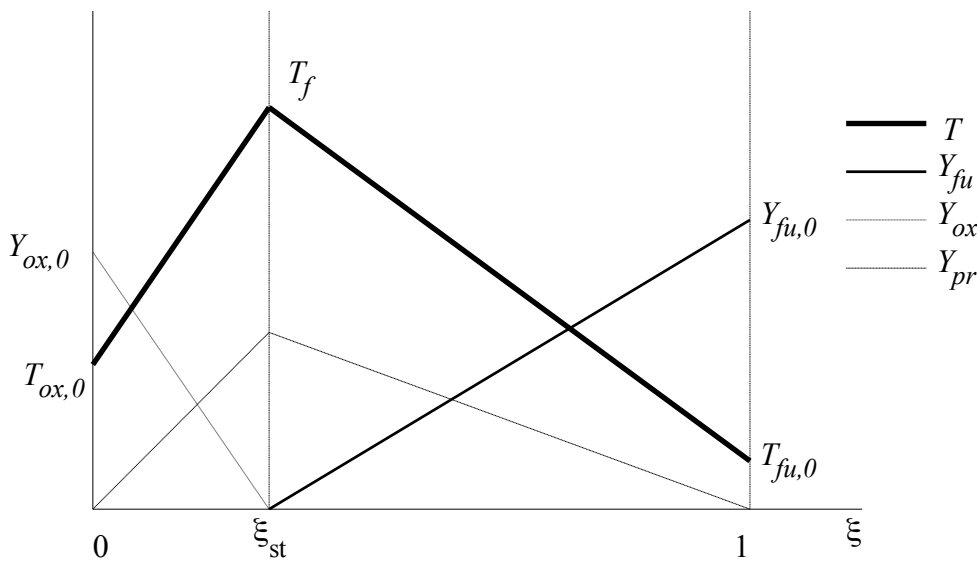

Figure 7-3. The mass fractions and temperature as a function of mixture fraction for a non-premixed flame.

For $\xi < \xi_{st}$: $\quad Y_{fu} = 0 \quad , \quad Y_{ox} = S Y_{fu,0} \dfrac{\xi_{st} - \xi}{1 - \xi_{st}} \quad , \quad T = (T_f - T_{ox,0}) \dfrac{\xi}{\xi_{st}} + T_{ox,0}$ (7.15)

For $\xi > \xi_{st}$: $\quad Y_{fu} = Y_{fu,0} \dfrac{\xi - \xi_{st}}{1 - \xi_{st}} \quad , \quad Y_{ox} = 0 \quad , \quad T = (T_f - T_{fu,0}) \dfrac{1 - \xi}{1 - \xi_{st}} + T_{fu,0}$ (7.16)

The flame temperature is given by

$$T_f = Y_{fu,0} \xi_{st} \frac{Q}{c_p} - \left(T_{ox,0} - T_{fu,0}\right)\xi_{st} + T_{ox,0}$$ (7.17)

Equations (7.15) and (7.16) are called *state relationships* and completely specify the mixture and the temperature as a function of the mixture fraction. We need not remember them, as they can be written down by inspection from Fig. 7-3. A subtle point is the calculation of T_f if the fuel and air streams have different temperatures, which is very common in practice: Eq. (7.17) comes from our well-known $T_f = Y_{fu,in} Q/c_p + T_{in}$, recognizing that $Y_{fu,in} = Y_{fu,0}\xi_{st}$ and that the "initial" temperature T_{in} is the temperature of the stoichiometric mixture (i.e. with $\xi = \xi_{st}$) for inert mixing, i.e. $T_{in} = T_{ox,0} - \xi_{st}(T_{ox,0} - T_{fu,0})$.

Equation (7.17) is important also from another point of view: the flame temperature of a non-premixed flame will always be equal to the adiabatic flame temperature of a premixed stoichiometric flame. Since high flame temperatures imply high potential for NO generation, reducing pollutants from non-premixed combustion is more difficult than from premixed combustion (which, for example, could be made to run very lean). Nevertheless, various successful techniques to reduce NO from non-premixed combustion have been developed (Chapter 8).

Final solution to the non-premixed flame problem

Hence, for any type of flow (steady or unsteady; one-, two-, or three-dimensional), if the mixture fraction spatial distribution is known, the fuel and oxygen mass fractions and the temperature can be calculated, as they are given functions (Fig. 7-3) of the mixture fraction. This method is called the "mixture fraction approach" and is used very often in non-premixed flame calculations. In most situations in practice with non-premixed flames, the requirement is to calculate the flame length and its location. The flame will be where the mixture fraction assumes its stoichiometric value. Therefore solving the mixing problem to find ξ and hence locate ξ_{st}, is very often sufficient for most information of engineering interest.

The major assumption behind the state relationships in Eqs. (7.15) (or Fig. 7-3) is that the chemistry is very fast. However, the state relationships in Fig. 7-3 are not qualitatively different even if the chemistry is reasonably slow, a fact that has been substantiated by experiments and by numerical solutions of the full governing equations (7.1-7.6). In the case of finite-rate chemistry, the straight lines in Fig. 7-3 obtain a small curvature around the stoichiometric mixture fraction and the state relationships are not substantially altered. Therefore we can say that, unlike premixed flames that are kinetically-controlled, non-premixed flames are very often *diffusion-* or *mixing-controlled*.
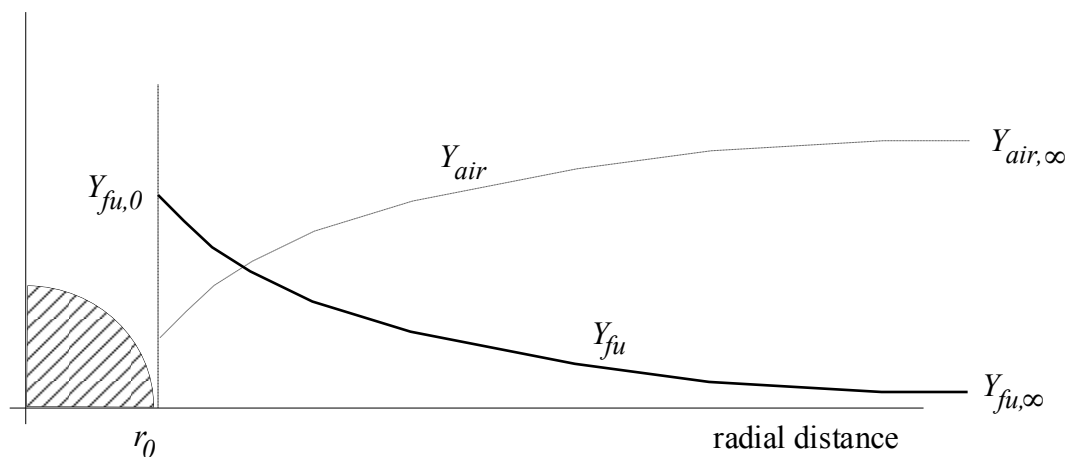
Droplet evaporation

Figure 7-4. Typical profiles of mass fractions around an evaporating spherical droplet of radius r_0.

Basic description, assumptions

When dealing with liquid fuels, we inject fuel droplets towards the flame. The reaction will proceed in the gaseous phase and hence the fuel must be evaporated first before it can be used. We

aim here to calculate the time taken for a fuel droplet to evaporate, as this time must be accounted for in the design of any combustion equipment.

Consider a spherical liquid droplet of radius r_0 in a quiescent environment of air (Fig. 7-4). It is our everyday experience that liquid vapour will form and hence mass will be lost from the droplet. Eventually, the droplet will disappear with all its mass being in vapour form. The vapour is diffused to the environment, which, for the sake of generality, can be taken to already contain liquid vapour at a mass fraction of $Y_{fu,\infty}$. We assume that the process is quasi-steady, i.e. at any particular instant in time, the process can be described as if it were in steady state. Also, that the temperature of the droplet is uniform and that the mass fraction of fuel vapour at the droplet surface ($Y_{fu,0}$) corresponds to the saturation point. We take the mass diffusivity ρD is uniform. Finally, we neglect chemical reactions because we want to understand the pure evaporation problem (e.g. unignited fuel-air mixing). The subscript 0 refers to conditions at the droplet surface.

Droplet mass conservation

$$\frac{dm_d}{dt} = -\dot{m}_{fu} \quad \Leftrightarrow \quad \frac{d}{dt}\left(\rho_L \frac{4}{3}\pi r_0^{\,3}\right) = -4\pi r_0^{\,2}\dot{m}''_{fu,0} \Leftrightarrow \rho_L 4\pi r_0^{\,2}\frac{dr_0}{dt} = -4\pi r_0^{\,2}\dot{m}''_{fu,0}$$

$$\Leftrightarrow \rho_L \frac{dr_0}{dt} = -\dot{m}''_{fu,0} \tag{7.18}$$

where:

$\dot{m}_{fu}$:   the mass flow rate of fuel evaporating from the droplet (kg/s);

ρ_L: the density of the liquid fuel (kg/m^3);

$\dot{m}''_{fu,0}$:   the mass flux of fuel at the droplet surface (kg/m^2/s);

Equation (7.18) relates the rate of change of the droplet mass to the rate of mass loss by evaporation from its surface. The latter quantity will be calculated by Mass Transfer considerations next.

Mass conservation in the gaseous phase

The vapour released by the droplet flows outwards in the gas. The mass conservation equation in spherical coordinates is:

$$4\pi r^2 \rho U = \text{constant} = 4\pi r_0^{\,2}\rho_0 U_0 = 4\pi r_0^{\,2}\dot{m}''_{fu,0} \tag{7.19}$$

where U is the local velocity of the gas (fuel+air mixture) and ρ is the mixture density. It is important to realize that since at the droplet surface the total mass flux is due to fuel only, the value of ρU at the surface is equal to $\dot{m}''_{fu,0}$ (no air flows into the droplet or out of it).

We can also write the species conservation equation in spherical coordinates (which was given as an exercise in Examples Paper) for steady flow without chemical reaction. This equation can be derived easily by performing a control volume analysis for a spherical shell. Hence, we can write for the fuel and air (the only components in the mixture):

$$\frac{d}{dr}\left(r^2 \dot{m}''_{fu}\right) = 0 \quad \Leftrightarrow \quad \frac{d}{dr}\left[r^2\left(Y_{fu}\rho U - \rho D\frac{dY_{fu}}{dr}\right)\right] = 0 \tag{7.20}$$

and

$$\frac{d}{dr}\left(r^2 \dot{m}''_{air}\right) = 0 \Leftrightarrow \frac{d}{dr}\left[r^2\left(Y_{air}\rho U - \rho D \frac{dY_{air}}{dr}\right)\right] = 0 \tag{7.21}$$

where we have employed the knowledge from Chapter 1 that the mass flux for each species is composed of an advective part ($Y\rho U$) and a diffusive part, the latter given by Fick's law. Of course, as the mixture is binary, we need not solve both species equations because $Y_{fu}+Y_{air}=1$. Integrating Eq. (7.20) once gives:

$$\frac{d}{dr}\left[r^2\left(Y_{fu}\rho U - \rho D \frac{dY_{fu}}{dr}\right)\right] = 0$$
$$\Leftrightarrow r^2\left(Y_{fu}\rho U - \rho D \frac{dY_{fu}}{dr}\right) = r_0^2\left(Y_{fu}\rho U - \rho D \frac{dY_{fu}}{dr}\right)_{r=r_0} = r_0^2 \dot{m}''_{fu,0} \tag{7.22}$$

where we have used the fact that the mass flux of fuel at r_0 is $\dot{m}''_{fu,0}$. Equation (7.22) becomes further:

$$r^2\left(Y_{fu}\rho U - \rho D \frac{dY_{fu}}{dr}\right) = r_0^2 \dot{m}''_{fu,0} \Leftrightarrow r^2\rho U Y_{fu} - r^2\rho D \frac{dY_{fu}}{dr} = r_0^2 \dot{m}''_{fu,0}$$

$$\Leftrightarrow r_0^2 \dot{m}''_{fu,0} Y_{fu} - r^2 \rho D \frac{dY_{fu}}{dr} = r_0^2 \dot{m}''_{fu,0}$$

$$\Leftrightarrow r^2 \rho D \frac{dY_{fu}}{dr} = r_0^2 \dot{m}''_{fu,0}(Y_{fu} - 1) \tag{7.23}$$

We have used Eq. (7.19) to relate $r^2\rho U$ to $r_0^2 \dot{m}''_{fu,0}$. Equation (7.23) can now be integrated subject to the boundary conditions $Y_{fu}(0)=Y_{fu,0}$ to give the spatial distribution of the fuel mass fraction.

Final results

The result of the integration of (7.23) is:

$$\frac{r_0^2 \dot{m}''_{fu,0}}{\rho D}\left(\frac{1}{r_0} - \frac{1}{r}\right) = \ln\left(\frac{Y_{fu}-1}{Y_{fu,0}-1}\right) \tag{7.24}$$

and employing the outer boundary condition that $Y_{fu}(\infty)= Y_{fu,\infty}$, gives the evaporation rate $\dot{m}''_{fu,0}$:

$$\dot{m}''_{fu,0} = \frac{\rho D}{r_0}\ln\left(\frac{1-Y_{fu,\infty}}{1-Y_{fu,0}}\right) \tag{7.25}$$

Therefore we see that the evaporation rate depends on:
- the fuel mass fraction at the surface of the droplet;
- the fuel mass fraction far from the droplet;
- the diffusion coefficient;
- the droplet radius.

The d^2-law

We now calculate the time needed for complete evaporation of the droplet. Using Eq. (7.25) in (7.18) yields:

$$\rho_L \frac{dr_0}{dt} = -\dot{m}''_{fu,0} = -\frac{\rho D}{r_0} \ln\left(\frac{1-Y_{fu,\infty}}{1-Y_{fu,0}}\right) \Leftrightarrow r_0 \, dr_0 = -\frac{\rho D}{\rho_L} \ln\left(\frac{1-Y_{fu,\infty}}{1-Y_{fu,0}}\right) \, dt \tag{7.26}$$

It is more common to calculate the evaporation time in terms of the droplet diameter, not radius. Therefore, if the initial diameter is d_{in}, integrating Eq. (7.26) gives that at any time instant the droplet diameter d is given by:

$$d_{in}{}^2 - d^2 = \beta \ t \tag{7.27}$$

Equation (7.27) is referred to as the "d^2-law" for droplet evaporation. The quantity β is called the *evaporation coefficient* (m^2/s) and is defined by

$$\beta = 8 \frac{\rho D}{\rho_L} \ln(1+B) \tag{7.28}$$

while B is called the *transfer number* given by

$$B = \frac{Y_{fu,0} - Y_{fu,\infty}}{1-Y_{fu,0}} \tag{7.29}$$

Finally, the time taken to completely evaporate a droplet is found by setting $d=0$:

$$t_{vap} = \frac{\rho_L}{8\rho D \ln(1+B)} d_{in}{}^2 \tag{7.30}$$

Equation (7.30) is a very important result. It shows that the evaporation time will be small if:
- the diffusion coefficient is high;
- the initial diameter is small;
- the transfer number is large.

The transfer number B can be understood as a "driving potential" for mass transfer. If there is a lot of fuel vapour far from the droplet, the transfer number will be small (slow evaporation). If the temperature is high, which implies that $Y_{fu,0}$ is high, then evaporation is fast.

Droplet combustion

When droplets burn, the flame is of the non-premixed type. There are many possibilities concerning the location of the flame, which are discussed very briefly below. The implications of our theory of droplet evaporation concerning the operation of some combustion devices are also discussed.

Individual droplet and group combustion

If each droplet is enclosed by a spherical flame sheet, we have *individual droplet combustion*. The above analysis can be repeated and the model of infinitely-fast chemistry (e.g. Fig. 7-3) can be employed. This situation is not often encountered in practice, but an individual droplet flame offers exciting possibilities for studying the fundamentals of flames (see "Internet-based

learning"). In contrast, if a cloud of droplets evaporate, a flame may surround the whole cloud and this is called *group combustion*. This situation may be closer to practice but is very difficult to analyse theoretically.

Implications for practice

Failure to achieve complete evaporation is a common problem in combustion devices. At high loads, gas turbines and diesel engines may emit smoke, which is partly attributed to incomplete evaporation of a few large droplets. In toxic waste incinerators, the similar problem of "rogue droplets", i.e. droplets of the liquid to be incinerated that are too large to evaporate on time, has plagued the operation of such devices. Industrial burners burning heavy oils use fuel preheat in order to decrease the evaporation time. The creation of a fine mist of liquid droplets is called *atomisation* and forms an important ancillary technology to combustion.

Summary

- It is very common in practice to supply the fuel and the oxidiser from different sides of the flame. These flames are called non-premixed flames.
- A non-premixed flame is formed at the interface of fuel and air streams and does not propagate.
- Very often, the rate of chemical reaction is very fast compared to the rate at which reactants are supplied to the flame by diffusion.
- This allows the development of the "flame sheet model", which is based on the concept of the mixture fraction (Eq. 7.10, 7.11).
- The flame sheet is located at the stoichiometric mixture fraction (Eq. 7.14).
- The relationships between the mass fractions and temperature to the mixture fraction are given in Fig. 7-3.
- The time needed for a fuel droplet to evaporate is useful because many fuels liquids.
- The evaporation time (Eq. 7.30) depends on the droplet diameter, the diffusion coefficient, and the transfer number B (Eq. 7.29).

Suggested reading

Turns:
Ch. 3 (pp. 98-105) for droplet evaporation; Ch. 9 (pp. 314-327) for non-premixed flames.

Glassman:
Ch. 6 (pp. 293-298) for droplet evaporation, but includes heat transfer (more complex treatment that ours). Ch. 6 (pp. 268-274) for structure of non-premixed flames.

Spalding:
Ch. 3 for droplet evaporation; Ch. 6 for non-premixed flames in general and state relationships in particular.

Worked examples

Example 7-1. Calculate the stoichiometric mixture fractions for combustion of (i) diesel in air; (ii) diesel in air diluted by an inert. (iii) Hence discuss how the flame length in a diesel engine is affected by exhaust gas recirculation (EGR).

We start from the combustion equation in order to find the oxygen to fuel mass ratio. For diesel (taken as dodecane) in <u>oxygen</u>, the combustion equation is: $C_{12}H_{26} + 18.5O_2 \rightarrow 12CO_2 + 13H_2O$. Therefore,

$S = 18.5 \cdot 31.999/170.337 = 3.375$. We use $Y_{fu,0} = 1$, as the fuel is undiluted.

(i) For diesel burning in air, $Y_{ox,0} = 0.233$ and hence $\xi_{st} = \dfrac{Y_{ox,0}/S}{Y_{fu,0} + Y_{ox,0}/S}$ (Eq. 7.14) gives for the

stoichiometric mixture fraction $\xi_{st} = 0.0646$.

(ii) Assume now that the air is diluted by an inert, for example the products of combustion (e.g. by employing exhaust gas recirculation). Say that in the new oxidiser stream the mass fraction of (inert) products is Y_{EGR}. Hence the mass fraction of air will be $1 - Y_{EGR}$. The mass fraction of <u>oxygen</u> in the new oxidiser stream will be $Y_{ox,0} = 0.233 \cdot (1 - Y_{EGR})$. (In reality, this is not exactly right as the EGR may already contain oxygen, but this is how we insisted in inert products.) Therefore the stoichiometric mixture fraction with the diluted air stream will be given by

$\xi_{st} = \dfrac{0.233(1 - Y_{EGR})/S}{Y_{fu,0} + 0.233(1 - Y_{EGR})/S}$. For example, if $Y_{EGR} = 0.2$ (a typical value), $\xi_{st} = 0.0523$.

(iii) Since with dilution the stoichiometric mixture fraction is smaller, the flame will be longer.

Notes:
- This example shows how the effects of dilution can be quantified. Equation (7.17) will give the flame temperature.
- EGR is a very common technique in combustion to reduce the flame temperature and hence reduce the generation of NO. However, the flame location <u>is</u> affected, because ξ_{st} changes, as this example showed.
- The state relationships (Fig. 7-3; Eq. 7.15, 7.16) are not affected by dilution.

Example 7-2. A particular droplet takes 1ms to evaporate in a pure air environment and $Y_{fu,0} = 0.8$.
(i) Calculate the evaporation time if the mass fraction of fuel vapour far from the droplet becomes $Y_{fu,0}/2$ with everything else staying the same. (ii) Calculate the evaporation time of a droplet with twice the diameter, all other quantities being unaffected.

(i) Starting from $t_{vap} = \dfrac{\rho_L}{8\rho D \ln(1+B)} d_{in}^{\,2}$ (Eq. 7.30) and denoting by a prime the condition with

elevated $Y_{fu,\infty}$, we have:

$$\frac{t_{vap}}{t'_{vap}} = \frac{\dfrac{\rho_L}{8\rho D \ln(1+B)} d_{in}^{\,2}}{\dfrac{\rho_L}{8\rho D \ln(1+B')} d_{in}^{\,2}} = \frac{\ln(1+B')}{\ln(1+B)} = \frac{\ln\left(1 + \dfrac{Y_{fu,0} - Y_{fu,\infty}}{1 - Y_{fu,0}}\right)}{\ln\left(1 + \dfrac{Y_{fu,0} - 0}{1 - Y_{fu,0}}\right)} = \frac{\ln\left(1 + \dfrac{0.8 - 0.4}{1 - 0.8}\right)}{\ln\left(1 + \dfrac{0.8 - 0}{1 - 0.8}\right)} = 0.68.$$

Therefore the evaporation time will be 1.46ms.

(ii) From Eq. (7.30) again, increasing the diameter by a factor of two, increases t_{vap} by a factor of 4, hence t_{vap} =4ms.

Notes:
- Large evaporation times are expected as the vapour content of the environment increases.
- The square dependence of evaporation time on initial diameter is the reason why fine droplets are needed in combustors.

Internet-based learning

http://science.nasa.gov/msl1/themes/micrograv_over.htm

   This is a NASA Web site that describes various scientific experiments performed on-board the Space Shuttle. Explore the material from the link on "Combustion Research". You will find photos of droplets burning in the absence of gravity, laminar jet diffusion flames, and others. Microgravity research is very useful because the data thereby collected can be used to validate our theoretical models for flames, such as the ones presented in this Chapter.