


# Cambridge University Engineering Department

## 4A13: Combustion & IC Engines

### Examples Paper

#### Chapter 1:

1. A 10 MW methane-air burner operates at an excess air factor of 0.1. (The excess air factor is defined as  $(1-\phi)/\phi$ ). Calculate: (i) the mass flow rates of fuel and air and the mass fractions of each constituent in the reactants stream. 

(Answer: (i)  $\dot{m}_f=0.2$  kg/s,  $\dot{m}_{air}=3.768$  kg/s;  $Y_{CH_4}=0.0504$ ,  $Y_{O_2}=0.2213$ ,  $Y_{N_2}=0.7283$ )

2. Derive the species conservation equation for a spherically symmetric system. You may use

$$\dot{m}_{i,DIFF}'' = -\rho D \frac{\partial Y_i}{\partial r}.$$


#### Chapter 2:

3. Consider the two elementary reactions for thermal nitric oxide formation:



with forward and backward rate constants  $k_{1f}$ ,  $k_{1b}$ ,  $k_{2f}$ , and  $k_{2b}$ . Assuming that initially  $[NO]=0$ , that all species except NO are in chemical equilibrium, that pressure and temperature are constant, and that the NO does not affect the concentration of the other species, show that

$$[NO] = [NO]_{eq} \left[ 1 - \exp\left(-\frac{At}{[NO]_{eq}}\right) \right]$$

where  $eq$  denotes equilibrium and  $A = k_{1f}[O]_{eq}[N_2]_{eq} + k_{2f}[O_2]_{eq}[N]_{eq}$ . 



4. In a refinery pollution-abatement program, the organic vapours released by the various processes are collected and are subsequently mixed very quickly with hot gases from a flame in order to destroy them before release to the atmosphere. Such devices are called “incinerators”. There are  $1.0 \times 10^{-5}$  kg/s of organic pollutants that must be so destroyed and legislation requires that the refinery should not emit more than  $1.0 \times 10^{-7}$  kg/s of pollutant at the exit of the incinerator. Assume that: (i) the hot gases are products of complete methane-air combustion at  $\phi=0.7$ , 1 bar and at 1700 K with a mass flow rate of  $1.0 \times 10^{-2}$  kg/s; (ii) the incinerator is a duct with diameter 0.1 m; (iii) the organic vapours can be taken as octane ( $C_8H_{18}$ ), obeying an one-step global reaction with an empirical rate expression

$$\omega = A_G \exp(-T_{act}/T) [C_8H_{18}][O_2] \quad \text{img alt="comment icon" data-bbox="748 740 778 760}}$$

where  $A_G=1.0 \times 10^9$  (kmol  $m^{-3}$ ) $^{-1}s^{-1}$  and  $T_{act}=16000$  K. Neglect the heat release associated with the incineration of the pollutant and assume that the oxygen concentration remains constant and that the mixture molecular weight is that of air. Calculate the length of duct required to destroy the vapours to the desired emission limit.

(Answer: 0.85 m)

### Chapter 3:

5. A spark-ignition engine burns octane in air at an air-fuel ratio of 12:1 (by mass). Determine the volumetric analysis of the combustion products at an exhaust temperature of 2400 K if no oxygen is present.  
(Answer: CO<sub>2</sub> 7.64%; H<sub>2</sub>O 14.26%; CO 7.06%; H<sub>2</sub> 2.27%; N<sub>2</sub> 68.79%). 
6. A rocket engine burns pure hydrogen with pure oxygen at  $\phi=1$ . The temperature at the exhaust is 3000 K and the pressure 100 bar. Assuming that the products contain only H<sub>2</sub>O, H<sub>2</sub>, O<sub>2</sub>, and OH, set up the system of equations needed to calculate the volume fractions of these species. (Perform the algebra, but only if you can!) 

### Chapter 4:

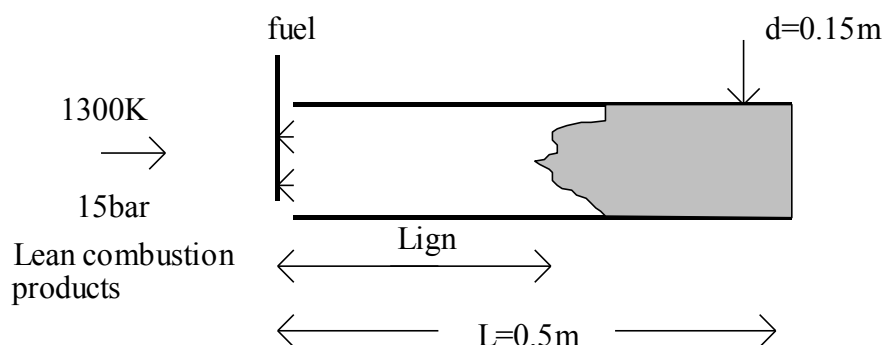
7. An industrial gas turbine has two combustion chambers. The first is conventional and the hot products are expanded in a turbine. The second combustor is shown in Fig. 1: extra fuel is mixed instantaneously with the exhaust gas coming from the turbine and the reactants flow in a straight duct of diameter  $d = 0.15$  m. Autoignition occurs somewhere along the duct, which can be taken as adiabatic. The gas entering this combustor has the composition of complete combustion products at  $\phi = 0.2$ , a temperature of 1300 K, a pressure of 15 bar and the mass flow rate is 10 kg/s. During tests with a methane flow rate of 0.1 kg/s, autoignition occurred halfway along the duct at  $L_{ign} = 0.25$  m.

(i) A user wants to switch to another fuel instead of methane in this autoignition combustor. This fuel is the gas coming off a municipal waste landfill and consists of 50% CH<sub>4</sub> and 50% CO<sub>2</sub> (by vol.). If the heat release (in kW) in the combustor is to stay the same, does the length of the duct need to be redesigned?

(ii) Where would autoignition occur with the new fuel if the temperature were to drop to 1200 K, with all other quantities staying the same? You may take  $E/R^0 = 20000$  K.



(Hint: Start with the expression for the autoignition delay time derived in the lectures and consider how the various parameters that appear in this expression change with the introduction of the new fuel. You don't need all of the quantities given in the question.)

(Answer: (i)  $L_{ign} = 0.27$  m, hence no redesign needed; (ii)  $L_{ign} = 0.71$  m, hence outside combustor) 





**Figure 1.** Duct autoignition combustor for Question 7. This combustor is used in the world's largest gas turbine, the Alstom GT26, rated at 240MWe.


## Chapter 5:

8. The flame zone of a burner is treated as a well-stirred reactor of volume  $1 \times 10^{-3} \text{ m}^3$ . The reactants enter at  $T_{in} = 300 \text{ K}$ . During stability tests, the burner extinguished at a temperature of  $1800 \text{ K}$  when the mass flow rate was  $0.1 \text{ kg/s}$ . When the initial temperature increased to  $600 \text{ K}$ , blow-off occurred at  $2000 \text{ K}$ . Calculate the mass flow rate at blow-off with the higher initial temperature. Take  $E/R^0 = 20000 \text{ K}$ . You may assume that the fuel and oxygen mass fractions at the blow-off point are approximately the same under both conditions.  
(Answer:  $0.26 \text{ kg/s}$ ) 
9. A friend from Chemical Engineering asks you the following question: “For my design project, I want to build a liquid reactor of volume  $1 \times 10^{-3} \text{ m}^3$ , where the reaction  $A+B \rightarrow C$  will take place. The production rate of either A or B is given by  $\dot{w} = -kY_A Y_B$ , where  $k$  is a constant equal to  $200 \text{ kg/m}^3/\text{s}$ . The reaction is exothermic and releases  $1000 \text{ kJ}$  per  $\text{kg}$  of A consumed. Yesterday, you mentioned that flames extinguish if the reactants supply is too high, so I presume that my reactor will also not be able to withstand too high a supply of reactants.” Is your friend right or can his/her reactor operate at any supply, nomatter how high? Calculate the reactor’s composition and temperature if A and B are diluted in water and enter into the reactor at  $20^\circ \text{C}$  with initial mass fractions  $0.01$  and  $0.09$  respectively. The total supply to the reactor has a mass flow rate of  $0.1 \text{ kg/s}$ . (Hint: Derive the species and temperature balance equations with this chemistry. Think about the existence or not of critical conditions.)  
(Answer:  $Y_A = 0.0085$ ;  $Y_B = 0.088$ ;  $T = 20.34^\circ \text{C}$ ) 

## Chapter 6:

10. From values for the burning velocity of atmospheric air-methane flames at  $\phi = 1$  and  $\phi = 0.6$ , calculate the flame thickness and the reaction time for both conditions. You may use the data in Fig. 6-3 in the Lecture Notes or other tabulated data and values for air at  $300 \text{ K}$  for the thermal diffusivity. 
11. Following step-by-step the procedure used in the Lecture Notes, derive an expression for the laminar burning velocity using a one-step chemical model given by  $\dot{w}_{fu} = A \exp(-E/R^0T)$ , i.e. a rate *independent* of the fuel and oxygen mass fractions. ( $A$  has units  $\text{kg/m}^3/\text{s}$ .)  
(Hint: Start from Eq. 6.18 and perform the high-activation energy expansion.) 

## Chapter 7:

12. A droplet of fuel evaporates in pure air at  $300 \text{ K}$  and  $1 \text{ bar}$ . Its initial diameter is  $25 \mu\text{m}$ , the transfer number is  $10.0$ , the density of the liquid is  $850 \text{ kg/m}^3$ , and the Lewis number of the fuel-air mixture is unity. Calculate the mass fraction of vapour at the droplet surface and the evaporation time.  
(Answer:  $0.91$ ;  $1.07 \text{ ms}$ )
13. Calculate the temperature of a non-premixed flame formed from a methane jet at  $300 \text{ K}$  and an oxidizer at  $400 \text{ K}$  if the oxidizer is (i) atmospheric air; (ii) a mixture of  $70\%$  atmospheric air and  $30\%$  inert exhaust products (by mass). You may take  $c_p = 1.35 \text{ kJ/kgK}$ .  
(Answer: (i)  $2440 \text{ K}$ ; (ii)  $1852 \text{ K}$ ) 

Chapter 8:

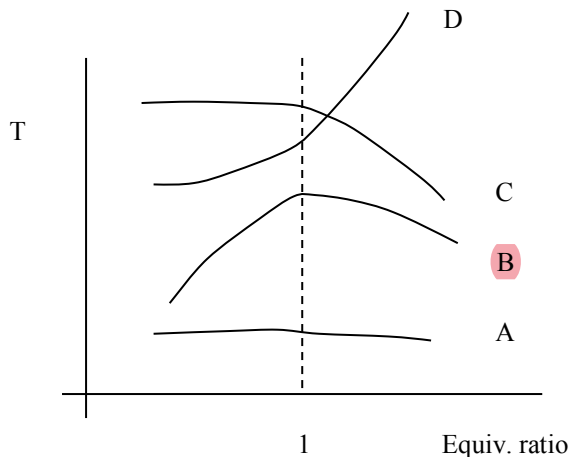
14. Calculate the amount of CO<sub>2</sub> generated per GJ of heat released from burning methane, propane, octane, and coal. You may assume that coal is described by the effective formula C<sub>20</sub>H<sub>20</sub>ON and that it has a lower calorific value of 32 MJ/kg. Do your results support the current policy of replacing coal with natural gas in power generation to reduce the danger of global warming?  
(Answer: kg CO<sub>2</sub> per GJ of heat: 54.8; 64.4; 68.6; 94.7)
15. Mention two techniques that can be employed to decrease the amount of NO emitted from a spark-ignition engine and discuss briefly how they work.

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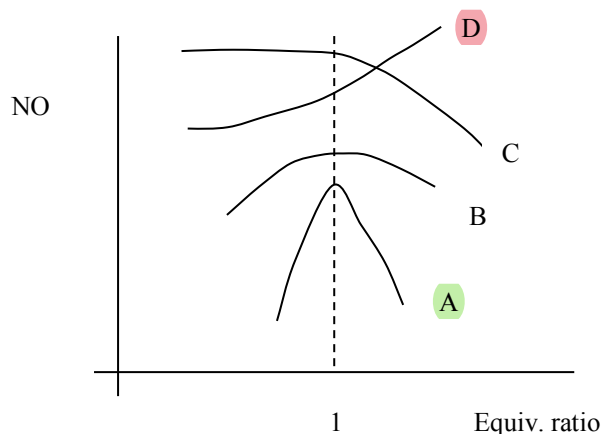
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#### Concept-enhancing questions

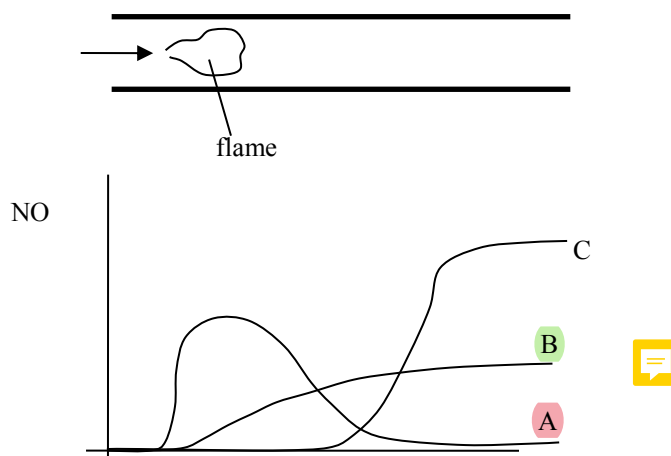
1. The constant-pressure adiabatic flame temperature of a stoichiometric methane-air flame with reactants initially at atmospheric conditions falls in which range:  
A: 600-900 K; B: 1000-1300 K; **C: 1800-2300 K;** D: 2500-3000 K
2. At stoichiometry, 1 kg of fuel needs 20 kg of air for complete combustion. What is the equivalence ratio of a mixture of 1 kg fuel and 10 kg air?  
A: 1 **B: 2** C: 4 D: 0.5
3. Which plot better represents the variation of the flame temperature with  $\phi$ ?



4. As the temperature increases, the reaction rate  
A: is constant B: increases slightly C: decreases **D: increases rapidly**
5. Which plot better represents NO formation with equivalence ratio in a flame?

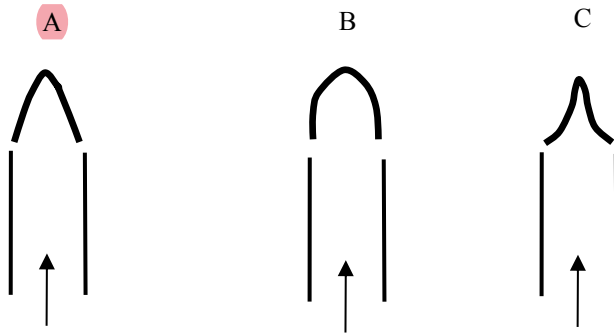


6. When we throw water on a wood fire, the flames extinguish because...
- A: water is decomposed and the flame becomes very rich as it now includes  $H_2$ .
  - B: presence of water decreases the temperature and hence reaction freezes.**
  - C: presence of water carries wood chips away from the flame.
  - D: water absorbs the radiation emitted from the flame.
7. When is dissociation of  $CO_2$  into  $CO$  and  $O_2$  more pronounced?
- A: As the pressure decreases and temperature increases.**
  - B: As the pressure increases and temperature decreases.
  - C: Is independent of pressure.
  - D: Is independent of temperature.
8. Which plot better represents the evolution of  $NO$  along a long, hot adiabatic furnace with stoichiometric reactants?



9. Autoignition time decreases when:
- A: the pressure and temperature increase;**
  - B: the pressure and temperature decrease.
  - C: Is independent of pressure.
  - D: Is independent of temperature.
10. A ship carrying coal is more likely to spontaneously catch fire if...
- A: the coal pile is small and well ventilated;
  - B: the coal is exposed to heat from the engines, is in large piles and not well-ventilated;**
  - C: the coal is of low heating value;
  - D: the coal is continuously kept wet.
11. When we double the size of the burner, the extinction mass flow rate
- A: increases slightly;
  - B: decreases significantly;
  - C: increases proportionately;**
  - D: does not change.
12. A well-stirred reactor is not adiabatic, but loses 10% of the heat energy of the fuel. Its extinction mass flow rate relative to the adiabatic reactor will be:
- A: the same;**
  - B: smaller by 5%;
  - C: higher;
  - D: smaller by more than 10%.**
13. What is a typical range of laminar burning velocities of hydrocarbon flames at atmospheric conditions?
- A: 0.01-0.1 m/s;
  - B: 0.1-1 m/s;**
  - C: 1-10 m/s;
  - D: 10-100 m/s

14. Which of the following statements for the laminar burning velocity  $S_L$  is false?  
 A: the reaction rate affects  $S_L$ .  
 B:  $S_L$  increases as the reactant temperature increases.  
 C:  $S_L$  decreases as the equivalence ratio drops from 1 to 0.5.  
 D: Replacing  $N_2$  by an equal amount of mass of He in the oxidizer does not affect  $S_L$ .
15. A tube carries a stoichiometric mixture and the velocity profile in the tube is top-hat (uniform velocity). Which of the plots below better represents the flame observed at the top of the tube?



16. Droplets of fuel in air evaporate fast when...  
 A: the droplets are big; B: the air is cold; C: the air already contains fuel vapour;  
 D: when the air has no vapour, is hot, and the droplets are small.
18. An engineer asked to reduce NO emission from an oil furnace suggested preheating of the air and suggested that in this way, oil droplets will evaporate faster and hence fuel will be better mixed with air. Is he/she right?  
 A: Yes, oil evaporates faster and the fuel vapour will react with NO.  
 B: Yes, but preheating will also have higher CO emission due to  $CO_2$  dissociation.  
 C: Yes in that oil will evaporate faster, but NO will probably be higher because the higher flame temperature will cause disproportionately higher NO.