Cribsheet

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1 Transport equations

1.1 Mass transfer

- **Mass (or species) transport equation**

$$\frac{\partial}{\partial t}(\rho Y_k) + \nabla.\mathcal{J}_\mathbf{k} = \dot{w}_k$$

$$\boxed{\underbrace{\frac{\partial}{\partial t}(\rho Y_k)}_{non-stationary} + \underbrace{\nabla.(\rho Y_k \mathbf{v})}_{convection} = \underbrace{-\nabla.\mathbf{J_k}}_{diffusion} + \underbrace{\dot{w}_k}_{production}}$$

where ρ is the density of the fluid, Y_k is the mass fraction of species k in the fluid, $\mathbf{v}$ the velocity vector of the fluid, $\mathcal{J}_\mathbf{k}$ the mass flux vector of species k, $\mathbf{J_k}$ the diffusive mass flux vector of species k, $\dot{w}_k$ the production rate of species k.

- **Fick's law:** (mass diffusion)

$$\mathbf{J_k} = -D_k \nabla C_k = -\rho D_k \nabla Y_k$$

We can then further develop the previous equation by introducing Fick's law and Einstein convention:

$$\rho(\frac{\partial Y_k}{\partial t} + \frac{\partial Y_k v_j}{\partial x_j}) = \rho D_k \frac{\partial^2 Y_k}{\partial x_j \partial x_j} + \dot{w}_k$$